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Final EXAM.

1.) a) TRUE. It was shown in SIPSER.

b). False. All multi-tape TMs have equivalent power.

c). True. Hampath is both undecidable and NP-compl

d.) True. Both are NP-complete so they must be polynomial time reducible to each other.

e.) $3n^2+5n+2 = O(\log n + \frac{n^2}{2})$ True.

$\Rightarrow 3n^2+5n+2 \leq (\log n + \frac{n^2}{2}) \cdot C$

$\Rightarrow \frac{3n^2+5n+2}{\log n + \frac{n^2}{2}} \leq C$

$\Rightarrow \lim_{n\to\infty} \frac{3n^2+5n+2}{\frac{n^2}{2}+\log n} = \frac{3}{\frac{1}{2}} = 6 \leq C$

Pick some $C > 6$.

2.) a.) $1q_1 1 \rightarrow q_2 1 0$

iv

b.) i

c.) ii

d.) iii

e.) i

3.) Given a DFA $D = (Q, \Sigma, \delta, q_0, F)$

Where Q = Set of States.

Σ = Alphabet of the DFA

δ = Transisition function

$\delta : Q \times \Sigma \to Q$

q_0 = Start State

F = Set of Final States $F \subseteq Q$.

Given a string $w = w_1 w_2 w_3 \dots w_n$

For D to accept w

$\delta(q_0, w_1)$ must be a transition to a non-trap State.

$\delta(q_k, w_n) \in F$ where $q_k \in Q$

that is to say there must be a valid Sequence of transitions Starting from q_0 with w_1 and ending with w_n and some $q_k \in F$.

4. $L_2 = \{ 0^n 1^n 2^n \mid n \geq 0 \}$

Assume L_2 is regular. By the Pumping Lemma there exists some pumping Length p.

Let $w = 0^p 1^p 2^p$. Clearly $w \in L_2$. Now, divide $w = xyz$. As $|xy| \leq p$ (by pumping Lemma), Then at most $xy = 0^p$ segment of w.

However $xy^2z \notin L_2$ as this would result in more 0's than 1's or 2's which is not in the Language L_2. Thus, L_2 cannot be a regular Language. $\square$

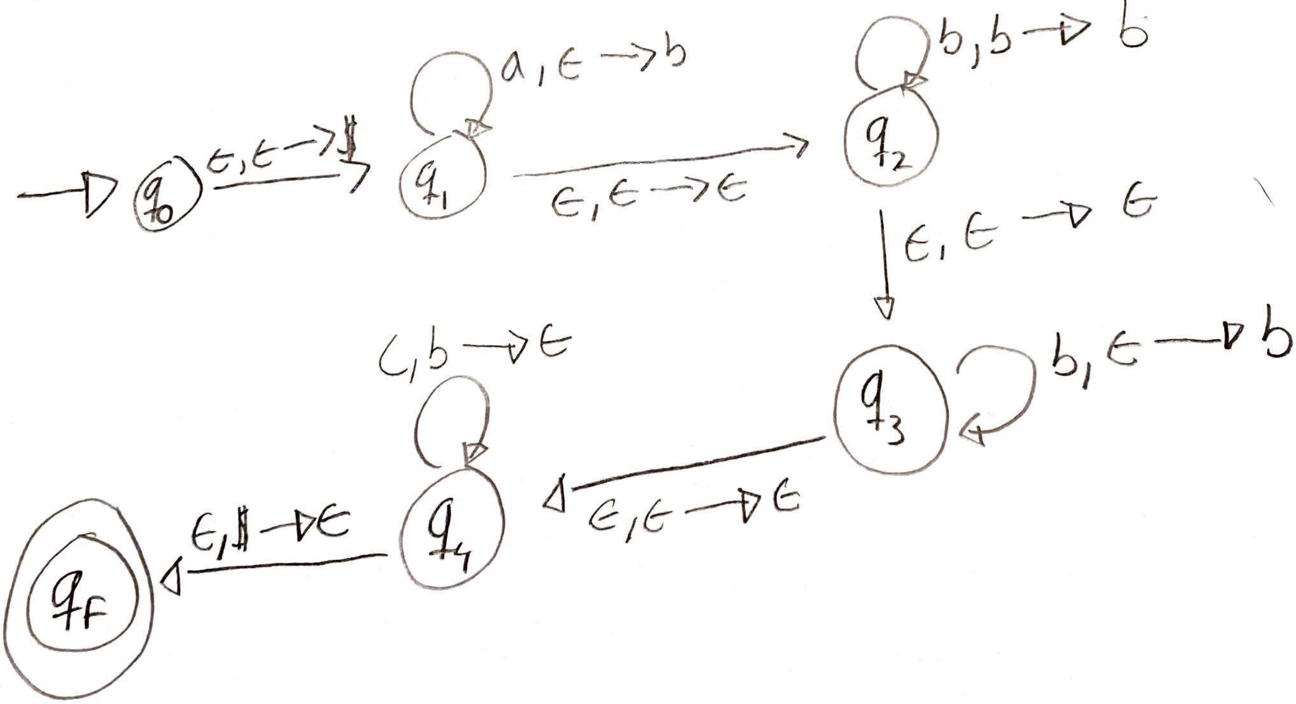
5.) a) $L_2 = \{a^i b^j c^k \mid i = 2j \quad i,j,k \geq 0\}$

