Florida International University
School of Computing and Information Sciences

Final Exam

COT 5310 Theory of Computation I
Spring 2020

Name:	
Netid:	

- Suggestions: Read through the entire exam first before starting work. Do not spend too much time on any single problem. If you get stuck, move on to something else and come back later.
- If you run short on time, remember that partial credit will be given.
- If any question is unclear, ask for clarification.

Question	Points	Score
Problem 1	10	
Problem 2	10	
Problem 3	5	
Problem 4	10	
Problem 5	10	
Problem 6	10	
Problem 7	10	
Problem 8	10	
Problem 9	10	
Problem 10	15	
Total	100	

1. True or False **(10 points)**

   Completely write out "True" if the statement is necessarily true. Otherwise, completely write "False". Include a **short** justification. Each question is worth 2 points.

   (a) The language A_{TM} is Turing-recognizable

   (b) For any $k > 1$, there is no language that is decided by a TM with k tapes, but it is undecidable by any TM having $k - 1$ (or less) tapes.

   (c) It is possible for some undecidable language to be NP-Complete.

   (d) If VERTEXCOVER is in P, then HAMILTONIANPATH is also in P.

   (e) $3n^2 + 5n + 2 = O(lgn + n^2/2)$

2. Multiple Choice (10 points)

   Pick the correct alternative from among the choices provided for each question below. Each question is worth 2 points.

   (a) Consider the following Turing Machine: $M = (\{q_0, q_1, q_2, q_{acc}, q_{rej}\}, \{0,1\}, \{0,1,\sqcup\}, \delta, q_0, q_{acc}, q_{rej})$, where
   $\delta(q_0, 0) = (q_1, 1, R)$ $\delta(q_2, 1) = (q_0, 1, R)$
   $\delta(q_1, 1) = (q_2, 0, L)$ $\delta(q_1, \sqcup) = (q_{acc}, \sqcup, R)$

   As always, we assume for cases not mentioned above, $\delta(q, a) = (q_{rej}, \sqcup, R)$. Suppose the current configuration is $1q_11$. The next configuration is:
   i. q_201
   ii. q_200
   iii. $1q_20$
   iv. q_210

   (b) Consider the following Turing Machine: $M = (q_0, q_1, q_2, q_{acc}, q_{rej}, \{0,1\}, \{0,1,\sqcup\}, \delta, q_0, q_{acc}, q_{rej})$, where
   $\delta(q_0, 0) = (q_1, 1, R)$ $\delta(q_2, 1) = (q_0, 1, R)$
   $\delta(q_1, 1) = (q_2, 0, L)$ $\delta(q_1, \sqcup) = (q_{acc}, \sqcup, R)$
   As always, we assume for cases not mentioned above, $\delta(q, a) = (q_{rej}, \sqcup, R)$. What can we say about the Turing machine M?
   i. M halts on all inputs
   ii. M never halts on some inputs
   iii. M does not halt on any input

   (c) Which of the following is true for the input alphabet Σ and the tape alphabet Γ of a Turing machine?
   i. It is possible for Σ and Γ to be equal.
   ii. Γ is always a strict superset of Σ.
   iii. It is possible for Σ and Γ to be disjoint.

   (d) If L is Turing-recognizable, then
   i. L and $\overline{L}$ must be decidable.
   ii. L must be decidable, but $\overline{L}$ need not be.
   iii. either L is decidable, or $\overline{L}$ is not Turing-recognizable.

   (e) Let A and B be any languages such that $A \leq_m B$. Under what conditions is it the case that $\overline{A} \leq_m \overline{B}$.
   i. Only when both A and B are decidable.
   ii. Only when both A and B are recognizable.
   iii. Always.

3. Remembering Definitions. (5 points)

   Define formally what it means for a DFA $(Q, \Sigma, \delta, q_0, F)$ to accept a string $w = w_1 w_2 \ldots w_n$.

4. Pumping Lemma (10 points)

   Prove that the following language is non-regular using the Pumping Lemma.

   $L_2 = \{0^n 1^n 2^n | n \geq 0\}$.

5. Context Free Languages (**10 points**)

   (a) (5 points) Give a context free grammar that generates the following language:
   $L_2 = \{a^i b^j c^k \mid i = 2j \;\text{ with } i, j, k \geq 0\}$

   (b) (5 points) Give a pushdown automaton that accepts the following language. Design the PDA directly, do not use the PDA to CFG conversion seen in class.
   $L_2 = \{a^i b^j c^k \mid j = i + k \;\text{ with } i, j, k \geq 0\}$

6. Turing Machines (10 points)

   Show that a language is decidable if and only if some enumerator enumerates it in lexicographic order.

7. Turing Machines (10 points)

   In this problem, you are required to give (draw) the transition diagram for a Turing machine that decides the language of binary strings ($\Sigma = \{0, 1\}$) such that the number of zeroes is equal to the number of ones. Briefly explain how your TM works.

8. Undecidability (10 points)

   Proof that $\mathrm{EQ}_{TM} = \{\langle M_1, M_2\rangle \mid M_1$ and M_2 are TMs and $L(M_1) = L(M_2)\}$ is undecidable by reduction from E_{TM}

9. Decidability (10 points)
Let $A = \{\langle M \rangle \mid M \text{ is a DFA and } L(M) = \Sigma^*\}$. Give a decider for this language.

10. NP-Completeness (15 points)

    (a) It is known that 3-SAT is NP-complete. Show that 4-SAT is NP-complete. (Don't forget to show that it is in NP.) (8 points)

    (b) Name four NP-Complete problems, do not include in your list satisfability problems (SAT, 3SAT, 4SAT,etc). Briefly describe each of these problems. (4 points)

    (c) Show that if there exists a language $L \in NP$ not in P, then no NP-complete language is in P. (3 points)