Assignment 2

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Problem 4.1.2

The diameter of a particle of contamination (in micrometers) is modeled with the probability density function $f(x) = 2\ /\ x^3$ for $x > 1$. Determine the following:

a. $P(X < 2)$

b. $P(X > 5)$

c. $P(4 < X < 8)$

d. $P(X < 4 \text{ or } X > 8)$

e. x such that $P(X < x) = 0.95$

Answers:

Since the diamter of a particle of contamination is modeled with a probability function, we can agree that $P(a \leq X \leq b) = \int_a^b f(x)dx$.

a. In this scenario,

$$P(X < 2) = P(1 < X < 2) = 1 - P(X \geq 2) = 1 - \int_2^\infty (2/x^3)dx = 1 - \left[1/x^2\right]_2^\infty = 1 - \frac{1}{4} = \frac{3}{4}$$

b. In this scenario,

$$P(X > 5) = 1 - P(X \leq 5) = 1 - \int_1^5 (2/x^3)dx = 1 - \left[1/x^2\right]_1^5 = 1 - \frac{24}{25} = \frac{1}{25}$$

c. In this scenario,

$$P(4 < X < 8) = 1 - \Big(P(X \geq 8) + P(X \leq 4)\Big) = 1 - \int_1^4 (2/x^3)dx - \int_8^\infty (2/x^3)dx = 1 - \frac{15}{16} - \frac{1}{64} = \frac{3}{64}$$

d. In this scenario,

$$P(X < 4 \text{ or } X > 8) = 1 - P(4 \leq x \leq 8) = 1 - \int_4^8 (2/x^3)dx = 1 - \frac{3}{64} = \frac{61}{64}$$

e. In this scenario,

$$P(X < x) = 1 - P(X \geq x) = 1 - \int_x^\infty (2/x^3)dx = 0.95 \rightarrow 1 - \Big[1/x^2\Big]_x^\infty = 0.95 \rightarrow 1 - 1/x^2 = 0.95 \rightarrow x = 2\sqrt{5}$$

Problem 4.2.1

Suppose that the cumulative distribution function of the random variable X is

$$F(x) = \begin{cases} 0, & x < 0 \\ 0.25x, & 0 \leq x < 5 \\ 1, & 5 \leq x \end{cases}$$

Determine the following:

a. $P(X < 2.8)$

b. $P(X > 1.5)$

c. $P(X < -2)$

d. $P(X > 6)$

Answers:

a. In this scenario,

$$P(X < 2.8) = F\Big(X \subset (-\infty, 0)\Big) + F\Big(X \subset [0, 5)\Big) = 0 + 0.25(2.8) = 0.7$$

b. In this scenario,

$$P(X > 1.5) = 1 - P(X \leq 1.5) = 1 - F\Big(X \subset (-\infty, 0)\Big) - F\Big(X \subset [0, 5)\Big) = 1 - 0 - 0.25(1.5) = 0.625$$

c. In this scenario,

$$P(X < -2) = F\Big(X \subset (-\infty, 0)\Big) = 0$$

d. In this scenario,

$$P(X > 6) = 1 - P(X \leq 6) = 0$$

Problem 4.3.1

Suppose that $f(x) = 1.5x^2$ for $-1 < x < 1$. Determine the mean and variance of X.

Answer:

The mean of X is determined as follows:

$$E(X) = \int_{\text{lower limit}}^{\text{upper limit}} x f(x) dx = \int_{-1}^{1} x(1.5x^2) dx = 0$$

The variance of X is determined as follows:

$$V(X) = \int_{\text{lower limit}}^{\text{upper limit}} x^2 f(x) dx - [E(X)]^2 = \int_{-1}^{1} x^2(1.5x^2) dx - 0^2 = 0.6$$

Problem 4.4.4

An adult can lose or gain two pounds of water in the course of a day. Assume that the changes in water weight are uniformly distributed between minus two and plus two pounds in a day. What is the standard deviation of a person's weight over a day?

