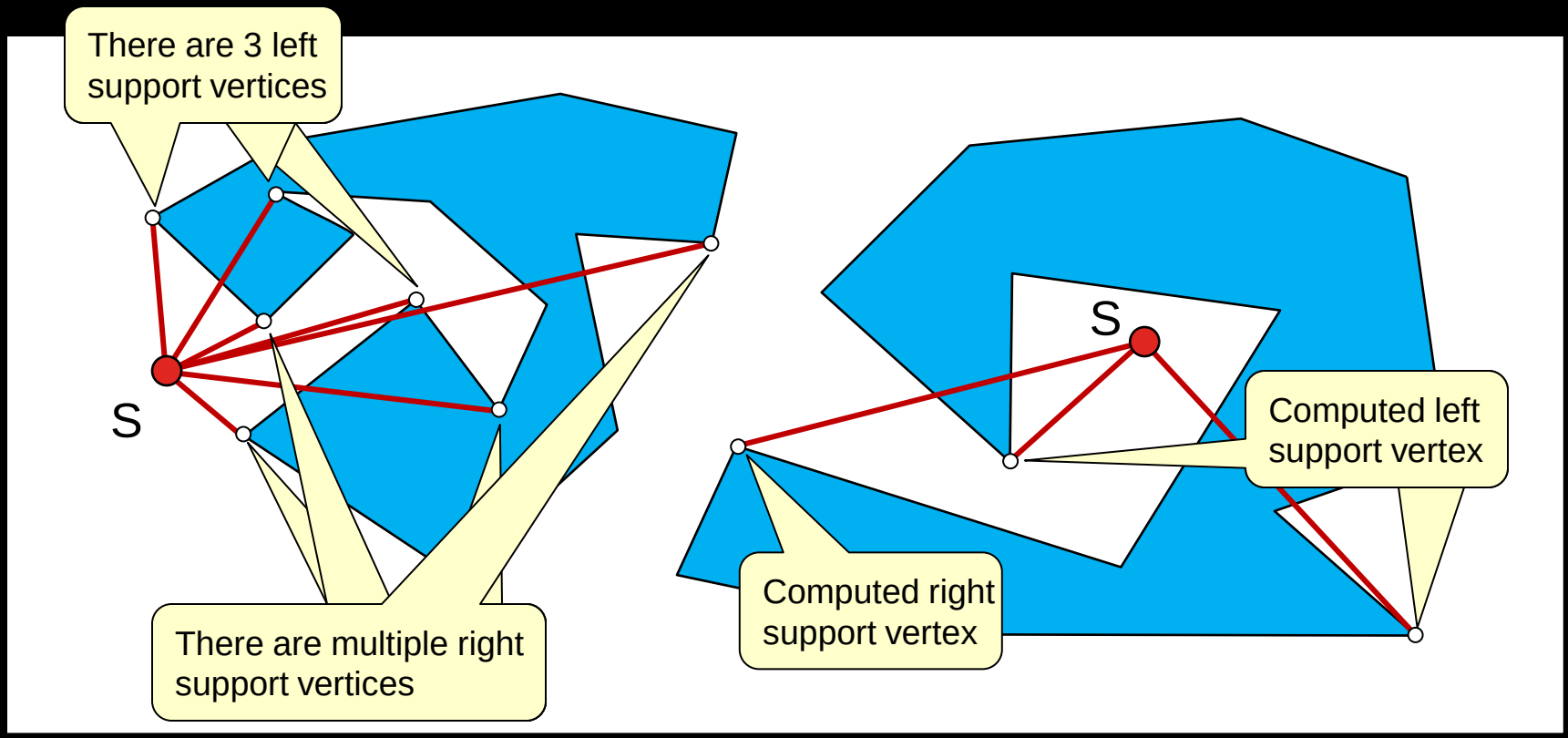
Path Planning
(Non-Convex Obstacles)

