Inverse Kinematics

Inverse Kinematics

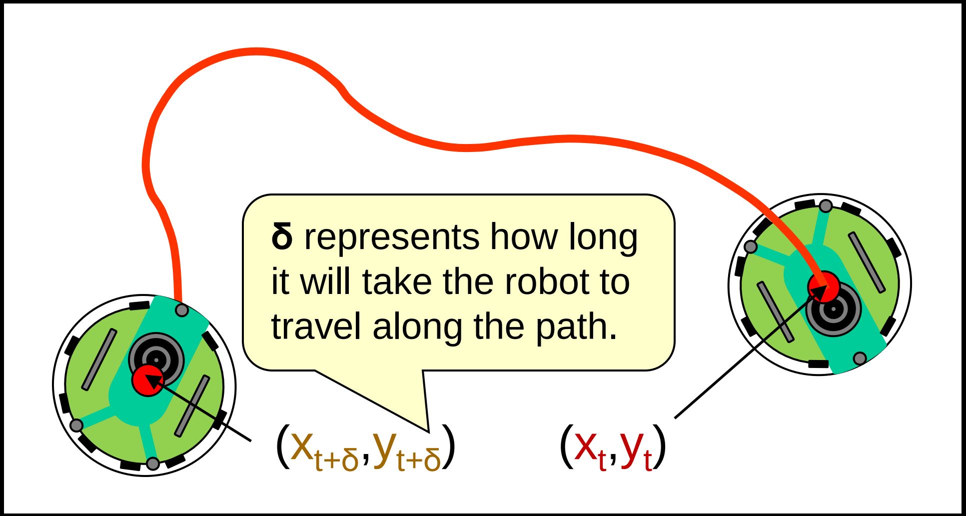
- Consider the ***inverse kinematics*** problem:

Given:

Sequence of robot positions (i.e. path) from (x_t, y_t, θ_t) to ($x_{t+\delta}$, $y_{t+\delta}$, $\theta_{t+\delta}$)

Find:

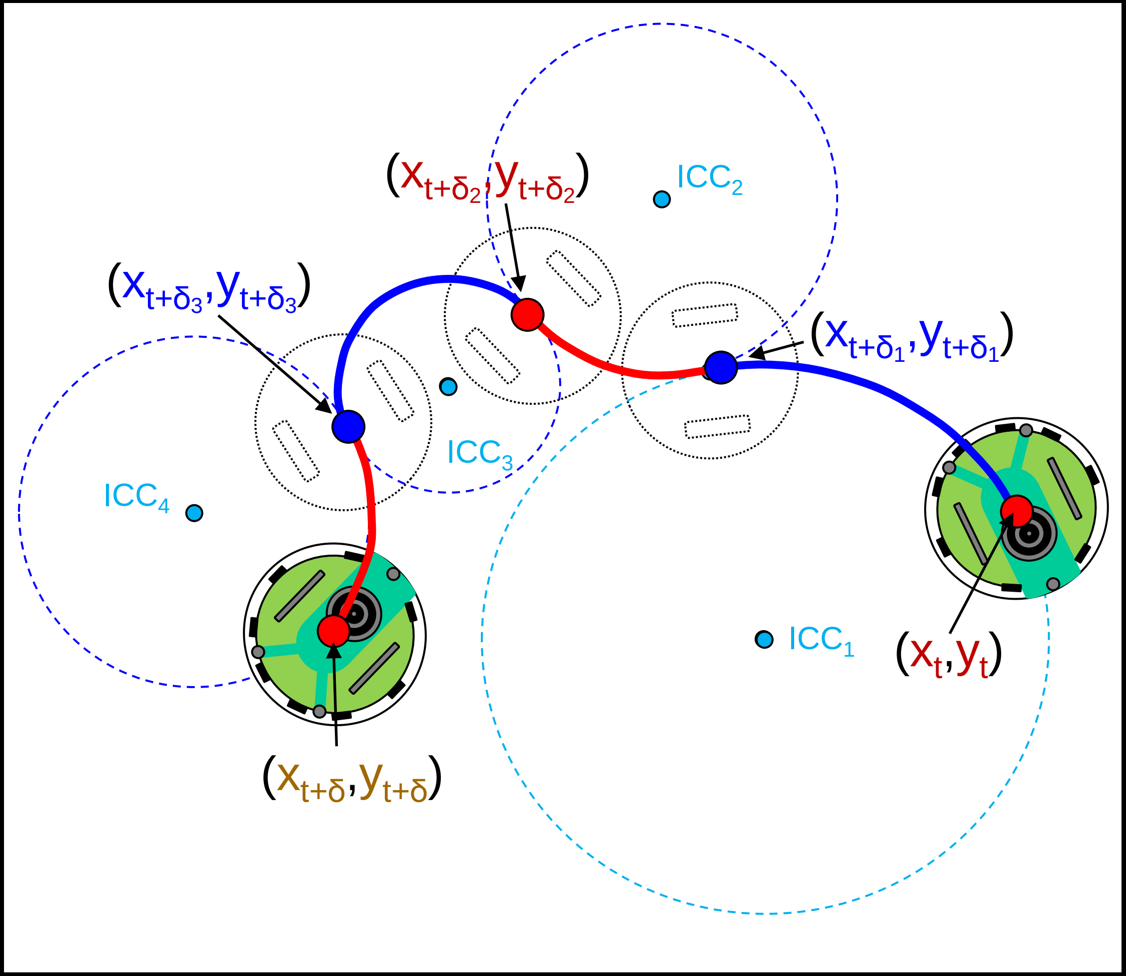
Speeds (over time) to set each motor to bring the robot along path from start position to end position over time δ (also need to find δ).



