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JEE MAIN 2021

ONLINE
16^{th} March
2^{nd} Shift

PHYSICS

SECTION-A (MULTIPLE CHOICE QUESTIONS)

1. Calculate the value of mean free path (λ) for oxygen molecules at temperature 27°C and pressure. 1.01×10^5 Pa. Assume the molecular diameter 0.3 nm and the gas is ideal. ($k = 1.38 \times 10^{-23}$ JK^{-1})
(a) 102 nm (b) 58 nm (c) 86 nm (d) 32 nm

2. The refractive index of a converging lens is 1.4. What will be the focal length of this lens if it is placed in a medium of same refractive index? Assume the radii of curvature of the faces of lens are R_1 and R_2 respectively.
(a) zero (b) $\dfrac{R_1R_2}{R_1 - R_2}$
(c) 1 (d) Infinite

3. Two identical antennas mounted on identical towers are separated from each other by a distance of 45 km. What should nearly be the minimum height of receiving antenna to receive the signals in line of sight? (Assume radius of earth is 6400 km)
(a) 79.1 m (b) 39.55 m
(c) 158.2 m (d) 19.77 m

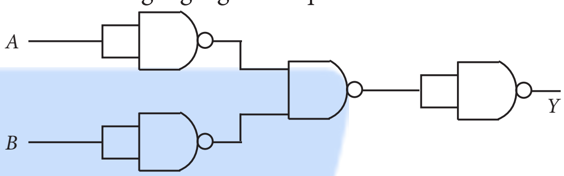
4. Red light differs from blue light as they have
(a) Different frequencies and different wavelengths
(b) Different frequencies and same wavelengths
(c) Same frequencies and different wavelengths
(d) Same frequencies and same wavelengths

5. In order to determine the Young's Modulus of a wire of radius 0.2 cm (measured using a scale of least count = 0.001 cm) and length 1 m (measured using a scale of least count = 1 mm), a weight of mass 1 kg (measured using a scale of least count = 1 g) was hanged to get the elongation of 0.5 cm (measured using a scale of least count 0.001 cm). What will be the fractional error in the value of Young's Modulus determined by this experiment?
(a) 1.4% (b) 0.14% (c) 9% (d) 0.9%

6. A resistor develops 500 J of thermal energy in 20 s when a current of 1.5 A is passed through it. If the current is increased from 1.5 A to 3 A, what will be the energy developed in 20 s.
(a) 1000 J (b) 500 J (c) 1500 J (d) 2000 J

7. The de-Broglie wavelength associated with an electron and a proton were calculated by accelerating them through same potential of 100 V. What should be the ratio of their wavelengths?
($m_p = 1.00727$ u, $m_e = 0.00055$ u)
(a) 1860 : 1 (b) 41.4 : 1
(c) 43 : 1 (d) $(1860)^2 : 1$

8. The following logic gate is equivalent to

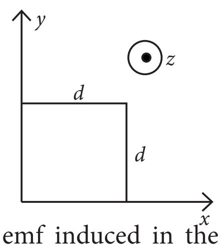


(a) NAND Gate (b) AND Gate
(c) NOR Gate (d) OR Gate

9. A charge Q is moving $d\vec{l}$ distance in the magnetic field $\vec{B}$. Find the value of work done by $\vec{B}$.
(a) Infinite (b) 1 (c) zero (d) −1

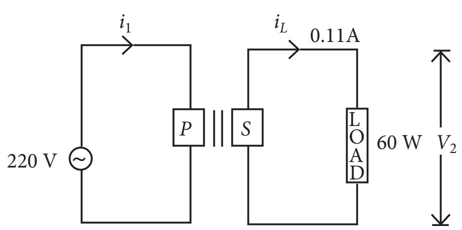
10. Find out the surface charge density at the intersection of point x = 3 m plane and x-axis, in the region of uniform line charge of 8 nC/m lying along the z-axis in free space.
(a) 47.88 C/m (b) 0.424 nC m^{-2}
(c) 0.07 nC m^{-2} (d) 4.0 nC m^{-2}

11. The magnetic field in a region is given by $\vec{B} = B_0\left(\dfrac{x}{a}\right)\hat{k}$.



A square loop of side d is placed with its edges along the x and y axes. The loop is moved with a constant velocity $\vec{v} = v_0\,\hat{i}$. The emf induced in the loop is
(a) $\dfrac{B_0v_0d^2}{2a}$ (b) $\dfrac{B_0v_0^2d}{2a}$
(c) $\dfrac{B_0v_0d}{2a}$ (d) $\dfrac{B_0v_0d^2}{a}$

12. For the given circuit, comment on the type of transformer used.



(a) Auxilliary transformer
(b) Step down transformer
(c) Auto transformer
(d) Step-up transformer

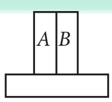
13. **Statement I :** A cyclist is moving on an unbanked road with a speed of 7 km h^{-1} and takes a sharp circular turn along a path of radius of 2 m without reducing the speed. The static friction coefficient is 0.2 . The cyclist will not slip and pass the curve. ($g = 9.8$ m/s^2)
Statement II : If the road is banked at an angle of 45°, cyclist can the curve of 2m radius with speed of 18.5 km h^{-1} without slipping.
In the light of the above statements, choose the correct answer from the options given below.
(a) Both statement I and statement II are false
(b) Statement I is incorrect and statement II is correct
(c) Statement I is correct and statement II is incorrect
(d) Both statement I and statement II are true

14. The half-life of Au^{198} is 2.7 days. The activity of 1.50 mg of Au^{198} if its atomic weight is 198 g mol^{-1} is, ($N_A = 6 \times 10^{23}$/mol).
(a) 357 Ci (b) 252 Ci (c) 240 Ci (d) 535 Ci

15. Calculate the time interval between 33% decay and 67% decay if half-life of a substance is 20 minutes.
(a) 40 minutes (b) 20 minutes
(c) 13 minutes (d) 60 minutes

16. Amplitude of a mass-spring system, which is executing simple harmonic motion decreases with time. If mass = 500 g, Decay constant = 20 g/s then how much time is required for the amplitude of the system to drop to half of its initial value? (ln 2 = 0.693)
(a) 17.32 s (b) 15.01 s
(c) 34.65 s (d) 0.034 s

17. A bimetallic strip consists of metals A and B. It is mounted rigidly as shown. The metal A has higher coefficient of expansion compared to that of metal B. When the bimetallic strip is placed in a cold bath, it will

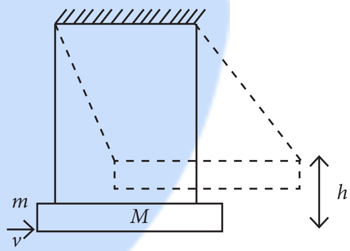


(a) Neither bend nor shrink
(b) Bend towards the right
(c) Not bend but shrink
(d) Bend towards the left

18. A mosquito is moving with a velocity $v = 0.5t^2\hat{i} + 3t\hat{j} + 9\hat{k}$ m/s and accelerating in uniform conditions. What will be the direction of mosquito after 2 s?
(a) $\tan^{-1}\left(\frac{5}{2}\right)$ from x-axis
(b) $\tan^{-1}\left(\frac{2}{3}\right)$ from y-axis
(c) $\tan^{-1}\left(\frac{5}{2}\right)$ from y-axis
(d) $\tan^{-1}\left(\frac{2}{3}\right)$ from x-axis

19. What will be the nature of flow of water from a circular tap, when its flow rate increased from 0.18 L/min to 0.48 L/min? The radius of the tap and viscosity of water are 0.5 cm and 10^{-3} Pa s, respectively.
(Density of water : 10^3 kg/m^3)
(a) Remains turbulent flow
(b) Steady flow to unsteady flow
(c) Unsteady to steady flow
(d) Remains steady flow

20. A large block of wood of mass $M = 5.99$ kg is hanging from two long massless cords. A bullet of mass $m = 10$ g, is fired into the block and gets embedded in it. The (block + bullet) then swing upwards their centre of mass rising a vertical distance $h = 9.8$ cm before the (block + bullet) pendulum comes momentarily to rest at the end of its arc. The speed of the bullet just before collision is (take $g = 9.8$ m s^{-2}).

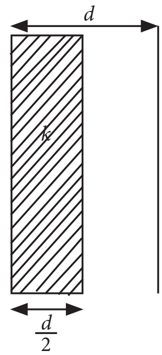


(a) 841.4 m/s (b) 821.4 m/s
(c) 831.4 m/s (d) 811.4 m/s

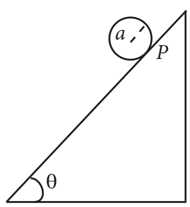
SECTION - B (NUMERICAL VALUE TYPE)

Attempt any 5 questions out of 10.

21. In a parallel plate capacitor set up, the plate area of capacitor is 2 m^2 and the plates are separated by 1 m. If the space between the plates are filled with a dielectric material of thickness 0.5 m and area 2 m^2 (see fig) the capacitance of the set-up will be _______ ε_0.
(Dielectric constant of the material = 3.2) (Round off to the nearest integer)



22. A solid disc of radius 'a' and mass 'm' rolls down without slipping on an inclined plane making an angle θ with the horizontal. The acceleration of the disc will be $\frac{2}{b} g \sin\theta$, where b is ______.
(Round off to nearest integer)
(g = acceleration due to gravity, θ = angles as shown in figure)



23. For an ideal heat engine, the temperature of the source is 127°C. In order to have 60% efficiency the temperature of the sink should be _______ °C. (Round off to nearest integer).

24. The energy dissipated by a resistor is 10 mJ in 1 s when an electric current of 2 mA flows through it. The resistance is ______ Ω. (Round off to the nearest integer)

25. A closed organ pipe of length L and an open organ pipe contain gases of densities ρ_1 and ρ_2 respectively. The compressibility of gases are equal in both the pipes. Both the pipes are vibrating in their first overtone with same frequency. The length of the open pipe is $\frac{x}{3}L\sqrt{\frac{\rho_1}{\rho_2}}$ where x is ______.
(Round off to the nearest integer)

26. A force $\vec{F}=4\hat{i}+3\hat{j}+4\hat{k}$ is applied on an intersection point of $x=2$ plane and x-axis. The magnitude of torque of this force about a point (2, 3, 4) is ______. (Round off to the nearest integer)

27. A body of mass 2 kg moves under a force of $(2\hat{i}+3\hat{j}+5\hat{k})$ N. It starts from rest and was at the origin intially. After 4 s, its new coordinates are (8, b, 20). The value of b is ______. (Round off to the nearest integer)

28. A deviation of 2° is produced in the yellow ray when prism of crown and flint glass are achromatically combined. Taking dispersive powers of crown and flint glass as 0.02 and 0.03 respectively. The refracting angles for crown glass prism will be ______ °(in degree). (Round off to the nearest integer)

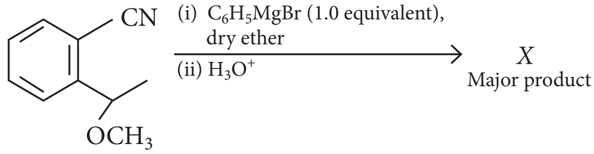
29. A swimmer can swim with velocity of 12 km/h in still water. Water flowing in a river has velocity 6 km/h. The direction with respect to the direction of flow of river water he should swim in order to reach the point on the other bank just opposite to his starting point is____ °. (Round off to the nearest integer) (Find the angle in degrees)

30. If one wants to remove all the mass of the earth to infinity in order to break it up completely. The amount of energy that needs to be supplied will be $\frac{x}{5}\frac{GM^2}{R}$ where x is ____ (Round off to the nearest integer) (M is the mass of Earth, R is the radius of Earth, G is the gravitational constant)

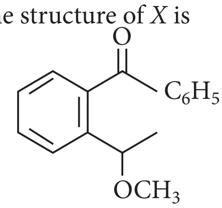
CHEMISTRY

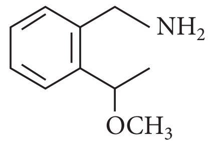
SECTION-A (MULTIPLE CHOICE QUESTIONS)

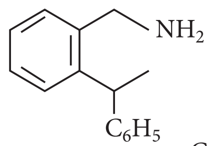
31. Statement I : Sodium hydride can be used as an oxidising agent.
Statement II : The lone pair of electrons on nitrogen in pyridine makes it basic.
Choose the correct answer from the options given below.
(a) Both statement I and statement II are false.
(b) Statement I is true but statement II is false.
(c) Statement I is false but statement II is true.
(d) Both statement I and statement II are true.

