ETH Zürich
#516

Visual Computing

Frédéric Vogel

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Convolution and Filtering

Correlation: $I' = K \circ I$, $I'(x,y) = \sum_{(i,j)\in N(x,y)} K(i,j)\cdot I(x+i, y+j)$

→ used for template matching (also towards slight areas → remove noise)

→ $I'(x,y) = \sum_{j=-k}^{k}\sum_{i=-k}^{k} K(i,j)\, I(x+i, y+j)$

Convolution: $I' = K * I$, $I'(x,y) = \sum_{(i,j)\in N(x,y)} K(i,j)\, I(x-i, y-j)$

→ same as correlation with reversed kernel

→ if $k(i,j) = k(-i,-j)$ then $K \circ I = K * I$

→ associative!

→ $I'(x,y) = \sum_{j=-k}^{k}\sum_{i=-k}^{k} K(-i,-j)\, I(x+i, y+j)$

Near the edge:

→ filter window falls off image

→ extrapolation:

- ↦ clip filter
- ↦ wrap around
- ↦ copy edge
- ↦ reflect across edge

Gaussian Kernel:

→ Weight contribution of neighboring pixels by nearness

→ $G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$

→ Separable: $g(x,y) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{-x^2}{2\pi\sigma^2}\right] \cdot \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[\frac{-y^2}{2\pi\sigma^2}\right]$

→ advantages:

- ↦ Rotationally symmetric
- ↦ Single lobe; neighbours influence decreases monotonically
- ↦ Still one lobe in frequency domain: No corruption from high freq.
- ↦ Simple relationship to σ
- ↦ Easy to implement efficiently

Morphological operation:

→ Fit: $\{y : y = x+s, s\in S\} \subset I$ $(\forall s\in S: \text{match})$

→ Hit: $\{y : y = x-s, s\in S\} \cap I \neq \emptyset$ $(\exists s\in S: \text{match})$

→ Miss: $\{y : y = x+s, s\in S\} \cap I = \emptyset$ $(\forall s\in S: \text{no match})$

→ Erosion: $E = I \ominus S = \{x : x+s \in I \text{ for every } s\in S\}$ $E(x) = \begin{cases} 1 \text{ if } S \subseteq I \text{ at } x \\ 0 \text{ otherwise} \end{cases}$

→ Dilation: $D = I \oplus S = \{x : x-s, y\in I \text{ and } s\in S\}$ $D(x) = \begin{cases} 1 \text{ if } S \text{ hits } I \text{ at } x \\ 0 \text{ otherwise} \end{cases}$

→ Opening: $I \circ S = (I \ominus S) \oplus S$

→ Closing: $I \bullet S = (I \oplus S) \ominus S$

Image Filtering:

Linear filtering: $g = f \otimes h$

→ $g(i,j) = \sum_{k,l} f(i+k, j+l)\, h(k,l)$

→ L is a linear operation if $L(\alpha I_1 + \beta I_2) = \alpha L(I_1) + \beta L(I_2)$

Example filters:

blur: $\frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$

sharper: $\begin{bmatrix}0&0&0\\0&2&0\\0&0&0\end{bmatrix} - \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$

gaussian (low pass): $G_\sigma = \frac{1}{2\pi\sigma^2} e^{\frac{(x^2+y^2)}{2\sigma^2}}$

differentiable: Prewitt $\begin{bmatrix}-1&0&1\\-1&0&1\\-1&0&1\end{bmatrix}$, Sobel $\begin{bmatrix}-1&0&1\\-2&0&1\\-1&0&1\end{bmatrix}$

high-pass: Laplacian $\begin{bmatrix}0&1&0\\1&-4&1\\0&1&0\end{bmatrix}$, high-pass $\begin{bmatrix}-1&-1&-1\\-1&8&-1\\-1&-1&-1\end{bmatrix}$

- gradient vs laplacian: ↦ gradient is isotropic, laplacian not
  ↦ laplacian gives more noise (uses 2nd derivative)

→ filter is separable if $h(u,v) = f(u)\,g(v)$

Image Features

Edge detection:

→ detect local gradient $|\text{grad}(f(x,y))| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$

→ use differential filters in x and y direction

→ detect discontinuities by considering 2nd derivative

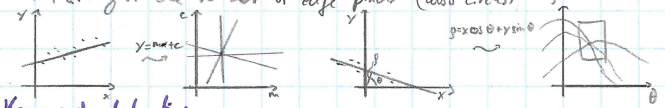
→ sensitive to fine details and noise → blur first

Canny edge detector:

1. Smooth image with gaussian filter
2. Find magnitude and orientation of gradient (Prewitt, Sobel)
   $M(x,y) = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$ $\alpha(x,y) = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$
3. Use non-maximal suppression on gradient magnitude image (thin edges)
4. Double thresholding to detect strong and weak edge pixels
   Strong: $M(x,y) \geq \theta_{high}$, weak: $\theta_{high} > M(x,y) \geq \theta_{low}$
5. Reject weak edge pixels not connected with strong edge pixels