Inverse Kinematics

- It is a difficult problem!
  - complicated math
  - motors change speeds often along path
  - multiple solutions

- To simplify, we can try to fit ICC circles along the desired path along the way and compute the equations from one point to another.



Inverse Kinematics

- Still involves solving for leftReading and rightReading in:

$$x_{t+\delta} = r \cdot \cos\theta^{\Delta} \cdot \sin\theta_t + r \cdot \cos\theta_t \cdot \sin\theta^{\Delta} + x_t - r \cdot \sin\theta_t$$

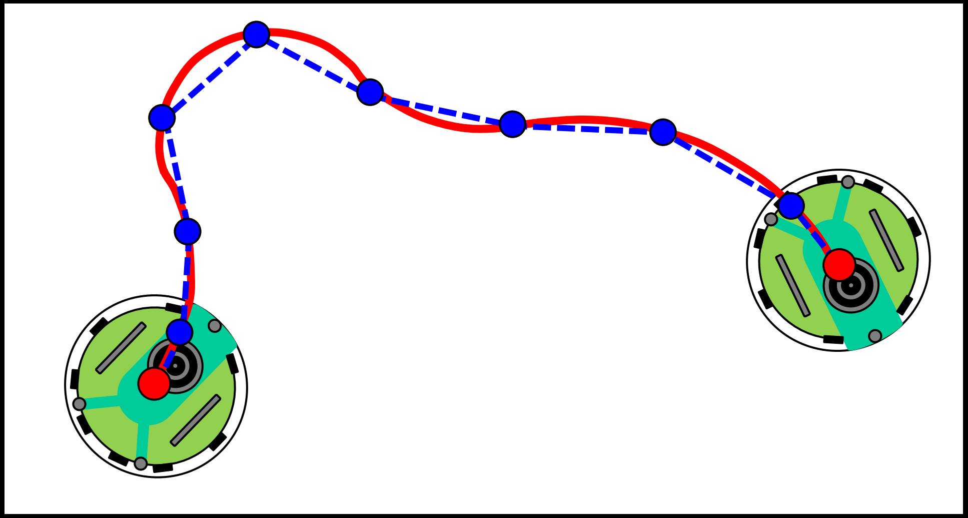
$$y_{t+\delta} = r \cdot \sin\theta^{\Delta} \cdot \sin\theta_t - r \cdot \cos\theta_t \cdot \cos\theta^{\Delta} + y_t + r \cdot \cos\theta_t$$

$$\theta_{t+\delta} = \theta_t + \theta^{\Delta}$$

$$r = [5.8 * (\text{leftReading} / (\text{rightReading} - \text{leftReading})) + 2.9]_{cm}$$