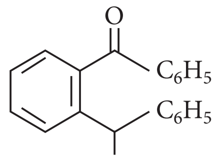
32. 

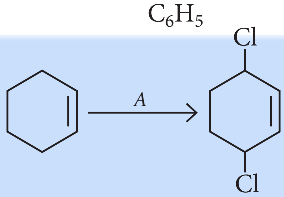
The structure of X is

(a) 

(b) 

(c) 

(d) 

33. 

Identify the reagent(s) 'A' and condition(s) for the reaction.
(a) A = HCl, $ZnCl_2$ (b) A = HCl ; Anhy. $AlCl_3$
(c) A = Cl_2 ; dark, Anhydrous $AlCl_3$
(d) A = Cl_2 ; UV light

34. Identify the elements X and Y using ionization energy values given below.
Ionization energy (kJ/ mol) :

	I[st]	2[nd]
X	495	4563
Y	731	1450

(a) X = Mg ; Y = Na (b) X = Mg ; Y = F
(c) X = Na ; Y = Mg (d) X = F ; Y = Mg

35. Which of the following reduction reaction cannot be carried out with coke?
(a) $ZnO \rightarrow Zn$ (b) $Al_2O_3 \rightarrow Al$
(c) $Cu_2O \rightarrow Cu$ (d) $Fe_2O_3 \rightarrow Fe$

36. The exact volumes of 1 M NaOH solution required to neutralise 50 mL of 1 M H_3PO_3 solution and 100 mL of 2 M H_3PO_2 solution respectively, are
(a) 100 mL and 100 mL (b) 100 mL and 200 mL
(c) 100 mL and 50 mL (d) 50 mL and 50 mL.

37. Match List-I with List-II.

	List-I Test/Reagents Observation(s)		**List-II Species detected**
(A)	Lassaigne's test	(i)	Carbon
(B)	Cu(II) oxide	(ii)	Sulphur
(C)	Silver nitrate	(iii)	N, S, P and halogen
(D)	The sodium fusion extract gives black precipitate with acetic acid and lead acetate	(iv)	Halogen specifically

The correct match is
(a) (A) – (iii), (B) – (i), (C) – (iv), (D) – (ii)
(b) (A) – (i), (B) – (iv), (C) – (iii), (D) – (ii)
(c) (A) – (iii), (B) – (i), (C) – (ii), (D) – (iv)
(d) (A) – (i), (B) – (ii), (C) – (iv), (D) – (iii)

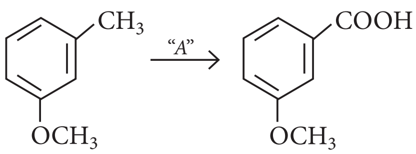
38. The incorrect statement below regarding colloidal solutions is
(a) a colloidal solution shows Brownian motion of colloidal particles
(b) a colloidal solution shows colligative properties
(c) the flocculating power of Al^{3+} is more than that of Na^+
(d) an ordinary filter paper can stop the flow of colloidal particles.

39. The correct statements about H_2O_2 are
(A) used in the treatment of effluents
(B) used as both oxidising and reducing agents
(C) the two hydroxyl group lie in the same plane
(D) miscible with water
Choose the correct answer from the options given below.
(a) (B), (C) and (D) only (b) (A), (B) and (D) only
(c) (A), (B), (C) and (D) (d) (A), (C) and (D) only.

40. The greenhouse gas/es is (are)
(A) carbon dioxide (B) oxygen
(C) water vapour (D) methane
Choose the most appropriate answer from the options given below.
(a) (A), (C) and (D) only (b) (A) and (C) only
(c) (A) only (d) (A) and (B) only.

41. Which of the following polymer is used in the manufacture of wood laminates?
(a) *cis*-Poly isoprene
(b) Urea formaldehyde resin
(c) Phenol and formaldehyde resin
(d) Melamine formaldehyde resin

42. Ammonolysis of alkyl halides followed by the treatment with NaOH solution can be used to prepare primary, secondary and tertiary amines. The purpose of NaOH in the reaction is
(a) to remove acidic impurities
(b) to activate NH_3 used in the reaction
(c) to increase the reactivity of alkyl halide
(d) to remove basic impurities.

43. 

In the above reaction the reagent "A" is
(a) Alkaline $KMnO_4$, H^+ (b) $NaBH_4$, H_3O^+
(c) HCl, Zn — Hg (d) $LiAlH_4$

44. The characteristics of elements X, Y and Z with atomic numbers respectively, 33, 53 and 83 are
(a) X is a metalloid, Y is a non-metal and Z is a metal
(b) X, Y and Z are metals
(c) X and Y are metalloids and Z is a metal
(d) X and Z are non-metals and Y is a metalloid.

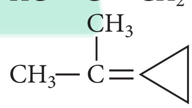
45. Which of the following is least basic?
(a) $(CH_3CO)\ddot{N}HC_2H_5$ (b) $(C_2H_5)_2\ddot{N}H$
(c) $(C_2H_5)_3\ddot{N}$ (d) $(CH_3CO)_2\ddot{N}H$

46. Fex_2 and Fey_3 are known when x and y are
(a) x = F, Cl, Br, I and y = F, Cl, Br
(b) x = F, Cl, Br, I and y = F, Cl, Br, I
(c) x = Cl, Br, I and y = F, Cl, Br, I
(d) x = F, Cl, Br and y = F, Cl, Br,I

47. The incorrect statement regarding the structure of C_{60} is
(a) the six-membered rings are fused to both six and five-membered rings
(b) the five-membered rings are fused only to six-membered rings
(c) each carbon atom forms three sigma bonds
(d) it contains 12 six-membered rings and 24 five-membered rings.

48. Arrange the following metal complex compounds in the increasing order of spin only magnetic moment. Presume all the three, high spin system.
(Atomic numbers Ce = 58, Gd = 64 and Eu = 63)
(A) $(NH_4)_2[Ce(NO_3)_6]$ (B) $Gd(NO_3)_3$ and
(C) $Eu(NO_3)_3$
(a) (C) < (A) < (B) (b) (B) < (A) < (C)
(c) (A) < (C) < (B) (d) (A) < (B) < (C)

49. The secondary structure of protein is stabilised by
(a) van der Waals forces (b) peptide bond
(c) hydrogen bonding (d) glycosidic bond.

50. An unsaturated hydrocarbon X on ozonolysis gives A. Compound A when warmed with ammonical silver nitrate forms a bright silver mirror along the sides of the test tube. The unsaturated hydrocarbon X is
(a) $HC\equiv C-CH_2-CH_3$
(b) 

(c) $CH_3-C\equiv C-CH_3$
(d) $CH_3-\underset{CH_3}{\underset{|}{C}}=\underset{CH_3}{\underset{|}{C}}-CH_3$

SECTION - B (NUMERICAL VALUE TYPE)

Attempt any 5 questions out of 10.

51. The number of orbitals with n = 5, m_l = +2 is ______.
(Round off to the Nearest Integer)

52. Sulphurous acid (H_2SO_3) has K_{a1} = 1.7×10^{-2} and K_{a2} = 6.4×10^{-8}. The pH of 0.588 M H_2SO_3 is _____.
(Round off to the Nearest Integer).

53. At 25°C, 50 g of iron reacts with HCl to form $FeCl_2$. The evolved hydrogen gas expands against a constant

pressure of 1 bar. The work done by the gas during this expansion is ______ J.
(Round off to the Nearest Integer).
[Given : $R = 8.314$ J mol^{-1} K^{-1}. Assume, hydrogen is an ideal gas]
[Atomic mass of Fe is 55.85 u]

54. A and B decompose via first order kinetics with half-lives 54.0 min and 18.0 min respectively. Starting from an equimolar non reactive mixture of A and B, the time taken for the concentration of A to become 16 times that of B is ______ min. (Round off to the Nearest Integer).

55. $[Ti(H_2O)_6]^{3+}$ absorbs light of wavelength 498 nm during a d-d transition. The octahedral splitting energy for the above complex is ________ $\times 10^{-19}$ J. (Round off to the Nearest Integer).
$h = 6.626 \times 10^{-34}$ Js; $c = 3 \times 10^8$ ms^{-1}.

56. A 5.0 mmol dm^{-3} aqueous solution of KCl has a conductance of 0.55 mS when measured in a cell of cell constant 1.3 cm^{-1}. The molar conductivity of this solution is _____ mS m^2 mol^{-1}. (Round off to the Nearest Integer).

57. In Duma's method of estimation of nitrogen, 0.1840 g of an organic compound gave 30 mL of nitrogen collected at 287 K and 758 mm of Hg pressure. The percentage composition of nitrogen in the compound is _______.
(Round off to the Nearest Integer).
[Given : Aqueous tension at 287 K = 14 mm of Hg]

58. Ga (atomic mass 70 u) crystallizes in a hexagonal close packed structure. The total number of voids in 0.581 g of Ga is _______ $\times 10^{21}$.
(Round off to the Nearest Integer).
[Given : $N_A = 6.023 \times 10^{23}$]

59. At 363 K the vapour pressure of A is 21 kPa and that of B is 18 kPa. One mole of A and 2 moles of B are mixed. Assuming that this solution is ideal, the vapour pressure of the mixture is _____ kPa.
(Round off to the Nearest Integer).

60. When 35 mL of 0.15 M lead nitrate solution is mixed with 20 mL of 0.12 M chromic sulphate solution, ______ $\times 10^{-5}$ moles of lead sulphate precipitate out.
(Round off to the Nearest Integer).

MATHEMATICS

SECTION-A (MULTIPLE CHOICE QUESTIONS)

61. Let $A(-1, 1)$, $B(3, 4)$ and $C(2, 0)$ be given three points. A line $y = mx$, $m > 0$, intersects lines AC and BC at points P and Q respectively. Let A_1 and A_2 be the areas of ΔABC and ΔPQC respectively, such that $A_1 = 3A_2$, then the value of m is equal to

(a) 2 (b) 1 (c) 3 (d) $\frac{4}{15}$

62. If (x, y, z) be an arbitrary point lying on a plane P which passes through the points (42, 0, 0), (0, 42, 0) and (0, 0, 42), then the value of the expression

$$3+\frac{x-11}{(y-19)^2(z-12)^2}+\frac{y-19}{(x-11)^2(z-12)^2}+\frac{z-12}{(x-11)^2(y-19)^2}-\frac{x+y+z}{14(x-11)(y-19)(z-12)}$$

is equal to

(a) 3 (b) 0 (c) -45 (d) 39

63. If the points of intersections of the ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ and the circle $x^2 + y^2 = 4b$, $b > 4$ lie on the curve $y^2 = 3x^2$, then b is equal to

(a) 10 (b) 6 (c) 5 (d) 12

64. Consider the integral $I=\int_0^{10}\frac{[x]e^{[x]}}{e^{x-1}}dx$, where $[x]$ denotes the greatest integer less than or equal to x. Then the value of I is equal to

(a) $9(e - 1)$ (b) $45(e - 1)$
(c) $45(e + 1)$ (d) $9(e + 1)$

65. Let A denote the event that a 6-digit integer formed by 0, 1, 2, 3, 4, 5, 6 without repetitions, be divisible by 3. Then probability of event A is equal to

(a) $\frac{3}{7}$ (b) $\frac{9}{56}$ (c) $\frac{11}{27}$ (d) $\frac{4}{9}$

66. Consider a rectangle $ABCD$ having 5, 7, 6, 9 points in the interior of the line segments AB, CD, BC, DA respectively. Let α be the number of triangles having these points from different sides as vertices and β be the number of quadrilaterals having these points from different sides as vertices. Then $(\beta - \alpha)$ is equal to

(a) 1173 (b) 795 (c) 1890 (d) 717

67. Let $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=2\hat{i}-3\hat{j}+5\hat{k}$.
If $\vec{r}\times\vec{a}=\vec{b}\times\vec{r}$, $\vec{r}\cdot(\alpha\hat{i}+2\hat{j}+\hat{k})=3$
and $\vec{r}\cdot(2\hat{i}+5\hat{j}-\alpha\hat{k})=-1$, $\alpha \in R$, then the value of $\alpha+|\vec{r}|^2$ is equal to

(a) 13 (b) 15 (c) 9 (d) 11

68. Let C_1 be the curve obtained by the solution of differential equation $2xy\frac{dy}{dx}=y^2-x^2, x>0$. Let the curve C_2 be the solution of $\frac{2xy}{x^2-y^2}=\frac{dy}{dx}$. If both the curves pass through (1, 1), then the area enclosed by the curves C_1 and C_2 is equal to