Hough Transform:

→ Fit straight line to set of edge pixels (also circles)

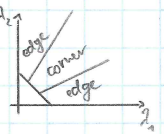


circle detection with $\|\vec{x} - \vec{m}\| - r = 0$ (midpoint, radius)

Keypoint detection

→ maximize eigenvalues of M in window w

$$M = \begin{bmatrix} \sum_{(x,y)\in w} f_x^2(x,y) & \sum_{(x,y)\in w} f_x(x,y) f_y(x,y) \\ \sum_{(x,y)\in w} f_x(x,y) f_y(x,y) & \sum_{(x,y)\in w} f_y^2(x,y) \end{bmatrix}$$



$C(x,y) = \det(M) - k\cdot(\text{trace}(M))^2$

$= \lambda_1\lambda_2 - k\cdot(\lambda_1+\lambda_2)$

→ local maximum of cornerness → key points

- using Gaussian weighting function to give more importance to central pixels

Harris corner detector

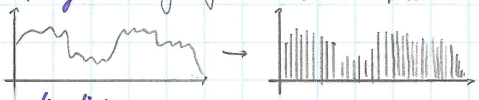
→ Invariant to brightness offset

→ Invariant to shift and rotation

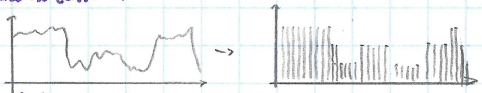
→ Not invariant to scaling

Computer Vision

Sampling: mapping signal to discrete representation



Quantization:



lossy conversion information is lost

Resolution:

→ geometric resolution: # pixels / area

→ radiometric resolution: # bits / pixel

Image Pyramid

→ search for correspondence: look at coarse scales, then refine with finer scales

→ edge tracking: a "good" edge at a fine scale has parents at a coarser scale

→ control of detail and comp. cost in matching (finding stripes → texture repres.)

Gaussian: gaussian² = gaussian, lowpass filter → redundant representation

Laplacian: Δ of upsampled and normal gaussian pyramid level → band-pass filter, each level has freq. not represented at other levels.

Pyramid: repeated smoothing and downsampling

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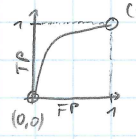
Image Segmentation

Thresholding: $B(x,y) = \begin{cases} 1 & \text{if } I(x,y) \geq T \\ 0 & \text{otherwise} \end{cases}$

Chromakeying: $I_\alpha = |I - g| > T$, $T = \sim 20$ e.g. $g = (0\ 255\ 0)$
↳ variation is NOT same in all 3 channels

ROC: characterizes performance of binary classifier



$TP = \frac{\text{True positives}}{\#\text{positives (P)}}$

$FP = \frac{\text{False positives}}{\#\text{negatives (N)}}$

$N = FP + TN \quad P = TP + FN$

↝ choose the point on the ROC curve with gradient $\beta = \frac{N}{P} \frac{V_{FN} + C_{FP}}{V_{FP} + C_{FN}}$, often $V_{TN} = V_{TP} = 0$ (V: value, C: cost)

Pixel connectivity:  : 4-neighbourhood : 8-neighbourhood

Connected components labelling: labels each connected component of a binary image with different number

(1)
1. Iterate through each element.
2. If ≠ background element
   i) get neighbouring elements
   ii) if ∄ neighbours → new unique label
   ↳ otherwise → assign label of smallest neighbour
   iii) store equivalence between neighbouring labels

(2)
1. Iterate through each element:
2. If the element is not background relabel with the lowest equivalent label

Fourier Transform

Aliasing: can't shrink image by taking every 2nd pixel

$$F(g)(u,v) = \iint_{\mathbb{R}^2} g(x,y)\, e^{-i2\pi(ux+vy)}\, dx\, dy$$

$$F(g)(u,v) = \frac{1}{N} \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} g(x,y)\, e^{-i2\pi\left(\frac{ux}{N} + \frac{vy}{M}\right)}$$

$e^{i\phi} = \cos\phi + i \sin\phi$

→ Phase: which basis has dominant, Magnitude: which basis how often

→ $\cos(iy) = \frac{e^{-y} + e^{y}}{2}$ $\quad \cos(y) = \frac{e^{iy} + e^{-iy}}{2}$

→ $\sin(iy) = \frac{e^{-y} - e^{y}}{2i}$ $\quad \sin(y) = \frac{e^{iy} - e^{-iy}}{2i}$

→ $\iint_{\mathbb{R}^2} \left(e^{-i2\pi(ax+by)} - e^{-i2\pi(ax+by)}\right) dx\, dy = \delta(a,b) - \delta(a,b)$

→ if image corrupted with $g(x,y)$ design band-pass at (u_0, v_0) to remove

→ $F \cdot G = \mathcal{F}(f ** g)$ and $F ** G = \mathcal{F}(f \cdot g)$ (conv. theorem)

Nyquist sampling theorem: Sampling frequency must be at least twice the highest frequency $\omega_s \geq 2\omega$, otherwise band-limit before sampling with low-pass

Optical Flow

→ OF = apparent motion of brightness patterns, used for tracking, segmentation, stabilization, compression, ...

