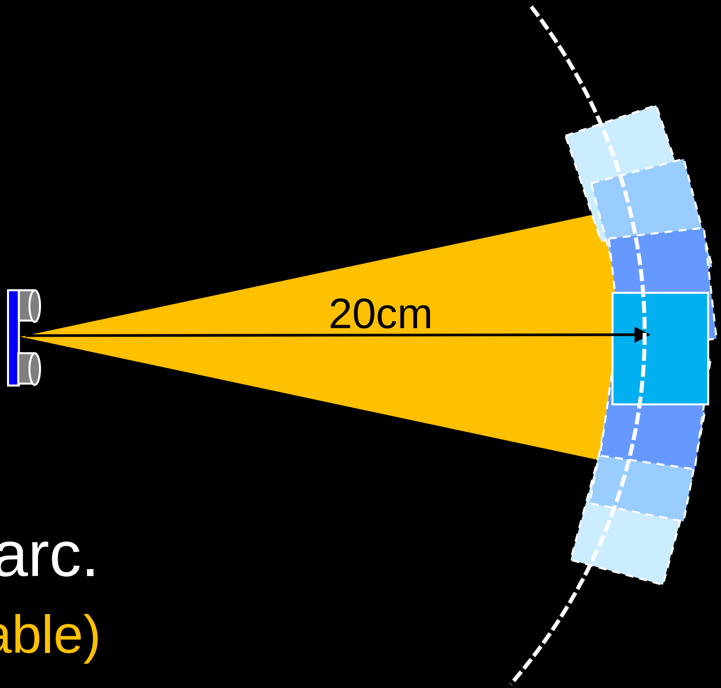
Improved Sensor Model Mapping

Error Distribution

- When object is detected at, say 20$_{cm}$, it can actually be anywhere within the beam arc defined by the 20$_{cm}$ radius.



- The likelihood (or probability) that the object is *centered* across the arc is greater than if the object was off to the side of the arc.
  (if location is considered to be a random variable)

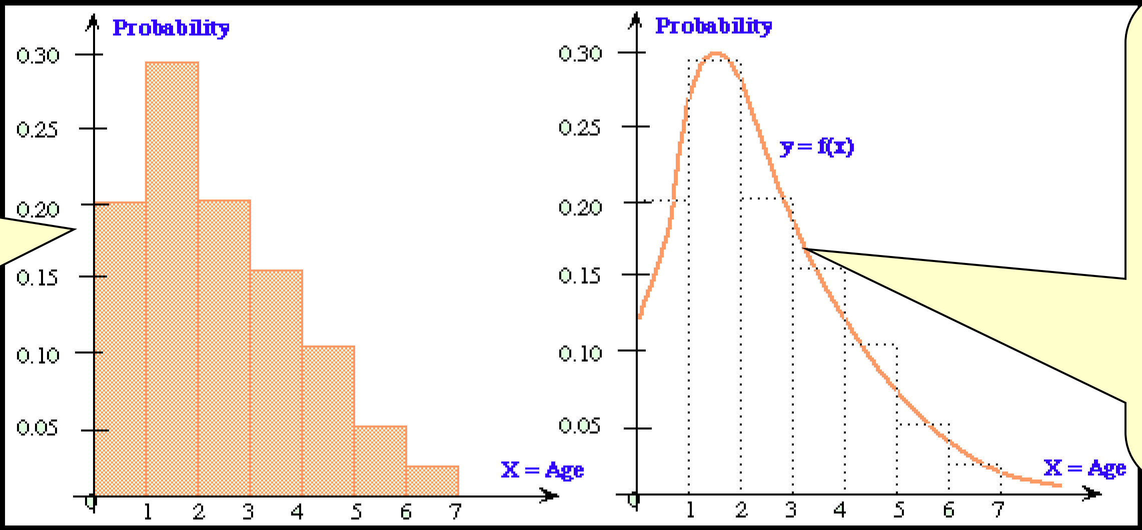
- We can thus express the sensor reading itself as a set of probabilities across the grid.

Probability Density Functions

- Random variables operating in continuous spaces are called *continuous random variables*.
- Assume that all continuous random variables posses a ***Probability Density Function*** (PDF).

E.g.,

Probability distribution of a car on the road being a certain age.



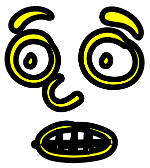
Probability Density Function for this distribution.

(Also known as the ***Probability Distribution Function***).

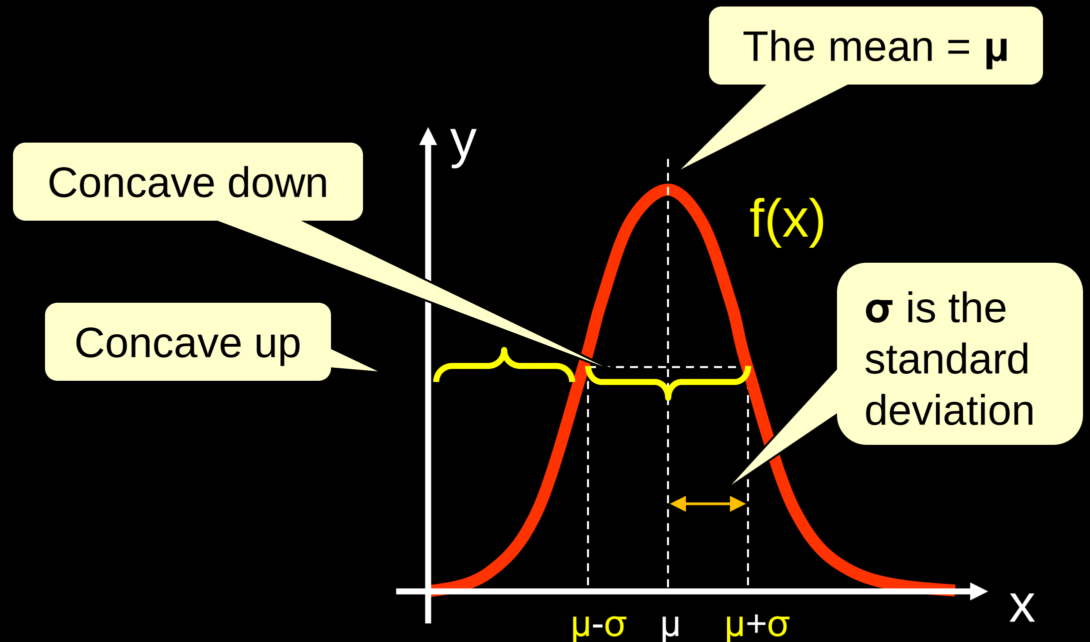
Probability Density Functions

- Common PDF is the ***normal distribution***:
  - given *mean* μ and *variance* σ^2 the normal distribution is …