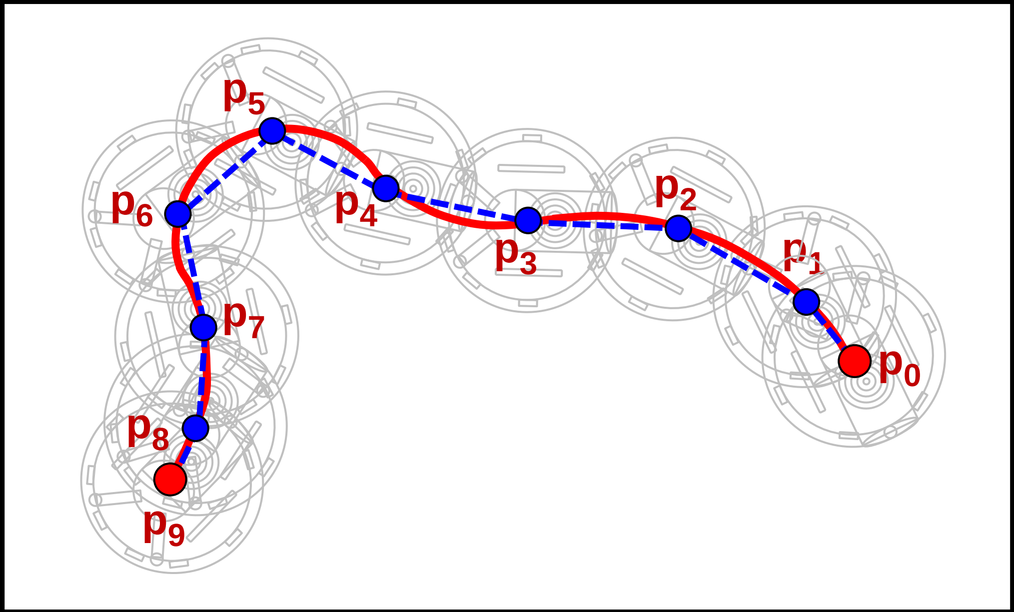
$$\theta^{\Delta} = (\text{rightReading} - \text{leftReading}) * 20.2510945\ ^{\circ}$$

- Easier to determine piecewise-linear approximation to path by travelling to intermediate points along the way.



Inverse Kinematics

- Can make robot travel to desired locations along approximated path by a series of spins and forward movements.
  - Spin at each vertex until facing desired angle
  - move forward until reaching next point



Inverse Kinematics - Spin

- How do we spin from angle θ_i to θ_{i+1} ?
- We need to compute θ_{i+1} from (x_i, y_i) to (x_{i+1}, y_{i+1}):

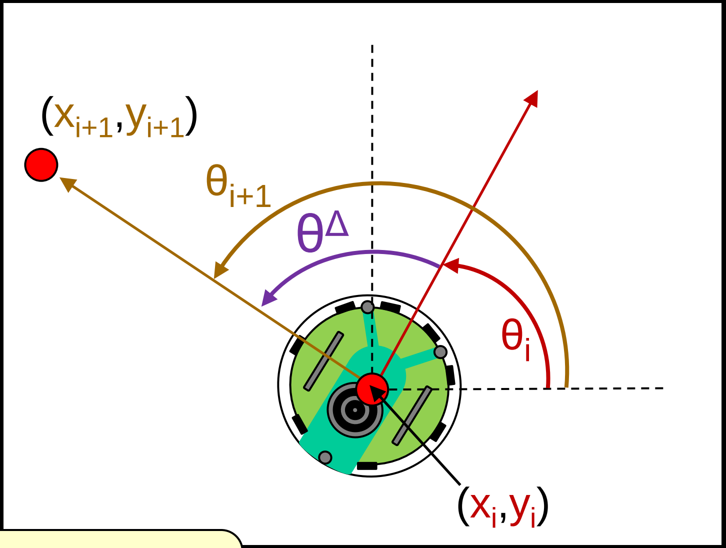
$$x^\Delta = x_{i+1} - x_i$$

$$y^\Delta = y_{i+1} - y_i$$

$$\theta_{i+1} = \arctan(y^\Delta / x^\Delta) * 180 / \pi$$

$$= \text{atan2}(y^\Delta, x^\Delta) * 180 / \pi$$

Function that handles special cases (e.g., $x^\Delta = 0$).



- Now compute amount of turn:

$$\theta^\Delta = (\theta_{i+1} - \theta_i) \,\%\, 360°$$

Modulus handles wraparound case of turning > 360°.

IF ($\theta^\Delta < -180°$) THEN $\theta^\Delta = \theta^\Delta + 360°$

ELSE IF ($\theta^\Delta > 180°$) THEN $\theta^\Delta = \theta^\Delta - 360°$

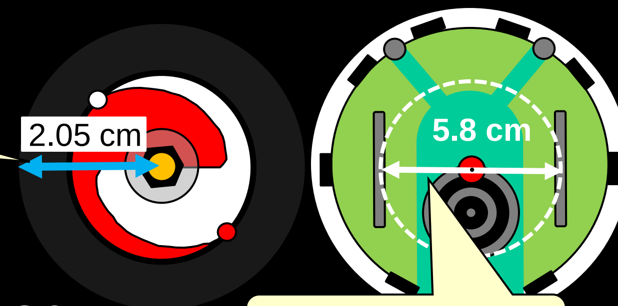
Do this to normalize so that all turning is within range of -180°and +180°.