Answers:

Denote K as an arbitrary weight of an adult. The standard deviation of a person's weight over a day is given as:

$$\sigma = \sqrt{V(X)} = \sqrt{\frac{\left((K+2)-(K-2)\right)^2}{12}} = \sqrt{\frac{4^2}{12}} = \frac{2}{\sqrt{3}} \approx 1.1547$$

Problem 4.5.2

Assume that Z has a standard normal distribution. Use Appendix Table III to determine the value for z that solves each of the following:

a. $P(-z < Z < z) = 0.95$

b. $P(-z < Z < z) = 0.99$

c. $P(-z < Z < z) = 0.68$

d. $P(-z < Z < z) = 0.9973$

Answers:

a. In this scenario, $P(-z < Z < z) = 0.95$ means that 95% of the total area under the standard normal curve lies between $-z$ and z. Thus, only an area of $(1-0.95)/2 = 0.025 = 2.5\%$ lies outside of the $[-z, z]$ region in each tail of the distribution. Therefore, by looking up the $0.95 + 0.025 = 0.9750$ value in the Cumulative Standard Normal Distribution Table, we find a z-score of approximately 1.96.

b. In this scenario, $P(-z < Z < z) = 0.99$ means that 99% of the total area under the standard normal curve lies between $-z$ and z. Thus, only an area of $(1-0.99)/2 = 0.005 = 0.5\%$ lies outside of the $[-z, z]$ region in each tail of the distribution. Thereofre, by looking up the $0.99 + 0.005 = 0.995$ value in the Cumulative Standard Normal Distribution Table, we find a z-score of approximately 2.33 to 2.58

c. In this scenario, $P(-z < Z < z) = 0.68$ means that 68% of the total area under the standard normal curve lies between $-z$ and z. Thus, only an area of $(1-0.68)/2 = 0.16 = 16\%$ lies outside of the $[-z, z]$ region in each tail of the distribution. Therefore, by ooking up the $0.68 + 0.16 = 0.84$ value in the Cumulative Standard Normal Distribution Table, we find a z-score of approximately 0.96 to 0.99.

d. In this scenario, $P(-z < Z < z) = 0.9973$ means that 99.73% of the total area under the standard normal curve lies between $-z$ and z. Thus, only an area of $(1-0.9973)/2 = 0.00135 = 0.135\%$ lies outside of the $[-z, z]$ region in each tail of the distribution. Therefore, by looking up the $0.9973 + 0.00135 = 0.99865$ value in the Cumulative Standard Normal Distribution Table, we find a z-score of approximately 2.98 to 3.0.

Problem 4.5.6

The time until recharge for a battery in a laptop computer under common conditions is normally distributed with a mean of 260 minutes and a standard deviation of 50 minutes.

a. What is the probability that a battery lasts more than four hours?

b. What are the quartiles (the 25% and 75% values) of battery life?

c. What value of life in minutes is exceeded with 95% probability?

Answers:

a. We can find the probability that a battery lasts more than four hours (e.g., $P(X > 240)$) by finding the z-score for 240 minutes and looking up the corresponding probability in the standard normal distribution table. The z-score is calculated as follows:

$$z = \frac{x - \mu}{\sigma} = \frac{240 - 260}{50} = -0.4$$

Therefore, this probability is given as:

$$P(X > 240) = P(Z > -0.4) = 1 - P(Z < -0.4) = 1 - 0.3446 = 0.6554$$

b. We can find the 1st and the 3rd quartiles by looking up the z-scores that correspond to probabilities of 0.25 and 0.75 in the standard normal distribution table. These z-scores are approximately -0.6745 and 0.6745, respectively. We can then convert these z-scores back to x-values using the formula:

$$x = \mu + z\sigma$$

Therefore, the first quartile (Q1) is $Q_1 = 260 + (-0.6745)(50) = 226.775$ minutes, and the third quartile (Q3) is $Q_3 = 260 + (0.6745)(50) = 293.225$ minutes.

c. To find this value, we need to find the z-score that corresponds to a probability of 0.95 then convert this z-score back to x. The corresponding z-score is -1.645, hence the equivalent x is: $x = \mu + z\sigma = 260 + (-1.645)(50) = 177.8$ minutes.

Problem 4.6.5

Hits to a high-volume Web site are assumed to follow a Poisson distribution with a mean of 10,000 per day. Approximate each of the following:

a. Probability of more than 20,000 hits in a day.

b. Probability of less than 9900 hits in a day.

c. Value such that the probability that the number of hits in a day exceeds the value is 0.01.

d. Expected number of days in a year (365 days) that exceed 10,200 hits.

e. Probability that over a year (365 days), each of the more than 15 days has more than 10,200 hits.

