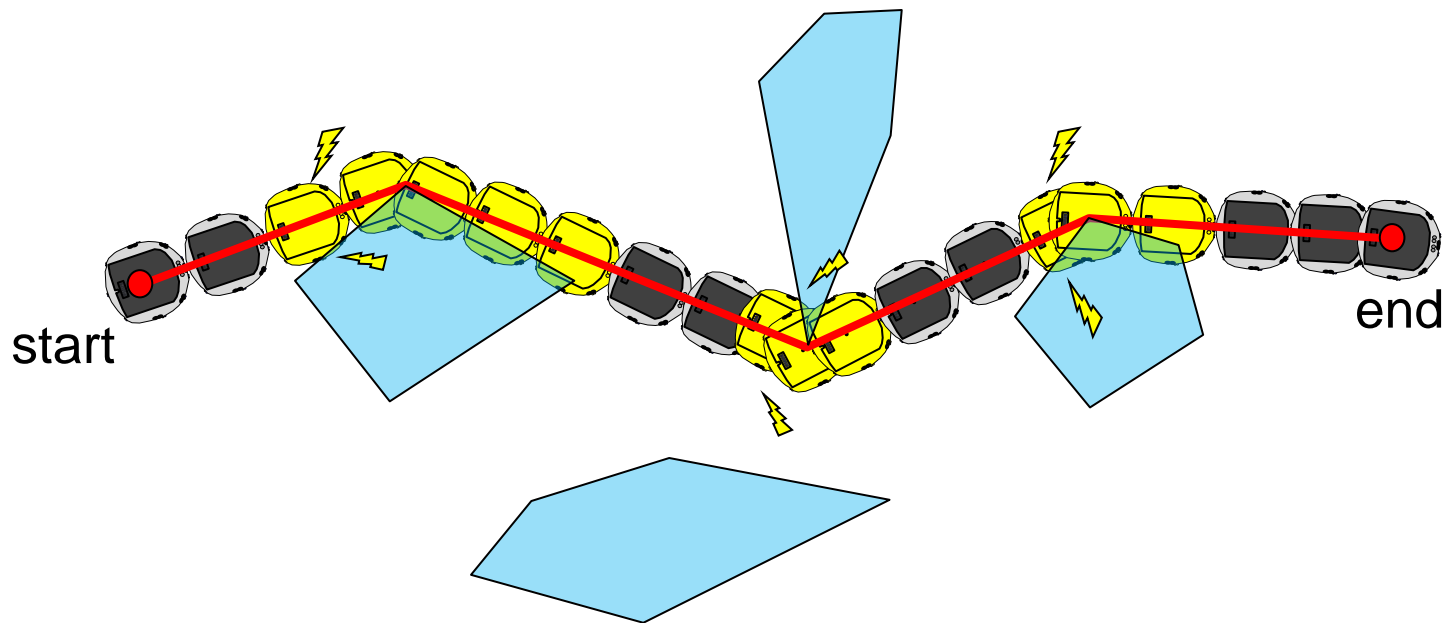


# Grown Obstacle Space

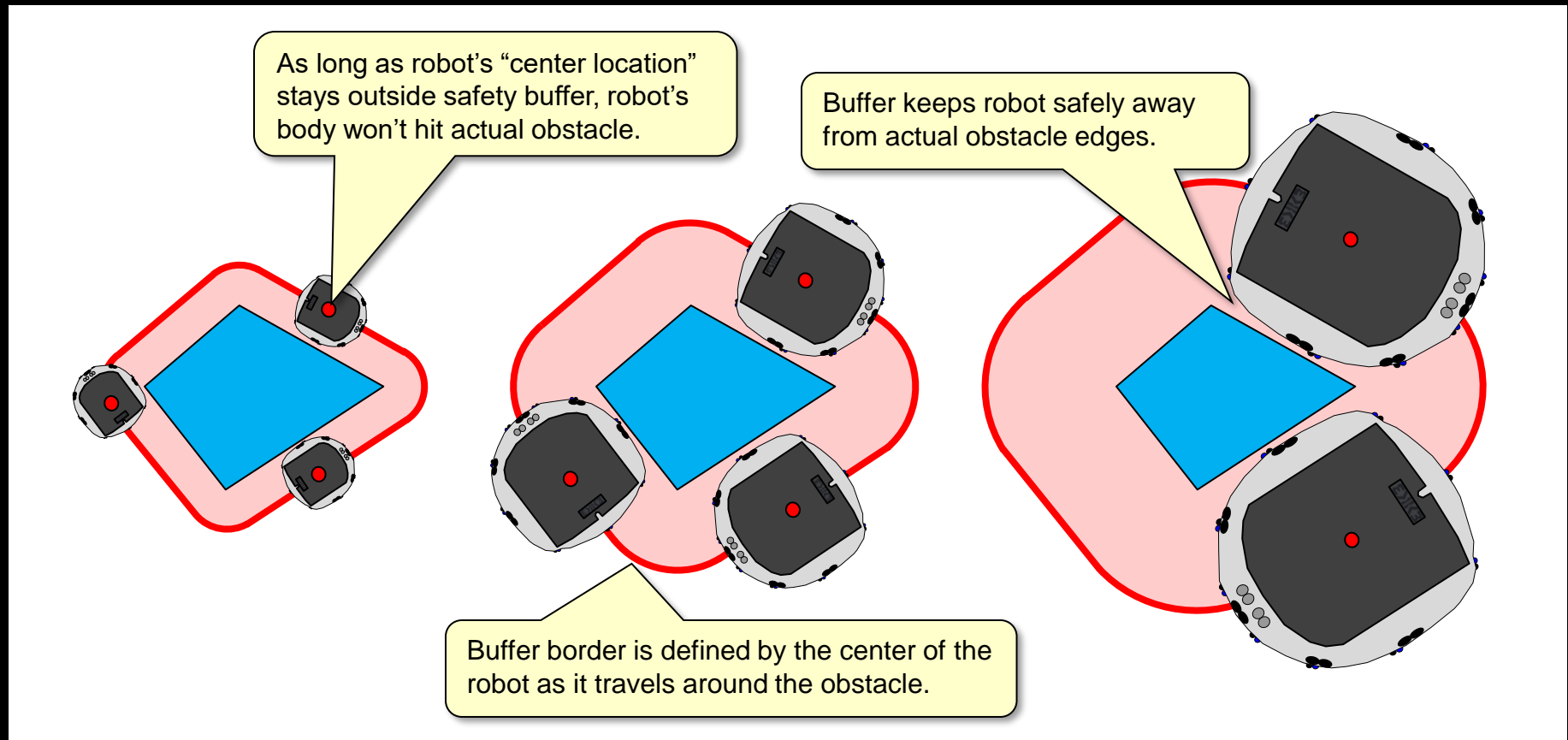
# Shortest Path Problems

- Any real robot will collide with obstacles if it travels along our computed shortest path because we assumed that the robot was just a point, but a robot has a shape:



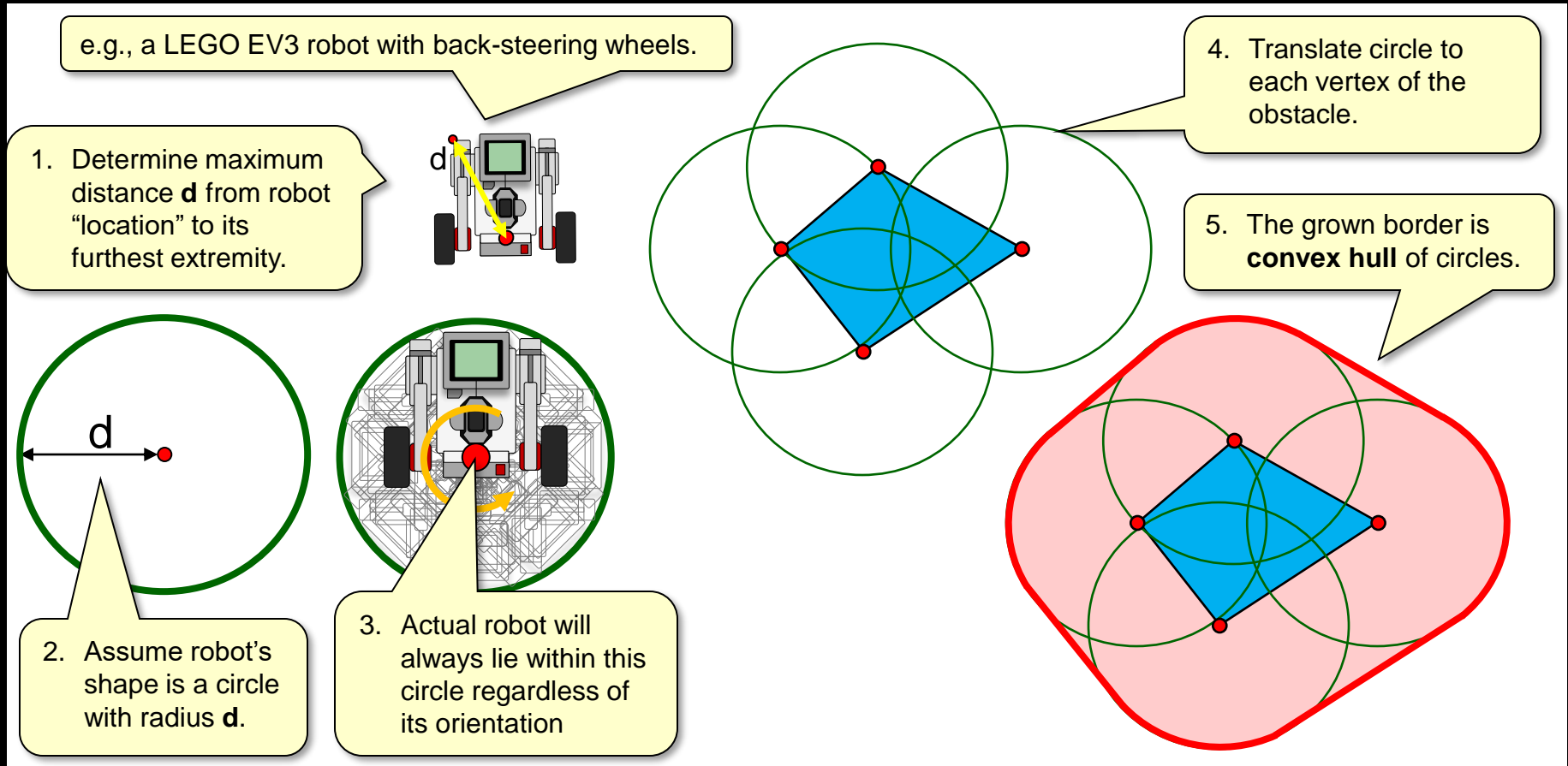
# Safety Buffers

- Need to have a “safety buffer” around the obstacles which takes into account robot’s size and shape.



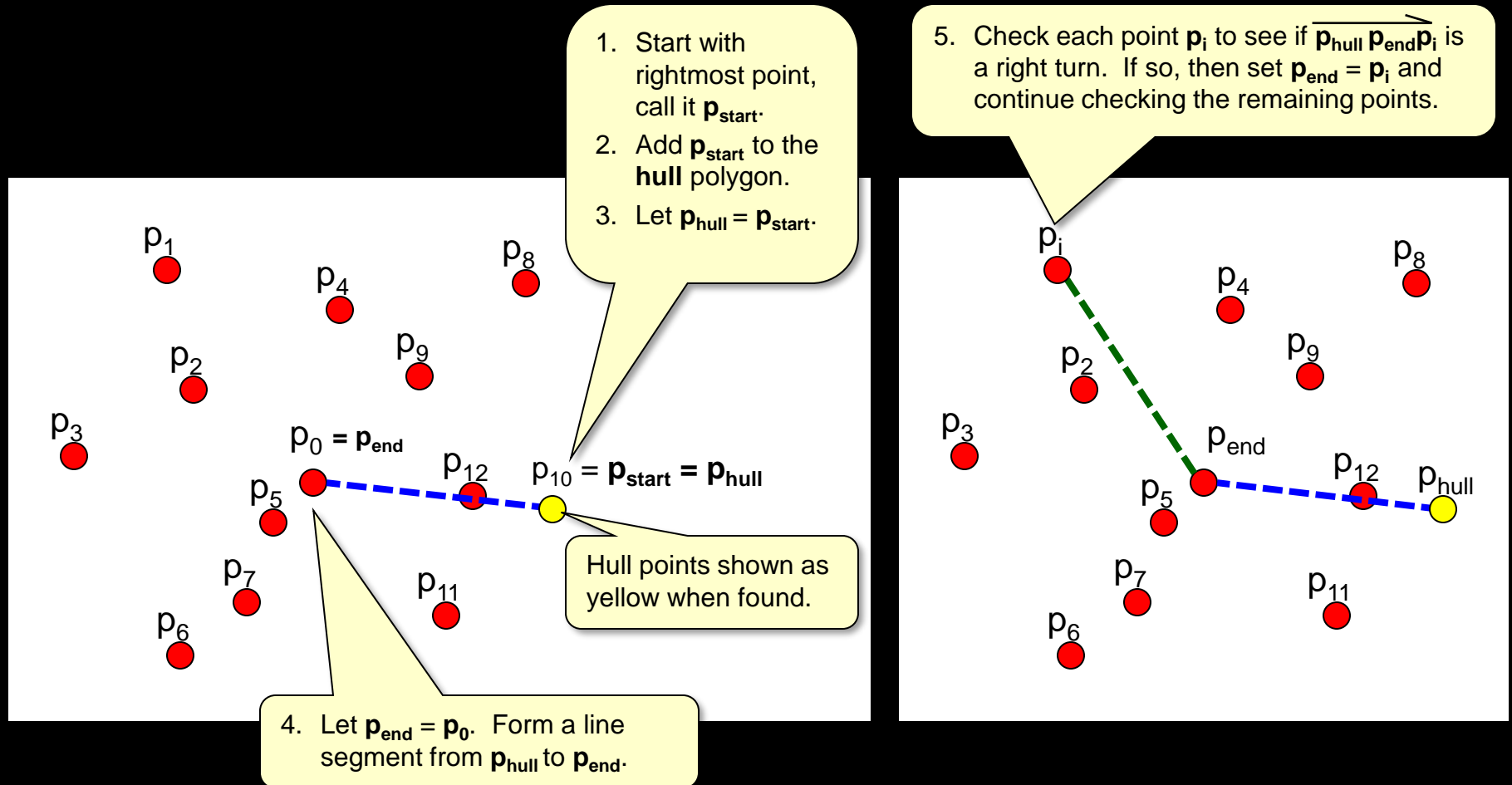
# Grown Obstacle Space

- Must “grow” obstacles by amount that considers **any orientation of robot** at any point on obstacle border:



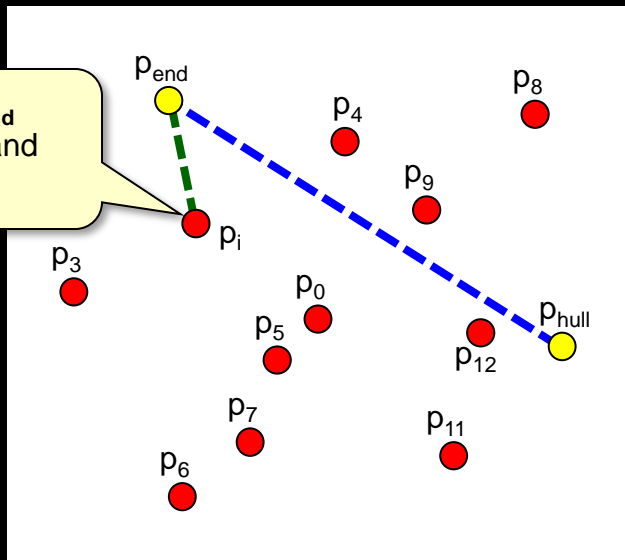
# Computing the Convex Hull

- Algorithms exist to find convex hull of a set of points
  - Graham Scan (a.k.a. “Gift-Wrapping” method) is a simple one:

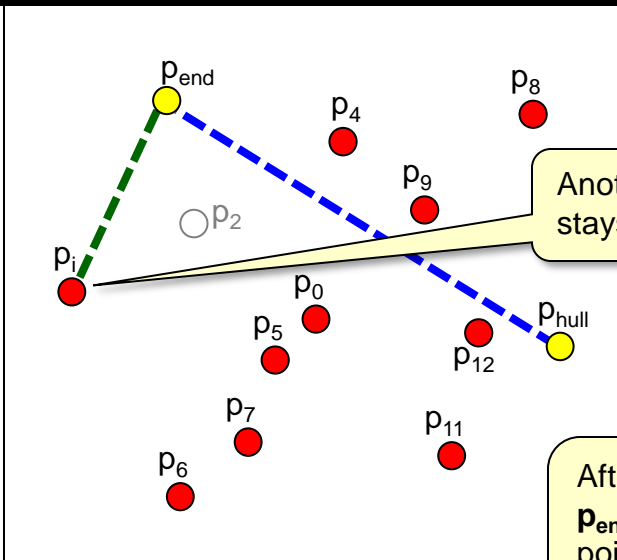


# Computing the Convex Hull

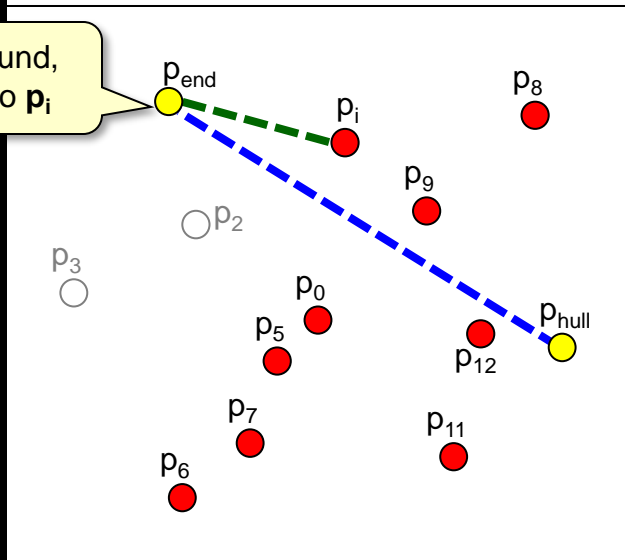
A left turn, so  $p_{end}$  stays the same and we continue



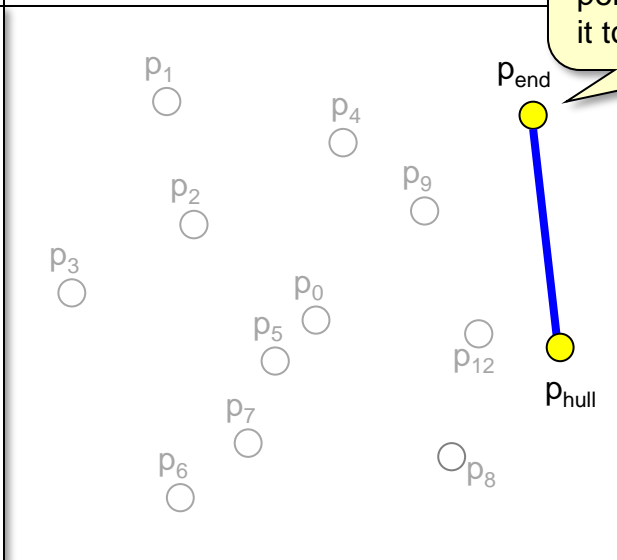
Another left turn, so  $p_{end}$  stays the same



