# 1 Determinante $A_{(n \times n)}$

$A_{1x1} = [a] \qquad det(A) = a$

$$A_{2\times 2} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad det(A) = a_{11} \cdot a_{22} - a_{12} \cdot a_{21}$$

$$A_{3\times 3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

Cofator $a_{ij} = (-1)^{i+j} \ \cdot \ det\tilde{A}_{ij}$
$\tilde{a}_{21} = (-1)^{2+1} \cdot det \begin{bmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{bmatrix}$

$det(A)$ = a soma dos produtos dos elementos de uma linha da matriz pelos seus cofatores $= a_{21} \cdot \tilde{a}_{21} + a_{22} \cdot \tilde{a}_{22} + a_{23} \cdot \tilde{a}_{23} +$

$$B = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 3 & 2 \\ 2 & 1 & -2 \end{bmatrix}$$

$$\tilde{a}_{11} = (-1)^{1+1} \cdot det \begin{bmatrix} 3 & 2 \\ 1 & -2 \end{bmatrix} = -8$$

$$\tilde{a}_{12} = (-1)^{1+2} \cdot det \begin{bmatrix} -1 & 2 \\ 2 & -2 \end{bmatrix} = -[-2] = 2$$

$$\tilde{a}_{13} = (-1)^{1+3} \cdot det \begin{bmatrix} -1 & 3 \\ 2 & 1 \end{bmatrix} = -7$$

$det(B) = 1 \cdot (-8) + 2 \cdot (2) + 3 \cdot (-7)$
$= -7$

$$C = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & -3 \\ -1 & 3 & 2 & 5 \\ 2 & 1 & -2 & 0 \end{bmatrix}$$

$detC =$

$$\tilde{a}_{24} = (-1)^{2+4} \cdot det \begin{bmatrix} 1 & 2 & 3 \\ -1 & 3 & -2 \\ 2 & 1 & -2 \end{bmatrix} = -25$$

$$A = \begin{bmatrix} a_{11} & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ a_{31} & a_{32} & a_{34} & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

$$det(A) = a_{11} \cdot \tilde{a}_{11} = a_{11} \cdot (-1)^{1+1} \cdot det \begin{bmatrix} a_{22} & 0 & 0 \\ a_{32} & a_{33} & 0 \\ a_{42} & a_{43} & a_{44} \end{bmatrix}$$

$$a_{11} \cdot [a_{11}\tilde{a}_{22}] = a_{11}a_{22} \cdot (-1)^{2+2} det \begin{bmatrix} a_{33} & 0 \\ a_{430} & a_{44} \end{bmatrix} = a_{11} \cdot a_{22} \cdot a_{33} \cdot a_{44}$$

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}$$

$$detA = 2$$
$$B = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$$

$$detB = -2$$
$$A + B = \begin{bmatrix} 1 & 3 \\ 1 & 1 \end{bmatrix}$$

$$det(A + B) = 2$$

$$det(A + B) \neq det(A) + det(B)$$

$$det \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ \alpha x + \beta y \\ \vdots \\ A_n \end{bmatrix} = det \begin{bmatrix} A_1 \\ \vdots \\ \alpha x \\ \vdots \\ A_n \end{bmatrix} + det \begin{bmatrix} A_1 \\ \vdots \\ \beta y \\ \vdots \\ A_n \end{bmatrix} = \alpha \cdot det \begin{bmatrix} A_1 \\ \vdots \\ x \\ \vdots \\ A_n \end{bmatrix} = \beta \cdot det \begin{bmatrix} A_1 \\ \vdots \\ y \\ \vdots \\ A_n \end{bmatrix}$$

$$det = \begin{bmatrix} \cos t & \sin t \\ 2\cos t - 3\sin t & \sin t + 3\cos t \end{bmatrix}$$

$$= det \begin{bmatrix} \cos t & \sin t \\ 2\cos t & 2\sin t \end{bmatrix} + det \begin{bmatrix} \cos t & \sin t \\ -3\sin t & 3\cos t \end{bmatrix} =$$

$$2 \cdot det \begin{bmatrix} \cos t & \sin t \\ \cos t & \sin t \end{bmatrix} + 3 \cdot det \begin{bmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{bmatrix} = 2 \cdot 0 + 3 \cdot 1 = 3$$
$$A = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}$$

$$detA = 2$$

$$B = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$$

$$detB = -2$$
$$A \cdot B = \begin{bmatrix} 4 & 3 \\ 0 & -1 \end{bmatrix}$$

$$det(A \cdot B) = -4 = det(A) \cdot det(B)$$

$$A^t = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$$

$$det(A^t) = 2$$

$$detA = det(A^t)$$