EXERCISE SET 1, DIFFERENTIAL AND INTEGRAL CALCULUS

The solutions to the problems should be handed in via MyCourses before 17:00, Friday 16.11.

You are allowed and encouraged to discuss the exercises with your fellow students, but every student should write down their own solutions. It is encouraged to solve MANY of the "additional exercises", and other exercises that you find in the textbook or elsewhere, in addition to the homework problems.

Problem 1

Let $p : \mathbb{R}^3 \to \mathbb{R}^2$ be the projection given by the matrix $P = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{pmatrix}$, let $r : \mathbb{R}^2 \to \mathbb{R}^2$ be rotation by θ radians counterclockwise around the origin, and let $f : \mathbb{R}^2 \to \mathbb{R}^3$ have transformation matrix P^T.

(1) Compute a matrix for the composition $f \circ r \circ p$.
(2) Is the matrix in the previous problem invertible?

Problem 2

What is the determinant of the linear map given by

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \mapsto \begin{pmatrix} x - y \\ x + y + z \\ y - 2z \end{pmatrix}.$$

Problem 3

Compute the inverse of

$$\begin{pmatrix} 1 & 1 & 2 & 2 \\ 1 & 2 & 1 & 3 \\ 0 & 0 & 1 & 2 \\ -1 & -1 & 0 & 0 \end{pmatrix}.$$

Problem 4

Compute the area of the parallelogram spanned by $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix}$ in $\mathbb{R}^3$.

Additional exercises

✓ **1.12** Multiply the matrix

$$\begin{pmatrix} 1 & 3 & 1 \\ 0 & -1 & 2 \\ 1 & 1 & 0 \end{pmatrix}$$

by each vector, or state "not defined."

(a) $\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$ **(b)** $\begin{pmatrix} -2 \\ -2 \end{pmatrix}$ **(c)** $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

✓ **2.14** Compute, or state "not defined".

(a) $\begin{pmatrix} 3 & 1 \\ -4 & 2 \end{pmatrix}\begin{pmatrix} 0 & 5 \\ 0 & 0.5 \end{pmatrix}$ **(b)** $\begin{pmatrix} 1 & 1 & -1 \\ 4 & 0 & 3 \end{pmatrix}\begin{pmatrix} 2 & -1 & -1 \\ 3 & 1 & 1 \\ 3 & 1 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 2 & -7 \\ 7 & 4 \end{pmatrix}\begin{pmatrix} 1 & 0 & 5 \\ -1 & 1 & 1 \\ 3 & 8 & 4 \end{pmatrix}$ **(d)** $\begin{pmatrix} 5 & 2 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} -1 & 2 \\ 3 & -5 \end{pmatrix}$

4.20 For each real number θ let $t_\theta : \mathbb{R}^2 \to \mathbb{R}^2$ be represented with respect to the standard bases by this matrix.

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

Show that $t_{\theta_1+\theta_2} = t_{\theta_1} \cdot t_{\theta_2}$. Show also that ${t_\theta}^{-1} = t_{-\theta}$.

1.8 Show this.

$$\det\left(\begin{pmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{pmatrix}\right) = (b-a)(c-a)(c-b)$$

✓ **1.9** Find the volume of the region defined by the vectors.

(a) $\langle \begin{pmatrix} 1 \\ 3 \end{pmatrix}, \begin{pmatrix} -1 \\ 4 \end{pmatrix} \rangle$

(b) $\langle \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}, \begin{pmatrix} 8 \\ -3 \\ 8 \end{pmatrix} \rangle$

(c) $\langle \begin{pmatrix} 1 \\ 2 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ 3 \\ 0 \\ 5 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 7 \end{pmatrix} \rangle$