Answers:

Recall that an approximation of a Poisson distribution using a standard normal distribution is given as:

$$Z = \frac{X - \lambda}{\sqrt{\lambda}}$$

a. In this scenario, we can translate $P(X > x)$ to $P(Z > z)$ as follows:

$$P(X > 20,000) = P(Z > \frac{20,000 - 10,000}{\sqrt{10,000}}) = P(Z > 100) \approx 0$$

b. In this scenario, we can translate $P(X \geq x)$ to $P(Z \geq z)$ as follows:

$$P(X < 9900) = P(Z < \frac{9900 - 10,000}{\sqrt{10,000}}) = P(Z < -1.005) = 0.1587$$

c. In this scenario, we can translate $P(X > t)$ as:

$$P(X > t) = P(Z > \frac{t - 10,000}{\sqrt{10,000}}) = 0.01 \rightarrow Z = 2.32 = \frac{t - 10,000}{\sqrt{10,000}} \rightarrow t = 10,232$$

d. To find the expected number of days in a year that exceed 10,200 hits, we first find the probability that a single day exceeds 10,200 hits. This probability can be computed as:

$$P(X > 10,200) = P(Z > \frac{10,200 - 10,000}{\sqrt{10,000}}) = P(Z > 2) = 0.0228$$

Denote D as the event where the number of hits in a single days exceed 10,200. We can agree that D follows a binomial distribution. Therefore, the expected number of days in a year can be computed as:

$$E(D) = 365 \times 0.0228 = 8.32$$

e. Denote E as the event where each of the more than 15 days has more than 10,200 hits. The probability that E occurs can be computed as:

$$P(X > 15) = P(Z > \frac{15 - 8.03}{\sqrt{8.03}}) = P(Z > 2.459) \approx 0.006947$$

Problem 4.7.4

The life of automobile voltage regulators has an exponential distribution with a mean life of 6 years. You purchase a 6-year-old automobile with a working voltage regulator and plan to own it for 6 years.

a. What is the probability that the voltage regulator fails during your ownership?

b. If your regulator fails after you own the automobile 3 years and it is replaced, what is the mean time until the next failure?

Answers:

Recall that an exponential distribution has a probability density function as follows:

$$f(x) = \lambda \exp^{-\lambda x}$$

In this case, the mean life is 6 years meaning $E(x) = \frac{1}{\lambda} = 6$. Therefore, the failure rate is given as $\lambda = \frac{1}{6}$.

a. Since the exponential distribution has memoryless property, the probability that the voltage regulator fails during the ownership even though it is six-years-old is the same as the probability of a new regulator fails for the next six years. Therefore, the probability that the voltage regulator fails during the ownership can be calculated as follows:

$$P(T \leq 6) = \int_{-\infty}^{6} (\lambda \exp^{-\lambda x}) dx = 1 - \exp^{(-\lambda x)} = 1 - \exp^{(-1/6 \times 6)} \approx 0.632$$

b. Suppose that the regulator fails after 3 years and it is replaced, the mean time until the next failure would still be 6 years. This is because the exponential distribution has the memoryless property, which means that the remaining time until an event occurs does not depend on how much time has already passed.

Problem 4.7.6

The number of stork sightings on a route in South Carolina follows a Poisson process with a mean of 2.3 per year.

a. What is the mean time between sightings?

b. What is the probability that there are no sightings within three months (0.25 years)?

c. What is the probability that the time until the first sighting exceeds six months?

d. What is the probability of no sighting within 3 years?

Answers:

Since the number of stork sightings follows a Poisson process, we can agree that its probability density function is given as:

$$f(x) = \lambda \exp^{-\lambda x}$$

In this case, the mean number of sightings is $E(x) = \frac{1}{\lambda} = 2.3$.

a. The mean time between sightings, which is a reciprocal of the mean number, can be computed as $\lambda = \frac{1}{2.3} = \frac{10}{23} \approx 0.4348$.

b. Given that the number of stork sightings follows a Poisson process, we can use the Poisson distribution to find the X number of stork sightings. Therefore, the probability that there are no sightings within three months (0.25 years) can be computed as:

$$P(X=0) = \frac{\exp^{-\lambda_{0.25} T_{0.25}} (\lambda_{0.25} T_{0.25})^0}{0!} = \exp^{-0.575}(0.575) \approx 0.5623$$

with $\lambda_{0.25} T_{0.25} = 2.3 \times 0.25 = 0.575$ is the mean number of sightings in three months.

