Dynamic Longitudinal Modeling

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SMIP Summer Workshop, Mannheim, 2025

HUMBOLDT-UNIVERSITÄT ZU BERLIN

Workshop Material

https://github.com/voelklem/smip2025

General introduction & overview of the workshop

Abstract: The goal of this workshop is to introduce participants to advanced modeling techniques, focusing on dynamic approaches to analyzing change and variability. We will begin by differentiating static and dynamic models, discussing their respective strengths and weaknesses, and exploring the fundamentals of dynamic modeling, including practical tools and software.

Throughout the workshop, we will cover key topics such as addressing heterogeneity, applying hierarchical models, analyzing individual-level data, and exploring innovative study designs. Participants will also be introduced to cutting-edge methods for causal inference and the integration of machine learning into dynamic modeling, with an emphasis on their practical applications and current limitations.

While examples will primarily draw from applied research, the workshop is designed for participants with an interest in quantitative methods. Prior experience with multivariate analysis is beneficial but not required, and familiarity with structural equation modeling and longitudinal data analysis is helpful. Emphasis will be placed on practical implementation using datasets and software tools.

Prerequisites: Participants should bring their own laptops with the latest versions of R and RStudio installed. Those new to R are encouraged to familiarize themselves with its basic functionality. Advanced knowledge of R is not necessary for participation.

A quick round of introductions

(2) What are your research interests?

(1) Who are you?

(3) What is your prior experience with (1) longitudinal data, (2) SEM, (3) R, OpenMx, Stan?

(4) I expect from this course…

(5) If I don't do research, I…

Outline Day 1

09:00 - 11:00	Static versus dynamic models & an overview of the next days
11:30 - 12:30	An introduction to continuous time dynamic modeling
14:00 - 17:15	A more solid introduction to the R package ctsem and the state space representation

Static versus dynamic longitudinal models

Rationales of Longitudinal Research

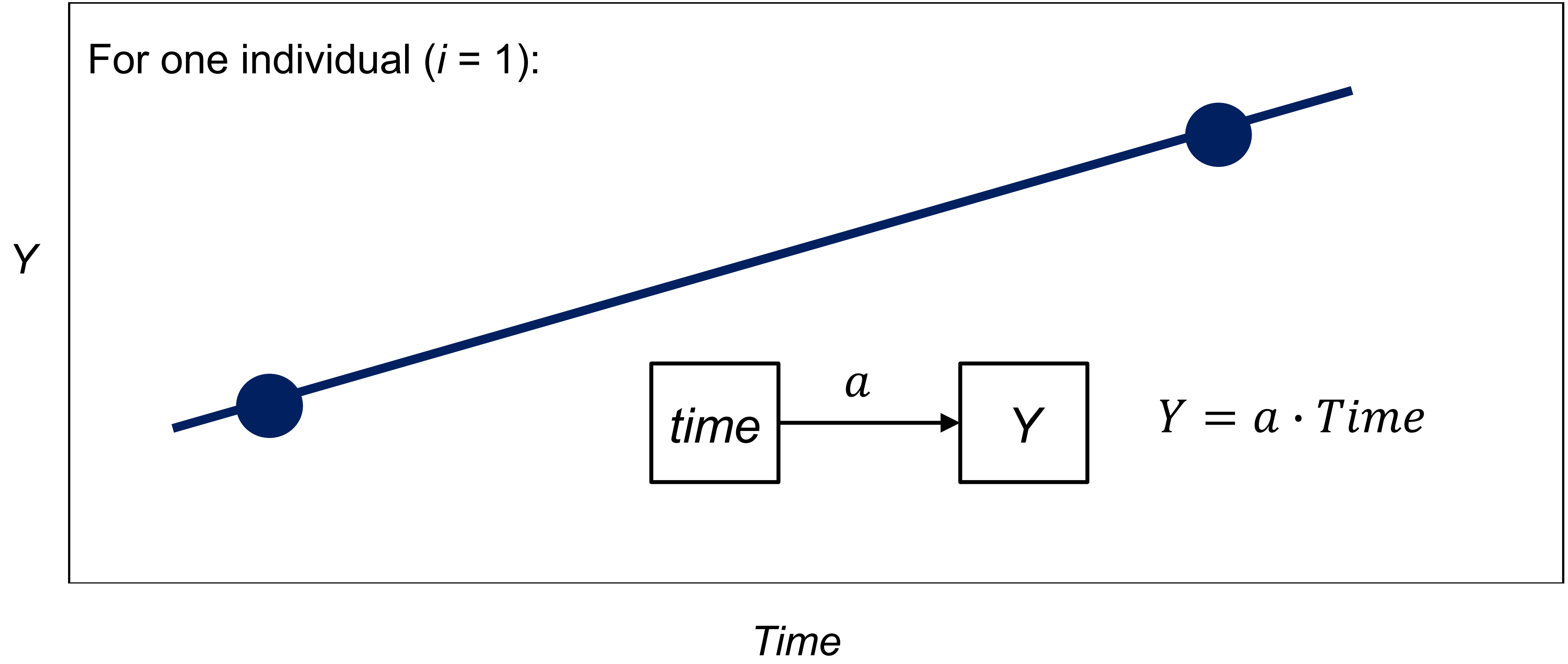
- Why longitudinal studies/data?

- Longitudinal research serves different goals/rationales.

- A famous systematization of **five rationales** for longitudinal research is provided by Baltes & Nesselroade (1979).

Rationales of Longitudinal Research

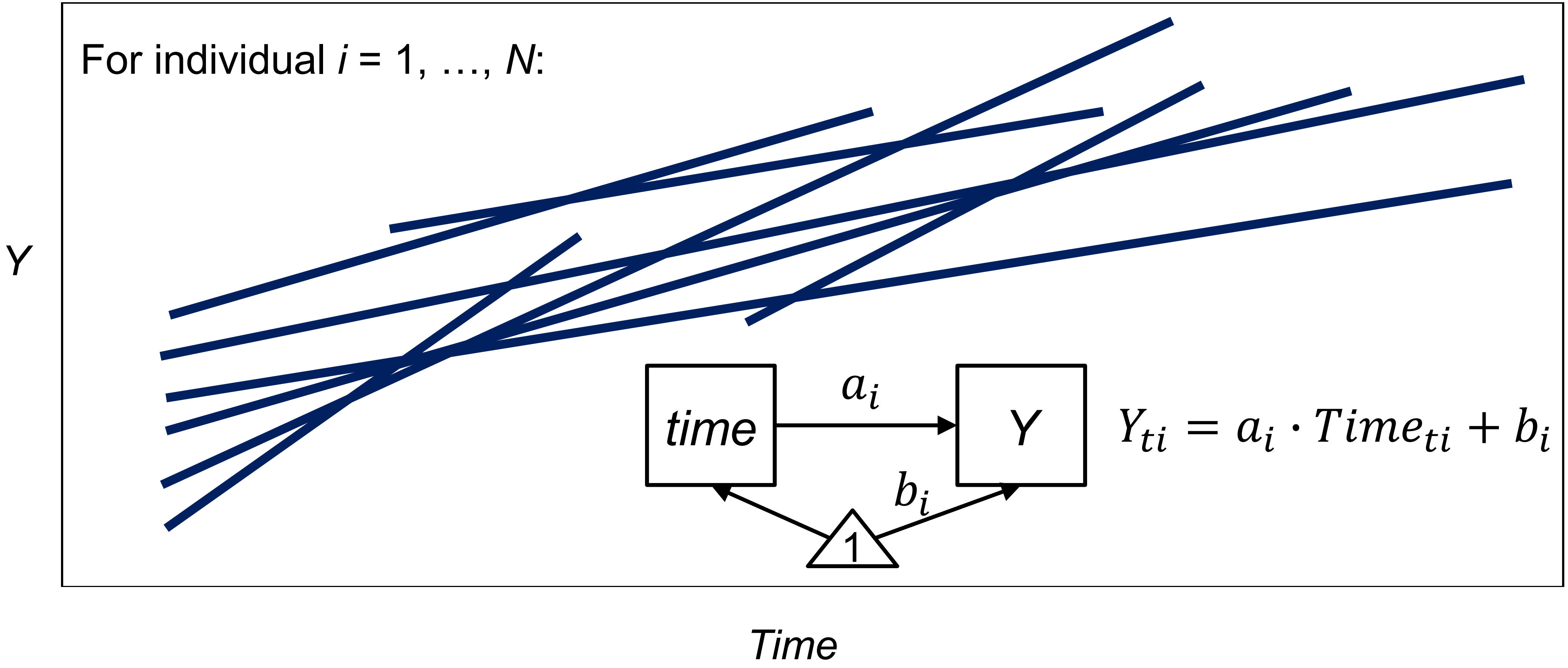
1. Direct identification of intraindividual change



Baltes & Nesselroade (1979)

Rationales of Longitudinal Research

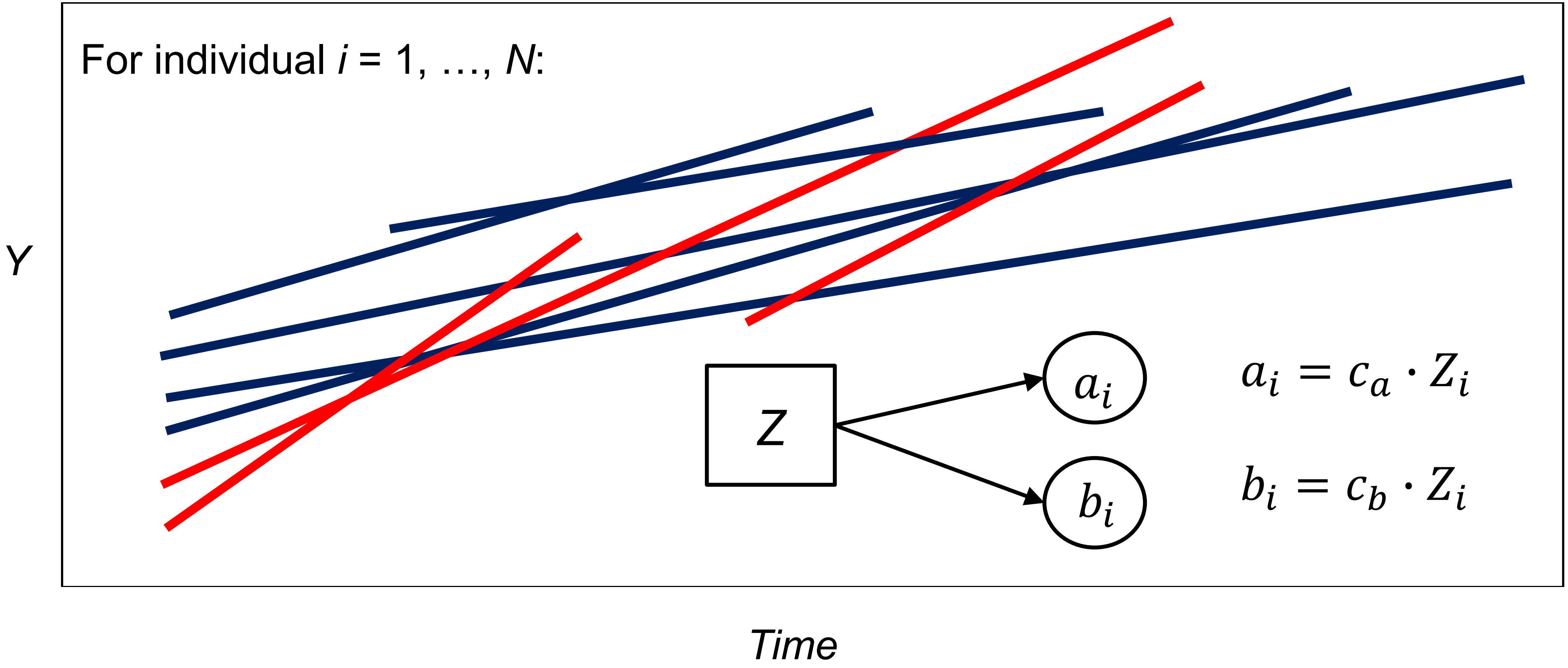
2. Direct identification of interindividual differences (similarity) in intraindividual change



Baltes & Nesselroade (1979)

Rationales of Longitudinal Research

3. Analysis of “causes” (determinants) of interindividual differences in intraindividual change

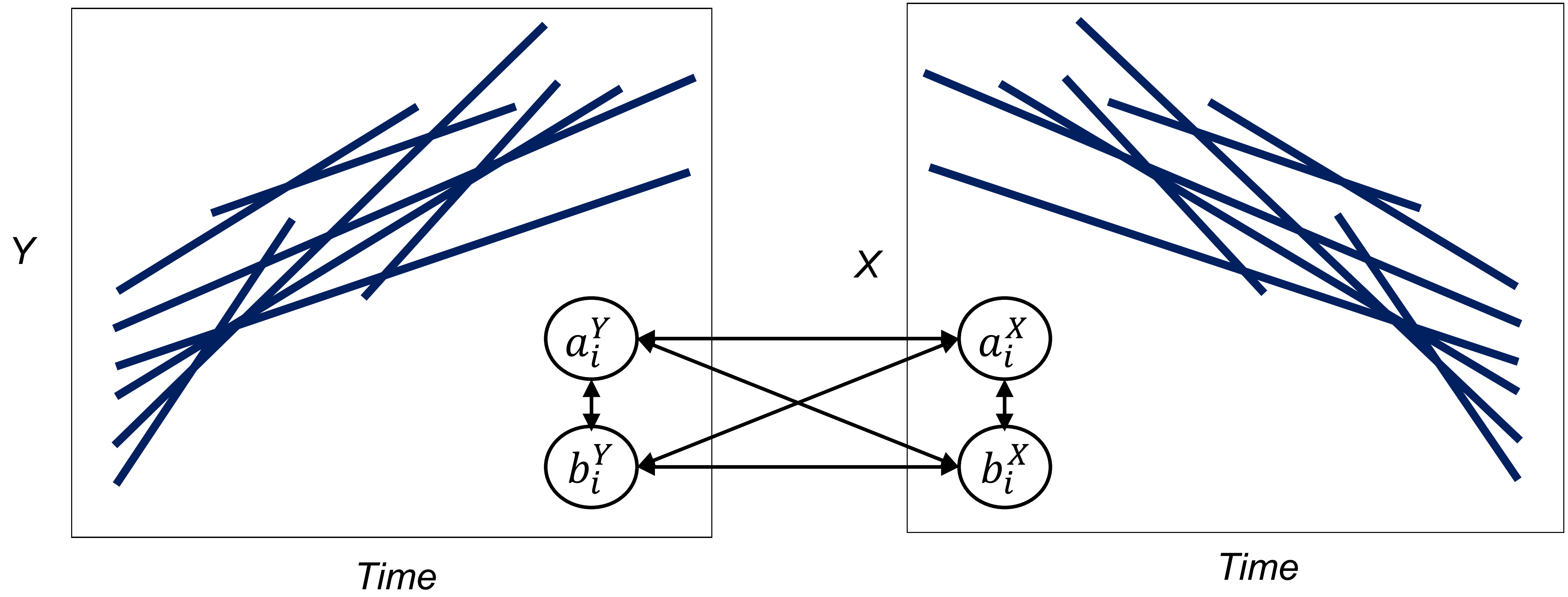


Baltes & Nesselroade (1979)

