EXERCISE SET 3, DIFFERENTIAL AND INTEGRAL CALCULUS

The solutions to the problems should be handed in via MyCourses before 17:00, Friday 30.11.

You are allowed and encouraged to discuss the exercises with your fellow students, but every student should write down their own solutions. It is encouraged to solve MANY of the "additional exercises", and other exercises that you find in the textbook or elsewhere, in addition to the homework problems.

Problem 1

Consider the space

$$P_2 = \{ax^2 + bx + c : a, b, c \in \mathbb{R}\}$$

of polynomials of degree ≤ 2. Let B be the basis $\{1, x, x^2\}$ and let B' be the basis $\{1, 1+x, (1+x)^2\}$.

(1) Compute the change of basis matrix from B to B'.
(2) Express the polynomial $x^2 + x + 1$ as a column matrix in the basis B.
(3) Express the polynomial $x^2 + x + 1$ as a column matrix in the basis B'.

Problem 2

Let P_2 be as in Problem 1, and let $D : P_2 \to P_2$ be the "differentiation map", so that $Df(x) = f'(x)$, *i.e.* $D(ax^2 + bx + c) = 2ax + b$.

(1) Compute the transformation matrix of D in the basis B.
(2) Compute the transformation matrix of D in the basis B'.

Problem 3

Compute the eigenvalues and corresponding eigenvectors of

$$A = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & -1 & 0 \end{vmatrix}.$$

Problem 4

Let A be as in Problem 3, and compute A^{100}.

Additional exercises

✓ **1.10** Find the change of basis matrix for each $B, D \subseteq \mathcal{P}_2$.

(a) $B = \langle 1, x, x^2 \rangle, D = \langle x^2, 1, x \rangle$ **(b)** $B = \langle 1, x, x^2 \rangle, D = \langle 1, 1+x, 1+x+x^2 \rangle$

(c) $B = \langle 2, 2x, x^2 \rangle, D = \langle 1+x^2, 1-x^2, x+x^2 \rangle$

1.13 For each space find the matrix changing a vector representation with respect to B to one with respect to D.

(a) $V = \mathbb{R}^3$, $B = \mathcal{E}_3$, $D = \langle \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \rangle$

(b) $V = \mathbb{R}^3$, $B = \langle \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \rangle$, $D = \mathcal{E}_3$

(c) $V = \mathcal{P}_2$, $B = \langle x^2, x^2+x, x^2+x+1 \rangle$, $D = \langle 2, -x, x^2 \rangle$

✓ **2.17** Show that matrices of this form are not diagonalizable.

$$\begin{pmatrix} 1 & c \\ 0 & 1 \end{pmatrix} \qquad c \neq 0$$

3.45 Diagonalize.

$$\begin{pmatrix} -1 & 2 & 2 \\ 2 & 2 & 2 \\ -3 & -6 & -6 \end{pmatrix}$$