c. The probability that the time until the first sighting exceeds six months (0.5 years) can be found using the exponential distribution, which is used to model the time between events in a Poisson proceess. Therefore, this probability computed as:

$$P(X > 0.5) = \int_{0.5}^{\infty} \exp^{-\lambda_{0.5} x}\, dx = \left[\exp^{-\lambda_{0.5} x}\right]_{0.5}^{\infty} = \exp^{-2.3 \times 0.5} \approx 0.2231$$

with $\lambda_{0.5} = 2.3$ (the same as in Section a).

d. The probability of no sighting within 3 years can be computed as:

$$P(X=0) = \frac{\exp^{-\lambda_3 T_3} (\lambda_3 T_3)^0}{0!} = \exp^{-6.9}(6.9)^0 \approx 0.0001$$

with $\lambda_3 T_3 = 2.3 \times 3 = 6.9$ is the mean number of sightings in three years.

Problem 4.8.5

The time between arrivals of customers at an automatic teller machine is an exponential random variable with a mean of 5 minutes.

a. What is the probability that more than three customers arrive in 10 minutes?

b. What is the probability that the time until the fifth customer arrives is less than 15 minutes?

Answers:

Since the time between arrivals of customers at an automatic teller machine is an exponential random variable, we can agree that its probability density function is given as:

$$f(x) = \lambda \exp^{-\lambda x}$$

where the mean time $E(x)$ is 5 minutes. Therefore, the number of customer per minute is given as: $\lambda = \frac{1}{5} = 0.2$.

a. The probability that more than three customers arrive in 10 minutes can be computed as:

$$P(X > 3) = 1 - P(X \leq 3) = 1 - P(X = 0) - P(X = 1) - P(X = 2) - P(X = 3)$$

with $P(X = 0)$, $P(X = 1)$, $P(X = 2)$, and $P(X = 3)$ are probabilities that there is no customer, only one customer, only two customers, and only three customers arriving in 10 minutes, respectively. These probabilities can be computed using Poisson distribution:

$$\begin{aligned}
P(X = 0) &= \frac{\exp^{-\lambda_{10} T_{10}} (\lambda_{10} T_{10})^0}{0!} \approx 0.1353 \\
P(X = 1) &= \frac{\exp^{-\lambda_{10} T_{10}} (\lambda_{10} T_{10})^1}{1!} \approx 0.2707 \\
P(X = 2) &= \frac{\exp^{-\lambda_{10} T_{10}} (\lambda_{10} T_{10})^2}{2!} \approx 0.2707 \\
P(X = 3) &= \frac{\exp^{-\lambda_{10} T_{10}} (\lambda_{10} T_{10})^3}{3!} \approx 0.1804
\end{aligned}$$

with $\lambda_{10} T_{10} = 0.2 \times 10 = 2$ is the number of customers per 10 minutes. Therefore, $P(X > 3)$ is computed as:

$$P(X > 3) = 1 - 0.1353 - 0.2707 - 0.2707 - 0.1804 = 0.1429$$

b. Denote B as the time until the fifth customer arrives is less than 15 minutes. The distribution of B can be modeled using a gamma distribution with shape parameter $k = 5$ (since we are considering the fifth customer) and rate parameter $\lambda = 0.2$ (per one minute, only 0.2 customers arriving at the teller). Therefore,

$$P(t < 15) = \int_0^{15} \frac{\lambda^k t^{k-1} \exp^{-\lambda t}}{\Gamma(k)} \approx 0.834$$

with $\Gamma(k) = (k-1)!$

Problem 5.1.1

Show that the following function satisfies the properties of a joint probability mass function.

x	y	f(x,y)
1.0	1	1/4
1.5	2	1/8
1.5	3	1/4
2.5	4	1/4
3.0	5	1/8

Determine the following:

a. $P(X < 2.5, Y < 3)$

b. $P(X < 2.5)$

c. $P(Y < 3)$

d. $P(X > 1.8, Y > 4.7)$

e. $E(X)$, $E(Y)$, $V(X)$, $V(Y)$

f. Marginal probability distribution of X

Answers:

Suppose $f_{x,y}(x,y)$ is joint probability mass function, it will have the following properties:

- $f_{x,y}(x,y) \geq 0 \quad \forall x, y.$
- $\sum_x \sum_y f_{x,y}(x,y) = 1.$

According to the given table, both properties are satisfied since

$$\begin{cases} f_{x,y}(X=1.0, Y=1) = 1/4 \geq 0 \\ f_{x,y}(X=1.5, Y=2) = 1/8 \geq 0 \\ f_{x,y}(X=1.5, Y=3) = 1/4 \geq 0 \\ f_{x,y}(X=2.5, Y=4) = 1/4 \geq 0 \\ f_{x,y}(X=3.0, Y=5) = 1/8 \geq 0 \\ \sum_x \sum_y f_{x,y} = 1/4 + 1/8 + 1/4 + 1/4 + 1/8 = 1 \end{cases}$$

Therefore, $f_{x,y}(x,y)$ is joint probability mass function is indeed a joint probability mass function.

a. In this scenario,

$$P(X < 2.5, Y < 3) = f_{X,Y}(1.0, 1) + f_{X,Y}(1.5, 2) = 1/4 + 1/8 = 3/8$$

b. In this scenario,

$$P(X < 2.5) = f_{X,Y}(1.0, 1) + f_{X,Y}(1.5, 2) + f_{X,Y}(1.5, 3) = 1/4 + 1/8 + 1/4 = 5/8$$

c. In this scenario,

$$P(Y < 3) = f_{X,Y}(1.0, 1) + f_{X,Y}(1.5, 2) = 1/4 + 1/8 = 3/8$$

d. In this scenario,

$$P(X > 1.8, Y > 4.7) = f_{X,Y}(3.0, 5) = 1/8$$

X

e. In this scenario,

$$E(X) = \sum_x x f_X(x,y) = (1.0)(1/4) + (1.5)(1/8 + 1/4) + (2.5)(1/4) + (3.0)(1/8) = 1.8125$$
$$V(X) = E(X^2) - [E(X)]^2 = (1.0)^2(1/4) + (1.5)^2(1/8 + 1/4) + (2.5)^2(1/4) + (3.0)^2(1/8) - 1.8125^2 \approx 0.4961$$
$$E(Y) = \sum_y y f_Y(x,y) = (1)(1/4) + (2)(1/8) + (3)(1/4) + (4)(1/4) + (5)(1/8) = 2.875$$
$$V(Y) = E(Y^2) - [E(Y)]^2 = (1)^2(1/4) + (2)^2(1/8) + (3)^2(1/4) + (4)^2(1/4) + (5)^2(1/8) - 2.875^2 \approx 1.8594$$

f. The marginal probability distribution of X is given as:

$$\begin{cases} P(X=1) = f_{1,1} = 1/4 \\ P(X=1.5) = f_{1.5,2} + f_{1.5,3} = 1/4 + 1/8 = 3/8 \\ P(X=2.5) = f_{2.5,4} = 1/4 \\ P(X=3.0) = f_{3.0,5} = 1/8 \end{cases}$$

Problem 5.2.2

Consider the joint distribution in Exercise 5.1.1. Determine the following:

a. Conditional probability distribution of Y given that $X = 1.5$

b. Conditional probability distribution of X given that $Y = 2$

c. $E(Y|X = 1.5)$

d. Are X and Y independent?

Answers:

We can construct the following table (x is on the horizontal axis, and y is on the vertical axis):

Y/X	1.0	1.5	2.5	3.0
1	1/4	0	0	0
2	0	1/8	0	0
3	0	1/8	0	0
4	0	0	1/4	0

Y/X	1.0	1.5	2.5	3.0
5	0	0	0	1/8

a. The conditional probability distribution of Y given that $X = 1.5$ is given as:

$$f_{Y|X}(Y|X = 1.5) = \frac{f_{X,Y}(X = 1.5, Y)}{f_X(X = 1.5)}$$

with $f_X(X = 1.5)$ is the marginal density of X at $X = 1.5$. In this case, $f_X(X = 1.5) = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$. Hence,

$$\begin{cases} f_{Y|X}(Y = 1|X = 1.5) = \frac{0}{1/4} = 0 \\ f_{Y|X}(Y = 2|X = 1.5) = \frac{1/8}{1/4} = 1/2 \\ f_{Y|X}(Y = 3|X = 1.5) = \frac{1/8}{1/4} = 1/2 \\ f_{Y|X}(Y = 4|X = 1.5) = \frac{0}{1/4} = 0 \\ f_{Y|X}(Y = 5|X = 1.5) = \frac{0}{1/4} = 0 \end{cases}$$

