Homework 1

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Problem 1

Answer:

In this problem:

- Decision variables: $x_{i,j}$ the number of j leasing months starting from month i.
- Objective function: we need to minimize the total leasing cost which is the sum of these five sets of sub-functions:
  - $65x_{1,1} + 100x_{1,2} + 135x_{1,3} + 160x_{1,4} + 190x_{1,5}$ (Started as Month $i = 1$, and lease up to $j = 1, 2, 3, 4, 5$ months).
  - $65x_{2,1} + 100x_{2,2} + 135x_{2,3} + 160x_{2,4}$ (Started as Month $i = 2$, and lease up to $j = 1, 2, 3, 4$ months).
  - $65x_{3,1} + 100x_{3,2} + 135x_{3,3}$ (Started as Month $i = 3$, and lease up to $j = 1, 2, 3$ months).
  - $65x_{4,1} + 100x_{4,2}$ (Started as Month $i = 4$, and lease up to $j = 1, 2$ months).
  - $65x_{5,1}$ (Started as Month $i = 5$, and lease up to $j = 1$ months).
- Constraints:
  - Each decision variable must be non-negative: $x_{i,j} \geq 0; \forall i, j = 1, 2, \ldots, 5$.
  - Minimum space requirements for each month:
    - $x_{1,1} + x_{1,2} + x_{1,3} + x_{1,4} + x_{1,5} \geq 30,000$
    - $x_{1,2} + x_{1,3} + x_{1,4} + x_{1,5} + x_{2,1} + x_{2,2} + x_{2,3} + x_{2,4} \geq 20,000$ (this constraint also includes the area leased in Month $i = 1$ that carries over).
    - $x_{1,3} + x_{1,4} + x_{1,5} + x_{2,2} + x_{2,3} + x_{2,4} + x_{3,1} + x_{3,2} + x_{3,3} \geq 40,000$ (this constraint also includes the area leased in Month $i = 1$ and Month $i = 2$ that carries over).

- $x_{1,4} + x_{1,5} + x_{2,3} + x_{2,4} + x_{3,2} + x_{3,3} + x_{4,1} + x_{4,2} \geq 10,000$ (this constraint also includes the area leased in Month $i = 1$, Month $i = 2$, Month $i = 3$ that carries over).
- $x_{1,5} + x_{2,4} + x_{3,3} + x_{4,2} + x_{5,1} \geq 50,000$ (this constraint also includes the area leased in Month $i = 1$, Month $i = 2$, Month $i = 3$, and Month $i = 4$ that carries over).

Therefore, the linear optimization model for this problem is:

$$\text{Min} \quad \begin{aligned} &65x_{1,1} + 100x_{1,2} + 135x_{1,3} + 160x_{1,4} + 190x_{1,5} \\ &+ 65x_{2,1} + 100x_{2,2} + 135x_{2,3} + 160x_{2,4} \\ &+ 65x_{3,1} + 100x_{3,2} + 135x_{3,3} \\ &+ 65x_{4,1} + 100x_{4,2} \\ &+ 65x_{5,1} \end{aligned}$$

s.t.

$$\begin{cases} x_{i,j} \geq 0; \forall i, j = 1, 2, \ldots, 5 \\ x_{1,1} + x_{1,2} + x_{1,3} + x_{1,4} + x_{1,5} \geq 30,000 \\ x_{1,2} + x_{1,3} + x_{1,4} + x_{1,5} + x_{2,1} + x_{2,2} + x_{2,3} + x_{2,4} \geq 20,000 \\ x_{1,3} + x_{1,4} + x_{1,5} + x_{2,2} + x_{2,3} + x_{2,4} + x_{3,1} + x_{3,2} + x_{3,3} \geq 40,000 \\ x_{1,4} + x_{1,5} + x_{2,3} + x_{2,4} + x_{3,2} + x_{3,3} + x_{4,1} + x_{4,2} \geq 10,000 \\ x_{1,5} + x_{2,4} + x_{3,3} + x_{4,2} + x_{5,1} \geq 50,000 \end{cases}$$

Problem 2

Answer:

In this problem:

- Decision variables: x_i the number of workers employed for two consecutive periods starting at the i-th period, and y_i the number of workers employed for three consecutive periods starting at the i-th period.
- Objective function: we need to minimize the total labour cost $F(x, y) = c_x \sum_i^6 x_i + c_y \sum_i^6 y_i$, with $c_y = 1.5c_x$ since the worker salary is proportional to the number of periods worked in a shift.
- Constraints:

- Each decision variables must be non-negative.
- For Period 1: $x_1 + x_6 + y_1 + y_6 + y_5 \geq 20$ (this constraint also includes workers from previous periods employed for another consecutive periods - namely Period 5, and Period 6).
- For Period 2: $x_2 + x_1 + y_2 + y_1 + y_6 \geq 50$ (this constraint also includes workers from previous periods employed for another consecutive periods - namely Period 6, and Period 1).
- For Period 3: $x_3 + x_2 + y_3 + y_2 + y_1 \geq 80$ (this constraint also includes workers from previous periods employed for another consecutive periods - namely Period 2, and Period 1)
- For Period 4: $x_4 + x_3 + y_4 + y_3 + y_2 \geq 100$ (this constraint also includes workers from previous periods employed for another consecutive periods - namely Period 3, and Period 2)
- For Period 5: $x_5 + x_4 + y_5 + y_4 + y_3 \geq 40$ (this constraint also includes workers from previous periods employed for another consecutive periods - namely Period 4, and Period 3)
- For Period 6: $x_6 + x_5 + y_6 + y_5 + y_4 \geq 30$ (this constraint also includes workers from previous periods employed for another consecutive periods - namely Period 5, and Period 4)

Therefore, the linear optimization model is:

$$\text{Min} \quad c_x \sum_i^6 x_i + c_y \sum_i^6 y_i$$

s.t.

$$\begin{cases} x_1 + x_6 + y_1 + y_6 + y_5 \geq 20 \\ x_2 + x_1 + y_2 + y_1 + y_6 \geq 50 \\ x_3 + x_2 + y_3 + y_2 + y1 \geq 80 \\ x_4 + x_3 + y_4 + y_3 + y_2 \geq 100 \\ x_5 + x_4 + y_5 + y_4 + y_3 \geq 40 \\ x_6 + x_5 + y_6 + y_5 + y_4 \geq 30 \end{cases}$$