Brightness constancy: for small dt brightness doesn't change

→ $I(x, y, t-1) \approx I(x, y, t-1) + \frac{\partial I}{\partial x}\delta x + \frac{\partial I}{\partial y}\delta y + \frac{\partial I}{\partial t}\delta t + \varepsilon$ (I_x, I_y, I_t)

→ with small δt: $I_x \frac{dx}{dt} + I_y \frac{dy}{dt} + I_t \approx 0$ (u, v)

Aperture problem: "The motion direction of a contour is ambiguous, because the motion component parallel to the line cannot be inferred based on visual input

Horn & Schunk: Introduce global smoothness to solve aperture problem

→ $e_s = \iint ((u_x^2 + u_y^2) + (v_x^2 + v_y^2))\, dx\, dy$
→ $e_c = \iint (I_x u + I_y v + I_t)^2\, dx\, dy$
} minimize $e_s + \lambda e_c$

Lucas-Kanade: Assume single velocity

$$M\vec{u} = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} \sum I_x t \\ \sum I_y t \end{bmatrix}$$
$A^T A$ $\qquad$ $A^T b$

→ solvable when: ↔ A^TA invertible (eigenvalues $\lambda_1 \geq \lambda_2$)
↦ A^TA not too small (λ_1, λ_2 not too small)
↦ A^TA well conditioned (λ_1 / λ_2 not too large)

→ Corners and textured areas OK

→ Problems:
↦ large motion (> 1 px): Iterative refinement, Coarse-to-fine estimation, Exhaustive neighbourhood search (feature matching)
↦ point does not move like neighbours: motion segment.
↦ brightness constancy doesn't hold: Exhaustive neighbourhood search with normalized correlation

Coarse-to-fine: Build pyramid and do LK or LK on each layer to get rid of distance constraint.

Parametric Motion Model: Global (non-local) motion model

→ 2D Models: Translation, Affine, Quadratic, Planar projective transform

→ 3D Models: Instantaneous camera motion models, Homography + epipole, Plane + Parallax

→ $E(\text{Transform}) = \sum \left[I(\text{Transform}) - I_0(x)\right]^2$ → least squares

→ Translation: $x + \begin{bmatrix} \delta x \\ \delta y \end{bmatrix}$ $\quad$ Affine: $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot x + \begin{bmatrix} \delta x \\ \delta y \end{bmatrix}$, $\begin{aligned} u &= a_0 + a_1 x + a_2 y \\ v &= a_0 + a_3 x + a_4 y \end{aligned}$ $\quad Err(\vec{a}) = \sum [I_x(w) + I_y(v) + I_t]^2$

→ Quadratic: instant approx. of planar motion, $\begin{aligned} u &= q_0 + q_1 x + q_2 y + q_3 x^2 + q_4 xy \\ v &= q_5 + q_6 x + q_7 y + q_8 xy + q_9 y^2 \end{aligned}$

→ Projective: exact planar motion $x' = \frac{h_0 + h_1 x + h_2 y}{h_6 + h_7 x + h_8 y}$, $y' = \frac{h_3 + h_4 x + h_5 y}{h_6 + h_7 x + h_8 y}$, $\begin{aligned} u &= x' - x \\ v &= y' - y \end{aligned}$

→ Instantaneous camera motion: $\begin{aligned} u &= -xy\,\Omega_x + (1+x^2)\Omega_y - y\,\Omega_z + (T_x - T_z x)/Z \\ v &= -(1+y^2)\Omega_x + xy\,\Omega_y - x\,\Omega_z + (T_y - T_z y)/Z \end{aligned}$ $\quad Z(x,y)$: local param., Ω_x, T_x: global param.