Rationales of Longitudinal Research

4. Analysis of interrelationships in behavioral change

   For individual *i* = 1, …, *N*:



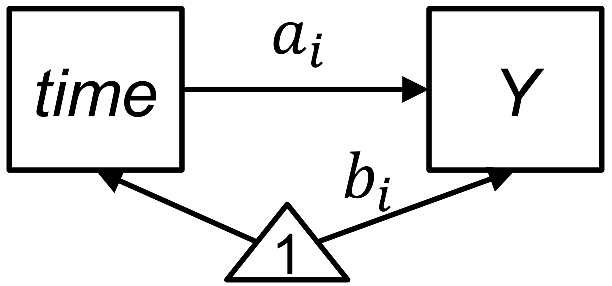
Baltes & Nesselroade (1979)

Rationales of Longitudinal Research

5. Analysis of causes (determinants) of intraindividual change

Baltes & Nesselroade (1979)

The Role of Time in Longitudinal Models



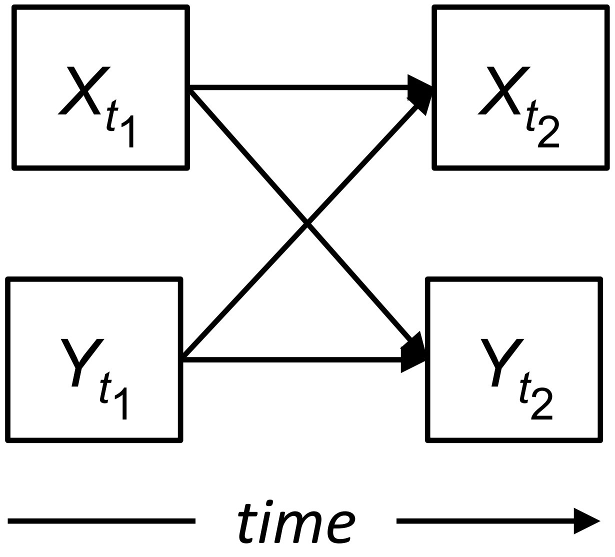
$$Y_{ti} = a_i \cdot Time_{ti} + b_i$$

The Role of Time in Longitudinal Models

"…although time is inextricably linked to the concept of development, in itself it cannot explain any aspect of developmental change…. Time, rather like the theatrical stage upon which the processes of development are played out, provides a necessary base upon which the description, explanation, and modification of development proceed."

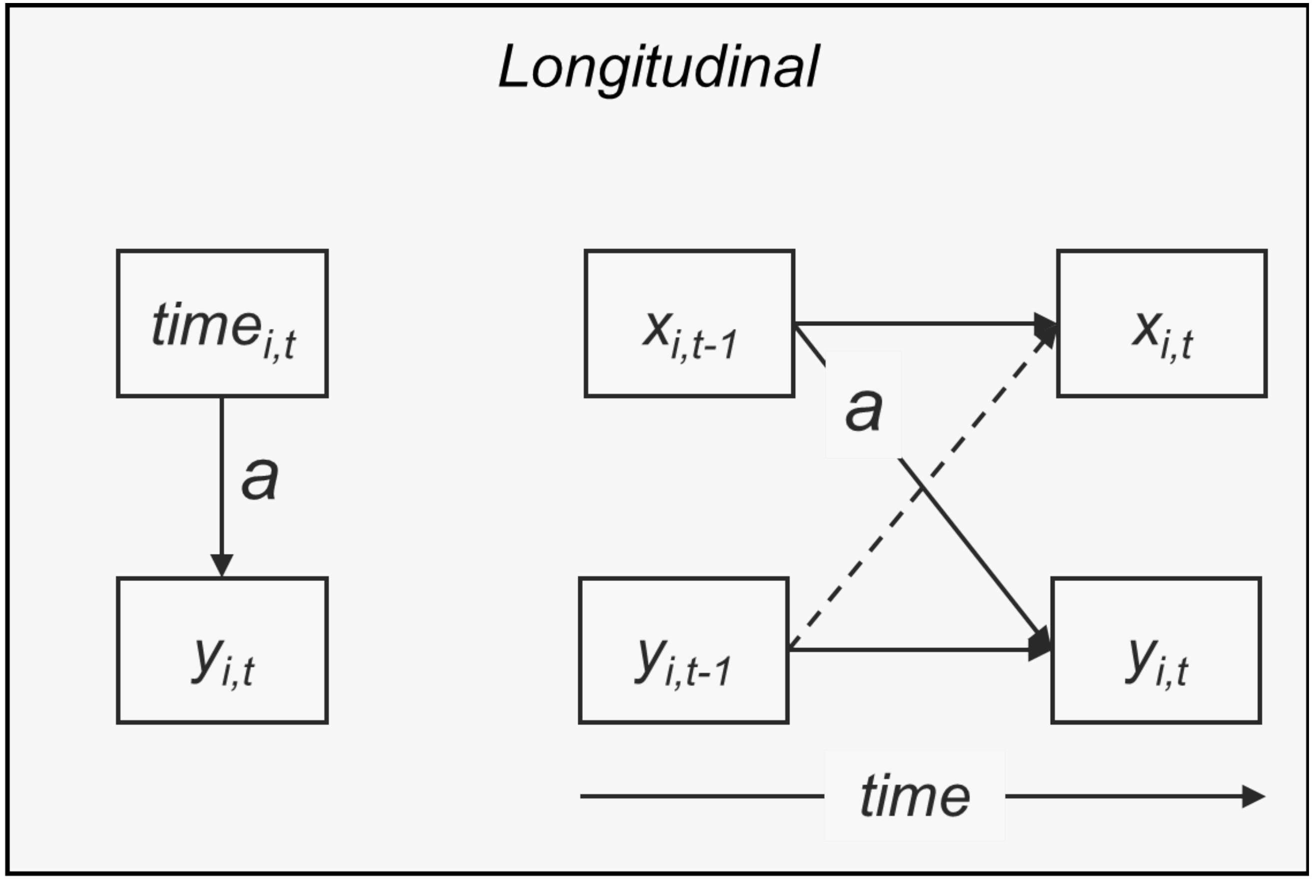
Baltes, Reese, Nesselroade (1988, p. 108)

The Role of Time in Longitudinal Models



$$\mathbf{x}_{t,i} = \mathbf{A} \cdot \mathbf{x}_{t-1,i}$$

The Role of Time in Longitudinal Models



Static

$$y_{i,t} = a \cdot time_{i,t} + w_{i,t}$$

Dynamic

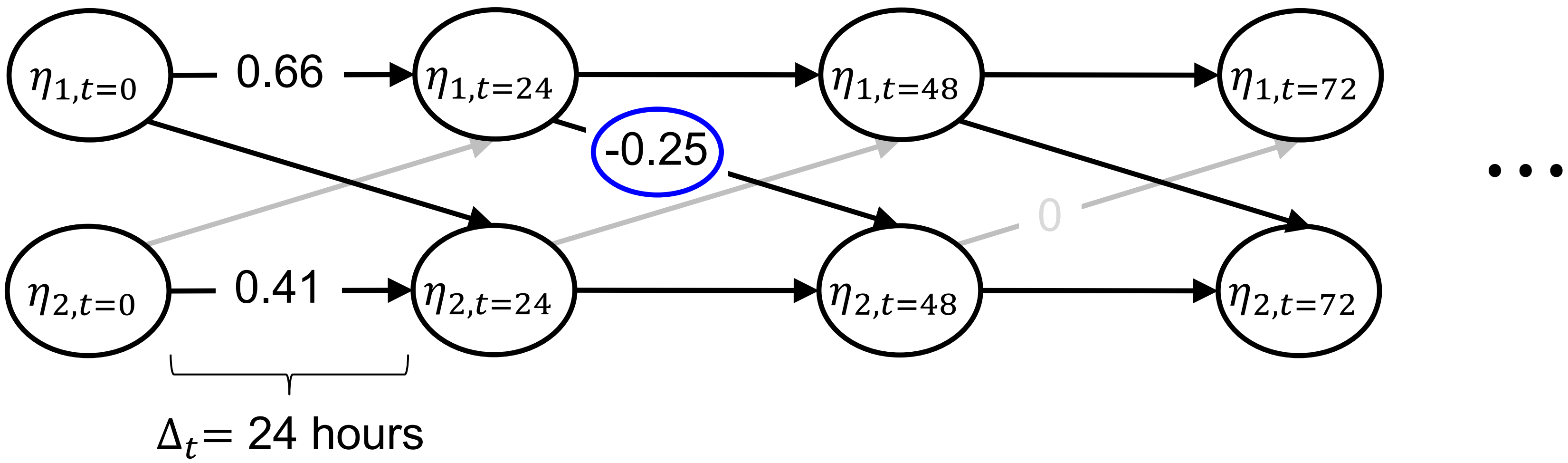
$$\mathbf{x}_{i,t} = \mathbf{A}\mathbf{x}_{i,t-1} + \mathbf{w}_{i,t}$$

Some challenges in the dynamic analysis of change

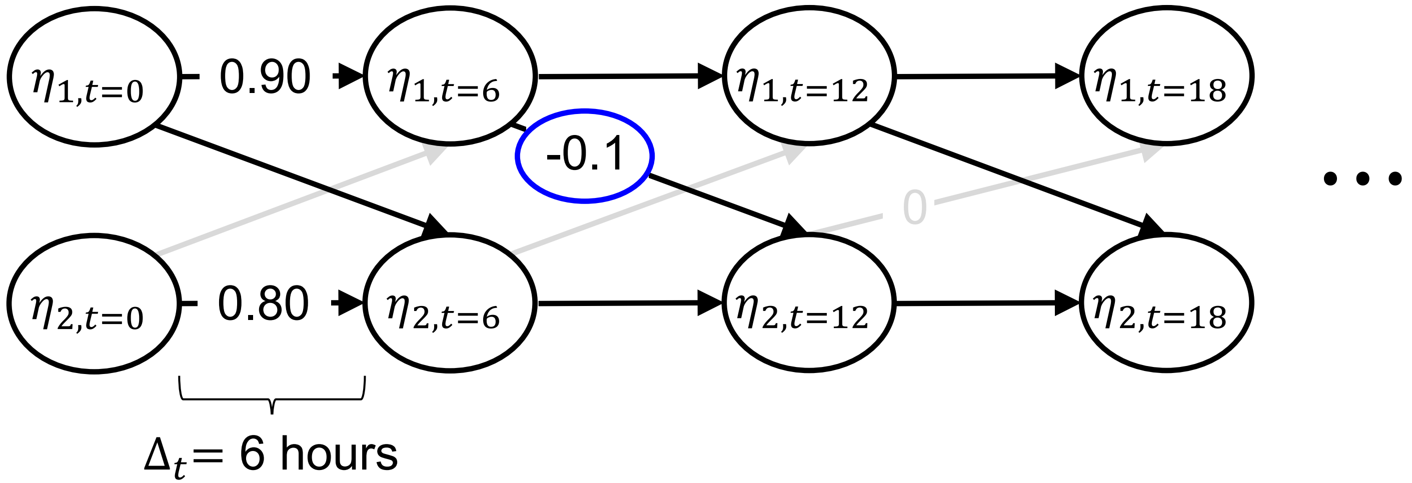
1.

Unequal time intervals – across studies

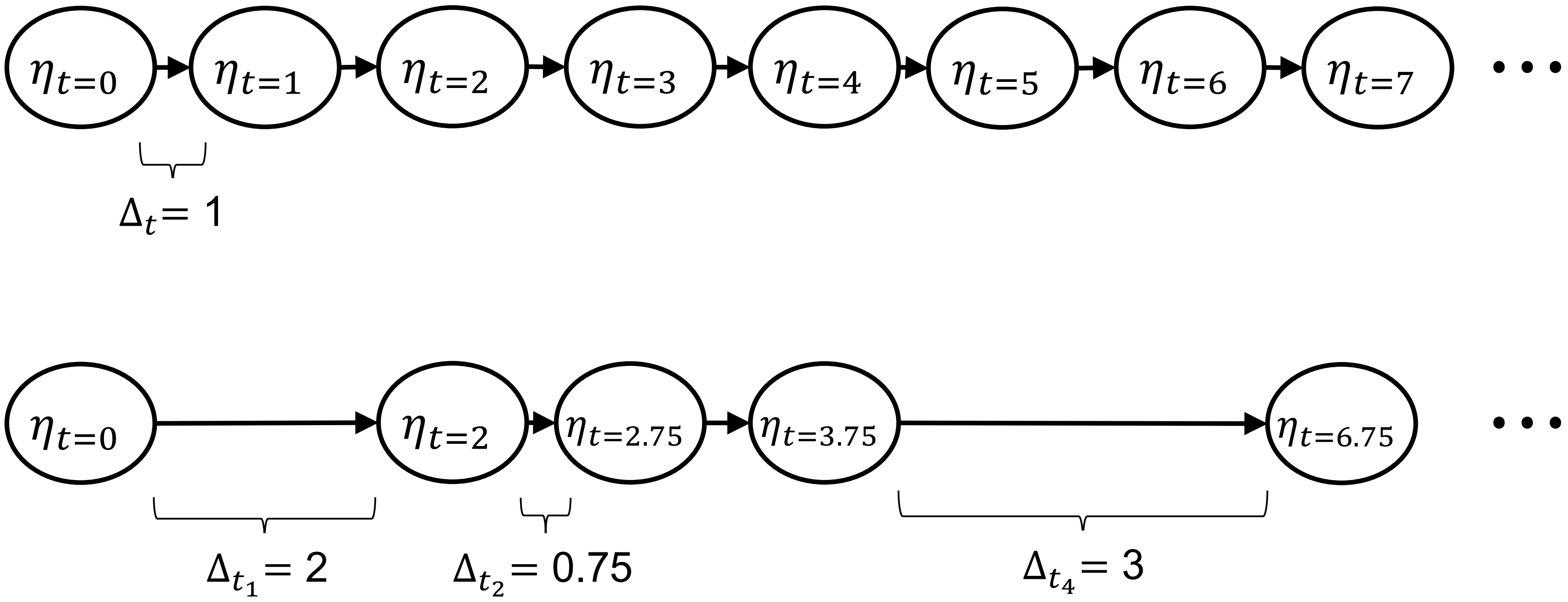
Researcher A:



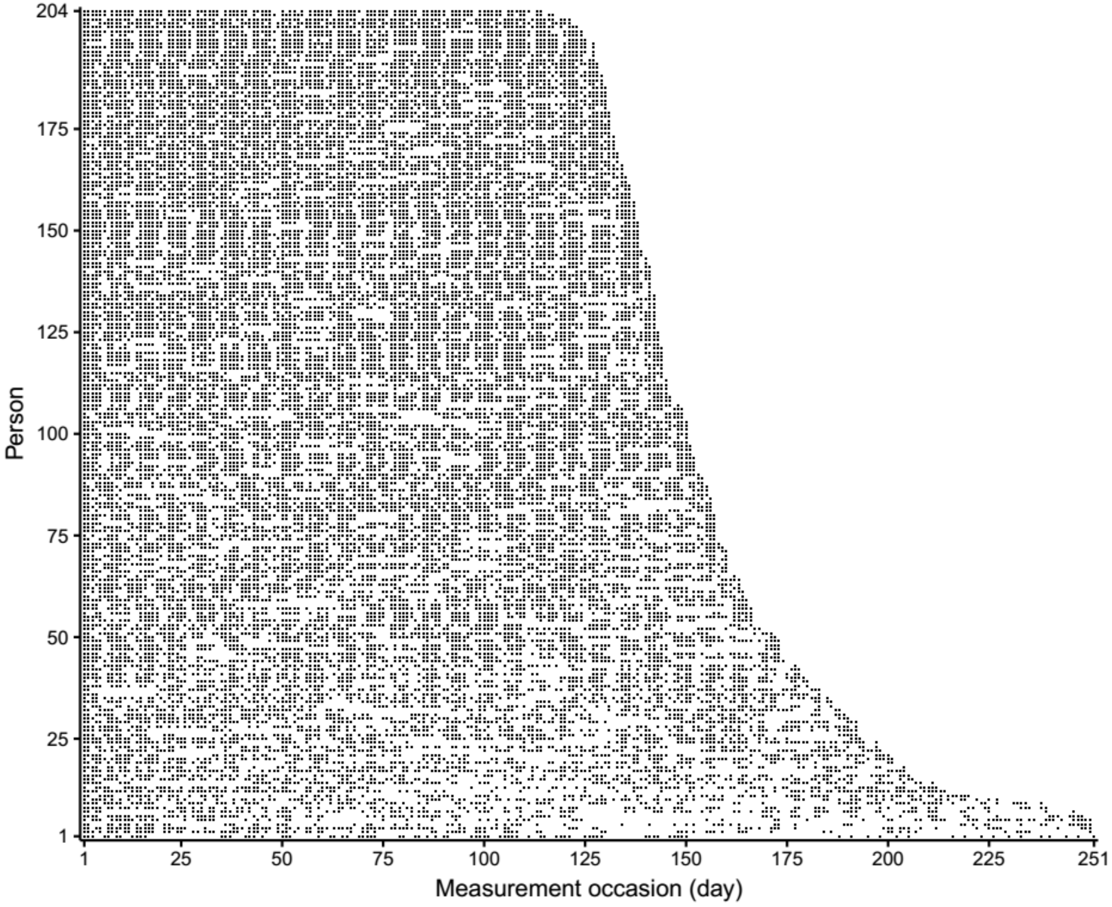
Researcher B:



1. Unequal time intervals – across measurement occasions



1. Unequal time intervals – across individuals



Time intervals (in days) between measurement occasions in the COGITO study (N = 204).

```
The decimal point is 1 digit(s) to the right of the |

14 | 7725577888
16 | 11377911249
18 | 02333334599188
20 | 22234458888001233344455558999
22 | 0112222233344455555555566667777788999000000111112233333444444455555+5
24 | 0022333344445555555555556677888888999990011222222222222333344444455+8
26 | 0000000011111111122223333334444444455555555666666666666666677778888+90
28 | 00000000000000000111111111112222222222222233333333333333444444444444+272
30 | 00000000000000000000000000000111111111111111111111111111111111222+585
32 | 0000000000000000000000000000000000000000000000001111111111111111+1281
34 | 00000000000000000000000000000000000000000000000000000000000000000+3215
36 | 00000000000000000000000000000000000000000000000000000000000000000+4837
38 | 00000000000000000000000000000000000000000000000000000000000000000+2108
40 | 0000000000000000000000000000000000000000000000000000000000000000+875
42 | 000000000000000000000000000000000000000000001111111111111111111111+460
44 | 0000000000011111111111111122222222222222223333333333334444444455555+187
46 | 000001111111122222222223333333334444555555666666666666777777778888888888+68
48 | 00000111222222223333344444556667788889001111111223334444455555555555
50 | 000011111222222334445555566667788000122222334555667777899999
52 | 1111122223335556666889000333444455666788
54 | 0011233336666788904669
56 | 555
58 | 67700
60 | 6
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Time intervals (in days) between measurement occasions in 2010 and 2011 in the SOEP (N = 15,300).

Voelkle & Oud (2013) *British Journal of Mathematical and Statistical Psychology*; Hecht, Hardt, Driver, & Voelkle (2019). *Psychological Methods.*

1. Unequal time intervals

Day 1 of this workshop will focus on the question of how to deal with unequal time intervals in dynamic modeling.

…and why this is important

…and why dynamic models are useful

…and how we can specify and estimate such models

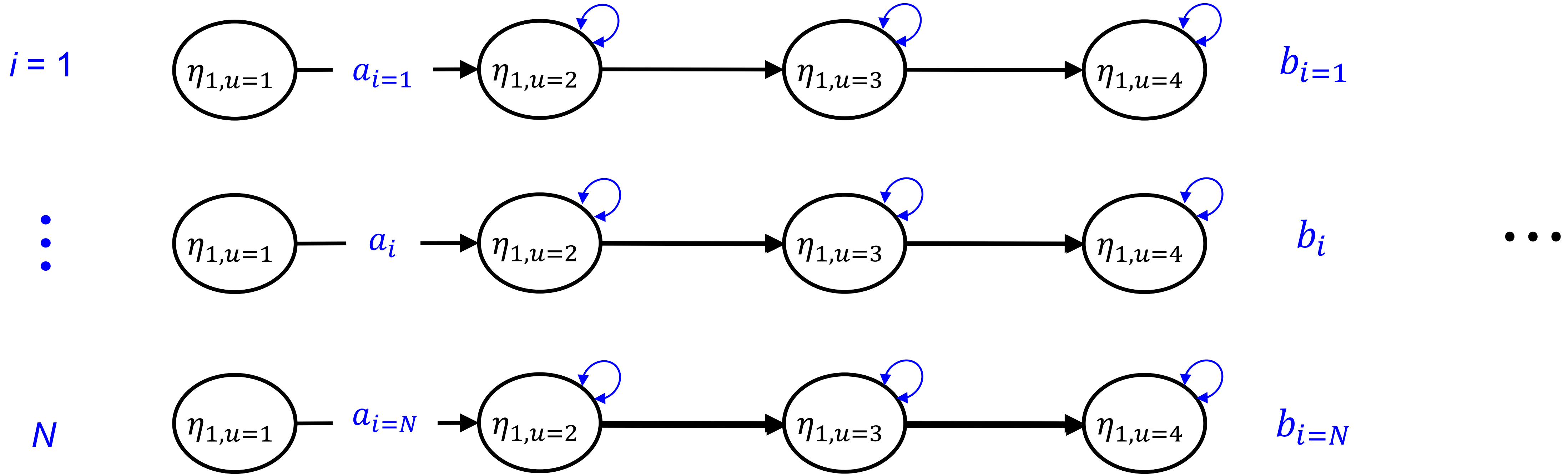
…and why it is worthwhile to adopt a dynamic systems perspective

…and why it is not really (just) about time intervals

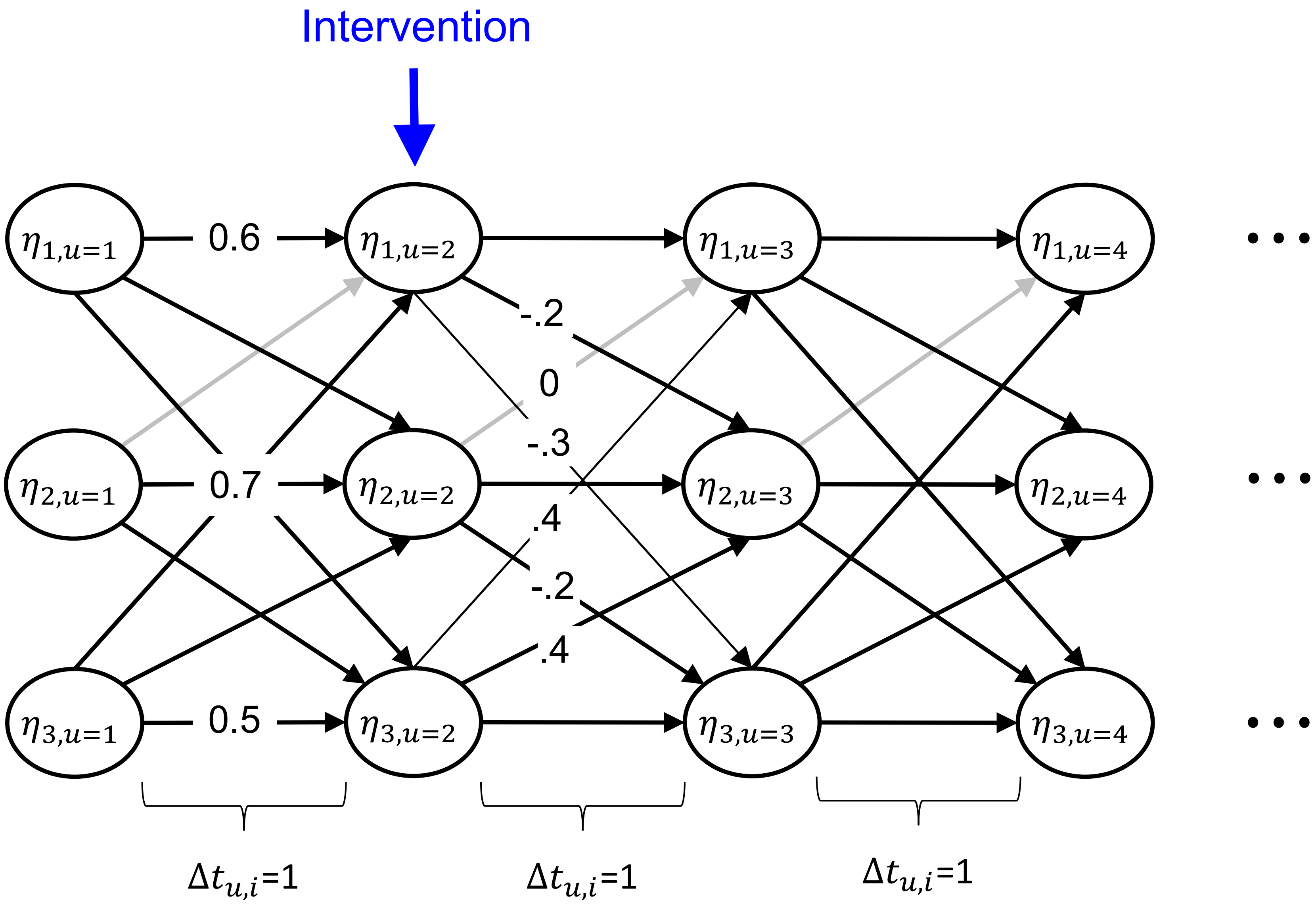
…

2. Heterogeneity

People differ – in level, process, variances…



3. Understanding the time course of input effects



Heterogeneity & input effects

