Homework 1

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Problem 1

Answers:

Denote the coordinates of the new machine as $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$. The distance from the new machine to each of the four machines is respectively given as:

Distance to Machine 1: $\begin{pmatrix} |x_1 - 3| \\ |x_2 - 0| \end{pmatrix} \rightarrow |x_1 - 3| + |x_2|$ or $z_{1,1}$ + $z_{1,2}$.

Distance to Machine 2: $\begin{pmatrix} |x_1 - 0| \\ |x_2 - (-3)| \end{pmatrix} \rightarrow |x_1| + |x_2 + 3|$ or $z_{2,1}$ + $z_{2,2}$.

Distance to Machine 3: $\begin{pmatrix} |x_1 - (-2)| \\ |x_2 - 1| \end{pmatrix} \rightarrow |x_1 + 2| + |x_2 - 1|$ or $z_{3,1}$ + $z_{3,2}$.

Distance to Machine 4: $\begin{pmatrix} |x_1 - 1| \\ |x_2 - 4| \end{pmatrix} \rightarrow |x_1 - 1| + |x_2 - 4|$ $z_{4,1}$ + $z_{4,2}$.

(a) In this scenario:

- Decision variables: the street distances between the new machine to the four pre-existing machines, respectively.
- Objective function: we need to minimize the sum of the street distances from the new machine to the four pre-existing machines, respectively. Hence, our objective function is given as:

$$\min z_{1,1} + z_{1,2} + z_{2,1} + z_{2,2} + z_{3,1} + z_{3,2} + z_{4,1} + z_{4,2} \equiv \min_{x_1, x_2} |x_1 - 3| + |x_2| + |x_1| + |x_2 + 3| + |x_1 + 2| + |x_2 - 1| + |x_1 - 1| \\ + |x_2 - 4|$$

- Constraints: there are only constraints for $z_{i,j}$ with $i = 1, 2, 3, 4$ and $j = 1, 2$.

Therefore, our linear optimization model is given as:

$$\min z_{1,1} + z_{1,2} + z_{2,1} + z_{2,2} + z_{3,1} + z_{3,2} + z_{4,1} + z_{4,2}$$

s.t.

$$\begin{cases} z_{1,1} \geq x_1 - 3; \quad z_{1,1} < -x_1 + 3 \\ z_{1,2} \geq x_2; \quad z_{1,2} < -x_2 \\ z_{2,1} \geq x_1; \quad z_{2,1} < -x_1 \\ z_{2,2} \geq x_2 + 3; \quad z_{2,2} < -x_2 - 3 \\ z_{3,1} \geq x_1 + 2; \quad z_{3,2} < -x_1 - 2 \\ z_{3,2} \geq x_2 - 1; \quad z_{3,2} < -x_2 + 1 \\ z_{4,1} \geq x_1 - 1; \quad z_{4,2} < -x_1 + 1 \\ z_{4,2} \geq x_2 - 4; \quad z_{4,2} < -x_2 + 4 \end{cases}$$

(b) In this scenario:

- Decision variables: the street distances between the new machine to the four pre-existing machines, respectively.
- Objective function: we need to minimize the sum of the weighted street distances from the new machine to the four pre-existing machines, respectively. Hence, our objective function is given as:

$$\begin{aligned} &\min w_1(z_{1,1} + z_{1,2}) + w_2(z_{2,1} + z_{2,2}) + w_3(z_{3,1} + z_{3,2}) + w_4(z_{4,1} + z_{4,2}) \\ &\equiv \min_{x_1, x_2} 5\Big(|x_1 - 3| + |x_2|\Big) + 7\Big(|x_1| + |x_2 + 3|\Big) + 3\Big(|x_1 + 2| + |x_2 - 1|\Big) + 1\Big(|x_1 - 1| + |x_2 - 4|\Big) \end{aligned}$$

- Constraints: there are only constraints for $z_{i,j}$ with $i = 1, 2, 3, 4$ and $j = 1, 2$.

Therefore, our linear optimization model is given as:

$$\min w_1(z_{1,1} + z_{1,2}) + w_2(z_{2,1} + z_{2,2}) + w_3(z_{3,1} + z_{3,2}) + w_4(z_{4,1} + z_{4,2})$$

with $w_1, w_2, w_3, w_4 = 5, 7, 3, 1$, respectively.

s.t.

$$\begin{cases} z_{1,1} \geq x_1 - 3; \quad z_{1,1} < -x_1 + 3 \\ z_{1,2} \geq x_2; \quad z_{1,2} < -x_2 \\ z_{2,1} \geq x_1; \quad z_{2,1} < -x_1 \\ z_{2,2} \geq x_2 + 3; \quad z_{2,2} < -x_2 - 3 \\ z_{3,1} \geq x_1 + 2; \quad z_{3,2} < -x_1 - 2 \\ z_{3,2} \geq x_2 - 1; \quad z_{3,2} < -x_2 + 1 \\ z_{4,1} \geq x_1 - 1; \quad z_{4,2} < -x_1 + 1 \\ z_{4,2} \geq x_2 - 4; \quad z_{4,2} < -x_2 + 4 \end{cases}$$

(c) In this scenario:

- Decision variables: the street distances between the new machine to the four pre-existing machines, respectively.
- Objective function: we need to minimize the sum of the street distances. Hence, our objective function is given as:

$$\min z_1 + z_2 + z_3 + z_4 \equiv \min_{x_1, x_2} |x_1 - 3| + |x_2| + |x_1| + |x_2 + 3| + |x_1 + 2| + |x_2 - 1| + |x_1 - 1| + |x_2 - 4|$$

- Constraints:
  - The new machine must be located in the square: $(x_1, x_2) : -1 \leq x_1 \leq 2, 0 \leq x_2 \leq 1$
  - There are also constraints for $z_{i,j}$ with $i = 1, 2, 3, 4$ and $j = 1, 2$.

