

Assignment 1

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0.1 Theoretically calculated $P(S_t = j)$

The initial probabilities of being in any of the states are given by q and the transition probabilities are given by A .

$$q = \begin{pmatrix} 0.75 \\ 0.25 \end{pmatrix} A = \begin{pmatrix} 0.99 & 0.01 \\ 0.03 & 0.97 \end{pmatrix}$$

When $t = 1$ then $P(S_t = j) = q$. When progressing to $t = 2$ you multiply the probabilities of being in any given state with the transition probabilities. This gives the new probabilities that you are in any given state.

$$q' * A = (0.75 \quad 0.25) * \begin{pmatrix} 0.99 & 0.01 \\ 0.03 & 0.97 \end{pmatrix} = (0.75 \quad 0.25)$$

Since the probabilities of being in any given state remain the same after the transition, the probability of being in any given state will remain stationary.

$$P(S_t = j) = P(S_{t+1} = j) \forall t \in \{1, 2, \dots\}$$

Our corresponding measured results for our MarkovChain code was that $P(S_t = 1) = 0.7341$ and $P(S_t = 2) = 0.2659$

0.2 Theoretically calculated $E[X_t]$

The mean depends on the probability of being in any given state and the output distribution. Since both of these will remain stationary when t changes, the expected output X will be the same for every t .

The expected output will be the probability of being in any given state times the mean of the output distribution for that state.

$$E[X] = P(S = 1) * E[b_1] + P(S = 2) * E[b_2] = 0.75 * 0 + 0.25 * 3 = 0.75$$

Our corresponding measured result was 0.7155.

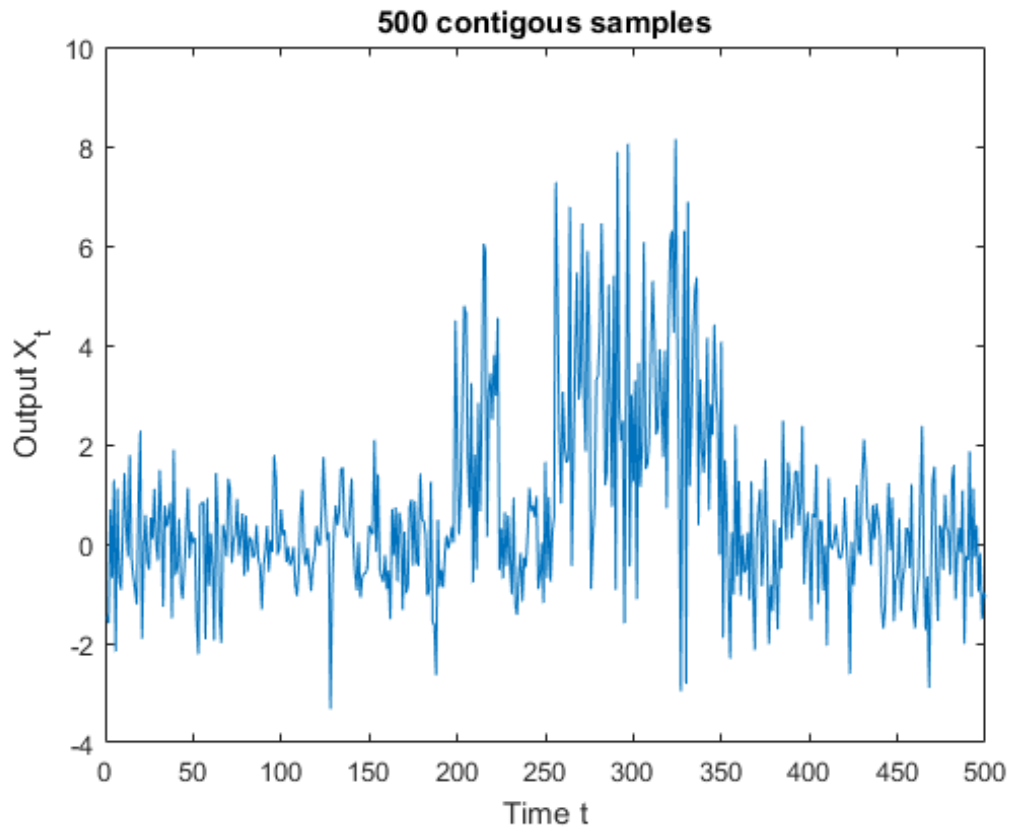
0.3 Theoretically calculated $var[X_t]$,

To calculate the variance the following formula can be used

$$\begin{aligned} var[X] &= E_Z[var_x[X|Z]] + var_Z[E_X[X|Z]] = \\ P(S = 1) * \sigma_1^2 + P(S = 2) * \sigma_2^2 + P(S = 1) * (\mu_1 - \mu)^2 + P(S = 2) * (\mu_2 - \mu)^2 = \\ 0.75 * 1^2 + 0.25 * 2^2 + 0.75 * (0 - 0.75)^2 + 0.25 * (3 - 0.75)^2 &= 3.4375 \end{aligned}$$