When right turn found, now change  $p_{end}$  to  $p_i$

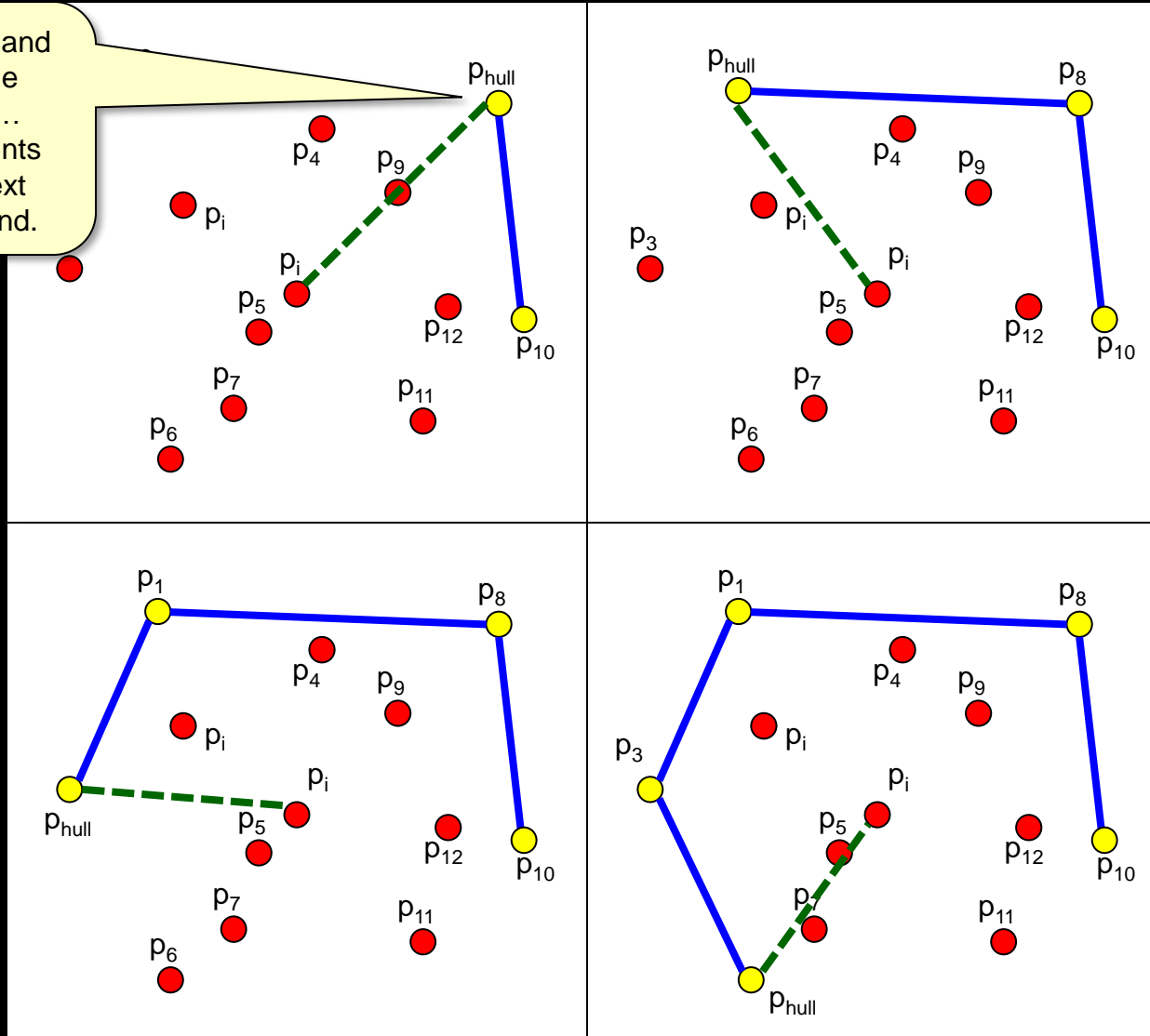


After all are checked,  $p_{end}$  will be the next point on the hull, so add it to the **hull** polygon.

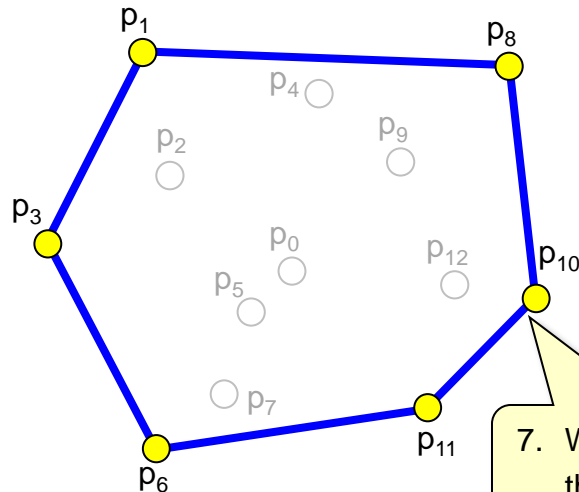
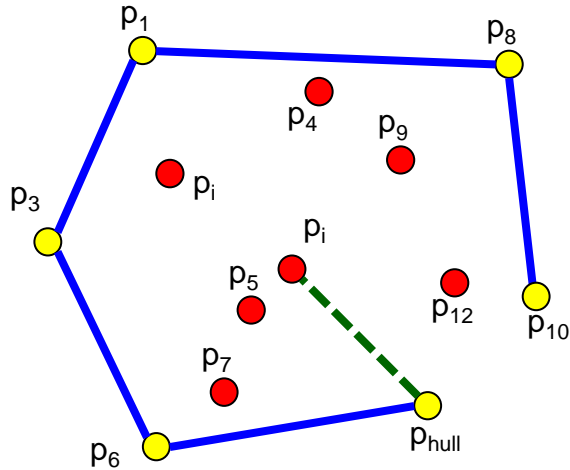


# Computing the Convex Hull

6. Set  $p_{\text{hull}} = p_{\text{end}}$  and repeat the whole process again ... checking all points in order until next hull point is found.



# Computing the Convex Hull



```

1  FUNCTION ConvexHull(PointList points)
2      hull = an empty list
3      pHull = the rightmost point in points
4      pStart = pHull
5      pEnd = NULL
6      WHILE (pEnd is not equal to pStart) DO
7          Add pHull to hull
8          pEnd = the first point in points
9          FOR (each point pi in points) DO
10             IF (pHull is the same as pEnd) THEN
11                 pEnd = pi
12             IF (angle pHull to pEnd to pi is a right turn) THEN
13                 pEnd = pi
14             pHull = pEnd
15  RETURN hull
    
```

Special case

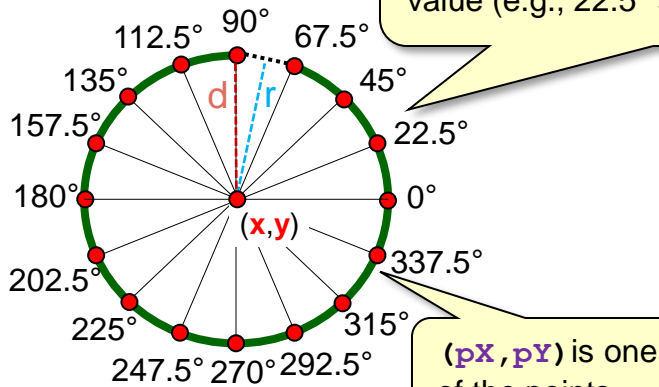
it is a right turn only if  
 $((p_{End_x} - p_{Hull_x}) * (p_{i_y} - p_{Hull_y}) - (p_{End_y} - p_{Hull_y}) * (p_{i_x} - p_{Hull_x})) < 0$

7. When  $p_{end} = p_{start}$   
 then we are done.



# Convex Hulls Around Obstacles

- Find Convex Hull of “points circles” around obstacle vertices:



Decide upon how many points you want on the circle by setting a **DEGREE\_UNIT** value (e.g.,  $22.5^\circ$  shown here).