(a) $\frac{\pi}{2}-1$ (b) $\frac{\pi}{4}+1$ (c) $\pi - 1$ (d) $\pi + 1$

69. Let $P(x) = x^2 + bx + c$ be a quadratic polynomial with real coefficients such that $\int_0^1 P(x)dx=1$ and $P(x)$ leaves remainder 5 when it is divided by $(x-2)$. Then the value of $9(b+c)$ is equal to

(a) 7 (b) 11 (c) 9 (d) 15

70. The maximum value of

$$f(x)=\begin{vmatrix} \sin^2 x & 1+\cos^2 x & \cos 2x \\ 1+\sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & \sin 2x \end{vmatrix}, x \in R \text{ is}$$

(a) $\sqrt{5}$ (b) 5 (c) $\sqrt{7}$ (d) $\frac{3}{4}$

71. If the foot of the perpendicular from point (4, 3, 8) on the line $L_1: \frac{x-a}{l}=\frac{y-2}{3}=\frac{z-b}{4}, l \neq 0$ is (3, 5, 7), then the shortest distance between the line L_1 and line $L_2: \frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$ is equal to

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{6}}$ (d) $\sqrt{\frac{2}{3}}$

72. Let $A = \{2, 3, 4, 5, ..., 30\}$ and '$\simeq$' be an equivalence relation on $A \times A$, defined by $(a, b) \simeq (c, d)$ if and only if $ad = bc$. Then the number of ordered pairs which satisfy this equivalence relation with ordered pair (4, 3) is equal to

(a) 5 (b) 6 (c) 7 (d) 8

73. Let $\alpha \in R$ be such that the function

$$f(x)=\begin{cases} \frac{\cos^{-1}(1-\{x\}^2)\sin^{-1}(1-\{x\})}{\{x\}-\{x\}^3}, & x \neq 0 \\ \alpha, & x=0 \end{cases}$$ is continuous

at $x = 0$, where $\{x\} = x - [x]$, $[x]$ is the greatest integer less than or equal to x. Then

(a) no such α exists (b) $\alpha=\frac{\pi}{4}$

(c) $\alpha=\frac{\pi}{\sqrt{2}}$ (d) $\alpha=0$

74. Given that the inverse trigonometric functions take principal values only. Then, the number of real values of x which satisfy $\sin^{-1}\left(\frac{3x}{5}\right)+\sin^{-1}\left(\frac{4x}{5}\right)=\sin^{-1} x$ is equal to

(a) 2 (b) 1 (c) 0 (d) 3

75. The least value of $|z|$, where z is complex number which satisfies the inequality

$$\exp\left(\frac{(|z|+3)(|z|-1)}{||z|+1|}\log_e 2\right) \geq \log_{\sqrt{2}} |5\sqrt{7}+9i|, \; i=\sqrt{-1}$$

is equal to

(a) 3 (b) 2 (c) $\sqrt{5}$ (d) 8

76. Let $f: S \to S$, where $S = (0, \infty)$ be a twice differentiable function such that $f(x + 1) = xf(x)$. If $g: S \to R$ be defined as $g(x) = \log_e f(x)$, then the value of $|g''(5) - g''(1)|$ is equal to

(a) $\frac{187}{144}$ (b) $\frac{197}{144}$ (c) 1 (d) $\frac{205}{144}$

77. Let C be the locus of the mirror image of a point on the parabola $y^2 = 4x$ with respect to the line $y = x$. Then the equation of tangent to C at $P(2, 1)$ is

(a) $x - y = 1$ (b) $x + 2y = 4$

(c) $2x + y = 5$ (d) $x + 3y = 5$

78. Let the lengths of intercepts on x-axis and y-axis made by the circle $x^2 + y^2 + ax + 2ay + c = 0$, $(a < 0)$ be $2\sqrt{2}$ and $2\sqrt{5}$, respectively. Then the shortest distance from origin to a tangent to this circle which is perpendicular to the line $x + 2y = 0$, is equal to

(a) $\sqrt{11}$ (b) $\sqrt{6}$ (c) $\sqrt{10}$ (d) $\sqrt{7}$

79. Let f be a real valued function, defined on $R - \{-1, 1\}$ and given by $f(x)=3\log_e\left|\frac{x-1}{x+1}\right|-\frac{2}{x-1}$. Then in which of the following intervals, function $f(x)$ is increasing?

(a) $\left(-1, \frac{1}{2}\right]$

(b) $(-\infty, -1)\cup\left(\left[\frac{1}{2}, \infty\right)-\{1\}\right)$

(c) $\left(-\infty, \frac{1}{2}\right]-\{-1\}$ (d) $(-\infty, \infty) - \{-1, 1\}$

80. If $y = y(x)$ is the solution of the differential equation $\frac{dy}{dx}+(\tan x)y=\sin x, 0 \leq x \leq \frac{\pi}{3}$, with $y(0) = 0$, then $y\left(\frac{\pi}{4}\right)$ equals to

(a) $\left(\frac{1}{2\sqrt{2}}\right)\log_e 2$ (b) $\frac{1}{4}\log_e 2$

(c) $\frac{1}{2}\log_e 2$ (d) $\log_e 2$

SECTION - B (NUMERICAL VALUE TYPE)

Attempt any 5 questions out of 10.

81. Let $f: R \to R$ and $g: R \to R$ be defined as

$$f(x)=\begin{cases} x+a, & x<0 \\ |x-1|, & x \geq 0 \end{cases}$$

and $g(x)=\begin{cases} x+1, & x<0 \\ (x-1)^2+b, & x \geq 0 \end{cases}$, where a, b are non-negative real numbers. If $(gof)(x)$ is continuous for all $x \in R$, then $a + b$ is equal to ________.

82. Let $\vec{c}$ be a vector perpendicular to the vectors $\vec{a}=\hat{i}+\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}+2\hat{i}+\hat{k}$. If $\vec{c}\cdot(\hat{i}+\hat{j}+3\hat{k})=8$, then the value of $\vec{c}\cdot(\vec{a}\times\vec{b})$ is equal to ________.

83. If the distance of the point (1, –2, 3) from the plane $x + 2y - 3z + 10 = 0$ measured parallel to the line, $\frac{x-1}{3}=\frac{2-y}{m}=\frac{z+3}{1}$ is $\sqrt{\frac{7}{2}}$, then the value of $|m|$ is equal to ________.

84. Let n be a positive integer. Let

$$A=\sum_{k=0}^{n}(-1)^k\,{}^nC_k\left[\left(\frac{1}{2}\right)^k+\left(\frac{3}{4}\right)^k+\left(\frac{7}{8}\right)^k+\left(\frac{15}{16}\right)^k+\left(\frac{31}{32}\right)^k\right]$$

If $63A=1-\frac{1}{2^{30}}$, then n is equal to ________.

85. Let $\frac{1}{16}$, a and b be in G.P. and $\frac{1}{a}$, $\frac{1}{b}$, 6 be in A.P., where $a, b > 0$. Then $72(a + b)$ is equal to ________.

86. In ΔABC, the lengths of sides AC and AB are 12 cm and 5 cm respectively. If the area of ΔABC is 30 cm^2 and R and r are respectively the radii of circumcircle and incircle of ΔABC, then the value of $2R + r$(in cm) is equal to ________.

87. Let $A=\begin{bmatrix}a_1\\a_2\end{bmatrix}$ and $B=\begin{bmatrix}b_1\\b_2\end{bmatrix}$ be two 2 × 1 matrices with real entries such that $A = XB$, where

$$X=\frac{1}{\sqrt{3}}\begin{bmatrix}1&-1\\1&k\end{bmatrix}, \text{ and } k\in R.$$

If $a_1^2+a_2^2=\frac{2}{3}(b_1^2+b_2^2)$ and $(k^2+1)b_2^2\neq -2b_1b_2$, then the value of k is ________.

88. Let $S_n(x)=\log_{a^{1/2}}x+\log_{a^{1/3}}x+\log_{a^{1/6}}x+\log_{a^{1/11}}x+\log_{a^{1/18}}x+\log_{a^{1/27}}x+....$ upto n-terms, where $a > 1$. If $S_{24}(x) = 1093$ and $S_{12}(2x) = 265$, then value of a is equal to ________.

89. Consider the statistics of two sets of observations as follows:

	Size	Mean	Variance
Observation I	10	2	2
Observation II	n	3	1

If the variance of the combined set of these two observations is $\frac{17}{9}$, then the value of n is equal to ________.

90. For real numbers α, β, γ and δ, if

$$\int\frac{(x^2-1)+\tan^{-1}\left(\frac{x^2+1}{x}\right)}{(x^4+3x^2+1)\tan^{-1}\left(\frac{x^2+1}{x}\right)}dx$$

$$=\alpha\log_e\left(\tan^{-1}\left(\frac{x^2+1}{x}\right)\right)+\beta\tan^{-1}\left(\frac{\gamma(x^2-1)}{x}\right)$$

$+\delta\tan^{-1}\left(\frac{x^2+1}{x}\right)+C$, where C is an arbitrary constant, then the value of $10(\alpha + \beta\gamma + \delta)$ is equal to ________.

HINTS & EXPLANATIONS

1. (a) : Given, temperature $T = 27°C = 27 + 273 = 300$ K
Pressure, $P = 1.01 \times 10^5$ Pa
Molecular diameter, $d = 0.3$ nm $= 0.3 \times 10^{-9}$ m
$k = 1.38 \times 10^{-23}$ J K^{-1}
As gas is ideal,

Mean free path, $\lambda=\frac{kT}{\sqrt{2}\pi d^2P}$

$$=\frac{1.38\times10^{-23}\times300}{\sqrt{2}\times3.14\times0.09\times10^{-18}\times1.01\times10^5}$$

$= 1025.6 \times 10^{-10}$ m $= 102.5 \times 10^{-9}$ m $= 102.5$ nm $\simeq$ 102 nm

2. (d) : Given, refractive index of lens, $\mu_2 = 1.4$
Refractive index of medium, $\mu_1 = 1.4$
From lens maker's formula,

$$\therefore\ \frac{1}{f}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\left(\frac{1.4}{1.4}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

or $\frac{1}{f}=0\Rightarrow f=\frac{1}{0}$

f = infinite

∴ Focal length of this lens would be infinite.

3. (b) : Given, distance between antennas,
$D = 45$ km $= 45 \times 10^3$ m
If h is the minimum height of receiving antenna to receive the signal, then,

$D=2\sqrt{2hR}$

$$\therefore\ h=\frac{D^2}{8R}=\frac{45\times45\times10^6}{8\times6400\times10^3}=39.55\text{ m}$$

4. (a) : Red light and blue light have different frequencies and different wavelengths as they have same speed in air.

5. (a) : Given, radius of wire, $r = 0.2$ cm, $\Delta r = 0.001$ cm
Length of wire, $L = 1$ m
Least count of scale in measuring, $\Delta L = 0.001$ m
Mass of wire, $m = 1$ kg
$\Delta m = 1$ g $= 0.001$ kg
Elongation, $l = 0.5$ cm
Least count of scale in measuring, $\Delta l = 0.001$ cm

Young's Modulus is given by, $Y=\frac{F/A}{l/L}=\frac{Mg/\pi r^2}{l/L}=\frac{MgL}{\pi r^2L}$

Taking log and differentiating, we get

$$\frac{\Delta Y}{Y}=\frac{\Delta m}{m}+\frac{\Delta L}{L}-\frac{2\Delta r}{r}-\frac{\Delta l}{l}\quad [g \text{ is constant}]$$

Error in Y,

$$\frac{\Delta Y}{Y}=\frac{\Delta m}{m}+\frac{2\Delta r}{r}+\frac{\Delta l}{l}+\frac{\Delta L}{L}$$

$$=\frac{0.001}{1}+\frac{2\times 0.001}{0.2}+\frac{0.001}{0.5}+\frac{0.001}{1}$$

$= 0.001 + 0.01 + 0.002 + 0.001 = 0.014$

Percentage error

$$\frac{\Delta Y}{Y}\times 100=0.014\times 100=1.4\ \%$$

Fractional error in the value of Y determined by this experiment is 1.4 %.

