

Reflections on causal inference from a continuous time perspective

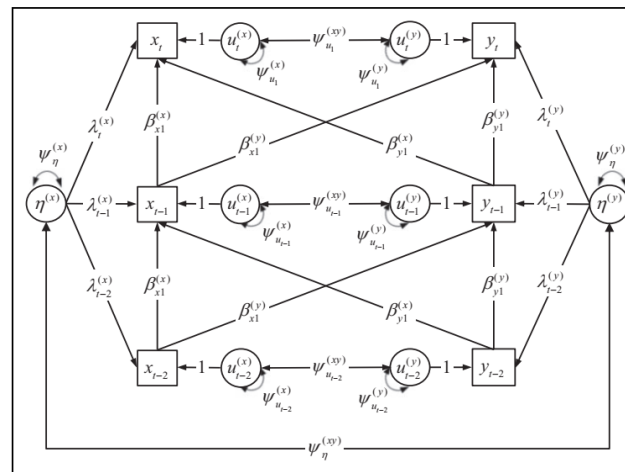
Outline

- On the role of time in graph-based causal models
- A mechanistic view on causality & local independence graphs
- Continuous time (structural) modeling
- Continuous time *structural* modeling in the social and behavioral sciences – quo vadis?

On the role of time in graph-based causal models

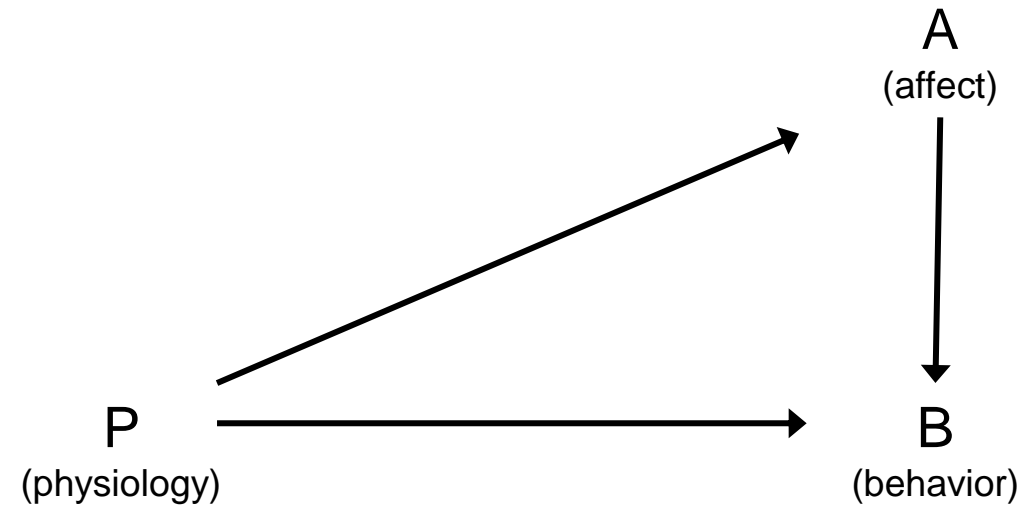
Recap:

- There are three key assumptions underlying graph-based causal models
 - Correctly encoded causal structure
 - Autonomy, stability, modularity, invariance
 - No interference
- We can use statistical approaches (e.g., the GCLPM) to infer causal effects from observational data (given some additional assumptions)



