JEE MAIN 2021

ONLINE
25th February
1st Shift

PHYSICS

SECTION-A (MULTIPLE CHOICE QUESTIONS)

1. Match List-I with List-II.

List-I	List-II
(A) h (Planck's constant)	(i) $[MLT^{-1}]$
(B) E (kinetic energy)	(ii) $[ML^2T^{-1}]$
(C) V (electric potential)	(iii) $[ML^2T^{-2}]$
(D) P (linear momentum)	(iv) $[ML^2I^{-1}T^{-3}]$

Choose the correct answer from the options given below.

(a) (A) → (ii), (B) → (iii), (C) → (iv), (D) → (i)
(b) (A) → (iii), (B) → (ii), (C) → (iv), (D) → (i)
(c) (A) → (iii), (B) → (iv), (C) → (ii), (D) → (i)
(d) (A) → (i), (B) → (ii), (C) → (iv), (D) → (iii)

2. A proton, a deuteron and an α-particle are moving with same momentum in a uniform magnetic field. The ratio of magnetic forces acting on them is _____ and their speed is ______, in the ratio.

(a) 4 : 2 : 1 and 2 : 1 : 1 (b) 1 : 2 : 4 and 1 : 1 : 2
(c) 2 : 1 : 1 and 4 : 2 : 1 (d) 1 : 2 : 4 and 2 : 1 : 1

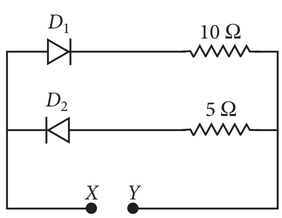
3. Two radioactive substances X and Y originally have N_1 and N_2 nuclei respectively. Half life of X is half of the half life of Y. After three half lives of Y, number of nuclei of both are equal. The ratio $\frac{N_1}{N_2}$ will be equal to

(a) 1/8 (b) 3/1 (c) 1/3 (d) 8/1

4. Two coherent light sources having intensity in the ratio $2x$ produce an interference pattern. The ratio $\frac{I_{max} - I_{min}}{I_{max} + I_{min}}$ will be

(a) $\frac{2\sqrt{2x}}{x+1}$ (b) $\frac{\sqrt{2x}}{x+1}$ (c) $\frac{\sqrt{2x}}{2x+1}$ (d) $\frac{2\sqrt{2x}}{2x+1}$

5. A 5 V battery is connected across the points X and Y. Assume D_1 and D_2 to be normal silicon diodes. Find the current supplied by the battery if the +ve terminal of the battery is connected to point X.



(a) ~ 0.86 A (b) ~ 0.5 A
(c) ~ 1.5 A (d) ~ 0.43 A

6. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : The escape velocities of planet A and B are same. But A and B are of unequal mass.

Reason R : The product of their mass and radius must be same. $M_1R_1 = M_2R_2$.

In the light of the above statements, choose the most appropriate answer from the options given below.

(a) A is correct but R is not correct.
(b) Both A and R are correct but R is not the correct explanation of A.
(c) A is not correct but R is correct.
(d) Both A and R are correct and R is the correct explanation of A.

7. Given below are two statements:

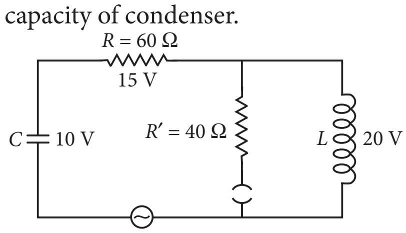
Statement I: A speech signal of 2 kHz is used to modulate a carrier signal of 1 MHz. The bandwidth requirement for the signal is 4 kHz.

Statement II : The side band frequencies are 1002 kHz and 998 kHz.

In the light of the above statements, choose the correct answer from the options given below.

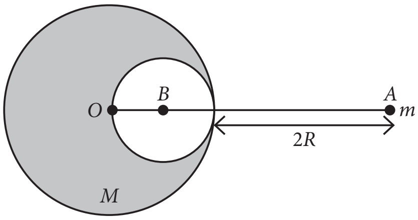
(a) Both statement I and statement II are true.
(b) Both statement I and statement II are false.
(c) Statement I is true but statement II is false.
(d) Statement I is false but statement II is true.

8. The angular frequency of alternating current in a LCR circuit is 100 rad/s. The components connected are shown in the figure. Find the value of inductance of the coil and capacity of condenser.