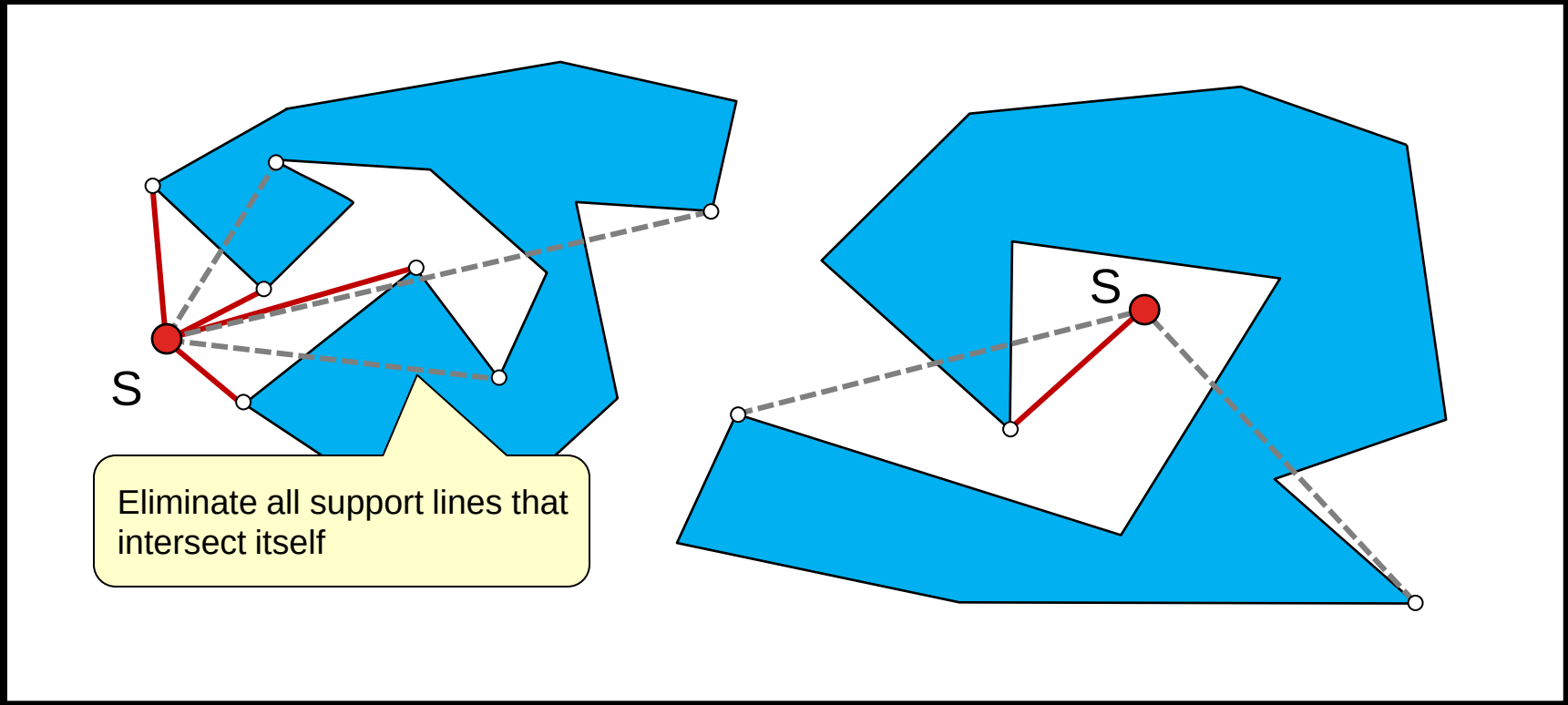
Non–Convex Obstacle Supports

- Our support-line algorithm can produce multiple support lines per polygon if it is non-convex:



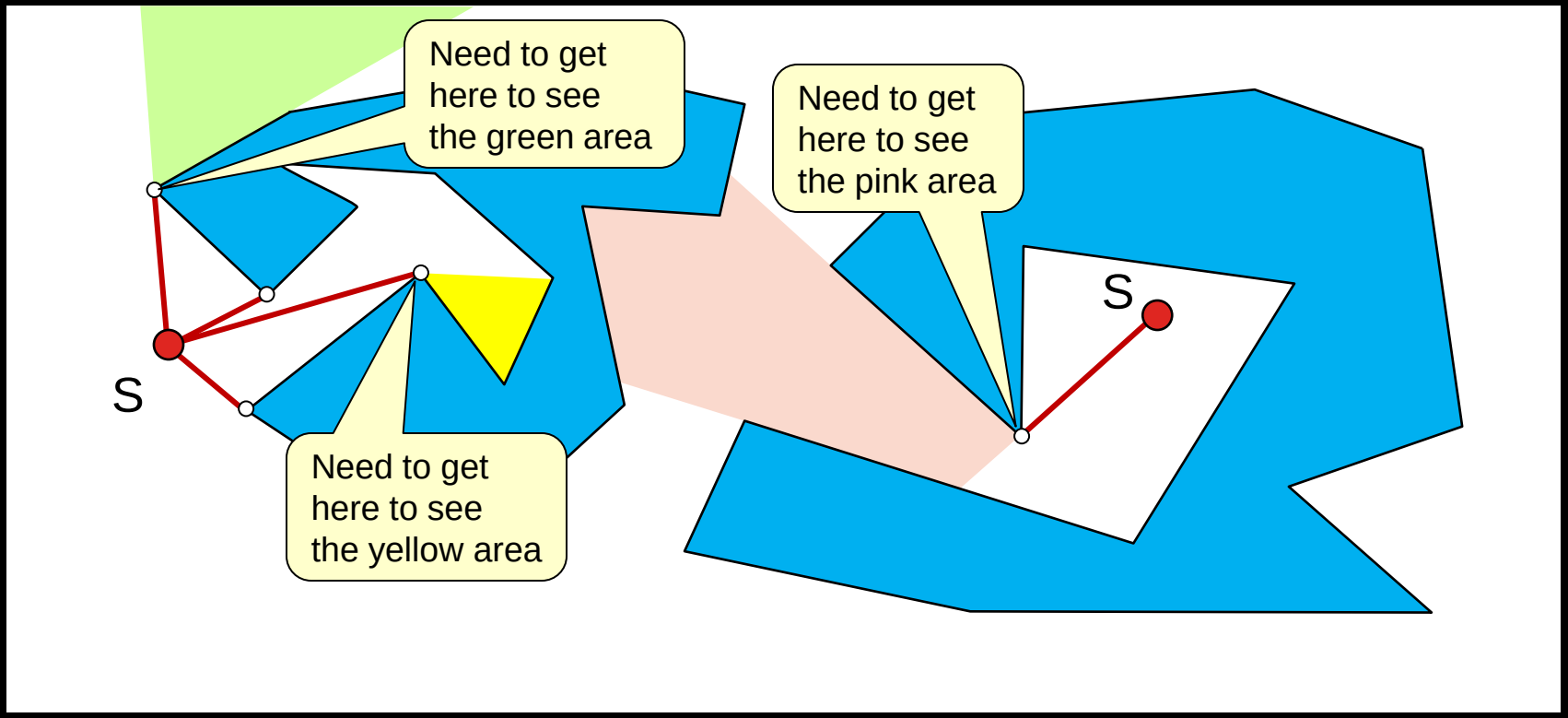
Non–Convex Obstacle Supports

- We can eliminate those that intersect itself as well as those that intersect other polygons.