$S \rightarrow \epsilon \mid VT$

$V \rightarrow aaVb \mid \epsilon$

$T \rightarrow cT \mid \epsilon$

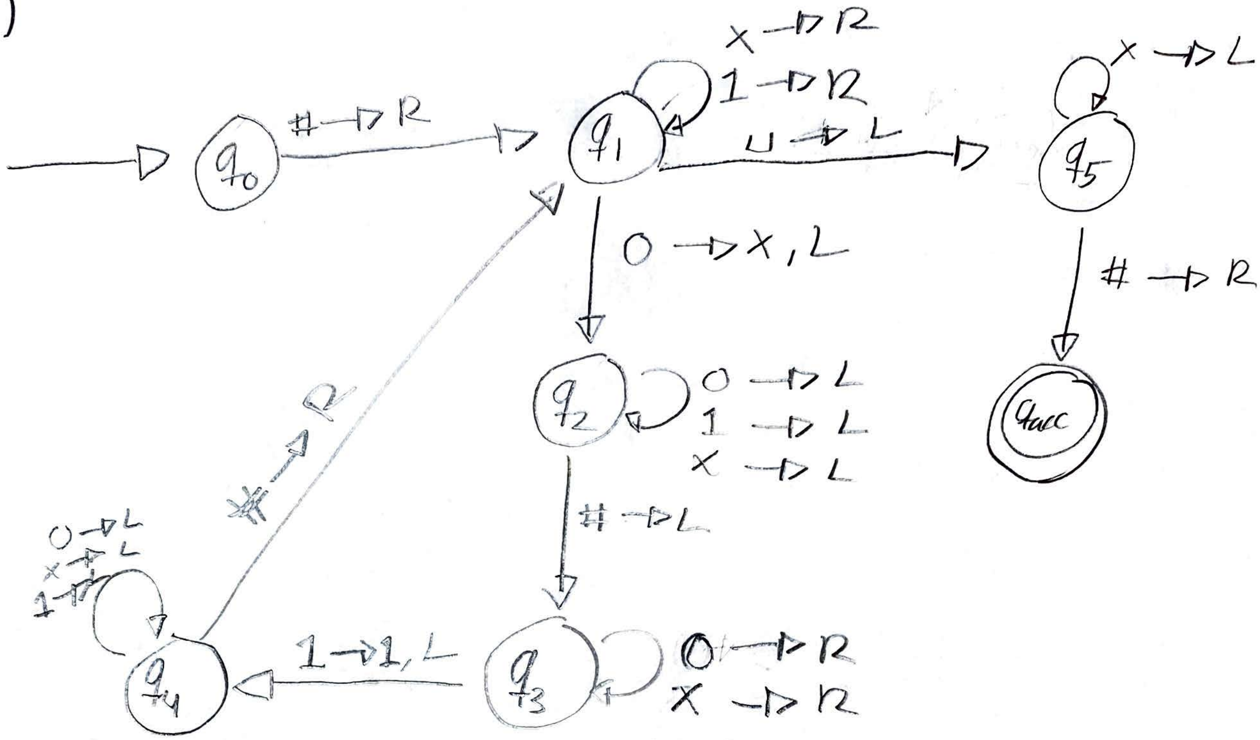
b) $L_2 = \{a^i b^j c^k \mid j = i + k, \quad i,j,k \geq 0\}$



