IEM 5013 Intro to Optimization
Homework 3

HW Policy: This assignment can be done individually or with a partner i.e., maximum team size is two. A student can be part of exactly one submission. In order to include your name in the team's submission you must have made a substantial contribution by actively and continuously participating in the development of your team's assignment submission. Any violation will be treated as academic dishonesty.

Permitted Resources: Course textbook and any other textbook, even if it is not listed among the references on the course syllabus. All materials provided on the Canvas course website. All materials available at Gurobi.com and at other links provided on the Canvas course website as learning resources for the Python programming language.

Prohibited Activities: Collaboration, using any means, with anyone other than your teammate. Searching the internet for answers. Accessing past assignment solutions with the help of former students or using websites that facilitate this access. Use of any resource outside the resources explicitly noted above as permissible. Anything else I haven't enumerated, but violates the spirit of academic integrity. The assignment must be completed using only your individual/team effort and no external assistance beyond the permissible resources.

Production Planning With Backorders. Jackson's Jungle Gyms is completing their sales and operations planning for the next six months. Forecasted demand for the six month planning horizon is shown in the table below.

Month, i	1	2	3	4	5	6
Demand, d_i	1055	700	1645	1600	1095	845

Operating during regular hours the production capacity is 1000 units/month (u_1). But overtime capacity is an additional 300 units/month (u_2). The selling price is the same irrespective of production mode, and the unit production costs are \$10 (c_1) and \$15 (c_2) for regular time production and overtime production, respectively. In addition, there is a inventory holding cost of \$2 (h) per unit per month for each unsold unit held over. Demand can be back-ordered at \$5 (b) per unit per month. Assume that the initial and closing inventory and back-orders are zero. This problem can be formulated as shown next.

Index sets:

- $I = \{1, \ldots, 6\}$ for the months
- $J = \{1, 2\}$ for production mode

Parameters:

- Demand: d_i for each $i \in I$
- Production capacity: u_j for each $j \in J$
- Inventory cost: h per unit per month

- Back-order cost: b per unit per month
- Production cost: c_j per unit for each $j \in J$
- Initial and closing inventory: $\bar{z}_0$ and $\bar{z}_n$
- Initial and closing back-orders: $\bar{w}_0$ and $\bar{w}_n$

Decision Variables:

- x_i denotes the number of units produced in month $i \in I$ in regular time
- y_i denotes the number of units produced in month $i \in I$ in overtime
- z_i denotes the number of units carried over in the inventory from month $i \in I$ to the next month and z_0 is the initial inventory
- w_i denotes the number of units back-ordered from month $i \in I$ to the next month and w_0 is the initial back-orders

$$\min c_1 \sum_{i \in I} x_i + c_2 \sum_{i \in I} y_i + h \sum_{i \in I} z_i + b \sum_{i \in I} w_i$$

$$\begin{aligned}
\textit{subject to:} \quad z_0 &= \bar{z}_0 \\
w_0 &= \bar{w}_0 \\
x_i + y_i + z_{i-1} - z_i &= d_i + w_{i-1} - w_i && \forall i \in I \\
0 \quad \leq x_i &\leq u_1 && \forall i \in I \\
0 \quad \leq y_i &\leq u_2 && \forall i \in I \\
z_{|I|} &= \bar{z}_n \\
w_{|I|} &= \bar{w}_n \\
x_i, y_i, z_i, w_i &\geq 0 && \forall\ i \in I
\end{aligned}$$

Note: $|I|$ denotes the number of elements of I.

Assignment: Implement this model in Python/Gurobi; upload your code (`*.py` file) to Canvas.

Recommendations: Try to separate the data and parameters from the model code; initialize the parameters/data before the model code starts. Check the status of Gurobi termination before you output your result. Look at the *flight attendant hiring LP code on Canvas* as an example.