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

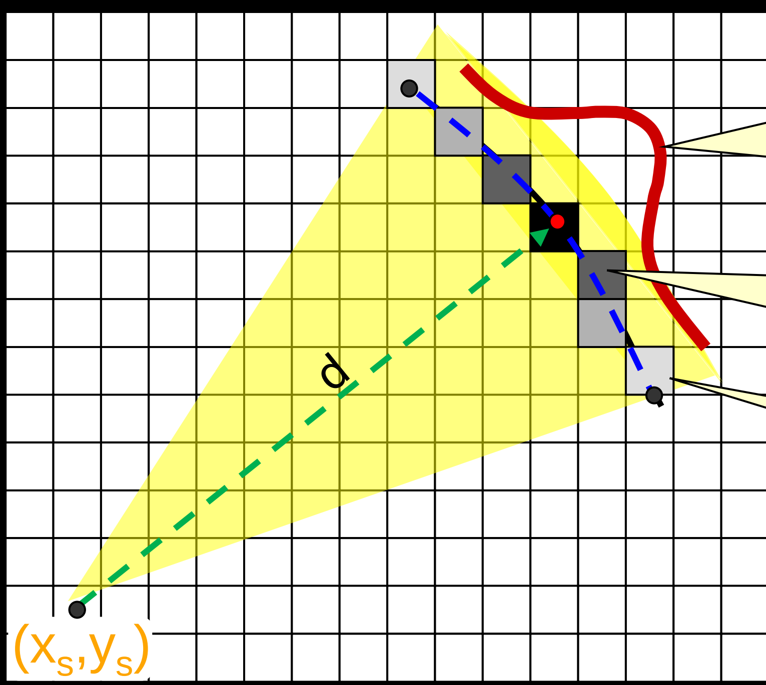


Don't worry, you do not need to understand this.



Gaussian Distribution

- A more realistic sensor model assigns probabilities to the cells according to some error distribution such as this ***Gaussian*** (or ***Normal***) distribution.



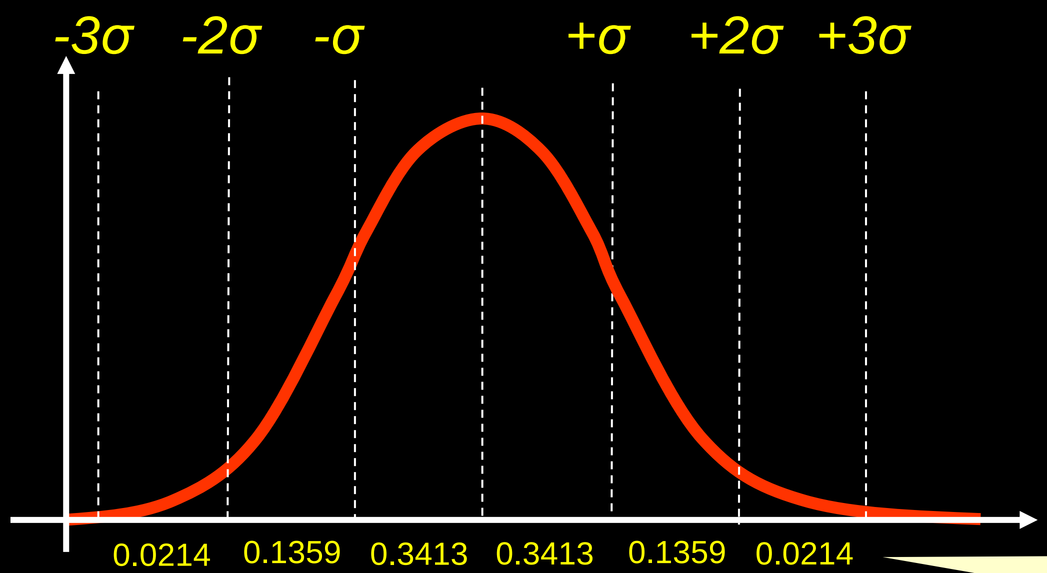
We will now apply this distribution across our arc to assign a "***probability that it is occupied***" value to each cell.

Darker means "**more** likely" that object was at this cell.

Lighter means "**less** likely" that object was at this cell.

Gaussian Distribution

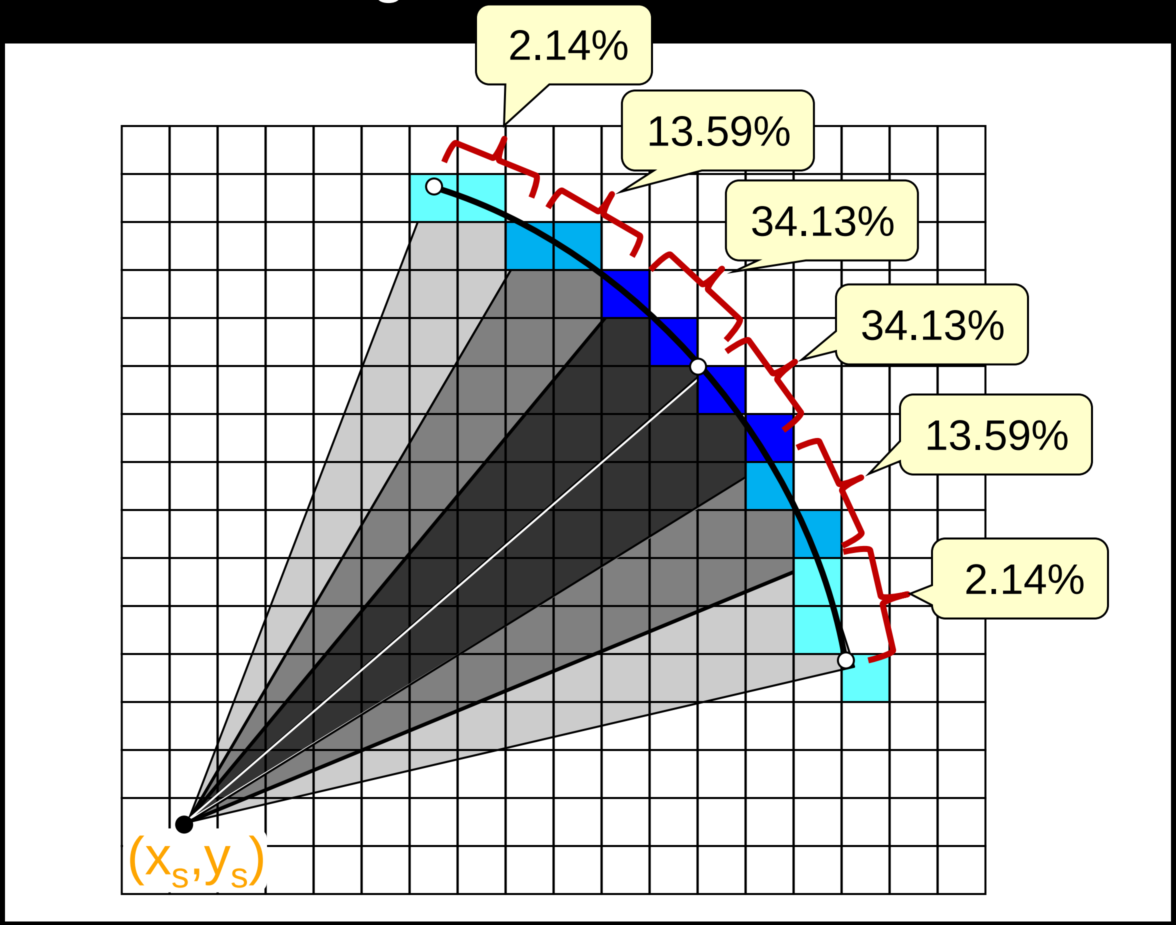
- How do we implement this on our occupancy grid ?
- Often the probabilities are <u>approximated</u> using what is known as the *six-sigma* rule. Which essentially divides the probabilities into 6 probability regions.