6. (d) : Given, energy, $E = 500$ J

Time, $t = 20$ s

Initial current, $I_1 = 1.5$ A

Final current, $I_2 = 3.0$ A

From Joule's law of heating effect of current, we know $H = I^2Rt$

Since time and resistance is same.

$$\text{So, } \frac{H_2}{H_1}=\left(\frac{I_2}{I_1}\right)^2=\left(\frac{3}{1.5}\right)^2=4$$

$H_2 = 4\,H_1 = 4 \times 500 \text{ J} = 2000$ J

7. (c) : Given, potential, $V = 100$ V

We know, de-Broglie wavelength is given by

$$\lambda=\frac{h}{\sqrt{2mk}}$$

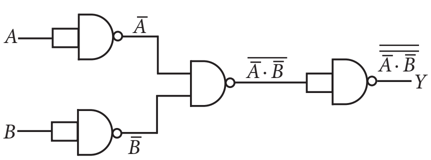
$$\lambda=\frac{h}{\sqrt{2mqV}}\quad [\because\ k=qV]$$

q and v is same for both electron and proton.

$$\text{So, } \frac{\lambda_e}{\lambda_p}=\sqrt{\frac{m_p}{m_e}}=\sqrt{\frac{1.00727\text{ u}}{0.00055\text{ u}}}=\sqrt{1831.4}=42.79\simeq 43$$

$$\therefore\ \frac{\lambda_e}{\lambda_p}=\frac{43}{1}$$

8. (c) :



$Y=\overline{\bar{A}\cdot\bar{B}}=\overline{A+B}$

This is NOR gate.

9. (c) : Force on a moving charge in magnetic field is given by $\vec{F}=q(\vec{v}\times\vec{B})$

Force is perpendicular to velocity and magnetic field.

Work done, $W = F.ds = 0$ as force is perpendicular to velocity so it is perpendicular to displacement as well.

10. (b) : Uniform line charge, $\lambda = 8$ nC/m $= 8 \times 10^{-9}$ C/m

Electric field due to wire is given by, $E=2\dfrac{k\lambda}{r}$

where $k=\dfrac{1}{4\pi\varepsilon_0}$.

Also, electric field due to surface charged density, $E'=\dfrac{\sigma}{\varepsilon_0}$.

where, σ is the surface charge density.

$$\text{So, } \frac{\sigma}{\varepsilon_0}=\frac{2k\lambda}{r}$$

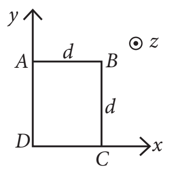
$$\sigma=\varepsilon_0\times 2\times\frac{1}{4\pi\varepsilon_0}\times\frac{\lambda}{r}=\frac{\lambda}{2\pi r}=\frac{8\times 10^{-9}}{2\times 3.14\times 3}$$

$= 0.424 \times 10^{-9}$ C m^{-2} $= 0.424$ nC m^{-2}

11. (d) : Magnetic field is given by $\vec{B}=B_0\left(\dfrac{x}{a}\right)\hat{k}$

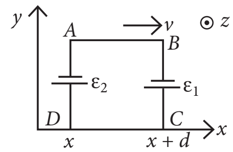
and loop is moving with velocity, $v=v_0\,\hat{i}$

Induced emf, $\varepsilon_1=B_0\left(\dfrac{x+d}{a}\right)v_0 d$



Induced emf, $\varepsilon_2=B_0\dfrac{x}{a}v_0 d$

Induced emf for side AB and CD would be zero as v_0 is parallel to d.



So net emf, $\varepsilon = \varepsilon_1 - \varepsilon_2$

$$=B_0\left(\frac{x+d}{a}\right)v_0 d-\frac{B_0 x}{a}v_0 d=\frac{B_0 v_0 d^2}{a}$$

12. (d) : In the given transformer,

For primary,

$V_p = 220$ V

For secondary,

$P = 60$ W

$i_L = 0.11$ A

$P = i_L \times V_2$

$$V_2=\frac{60}{0.11}=545.4\text{ V}$$

Since, voltage in secondary > voltage in primary.

This is step-up transformer.

13. (d) : On horizontal ground,

$$v_{\max}=\sqrt{\mu Rg}=\sqrt{0.2\times 2\times 9.8}=1.97\text{ m/s}$$

$$=\frac{1.97\times 60\times 60}{1000}=7.092\text{ km/h}\simeq 7\text{ km/h}$$

Cyclist will not slip and he can turn safely.

So, statement I is true.

If road is banked at $\theta = 45°$, then

$$v_{\max}=\sqrt{gr\left(\frac{\mu+\tan\theta}{1-\mu\tan\theta}\right)}=\sqrt{2\times 9.8\left(\frac{0.2+1}{1-0.2\times 1}\right)}$$

$$=\sqrt{29.4}=5.4\text{ m/s}\quad\text{or}\quad\frac{5.4\times 60\times 60}{1000}=19.5\text{ km/h}$$

$$\text{Similarly, } v_{\min}=\sqrt{gr\frac{(\tan\theta-\mu)}{(1+\mu\tan\theta)}}=\sqrt{2\times 9.8\left(\frac{1-0.2}{1+0.2}\right)}$$

$$=3.6\text{ m/s}\quad\text{or}\quad\frac{3.6\times 60\times 60}{1000}=12.966\text{ km/h}$$

Since, speed of cyclist (18 km h^{-1}) is between v_{max} and v_{min}, he can safely turn.
Statements II is true.

14. (a) : Given, half life of Au^{198} = 2.7 days
Atomic weight of Au = 198 g mol^{-1}
Half life and decay constant are related as

$$\lambda=\frac{\ln 2}{t_{1/2}}=\frac{0.693}{t_{1/2}}=\frac{0.693}{2.7\times 24\times 3600}=2.9\times 10^{-6}\,s^{-1}$$

Activity, $A = \lambda N$

As, $N=\frac{1.5\times 10^{-3}}{198}\times 6\times 10^{23}=4.5\times 10^{18}$

So, $A = 2.9 \times 10^{-6} \times 4.5 \times 10^{18}$
$= 1.3 \times 10^{13}$ disintegration/sec.

$$=\frac{1.3\times 10^{13}}{3.7\times 10^{10}}\text{Ci}$$

= 351 Ci (1 curie = 3.7×10^{10} Bq)
It is nearest to the option (a).

15. (b) : Given, half life of substance = 20 minutes
= 20 × 60 s = 1200 s

Decay constant, $\lambda=\frac{0.693}{t_{1/2}}$

or $t_{1/2}=\frac{0.693}{\lambda}=\frac{\ln 2}{\lambda}$ ...(i)

We know, $N=N_0e^{-\lambda t}$

$\frac{N}{N_0}=e^{-\lambda t}\Rightarrow 0.67=e^{-\lambda t_1}$

$-\lambda t_1 = \ln(0.67)$

or $t_1=\frac{-\ln(0.67)}{\lambda}$

Similarly, $t_2=\frac{-\ln(0.33)}{\lambda}$

Required time interval, $t_2-t_1=\frac{-\ln(0.33)}{\lambda}+\ln\frac{(0.67)}{\lambda}$

$=\frac{1}{\lambda}\left[\ln\left(\frac{0.67}{0.33}\right)\right]=\frac{\ln 2}{\lambda}=t_{1/2}$ [From (i)]

= 20 minutes

Therefore, required time interval is, $t_2 - t_1$ = 20 minutes.

16. (c) : Given, mass, M = 500 g = 0.5 kg
Decay constant = 20 g/s
In damping SHM, amplitude is given by

$A=A_0\,e^{-bt/2m}$

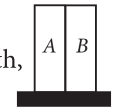
$\frac{A}{A_0}=e^{-bt/2m}$

As per the question, $\frac{1}{2}=e^{-bt/2m}$

$\frac{-bt}{2m}=\ln\left(\frac{1}{2}\right)\Rightarrow\frac{bt}{2m}=\ln 2=0.693$

or $t=\frac{2m\times 0.693}{b}=\frac{2\times 0.5\times 0.693}{20\times 10^{-3}}$ = 34.65 s

17. (d) : Given, $\alpha_A > \alpha_B$



where α is coefficient of expansion.
When both metals A and B are placed in cold bath,
$\Delta L_A > \Delta L_B$
So, length of strip A decreases more in comparison to that of B.
∴ It will bend towards left.

18. (*) : Given, velocity, $\vec{v}=0.5t^2\,\hat{i}+3t\,\hat{j}+9\hat{k}$ m s^{-1}
time, t = 2 s

Velocity after 2 s is $v_{|t=2}=0.5\times(2)^2\,\hat{i}+3\times 2\,\hat{j}+9\hat{k}$

$$=2\hat{i}+6\hat{j}+9\hat{k}$$

Magnitude of velocity, $|\vec{v}|=\sqrt{4+36+81}=11$ m s^{-1}

Angle made by direction of mosquito will be

$\cos^{-1}\left(\frac{2}{11}\right)$ from x axis or $\tan^{-1}\left(\frac{\sqrt{117}}{2}\right)$

$\cos^{-1}\left(\frac{6}{11}\right)$ from y axis or $\tan^{-1}\left(\frac{\sqrt{85}}{6}\right)$

$\cos^{-1}\frac{9}{11}$ from z axis or $\tan^{-1}\frac{\sqrt{40}}{9}$

*None of the given options is correct and the official answer key is given by NTA is option (b).

19. (b) : Nature of flow is denoted by Relynold's number

$R_e=\frac{\rho v D}{\eta}$ where, $v=\frac{\text{Rate of flow}}{\pi r^2}$

ρ = density of fluid
$D \longrightarrow$ diameter
η = coefficient of viscosity
If $R_e < 1000$, flow is steady.
$1000 < R_e < 2000$, flow becomes unsteady.
$R_e > 2000$, flow is turbulent.

$$(R_e)_{\text{Initial}}=\frac{10^3\times 0.18\times 10^{-3}\times 10^{-2}}{\pi\times(0.5\times 10^{-2})^2\times 60\times 10^{-3}}=382.16$$

$$(R_e)_{\text{Final}}=10^3\times\frac{0.48\times 10^{-3}\times 10^{-2}}{\pi\times(0.5\times 10^{-2})^2\times 60\times 10^{-3}}=1019.09$$

Steady flow to unsteady flow will be the nature of flow of water from a circular tap.

20. (c) : Given, mass of wooden block, M = 5.99 kg
Mass of bullet, M = 10 g = 10×10^{-3} kg = 10^{-2} kg
Initial momentum of wood + bullet, $p_i = 0.01 \times u + 0$
Final momentum of wood + bullet, $p_f = (m + M)\,v$
$= (0.01 + 5.99) = 6\,v$
where v is the final velocity after collision.
As no external force is acting, $p_i = p_f$
$0.01\,u = 6\,v$

$v=\frac{0.01\,u}{6}$ ...(i)

By energy conservation, P.E. = K.E.

$$(m+M)gh = \frac{1}{2}(m+M)v^2$$

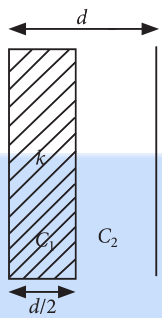
$$6\times 9.8\times 9.8\times 10^{-2} = \frac{1}{2}\times 6\times v^2 \Rightarrow v^2 = \frac{2}{100}\times 9.8\times 9.8$$

$$\therefore\ v = \frac{9.8}{\sqrt{50}}\ \text{m s}^{-1}$$

Initial velocity of bullet, $u = \dfrac{6v}{0.01}$ [From (i)]

$$= \frac{6\times 9.8}{\sqrt{50}\times 0.01} = 831.4\ \text{m s}^{-1}$$

21. (3) : Given, area of plate, $A = 2\ \text{m}^2$
Distance between plates, $d = 1$ m,
Dielectric constant, $k = 3.2$
As this system can be considered as the combination of two capacitors in series.



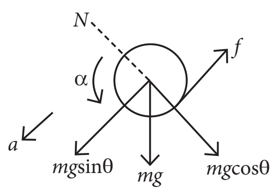
So, equivalent capacitance, $C_{eq} = \dfrac{C_1C_2}{C_1+C_2}$

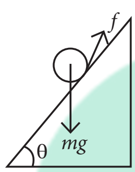
$$= \frac{k\varepsilon_0\dfrac{A}{d/2}\cdot\varepsilon_0\dfrac{A}{d/2}}{\dfrac{k\varepsilon_0 A}{d/2}+\dfrac{\varepsilon_0 A}{d/2}} = \frac{k\varepsilon_0\dfrac{A}{d/2}}{k+1} = 2\frac{k}{d}\frac{\varepsilon_0 A}{(k+1)}$$

$$\text{or}\quad C_{eq} = \frac{2k\varepsilon_0 A}{d(k+1)} = \frac{2\times 3.2\times\varepsilon_0\times 2}{1(3.2+1)} \simeq 3\varepsilon_0$$

$$\therefore\ C_{eq} = 3\,\varepsilon_0$$

22. (3) : Free body diagram of a solid disc is





If a is the acceleration of the disc, then, $a = \dfrac{g\sin\theta}{1+\dfrac{k^2}{r^2}}$

For solid disc, $\dfrac{k^2}{r^2} = \dfrac{1}{2}$

$$\text{So, } a = \frac{g\sin\theta}{1+\frac{1}{2}} = \frac{g\sin\theta}{\frac{3}{2}} \quad\text{or}\quad a = \frac{2}{3}g\sin\theta$$

On comparing with, $a = \dfrac{2}{b}g\sin\theta$ we get, $b = 3$

23. (113*): Given, temperature of source,
$T_1 = 127°\text{C}$
$= 127 + 273 = 400$ K, $\eta = 0.6$
We know that for Carnot engine,

$$\eta = 1 - \frac{T_2}{T_1}$$

where T_2 is the sink temperature.

$$0.6 = 1 - \frac{T_2}{400} \Rightarrow \frac{T_2}{400} = 0.4$$

$\therefore$ $T_2 = 160\ \text{K} = 160 - 273 = -113°\text{C}$
*This question has been dropped by NTA.