(a) 0.8 H and 150 μF (b) 1.33 H and 250 μF
(c) 1.33 H and 150 μF (d) 0.8 H and 250 μF

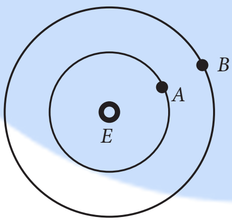
9. A solid sphere of radius R gravitationally attracts a particle placed at $3R$ from its centre with a force F_1. Now a spherical cavity of radius $(R/2)$ is made in the sphere (as shown in figure) and the force becomes F_2. The value of $F_1 : F_2$ is



(a) 41 : 50 (b) 25 : 36
(c) 36 : 25 (d) 50 : 41

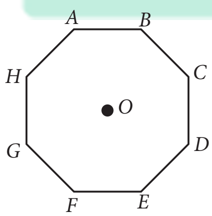
10. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.
Assertion A : When a rod lying freely is heated, no thermal stress is developed in it.
Reason R : On heating, the length of the rod increases.
In the light of the above statements, choose the correct answer from the options given below.
(a) A is true but R is false.
(b) Both A and R are true and R is the correct explanation of A.
(c) Both A and R are true but R is not the correct explanation of A.
(d) A is false but R is true.

11. Two satellites A and B of masses 200 kg and 400 kg are revolving round the earth at height of 600 km and 1600 km respectively. If T_A and T_B are the time periods of A and B respectively then the value of $T_B - T_A$
(Given : radius of earth = 6400 km, mass of earth = 6×10^{24} kg)



(a) 4.24×10^3 s (b) 1.33×10^3 s
(c) 4.24×10^2 s (d) 3.33×10^2 s

12. In an octagon $ABCDEFGH$ of equal side, what is the sum of $\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}+\overrightarrow{AG}+\overrightarrow{AH}$, if $\overrightarrow{AO} = 2\hat{i}+3\hat{j}-4\hat{k}$?



(a) $16\hat{i}+24\hat{j}+32\hat{k}$
(b) $16\hat{i}+24\hat{j}-32\hat{k}$
(c) $-16\hat{i}-24\hat{j}+32\hat{k}$
(d) $16\hat{i}-24\hat{j}+32\hat{k}$

13. The pitch of the screw gauge is 1 mm and there are 100 divisions on the circular scale. When nothing is put in between the jaws, the zero of the circular scale lies 8 divisions below the reference line. When a wire is placed between the jaws, the first linear scale division is clearly visible while 72nd division on circular scale coincides with the reference line. The radius of the wire is
(a) 0.90 mm (b) 0.82 mm
(c) 1.64 mm (d) 1.80 mm

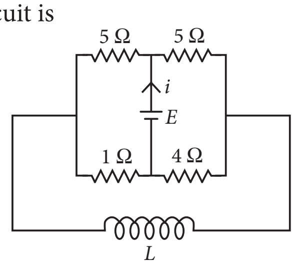
14. A student is performing the experiment of resonance column. The diameter of the column tube is 6 cm. The frequency of the tuning fork is 504 Hz. Speed of the sound at the given temperature is 336 m/s. The zero of the metre scale coincides with the top end of the resonance column tube. The reading of the water level in the column when the first resonance occurs is
(a) 14.8 cm (b) 13 cm
(c) 16.6 cm (d) 18.4 cm

15. A diatomic gas having $C_P = \frac{7}{2}R$ and $C_V = \frac{5}{2}R$, is heated at constant pressure. The ratio $dU : dQ : dW$
(a) 3 : 7 : 2 (b) 5 : 7 : 2
(c) 3 : 5 : 2 (d) 5 : 7 : 3

16. An α-particle and a proton are accelerated from rest by a potential difference of 200 V. After this, their de-Broglie wavelengths are λ_α and λ_P respectively. The ratio $\frac{\lambda_P}{\lambda_\alpha}$ is
(a) 8 (b) 3.8 (c) 7.8 (d) 2.8

17. If the time period of a two meter long simple pendulum is 2 s, the acceleration due to gravity at the place where pendulum is executing simple harmonic motion is
(a) $2\pi^2$ m s^{-2} (b) 9.8 m s^{-2}
(c) π^2 m s^{-2} (d) 16 m s^{-2}