6.) If some enumerator enumerates a Language in Lexicographic order then this is a countably infinite list of all possible words in the language. So deciding the language would be iterrating through the words to find a match. If the list is exhausted the word is not in the Language.

If a Language is decidable then construct a TM that enumerates all valid paths through the states $q_0 \cdots q_{accept}$ and output them to a k-tape TM. This will be an enumerator that enumerates a Language. Preform this enumeration by path length to obtain lexicographical ordering.

7.)



This machine will first begin by scanning for a 0. If a 0 is found it is replaced by an X then the head moves back to the start of the tape to find a 1 and replace it with an X.

This process is repeated until no more 0's are found. Finally the tape will seek through only moving the tape head left if an X is read. If the # symbol is reached the TM accepts.

8.) Assume EQ_{TM} is is decidable. Then, there exists a decider R for EQ_{TM}.

Build a new TM M as follows:

$M =$ "On input $\langle N \rangle$ where N is a turing machine.

1. Run decider R on $\langle N, N_0 \rangle$ where $L(N_0) = \emptyset$.
2. If R accepts, accept. If R rejects, reject.

Clearly M will accept iff $L(N) = \emptyset$. However, this decides E_{TM} and contradicts E_{TM} being undecidable. So, EQ_{TM} must also be undecidable $\square$.

9) N = "On input $\langle M \rangle$ where M is a DFA

1. Create a new DFA D by constructing the compliment of M.
2. As E_{DFA} is decidable, use the TM for it on $\langle D \rangle$.
3. If E_{DFA} TM rejects, accept. If E_{DFA} TM accept, reject."

N will accept iff $L(M) = \overline{L(\emptyset)} = \Sigma^*$, thus N Decides $A_{\square}$.

10.) a.) Let $\phi = (q_1 \vee r_1 \vee s_1 \vee t_1) \wedge \dots \wedge (q_n \vee r_n \vee s_n \vee t_n)$

Define $\phi' = (\phi) \wedge (y \vee \bar{y} \vee y \vee \bar{y})$

Clearly $\phi \in$ 4-SAT and $\phi' \in$ 4-SAT.

If ϕ has one agreeable claim then $\phi \in SAT$

make $y = 1$ so $\phi' \in SAT$.

If ϕ has no agreeable claims then $\phi' \notin$ Satisfiable.

So $SAT \leq_p$ 4-SAT.

To construct a verifier for 4-SAT give as certificate configuration for the values for q, r, s, t. If the configuration is satifiable accept, else reject. As this verifier is polynomial, then 4-SAT $\in$ NP.

10b) HAMPATH - Is there a path in a graph that visits every node exactly Once.

SUBSETSUM - Given a set and an integer can that integer be made from a sum of any subset of values from the set.

Clique - Given a graph G and value k does G contain a complete subgraph with k-nodes.

HALFClique - Given a graph G does it contain a complete subgraph with $\frac{m}{2}$ nodes where m = total nodes of the graph.

10c) If a Language $L \in NP$, $L \notin P$

then by the definition of NP - Complete

no other NP - complete language could be

in P because for a Language to

be NP - Complete every Language $A \in NP$

must be polynomial time reducible to it.