24. (2500) : Given, energy dissipated,
$H = 10\ \text{mJ} = 10\times 10^{-3}\ \text{J} = 10^{-2}\ \text{J}$
Time, $t = 1$ s
Electric current, $I = 2\ \text{mA} = 2\times 10^{-3}$ A
Let R be the resistance of resistor,
From Joule's law,
$H = I^2Rt$
$\Rightarrow$ $10^{-2} = (2\times 10^{-3})^2\times R\times 1$

$$\therefore\ R = \frac{1\times 10^6}{100\times 4} = 2500\ \Omega$$

25. (4) : We know, $V = \sqrt{\dfrac{\beta}{\rho}} = \sqrt{\dfrac{1}{K\rho}}$

where K is the compressibility $= \dfrac{1}{\beta}$.

As both the pipes are vibrating in first overtone.

For closed pipe, $f = \dfrac{3}{4L}\sqrt{\dfrac{1}{K_1\rho_1}}$

For open pipe, $f = \dfrac{1}{L_2}\sqrt{\dfrac{1}{K_2\rho_2}}$

Frequency of both the pipes is given as same.

$$\frac{3}{4L}\sqrt{\frac{1}{K_1\rho_1}} = \frac{1}{L_2}\sqrt{\frac{1}{K_2\rho_2}}$$

$K_1 = K_2$ (given)

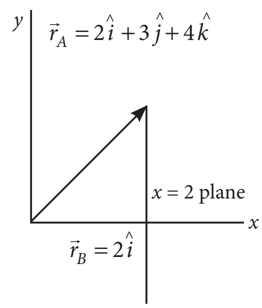
$$\frac{3}{4L}\sqrt{\frac{1}{\rho_1}} = \frac{1}{L_2}\sqrt{\frac{1}{\rho_2}} \quad\text{or}\quad L_2 = \frac{4L}{3}\sqrt{\frac{\rho_1}{\rho_2}}$$

On comparing the given value with $\dfrac{x}{3}L\sqrt{\dfrac{\rho_1}{\rho_2}}$ we get, $x = 4$.

26. (20) : Given, $\vec{F} = 4\hat{i}+3\hat{j}+4\hat{k}$

$$\vec{r} = \vec{r}_B - \vec{r}_A = 2\hat{i} - (2\hat{i}+3\hat{j}+4\hat{k}) = -3\hat{j}-4\hat{k}$$

Torque is given by, $\vec{\tau} = \vec{r}\times\vec{F}$



$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -3 & -4 \\ 4 & 3 & 4 \end{vmatrix}$$

$$\vec{\tau} = \hat{i}(-12+12) - \hat{j}(16) + \hat{k}(12)$$

$$\vec{\tau} = -16\hat{j}+12\hat{k}$$

Magnitude of torque,

$$|\vec{\tau}| = \sqrt{(-16)^2+(12)^2} = \sqrt{256+144} = \sqrt{400}$$

$$|\vec{\tau}| = 20\ \text{N m}$$

27. (12) : Given, mass of body, $m = 2$ kg
Force, $\vec{F} = (2\hat{i}+3\hat{j}+5\hat{k})$ N

Initial velocity, $u = 0$

Time, $t = 4$ s

Acceleration of body, $\vec{a} = \frac{\vec{F}}{m} = \left(\frac{2\hat{i}+3\hat{j}+5\hat{k}}{2}\right)\text{m s}^{-2}$

Initial position vector, $\vec{r}_i = 0\hat{i}+0\hat{j}+0\hat{k}$ [Object is at origin]

Final position vector, $\vec{r}_f = 8\hat{i}+b\hat{j}+20\hat{k}$

Displacement, $\vec{S} = \vec{r}_f - \vec{r}_i = 8\hat{i}+b\hat{j}+20\hat{k}$

From second equation of motion,

$S = ut + \frac{1}{2}at^2$

$8\hat{i}+b\hat{j}+20\hat{k} = \frac{1}{2}\left(\frac{2\hat{i}+3\hat{j}+5\hat{k}}{2}\right)\times(4)^2$

$8\hat{i}+b\hat{j}+20\hat{k} = (2\hat{i}+3\hat{j}+5\hat{k})\cdot 4 = 8\hat{i}+12\hat{j}+20\hat{k}$

On comparing, we get $b = 12$.

28. (12) : Given, deviation, $\delta_{net} = 2°$

Dispersive power of crown glass $(\omega_1) = 0.02$

Dispersive power of flint glass $(\omega_2) = 0.03$

Refractive index of yellow light for crown glass, $\mu_1 = 1.5$

Refractive index of yellow light for flint glass, $\mu_2 = 1.6$

Let A_1 be the refractive angle for crown glass and A_2 be the refractive angle for flint glass.

Deviation is given by,

$\delta_{net} = (\mu_1 - 1)A_1 - (\mu_2 - 1)A_2$

$2° = (1.5 - 1)A_1 - (1.6 - 1)A_2$

$2° = 0.5A_1 - 0.6\,A_2$ ...(i)

Also, for no dispersion, we have

$\omega_1\ (\mu_1 - 1)A_1 = \omega_2(\mu_2 - 1)A_2$

$0.02(1.5 - 1)\,A_1 = 0.03(1.6 - 1)A_2$

$0.02 \times 0.5\,A_1 = 0.03 \times 0.6\,A_2$

$0.01\,A_1 = 0.018\,A_2$

$10\,A_1 = 18\,A_2$ ...(ii)

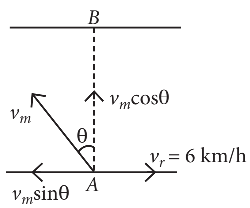
From equation (i) and (ii), we get

$2° = 0.5\,A_1 - 0.6 \times \frac{10\,A_1}{18\times 10} \Rightarrow A_1 = 12°$

29. (120) : Velocity of swimmer in still water, $v_m = 12$ km/h

Velocity of water flowing, $v_r = 6$ km/h

From diagram,



$v_m\sin\theta = v_r$

$v_m\sin\theta = 6$

$12\sin\theta = 6$ or $\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6} = 30°$

$\therefore$ Direction with respect to flow of water

$= 90° + 30° = 120°$

30. (3) : We know, self energy of Earth $= -\frac{3}{5}\frac{GM^2}{R}$

Energy required to break Earth into pieces = –self energy

$= -\left(-\frac{3}{5}\frac{GM^2}{R}\right) = \frac{3}{5}\frac{GM^2}{R}$

On comparing with $\frac{x}{5}\frac{GM^2}{R}$ we get, $x = 3$.

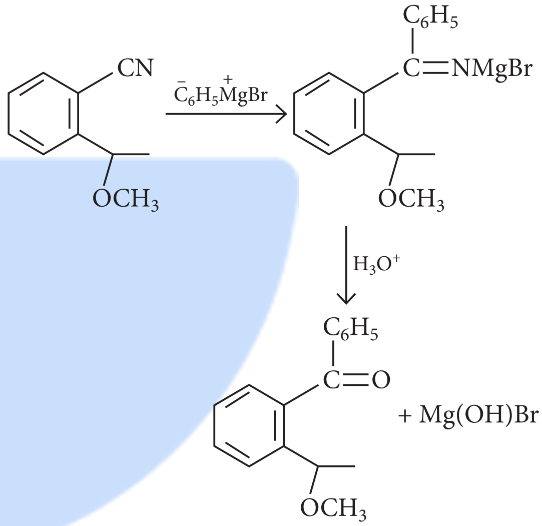
31. (c) : NaH acts as a reducing agent.

The lone pair on nitrogen in pyridine does not get delocalised and is available easily for donation, making it basic.

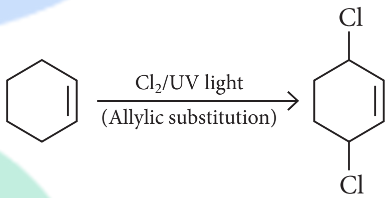
$NaH \rightleftharpoons Na^+ + H^-$

 (Pyridine)

32. (a) :



33. (d) :



34. (c) : The first ionisation enthalpy of X is less than that of Y. For Na, the electron is removed from $3s^1$ and for Mg, the electron is removed from $3s^2$. Besides, a high jump is seen in first and second ionisation enthalpies of X, which is seen in the case of alkali metals (like Na) as well since the second electron needs to be removed from noble gas core configuration. Hence, X is Na and Y is Mg.

35. (b) : The metals which occupy higher positions in the reactivity series are reduced by electrolytic reduction method, for example, Al.

36. (b) : For H_3PO_3,

$H_3PO_3 + 2NaOH \longrightarrow Na_2HPO_3 + 2H_2O$

Number of moles of $H_3PO_3 = 0.05$ mol

Number of moles of NaOH required $= 0.05 \times 2 = 0.1$ mol

Volume of NaOH = 0.1 L = 100 mL

For H_3PO_2,

$NaOH + H_3PO_2 \longrightarrow NaH_2PO_2 + H_2O$

Number of moles of $H_3PO_2 = 0.2$ mol

Number of moles of NaOH required $= 0.2 \times 1 = 0.2$ mol

Volume of NaOH = 0.2 L = 200 mL

37. (a) : (A) → (iii), (B) → (i), (C) → (iv), (D) → (ii)

38. (d) : An ordinary filter paper cannot stop the flow of colloidal particles.

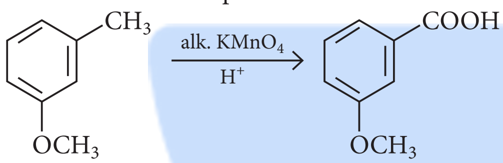
39. (b) : H_2O_2 is used in the treatment of effluents. The oxidation state of O in H_2O_2 is –1. Hence, it acts as both oxidising and reducing agent. It is miscible with water due to the formation of intermolecular H-bonding. Since, it has an open book structure, the two hydroxyl group do not lie in same plane.

40. (a) : CO_2, CH_4 and water vapour are considered as greenhouse gases because they have the property of absorbing IR radiations and re-radiating them back to the earth's surface.

41. (b) : Urea formaldehyde polymer is used in the manufacture of wood laminates.

42. (a) : The purpose of NaOH in ammonolysis of alkyl halides is to remove acidic impurities.

43. (a) :



44. (a) : Atomic number 33 ⇒ Arsenic (As) ⟶ Metalloid (Group 15)
Atomic number 53 ⇒ Iodine (I) ⟶ Non-metal (Group 17)
Atomic number 83 ⇒ Bismuth (Bi) ⟶ Metal (Group 15)

45. (d) : Higher resonance in $(CH_3CO)_2\ddot{N}H$ leads to less availability of lone pair on N, making it least basic.

46. (a) : x = F, Cl, Br, I and y = F, Cl, Br because I^- is a good reducing agent and does not form FeI_3.

47. (d) : It contains 12 five membered rings and 20 six membered rings.

48. (c) : The spin only magnetic moment is directly proportional to the number of unpaired electrons in the complex.
$(NH_4)_2[Ce(NO_3)_6]$, Ce^{4+}, $n = 0$, $\mu = 0$
$Eu(NO_3)_3$, Eu^{3+}, $n = 6$, $\mu = 6.93$ BM
$Gd(NO_3)_3$, Gd^{3+}, $n = 7$, $\mu = 7.94$ BM

49. (c) : The secondary structure of protein is stabilised by H-bonding.