Given a robot radius of  $r$ , then points must be at least distance  $d$  from  $(x,y)$  so that robot does not collide when travelling from one vertex to another, where  $d$  is

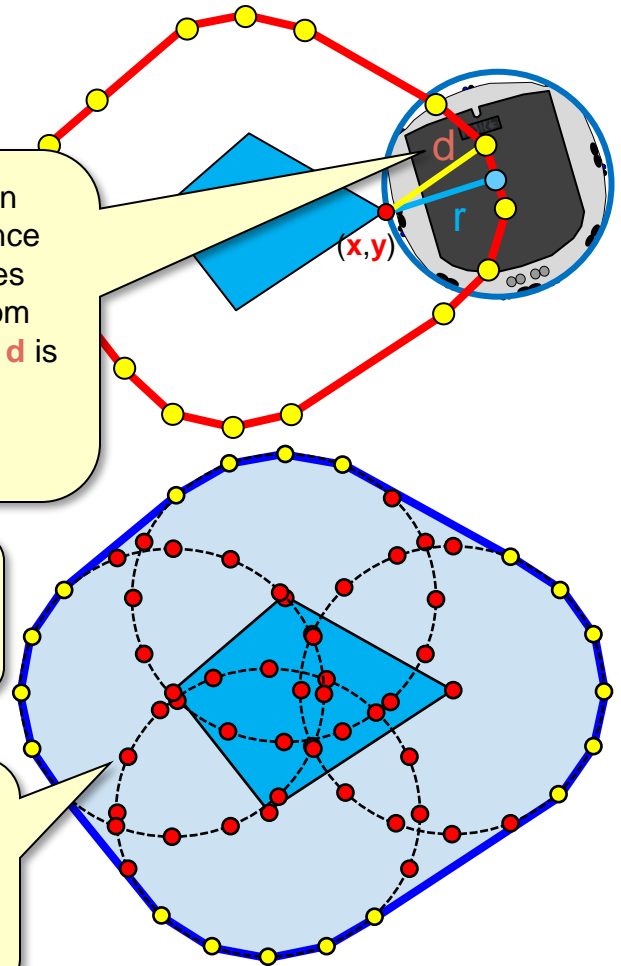
$$\frac{r}{\cos\left(\frac{\text{DEGREE\_UNIT}}{2}\right)}$$

$(pX, pY)$  is one of the points.

```
circlePoints = empty list
FOR (a=0; a<360; a+=DEGREE_UNIT) DO {
  xoff = d*cos(a)
  yoff = d*sin(a)
  if (xoff > 0) pX = x + ceil(xoff)
  otherwise pX = x + floor(xoff)
  if (yoff > 0) pY = y + ceil(yoff)
  otherwise pY = y + floor(yoff)
  add point (pX, pY) to circlePoints
}
```

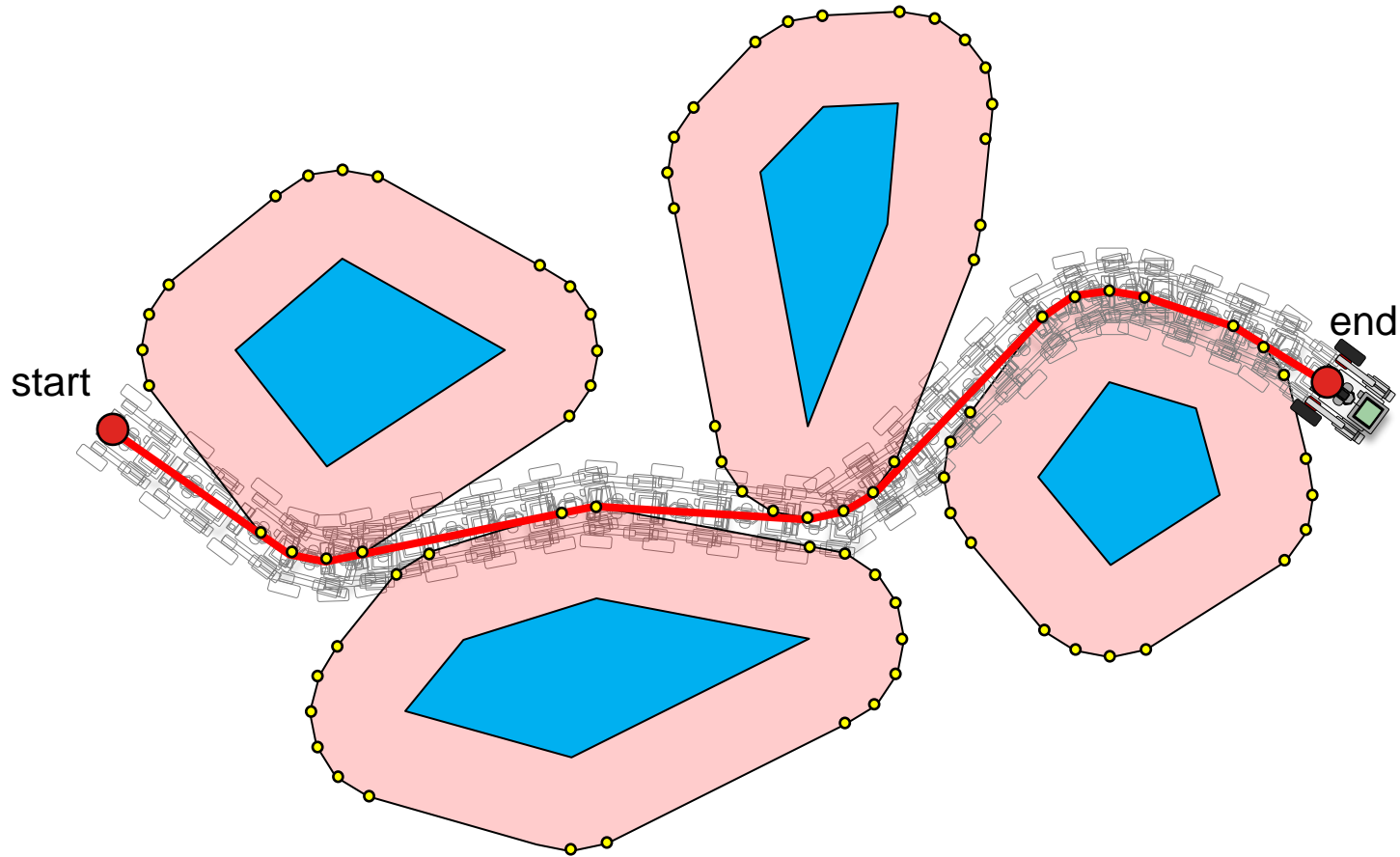
Create a list of **circlePoints** centered around an obstacle vertex  $(x, y)$  as shown here.

Merge all **circlePoints** lists from each vertex  $(x, y)$  of the obstacle. Using this “merged” list of points ... find the convex hull.