Total of probabilities add to 0.9972 ≈ 1.0

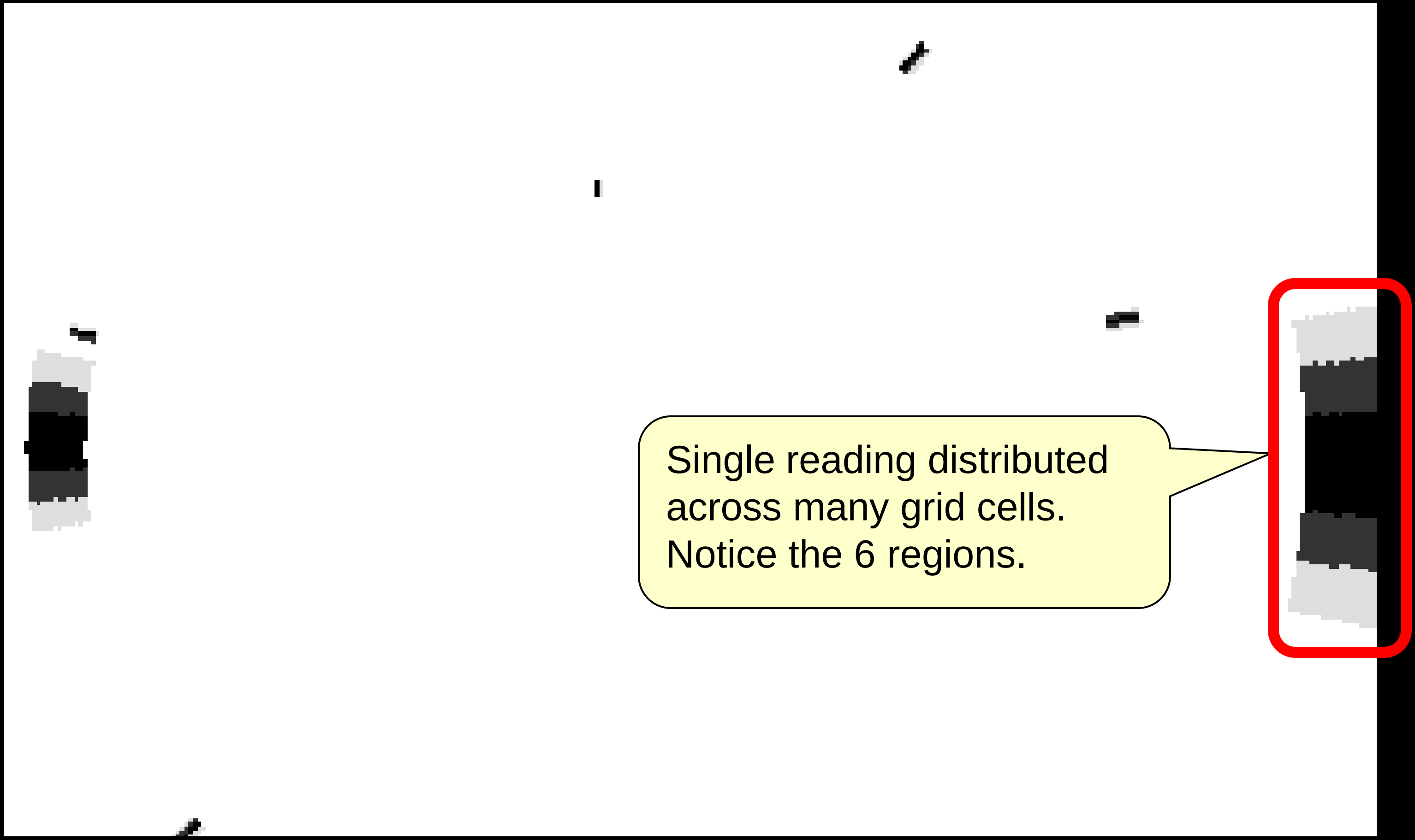
Applying Gaussian Distribution

- Divide arc into 6 “wedges” and apply the specific probabilities to the cells in each wedge.



Applying Gaussian Distribution

- Here is the result of applying the Gaussian distribution across the angle:

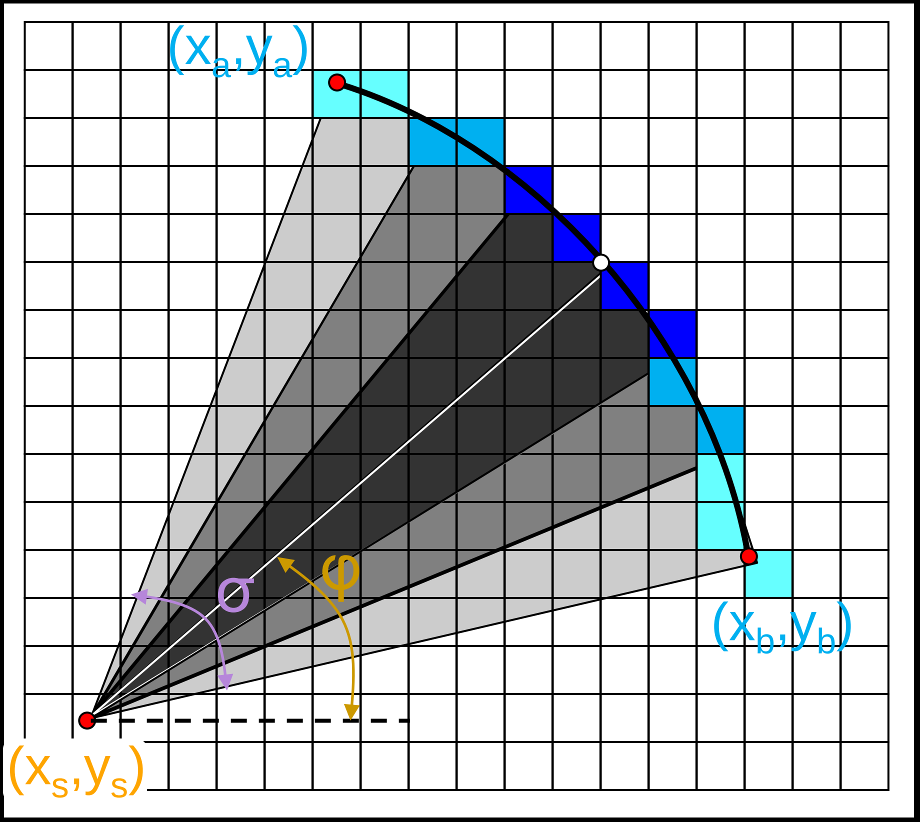


The code

- Recall the code for filling in grid cells along the arc:

$$\omega = \sigma / \sqrt{(x_a - x_b)^2 + (y_a - y_b)^2}$$

```
FOR a = -σ/2 TO σ/2 BY ω DO {
  objX = xs + (d * cos(φ + a))
  objY = ys + (d * sin(φ + a))
  grid[objX][obyY] = 1
}
```



- We need to set the probability now instead of setting to 1.

- Can grab the probability from a hard-coded array:

```
static final float[] SIGMA_PROB =
    {0.0214f, 0.1359f, 0.3413f, 0.3413f, 0.1359f, 0.0214f};
```

The Code

- Just need to find the index i to look up into array:

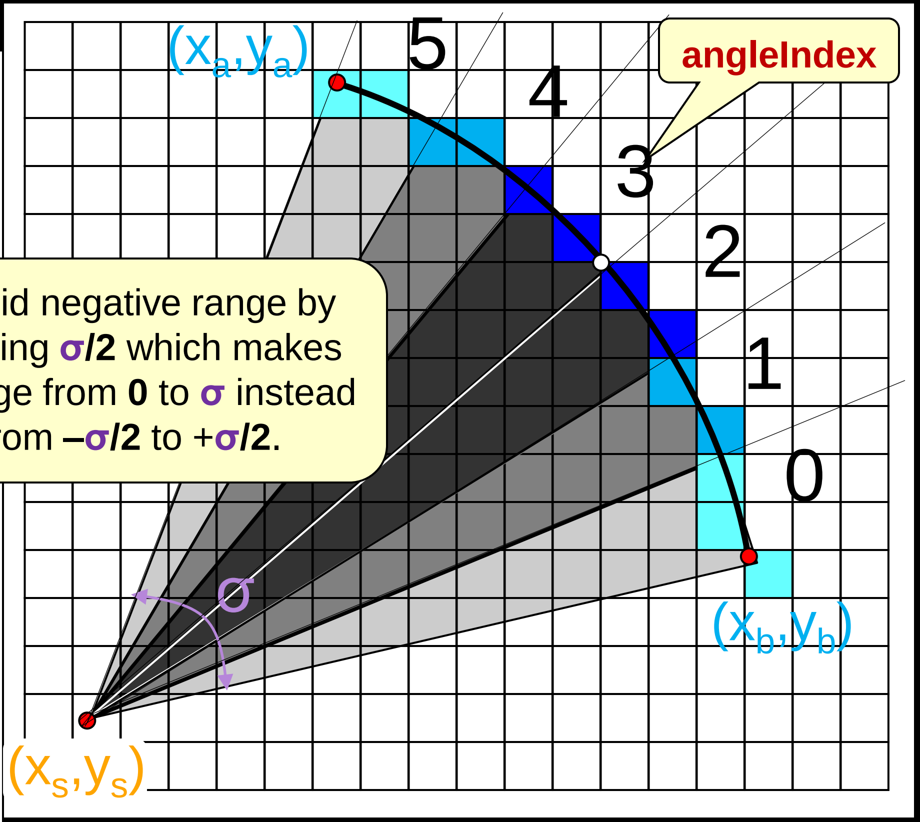
```
SIGMA_PROB = {0.0214, 0.1359, 0.3413,
              0.3413, 0.1359, 0.0214}
```

$\omega = \sigma / \sqrt{(x_a - x_b)^2 + (y_a - y_b)^2}$

```
FOR a = -σ/2 TO σ/2 BY ω DO {
    objX = xs + (d * cos(φ + a))
    objY = ys + (d * sin(φ + a))

    percentArc = (a + σ/2)/σ

    angleIndex =(int)(percentArc*5.99)

    grid[objX][obyY] =
            SIGMA_PROB[angleIndex]
  }
```

This is the amount of processing so far that we reached during the FOR loop (i.e., 0% to 100%)

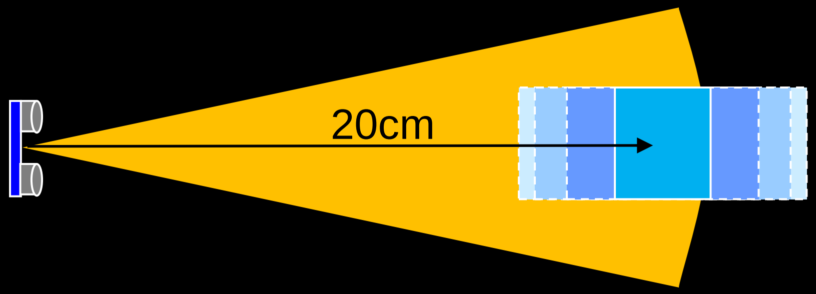
Avoid negative range by adding **σ/2** which makes range from **0** to **σ** instead of from **–σ/2** to +**σ/2**.

