EXERCISE SET 3, DIFFERENTIAL AND INTEGRAL CALCULUS

The solutions to the problems should be handed in via MyCourses before 17:00, Friday 23.11.

You are allowed and encouraged to discuss the exercises with your fellow students, but every student should write down their own solutions. It is encouraged to solve MANY of the "additional exercises", and other exercises that you find in the textbook or elsewhere, in addition to the homework problems.

Problem 1

Make up a system of four linear equations in four variables that has

(a) no solution.
(b) a unique solution.
(c) a one-parameter solution set.
(d) a three-parameter solution set.

Problem 2

Let a be a constant. Consider the system of equations

$$\begin{cases} ax + y & = a^2 \\ x + ay & = 1 \end{cases}.$$

For what values of a does the system have

(a) no solution.
(b) a unique solution.
(c) infinitely many solutions.

Problem 3

Find a lower triangular matrix L and an upper triangular matrix U such that

$$LU = \begin{pmatrix} 1 & 1 & 2 & 2 \\ 1 & 2 & 1 & 3 \\ 0 & 0 & 1 & 2 \\ -1 & -1 & 0 & 0 \end{pmatrix}.$$

Problem 4

Find the null space of the matrix

$$\begin{pmatrix} 1 & 1 & 2 & 2 \\ 1 & 2 & 1 & 3 \end{pmatrix}.$$

Additional exercises

1.23 This system is not linear in that it says $\sin\alpha$ instead of α

$$\begin{aligned} 2\sin\alpha - \cos\beta + 3\tan\gamma &= 3 \\ 4\sin\alpha + 2\cos\beta - 2\tan\gamma &= 10 \\ 6\sin\alpha - 3\cos\beta + \tan\gamma &= 9 \end{aligned}$$

and yet we can apply Gauss's Method. Do so. Does the system have a solution?

? 1.39 [Am. Math. Mon., Jan. 1935] Laugh at this: AHAHA + TEHE = TEHAW. It resulted from substituting a code letter for each digit of a simple example in addition, and it is required to identify the letters and prove the solution unique.

2.20 Solve each system using matrix notation. Express the solution set using vectors.

(a) $\begin{aligned} 3x + 2y + z &= 1 \\ x - y + z &= 2 \\ 5x + 5y + z &= 0 \end{aligned}$

(b) $\begin{aligned} x + y - 2z &= 0 \\ x - y &= -3 \\ 3x - y - 2z &= -6 \\ 2y - 2z &= 3 \end{aligned}$

(c) $\begin{aligned} 2x - y - z + w &= 4 \\ x + y + z &= -1 \end{aligned}$

(d) $\begin{aligned} x + y - 2z &= 0 \\ x - y &= -3 \\ 3x - y - 2z &= 0 \end{aligned}$