On the role of time in graph-based causal models

Example 1



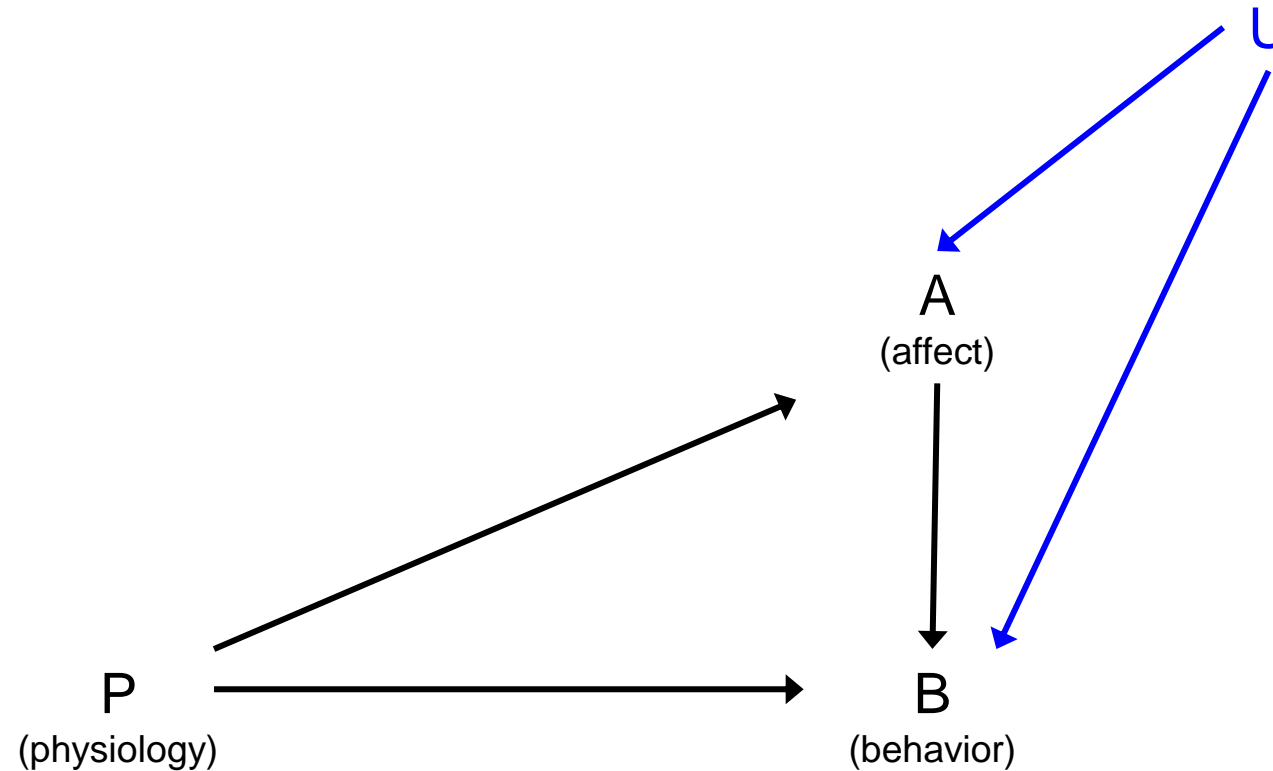
physiology = serotonin

affect = social anxiety

behavior = social withdrawal behavior

On the role of time in graph-based causal models

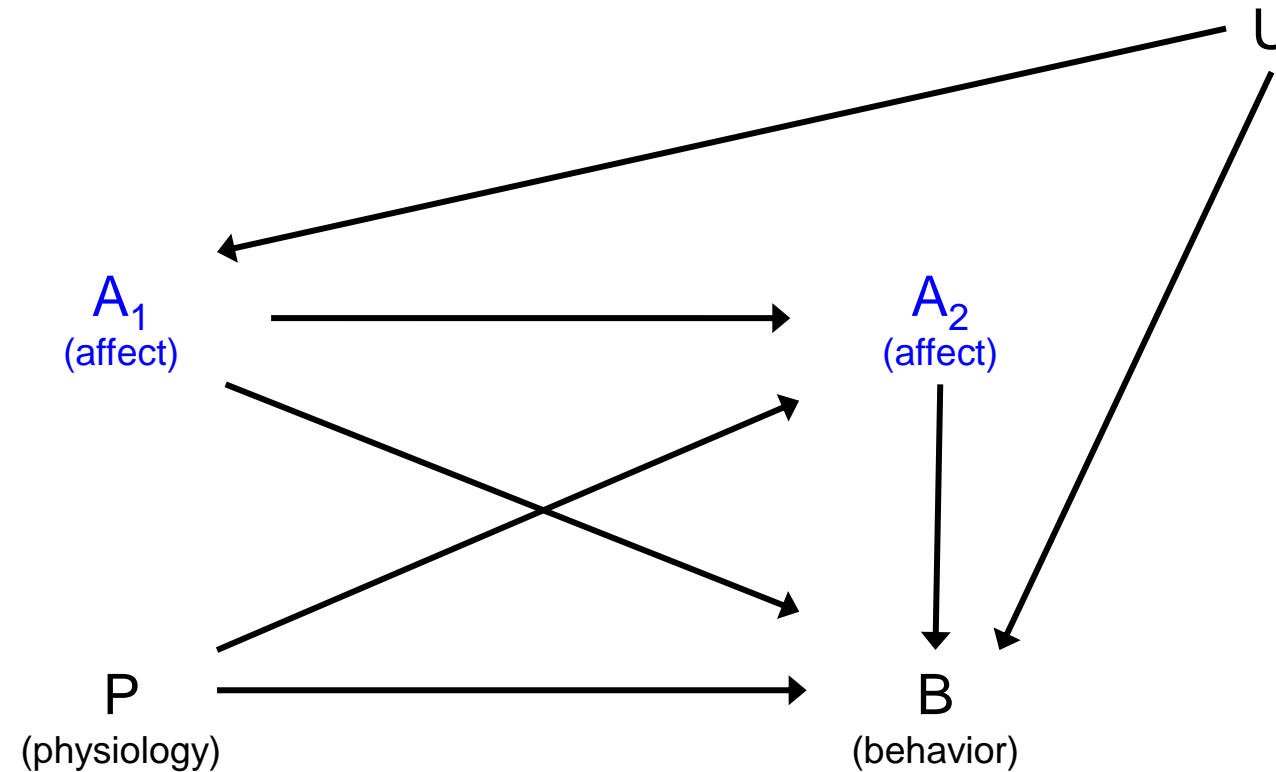
Example 1



In the presence of an **unmeasured confounder U** neither the direct nor the indirect effect of P on B is identified (A is a collider and U opens a backdoor path).