50. (a) : $HC{\equiv}C{-}CH_2{-}CH_3 \xrightarrow[H_2O]{O_3} \underset{(A)}{CH_3CH_2COOH + HCOOH}$

$$HCOOH \xrightarrow[{[Ag(NH_3)_2]^+}]{\text{Tollen's test}} CO_2 + H_2O + 2Ag\downarrow$$

51. (3) : For $n = 5$
$l = 0, 1, 2, 3, 4$
For $l = 2$, $m = -2, -1, 0, 1, 2$
For $l = 3$, $m = -3, -2, -1, 0, 1, 2, 3$
For $l = 4$, $m = -4, -3, -2, -1, 0, 1, 2, 3, 4$
Hence, the number of orbitals with $n = 5$, $m_l = +2$ is 3.

52. (1) : $K_{a_2} << K_{a_1}$, hence H^+ ions are mainly released from first dissociation.

$$[H^+] = \sqrt{K_{a_1} \times c} = \sqrt{1.7\times10^{-2}\times0.588}$$

$[H^+] = 0.9998 \times 10^{-1} = 0.09998$
pH = –log $[H^+]$ = –log [0.09998] = 1.000087
Hence, pH (to nearest integer value) = 1

53. (–2218) : $Fe + 2HCl \longrightarrow FeCl_2 + H_2$
One mole of Fe forms one mole of H_2
$\frac{50}{55.85}$ mol of Fe = 0.89525 mol of Fe forms 0.89525 mol of H_2 gas.

$$W_{\text{irreversible}} = -P_{\text{ext.}}\,\Delta V = -n_{H_2}RT$$
$$= -0.89525 \times 8.314 \times 298 \text{ J} = -2218.05 \text{ J}$$

54. (108) : $A \xrightarrow{54\text{ min}} \frac{A}{2} \xrightarrow{54\text{ min}} \frac{A}{4}$

$$B \xrightarrow{18\text{ min}} \frac{B}{2} \xrightarrow{18\text{ min}} \frac{B}{4} \xrightarrow{18\text{ min}} \frac{B}{8} \xrightarrow{18\text{ min}} \frac{B}{16} \xrightarrow{18\text{ min}} \frac{B}{32} \xrightarrow{18\text{ min}} \frac{B}{64}$$

If $[A]_0 = [B]_0 = x$ then,
Conc. of A after two half lives $= \frac{x}{4}$
Conc. of B after six half lives $= \frac{x}{64}$
Hence, after 108 minutes conc. of A *i.e.*, $\frac{x}{4}$ will be 16 times the conc. of B *i.e.*, $\frac{x}{64}$.

55. (4) : $\Delta_0 = \frac{hc}{\lambda_{\text{abs.}}} = \frac{6.626\times10^{-34}\times3\times10^8}{498\times10^{-9}} = 4\times10^{-19}$ J

56. (14) : $G = 0.55$ mS

$$\lambda_m = \frac{\kappa\times1000}{c\,(\text{mol / mL})}$$

$$\kappa = \frac{Gl}{A} = 0.55 \times 1.3 \times 10^2 \text{ mS m}^{-1}$$

$$\lambda_m = \frac{0.55\times1.3\times10^2\times10^{-3}}{5\times10^{-3}}$$

$\lambda_m = 14.3\ (\text{mS})\text{m}^2\text{ mol}^{-1}$
$\approx 14\ (\text{mS})\text{m}^2\text{ mol}^{-1}$

57. (19) : Number of moles of $N_2 = \frac{758-14}{760}\times\frac{30\times10^{-3}}{0.0821\times287}$
$= 1.246 \times 10^{-3}$ mol

Mass of $N_2 = 1.246 \times 10^{-3} \times 28$ g

$$\%\text{ of }N_2 = \frac{1.246\times10^{-3}\times28}{0.1840}\times100 = 18.96\% \approx 19\%$$

58. (15) : Number of moles of Ga $= \frac{0.581}{70}$

Number of atoms of Ga $= \frac{0.581\,N_A}{70}$

Total number of voids

$$= \frac{3\times0.581\times6.023\times10^{23}}{70} = 15\times10^{21} \text{ voids}$$

59. (19) : $P_A^\circ = 21$ kPa

$P_B^\circ = 18$ kPa

$x_A = \frac{1}{3}$

$x_B = \frac{2}{3}$

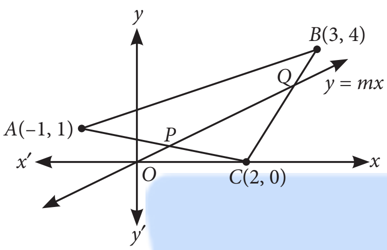
$P = P_A^\circ x_A + P_B^\circ x_B = 21 \times \frac{1}{3}$ kPa $+ 18 \times \frac{2}{3}$ kPa

$= 7$ kPa $+ 12$ kPa $= 19$ kPa

60. (525) : $3Pb(NO_3)_2 + Cr_2(SO_4)_3 \longrightarrow 3PbSO_4\downarrow + 2Cr(NO_3)_3$

Number of moles of lead nitrate = 0.00525 mol
Number of moles of chromic sulphate = 0.0024 mol
Limiting reagent $\rightarrow Pb(NO_3)_2$
0.00525 mol of $Pb(NO_3)_2$ produces 0.00525 mol of $PbSO_4$
0.00525 mol of $PbSO_4 = 525 \times 10^{-5}$ mol of $PbSO_4$

61. (b) :



$$A_1 = \frac{1}{2}\begin{vmatrix} -1 & 1 & 1 \\ 3 & 4 & 1 \\ 2 & 0 & 1 \end{vmatrix} = \frac{13}{2}$$

Let coordinates of points P and Q respectively are (x_1, mx_1) and (x_2, mx_2).

$$A_2 = \frac{1}{2}\begin{vmatrix} 2 & 0 & 1 \\ x_1 & mx_1 & 1 \\ x_2 & mx_2 & 1 \end{vmatrix} = \frac{1}{2}\left|2(mx_1 - mx_2)\right| = m|x_1 - x_2|$$

Equation of line AC, $x + 3y - 2 = 0$

and $P \equiv (x_1, mx_1) \Rightarrow x_1 = \dfrac{2}{1+3m}$

Equation of line BC, $4x - y - 8 = 0$

and $Q \equiv (x_2, mx_2) \Rightarrow x_2 = \dfrac{8}{4-m}$

Now, $A_1 = 3A_2 \Rightarrow \dfrac{13}{2} = 3m\left|\dfrac{2}{1+3m} - \dfrac{8}{4-m}\right|$

$$\Rightarrow \frac{13}{6m} = \frac{26m}{(3m+1)(4-m)}$$

Only option (b) *i.e.*, $m = 1$ satisfies it.

62. (a) : The equation of plane be $x + y + z - 42 = 0$
Let $x - 11 = p$, $y - 19 = q$ and $z - 12 = r$

$\Rightarrow p + q + r = 0 \Rightarrow p^3 + q^3 + r^3 = 3pqr$ ...(i)

Now, $3 + \dfrac{p}{q^2r^2} + \dfrac{q}{p^2r^2} + \dfrac{r}{p^2q^2} - \dfrac{42}{14pqr}$

$$= 3 + \frac{p^3 + q^3 + r^3}{p^2q^2r^2} - \frac{3}{pqr}$$

$$= 3 + \frac{3pqr}{p^2q^2r^2} - \frac{3}{pqr} \quad \text{[Using (i)]} = 3$$

63. (d) : Let $(t, \pm\sqrt{3}t)$ be the point of intersection.
So, $t^2 + 3t^2 = 4b \Rightarrow t^2 = b$

Also, $\dfrac{t^2}{16} + \dfrac{3t^2}{b^2} = 1 \Rightarrow \dfrac{b}{16} + \dfrac{3b}{b^2} = 1$

$\Rightarrow b^2 - 16b + 48 = 0 \Rightarrow b = 4, 12$

But $b > 4 \Rightarrow b = 12$

64. (b) : We have, $I = \displaystyle\int_0^{10} [x]\, e^{[x]-x+1} dx$

$$= \int_0^1 0\,dx + \int_1^2 1\cdot e^{1-x+1}\,dx + \int_2^3 2\cdot e^{2-x+1}\,dx + \ldots\ldots + \int_9^{10} 9\cdot e^{9-x+1}\,dx$$

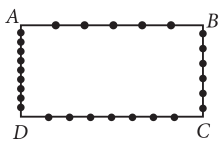
$$= \sum_{n=0}^{9}\int_n^{n+1} n\cdot e^{n+1-x}\,dx = -\sum_{n=0}^{9} n\left(e^{n+1-x}\right)_n^{n+1}$$

$$= -\sum_{n=0}^{9} n\,(e^0 - e^1) = (e-1)\cdot\frac{9\times 10}{2} = 45(e-1)$$

65. (d) : Total cases are $6\,[6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1] = 6\cdot 6!$
For favourable cases :
Case I : When 1, 2, 3, 4, 5, 6 are included
$\therefore$ Number of cases = 6!
Case II : When 0, 1, 2, 4, 5, 6 are included
$\therefore$ Number of cases = $5\cdot 5!$
Case III : When 0, 1, 2, 3, 4, 5 are included
$\therefore$ Number of cases = $5\cdot 5!$

$\therefore$ Required probability $= \dfrac{6! + 5\cdot 5! + 5\cdot 5!}{6\cdot 6!} = \dfrac{6+10}{36} = \dfrac{4}{9}$

66. (d) : α = Number of triangles formed
$= 5 \times 7 \times 6 + 5 \times 9 \times 7 + 5 \times 9 \times 6 + 7 \times 9 \times 6$
$= 210 + 315 + 270 + 378 = 1173$
β = Number of quadrilateral formed
$= 5 \times 6 \times 7 \times 9 = 1890$
$\Rightarrow \beta - \alpha = 1890 - 1173 = 717$



67. (b) : We have, $\vec{r}\times\vec{a} = \vec{b}\times\vec{r} \Rightarrow \vec{r}\times\vec{a} + \vec{r}\times\vec{b} = 0$

$\Rightarrow \vec{r}\times(\vec{a}+\vec{b}) = 0$

$\Rightarrow \vec{r} = \lambda(\vec{a}+\vec{b}) \Rightarrow \vec{r} = \lambda(3\hat{i} - \hat{j} + 2\hat{k})$

Also, $\vec{r}\cdot(\alpha\hat{i} + 2\hat{j} + \hat{k}) = 3$

$\Rightarrow \lambda(3\hat{i} - \hat{j} + 2\hat{k})\,(\alpha\hat{i} + 2\hat{j} + \hat{k}) = 3$

$\Rightarrow \lambda(3\alpha - 2 + 2) = 3 \Rightarrow \lambda = 1/\alpha$

Also, $\vec{r}\cdot\left(2\hat{i} + 5\hat{j} - \alpha\hat{k}\right) = -1$

$\Rightarrow \lambda\left(3\hat{i} - \hat{j} + 2\hat{k}\right)\left(2\hat{i} + 5\hat{j} - \alpha\hat{k}\right) = -1 \Rightarrow \lambda(6 - 5 - 2\alpha) = -1$

$$\Rightarrow \lambda\left(1 - \frac{2}{\lambda}\right) = -1 \Rightarrow \lambda - 2 = -1 \Rightarrow \lambda = 1$$

So, $\lambda = 1$, $\alpha = 1$

Hence, $\alpha + |\vec{r}|^2 = 1 + (3^2 + 1^2 + 2^2) = 15$

68. (a) : Given, $2xy\dfrac{dy}{dx} = y^2 - x^2 \Rightarrow \dfrac{dx}{dy} = \dfrac{2xy}{y^2 - x^2}$

Putting $x = vy \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$, we get

$$v + y\frac{dv}{dy} = \frac{2v}{1-v^2} \Rightarrow y\frac{dv}{dy} = \frac{v^3+v}{1-v^2}$$

$$\Rightarrow \frac{1-v^2}{v(v^2+1)}dv = \frac{1}{y}dy \Rightarrow \int \frac{1-v^2}{v(v^2+1)}\,dv = \int \frac{1}{y}\,dy$$

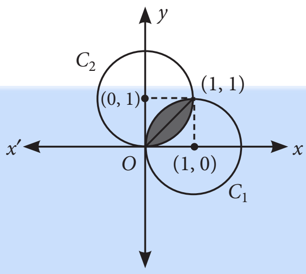
$$\Rightarrow \log\left(\frac{v}{v^2+1}\right) = \log(y) + \log c$$

$$\Rightarrow \frac{\frac{x}{y}}{\left(\frac{x}{y}\right)^2+1} = yc \Rightarrow \frac{x}{x^2+y^2} = c$$

This passes through (1, 1)