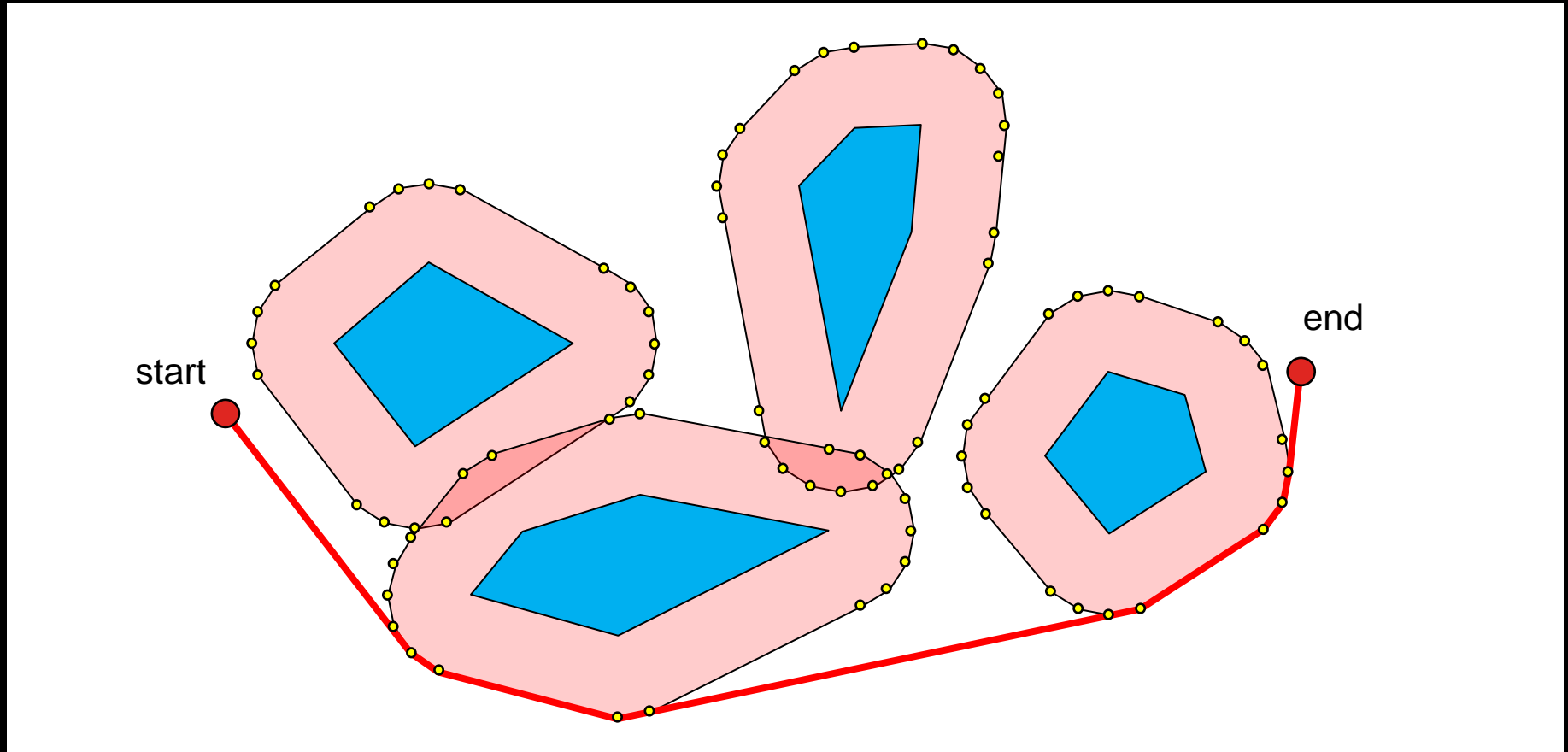
# Real Robot Shortest Path

- Now the robot has collision-free travel



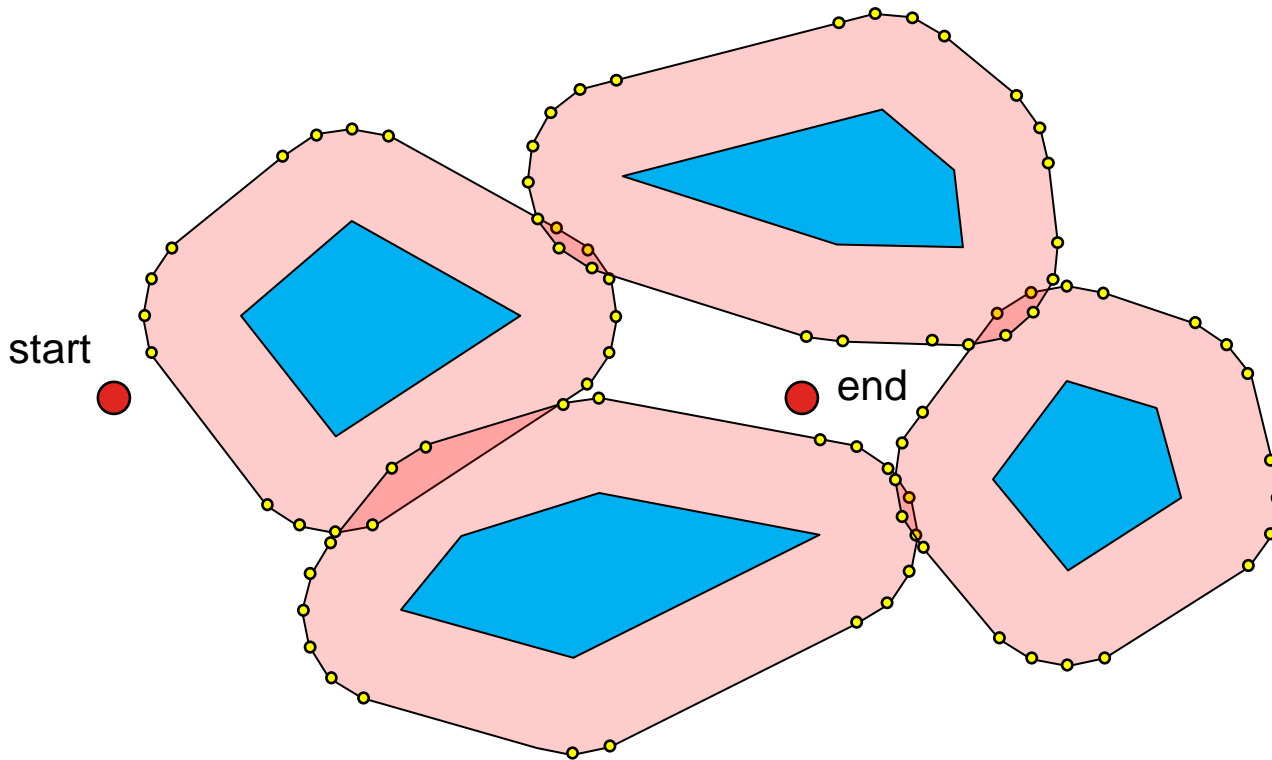
# Real Robot Shortest Path

- In some cases, the grown obstacles will intersect, resulting in a different solution path.