On the role of time in graph-based causal models

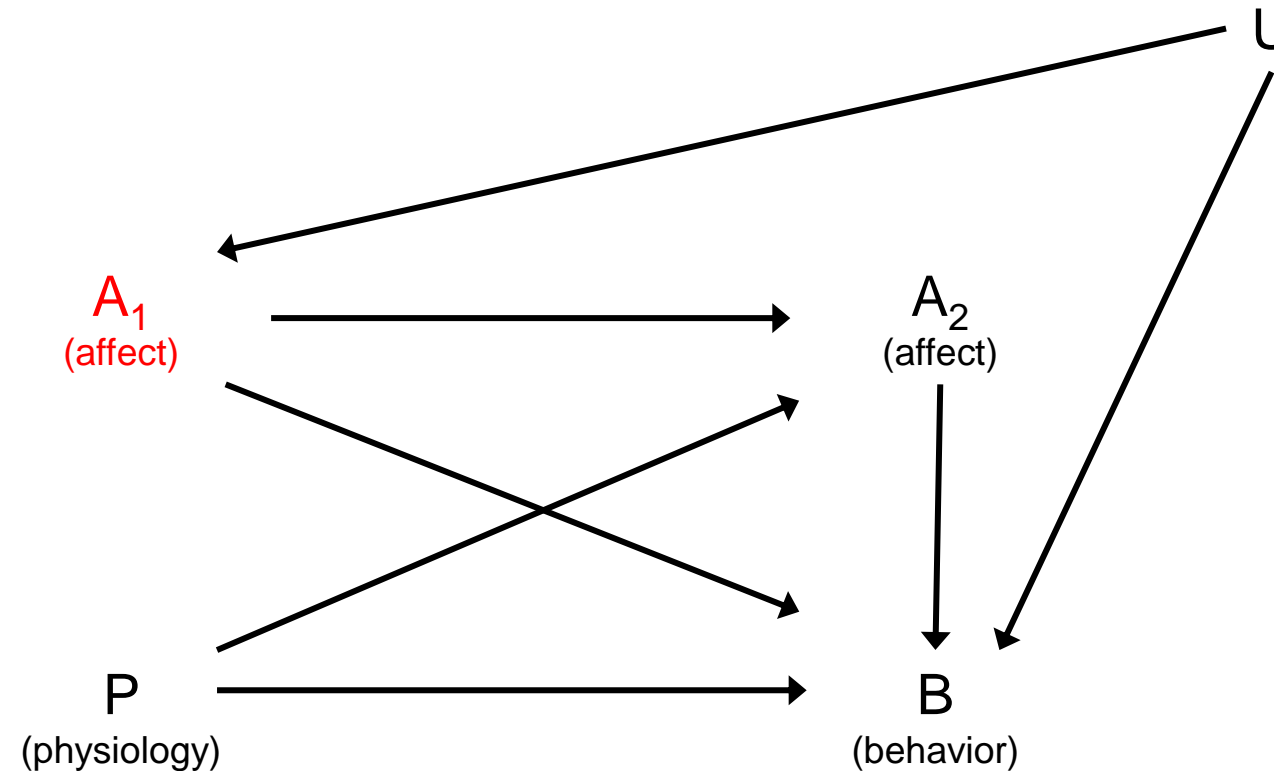
Example 1



Including repeated measurements, changes the DAG..

On the role of time in graph-based causal models

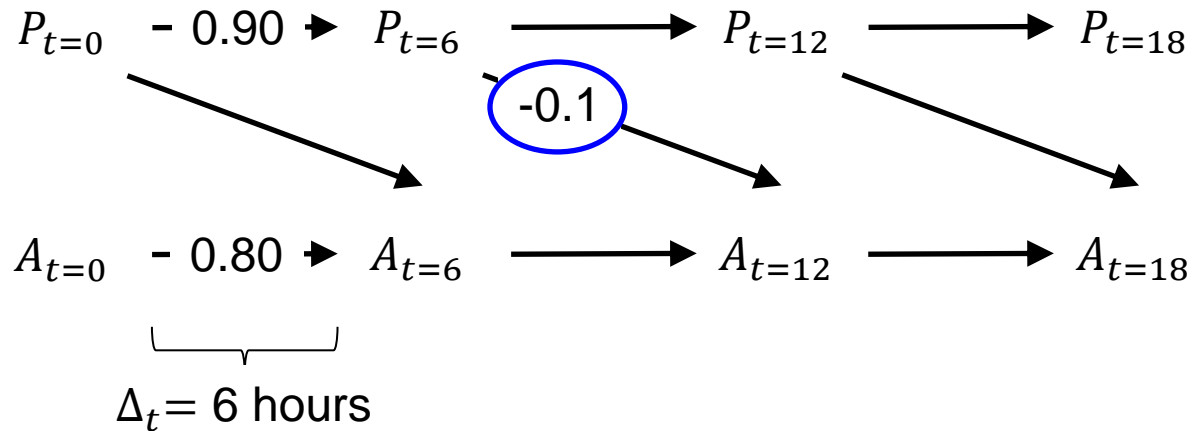
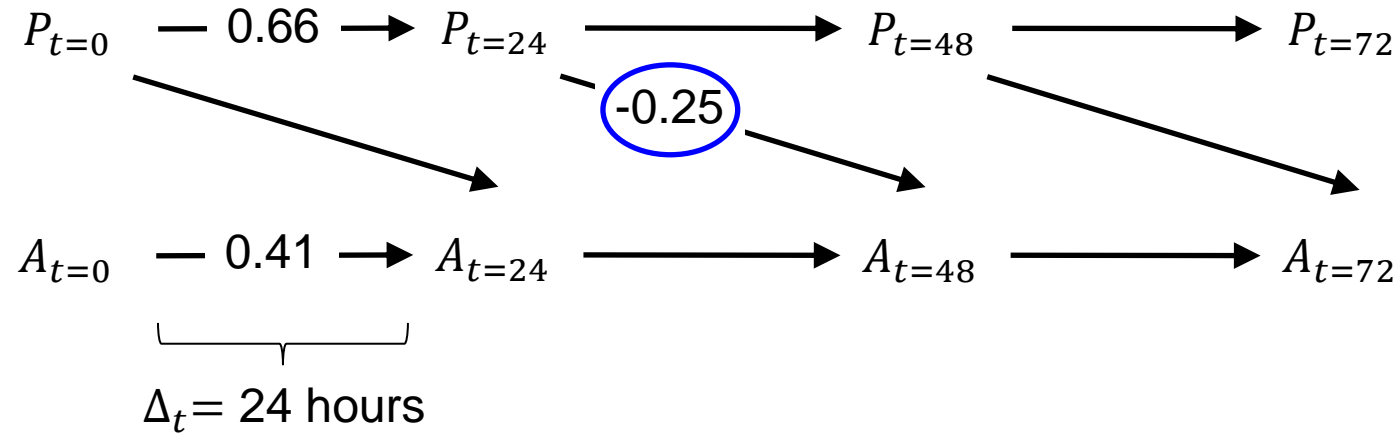
Example 1



Conditioning on A_1 permits the identification of direct and indirect causal effects of P (via A_2) on B

On the role of time in graph-based causal models

Example 2

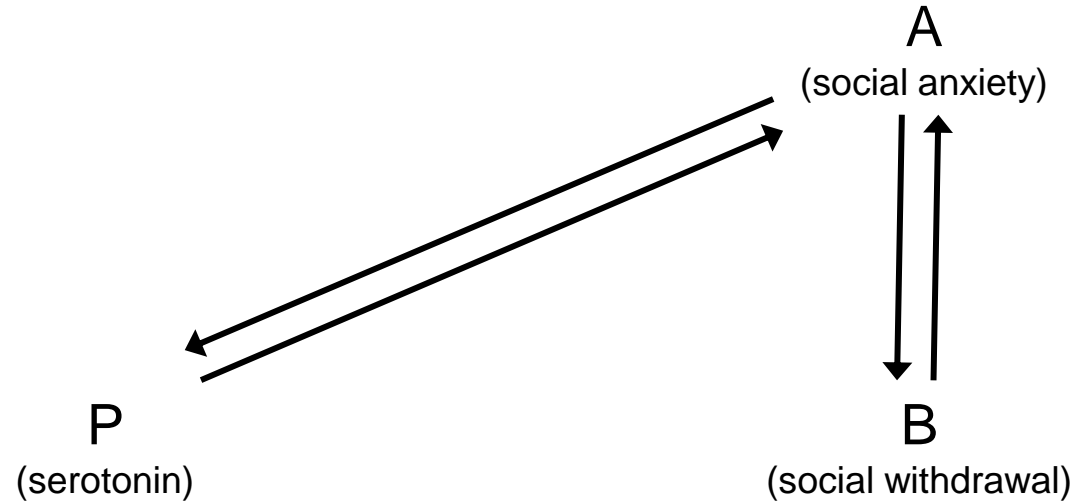


On the role of time in graph-based causal models

- Changing the number of measurement occasions changes the DAG
 - Should the causal model depend on the *measurement*?
- Changing the time intervals between measurement occasions does not change the DAG (but will almost always change the estimates of causal effects)
 - Should not time matter for the inference of causal effects?