Day 2 of this workshop will focus on the question of how to deal with (between-person) heterogeneity and input effects

…and why/when it is important to separate BP and WP sources of variance

…and how to control for different confounds in the hunt for causal effects

…and how to interpret dynamic parameters

…and the basics of Bayesian continuous time dynamic modeling

…and the basics of fully hierarchical models, including conditional level 2 models

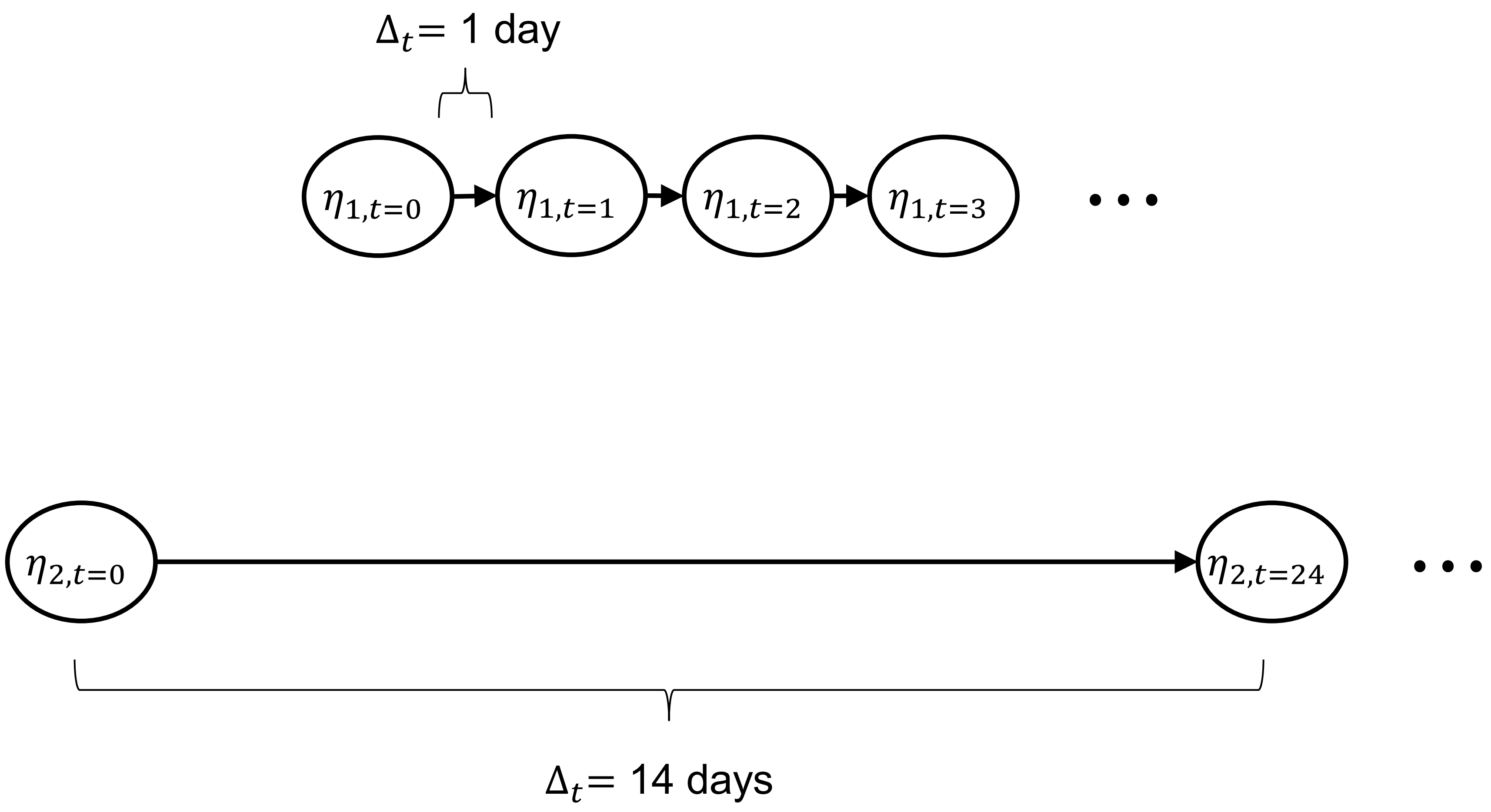
…and how to model (complex) input effects

…

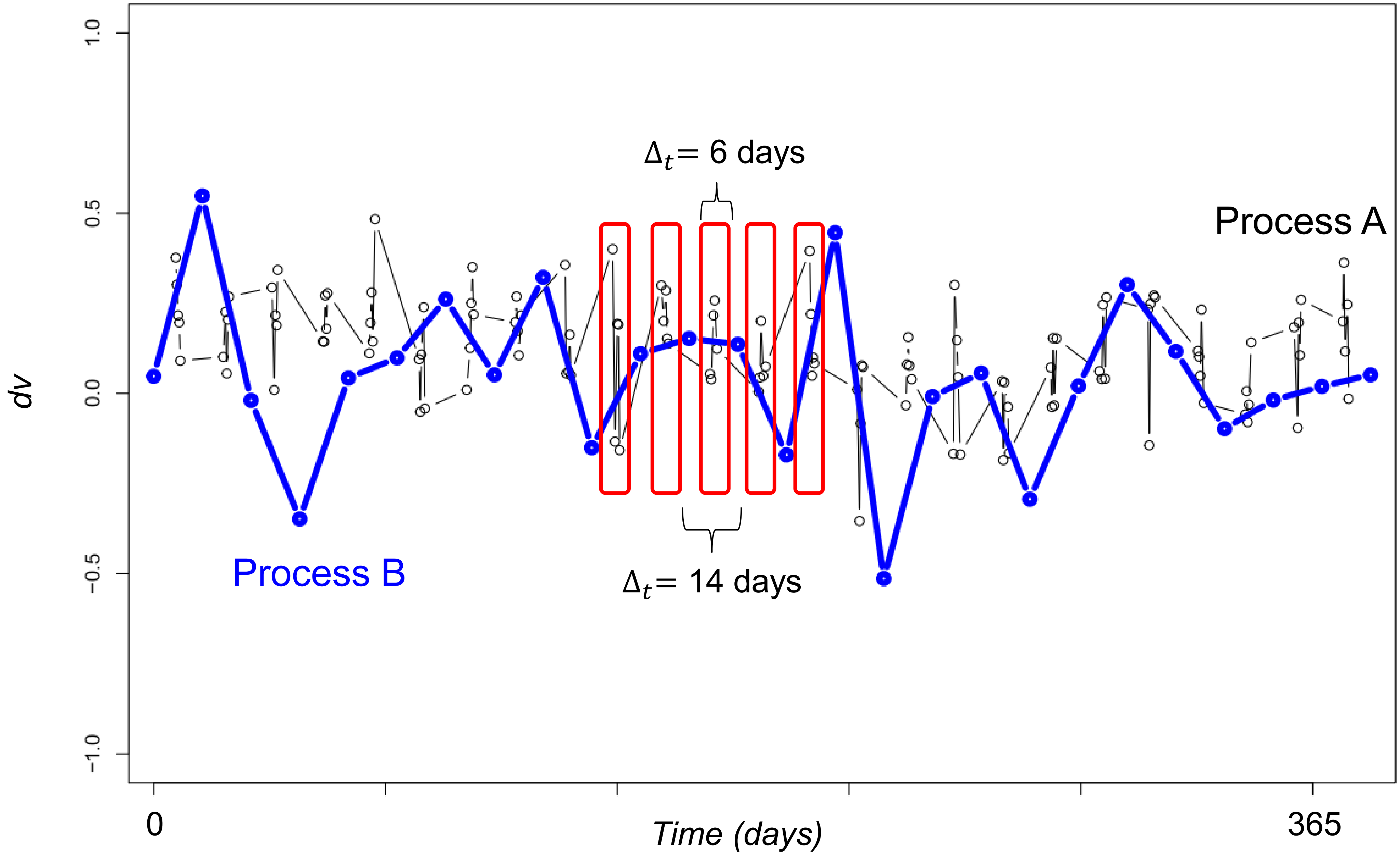
4. Complex processes at different time scales

Process A:

Process B:



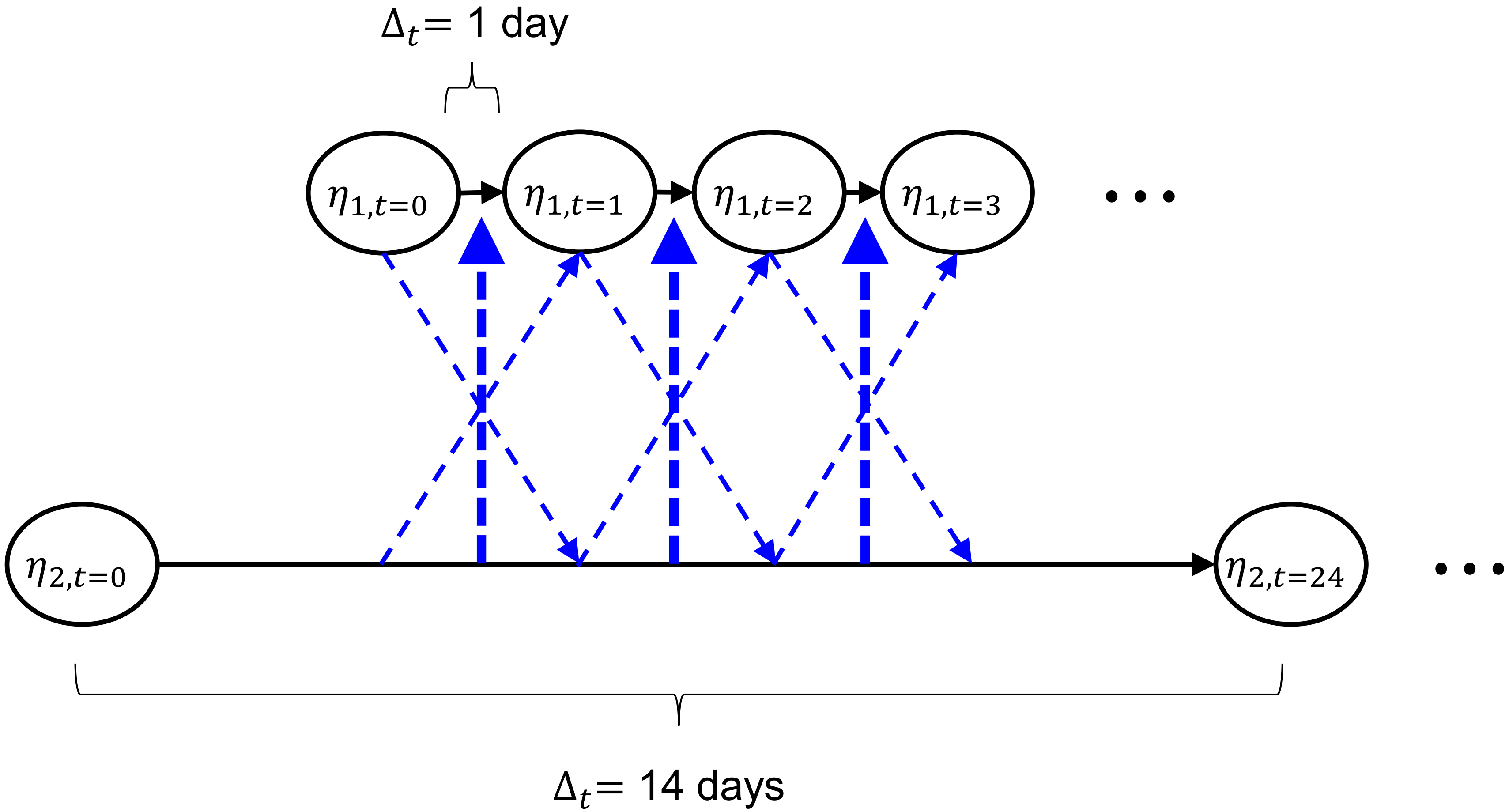
4. Complex processes at different time scales



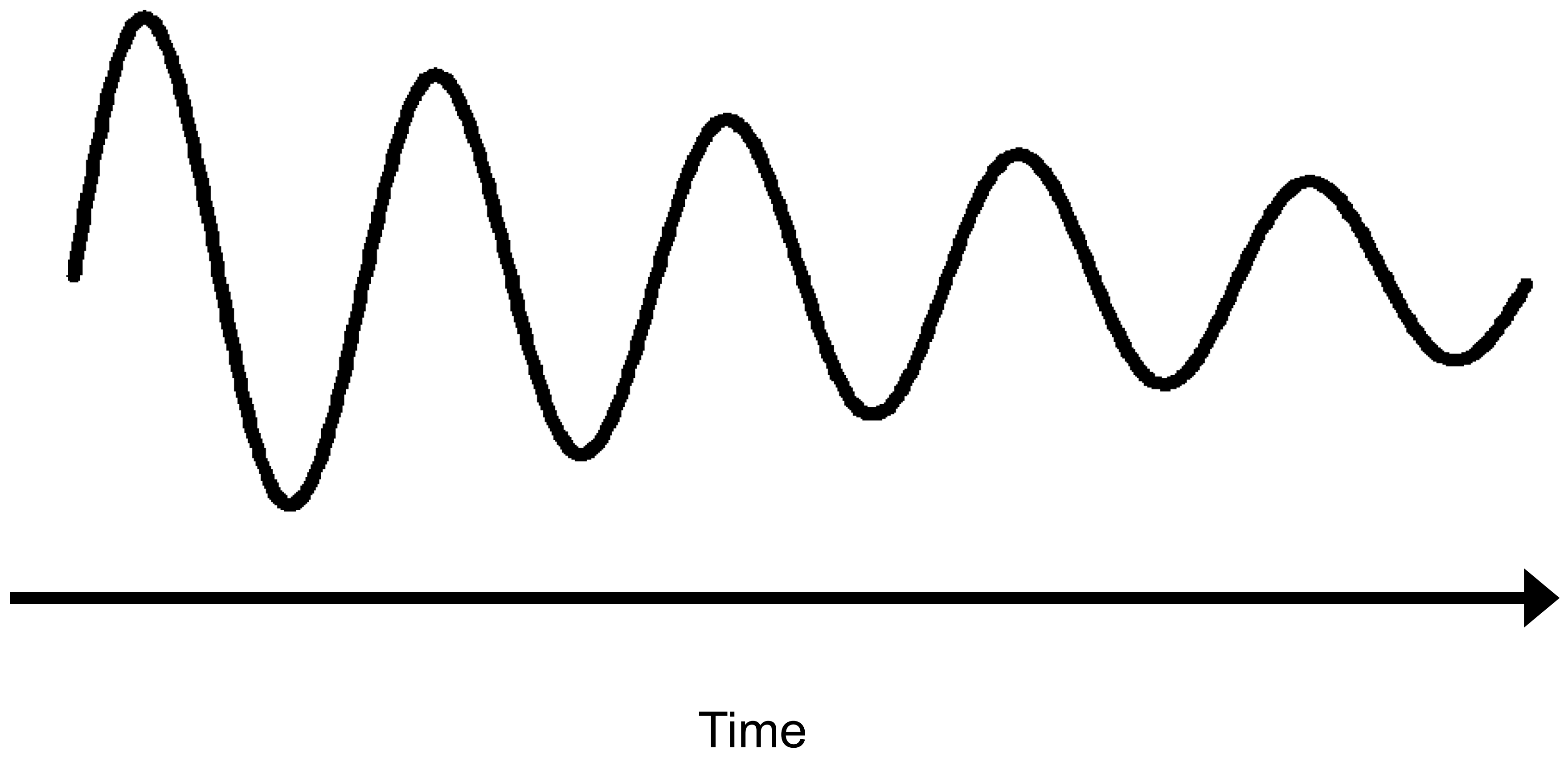
4. Complex processes at different time scales

Process A:

Process B:



4. Complex processes at different time scales



4. Complex processes at different time scales

Day 3 of this workshop will focus on the question of how to deal with more complex processes and different time scales, including

…time series analyses (large *T*, small *N* or *N* = 1)

…the trick of the state expansion

…higher order models

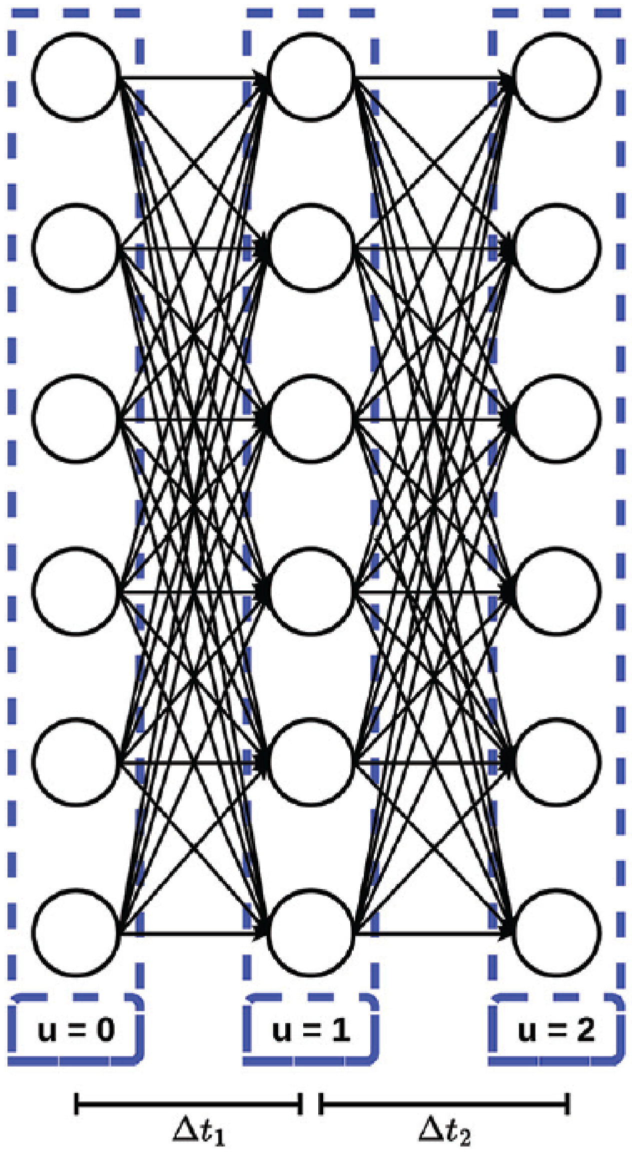
…oscillating processes

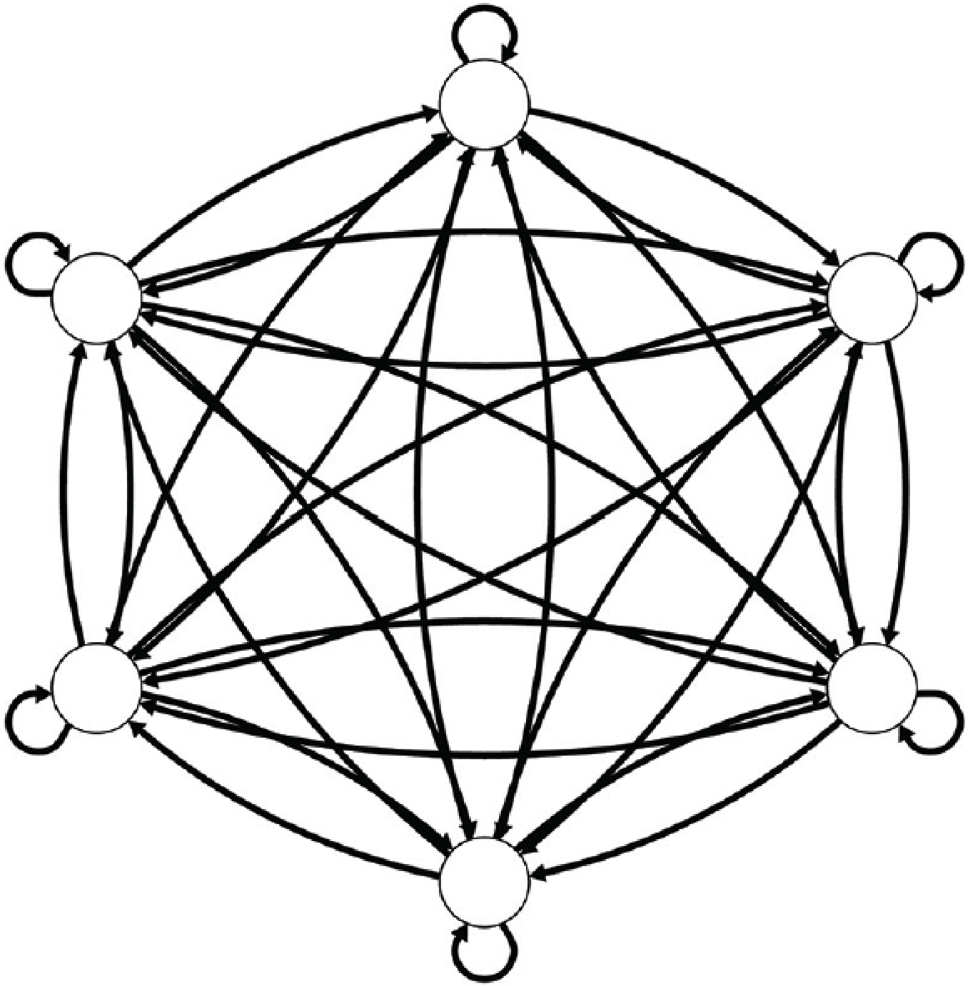
…heterogeneity in time

…multiple time scales

…special approaches (LCS) designs (ALD)

5. Dealing with complexity and causal inference





5. Dealing with complexity and causal inference

Day 4 of this workshop will focus on the question of how to deal with the curse of dimensionality (complexity) and how to use ct models to study psychological mechanisms. We will talk about

…LASSO regularization

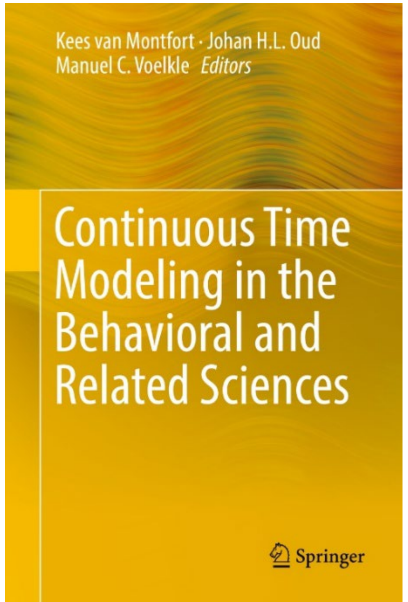
…causal inference

…the many limitations and open questions we are facing

…the many opportunities to advance things if you want to get involved.

What are continuous time dynamic models?

– a non-technical "birds-eye" introduction to get you motivated –



What are continuous time dynamic models?

…and why the use of the term is sometimes confusing.

	Discrete time	Continuous time
Static models	e.g., „standard“ latent growth curve models	e.g., linear mixed models (latent growth curve models with definition variables)
Dynamic models	e.g., (vector) autoregressive cross-lagged models, “DSEM”	**continuous time** (dynamic) **models**

What are continuous time dynamic models?

An “intuitive” introduction:

$$\mathbf{x}_{i,u} = \mathbf{A}_u \cdot \mathbf{x}_{i,u-1} + \mathbf{w}_{i,u}$$

$$\mathbf{x}_{i,u} - \mathbf{x}_{i,u-1} = \mathbf{A}_u \cdot \mathbf{x}_{i,u-1} - \mathbf{x}_{i,u-1} + \dots$$

$$\underbrace{\mathbf{x}_{i,u} - \mathbf{x}_{i,u-1}}_{} = (\mathbf{A} - \mathbf{I}) \cdot \mathbf{x}_{i,u-1}$$

$$\frac{\Delta \mathbf{x}_{i,u}}{\Delta t_u} = \underbrace{\frac{(\mathbf{A}_u - \mathbf{I})}{\Delta t_u}}_{} \cdot \mathbf{x}_{i,u-1}$$

$$\frac{\Delta \mathbf{x}_{i,u}}{\Delta t_u} = \boxed{\mathbf{A}_*} \cdot \mathbf{x}_{i,u-1}$$

“approximate” drift matrix

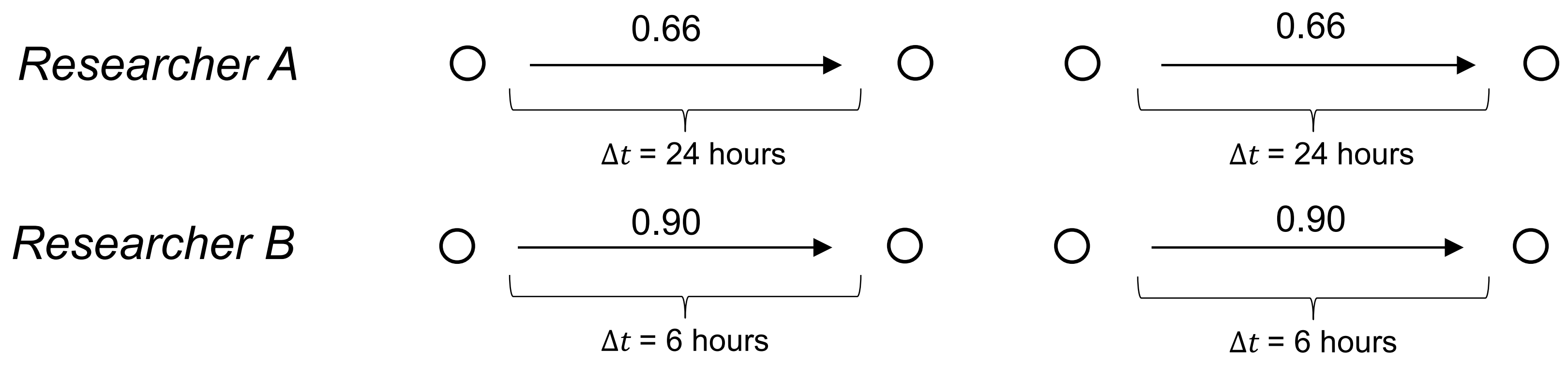
What are continuous time dynamic models?

An “intuitive” introduction:

This approach permits an approximate ad-hoc comparison of parameters:

$$\mathbf{A}_* = \frac{(\mathbf{A}_u - \mathbf{I})}{\Delta t_u}$$

In our example



Researcher A: A_{*1} = (0.66-1)/24 = -0.014 (more stable)
Researcher B: A_{*2} = (0.90-1)/6 = -0.017 (less stable)

What are continuous time dynamic models?

An „exact“ introduction:

$$\lim_{\Delta t \to 0}\left(\frac{\Delta \mathbf{x}}{\Delta t}\right) = \frac{\mathrm{d}\mathbf{x}(t)}{\mathrm{d}t}$$

$$\frac{\mathrm{d}\boldsymbol{\eta}(t)}{\mathrm{d}t} = \boxed{\mathbf{A}}\,\boldsymbol{\eta}(t)$$

drift matrix

What are continuous time dynamic models?

1. Latent dynamic model (extended Ornstein-Uhlenbeck process)

$$\mathrm{d}\boldsymbol{\eta}(t) = \big(\mathbf{A}\boldsymbol{\eta}(t) + \mathbf{b} + \mathbf{M}\boldsymbol{\chi}(t)\big)\mathrm{d}t + \mathbf{G}\mathrm{d}\mathbf{W}(t)\big)$$

including the measurement part

$$\mathbf{y}(t) = \boldsymbol{\Lambda}\boldsymbol{\eta}(t) + \boldsymbol{\tau} + \boldsymbol{\varepsilon}(t) \quad \text{with} \quad \boldsymbol{\varepsilon}(t) \sim \mathrm{N}(\mathbf{0}, \boldsymbol{\Theta})$$

and $\boldsymbol{\chi}(t) = \sum_{u \in U} \mathbf{x}_u \delta(t - t_u)$ with $\delta()$ denoting the Dirac delta function

What are continuous time dynamic models?

2. Discrete time solution of the stochastic differential equation and (3.) imposing constraints

$$\boldsymbol{\eta}_u = \mathbf{A}^*_{\Delta t_u}\boldsymbol{\eta}_{u-1} + \mathbf{b}^*_{\Delta t_u} + \mathbf{M}\mathbf{x}_u + \boldsymbol{\zeta}_u \quad \text{with} \quad \boldsymbol{\zeta}_u \sim \mathrm{N}(\mathbf{0}, \mathbf{Q}^*_{\Delta t_u})$$

and

$$\mathbf{A}^*_{\Delta t_u} = e^{\mathbf{A}(t_u - t_{u-1})}$$

$$\mathbf{b}^*_{\Delta t_u} = \mathbf{A}^{-1}\left(\mathbf{A}^*_{\Delta t_u} - \mathbf{I}\right)\mathbf{b} \quad \text{thus} \quad \mathbf{b}^*_{\Delta t_\infty} = -\mathbf{A}^{-1}\mathbf{b}$$