18. The current (i) at time $t = 0$ and $t = \infty$ respectively for the given circuit is



(a) $\frac{18E}{55}, \frac{5E}{18}$ (b) $\frac{10E}{33}, \frac{5E}{18}$
(c) $\frac{5E}{18}, \frac{18E}{55}$ (d) $\frac{5E}{18}, \frac{10E}{33}$

19. An engine of a train moving with uniform acceleration, passes the signal-post with velocity u and the last compartment with velocity v. The velocity with which middle point of the train passes the signal post is
(a) $\frac{v-u}{2}$ (b) $\sqrt{\frac{v^2-u^2}{2}}$
(c) $\frac{u+v}{2}$ (d) $\sqrt{\frac{v^2+u^2}{2}}$

20. Magnetic fields at two points on the axis of a circular coil at a distance of 0.05 m and 0.2 m from the centre are in the ratio 8 : 1. The radius of the coil is ______.
(a) 1.0 m (b) 0.15 m (c) 0.2 m (d) 0.1 m

SECTION-B (NUMERICAL VALUE TYPE)

Attempt any 5 questions out of 10.

21. A coil of inductance 2 H having negligible resistance is connected to a source of supply whose voltage is given by $V = 3t$ volt (where t is in second). If the voltage is applied when $t = 0$, then the energy stored in the coil after 4 s is _______ J.

22. A monoatomic gas of mass 4.0 u is kept in an insulated container. Container is moving, with velocity 30 m/s. If container is suddenly stopped then change in temperature of the gas (R = gas constant) is $\frac{x}{3R}$. Value of x is _______.

23. A small bob tied at one end of a thin string of length 1 m is describing a vertical circle so that the maximum and minimum tension in the string are in the ratio 5 : 1. The velocity of the bob at the highest position is _______ m/s. (Take $g = 10$ m/s^2)

24. In a certain thermodynamical process, the pressure of a gas depends on its volume as kV^3. The work done when the temperature changes from 100°C to 300°C will be ______ nR, where n denotes number of moles of a gas.

25. 512 identical drops of mercury are charged to a potential of 2 V each. The drops are joined to form a single drop. The potential of this drop is _______ V.

26. A transmitting station releases waves of wavelength 960 m. A capacitor of 2.56 μF is used in the resonant circuit. The self inductance of coil necessary for resonance is ______ $\times 10^{-8}$ H.

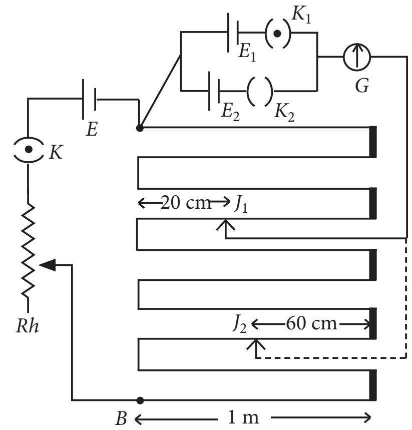
27. The same size images formed by a convex lens when the object is placed at 20 cm or at 10 cm from the lens. The focal length of convex lens is _______ cm.

28. The potential energy U of a diatomic molecule is a function dependent on r (interatomic distance) as

$$U = \frac{\alpha}{r^{10}} - \frac{\beta}{r^5} - 3$$

where, α and β are positive constants. The equilibrium distance between two atoms will be $\left(\frac{2\alpha}{\beta}\right)^{a/b}$, where a = _______.

29. In the given circuit of potentiometer, the potential difference E across AB (10 m length) is larger than E_1 and E_2 as well. For key K_1 (closed), the jockey is adjusted to touch the wire at point J_1 so that there is no deflection in the galvanometer. Now the first battery (E_1) is replaced by second battery (E_2) for working by making K_1 open and K_2 closed. The galvanometer gives then null deflection at J_2. The value of $\frac{E_1}{E_2}$ is $\frac{a}{b}$, where a = _______.



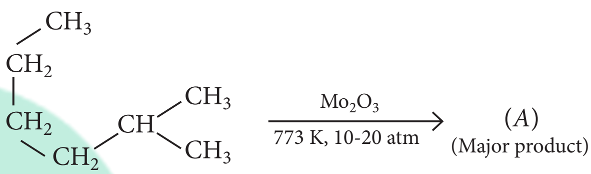
30. The electric field in a region is given by $\vec{E} = \left(\frac{3}{5}E_0\,\hat{i} + \frac{4}{5}E_0\,\hat{j}\right)$ N/C. The ratio of flux of reported field through the rectangular surface of area 0.2 m^2 (parallel to y-z plane) to that of the surface of area 0.3 m^2 (parallel to x-z plane) is $a : b$, where a = _______.