A mechanistic view on causality & local independence graphs

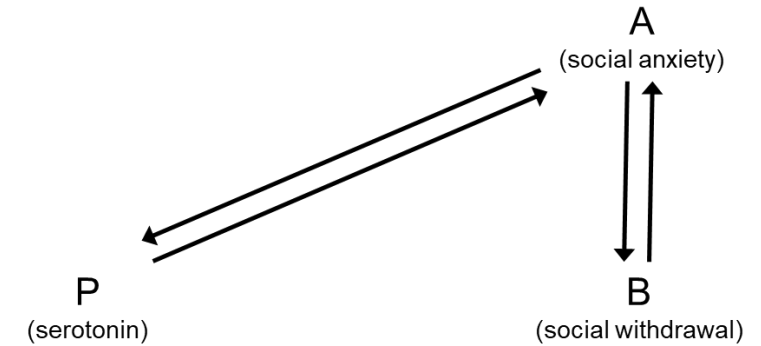
- In local (in)dependence graphs, nodes represent processes and arrows local dependencies.



- A missing edge implies local independence.
- Graphs are (usually) not acyclic.
- May be seen as a “natural extension of conditional independence (Bayesian networks) to a **dynamic time-continuous framework**” (Aalen et al., 2016, p. 2299)

Continuous time (structural) modeling: Theory

$$d\mathbf{x}(t) = \mathbf{A}\mathbf{x}(t)dt + \mathbf{G}d\mathbf{W}(t)$$



Local (in)dependencies are captured in the \mathbf{A} matrix

$$\mathbf{A} = \begin{pmatrix} a_{PP} & a_{PA} & 0 \\ a_{AP} & a_{AA} & a_{AB} \\ 0 & a_{BA} & a_{BB} \end{pmatrix}$$

“If the differential equation is structural in the sense that changing an element of \mathbf{A} does not change the other elements, then the local (in)dependence relationships are causal” (Aalen et al. 2016, p. 2302).

Continuous time (structural) modeling: Theory

- As discussed during the last couple of days, the statistical theory and tools for continuous time modeling are well established.

$$\frac{\Delta \mathbf{x}_{t_u}}{\Delta time} = (\mathbf{A}_{\dagger} - \mathbf{I}) \mathbf{x}_{t_{u-1}} + \mathbf{w}_{t_u}^*$$

$$\lim_{\Delta time \rightarrow 0} \left(\frac{\Delta \mathbf{x}_{t_u}}{\Delta time} \right) = \frac{d\mathbf{x}(t)}{dt}$$

$$d\mathbf{x}(t) = \mathbf{A}\mathbf{x}(t)dt + \mathbf{G}d\mathbf{W}(t)$$

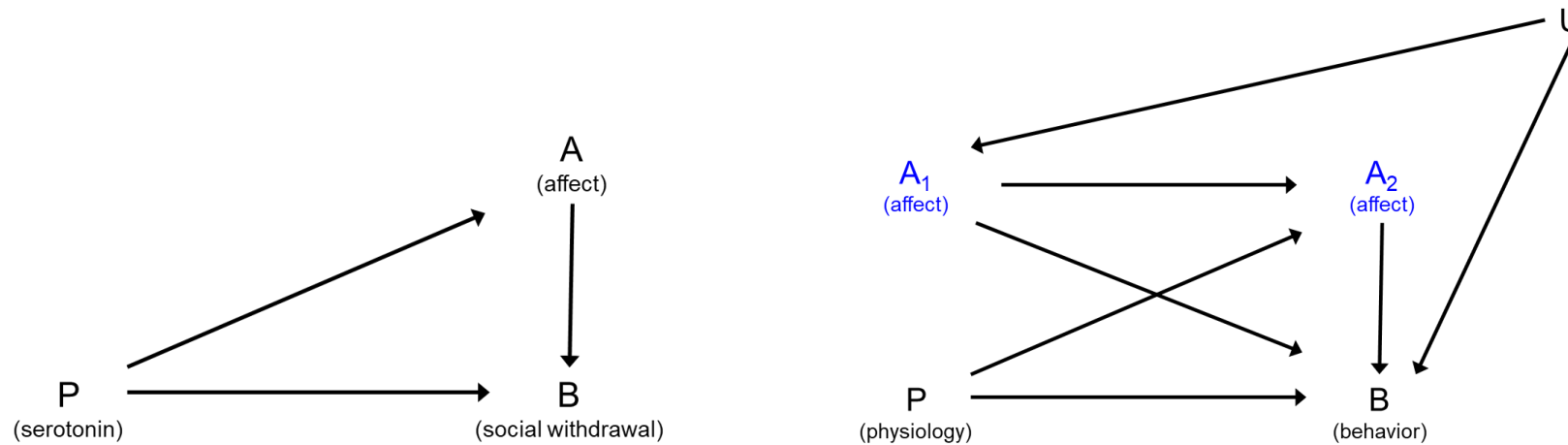
- But what about their role in (applied) causal inference from observational data?

Ct *structural* modeling in the social and behavioral sciences – quo vadis?

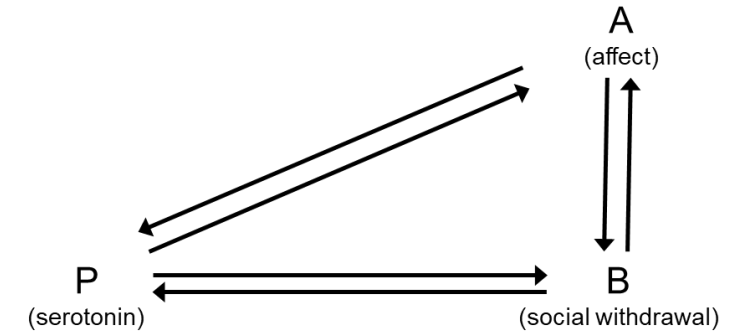


Ct *structural* modeling in the social and behavioral sciences – quo vadis?