→ Homography + Epipole: same as Projective with $x' = \frac{\cdots + \gamma_3 t_x}{\cdots + \gamma_3 t_z}$ and $y' = \frac{\cdots + \gamma_3 t_y}{\cdots + \gamma_3 t_z}$

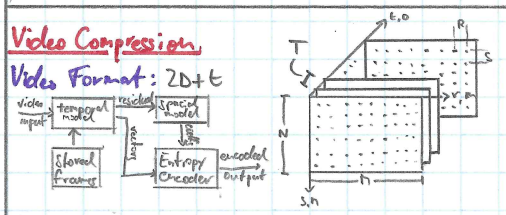
→ Plane + Parallax: $u = x' - x = \frac{\gamma}{1 + \gamma t_z}(t_z x - t_x)$, $v = y' - y = \frac{\gamma}{1 + \gamma t_z}(t_z y - t_y)$

Correlation and SSD: For large displacements: do template matching

Bayesian Flow: Add uncertainty model to Lucas-Kanade tracking (Kalman)

Video Compression

Video Format: 2D+t

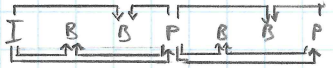


Frames:

→ I-Frame: intra-coded, independent of all other frames

→ P-Frame: coded based on previously coded Po- I Frame

→ B-Frame: bi-directional predicted, based on previous and future coded frames



→ take advantage of temporal redundancy

→ predict current frame based on previously coded frames

→ Ineffective if many scene changes, high motion

Block-matching motion estimation:

→ Partition each frame into blocks, describe motion of each block
↳ assume same motion within block
+ no object identification required

→ Vblocks search for best matching block in reference frame (MAE, MSE) (Mean absolute / squared Error)
↳ Full Search: Examine all candidate blocks
↳ Partial search: Examine carefully selected subset

Summary: → RGB → YUV: small color differences not noticable, brightness more important
→ temporal redundancy: use MC and ME, drop B Frame
→ spatial redundancy: use DCT and get rid of high frequencies
} quantize

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General formulas

sin/cos table:

	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π
	$0°$	$30°$	$45°$	$60°$	$90°$	$120°$	$135°$	$150°$	$180°$
sin	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	0
cos	1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}\sqrt{2}$	$-\frac{1}{2}\sqrt{3}$	-1

$\sin(x) = -\sin(x-180)$

$\cos(x) = -\cos(x-180)$

$180° \leq x \leq 360°$

skalar product:

$$\begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{pmatrix} = \sum_{i=1}^{n} a_i \cdot b_i \qquad \vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos(\gamma)$$

vector product:

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix} \qquad \vec{a} \times \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \sin(\gamma)$$

sin & cos:

$\sin(-\alpha) = -\sin(\alpha)$ $\qquad \sin(2\alpha) = 2\sin(\alpha)\cos(\alpha)$

$\cos(\alpha) = \sin(90° - \alpha)$ $\qquad \sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos^2\alpha + \sin^2\alpha = 1$ $\qquad \cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha = 2\cos^2\alpha - 1 = 1 - 2\sin^2\alpha$

polar form

$z = r \cdot e^{i\varphi} = r(\cos\varphi + i\sin\varphi), \quad x = r\cos\varphi, \; y = r\sin\varphi$

$r = \sqrt{x^2 + y^2}$

least squares

$A^T A x = A^T b \Rightarrow x = (A^T A)^{-1} A^T b$

Radon Transform

Mathematical formulation of taking projections of an object at different angles, i.e. take the line integral of a function at different angles and plot it.

$$\mathcal{R}(t(x,y))(\theta, s) = \int f((t\sin\theta + s\cos\theta)(t\cos\theta + \sin\theta))\,dt$$

The reconstruction of the original image from the radon transform happens through Fourier. Because Fourier transform of a radon slice corresponds to a slice in the fourier transform of $f(x,y)$.

Textures

Chaos Mosaic:

1. tile input image
2. pick random blocks and place them in random location
3. smooth edges

} doesn't work for structured textures

Efros & Leung:

→ to synthesize new pixel search input image for similar neighbourhood, pick match at random

→ better: combine with blocks and synthesize blocks, not pixels

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Colour

Human perception:
- → S-cone: 420nm
- → M-cone: 530nm
- → L-cone: 560nm
} Colour vision
- → Rod: 496nm } Brightness vision
- → Metamer: actually different colours that look same to us

CIE Primary System: blue green red
- → three light sources: 435.8nm, 546.1nm, 700.0nm
- → find weights so that resulting color is same as given spectral light
- → if not possible, add red to spectral light → "negative" red value

CIE XYZ:
- → Y is luminance
- → Z is similar to blue (S-cone)
- → X is mix of cone response curves that are non-negative
} analogous to LMS cone response of human eye

CIE xyY:
- → Y is luminance $x = \frac{X}{X+Y+Z}$, $y = \frac{Y}{X+Y+Z}$, $Y = Y$
- → x, y define chromacity $X = x\frac{Y}{y}$, $Y = Y$, $Z = \frac{Y}{y} - x\frac{Y}{y} - Y$

CIE xyY Chromacity diagram:
- → horseshoe = spectral colors, full saturation
- → line between ends of horseshoe: line of purples, not spectral
- → white point at $x = y = \frac{1}{3}$
- → saturation: distance to whitepoint / distance whitepoint spectral color