$$\mathbf{Q}^*_{\Delta t_u} = \mathbf{Q}_{\Delta t_\infty} - \mathbf{A}^*_{\Delta t_u}\mathbf{Q}_{\Delta t_\infty}(\mathbf{A}^*_{\Delta t_u})^{\mathrm{T}}$$

$$\text{with} \quad \mathbf{Q}_{\Delta t_\infty} = \mathrm{irow}(-(\mathbf{A}\otimes\mathbf{I} + \mathbf{I}\otimes\mathbf{A})^{-1} row(\mathbf{Q}))$$

What are continuous time dynamic models?

4. Unit level log likelihood

$$ll = \sum^{U} \left(-\frac{1}{2} (n \ln(2\pi) + \ln|\mathbf{V}_u| + \left(\hat{\mathbf{y}}_{u|u-1} - \mathbf{y}_u\right) \mathbf{V}_u^{-1} \left(\hat{\mathbf{y}}_{u|u-1} - \mathbf{y}_u\right)^{\mathrm{T}}) \right)$$

What are continuous time dynamic models?

1. take derivative with respect to time

$$\mathrm{d}\boldsymbol{\eta}(t) = \big(\mathbf{A}\boldsymbol{\eta}(t) + \mathbf{b} + \mathbf{M}\boldsymbol{\chi}(t)\big)\mathrm{d}t + \mathbf{G}\mathrm{d}\mathbf{W}(t)\big)$$

2. solve differential equation for initial time point and given time interval

3. constrain discrete time parameters to the underlying continuous time parameters

$$\boldsymbol{\eta}_u = \mathbf{A}^*_{\Delta t_u}\boldsymbol{\eta}_{u-1} + \mathbf{b}^*_{\Delta t_u} + \mathbf{M}\mathbf{x}_u + \boldsymbol{\zeta}_u$$

4. estimate parameters
(using either frequentist or Bayesian methods)

ctsem & ctsemOMX – two R packages for continuous time dynamic modeling

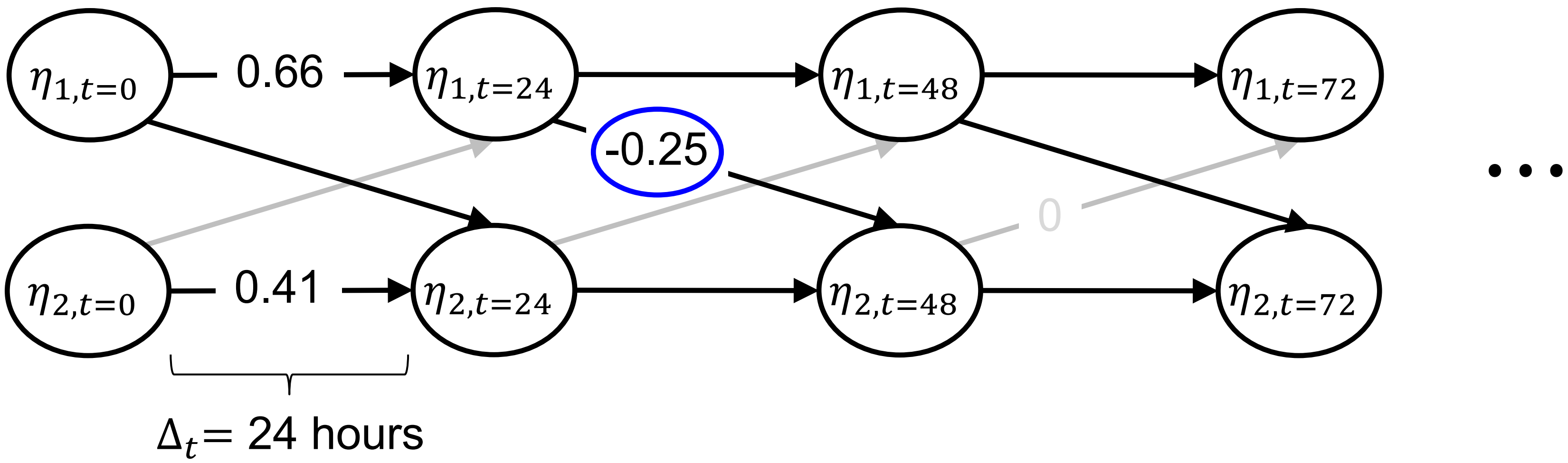
Driver, Oud, & Voelkle (2017). *Journal of Statistical Software.*

Revisiting problems 1 to 5 from a continuous time perspective

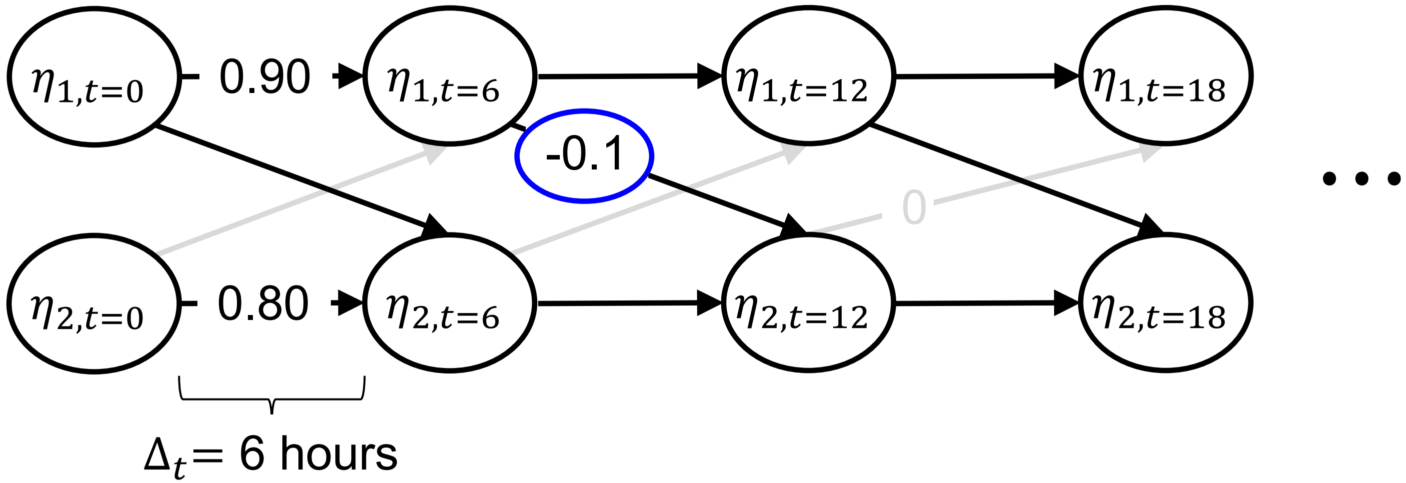
1.

Unequal time intervals – across studies

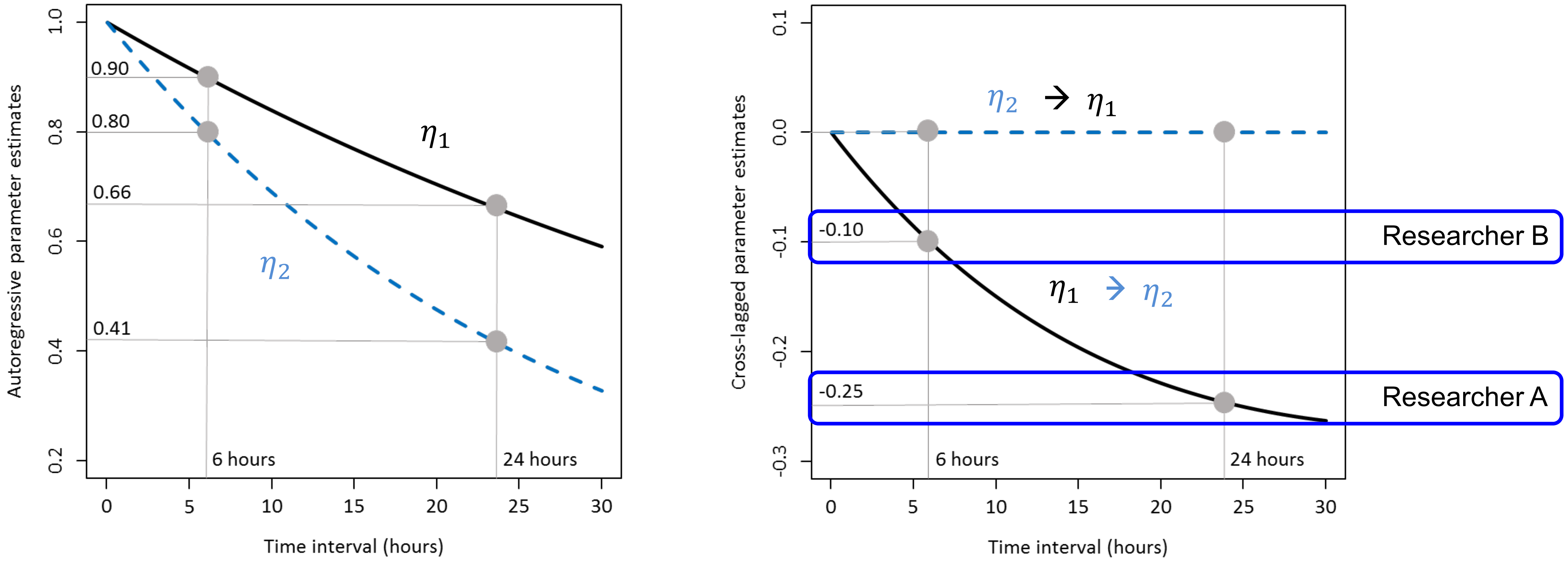
Researcher A:



Researcher B:



1. Unequal time intervals – across studies

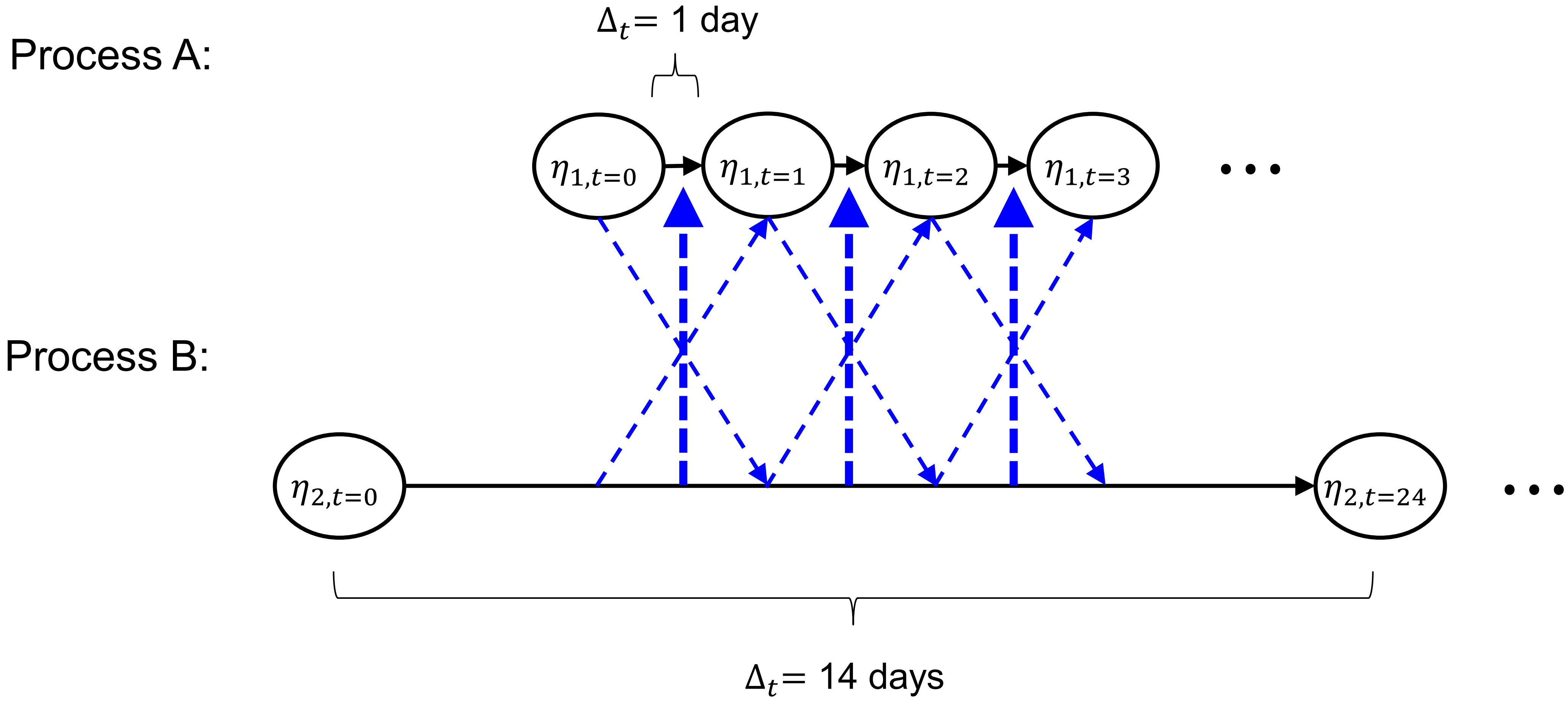


$$\mathbf{A}^*_{\Delta t=6} = e^{\begin{pmatrix} -0.0176 & 0 \\ -0.0196 & -0.0372 \end{pmatrix}\cdot 6} = \begin{pmatrix} 0.90 & 0 \\ \boxed{-0.10} & 0.80 \end{pmatrix}$$

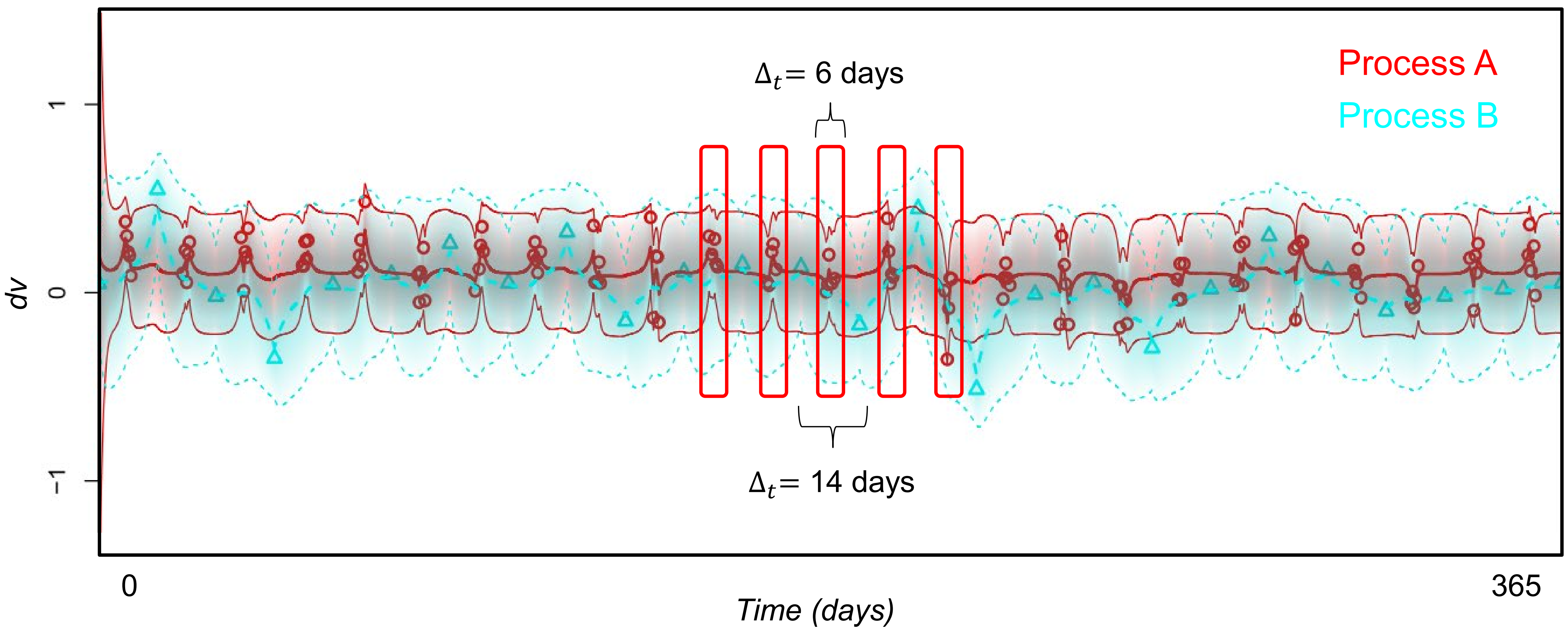
$$\mathbf{A}^*_{\Delta t=24} = e^{\begin{pmatrix} -0.0176 & 0 \\ -0.0196 & -0.0372 \end{pmatrix}\cdot 24} = \begin{pmatrix} 0.66 & 0 \\ \boxed{-0.25} & 0.41 \end{pmatrix}$$

Voelkle, Oud, Davidov, & Schmidt (2012). *Psychological Methods*

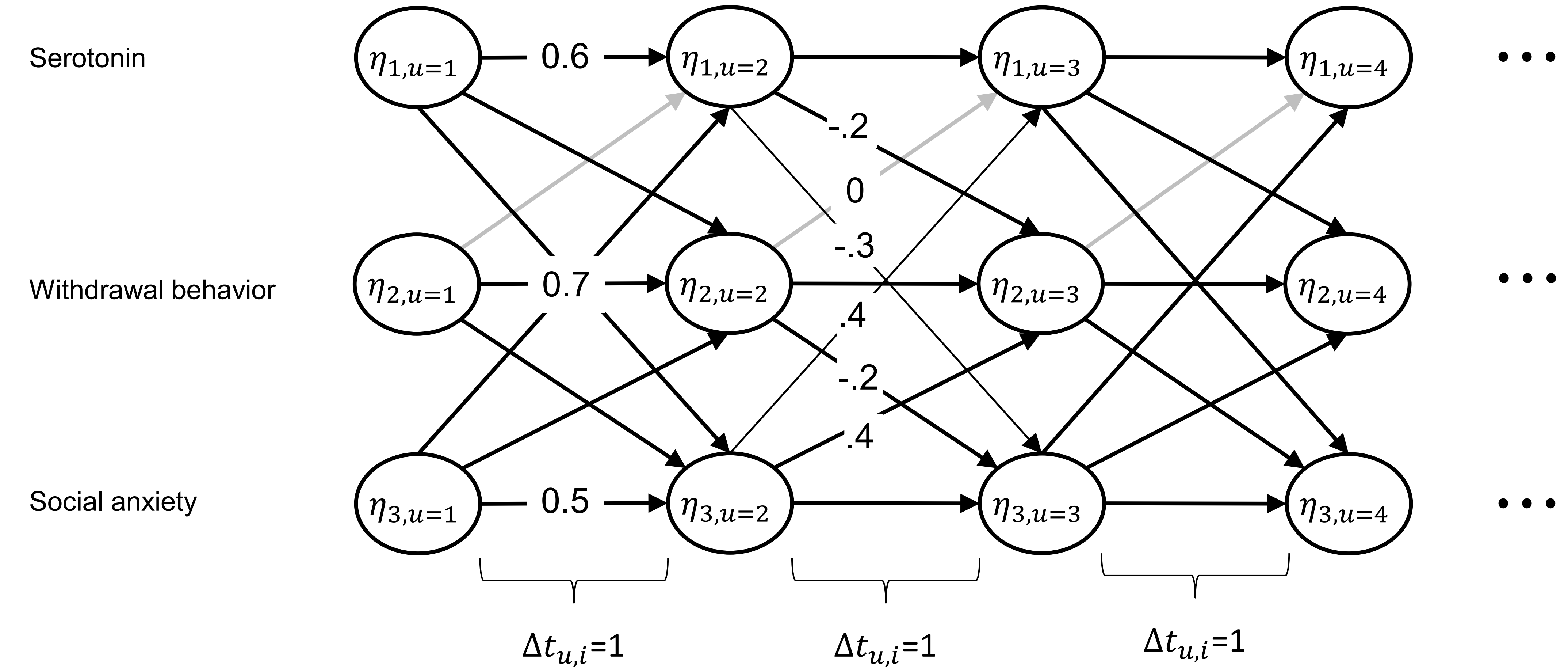
Processes at different time scales



2. Processes at different time scales



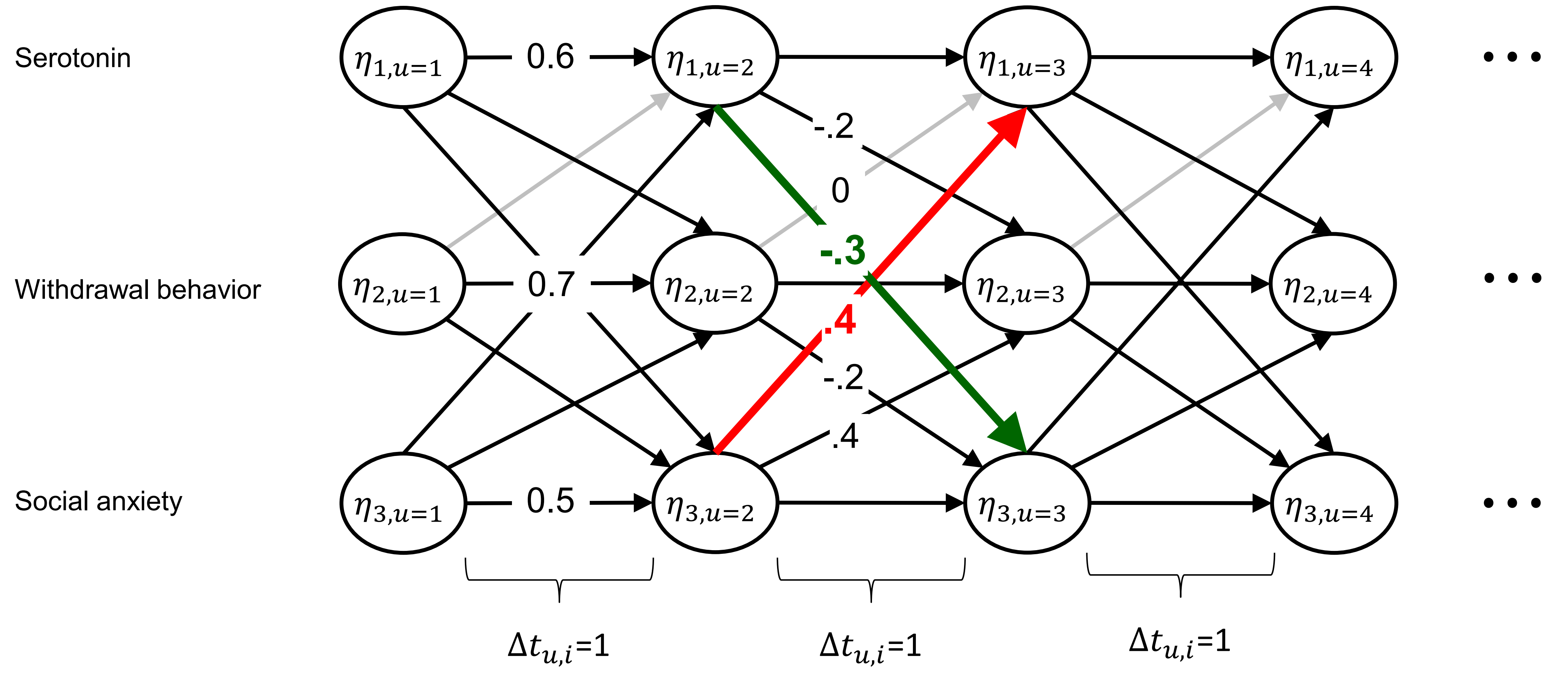
3. Understanding the time course of input effects