1. We need to better clarify the role of time in our causal models.



Directed Acyclic Graphs



Local independence graph

Ct *structural* modeling in the social and behavioral sciences – quo vadis?

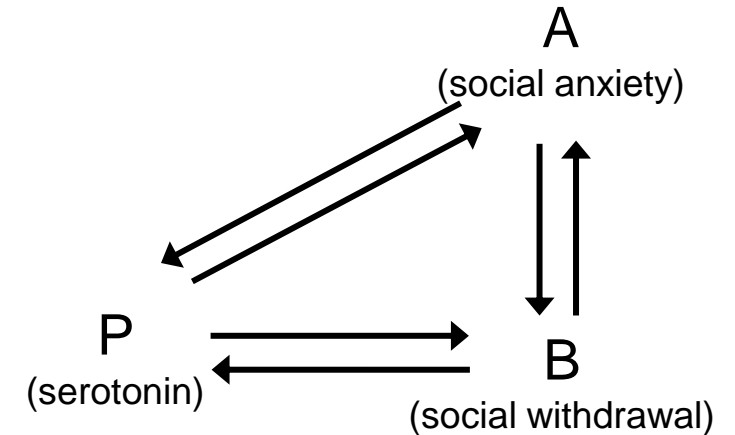
2. We need to better separate the (discrete time) *measurement* from the (continuous time) causal *mechanisms*.

$P_{u=1} \longrightarrow P_{u=2} \longrightarrow P_{u=3} \longrightarrow P_{u=4}$

$B_{u=1} \longrightarrow B_{u=2} \longrightarrow B_{u=3} \longrightarrow B_{u=4}$

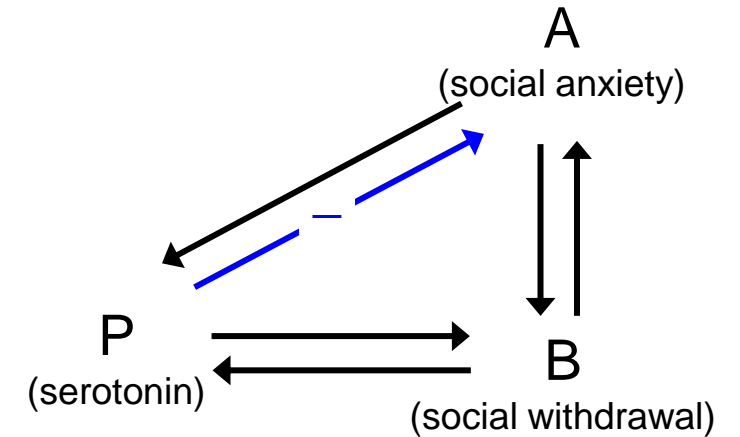
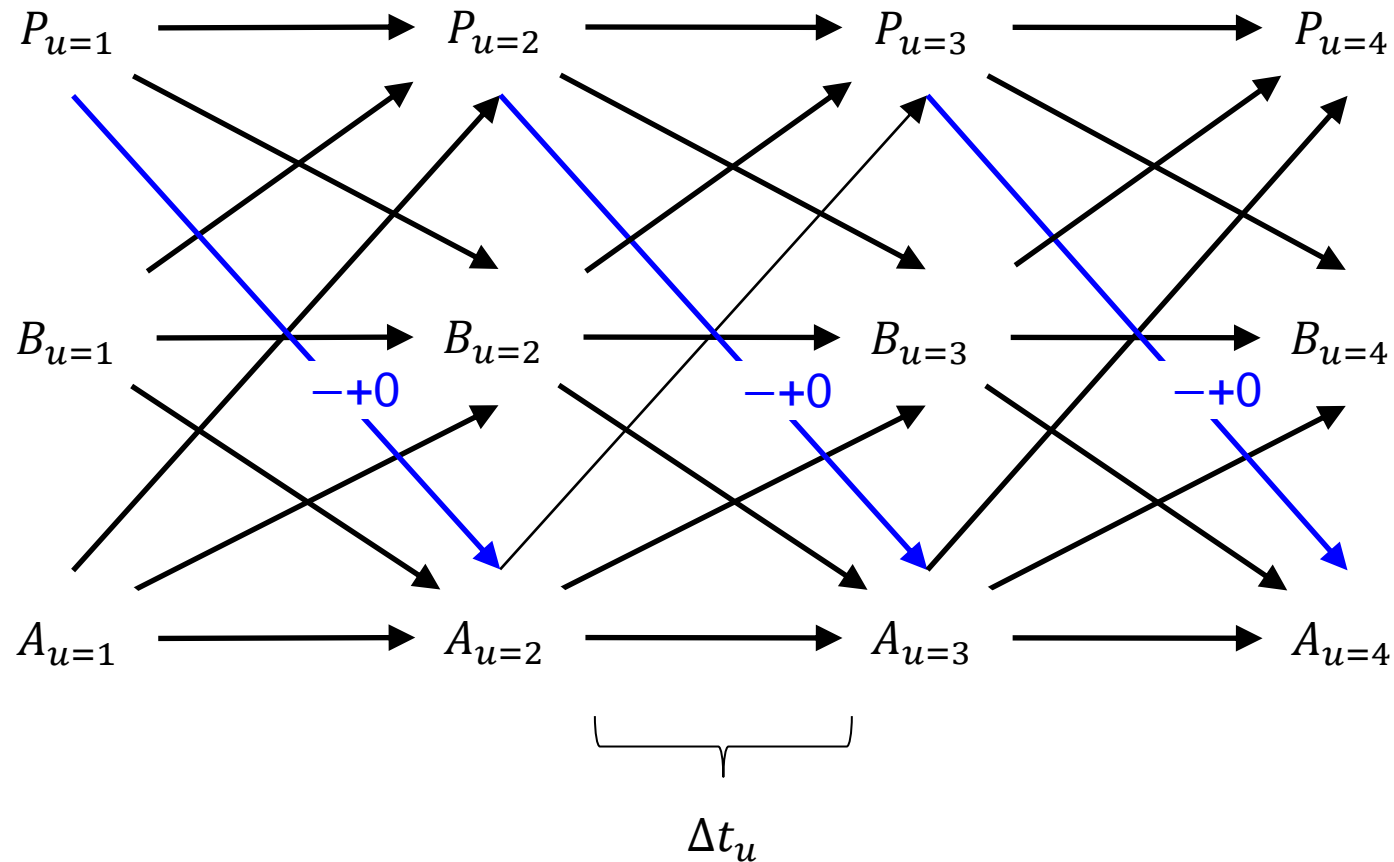
$A_{u=1} \longrightarrow A_{u=2} \longrightarrow A_{u=3} \longrightarrow A_{u=4}$