CMY: subtractive, used in passive color system (printer) non-pos. linear combi

RGB: additive, used in lit and monitors $\begin{bmatrix}R\\G\\B\end{bmatrix} + \begin{bmatrix}C\\M\\Y\end{bmatrix} = \begin{bmatrix}1\\1\\1\end{bmatrix}$ non-neg linear combi

YIQ: based on psychophysical properties of human eye
- → Y: Luminance, I: In-Phase (orange-blue), Q: Quadrature (purple-green)
- → advantage for natural and skin colors, used in NSTV NTSC

$$\begin{bmatrix}Y\\I\\Q\end{bmatrix} = \begin{pmatrix}0.299 & 0.587 & 0.114\\0.596 & -0.275 & -0.321\\0.212 & -0.528 & 0.311\end{pmatrix}\begin{bmatrix}R\\G\\B\end{bmatrix}$$

HSV/HSL: user-oriented color space, intuitive for color picking
- → H: Hue (base color) S: saturation, L: Luminance (brightness)
  V: Value (brightness)

McAdams Color Ellipses: region in chromaticity diagram where colours are indistinguishable for human. Gets almost round with CIELAB and CIELUV

Transformations

2D Transformations: $p = (x, y)^T$
- → Translation: $\begin{pmatrix}x'\\y'\end{pmatrix} = \begin{pmatrix}x\\y\end{pmatrix} + \begin{pmatrix}t_x\\t_y\end{pmatrix} \Rightarrow p' = p + t$
- → Scaling: $\begin{pmatrix}x'\\y'\end{pmatrix} = \begin{bmatrix}s_x & 0\\0 & s_y\end{bmatrix}\begin{pmatrix}x\\y\end{pmatrix} \Rightarrow p' = Sp$
- → Rotation θ: $\begin{pmatrix}x'\\y'\end{pmatrix} = \begin{bmatrix}\cos\theta & -\sin\theta\\\sin\theta & \cos\theta\end{bmatrix}\begin{pmatrix}x\\y\end{pmatrix} \Rightarrow p' = Rp$

Homogeneous coordinates: $p = (x, y, 1)^T$ affine map is linear map
- → Translation: $\begin{pmatrix}x'\\y'\\1\end{pmatrix} = \begin{bmatrix}1 & 0 & t_x\\0 & 1 & t_y\\0 & 0 & 1\end{bmatrix}\begin{pmatrix}x\\y\\1\end{pmatrix} \Rightarrow p' = Tp$
- → Rotation: $R = \begin{bmatrix}R & 0\\0\,0 & 1\end{bmatrix}$
- → Scaling: $S = \begin{bmatrix}S & 0\\0\,0 & 1\end{bmatrix}$
- → Shear along x and y-axis
  $SH_x = \begin{bmatrix}1 & a & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$, $SH_y = \begin{bmatrix}1 & 0 & 0\\b & 1 & 0\\0 & 0 & 1\end{bmatrix}$

Combination of 2D Transformations:
- → Combine via matrix multiplication
- → Commutativity $M_1 M_2 = M_2 M_1$
  - ↦ Translation – Translation
  - ↦ Rotation – Rotation
  - ↦ Scaling – Scaling
  - ↦ Scaling – Rotation

3D Transformations: $p = (x, y, z, w)^T$

→ Translation
$\begin{bmatrix}1 & 0 & 0 & t_x\\0 & 1 & 0 & t_y\\0 & 0 & 1 & t_z\\0 & 0 & 0 & 1\end{bmatrix}$

→ Scaling
$\begin{bmatrix}s_x & 0 & 0 & 0\\0 & s_y & 0 & 0\\0 & 0 & s_z & 0\\0 & 0 & 0 & 1\end{bmatrix}$

→ Rotation around x-, y-, z-axis by angle θ

$\begin{bmatrix}1 & 0 & 0 & 0\\0 & \cos\theta & -\sin\theta & 0\\0 & \sin\theta & \cos\theta & 0\\0 & 0 & 0 & 1\end{bmatrix}$ x
$\begin{bmatrix}\cos\theta & 0 & \sin\theta & 0\\0 & 1 & 0 & 0\\-\sin\theta & 0 & \cos\theta & 0\\0 & 0 & 0 & 1\end{bmatrix}$ y
$\begin{bmatrix}\cos\theta & -\sin\theta & 0 & 0\\\sin\theta & \cos\theta & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}$ z

→ Shearing parallel to principal planes

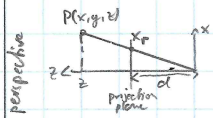
$\begin{bmatrix}1 & 0 & sh_x & 0\\0 & 1 & sh_y & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}$ SH_{xy}
$\begin{bmatrix}1 & sh_x & 0 & 0\\0 & 1 & 0 & 0\\0 & sh_z & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}$ SH_{xz}
$\begin{bmatrix}1 & 0 & 0 & 0\\sh_y & 1 & 0 & 0\\sh_z & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}$ SH_{yz}