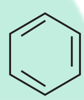
[Here $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axes respectively

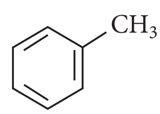
CHEMISTRY

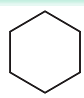
SECTION-A (MULTIPLE CHOICE QUESTIONS)

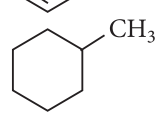
31. Identify A in the given chemical reaction.



(a) 

(b) 

(c) 

(d) 

32. The solubility of AgCN in a buffer solution of pH = 3 is x. The value of x is
[Assume : No cyano complex is formed : K_{sp}(AgCN) = 2.2×10^{-16} and K_a(HCN) = 6.2×10^{-10}]

(a) 0.625×10^{-6}
(b) 2.2×10^{-16}
(c) 1.6×10^{-6}
(d) 1.9×10^{-5}

33. Which of the following equations depicts the oxidizing nature of H_2O_2?

(a) $I_2 + H_2O_2 + 2OH^- \longrightarrow 2I^- + 2H_2O + O_2$
(b) $KIO_4 + H_2O_2 \longrightarrow KIO_3 + H_2O + O_2$
(c) $Cl_2 + H_2O_2 \longrightarrow 2HCl + O_2$
(d) $2I^- + H_2O_2 + 2H^+ \longrightarrow I_2 + 2H_2O$

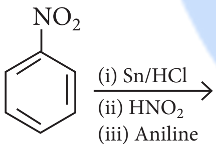
34. The correct statement about B_2H_6 is
(a) terminal B — H bonds have less p-character when compared to bridging bonds
(b) the two B — H — B bonds are not of same length
(c) its fragment BH_3, behaves as a Lewis base
(d) all B — H — B angles are of 120°.

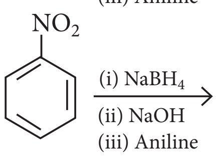
35. According to molecular orbital theory, the species among the following that does not exist is
(a) Be_2 (b) He_2^+ (c) O_2^{2-} (d) He_2^-

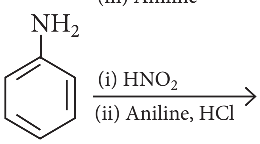
36. In which of the following pairs, the outermost electronic configuration will be the same?
(a) Cr^+ and Mn^{2+} (b) Ni^{2+} and Cu^+
(c) V^{2+} and Cr^+ (d) Fe^{2+} and Co^+

37. In Freundlich adsorption isotherm at moderate pressure, the extent of adsorption $\left(\frac{x}{m}\right)$ is directly proportional to P^x. The value of x is
(a) zero (b) 1 (c) $\frac{1}{n}$ (d) ∞

38. Which of the following reaction/s will not give p-aminoazobenzene?

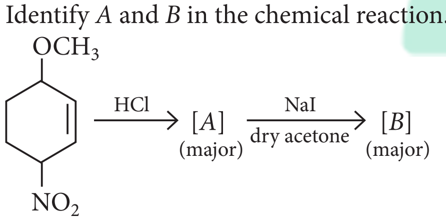
A.


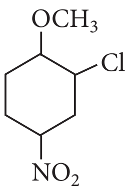
B.


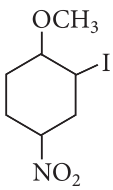
C.


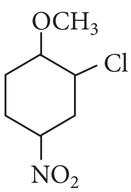
(a) *A* only (b) *A* and *B* (c) *B* only (d) *C* only

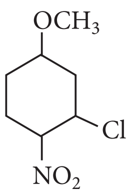
39. Identify *A* and *B* in the chemical reaction.

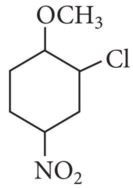


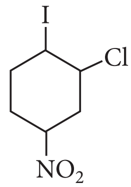
(a) $A =$


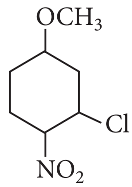
$B =$


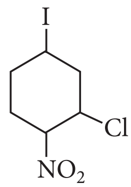
(b) $A =$


$B =$


(c) $A =$


$B =$


(d) $A =$


$B =$


40. The hybridization and magnetic nature of $[Mn(CN)_6]^{4-}$ and $[Fe(CN)_6]^{3-}$, respectively are
(a) d^2sp^3 and diamagnetic
(b) d^2sp^3 and paramagnetic
(c) sp^3d^2 and diamagnetic
(d) sp^3d^2 and paramagnetic.

