

IEM 5013 Intro to Optimization

Homework 2

HW Policy: This assignment can be done individually or with a partner i.e., maximum team size is two. A student can be part of exactly one submission. In order to include your name in the team's submission you must have made a substantial contribution by **actively and continuously participating** in the development of your team's assignment submission. Any violation will be treated as academic dishonesty.

Problems:

1. Consider the problem of locating a new machine to an existing layout consisting of four machines. These four machines (approximated as points) are located at the following (x_1, x_2) coordinates: $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ -3 \end{pmatrix}$, $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$. Let the coordinates of the new machine be $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$. Formulate the problem of finding an optimal location as a linear optimization model for each of the following cases.
 - (a) The sum of the distances from the new machine to the four existing machines is minimized. Use street distance; for example, the distance from $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ to the first machine located at $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ is $|x_1 - 3| + |x_2 - 0|$ (note that absolute value function is not linear!).
 - (b) Because of various amounts of flow between the new machine and the existing machines, reformulate the problems where the sum of the weighted street distances is minimized, where weights corresponding to the four machines are 5,7,3 and 1 respectively.
 - (c) In order to avoid congestion, suppose that the new machine must be located in the square $\{(x_1, x_2) : -1 \leq x_1 \leq 2, 0 \leq x_2 \leq 1\}$. Formulate (a) with this additional restriction.
 - (d) Suppose that the new machine must be located so that its distance from the first machine does not exceed $3/2$. Formulate (a) with this additional restriction.
2. Completing a project X requires tasks A, B, C, D, and E to be completed, and for each of these tasks, the normal duration to complete them are known. Some of these tasks have predecessors that must be completed before the task can be started. Formulate a linear optimization model to find out the earliest completion date of project X if all tasks are completed according to their regular duration.

Task	Duration (weeks)	Predecessors
A	2	-
B	13	A
C	7	A
D	8	A,B
E	10	B,C,D