Change of coordinate system:
- → $p' = \begin{bmatrix}\vec{r}_1 & \vec{r}_2 & \vec{r}_3 & \vec{t}\\0 & 0 & 0 & 1\end{bmatrix} \cdot [p_x, p_y, p_z, 1]^T$
  $= TRp$

$\vec{r}_i$: ith new box vector
$\vec{r}_{1,2,3}$: Rotation
$\vec{t}$: Translation

Projection:

perspective



$x_p = dx/z \quad y_p = dy/z$

$M_{persp} = \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 1/d & 0\end{bmatrix}\begin{bmatrix}x\\y\\z\\1\end{bmatrix} = \begin{bmatrix}x\\y\\z\\z/d\end{bmatrix} \rightarrow \begin{bmatrix}\frac{dx}{z}\\\frac{dy}{z}\\d\end{bmatrix}$ 3D-coordinate

divide by last component ⇝ 2D

parallel

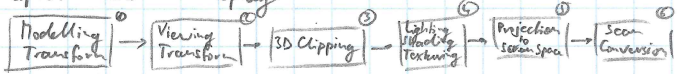
$M_{ort} = \begin{bmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & 1\end{bmatrix}$

→ z-buffer value is like d

The Graphics Pipeline

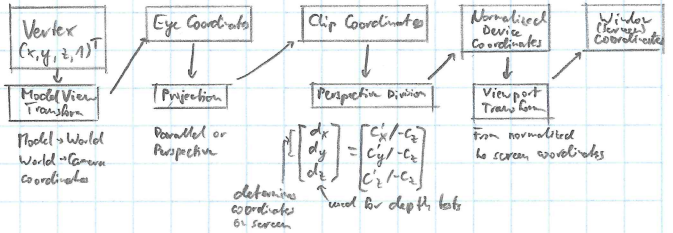
Inputs: Geometry, Materials & Lighting, Virtual Camera

Output: Colors for Display



① From object to world space
② From world space to camera space
③ Remove parts outside of frustum (avoid ← unnecessary computations, numerical instabilities)
④ Compute color based on lighting, materials, surface normals
⑤ Project from 3D to 2D space
⑥ Compute fragment attributes, interpolate per-vertex values

OpenGL Transformations:



ETH Zürich
HS16

Visual Computing

Frédéric Vogel

12-929-246

Quaternions

→ $q = a + ib + jc + kd$ $\quad i^2 = j^2 = k^2 = -1$

$ij = k$ $\quad ji = -k$ $\quad \|q\| = \sqrt{a^2+b^2+c^2+d^2}$ magnitude

$jk = i$ $\quad kj = -i$ $\quad \frac{q}{\|q\|}$ unit form

$ki = j$ $\quad ik = -j$

$ijk = -1$

scalar ↓ vector ↓

→ $z = s + ix + jy + kz = s + \vec{v}$

$z_1 = s_1 + v_1$, $z_2 = s_2 + v_2$ | $z_1 \cdot z_2 = s_1 \cdot s_2 - v_1 v_2 + s_1 v_2 + s_2 v_1 + v_1 \times v_2$

$z_1 z_2 \neq z_2 z_1$, $\bar{z} = s - v$, $z\bar{z} = \|z\|^2$, $z^{-1} = \frac{\bar{z}}{\|z\|^2}$, $1 = z\bar{z} = z^{-1}z$

→ $P = (x, y, z)^T$ → Quaternion $p = 0 + ix + jy + kz$

Rotation: $q = \cos(\theta/2) + \sin(\theta/2)u$ axis of rotation $\quad p' = qpq^{-1}$

$q^{-1} = \cos(\theta/2) - \sin(\theta/2)u$ — normalized and ok quaternion

→ Why?

↳ efficient implementation ↳ Translation simple addition

↳ Easy interpolation ↳ Rotation super easy

↳ No Gimbal lock

Radiometry

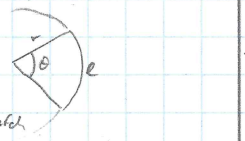
Basics:

→ Angle in 2D: $\theta = \frac{l}{r}$

circle: 2π radians

→ Solid Angle in 3D: $\Omega = \frac{A}{r^2}$ ← surface patch on sphere

sphere: 4π steradians



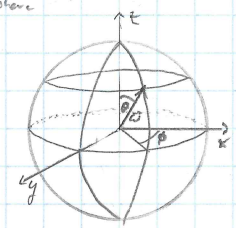
Direction:

→ point on unit sphere

→ parametrized by two angles:

$\vec{\omega} = (\theta, \phi)$

zenith azimuth



$\vec{\omega}_x = \sin\theta\cos\phi$ $\quad \vec{\omega}_y = \sin\theta\sin\phi$, $\quad \vec{\omega}_z = \cos\theta$

latitude $= \frac{90}{\pi}(\pi - \theta)$, $\quad$ longitude $= \frac{90}{\pi}\phi$

Flux: total amount of energy passing through surface per unit time $\quad \Phi(A)$ [W]

Irradiance: Flux per unit area arriving at surface

$E(x) = \frac{d\Phi(A)}{dA(x)}$ $\left[\frac{W}{m^2}\right]$ $\quad E = \frac{\Phi}{A}\cos\theta$ (Lambert's Cosine Law)

Radiosity: $B(x) = \frac{d\Phi(A)}{dA(x)}$ $\left[\frac{W}{m^2}\right]$ → leaving surface (reflection)

Radiant intensity: Flux per solid angle, directional density

$I(\vec{\omega}) = \frac{d\Phi}{d\vec{\omega}}$ $\left[\frac{W}{sr}\right]$

Radiance: Intensity per unit area

$L(x, \vec{\omega}) = \frac{dI(\vec{\omega})}{dA(x)} = \frac{d^2\Phi(A)}{d\vec{\omega}\, dA^2(x,\vec{\omega})} = \frac{d^2\Phi(A)}{d\vec{\omega}\, dA(x)\cos\theta}$ $\left[\frac{W}{m^2 sr}\right]$

→ most fundamental for raytracing, constant along ray

Reflection

BRDF: bidirectional reflectance distribution function

→ $f_r(x, \vec{\omega}_i, \vec{\omega}_r) = \frac{dL_r(x,\vec{\omega}_r)}{dE_i(x,\vec{\omega}_i)} = \frac{dL_r(x,\vec{\omega}_r)}{L_i(x,\vec{\omega}_i)\cos\theta_i d\vec{\omega}_i}$ $\left[\frac{1}{sr}\right]$

Differential irradiance due to a cone of direction around $\vec{\omega}_i$

Reflection Equation: derived from BRDF

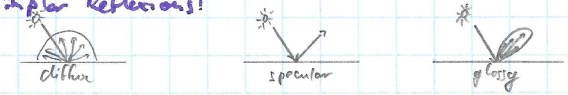
→ $L_r(x, \vec{\omega}_r) = \int_{\Omega^2} f_r(x, \vec{\omega}_i, \vec{\omega}_r) L_i(x, \vec{\omega}_i)\cos\theta_i\, d\vec{\omega}_i$

→ local illumination model, reflected radiance

Complex Reflections:

→ layered materials, anisotropic reflections, subsurface scattering, volumetric structure

Simple Reflections:



→ Ambient light: scattered by environment, coming from all directions

↳ reflection independent of: Camera and light position, surface orientation

↳ reflected intensity: $I = I_a k_a$, I_a = light source, k_a mat param

→ Diffuse Reflection: directed light I_p, dependent on surface orientation, light source position

↳ independent of: camera position (reflects equally in all directions)

↳ reflected intensity: $I = I_p k_d \cos\theta = I_p k_d (\underbrace{N}_{\text{surface normal}} \cdot \underbrace{L}_{\text{vector to src}})$

→ Simple Model: $I = I_a k_a + I_p k_d (N \cdot L) \cdot f_{att}$

Quadratic attenuation due to spacial radiation $f_{att} = \frac{1}{d_L^2}$

→ Specular Reflection: depends on angle between reflection and viewing ray

$R = N\cos\theta + S = 2N\cos\theta - L = 2N(N \cdot L) - L$

$\cos\alpha = R \cdot V = (2N(N \cdot L) - L) \cdot V$

Phong Illumination Model: Approx specular reflection by wide power

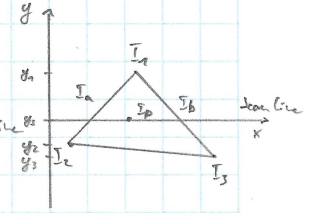
→ $I_\lambda = I_{a_\lambda} k_a O_{d_\lambda} + f_{att} I_{p_\lambda}[k_d O_{d_\lambda}(N \cdot L) + k_s (R \cdot V)^n]$

Shading

Flat Shading: One color per primitive

Gouraud Shading: linear interpolation of vertex intensities } in screen space

1. Calculate face normals
2. Calculate vertex normals by averaging
3. Eval. illumination model for each vertex
4. Interpolate vertex color bilinearly on current line