$\underbrace{\hspace{10em}}_{\Delta t_u}$



Ct *structural* modeling in the social and behavioral sciences – quo vadis?

2. We need to better separate the (discrete time) *measurement* from the (continuous time) causal *mechanisms*.

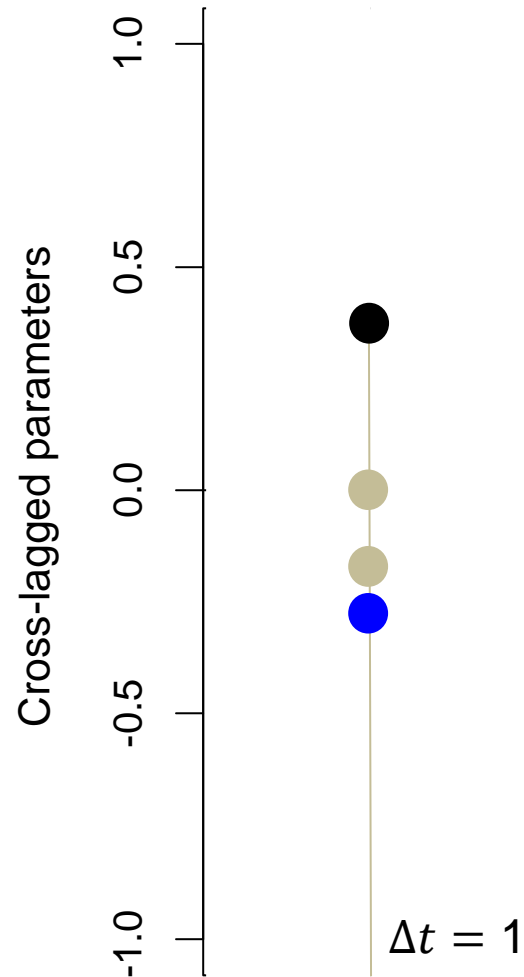


Ct *structural* modeling in the social and behavioral sciences – quo vadis?

2. We need to better separate the (discrete time) *measurement* from the (continuous time) causal *mechanisms*.

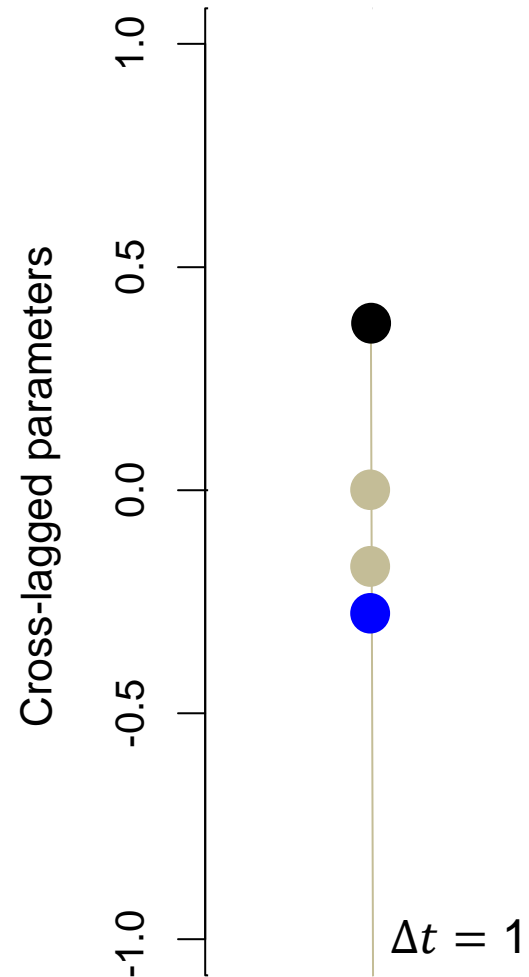
	P (serotonin)	B (social withdrawal)	A (social anxiety)
$\mathbf{A} =$	$\begin{pmatrix} -0.3496 \\ -0.1315 \\ -0.4992 \end{pmatrix}$	$\begin{pmatrix} +0.0863 \\ -0.2623 \\ -0.2825 \end{pmatrix}$	$\begin{pmatrix} +0.6081 \\ +0.6512 \\ -0.4153 \end{pmatrix}$

Ct *structural* modeling in the social and behavioral sciences – quo vadis?