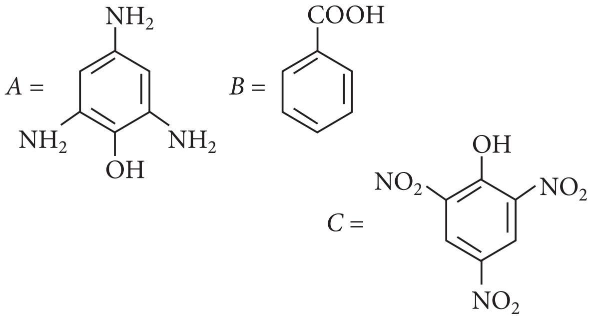
41. Which statement is correct?
(a) Neoprene is an addition copolymer used in plastic bucket manufacturing.
(b) Buna-N is a natural polymer.
(c) Buna-S is a synthetic and linear thermosetting polymer.
(d) Synthesis of Buna-S needs nascent oxygen.

42. Ellingham diagram is a graphical representation of
(a) ΔG *vs* P (b) ΔH *vs* T
(c) $(\Delta G - T\Delta S)$ *vs* T (d) ΔG *vs* T.

43. Given below are two statements :
Statement I : An allotrope of oxygen is an important intermediate in the formation of reducing smog.
Statement II : Gases such as oxides of nitrogen and sulphur present in troposphere contribute to the formation of photochemical smog.
In the light of the above statements, choose the correct answer from the options given below.
(a) Statement I is true but statement II is false.
(b) Statement I is false but statement II is true.
(c) Both statement I and statement II are true.
(d) Both statement I and statement II are false.

44. Which one of the following reactions will not form acetaldehyde?
(a) $CH_3CH_2OH \xrightarrow{CrO_3-H_2SO_4}$
(b) $CH_2{=}CH_2 + O_2 \xrightarrow[H_2O]{Pd(II)/Cu(II)}$
(c) $CH_3CH_2OH \xrightarrow[573\ K]{Cu}$
(d) $CH_3CN \xrightarrow[(ii)\ H_2O]{(i)\ DIBAL\text{-}H}$

45. Compound(s) which will liberate carbon dioxide with sodium bicarbonate solution is/are



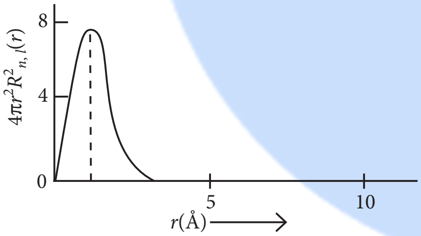
(a) A and B only (b) C only
(c) B and C only (d) B only.

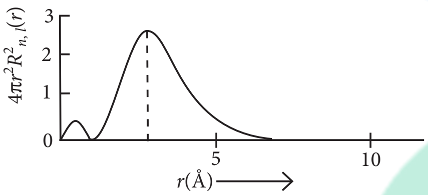
46. The major product of the following chemical reaction is

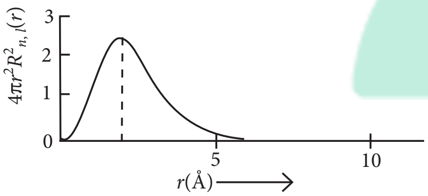
$$CH_3CH_2CN \xrightarrow[\text{(3) Pd/BaSO}_4\text{, H}_2]{\text{(1) H}_3\text{O}^+\text{, }\Delta \quad \text{(2) SOCl}_2} ?$$

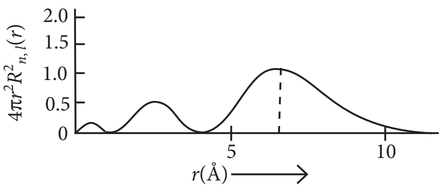
(a) $(CH_3CH_2CO)_2O$ (b) $CH_3CH_2CH_2OH$
(c) $CH_3CH_2CH_3$ (d) CH_3CH_2CHO

47. The plots of radial distribution functions for various orbitals of hydrogen atom against 'r' are given below:

(A)


(B)


(C)


(D)


The correct plot for 3s-orbital is

(a) (C) (b) (A) (c) (B) (d) (D)