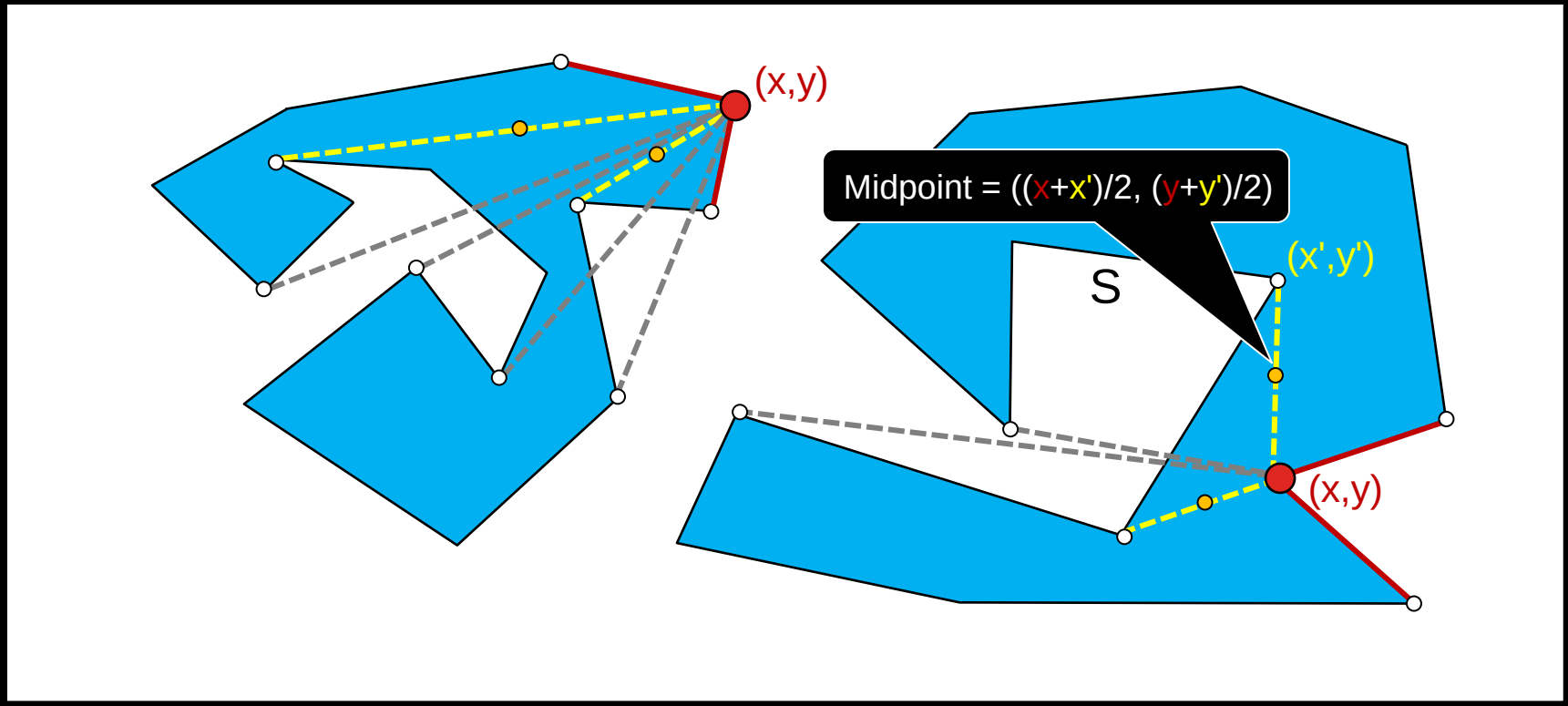
Non–Convex Visibility

- A support vertex represents an “observation point” that robot must be at in order to “see” (i.e., have visibility) around a corner. They are all necessary.