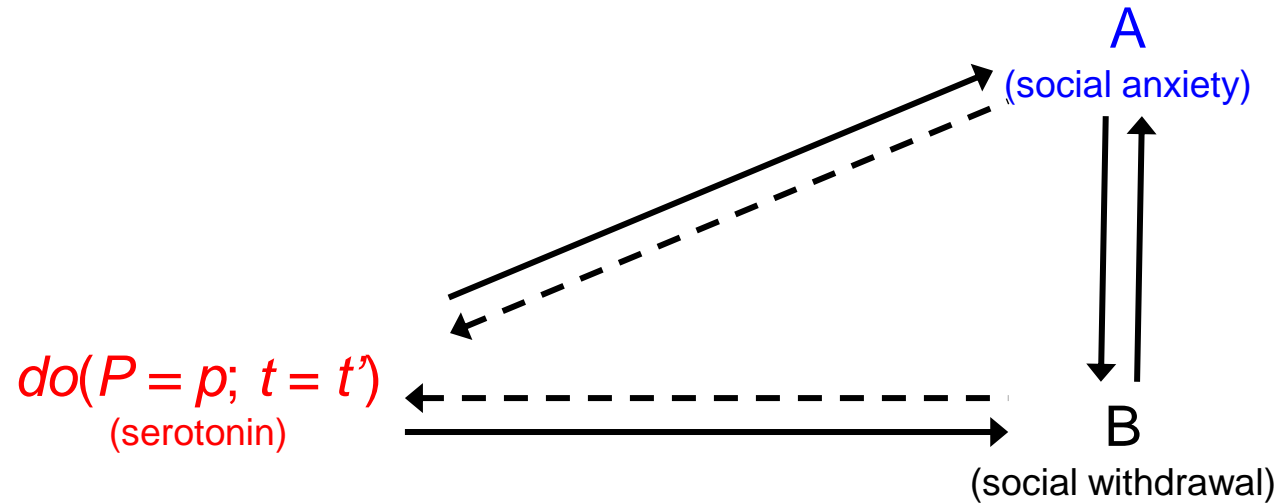
$$\mathbf{A}_{\Delta t=1}^* = \begin{pmatrix} +0.6 & \pm 0.0 & +0.4 \\ -0.2 & +0.7 & +0.4 \\ -0.3 & -0.2 & +0.5 \end{pmatrix}$$

Ct *structural* modeling in the social and behavioral sciences – quo vadis?

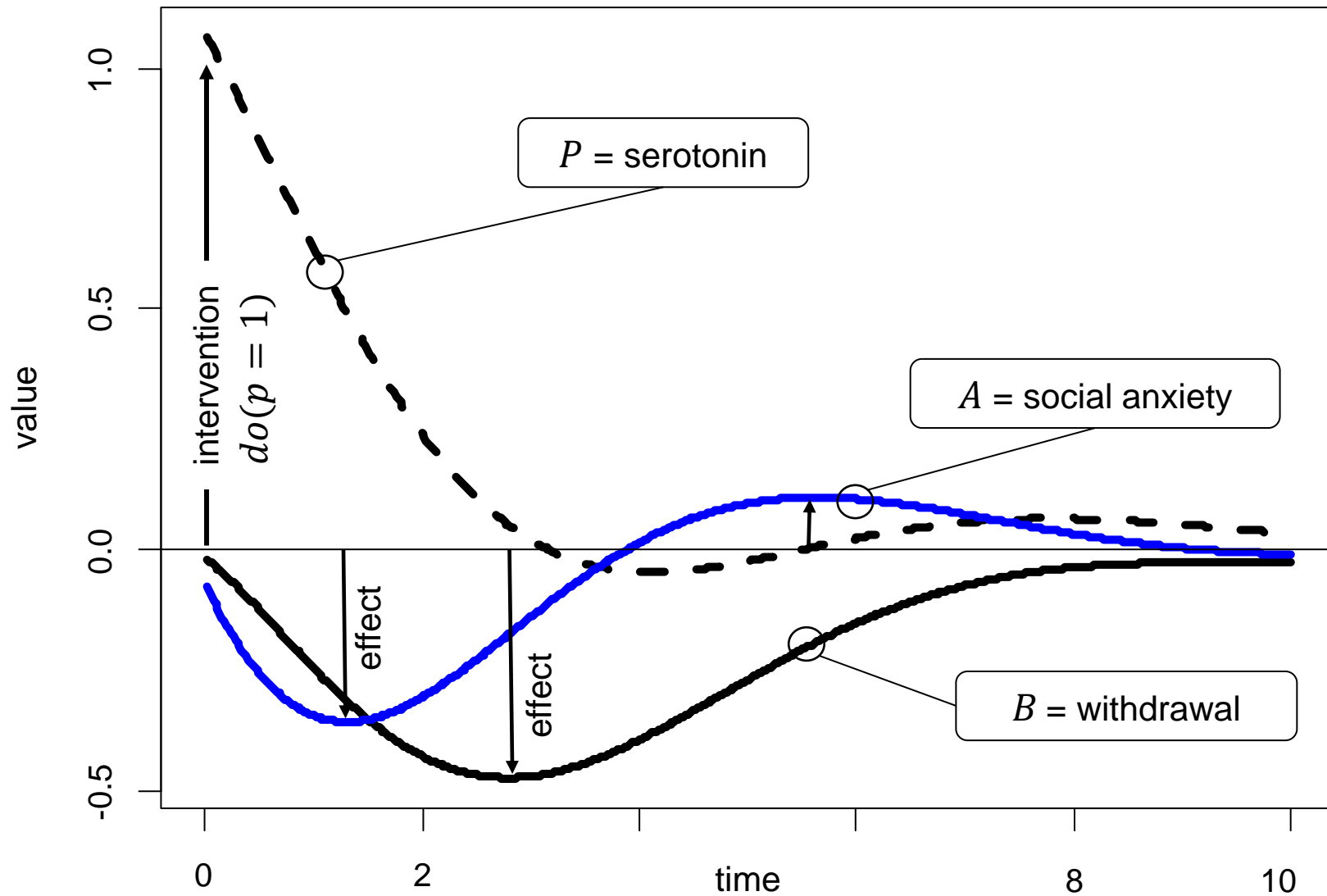


Ct *structural* modeling in the social and behavioral sciences – quo vadis?

3. Time is important for understanding effects of interventions.



Ct *structural* modeling in the social and behavioral sciences – quo vadis?



Ct *structural* modeling in the social and behavioral sciences – quo vadis?

- “Statistical” issues:
 - from linear dynamics to non-linear dynamics (non-linear SDEs)?
 - from few variables to many variables and back (regularization, partitioning)?
 - from fast (frequentist) estimation to slow (Bayesian) estimation and back?
- “Causal” issues:
 - Mechanistic vs. interventionist approach vs. both?
 - Potentials and limits of local independence graphs?
 - Lost in translation?
- Practical issues:
 - What’s in for our discipline?
 - How to (better) get from data to causes?
 - Will it improve science?