48. Complete combustion of 1.80 g of an oxygen containing compound ($C_xH_yO_z$) gave 2.64 g of CO_2 and 1.08 g of H_2O. The percentage of oxygen in the organic compound is

(a) 50.33 (b) 53.33 (c) 51.63 (d) 63.53

49. Which of the glycosidic linkage between galactose and glucose is present in lactose?

(a) C–1 of glucose and C–4 of galactose
(b) C–1 of galactose and C–6 of glucose
(c) C–1 of glucose and C–6 of galactose
(d) C–1 of galactose and C–4 of glucose

50. Given below are two statements :

Statement I : CeO_2 can be used for oxidation of aldehydes and ketones.

Statement II : Aqueous solution of $EuSO_4$ is a strong reducing agent.

In the light of the above statements, choose the correct answer from the options given below.

(a) Both statement I and statement II are true.
(b) Statement I is true but statement II is false.
(c) Both statement I and statement II are false.
(d) Statement I is false but statement II is true.

SECTION-B (NUMERICAL VALUE TYPE)

Attempt any 5 questions out of 10.

51. Consider the following chemical reaction :

$$CH{\equiv}CH \xrightarrow[\text{(2) CO, HCl, AlCl}_3]{\text{(1) Red hot Fe tube, 873 K}} \text{Product}$$

The number of sp^2-hybridized carbon atom(s) present in the product is ________.

52. Among the following, the number of halide(s) which is/are inert to hydrolysis is ________.

A. BF_3 B. $SiCl_4$ C. PCl_5 D. SF_6

53. In basic medium CrO_4^{2-} oxidises $S_2O_3^{2-}$ to form SO_4^{2-} and itself changes into $Cr(OH)_4^-$. The volume of 0.154 M CrO_4^{2-} required to react with 40 mL of 0.25 M $S_2O_3^{2-}$ is ________ mL. (Rounded-off to the nearest integer)

54. Using the provided information in the following paper chromatogram :

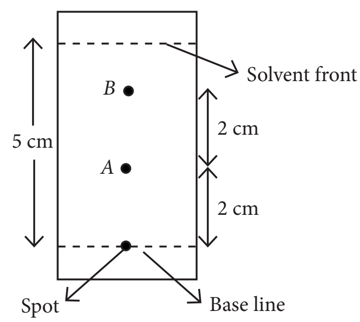


Fig : Paper chromatography for compounds A and B

The calculated R_f value of A is ________ $\times 10^{-1}$.

55. 0.4 g mixture of NaOH, Na_2CO_3 and some inert impurities was first titrated with $\frac{N}{10}$ HCl using phenolphthalein as an indicator, 17.5 mL of HCl was required at the end point. After this methyl orange was added and titrated. 1.5 mL of same HCl was required for the next end point. The weight percentage of Na_2CO_3 in the mixture is _______. (Rounded-off to the nearest integer)

56. 1 molal aqueous solution of an electrolyte A_2B_3 is 60% ionised. The boiling point of the solution at 1 atm is _______ K. (Rounded-off to the nearest integer) [Given : K_b for (H_2O) = 0.52 K kg mol^{-1}]

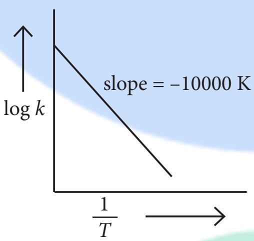
57. A car tyre is filled with nitrogen gas at 35 psi at 27°C. It will burst if pressure exceeds 40 psi. The temperature in °C at which the car tyre will burst is _______. (Rounded-off to the nearest integer)

58. The reaction of cyanamide, $NH_2CN_{(s)}$ with oxygen was run in a bomb calorimeter and ΔU was found to be –742.24 kJ mol^{-1}. The magnitude of ΔH_{298} for the reaction

$$NH_2CN_{(s)} + \frac{3}{2}O_{2(g)} \longrightarrow N_{2(g)} + O_{2(g)} + H_2O_{(l)}$$

is _______ kJ. (Rounded-off to the nearest integer)
[Assume ideal gases and R = 8.314 J mol^{-1} K^{-1}]

59. For the reaction, $aA + bB \longrightarrow cC + dD$, the plot of $\log k$ vs $\frac{1}{T}$ is given below :



The temperature at which the rate constant of the reaction is 10^{-4} s^{-1} is _______ K. (Rounded-off to the nearest integer)
[Given : The rate constant of the reaction is 10^{-5} s^{-1} at 500 K]

60. The ionization enthalpy of Na^+ formation from $Na_{(g)}$ is 495.8 kJ mol^{-1}, while the electron gain enthalpy of Br is –325.0 kJ mol^{-1}. Given the lattice enthalpy of NaBr is –728.4 kJ mol^{-1}. The energy for the formation of NaBr ionic solid is (–) _______ × 10^{-1} kJ mol^{-1}.