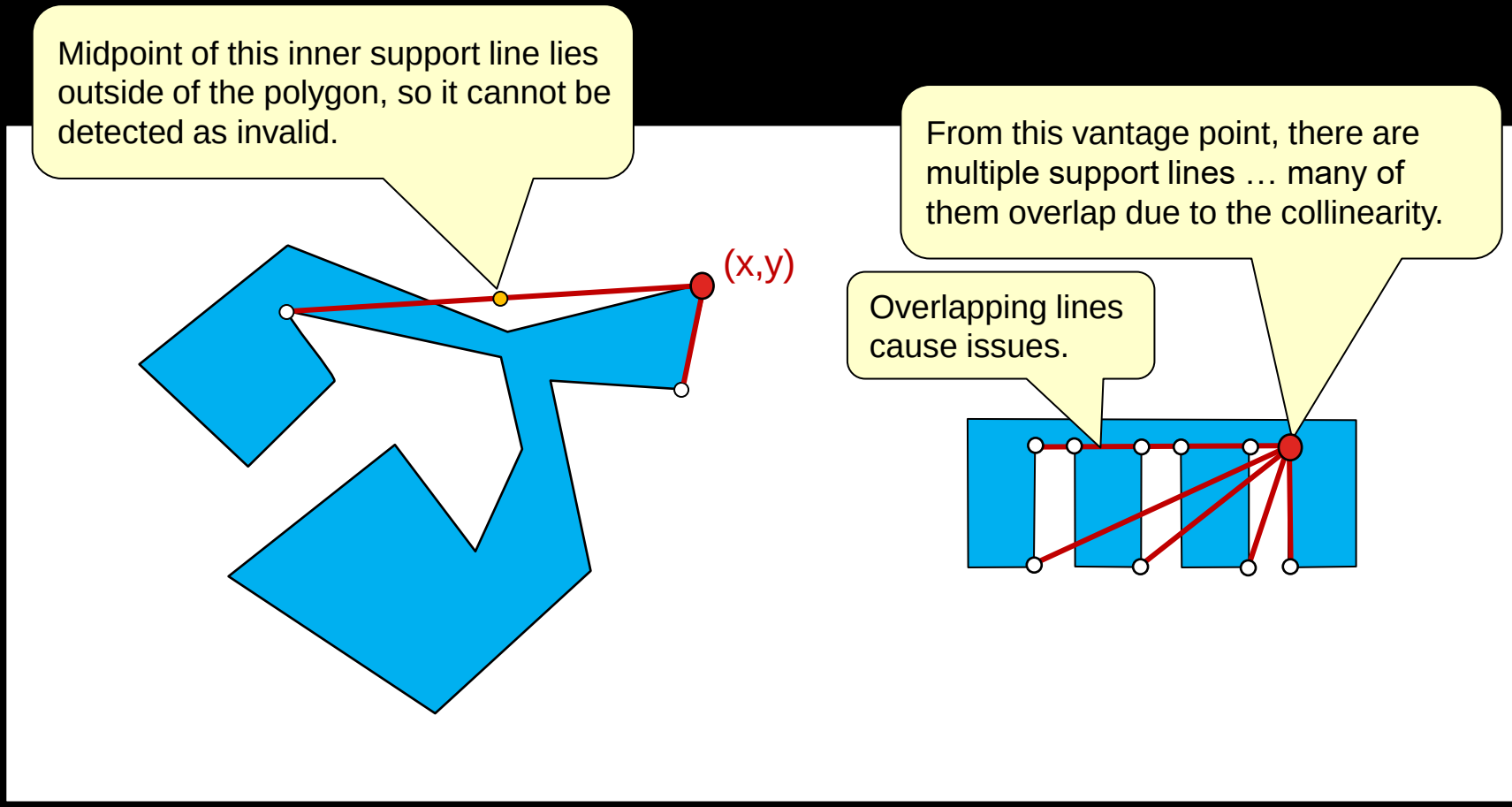
Non–Convex Problems

- Some “inner” support lines intersect edges only at vertices (see yellow below) and these are invalid.
- Perhaps we could check if midpoint of line lies within obstacle



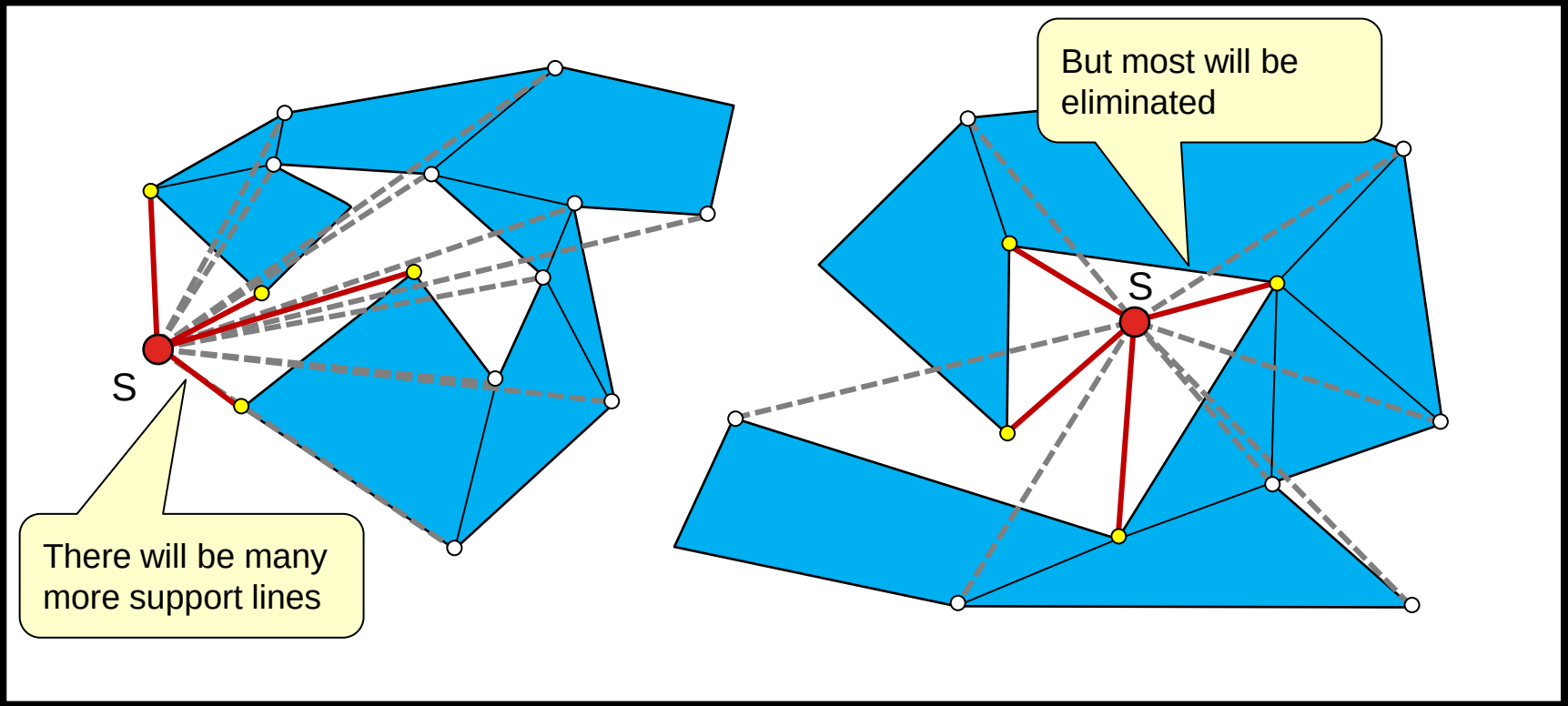
Non–Convex Problems

- But there are cases where this will not work:



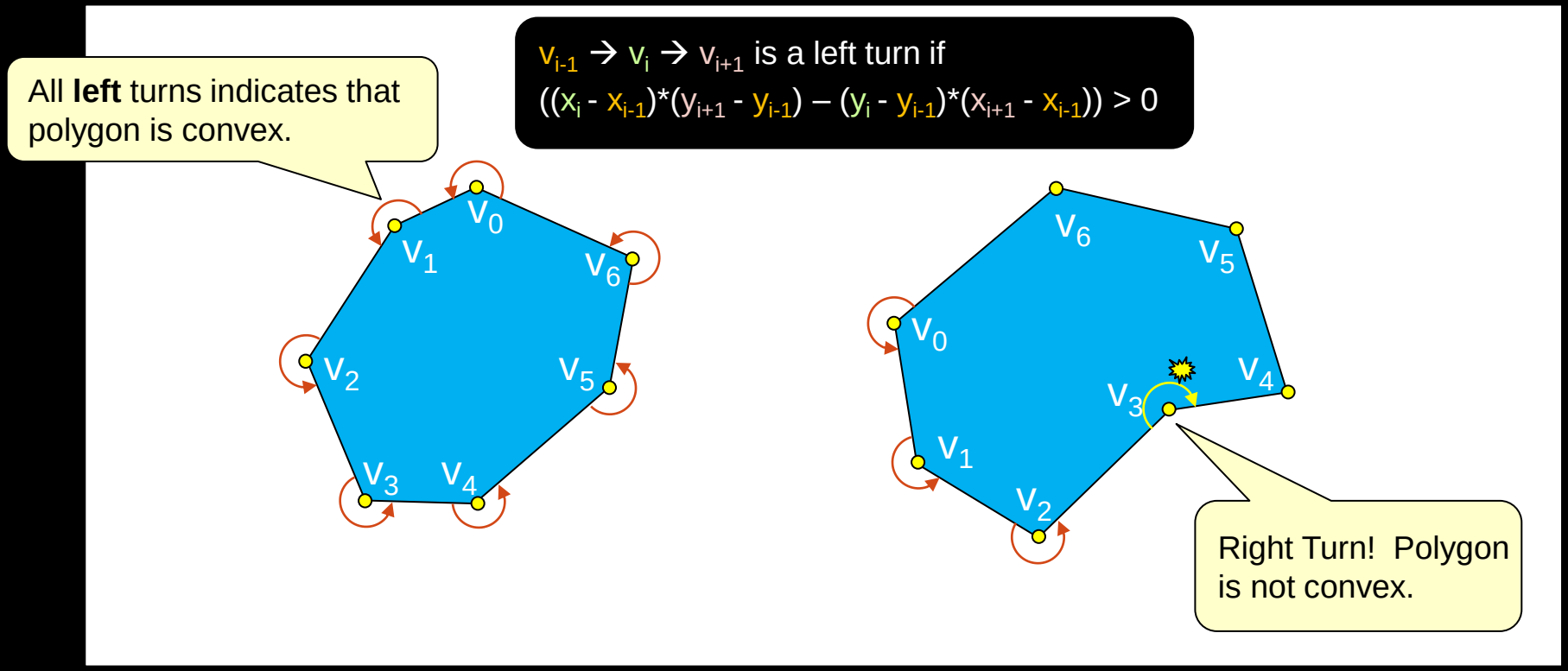
Non–Convex Easy Solution

- Easiest solution is to break up the non-convex polygons into convex pieces. Then the algorithm from before should work.