E-Puck - Inverse Kinematics

- If ($\theta^{\Delta} > 0$) then right wheel should go forward and left backwards otherwise left should go forward and right backwards.

- Need to determine # encoder steps required to make the spin based on this formula (from before):

$$\theta^{\Delta} = (D_R - D_L) / L$$
$$= (R_{cm} * (rightReading - leftReading)) / L_{cm}$$



- Since rightReading = - leftReading when spinning:

$$\theta^{\Delta} = (R_{cm} *(2*rightReading)) / L_{cm} = (2R_{cm} / L_{cm}) * rightReading$$

And so…

$$rightReading = \theta^{\Delta} * (L_{cm} / R_{cm}) / 2$$

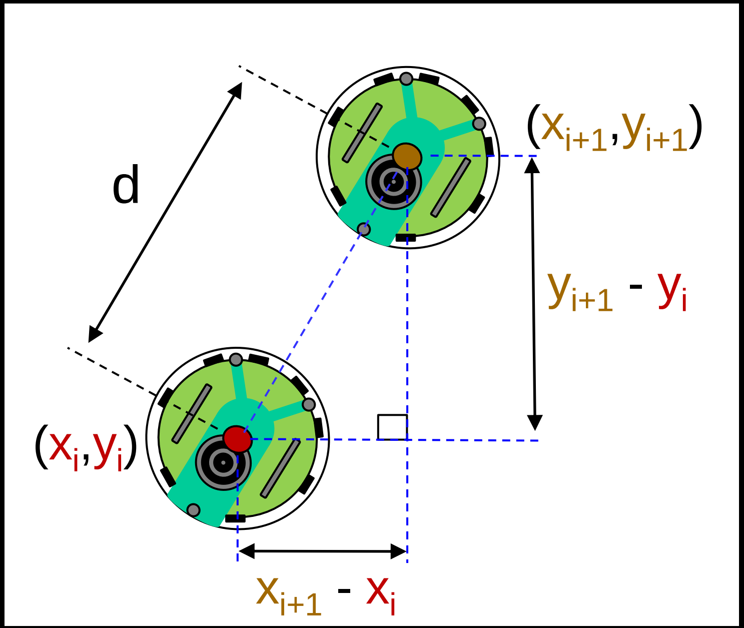
θ^{Δ} is in radians here !

- Equations work for any speed.

Inverse Kinematics - Forward

- How do we move forward from (x_i, y_i) to (x_{i+1}, y_{i+1}) ?
- Length of time to move depends on wheel speed and distance to be travelled:

$$d = \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2}$$



- Can express in terms of motor steps (where rightReading = leftReading)
- Solve for rightReading to move forward until:

Wheel circumference

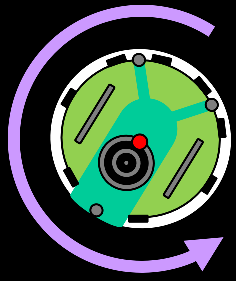
$$\text{rightReading} = (d / (2\pi * R_{cm})) * 2\pi$$

$$= d / R_{cm} = \left(\sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2}\right) / R_{cm}$$

radians of rotation

Summary

- To spin from (x_i, y_i) to (x_{i+1}, y_{i+1}), starting at angle θ_i
  - we determine the amount of turning θ^Δ to do:



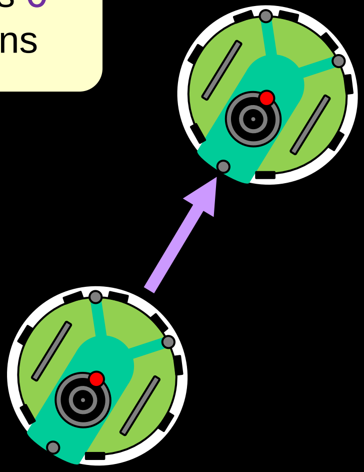
$$\theta^\Delta = (\theta_{i+1} - \theta_i) \ \% \ 360°$$

$$\theta_{i+1} = \underline{\textbf{atan2}}((y_{i+1} - y_i), (x_{i+1} - x_i)) * 180° / \pi$$

  - We then spin left if $\theta^\Delta > 0$ (or right $\theta^\Delta < 0$) and wait until the right wheel encoder has moved this much:

$$\text{rightReading} = \underbrace{\theta^\Delta * \pi/180°}_{\text{converts } \theta^\Delta \text{ to radians}} * (L_{cm} / R_{cm} / 2)$$

- To move forward from (x_i, y_i) to (x_{i+1}, y_{i+1})
  - we move straight until the right wheel encoder has moved this much:



$$\text{rightReading} = \left(\sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2}\right) / R_{cm}$$

Start the Lab …