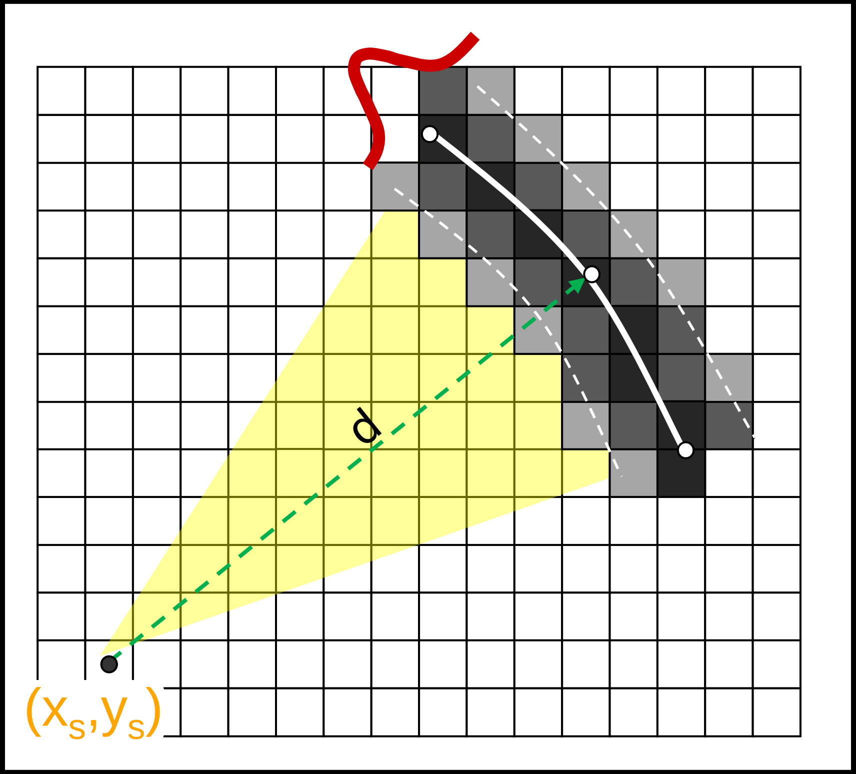


Multiplying by 5.99 and then truncating to an integer, will ensure indices in the 0 to 5 range).

Applying Gaussian Distribution

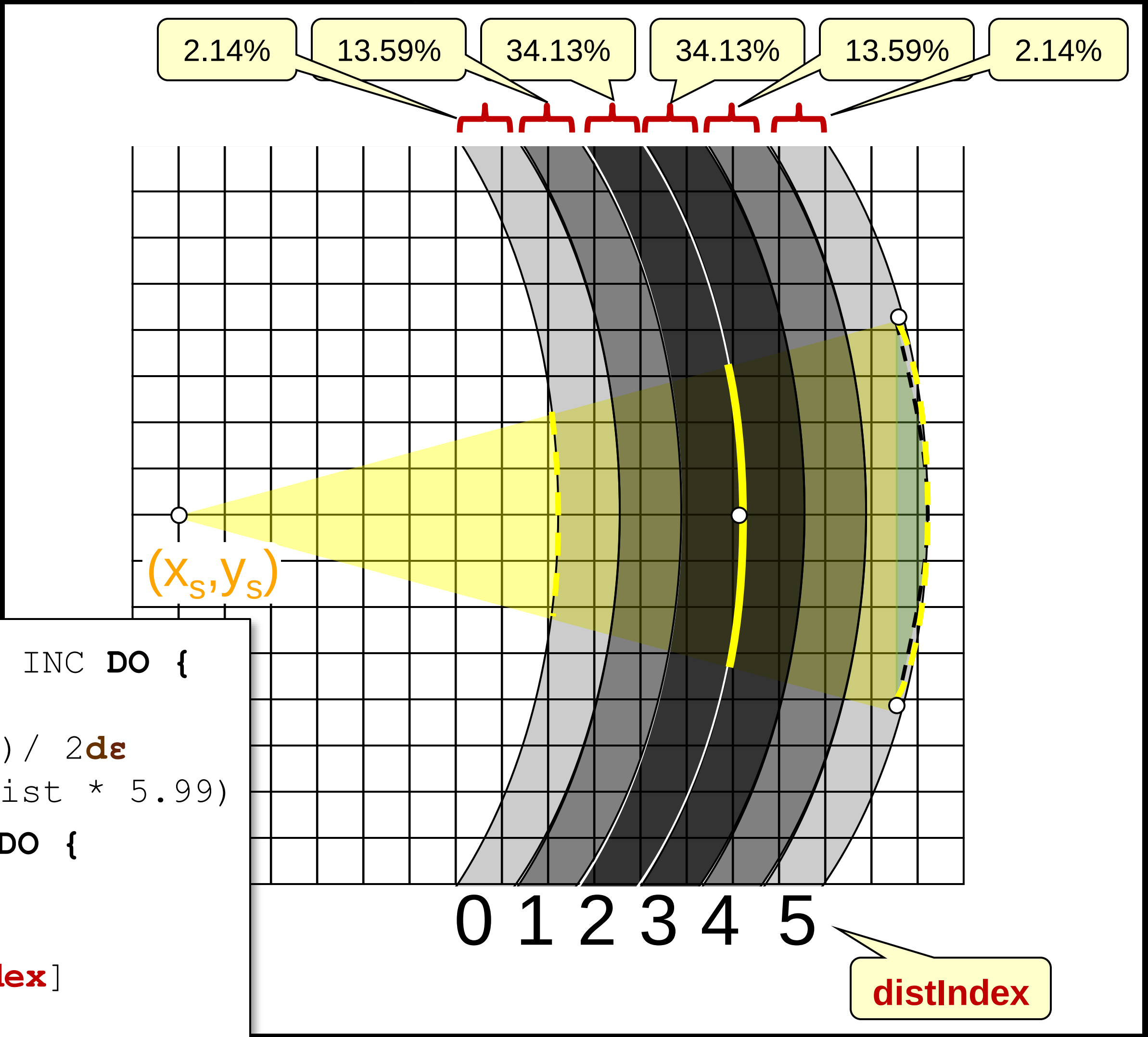
- Should also apply the distribution to distance since object is more likely at the distance range measured than closer or further.





