EXERCISE SET 1, DIFFERENTIAL AND INTEGRAL CALCULUS

The solutions to the problems should be handed in via MyCourses before 17:00, Friday 9.11.

You are allowed and encouraged to discuss the exercises with your fellow students, but every student should write down their own solutions. It is encouraged to solve MANY of the "additional exercises", and other exercises that you find in the textbook or elsewhere, in addition to the homework problems.

Problem 1

Are the following triples linearly dependent or independent?

(1) $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$, and $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ in $\mathbb{R}^3$.

(2) $\begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix}$, $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$, and $\begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$ in $\mathbb{R}^3$.

(3) $\begin{pmatrix} \pi \\ 19 \end{pmatrix}$, $\begin{pmatrix} \mathrm{e} \\ 1 \end{pmatrix}$, and $\begin{pmatrix} 0 \\ -17 \end{pmatrix}$ in $\mathbb{R}^2$.

Problem 2

Find a vector of length 1, whose angle with $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ is $60°$.

Problem 3

Find an equation for the plane through the origin and the points $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$.

Problem 4

For the following functions $\mathbb{R}^2 \to \mathbb{R}^2$, determine if they are linear or not. If they are linear, write down a matrix representation.

(1) $\begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} -x \\ -y \end{pmatrix}$

(2) $\begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x^2 \\ y^2 \end{pmatrix}$

(3) $\begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x+1 \\ y+1 \end{pmatrix}$

(4) $\begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} y \\ x \end{pmatrix}$

Additional exercises

From Stewart:

1. Which of the following expressions are meaningful? Which are meaningless? Explain.

(a) $(\mathbf{a} \cdot \mathbf{b}) \cdot \mathbf{c}$ (b) $(\mathbf{a} \cdot \mathbf{b})\mathbf{c}$

(c) $|\mathbf{a}|(\mathbf{b} \cdot \mathbf{c})$ (d) $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c})$

(e) $\mathbf{a} \cdot \mathbf{b} + \mathbf{c}$ (f) $|\mathbf{a}| \cdot (\mathbf{b} + \mathbf{c})$

2–10 Find $\mathbf{a} \cdot \mathbf{b}$.

2. $\mathbf{a} = \langle -2, 3 \rangle, \quad \mathbf{b} = \langle 0.7, 1.2 \rangle$

3. $\mathbf{a} = \left\langle -2, \frac{1}{3} \right\rangle, \quad \mathbf{b} = \langle -5, 12 \rangle$

4. $\mathbf{a} = \langle 6, -2, 3 \rangle, \quad \mathbf{b} = \langle 2, 5, -1 \rangle$

5. $\mathbf{a} = \left\langle 4, 1, \frac{1}{4} \right\rangle, \quad \mathbf{b} = \langle 6, -3, -8 \rangle$

1–7 Find the cross product $\mathbf{a} \times \mathbf{b}$ and verify that it is orthogonal to both $\mathbf{a}$ and $\mathbf{b}$.

1. $\mathbf{a} = \langle 6, 0, -2 \rangle, \quad \mathbf{b} = \langle 0, 8, 0 \rangle$

2. $\mathbf{a} = \langle 1, 1, -1 \rangle, \quad \mathbf{b} = \langle 2, 4, 6 \rangle$