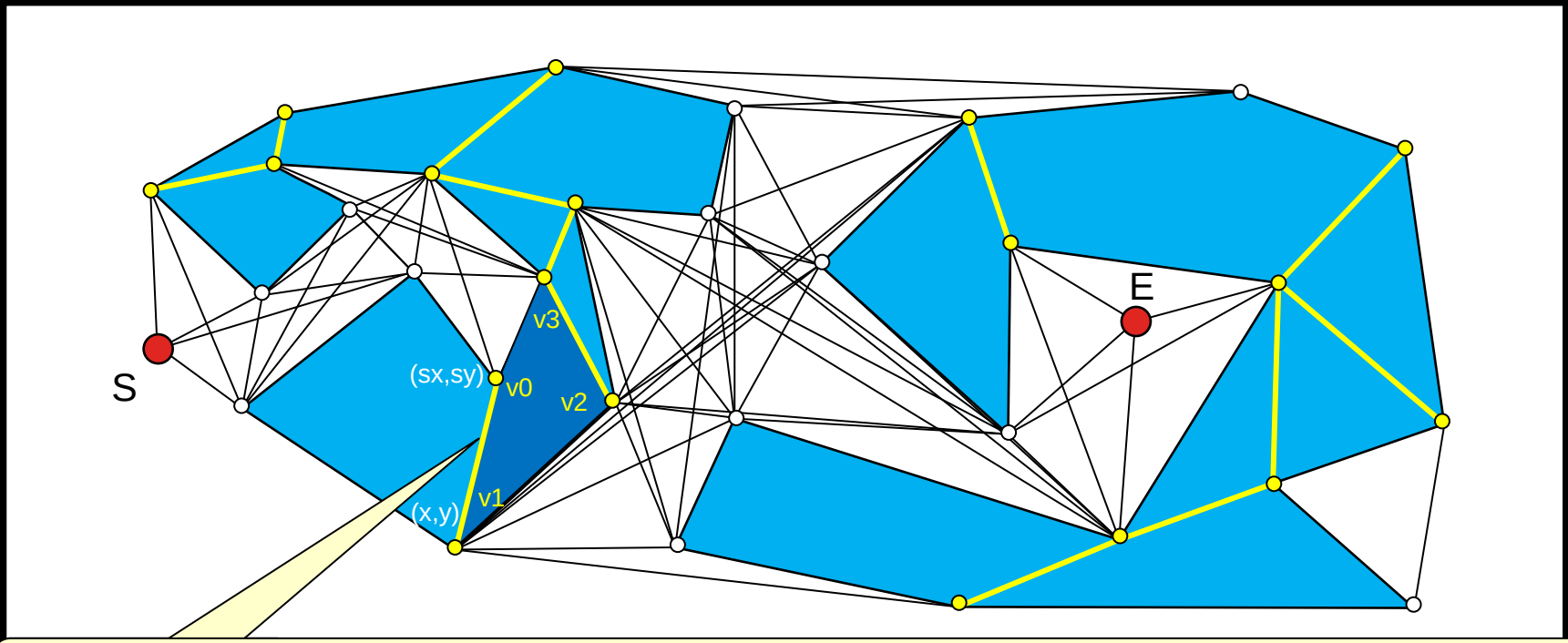
Convexity Checking

- How do we know if a polygon is convex or not ?
  - traverse vertices CCW … all must make **left** turns:



Non–Convex Easy Solution

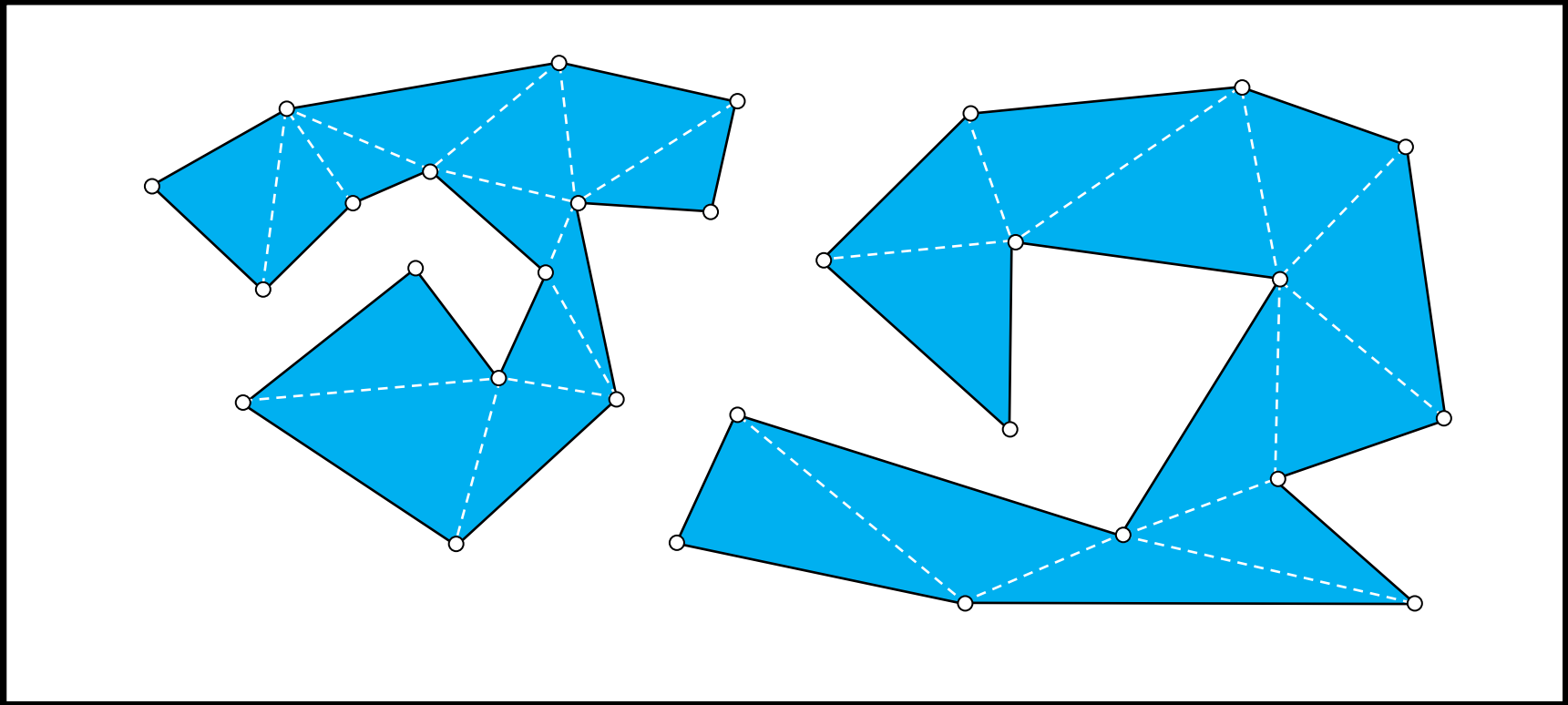
- There will be special cases for shared edges between convex pieces that need to be eliminated when computing the visibility graph:



Must make sure that none of these (yellow) *interior* bordering support lines are in the graph. Just need to check if support line (x,y)→(sx,sy) is the same as edge(v0,v1) … or (v1,v0)… of a "different" obstacle.