b. The conditional probability distribution of X given that $Y = 2$ is given as:

$$f_{X|Y}(X|Y = 2) = \frac{f_{X,Y}(X, Y = 2)}{f_Y(Y = 2)}$$

with $f_Y(Y = 2)$ is the marginal density of Y at $Y = 2$. In this case, $f_Y(Y = 2) = \frac{1}{8}$. Hence,

$$\begin{cases} f_{X|Y}(X = 1.0|Y = 2) = \frac{0}{1/8} = 0 \\ f_{X|Y}(X = 1.5|Y = 2) = \frac{1/8}{1/8} = 1 \\ f_{X|Y}(X = 2.5|Y = 2) = \frac{0}{1/8} = 0 \\ f_{X|Y}(X = 3.0|Y = 2) = \frac{0}{1/8} = 0 \end{cases}$$

c. The expectation $E(Y|X = 1.5)$ is given as:

$$E(Y|X = 1.5) = \sum_y y f_{Y|X}(Y|X = 1.5) = 2 \times \frac{2}{3} + 3 \times \frac{2}{3} = \frac{10}{3} \approx 3.3333$$

d. For X and Y to be independent these relationships must hold true:

- $P(X, Y) = P(X) \times P(Y)$
- $P(X|Y) = P(X)$

In this scenario, $P(X = 1.5|Y = 2) = 1$ which is not equal to $P(X = 1.5) = 1/8 + 1/4 = 3/8$. On the other hand, when $X = 1.0$ and $Y = 1$, $f_{X,Y}(X = 1.0, Y = 1) = 1/4$. This is not equal to $f_X(X = 1.0) \times f_Y(Y = 1) = (1/4)(1/4) = 1/16$. Therefore, X and Y are not independent.

Problem 5.4.1

Determine the covariance and correlation for the following joint probability distribution:

x	y	f(x,y)
1	3	1/8
1	4	1/4
2	5	1/2
4	6	1/8

Answers

1. The covariance of the joint distribution is given as:

$$\text{Cov}(X, Y) = E\Big[\big(X - E(X)\big)\big(Y - E(Y)\big)\Big] = E(X, Y) - E(X)E(Y)$$

with $E(X)$, $E(Y)$, and $E(X, Y)$ can be computed as follows:

$$E(X) = \sum_x xP(X = x) = 1 \times (\frac{1}{8} + \frac{1}{4}) + 2 \times \frac{1}{2} + 4 \times \frac{1}{8} = 1.875$$
$$E(Y) = \sum_y yP(Y = y) = 3 \times (\frac{1}{8}) + 4 \times (\frac{1}{4}) + 5 \times \frac{1}{2} + 6 \times \frac{1}{8} = 4.625$$
$$E(X, Y) = \sum_x \sum_y P(X = x, Y = y) = 1 \times 3 \times \frac{1}{8} + 1 \times 4 \times \frac{1}{4} + 2 \times 5 \times \frac{1}{2} + 4 \times 6 \times \frac{1}{8} = 9.375$$

Therefore, $\text{Cov}(x, y)$ is computed as:

$$\mathrm{Cov}(x, y) = 9.375 - 1.875 \times 4.625 = 0.703125 \approx 0.7031$$

2. The correlation of the joint distribution is given as:

$$\mathrm{Corr}(x, y) = \frac{\mathrm{Cov}(x, y)}{\sqrt{V(x)V(y)}}$$

with $V(x)$ and $V(y)$ can be computed as follows:

$$V(x) = \sum_x x^2 P(X = x) - \left[E(X)\right]^2 = 1^2 \times (\frac{1}{8} + \frac{1}{4}) + 2^2 \times \frac{1}{2} + 4^2 \times \frac{1}{8} - 1.875^2 \approx 0.8594$$

$$V(y) = \sum_y y^2 P(Y = y) - \left[E(Y)\right]^2 = 3^2 \times (\frac{1}{8}) + 4^2 \times (\frac{1}{4}) + 5^2 \times \frac{1}{2} + 6^2 \times \frac{1}{8} - 4.625^2 \approx 0.7344$$

Therefore, $\mathrm{Cov}(x, y)$ is computed as:

$$\mathrm{Corr}(x, y) = \frac{0.7031}{\sqrt{0.8594 \times 0.7344}} \approx 0.8850$$