Selected References

- Aalen, O. O., Røysland, K., Gran, J. M., Kouyos, R., & Lange, T. (2016, Oct). Can we believe the DAGs? A comment on the relationship between causal DAGs and mechanisms. *Stat Methods Med Res*, 25(5), 2294-2314. <https://doi.org/10.1177/0962280213520436>
- Arnold, M., Oberski, D. L., Brandmaier, A. M., & Voelkle, M. C. (2019). Identifying Heterogeneity in Dynamic Panel Models with Individual Parameter Contribution Regression. *Structural Equation Modeling: A Multidisciplinary Journal*, 1-16. <https://doi.org/10.1080/10705511.2019.1667240>
- Brandmaier, A. M., Driver, C. C., & Voelkle, M. C. (2018). Recursive partitioning in continuous time analysis. In K. v. Montfort, J. H. L. Oud, & M. C. Voelkle (Eds.), *Continuous Time Modeling in the Behavioral and Related Sciences* (pp. 259-282). Springer Nature.
- Didelez, V. (2008). Graphical models for marked point processes based on local independence. *Journal of the Royal Statistical Society: Series B (Statistical Methodology)*, 70(1), 245-264. <https://doi.org/https://doi.org/10.1111/j.1467-9868.2007.00634.x>
- Driver, C. C., Oud, J. H. L., & Voelkle, M. C. (2017). Continuous time structural equation modeling with R package ctsem. *Journal of Statistical Software*, 77(5), 1-35. doi:10.18637/jss.v077.i05
- Driver, C. C., & Voelkle, M. C. (2018). Hierarchical Bayesian continuous time dynamic modeling. *Psychological Methods*, 23(4), 774-799. <https://doi.org/10.1037/met0000168>
- Driver, C. C., & Voelkle, M. C. (2018). Understanding the time course of interventions with continuous time dynamic models. In K. v. Montfort, J. H. L. Oud, & M. C. Voelkle (Eds.), *Continuous time modeling in the behavioral and related sciences* (pp. 79-109). Springer.
- Gische, C., West, S. G., & Voelkle, M. C. (2020). Forecasting Causal Effects of Interventions versus Predicting Future Outcomes. *Structural Equation Modeling: A Multidisciplinary Journal*, 1-18. <https://doi.org/10.1080/10705511.2020.1780598>
- Guthier, C., Dormann, C., & Voelkle, M. C. (2020). Reciprocal effects between job stressors and burnout: A continuous time meta-analysis of longitudinal studies. *Psychological Bulletin*, 146(12), 1146-1173. <https://doi.org/10.1037/bul0000304>
- Hecht, M., Hardt, K., Driver, C. C., & Voelkle, M. C. (2019). Bayesian continuous-time Rasch models. *Psychological Methods*, 24(4), 516-537. <https://doi.org/10.1037/met0000205>
- Mogensen, S. W., & Hansen, N. R. (2020). Markov equivalence of marginalized local independence graphs. *The Annals of Statistics*, 48(1), 539-559, 521. <https://doi.org/10.1214/19-AOS1821>
- Oud, J. H. L., & Voelkle, M. C. (2013). Do missing values exist? Incomplete data handling in cross-national longitudinal studies by means of continuous time modeling. *Quality & Quantity*, 1-18. doi:10.1007/s11135-013-9955-9
- Oud, J. H. L., Voelkle, M. C., & Driver, C. C. (2018). SEM Based CARMA Time Series Modeling for Arbitrary N. *Multivariate Behavioral Research*, 53(1), 36-56. doi:10.1080/00273171.2017.1383224
- Sokol, A., & Hansen, N. (2014). Causal interpretation of stochastic differential equations. *Electronic Journal of Probability*, 19(none), 1-24, 24. <https://doi.org/10.1214/EJP.v19-2891>
- van Montfort, K., Oud, J. H. L., & Voelkle, M. C. (Eds.). (2018). *Continuous Time Modeling in the Behavioral and Related Sciences*. Springer.
- Voelkle, M. C. (2017). A new perspective on three old methodological issues: The role of time, missing values, and cohorts in longitudinal models of youth development. In A. C. Petersen, S. H. Koller, F. Motti-Stefanidi, & S. Verma (Eds.), *Positive youth development in global contexts of social and economic change* (pp. 110-136). New York, NY: Taylor & Francis.
- Voelkle, M. C., Gische, C., Driver, C. C., & Lindenberger, U. (2018). The role of time in the quest for understanding psychological mechanisms. *Multivariate Behavioral Research*, 53(6), 782-805. <https://doi.org/10.1080/00273171.2018.1496813>
- Voelkle, M. C., & Oud, J. H. L. (2015). Relating latent change score and continuous time models. *Structural Equation Modeling: A Multidisciplinary Journal*, 22(3), 366-381. doi:10.1080/10705511.2014.935918
- Voelkle, M. C., & Oud, J. H. L. (2013). Continuous time modelling with individually varying time intervals for oscillating and non-oscillating processes. *British Journal of Mathematical and Statistical Psychology*, 103-126. doi:10.1111/j.2044-8317.2012.02043.x
- Voelkle, M. C., Oud, J. H. L., Davidov, E., & Schmidt, P. (2012). An SEM approach to continuous time modeling of panel data: Relating authoritarianism and anomia. *Psychological Methods*, 17, 176-192. doi:10.1037/a0027543

Study Questions

Question 1:

Discuss the importance of correctly specifying causal structures in DAGs and explain how the temporal dimension complicates this process in longitudinal psychological research.

Question 2:

What is a local independence graph and how does it differ from a traditional DAG in terms of causal interpretation?

Question 3:

Discuss the phrase "we need to better separate the measurement from the mechanism" in the context of CT modeling. What does it mean? Do you agree or disagree? What are your thoughts?

Study Questions

Question 4:

Form a group with your fellow students. As a group, reflect on the following:

1. How can we responsibly make causal claims about latent processes?
2. What role should theory, measurement design, and modeling techniques (e.g., continuous-time models, latent variables, local independence graphs) play in bridging the gap between what we can observe and what we want to explain?
3. Are there limits to what formal models can tell us about causal psychological mechanisms?

Use a real or hypothetical example from your field (e.g., therapy, development, social interaction, cognition) to ground your discussion.

Thank you very much for your attention!

...and acknowledgements to the many people who have contributed to the workshop in one way or another

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Continuous Time
Modeling in the
Behavioral and
Related Sciences