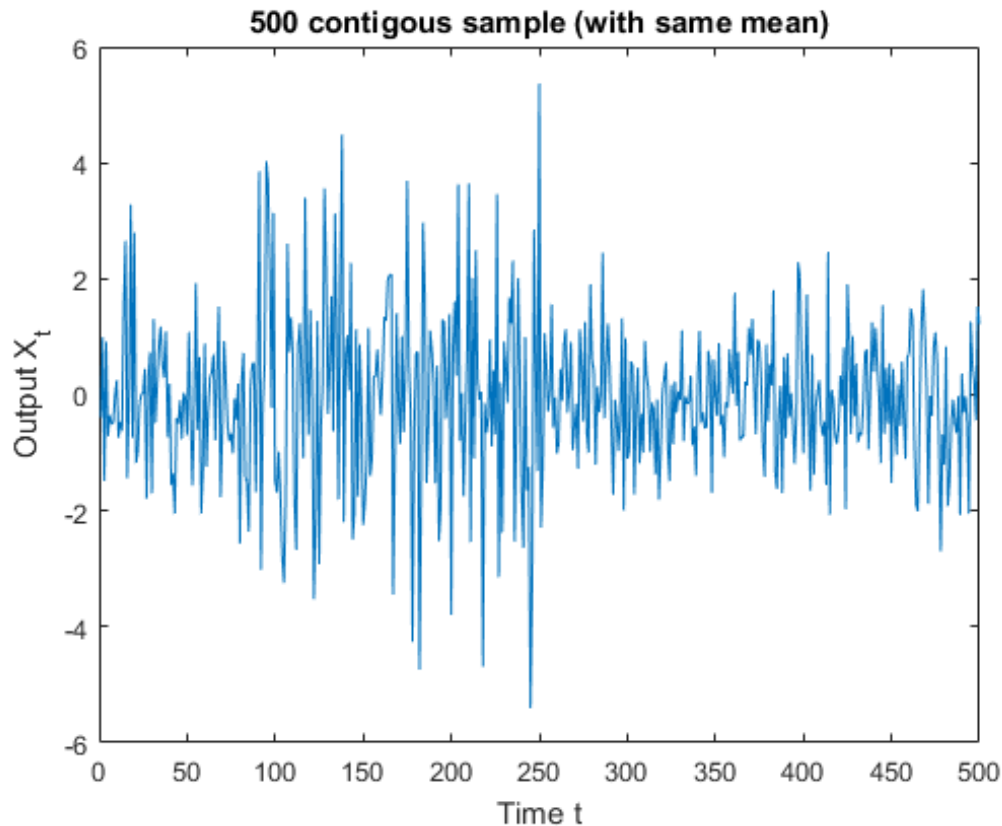
Our corresponding measured result was 3.3391.

0.4 Plotting 500 contiguous samples (with different means)



It can be seen that when the mean is $\tilde{0}$ and the variance is low we are in state 1 and when the mean is 3 and the variance is high we are in state 2. From this observation it can be seen that state 1 has been generated approximately 75% of the time.

0.5 Plotting 500 contiguous samples (with same means)



The two plots are similar in that a difference in variance can be seen between the two states. The difference between the plots is that it is harder to distinguish the two states in the second plot due to the same mean of the output distributions. In some plots it was very difficult to estimate the state sequence, but in this plot it was possible.

0.6 Finite duration test HMM

In our finite duration test HMM we used the same q and B matrices as in the previous section. The A matrix used was the following:

$$A = \begin{pmatrix} 0.5 & 0.4 & 0.1 \\ 0.2 & 0.5 & 0.3 \end{pmatrix}$$

When running this HMM 100 times we got sequences of length ranging from 1 to 19, with an average of 4.56. Since we have quite high probabilities of reaching the end state we believe these results are reasonable.

Relevant testing code:

```
lens = zeros(1, 100);
for i=1:100
    res = rand(h, 500);
    lens(i)=numel(res);
end
mean(lens)
min(lens)
max(lens)
```

0.7 2D output distributions

When testing if our code could handle multidimensional output distribution we used the same A and q matrices as the one given in the beginning of the lab. Our two output distributions were the

following:

$$\mu_{b_1} = \begin{pmatrix} -5 \\ 5 \end{pmatrix} \Sigma_{b_1} = \begin{pmatrix} 2 & 1 \\ 1 & 4 \end{pmatrix}$$
$$\mu_{b_2} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Sigma_{b_2} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

To verify that our code worked for multidimensional data we ran our code and plotted 500 contiguous data samples. Since the distributions behaved as expected and the distributions took up roughly 75% and 25% respectively we assumed that our code was correct.

