# Carleton University
# MATH 1104 Linear Algebra
# Lecture Notes

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# Contents

# 1 Lectures

Notes from lecture classes are shown in the following pages.

## 1.1 Square Matrices

A square matrix whose non-diagonal entires are all zero is called a diagonal matrix.
Examples of **diagonal matrices**:

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -7 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

An identity matrix is a diagonal matrix whose diagonal entries are all 1s. In other words, an identity matrix is a square matrix such that all entries on its main diagonal are 1, and all other entries are zero. Denote an $n * n$ identity matrix by $I_{n\times n}$
Zero Matrix
Matrix Operations: 1. Addition, Subtraction 2. Scalar Multiplication 3. Matrix Multiplication

$$A_{m\times n} \cdot B_{n\times p}$$

$$AB = C = [C_{ij}]_{m\times p}$$

# 2 Tutorials

Notes from tutorial classes are shown in the following pages.

## 2.1 Tutorial 1 - Jan 18

### 2.1.1 Important notes

- No unique solutions if # of equations < # of variables (or if a free variable exists).
- Reduced row echelon form (RREF) is always unique.
- Homogeneous systems always has at least one unique solution.

### 2.1.2 Question 1

Solving systems of linear equations using matrices.

| | |
|---|---|
| **Step 1**: | Write the system of equations in augmented matrix form. |
| **Step 2**: | Use elementary row operations to put the augmented matrix in row echelon form (REF). |
| **Step 3**: | Either (1)continue to reduced row echelon form (RREF) or (2)use back-substitution. |

### 2.1.3 Question 2

For what values of $h$ and $k$ will the system have:

$$\left.\begin{array}{ll} \textbf{No solution} & [0\ldots0|A\neq0] \\ \textbf{One unique solution} & [A\neq0|*] \\ \textbf{Infinite solutions} & [0\ldots0|0] \end{array}\right\}\text{Last row of REF matrix}$$

*Note: Perform elementary row operations to put the augmented matrix in REF first.

### 2.1.4 Question 3

$$\left[\begin{array}{cccc|c} 1 & \mathbf{0} & 2 & 1 & 7 \\ 0 & 1 & -3 & 4 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Always reduce to RREF first before converting back to equations. Notice the bold **0**, that is because there is already a leading 1 in a different row. When there is a leading 1, other elements in the same column has to be 0 in RREF matrices.

Converting back to equations:

$$\begin{cases} x_1 + 2x_3 + x_4 = 7 \\ x_2 - 3x_3 + 4x_4 = 1 \\ \textbf{Free Vars} \begin{cases} \text{Let } s = x_3 \\ \text{Let } t = x_4 \end{cases} \end{cases}$$

$$x_1 + 2s + t = 7, x_2 - 3s + 4t = 1$$
$$x_1 = -2s - t + 7, x_2 = 3s - 4t + 1$$
$$s, t \in \mathbb{R}$$