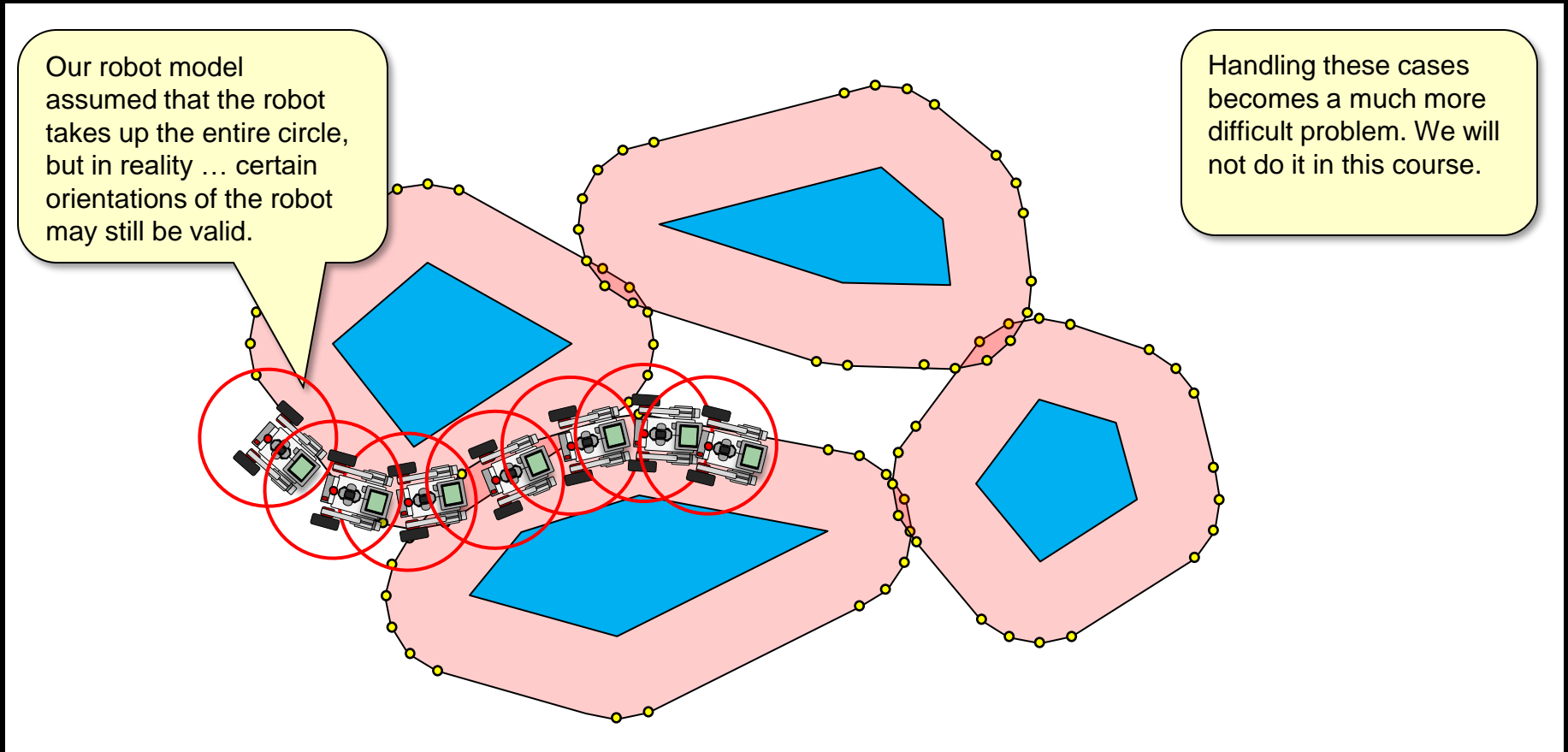
# Real Robot Shortest Path

- In some cases, there may even be no solution !



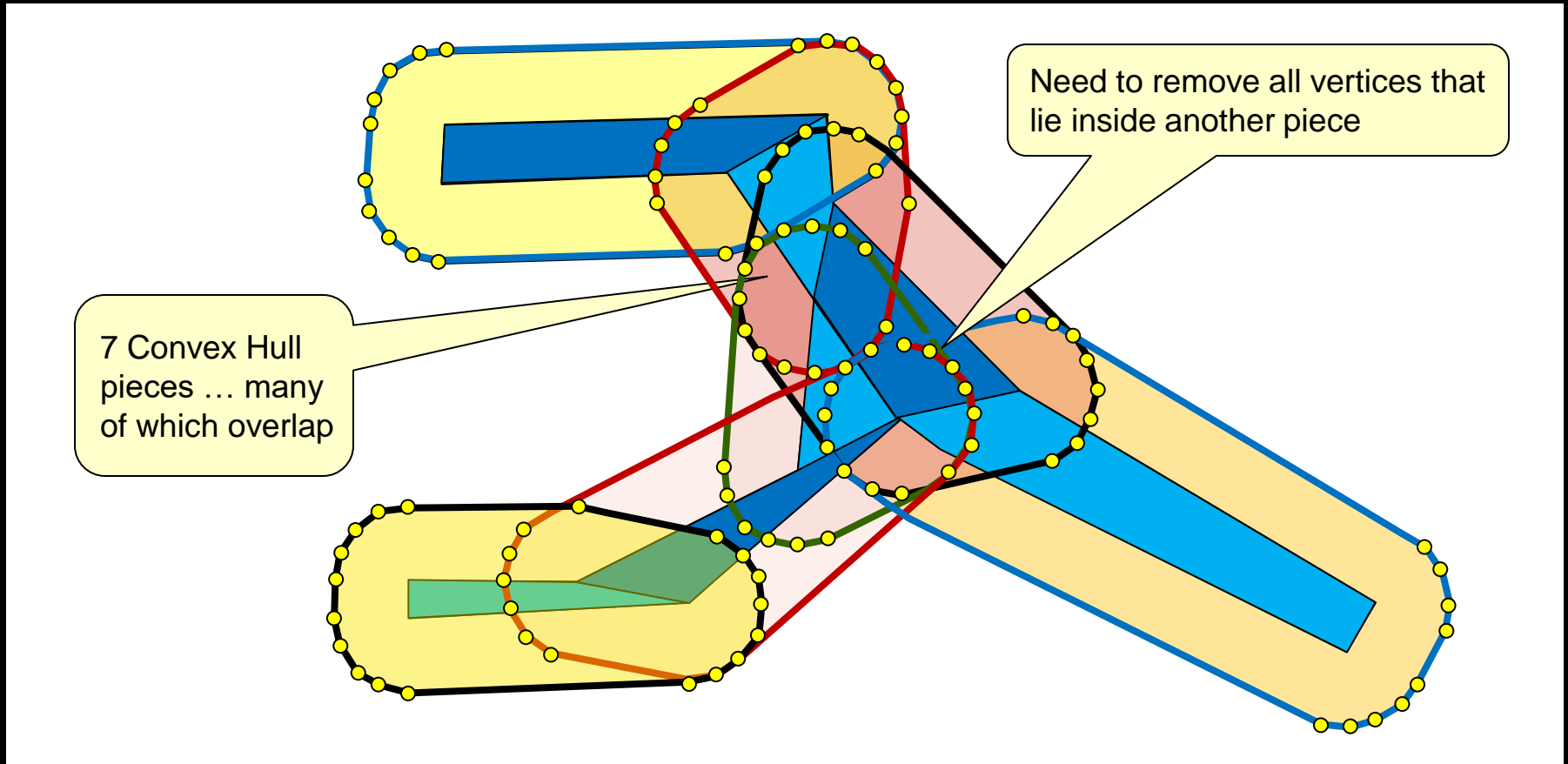
# Real Robot Shortest Path

- However, this is a restriction on our robot model and algorithm, as actual robot can “fit”:



# Non-Convex Solution

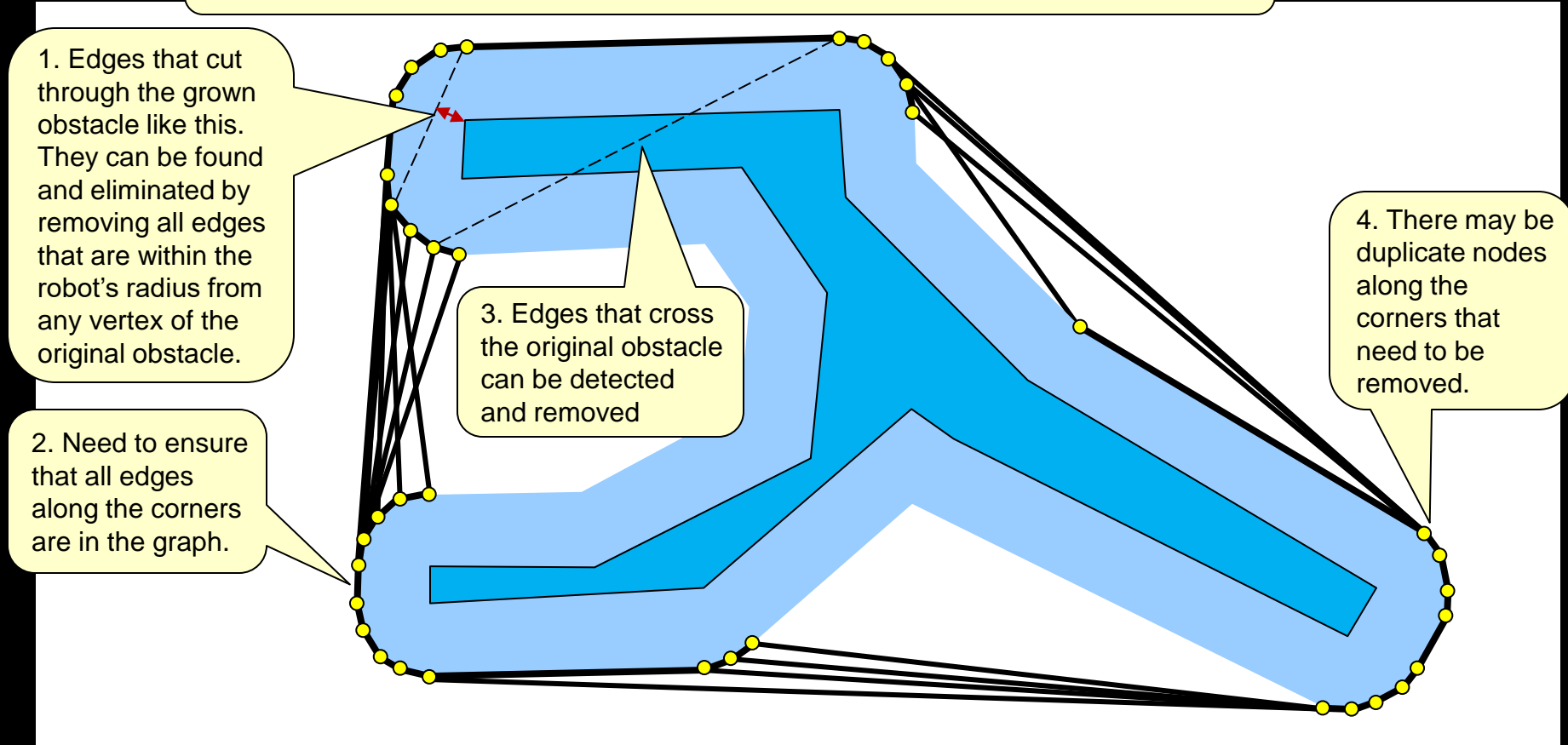
- Compute Convex Hull for each piece:



# Non-Convex Challenges

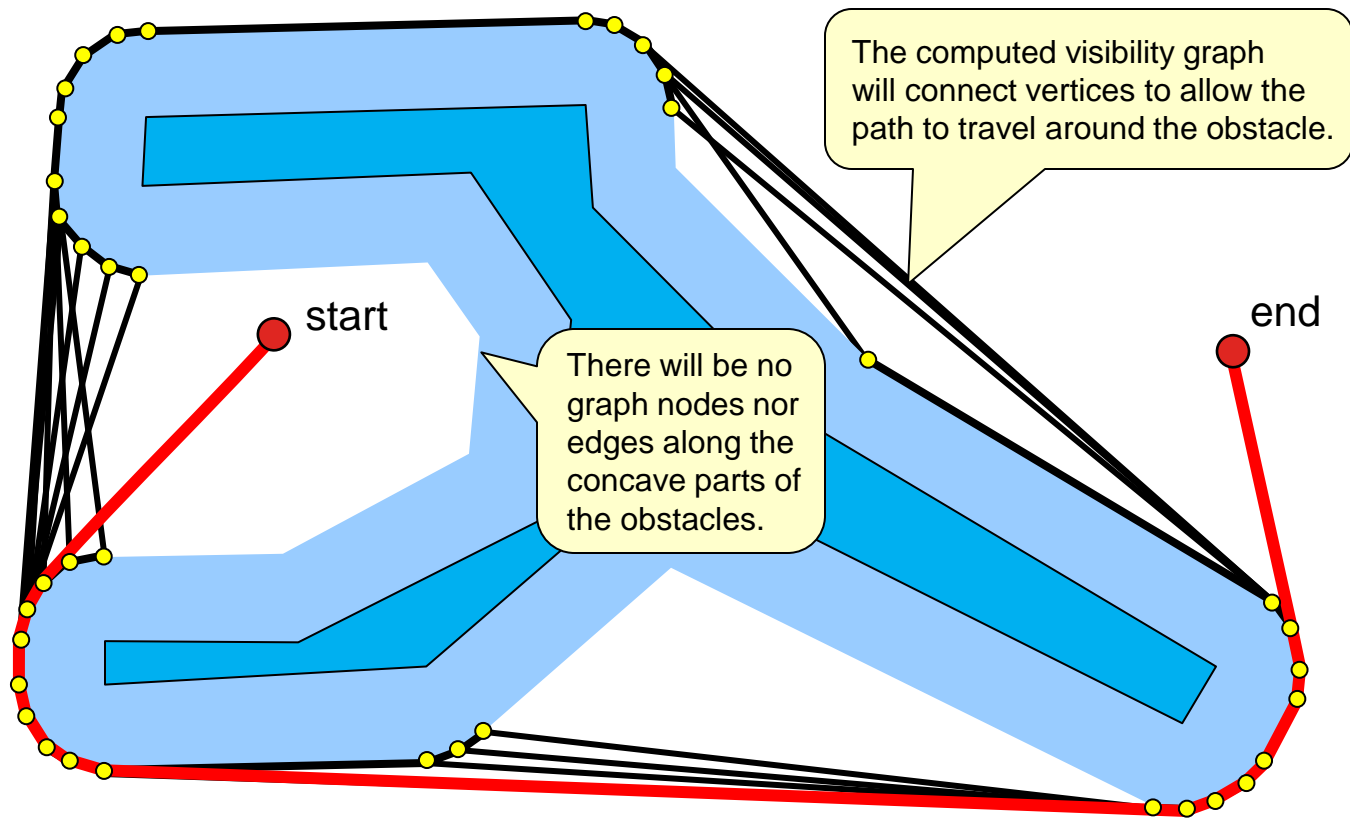
## ■ Compute visibility graph ... but it is tricky:

Unfortunately, there are a lot of intersection-related problems that pop up due to the overlapping of the various convex pieces. This will require the coding of special cases.



# Non-Convex Shortest Path

- If done correctly, the shortest path can be found:







**Start the  
Lab ...**