COMP/MATH 3804
Design and Analysis of Algorithms I -
Assignment 4

Hand in your assignments on, or before Dec 3^{rd} 23:59. No late assignment will be accepted. Your assignment should be submitted online on Brightspace as a single .pdf file. The filename should contain your name and student number. You can type your assignment or you can upload a scanned copy of it. Please, use a good image capturing device. Make sure that your upload is clearly readable. If it is difficult to read, it will not be graded. Whenever you are designing an algorithm you must address the three questions we are typically posing (correctness, complexity and improvement potential). The faster your algorithm, the better your mark.

Question 1:[15 points]
You and $m-1$ of your friends live all in different cities ($m > 1$). Each of you has a car and can start driving right now (after determining the meeting location). You drive on a road network, i..e, you have cities as vertices and two cities are connected via a directed edge if there is a road between them; the weight of the directed edge (u, v) is the time it takes you to get from u to v. The number of vertices is n and you may assume that the number of edges is also $O(n)$. You want to meet as soon as possible. How would you select a meeting location from one of your k favourite hang-out spot known to all of you? Which algorithm would you use and how is this done most efficiently. The more efficient the solution, the better the mark. State the complexities in terms of m, k, and n, you can make case distincations between m and k. Sometimes an algorothm can be stopped while it is stil running to make it a bit

Question 2:[10 points]
Find an optimal parameterization of a matrix-chain product whose sequence of dimensions is (3, 10, 2, 120, 5, 100, 4). Show the two solution matrices. How many different ways are there to evaluate this(!) matrix-chain product (i.e., different bracketings)?

Question 3:[15 points]
Consider the following linear program.

$$\begin{array}{lr} \text{minimize} & 3x_1 + 5x_2 \\ \text{subject to} & x_1 + 2x_2 \leq 12 \\ & x_1 + x_2 \leq 10 \\ & x_2 \leq 5 \\ & x_1, x_2 \geq 0. \end{array}$$

- Show the feasible region by plotting the constraints on the (x_1, x_2)-Cartesian coordinate system.
- Using your feasible region, find the optimal solution for this linear program. Is this the only solution? If yes, then explain why. If no, then state how many optimal solutions are there and justify your answer.
- Enter the LP into an LP solver that you can get from the internet and show us the input and solution pages (via screen captures).

Question 4:[10 points]
Suppose we have a polynomial-time reducution from Problem A to Problem B. Suppose further we know that Problem B has a polynomial-time solution.

- What can we conclude about Problem A, if anything?
- Suppose the polynomial-time reduction is now from Problem B to Problem A. What can we conclude about Problem A, if anything?

Question 5:[10 points]
We are considering the following problem: *Does a given directed graph G have a spiked Hamiltonian cycle?*. A spiked Hamiltonian cycle is a Hamiltonian cycle of all vertices of graph G except for one which must be joint to the cycle via a single edge (say directed from the vertex to the cycle). (So it sticks out)

Reduce this problem to the problem of finding Hamiltonian cycles in directed graphs. The reduction needs to be polynomial; note you can call Hamiltonian Cycle problem a polynomial number of times. Carefully argue the correctness of your reduction. You are allowed to assume for this question that in any graph you consider the Hamiltonian cycle (if it exists) is unique (obviosuly, that is not true in general.)

End of Assignment