Applying Gaussian Distribution

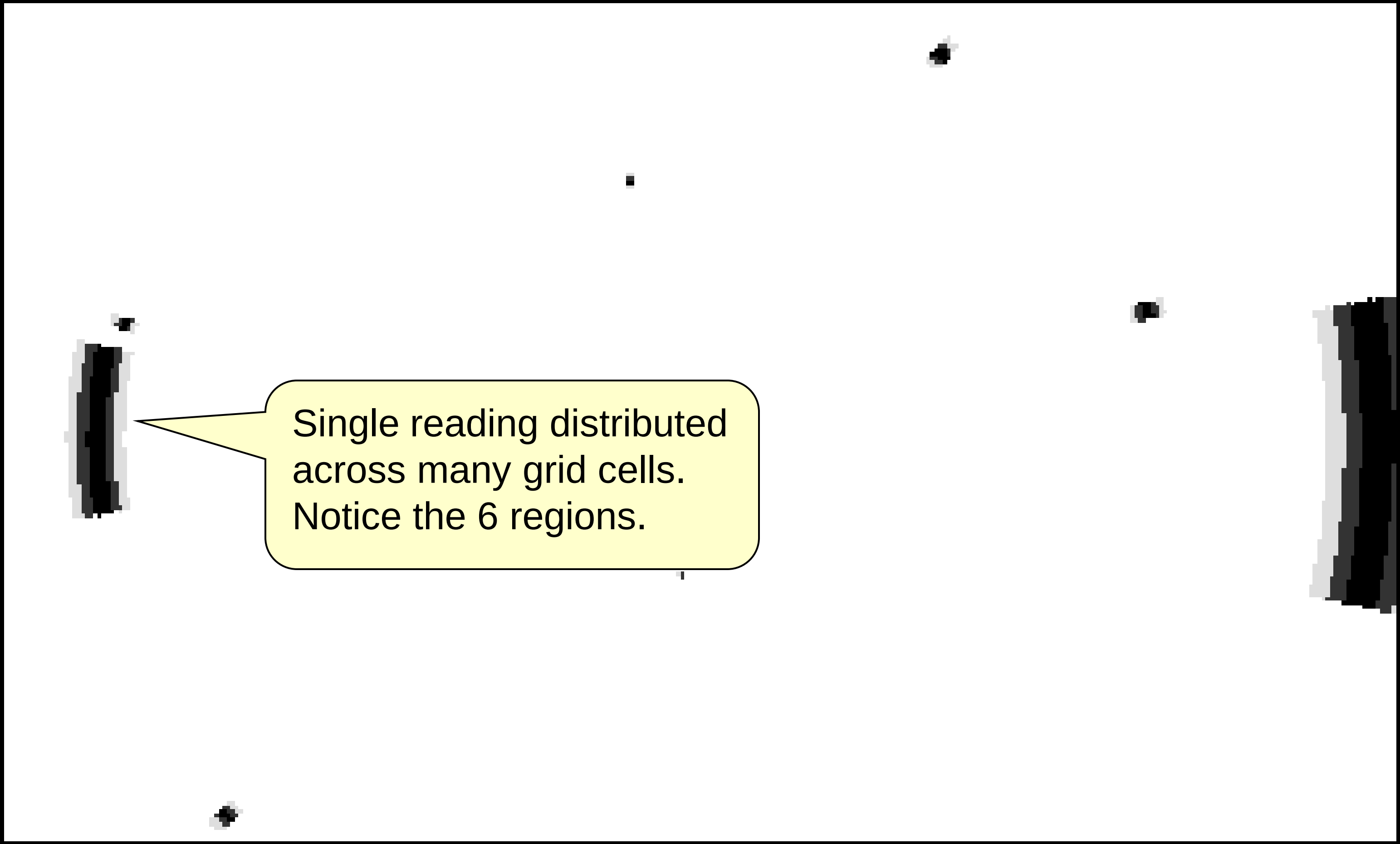
- Divide distance range into 6 “bands” and apply the specific probabilities to the cells in each band.




```
FOR r = d*(1-ε) TO d*(1+ε) BY INC DO {
   ...
   percentDist = (r - d*(1-ε))/ 2dε
   distIndex = (int)(percentDist * 5.99)
   FOR a = -σ/2 TO σ/2 BY ω DO {
      ...
      grid[objX][obyY] =
            SIGMA_PROB[distIndex]
   }
}
```

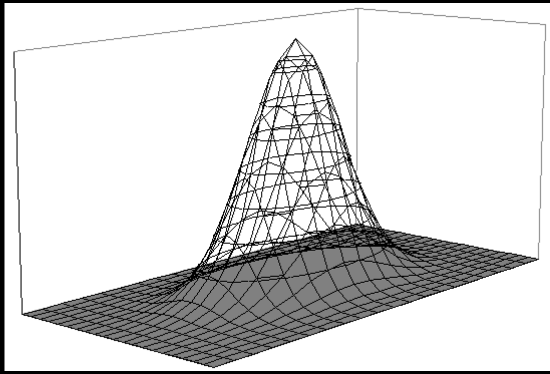
Applying Gaussian Distribution

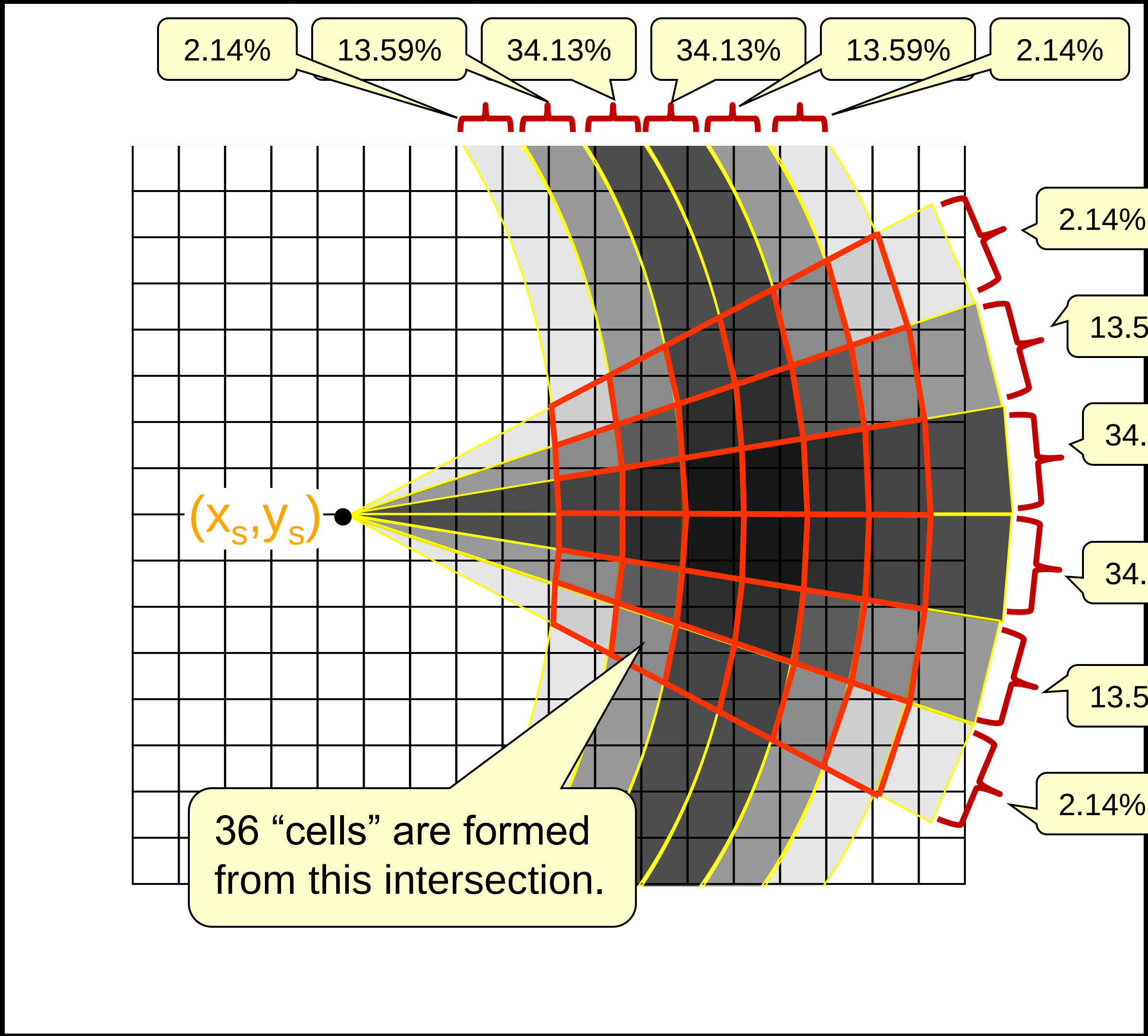
- Here is the result of applying the Gaussian distribution to the distance:



Applying Gaussian Distribution

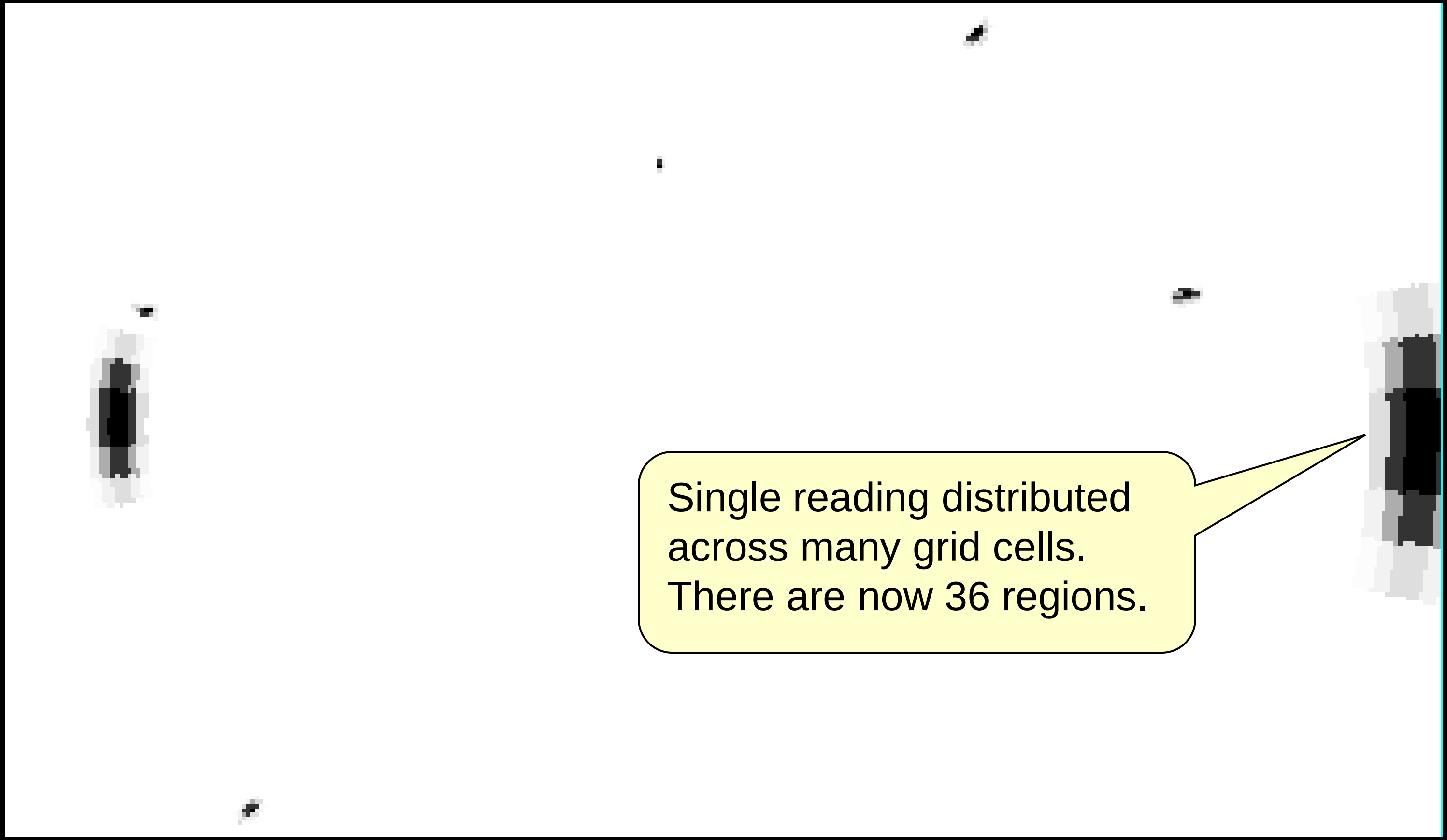
- Finally, apply probabilities along both angle as well as distance:





Applying Gaussian Distribution

- Here is the result of applying the Gaussian distribution to both angle and distance.



Applying Gaussian Distribution

- Here are the probabilities that are to be assigned to each of the 36 cells:

0.05%	0.29%	0.73%	0.73%	0.29%	0.05%
0.29%	1.85%	4.64%	4.64%	1.85%	0.29%
0.73%	4.64%	11.65%	11.65%	4.64%	0.73%
0.73%	4.64%	11.65%	11.65%	4.64%	0.73%
0.29%	1.85%	4.64%	4.64%	1.85%	0.29%
0.05%	0.29%	0.73%	0.73%	0.29%	0.05%

- Can just store the probabilities in a 1D array:

```
static final float[] SIGMA_PROB =
    {0.0214f, 0.1359f, 0.3413f, 0.3413f, 0.1359f, 0.0214f};
```

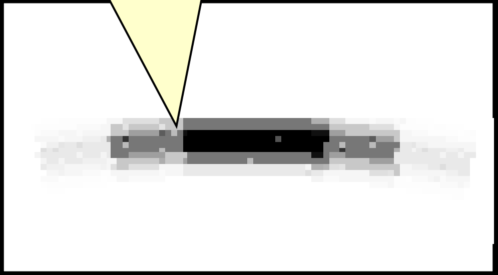
- Then combine both directions through multiplication:

```
probability = SIGMA_PROB[angIndex] * SIGMA_PROB[distIndex];
```

Ensuring Consistency

- The technique just shown, if not careful, does not properly assign probabilities across the wedge for a single sensor reading.