→ $I_a = I_1 - (I_1 - I_2)\frac{(y_1 - y_a)}{(y_1 - y_2)}$

→ $I_b = I_1 - (I_1 - I_3)\frac{(y_1 - y_b)}{(y_1 - y_3)}$

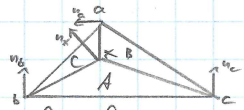
→ $I_p = I_b - (I_b - I_a)\frac{(x_b - x_p)}{(x_b - x_a)}$

→ Problems: perspective distortion, orientation dependence, shared vertices

Phong shading:

→ $n_x = \lambda_a n_a + \lambda_b n_b + \lambda_c n_c$

→ $\lambda_a = \frac{A}{ABC}$, $\lambda_b = \frac{B}{ABC}$, $\lambda_c = \frac{C}{ABC}$



→ $1 = \lambda_a + \lambda_b + \lambda_c$, $\quad x = a \Rightarrow n_x = n_a$, $\quad \lambda_a \cdot a + \lambda_b \cdot b + \lambda_c \cdot c = x$

→ Color of x determined by the interpolated normal

→ Problem: normal not defined / representative

Transparency: Alpha channel: $I_\lambda = I_{\lambda_1}\alpha_n + I_{\lambda_2}(1 - \alpha_n)$

→ Problem: rendering order

↳ Back 2 Front: simple, ✗ overlap or crossing

↳ Depth Peeling: multiple passes, each pass renders next closest fragment

Texture Filtering

Aliasing: use bandpass filter to avoid aliasing by reducing high freqs

→ in screen space: $f(x,y) = \frac{1}{2\pi\sigma^2} e^{-\left(\frac{x^2+y^2}{2\sigma^2}\right)}$ isotropic 2D Gaussian

Light Map: simulates effect of a local light source by an alpha mask, can be pre-computed and adapted dynamically

Environment Map: rendering reflective objects effectively

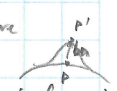
Texture coordinates are computed by intersecting reflected ray with surrounding sphere / box

Bump Map: perturb surface normal according to texture

→ $n = \frac{\partial p}{\partial u} \times \frac{\partial p}{\partial v} = p_u \times p_v$, $\quad p' = p + \frac{b_n}{|n|}$, $\quad n' = \frac{\partial p'}{\partial u} \times \frac{\partial p'}{\partial v}$

parameter domain

$= n + b_v(n \times p_u) + b_u(n \times p_v)$



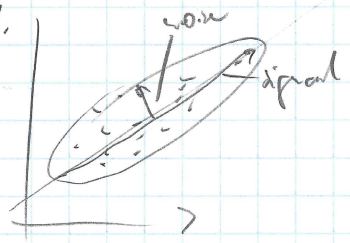
→ no bumps silhouette, no self-shading, no self-occlusion

Mip Map: Downsample texture for far away objects

→ improved efficiency, avoids aliasing

PCA

1.



- Calculate mean for each column
- Substract mean vector from each row in X
- Covariance Matrix $C = \frac{1}{n-1} B^T \cdot B$
- Eigenvalues and Eigenvectors of C
- Project data using truncated version of EC

PY

lin. $v(t) = \frac{d}{dt} x(t)$

sim. $\omega(t) = \dot{R}(t) = \omega(t) \cdot R(t)$ → $\omega(t) \otimes x'(t)$, $\omega(t) \otimes y'(t)$, $\omega(t) \otimes z'(t)$

gen. $\dot{v}(t) = \omega(t) \otimes r(t) \otimes v(t)$

Rigid body

$\dot{x}(t) = \frac{p(t)}{m}$

$\dot{p}(t) = F_i(t)$

ODEs

$\dot{q}(t) = \frac{1}{2}(\hat{I}^{-1}(t) L(t)) q(t)$

$\dot{L}(t) = (r_i(t) - x(t)) \otimes F_i(t)$

Governing equation

$\frac{d}{dt} \Sigma(t) = \frac{d}{dt} (x(t), R(t), P(t), L(t))^T = \begin{pmatrix} v(t) \\ \omega(t) \cdot R(t) \\ F(t) \\ \tau(t) \end{pmatrix}$

Geometry

Circle: $r: \mathbb{R} \to \mathbb{R}^2$ $t \mapsto p(t) = (x(t), y(t))$ $p(t) = r(\cos(t), \sin(t))$

Bezier: $x(t) = \sum_{i=0}^{n} p_i B_i^n(t)$ $B_i^n(t) = \binom{n}{i} t^i (1-t)^{n-i}$

Space Curves: $f: X \to Y$ $X \subset \mathbb{R}^{n=1}$ $Y \subseteq \mathbb{R}^{n=3}$

$S(t) = (x(t), y(t), z(t))$

Normal / Tangent Plane: $S_u = \frac{\partial S(u,v)}{\partial u}$ $S_v = \frac{\partial S(u,v)}{\partial v}$ $n = \frac{S_u \otimes S_v}{\|S_u \otimes S_v\|}$