$\Rightarrow \; c = 1/2$

So, $C_1 \equiv x^2 + y^2 - 2x = 0$



Similarly, after solving

$\frac{2xy}{x^2-y^2} = \frac{dy}{dx}$, we get

$C_2 \equiv x^2 + y^2 - 2y = 0$

$\therefore$ Required area $= 2\left[\frac{\pi}{4}(1)^2\right] - (1)^2 = \frac{\pi}{2} - 1$

69. (a) : We have, $P(x) = x^2 + bx + c$

$P(2) = 5 \Rightarrow 2b + c = 1$ ...(i)

Also, $\int_0^1 P(x)\,dx = 1 \Rightarrow \int_0^1 (x^2 + bx + c)dx = 1$

$\Rightarrow \left[\frac{x^3}{3} + \frac{bx^2}{2} + cx\right]_0^1 = 1 \Rightarrow 3b + 6c = 4$ ...(ii)

Solving (i) and (ii), we get $b = \frac{2}{9}$ and $c = \frac{5}{9}$

So, $9(b+c) = 9\left(\frac{2}{9} + \frac{5}{9}\right) = 7$

70. (a) : We have, $f(x) = \begin{vmatrix} \sin^2 x & 1+\cos^2 x & \cos 2x \\ 1+\sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & \sin 2x \end{vmatrix}$

Applying $R_1 \to R_1 - R_2$, we get

$$\begin{vmatrix} -1 & 1 & 0 \\ 1+\sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & \sin 2x \end{vmatrix}$$

Applying $C_1 \to C_1 + C_2$, we get

$$\begin{vmatrix} 0 & 1 & 0 \\ 2 & \cos^2 x & \cos 2x \\ 1 & \cos^2 x & \sin 2x \end{vmatrix} = \cos 2x - 2\sin 2x$$

$\therefore$ Maximum value of $f(x) = \sqrt{1^2 + 2^2} = \sqrt{5}$

71. (c) : Let $P(4, 3, 8)$ and $Q(3, 5, 7)$

Dr's of PQ are $(1, -2, 1)$

$\because \; PQ \perp L_1 \Rightarrow l - 6 + 4 = 0 \Rightarrow l = 2$

So, line $L_1 : \frac{x-a}{2} = \frac{y-2}{3} = \frac{z-b}{4} = k$ (say)

$\Rightarrow \; 2k + a = 3, \; 3k + 2 = 5$ and $4k + b = 7$

$\Rightarrow \; a = 1, b = 3$

$\therefore$ Equation of L_1 be $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$

and L_2 be $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$

$\therefore$ Shortest distance between lines L_1 and L_2 is given by

$$\text{S.D.} = \frac{|(\hat{i}+2\hat{j}+2\hat{k})\cdot(-\hat{i}+2\hat{j}-\hat{k})|}{|-\hat{i}+2\hat{j}-\hat{k}|} = \frac{|-1+4-2|}{\sqrt{6}} = \frac{1}{\sqrt{6}}$$

72. (c) : $(a, b) \simeq (c, d) \Leftrightarrow ad = bc$

$\Rightarrow \frac{a}{c} = \frac{b}{d} \Rightarrow \frac{4}{c} = \frac{3}{d} \Rightarrow \frac{c}{d} = \frac{4}{3}$

$(c, d) \in (2, 3, 4, ..., 30)$

$\therefore \; (c, d) = (4, 3), (8, 6), (12, 9), (16, 12), (20, 15), (24, 18), (28, 21)$

$\therefore$ 7 ordered pair exist.

73. (a) : L.H.L. $= \lim_{x \to 0^-} \frac{\cos^{-1}\left(1-(1+x)^2\right)\sin^{-1}(-x)}{(1+x)-(1+x)^3}$

$$= \lim_{h\to 0} \frac{\cos^{-1}(1-(1-h)^2) - \sin^{-1}(h)}{(1-h)-(1-h)^3}$$

$$= \lim_{h\to 0} \frac{\cos^{-1}(1-(1-h)^2)\sin^{-1}(h)}{(1-h)(h)(2-h)} = \frac{\cos^{-1}(0)\cdot 1}{1\cdot 2} = \frac{\frac{\pi}{2}\cdot 1}{2} = \frac{\pi}{4}$$

R.H.L. $= \lim_{x\to 0^+} \frac{\cos^{-1}(1-x^2)\sin^{-1}(1-x)}{x-x^3}$

$$= \lim_{h\to 0} \frac{\cos^{-1}(1-h^2)\sin^{-1}(1-h)}{h-h^3}$$

$$= \lim_{h\to 0} \frac{\sin^{-1}\sqrt{1-(1-h^2)^2}\,\sin^{-1}(1-h)}{h(1-h^2)}$$

$$= \lim_{h\to 0} \frac{\sin^{-1}(h\sqrt{2-h^2})\,\sin^{-1}(1-h)}{h(1-h^2)}$$

$$= \lim_{h\to 0} \frac{\sin^{-1}(h\sqrt{2-h^2})}{h\sqrt{2-h^2}} \cdot \frac{h\sqrt{2-h^2}}{h(1-h^2)} \sin^{-1}(1-h)$$

$$= 1 \times \sqrt{2} \times \frac{\pi}{2} = \frac{\pi}{\sqrt{2}}$$

$\therefore$ L.H.L. $\neq$ R.H.L.

$\therefore$ $f(x)$ is not continuous at $x = 0$.

$\therefore$ No such α exists.

74. (d) : We have, $\sin^{-1}\left(\frac{3x}{5}\right) + \sin^{-1}\left(\frac{4x}{5}\right) = \sin^{-1} x$

$$= \sin^{-1}\left(\frac{3x}{5}\sqrt{1-\left(\frac{4x}{5}\right)^2} + \frac{4x}{5}\sqrt{1-\left(\frac{3x}{5}\right)^2}\right) = \sin^{-1} x$$

$\Rightarrow \quad 3x\sqrt{25-16x^2}+4x\sqrt{25-9x^2}=25x$

$\Rightarrow \quad x=0 \text{ or } 3\sqrt{25-16x^2}+4\sqrt{25-9x^2}=25$

$\Rightarrow \quad 4\sqrt{25-9x^2}=25-3\sqrt{25-16x^2}$

Squaring both sides, we get

$16\left(25-9x^2\right)=625+9(25-16x^2)-150\sqrt{25-16x^2}$

$\Rightarrow \quad 150\sqrt{25-16x^2}=625+225-144x^2-400+144x^2$

$\Rightarrow \quad 150\sqrt{25-16x^2}=450 \Rightarrow \sqrt{25-16x^2}=3$

Again squaring both sides, we get

$25-16x^2=9 \Rightarrow 16x^2=16 \Rightarrow x^2=1$

$\Rightarrow \quad x=\pm 1$

$\therefore \quad x=0, \pm 1$ are three real values of x.

75. (a) : Let $|z|=t,\ t\geq 0$

$\therefore \quad e^{\frac{(t+3)(t-1)}{(t+1)}\log_e 2} \geq \log_{\sqrt{2}} 16$

$\Rightarrow \quad 2^{\frac{(t+3)(t-1)}{(t+1)}} \geq 2^3 \Rightarrow \dfrac{(t+3)(t-1)}{t+1} \geq 3$

$\Rightarrow \quad t^2+2t-3 \geq 3t+3 \Rightarrow t^2-t-6 \geq 0$

$\Rightarrow \quad t \in (-\infty, -2] \cup [3, \infty)$

But $t \geq 0$, $\therefore\ t \in [3, \infty)$

$\Rightarrow$ The least value of $t=|z|$ is 3.

76. (d) : $g(x+1)=\log_e f(x+1)=\log_e x+\log_e f(x)$

$\Rightarrow \quad g(x+1)=\log_e x+g(x)$

$\Rightarrow \quad g'(x+1)=\dfrac{1}{x}+g'(x)$

$\Rightarrow \quad g''(x+1)=\dfrac{-1}{x^2}+g''(x) \Rightarrow g''(x+1)-g''(x)=\dfrac{-1}{x^2}$

Now, put $x=4, 3, 2, 1$ we get

$g''(5)-g''(4)=\dfrac{-1}{16}$

$g''(4)-g''(3)=\dfrac{-1}{9}$

$g''(3)-g''(2)=\dfrac{-1}{4}$

$g''(2)-g''(1)=-1$

By adding above equations, we get

$g''(5)-g''(1)=\left(\dfrac{-1}{16}\right)+\left(\dfrac{-1}{9}\right)+\left(\dfrac{-1}{4}\right)+(-1)$

$\therefore \quad |g''(5)-g''(1)|=\dfrac{205}{144}$

77. (a) : Equation of given parabola is $y^2=4x$.
Curve C is mirror image with respect to line $y=x$.
Equation of curve C is $x^2=4y$
Differentiating w.r.t. 'x', we get

$2x=4\dfrac{dy}{dx}$

$\therefore \quad \left(\dfrac{dy}{dx}\right)_{(2,1)}=1$

So, equation of tangent to curve C at point (2, 1) is
$(y-1)=1(x-2) \Rightarrow x-y=1$

78. (b) : Given equation of circle is
$x^2+y^2+ax+2ay+c=0$

We have, $2\sqrt{g^2-c}=2\sqrt{2} \Rightarrow 2\sqrt{\dfrac{a^2}{4}-c}=2\sqrt{2}$

and $2\sqrt{f^2-c}=2\sqrt{5} \Rightarrow 2\sqrt{a^2-c}=2\sqrt{5}$

$\Rightarrow \quad \dfrac{a^2}{4}-c=2$ and $a^2-c=5$

Solving these equations, we get $c=-1,\ a^2=4$

$\Rightarrow \quad a=-2 \qquad (\because a<0)$

$\therefore$ Equation of circle is $x^2+y^2-2x-4y-1=0$

$\Rightarrow \quad (x-1)^2+(y-2)^2=6$

$\Rightarrow$ Centre $\equiv (1, 2)$ and Radius $=\sqrt{1+4+1}=\sqrt{6}$

Given, $x+2y=0$

$\therefore$ Slope of this line $=-1/2$

$\therefore$ Slope of tangent, $m=2$

Now, equation of tangent with slope m to the circle $(x-1)^2+(y-2)^2=6$ is given by

$y-2=2(x-1) \pm \sqrt{6}\sqrt{1+4}$

$\Rightarrow \quad 2x-y \pm \sqrt{30}=0$

$\therefore$ Shortest distance from origin

$=\left|\dfrac{0+0\pm\sqrt{30}}{\sqrt{4+1}}\right|=\sqrt{6}$

79. (b) : $f'(x)=3\left(\dfrac{x+1}{x-1}\right)\cdot\dfrac{2}{(x+1)^2}+\dfrac{2}{(x-1)^2}$

$=\dfrac{6}{x^2-1}+\dfrac{2}{(x-1)^2}=\dfrac{6(x-1)+2(x+1)}{(x-1)^2(x+1)}$

$\Rightarrow \quad f'(x)=\dfrac{4(2x-1)}{(x+1)(x-1)^2}$

$\therefore \quad f'(x) \geq 0\ \forall\ x \in (-\infty, -1) \cup \left(\left[\dfrac{1}{2}, \infty\right)-\{1\}\right)$

80. (a) : We have, $\dfrac{dy}{dx}+(\tan x)\,y=\sin x$

$\text{I.F.}=e^{\int \frac{\sin x}{\cos x}dx}=\sec x$

$\therefore$ Solution of differential equation is given by

$y\cdot\sec x=\int \dfrac{\sin x}{\cos x}dx+c$

$\Rightarrow \quad y\cdot\sec x=\log_e \sec x+c$

At $y(0)=0 \Rightarrow c=0$

$\therefore \quad y\cdot\sec x=\log_e \sec x$

$\therefore \quad y\left(\dfrac{\pi}{4}\right)\sec\dfrac{\pi}{4}=\log_e \sec\left(\dfrac{\pi}{4}\right)$

$\Rightarrow \quad y\left(\dfrac{\pi}{4}\right)=\dfrac{1}{\sqrt{2}}\log_e\sqrt{2}=\dfrac{1}{2\sqrt{2}}\log_e 2$.