Due to round-off inaccuracies, there will likely be some grid cells counted twice and some not counted during a single update. This may lead to a **speckled** pattern.

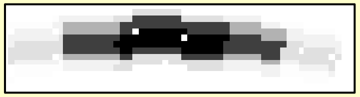


This can also occur if the increment on the **FOR** loop for the distance is not small enough. It should be smaller than the grid's precision to ensure that no grid cells are missed:

```
for (double r=0; r<limit; r+=INC) {
     ....
}
```

e.g., **INC** = **1**

INC = **0.75**



INC = **0.5**

INC = **0.25**

0.25 avoids speckled pattern.

Ensuring Consistency

- To avoid speckled pattern, create a temporary grid

1. Create to be same size as entire grid

```
temp = new float[width][height];
```

2. Initialize all values to 0

```
temp[i][j] = 0;
```

3. Apply all readings to the temporary grid by *setting* the cell values (i.e., not adding them)

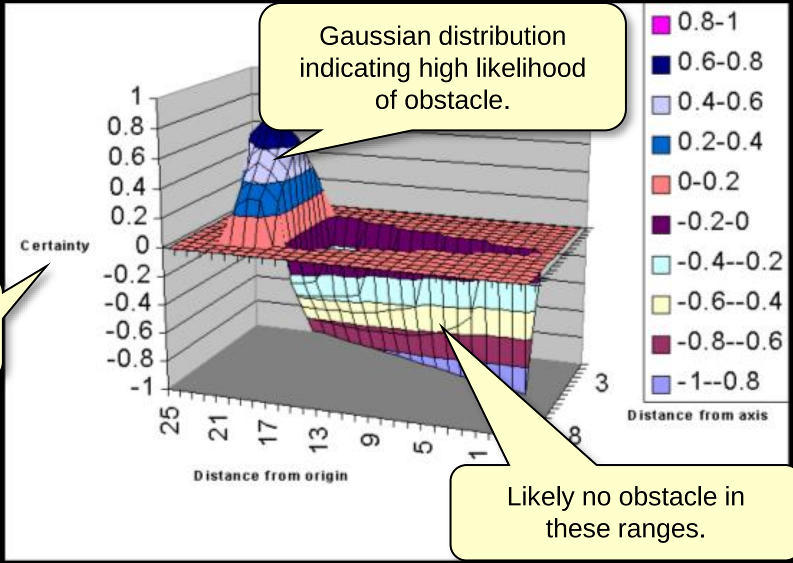
```
temp[i][j] = SIGMA_PROB[angIndex] * SIGMA_PROB[distIndex];
```

4. Merge temporary grid with complete map once reading probabilities have been completed

```
grid[i][j] += temp[i][j]
```

Non-Obstacle Certainty

- Another way to refine the grid is to say something about the certainty that an obstacle is NOT there.



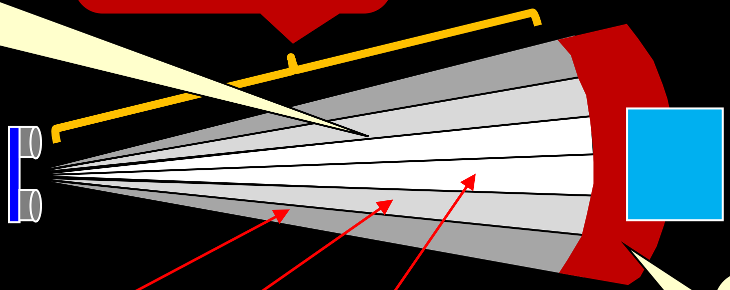
Gaussian distribution indicating high likelihood of obstacle.

Our scale goes from 0 to 1, not -1 to 1 as shown here.

Likely no obstacle in these ranges.

Obstacle CANNOT lie in here otherwise distance reading would have been smaller.

We won't apply any distance distribution.



We can **decrease** occupancy grid values here according to the angular distribution.

Obstacle lies in here somewhere according to a Gaussian distribution.

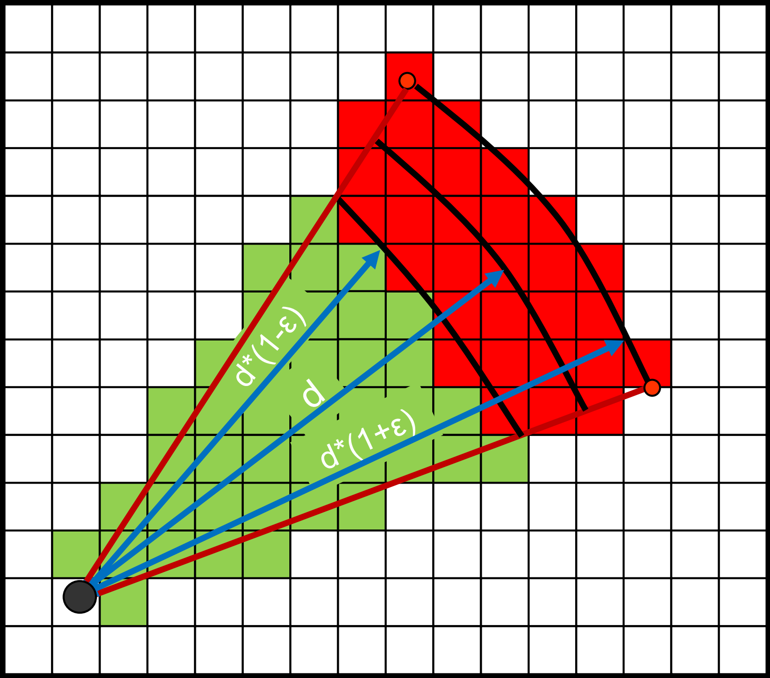
```
{0.0214f, 0.1359f, 0.3413f, 0.3413f, 0.1359f, 0.0214f};
```

Non-Obstacle Certainty Range

- Currently, the current FOR loop code only updates radius values that are between d*(1-ε) and d*(1+ε) … red cells.
- But now we need to update cells with radius values from 0 up to d*(1- ε) as well … green cells:

Start at **0** now instead of **d***(1−**ε**)

```
FOR r = 0 TO d*(1+ε) BY INC DO {
    ...
    FOR a = -σ/2 TO σ/2 BY ω DO {
        ...
        IF (r < d*(1-ε)) THEN
            lighten the cell using angular
            distribution only (not distance)
        ELSE
            darken the cell as before using
            both angular and distance
    }
}
```



Start the Lab …