Therefore, our linear optimization model is given as:

$$\min z_{1,1} + z_{1,2} + z_{2,1} + z_{2,2} + z_{3,1} + z_{3,2} + z_{4,1} + z_{4,2}$$

s.t.

$$\begin{cases} -1 \leq x_1 \leq 2 \\ 0 \leq x_2 \leq 1 \\ z_{1,1} \geq x_1 - 3; \quad z_{1,1} < -x_1 + 3 \\ z_{1,2} \geq x_2; \quad z_{1,2} < -x_2 \\ z_{2,1} \geq x_1; \quad z_{2,1} < -x_1 \\ z_{2,2} \geq x_2 + 3; \quad z_{2,2} < -x_2 - 3 \\ z_{3,1} \geq x_1 + 2; \quad z_{3,2} < -x_1 - 2 \\ z_{3,2} \geq x_2 - 1; \quad z_{3,2} < -x_2 + 1 \\ z_{4,1} \geq x_1 - 1; \quad z_{4,2} < -x_1 + 1 \\ z_{4,2} \geq x_2 - 4; \quad z_{4,2} < -x_2 + 4 \end{cases}$$

(d) In this scenario:

- Decision variables: the street distances between the new machine to the four pre-existing machines, respectively.
- Objective function: we need to minimize the sum of the street distances. Hence, our objective function is given as:

$$\min z_1 + z_2 + z_3 + z_4 \equiv \min_{x_1, x_2} |x_1 - 3| + |x_2| + |x_1| + |x_2 + 3| + |x_1 + 2| + |x_2 - 1| + |x_1 - 1| + |x_2 - 4|$$

- Constraints:
  - The new machine must be located so that its distance from the first machine does not exceed 3/2. Thus, $|x_1 - 3| + |x_2| \leq 3/2$
  - There are also constraints for $z_{i,j}$ with $i = 1, 2, 3, 4$ and $j = 1, 2$.

Therefore, our linear optimization model is given as:

$$\min z_{1,1} + z_{1,2} + z_{2,1} + z_{2,2} + z_{3,1} + z_{3,2} + z_{4,1} + z_{4,2}$$

s.t.

$$\begin{cases} |x_1 - 3| + |x_2| \leq 3/2 \\ z_{1,1} \geq x_1 - 3; \quad z_{1,1} < -x_1 + 3 \\ z_{1,2} \geq x_2; \quad z_{1,2} < -x_2 \\ z_{2,1} \geq x_1; \quad z_{2,1} < -x_1 \\ z_{2,2} \geq x_2 + 3; \quad z_{2,2} < -x_2 - 3 \\ z_{3,1} \geq x_1 + 2; \quad z_{3,2} < -x_1 - 2 \\ z_{3,2} \geq x_2 - 1; \quad z_{3,2} < -x_2 + 1 \\ z_{4,1} \geq x_1 - 1; \quad z_{4,2} < -x_1 + 1 \\ z_{4,2} \geq x_2 - 4; \quad z_{4,2} < -x_2 + 4 \end{cases}$$

Problem 2

Answers:

- Decision variables: denote t_i is the start time of task i, and d_i is the duration needed to complete task i. Hence, our decision variables are:
  - Start time and duration needed to complete task A: t_A, d_A.
  - Start time and duration needed to complete task B: t_B, d_B.
  - Start time and duration needed to complete task C: t_C, d_C.

- Start time and duration needed to complete task D: t_D, d_D.
- Start time and duration needed to complete task E: t_E, d_E.

- Objective function: we need to find the earliest completion date of the X project if all tasks are completed according to their regular duration. Hence, our objective function is given as:

$$\min t_E$$

where $d_A = 2, d_B = 13, d_C = 7, d_D = 8, d_E = 10$

- Constraints:

  - All start time t_A, t_B, t_C, t_D, t_E must be non-negative.
  - Some of these tasks have predecessors that must be completed before the task can be started. Therefore, our constraints are given as:
    - For the start time of task B: $t_B \geq t_A + d_A$.
    - For the start time of task C: $t_C \geq t_A + d_A$.
    - For the start time of task D: $\max\left(t_A + d_A, t_B + d_B\right)$.
    - For the start time of task E: $\max\left(t_B + d_B, t_C + d_C, t_D + d_D\right)$

Therefore, our linear optimization problem is given as:

$$\min t_E$$

s.t

$$\begin{cases} t_i \geq 0, \forall i = \{A, B, C, D, E\} \\ t_B \geq t_A + d_A \\ t_C \geq t_A + d_A \\ t_D \geq \max\left(t_A + d_A, t_B + d_B\right) \\ t_E \geq \max\left(t_B + d_B, t_C + d_C, t_D + d_D\right) \end{cases}$$