81. (1) : $g(f(x))=\begin{cases} f(x)+1 & , f(x)<0 \\ (f(x)-1)^2+b & , f(x)\geq 0 \end{cases}$

$$\Rightarrow g(f(x))=\begin{cases} x+a+1 & , x\in(-\infty,-a) \text{ and } x\in(-\infty,0) \\ |x-1|+1 & , \quad x\in\phi \\ (x+a-1)^2+b & , x\in[-a,\infty) \text{ and } x\in(-\infty,0) \\ (|x-1|-1)^2+b & , \quad x\in[0,\infty) \end{cases}$$

$$\Rightarrow g(f(x))=\begin{cases} x+a+1 & , x\in(-\infty,-a) \\ (x+a-1)^2+b & , x\in[-a,0) \\ (|x-1|-1)^2+b & , x\in[0,\infty) \end{cases}$$

$\because$ $g(f(x))$ is continuous at $x = -a$ and at $x = 0$

$\Rightarrow$ $1 = b + 1$ and $(a - 1)^2 + b = b$

$\Rightarrow$ $b = 0$ and $a = 1$

$\Rightarrow$ $a + b = 1$.

82. (28) : Given, $\vec{c}=\lambda(\vec{a}\times\vec{b})$,

where, $\vec{a}\times\vec{b}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix}=3\hat{i}-2\hat{j}+\hat{k}$

$\Rightarrow$ $\vec{c}=\lambda(3\hat{i}-2\hat{j}+\hat{k})$

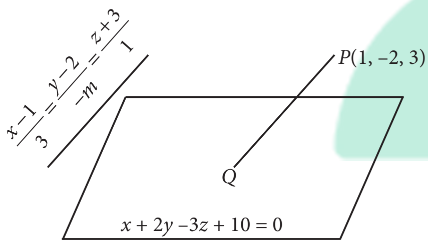
Now, $\vec{c}\cdot(\hat{i}+\hat{j}+3\hat{k})=8$

$\Rightarrow$ $\lambda(3 - 2 + 3) = 8 \Rightarrow \lambda = 2$

$\Rightarrow$ $\vec{c}=2(3\hat{i}-2\hat{j}+\hat{k})$

Hence, $\vec{c}\cdot(\vec{a}\times\vec{b})=2\begin{vmatrix} 3 & -2 & 1 \\ 1 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix}=28$

83. (2) :



Equation of line PQ is

$\dfrac{x-1}{3}=\dfrac{y+2}{-m}=\dfrac{z-3}{1}=r$ (say)

Any point on line PQ is

$((3r + 1), (-mr - 2), r + 3)$

Since, Q lies on $x + 2y - 3z + 10 = 0$

$\therefore$ $3r + 1 - 4 - 2mr - 3r - 9 + 10 = 0$

$\Rightarrow$ $-2 - 2mr = 0 \Rightarrow mr = -1$

$\therefore$ $Q \equiv (3r + 1, -1, r + 3)$

Given, $PQ^2=\dfrac{7}{2}$

$\Rightarrow$ $(3r + 1 - 1)^2 + (-1 + 2)^2 + (r + 3 - 3)^2 = \dfrac{7}{2}$

$\Rightarrow$ $9r^2 + 1 + r^2 = \dfrac{7}{2}$

$\Rightarrow$ $10r^2=\dfrac{5}{2} \Rightarrow r^2=\dfrac{1}{4} \Rightarrow r=\pm\dfrac{1}{2}$

$\therefore$ $mr=-1 \Rightarrow m=\dfrac{-1}{r}=\pm 2$

$\Rightarrow$ $|m| = 2$.

84. (6) : $A=\left(1-\dfrac{1}{2}\right)^n+\left(1-\dfrac{3}{4}\right)^n+\left(1-\dfrac{7}{8}\right)^n+\left(1-\dfrac{15}{16}\right)^n+\left(1-\dfrac{31}{32}\right)^n$

$\Rightarrow$ $A=\dfrac{1}{2^n}+\dfrac{1}{4^n}+\dfrac{1}{8^n}+\dfrac{1}{16^n}+\dfrac{1}{32^n}$

$\Rightarrow$ $A=\dfrac{\frac{1}{2^n}\left(1-\left(\frac{1}{2^n}\right)^5\right)}{1-\frac{1}{2^n}} \Rightarrow (2^n-1)A=1-\dfrac{1}{2^{5n}}$

$\Rightarrow$ $2^n - 1 = 63$ and $5n = 30$ $\quad\left(\because 63A=1-\dfrac{1}{2^{30}}\right)$

$\Rightarrow$ $n = 6$

85. (14) : Given, $\dfrac{1}{16}$, a, b are in G.P. $\Rightarrow 16a^2 = b$

Also, $\dfrac{1}{a},\dfrac{1}{b}, 6$ are in A.P. $\Rightarrow \dfrac{2}{b}=\dfrac{1}{a}+6$

$\Rightarrow$ $\dfrac{2}{16a^2}=\dfrac{1}{a}+6 \Rightarrow \dfrac{1}{8a^2}-\dfrac{1}{a}-6=0$

$\Rightarrow$ $\dfrac{1}{a^2}-\dfrac{8}{a}-48=0 \Rightarrow \dfrac{1}{a}=12,-4 \Rightarrow a=\dfrac{1}{12},\dfrac{-1}{4}$

But $a > 0 \Rightarrow a=\dfrac{1}{12}$

$\therefore$ $b=16\left(\dfrac{1}{12}\right)^2=\dfrac{1}{9}$

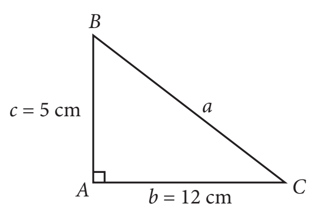
$\therefore$ $72(a + b) = 72\left(\dfrac{1}{12}+\dfrac{1}{9}\right)=14$

86. (15) : Let Δ = Area of $\Delta ABC = 30\text{ cm}^2$

$\Rightarrow$ $\dfrac{1}{2}bc\sin A = 30$

$\Rightarrow$ $\dfrac{1}{2}\times 12\times 5\sin A = 30$

$\Rightarrow$ $\sin A = 1 \Rightarrow A=\dfrac{\pi}{2}$



$\therefore$ $a=\sqrt{12^2+5^2}=13$ cm

Now, $R=\dfrac{a}{2\sin A}=\dfrac{13}{2\times 1}=\dfrac{13}{2}$ cm

Now, $S=\dfrac{a+b+c}{2}=15$ cm

and $r=\dfrac{\Delta}{S}=\dfrac{30}{15}=2$ cm

Hence, $2R + r = 2\times\dfrac{13}{2}+2 = 15$ cm

87. (1) : We have, $A = XB \Rightarrow \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

$\Rightarrow \quad a_1 = \frac{1}{\sqrt{3}}(b_1 - b_2)$ and $a_2 = \frac{1}{\sqrt{3}}(b_1 + kb_2)$

$\therefore \quad a_1^2 + a_2^2 = \frac{1}{3}(b_1 - b_2)^2 + \frac{1}{3}(b_1 + kb_2)^2$

$\Rightarrow \quad \frac{2}{3}b_1^2 + \frac{2}{3}b_2^2 = \frac{2}{3}b_1^2 + \frac{1}{3}(1+k^2)\,b_2^2 + \frac{2}{3}(k-1)b_1 b_2$

On comparing, $\frac{2}{3} = \frac{1}{3}(1+k^2) \Rightarrow k = \pm 1$

and $\frac{2}{3}(k-1) = 0 \Rightarrow k = 1$.

88. (16) : $S_n(x) = (2 + 3 + 6 + 11 + 18 + 27 + \ldots\ldots + T_n)\log_a x$

Let $S = 2 + 3 + 6 + 11 + 18 + 27 + \ldots\ldots + T_{n-1} + T_n$ ...(i)

Also, $S = 2 + 3 + 6 + 11 + 18 + \ldots\ldots + T_n$ ...(ii)

Subtracting (i) from (ii), we get

$T_n = 2 + [1 + 3 + 5 + \ldots\ldots + (n-1)\text{terms}]$

$\therefore \quad T_n = 2 + (n-1)^2$

$$S = \sum T_n = 2n + \frac{(n-1)\,n\,(2n-1)}{6}$$

Thus, $S_n(x) = \left(2n + \frac{n(n-1)(2n-1)}{6}\right)\log_a x$

Also, $S_{24}(x) = 1093$

$\Rightarrow \quad \log_a(x)\left(48 + \frac{23\times24\times47}{6}\right) = 1093$

$\Rightarrow \quad \log_a(x) = \frac{1}{4}$ ...(iii)

Also, $S_{12}(2x) = 265$

$\Rightarrow \quad \log_a(2x)\left(24 + \frac{11\times12\times23}{6}\right) = 265$

$\Rightarrow \quad \log_a 2x = \frac{1}{2}$ ...(iv)

Subtracting (iii) from (iv), we get

$\log_a 2x - \log_a x = \frac{1}{4} \Rightarrow \log_a 2 = \frac{1}{4} \Rightarrow a = 16$

89. (5) : We have, $\bar{x}_1 = 2,\ n_1 = 10,\ \sigma_1^2 = 2;$

$\bar{x}_2 = 3,\ n_2 = n,\ \sigma_2^2 = 1$

$$\bar{x} = \frac{10\times2 + n\times3}{10+n} = \frac{20+3n}{10+n}$$

So, $d_1^2 = (\bar{x} - \bar{x}_1)^2 = \frac{n^2}{(10+n)^2}$

and $d_2^2 = (\bar{x} - \bar{x}_2)^2 = \left(\frac{-10}{10+n}\right)^2 = \frac{100}{(10+n)^2}$

Now, $\sigma^2 = \frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}$

$$\Rightarrow \quad \frac{17}{9} = \frac{10\left(2 + \frac{n^2}{(10+n)^2}\right) + n\left(1 + \frac{100}{(10+n)^2}\right)}{10+n}$$

$\Rightarrow \quad 17(10+n) = \left[20 + n + \frac{10n^2 + 100n}{(10+n)^2}\right]\times 9$

$\Rightarrow \quad 17(10+n)^2 = [(20+n)(10+n) + 10n]\times 9$

$\Rightarrow \quad 17n^2 + 1700 + 340n = 9n^2 + 360n + 1800$

$\Rightarrow \quad 8n^2 - 20n - 100 = 0 \Rightarrow 2n^2 - 5n - 25 = 0$

$\Rightarrow \quad 2n^2 - 10n + 5n - 25 = 0 \Rightarrow (n-5)(2n+5) = 0$

$\Rightarrow \quad n = 5$ ($\because$ n can't be negative)

90. (6) : Let $I = \displaystyle\int \frac{(x^2-1) + \tan^{-1}\left(\frac{x^2+1}{x}\right)}{(x^4+3x^2+1)\tan^{-1}\left(\frac{x^2+1}{x}\right)}dx$

$$= \int \frac{x^2-1}{(x^4+3x^2+1)\tan^{-1}\left(\frac{x^2+1}{x}\right)}dx + \int \frac{1}{x^4+3x^2+1}dx$$

$$= \int \frac{\left(1-\frac{1}{x^2}\right)}{\left(\left(x+\frac{1}{x}\right)^2+1\right)\tan^{-1}\left(x+\frac{1}{x}\right)}dx + \frac{1}{2}\int\frac{(x^2+1)-(x^2-1)}{x^4+3x^2+1}dx$$

$\Rightarrow \quad I = I_1 + I_2$

Now, for, I_1 put $\tan^{-1}\left(x+\frac{1}{x}\right) = t$

Thus, $I_1 = \int\frac{1}{t}dt = \log t = \log\left(\tan^{-1}\left(x+\frac{1}{x}\right)\right) + C_1$

and $I_2 = \frac{1}{2}\int\frac{(x^2+1)}{(x^4+3x^2+1)}dx - \frac{1}{2}\int\frac{(x^2-1)}{(x^4+3x^2+1)}dx$

$$= \frac{1}{2}\int\frac{\left(1+\frac{1}{x^2}\right)}{\left(x-\frac{1}{x}\right)^2+5}dx - \frac{1}{2}\int\frac{\left(1-\frac{1}{x^2}\right)}{\left(x+\frac{1}{x}\right)^2+1}dx$$

$$= \frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{5}\,x}\right) - \frac{1}{2}\tan^{-1}\left(\frac{x^2+1}{x}\right) + C_2$$

$$\therefore \quad I = \log\left(\tan^{-1}\left(x+\frac{1}{x}\right)\right) + \frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{5}\,x}\right) - \frac{1}{2}\tan^{-1}\left(\frac{x^2+1}{x}\right) + C$$

$\Rightarrow \quad \alpha = 1,\ \beta = \frac{1}{2\sqrt{5}},\ \gamma = \frac{1}{\sqrt{5}},\ \delta = \frac{-1}{2}$

$\therefore \quad 10(\alpha + \beta\gamma + \delta) = 10\left(1 + \frac{1}{10} - \frac{1}{2}\right) = 10\left(\frac{11-5}{10}\right) = 6$

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