EXERCISE SET 5, DIFFERENTIAL AND INTEGRAL CALCULUS

The solutions to the problems should be handed in via MyCourses before 17:00, Friday 7.12.

You are allowed and encouraged to discuss the exercises with your fellow students, but every student should write down their own solutions. It is encouraged to solve MANY of the "additional exercises", and other exercises that you find in the textbook or elsewhere, in addition to the homework problems.

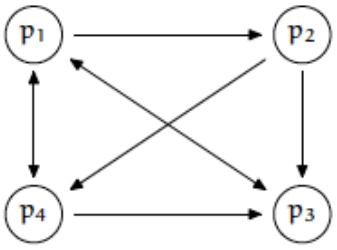
Problem 1

Find an orthonormal basis for the space

$$V = \left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} : x + y + z + w = 0 \right\}.$$

Problem 2

Give the importance ranking for this web of pages, using the PageRank algorithm:



Problem 3

Compute a singular value decomposition of $\begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 1 \end{pmatrix}$.

Additional exercises

✓ **2.15** **Find an orthonormal basis for this subspace of $\mathbb{R}^3$: the plane $x - y + z = 0$.**

1. Factor the following matrices into $S\Lambda S^{-1}$:

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \qquad \text{and} \qquad A = \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}.$$

2. Find the matrix A whose eigenvalues are 1 and 4, and whose eigenvectors are $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$, respectively. (*Hint*: $A = S\Lambda S^{-1}$.)

3. Find *all* the eigenvalues and eigenvectors of

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

and write two different diagonalizing matrices S.

4. If a 3 by 3 upper triangular matrix has diagonal entries 1, 2, 7, how do you know it can be diagonalized? What is Λ?

5. Which of these matrices cannot be diagonalized?

$$A_1 = \begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix} \qquad A_2 = \begin{bmatrix} 2 & 0 \\ 2 & -2 \end{bmatrix} \qquad A_3 = \begin{bmatrix} 2 & 0 \\ 2 & 2 \end{bmatrix}.$$