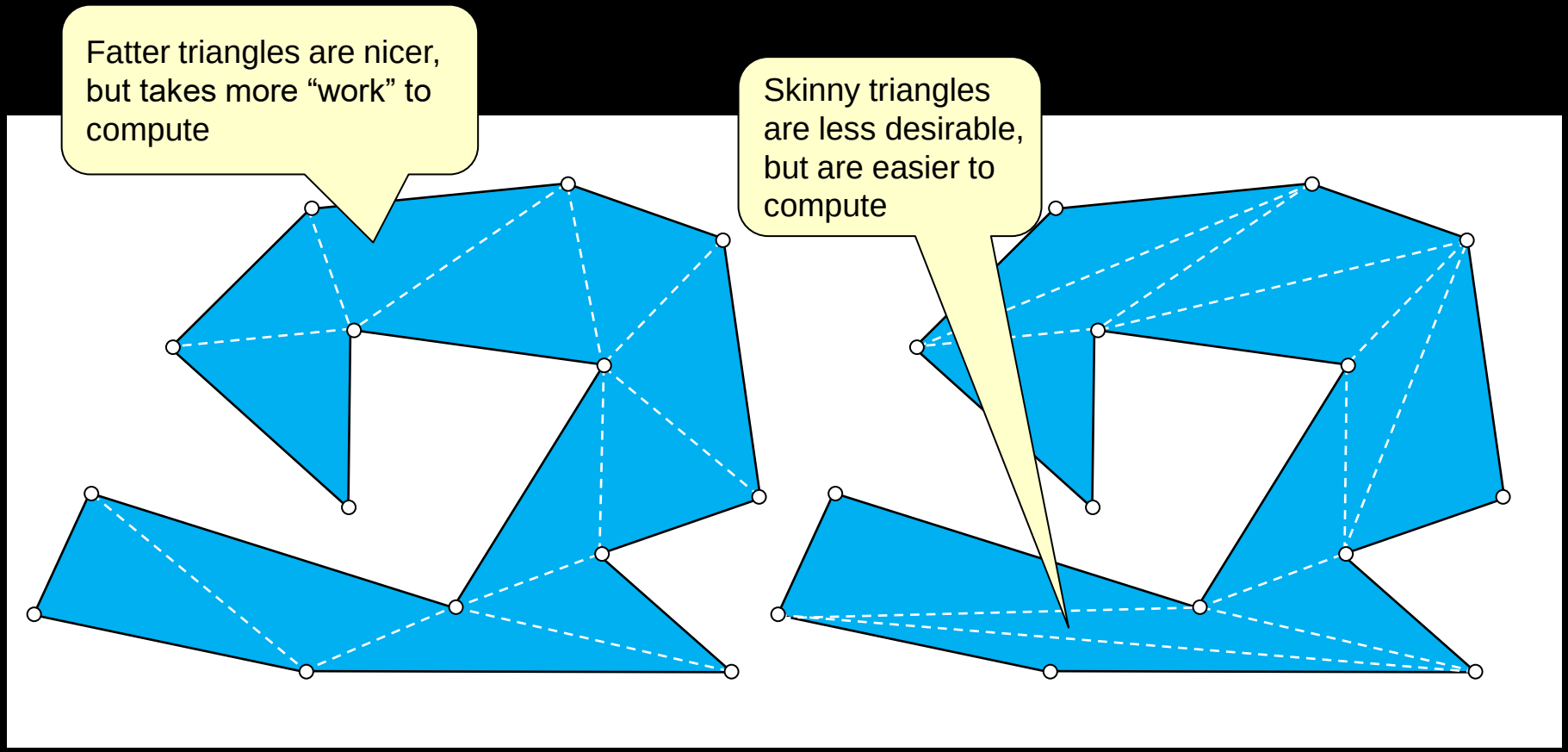
Convert Non–Convex To Convex

- How do we break a polygon into convex pieces ?
  - There are many algorithms. The most popular is to break into triangles. This is known as *triangulation*.



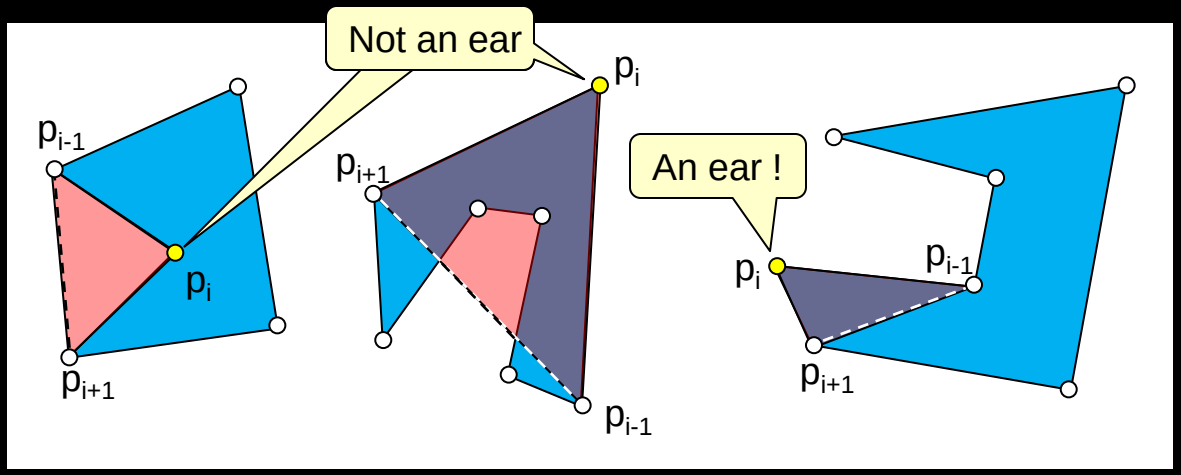
Triangulation

- How do we triangulate a polygon ?
  - There are many ways/algorithms to do this as well.