Voelkle, Gische, Driver & Lindenberger (2018). *Multivariate Behavioral Research*.

3.

Understanding the time course of input effects



Voelkle, Gische, Driver & Lindenberger (2018). *Multivariate Behavioral Research*.

3. Understanding the time course of input effects

-.2

0

-.3

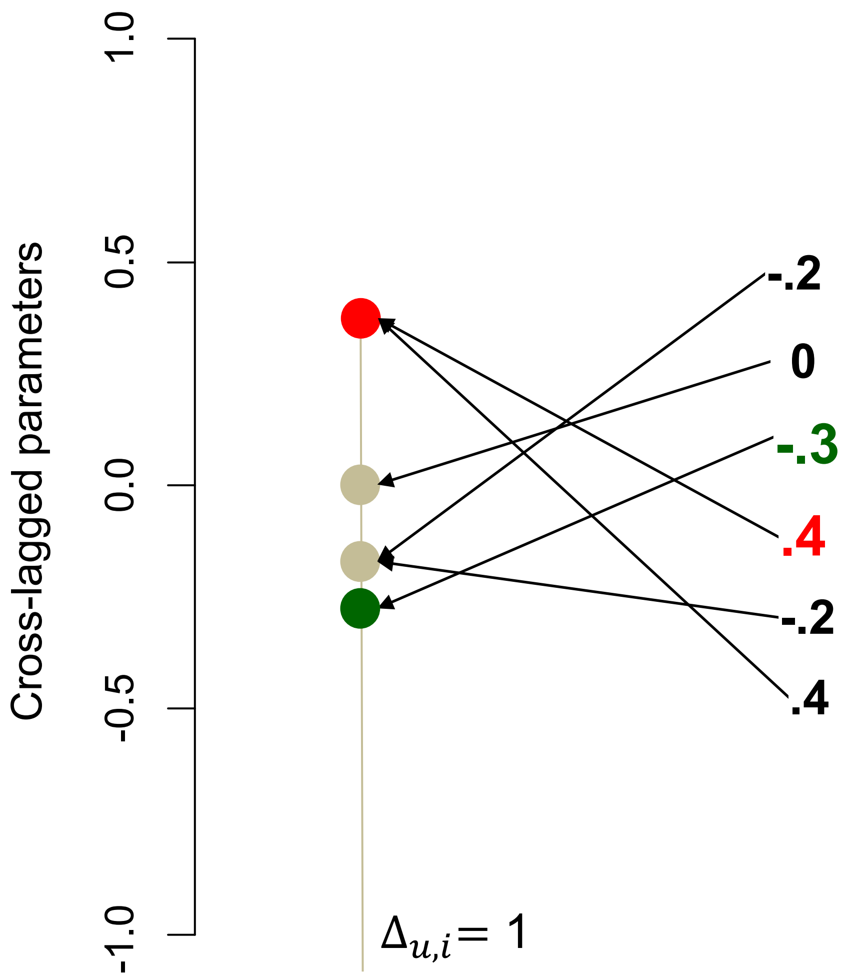
.4

-.2

.4

$$\underbrace{\qquad\qquad}_{\Delta t_{u,i}=1}$$

3. Understanding the time course of input effects



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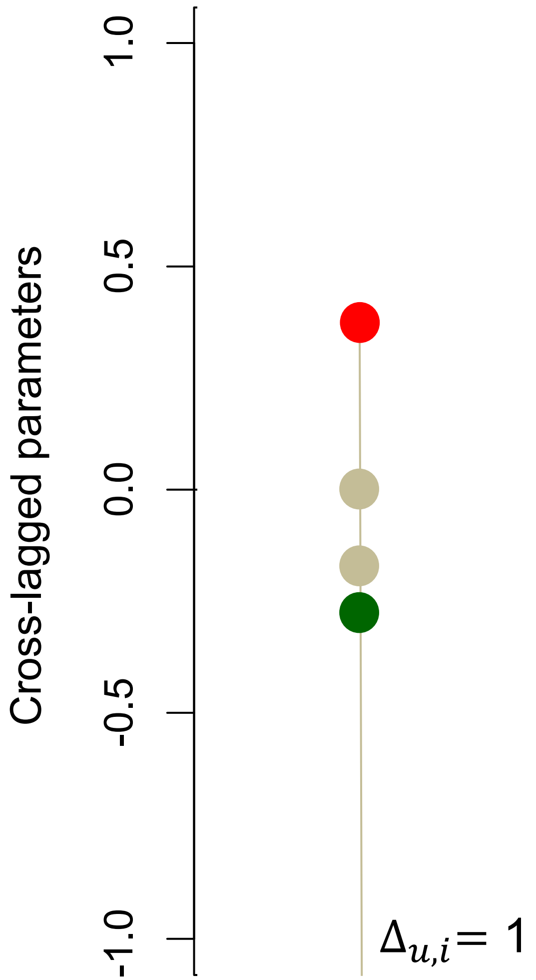

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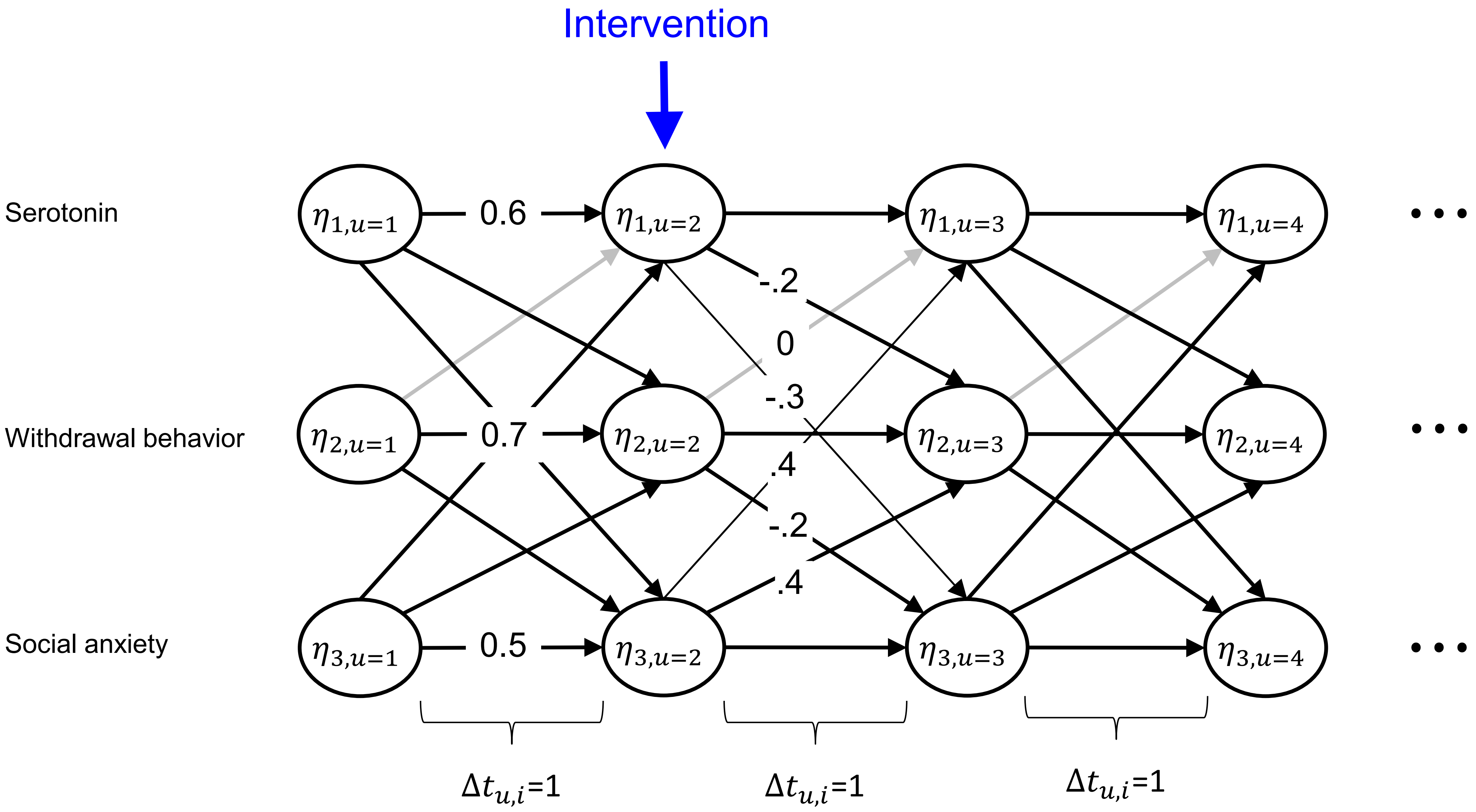
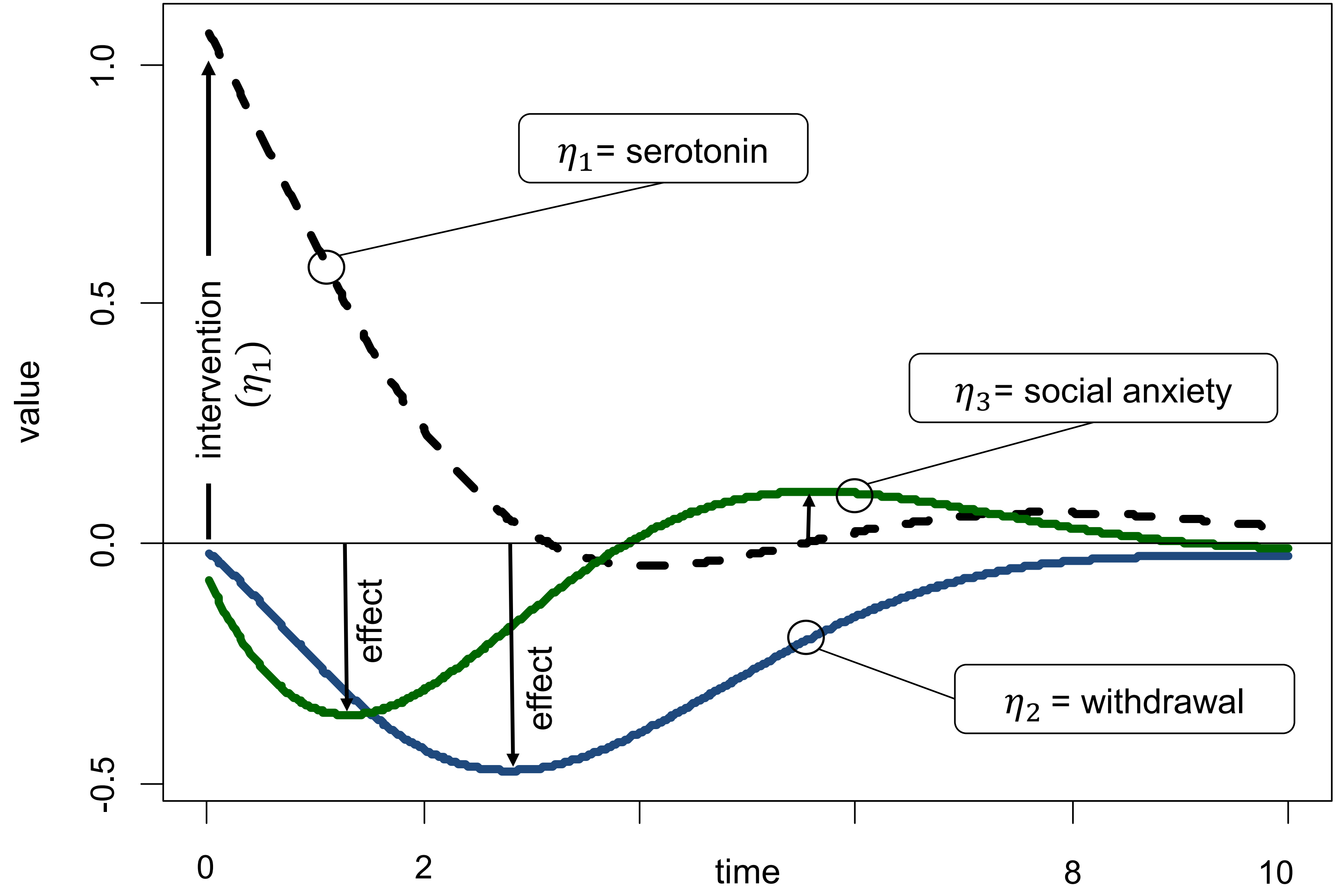


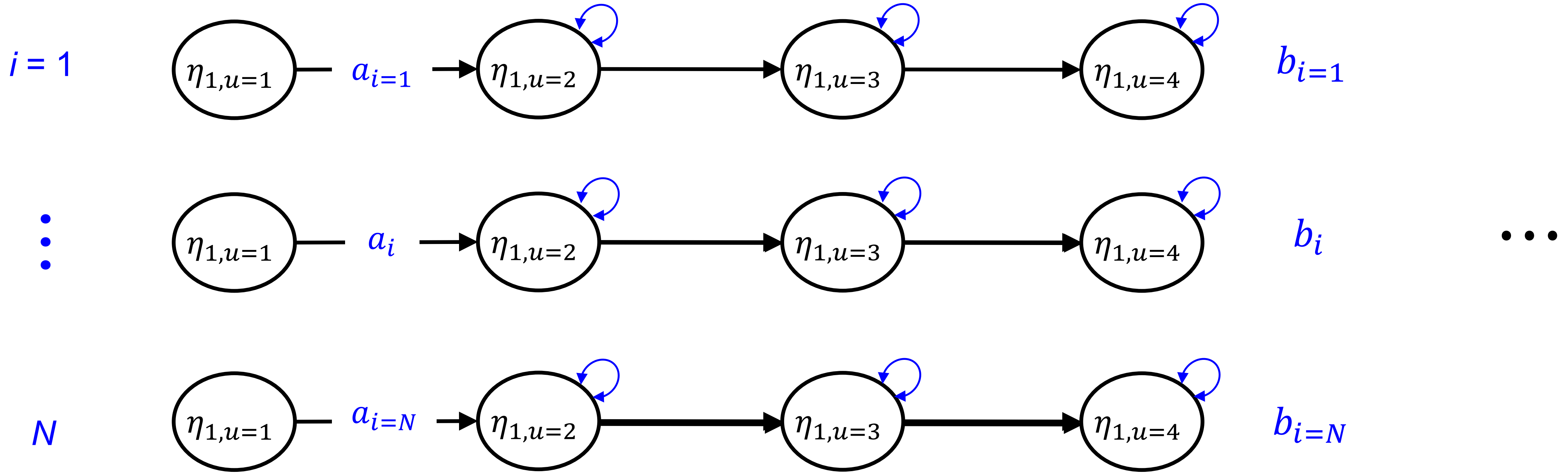
Understanding the time course of input effects



Voelkle, Gische, Driver & Lindenberger (2018). *Multivariate Behavioral Research*.

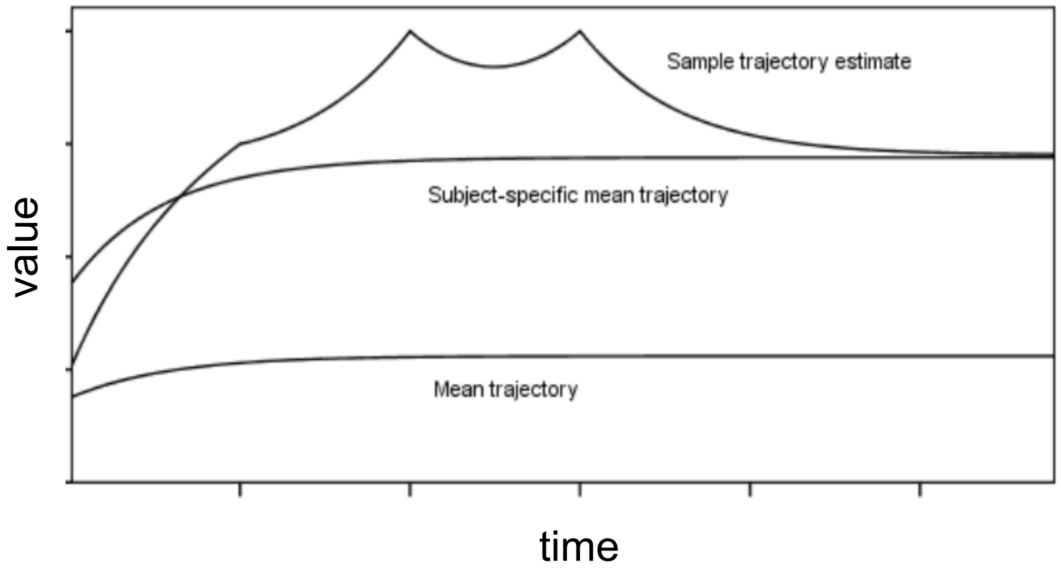
Heterogeneity

People differ – in level, process, variances…

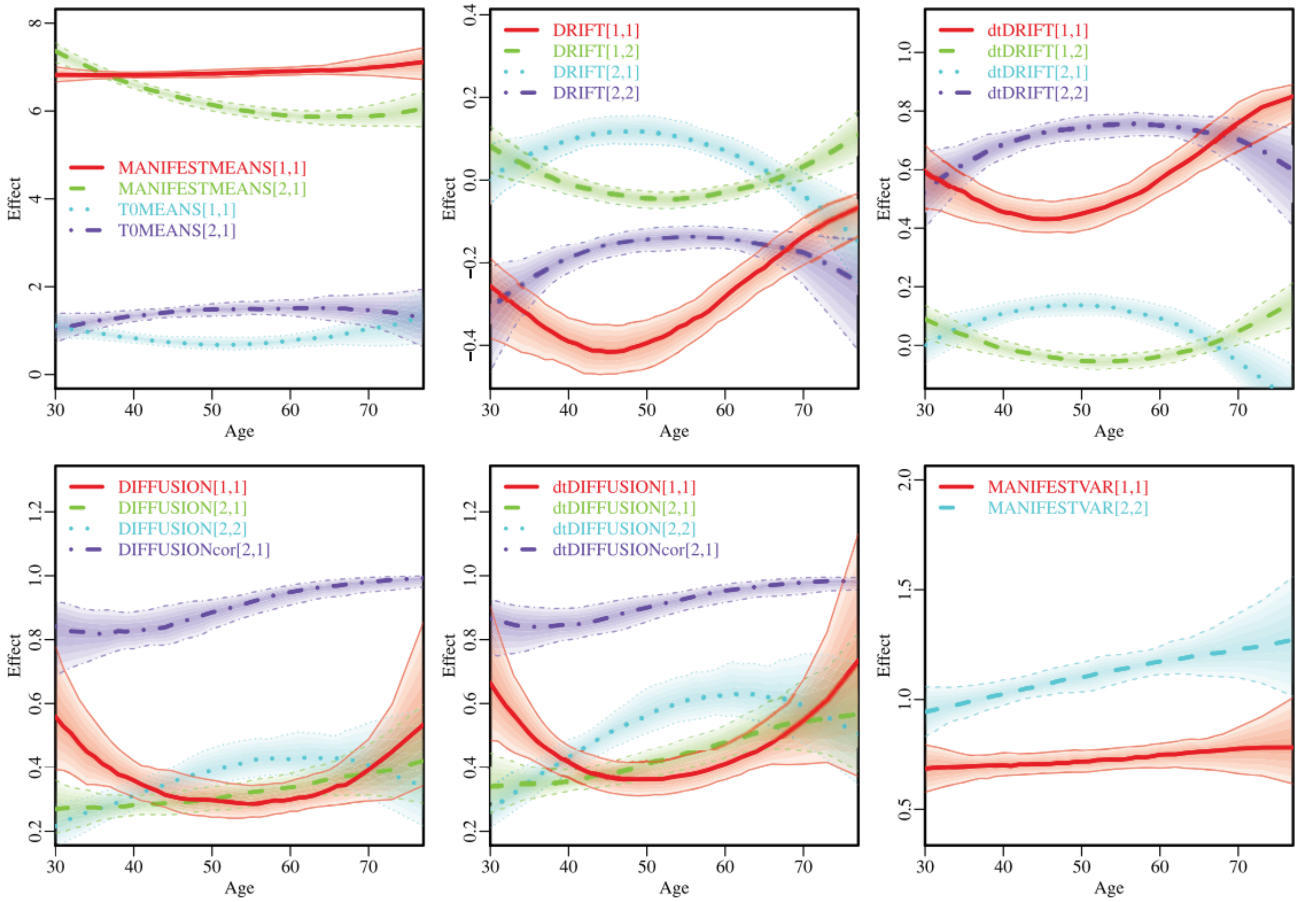


Heterogeneity

Frequentist Approach

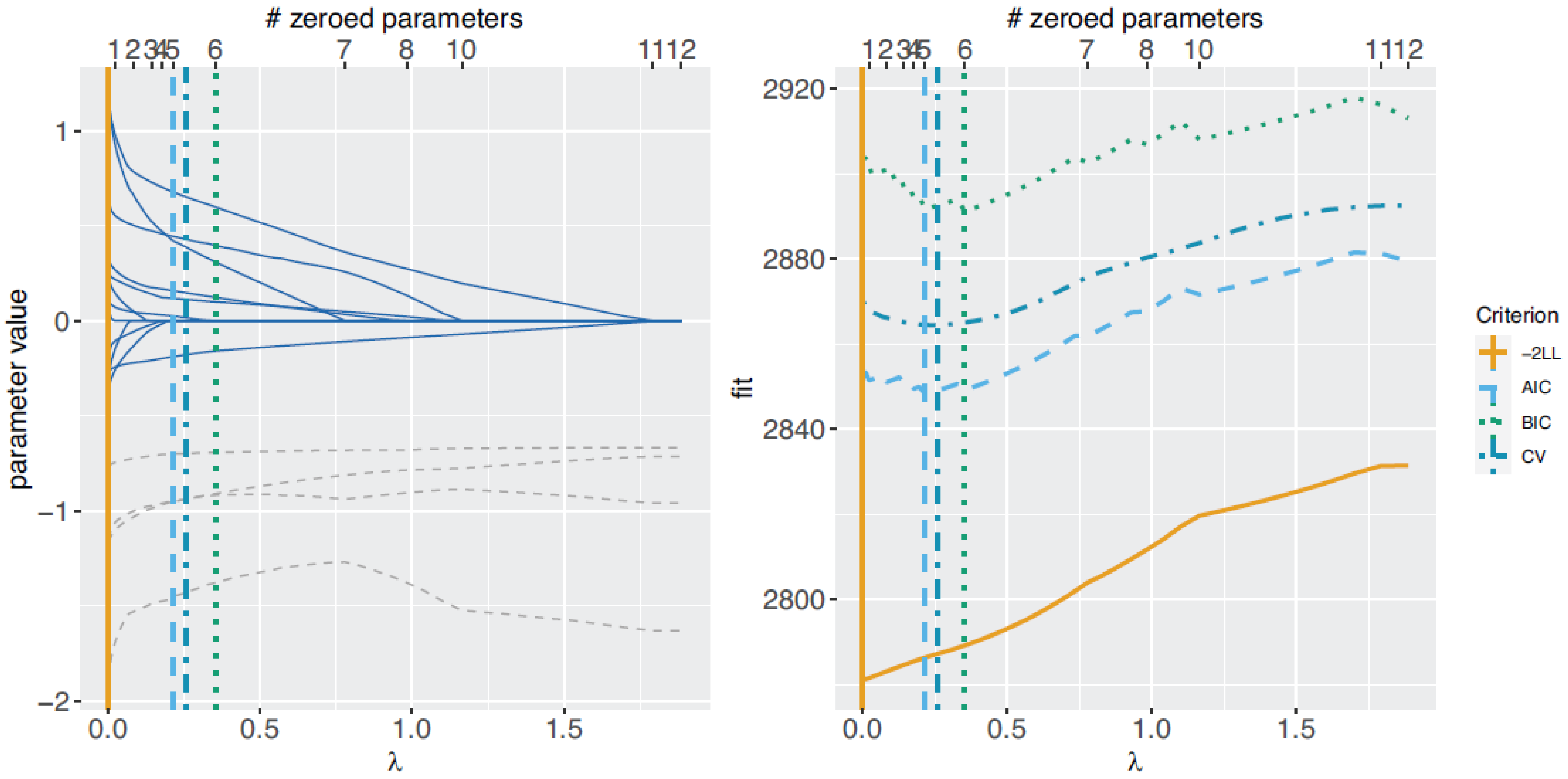


Bayesian Approach (fully hierarchical)



Oud & Delsing (2010). van Montfort, Oud, & Satorra (Eds.), *Longitudinal research with latent variables*. New York: Springer;
Driver & Voelkle (2018). *Psychological Methods.*

Dealing with complexity and causal inference



A somewhat biased list of references

Arnold, M., Oberski, D. L., Brandmaier, A. M., & Voelkle, M. C. (2019). Identifying Heterogeneity in Dynamic Panel Models with Individual Parameter Contribution Regression. Structural Equation Modeling: A Multidisciplinary Journal, 1-16. https://doi.org/10.1080/10705511.2019.1667240

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Study Questions

Question 1:
Explain the key differences between static and dynamic longitudinal models. Why is this distinction useful in psychological research? What are limits and problems of this distinction?

Question 2:
According to Baltes, Reese, and Nesselroade (1988), why is time described as a "theatrical stage" in longitudinal modeling? What does this metaphor imply for model specification?

Question 3:
Refer to the example of unequal time intervals across studies. What problem arises when comparing parameter estimates from data sampled at different intervals? How do continuous time models address this?

Question 4:
What is the role of the drift matrix in continuous time models? Explain why it is important for the interpretation of effects across different time intervals.