PhD Math Camp

August 2023

Instructor: Shuoqi Sun(shuoqis@andrew.cmu.edu)

1 Overview

This is a nine-day mini course in mathematics for incoming PhD students at Tepper school. It has three aims:

- To prepare you with the mathematical tools needed in your first year graduate coursework in economics;
- To get you a first understanding of the ideas underpinning these tools;
- To familiarize you with "theorem-proof" method of analysis.

2 Textbooks

I try to make the course self-contained, but you may find the following textbooks useful and may want to keep them as references.

- For real analysis, *Real analysis with economic applications* by Efe Ok, Princeton University Press, 2007.
- For linear algebra,*Linear Algebra Done Right* by Sheldon Axler, Springer 2015
- For optimization theory, *A First Course in Optimization Theory* by Rangarajan K. Sundaram, Cambridge 1996

3 Course outline

Below are the topics I plan to cover in the math camp.

Module I: Logic and methods of proof(0.5 sessions)

Module II: Real analysis(3 sessions)

- Metric spaces
- Sets and functions
- Basic topology
- Real sequences, Bolzano-Weierstrass theorem
- Cauchy sequence and completeness;
- Compactness
- Continuity of functions
- Correspondences
- Applications: Weierstrass theorem; Theorem of the maximum; Banach fixed-point theorem

Module III: Linear Algebra(1.5 sessions)

- Linear spaces and subspaces
- Spans, basis and dimensions
- Norms, inner products and Cauchy-Schwartz inequality
- Affinity
- Linear operators and functionals
- Rank-nullity theorem
- Review of matrix algebra

Module IV: Convex Analysis(1.5 sessions)

- Convex sets
- Caratheodory's theorem and Krein-Milman theorem
- Hilbert projection theorem
- Hyperplane theorems
- Farka's lemma(if time permits)

Module V: Optimization(2 sessions)

- Unconstrained optimization: first order and second order conditions
- Optimization with equality constraints
- Optimization with inequality constraints
- Existence and uniqueness of solution
- Comparative statics(implicit function theorem,envelope theorem)
- Calculus of variations(Euler-Lagrange equation)

Module VI: Introduction to Lebesgue measure(if time permits)