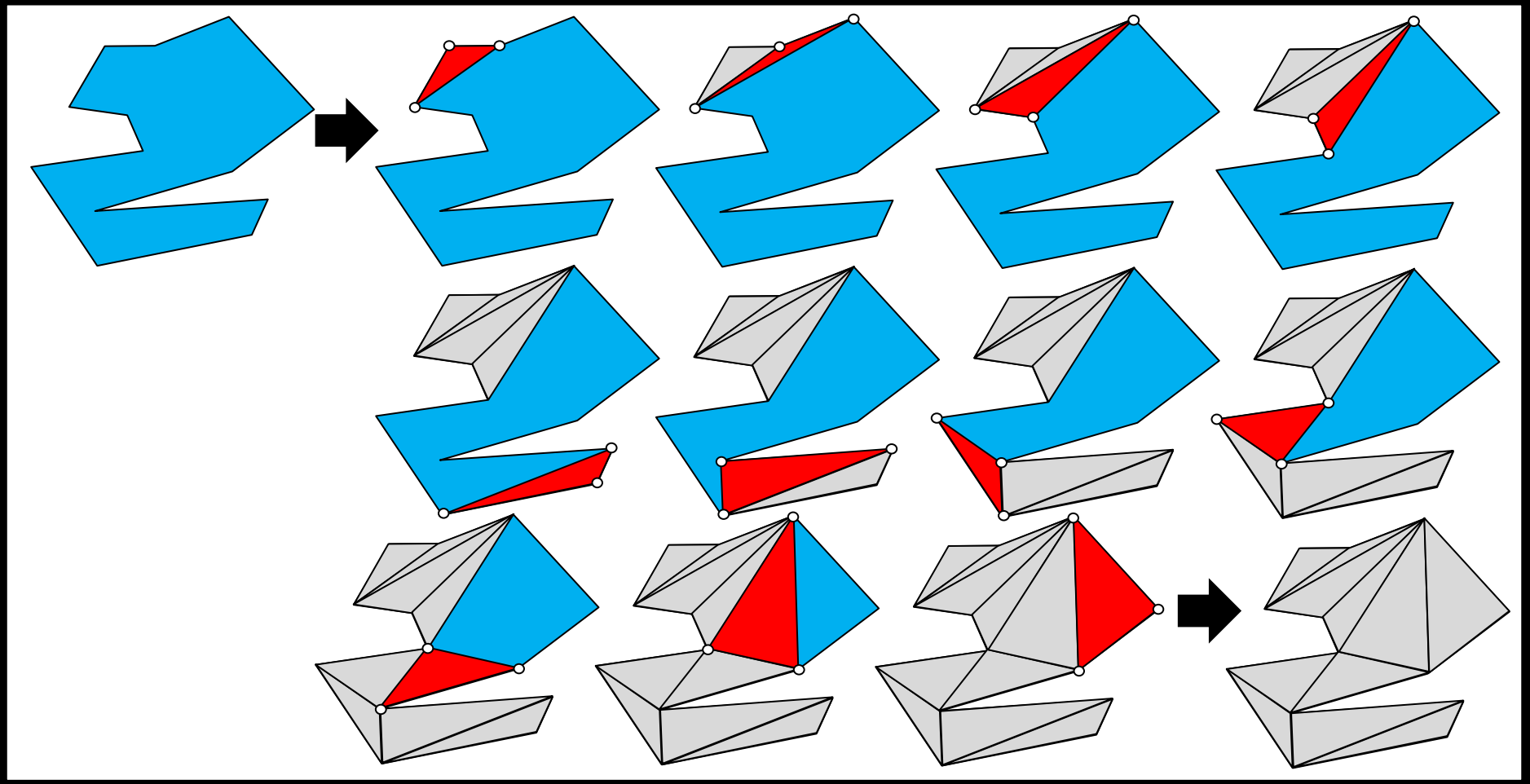
Ear-Cutting Triangulation

- We will use an “ear-cutting” algorithm since it is easy.
  - Idea is to repeatedly cut off “ears” of the polygon, making a set of triangles along the way.
  - An “ear” is a vertex p_i such that Line segment $\overline{p_{i-1}p_{i+1}}$ …
    1. intersects the polygon boundary only at points p_{i-1} and p_{i+1} AND…
    2. it lies entirely inside the polygon.



Ear-Cutting Triangulation

- Repeatedly cut off “ears” of the polygon, making a set of triangles along the way.



Ear-Cutting Algorithm

```
1  function EarCut(Obstacle P)
2       triangles = an empty list
3       if (P has only 3 vertices) then
4            Add P to triangles and return triangles
5       Copy all vertices of P into a list of points Q
6       while (Q has more than 3 points) do
7            earIndex = -1
8            for each point p_i in Q do
9                 if (p_{i-1}p_ip_{i+1} is a left turn) then
10                     ear = a new obstacle with vertices p_{i-1}p_i and p_{i+1}
11                     earIndex = i
12                     for each point p_k in Q (such that k≠i-1, k≠i, k≠i+1) do
13                          if (point p_k lies inside or on boundary of the ear) then
14                               earIndex = -1
15                     if (earIndex != -1) then
16                          Break out of the FOR loop at line 8 … we found an ear
17           if (earIndex == -1) quit and return triangles, since no more ears were found
18           Remove p_earIndex from Q
19           Add ear to triangles
20      Add (a new obstacle with vertices p_0, p_1 and p_2) to triangles
21      return triangles
```

Nothing to do if only 3 vertices

Copy obstacle vertices into a new list that we can add/remove from without destroying the original obstacle.

Find an ear

Left turn if CCW ordering and

$((x_i - x_{i-1})*(y_{i+1} - y_{i-1}) - (y_i - y_{i-1})*(x_{i+1} - x_{i-1})) > 0$

reject $\mathbf{p_i}$ as an ear since it is invalid

Cut the ear off and add to the solution

The last 3 points form an ear, so add it too

Start the Lab …