MATHEMATICS

SECTION-A (MULTIPLE CHOICE QUESTIONS)

61. If $0 < \theta, \phi < \pi/2$, $x = \sum_{n=0}^{\infty} \cos^{2n}\theta$, $y = \sum_{n=0}^{\infty} \sin^{2n}\phi$ and

$z = \sum_{n=0}^{\infty} \cos^{2n}\theta \cdot \sin^{2n}\phi$, then

(a) $xy - z = (x + y)z$ (b) $xy + z = (x + y)z$
(c) $xyz = 4$ (d) $xy + yz + zx = z$

62. All possible values of $\theta \in [0, 2\pi]$ for which $\sin 2\theta + \tan 2\theta > 0$ lie in

(a) $\left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{7\pi}{6}\right)$

(b) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{11\pi}{6}\right)$

(c) $\left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$

(d) $\left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

63. The total number of positive integral solutions (x, y, z) such that $xyz = 24$ is

(a) 45 (b) 30 (c) 24 (d) 36

64. A tangent is drawn to the parabola $y^2 = 6x$ which is perpendicular to the line $2x + y = 1$. Which of the following points does NOT lie on it?

(a) (4, 5) (b) (5, 4) (c) (–6, 0) (d) (0, 3)

65. A man is observing, from the top of a tower, a boat speeding towards the tower from a certain point A, with uniform speed. At that point, angle of depression of the boat with the man's eye is 30° (Ignore man's height). After sailing for 20 seconds, towards the base of the tower (which is at the level of water), the boat has reached a point B, where the angle of depression is 45°. Then the time taken (in seconds) by the boat from B to reach the base of the tower is

(a) $10(\sqrt{3}+1)$ (b) $10\sqrt{3}$
(c) 10 (d) $10(\sqrt{3}-1)$

66. If the curves, $\frac{x^2}{a} + \frac{y^2}{b} = 1$ and $\frac{x^2}{c} + \frac{y^2}{d} = 1$ intersect each other at an angle of 90°, then which of the following relations is TRUE?

(a) $ab = \frac{c+d}{a+b}$ (b) $a - b = c - d$
(c) $a - c = b + d$ (d) $a + b = c + d$

67. When a missile is fired from a ship, the probability that it is intercepted is 1/3 and the probability that the missile hits the target, given that it is not intercepted, is 3/4. If three missiles are fired independently from the ship, then the probability that all three hit the target, is

(a) 1/8 (b) 3/8 (c) 3/4 (d) 1/27

68. Let $f, g : N \to N$ such that $f(n + 1) = f(n) + f(1)\ \forall\ n \in N$ and g be any arbitrary function. Which of the following statements is NOT true?

(a) If f is onto, then $f(n) = n\ \forall\ n \in N$
(b) f is one-one
(c) If fog is one-one, then g is one-one.
(d) If g is onto, then fog is one-one.

69. Let the lines $(2-i)z = (2+i)\bar{z}$ and $(2+i)z + (i-2)\bar{z} - 4i = 0$, (here $i^2 = -1$) be normal to a circle C. If the line $iz + \bar{z} + 1 + i = 0$ is tangent to this circle C, then its radius is

(a) $\frac{3}{2\sqrt{2}}$ (b) $\frac{3}{\sqrt{2}}$ (c) $\frac{1}{2\sqrt{2}}$ (d) $3\sqrt{2}$

70. The equation of the line through the point (0, 1, 2) and perpendicular to the line $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{-2}$ is

(a) $\frac{x}{3}=\frac{y-1}{4}=\frac{z-2}{-3}$ (b) $\frac{x}{3}=\frac{y-1}{4}=\frac{z-2}{3}$

(c) $\frac{x}{-3}=\frac{y-1}{4}=\frac{z-2}{3}$ (d) $\frac{x}{3}=\frac{y-1}{-4}=\frac{z-2}{3}$

71. The integer k, for which the inequality $x^2 - 2(3k - 1)x + 8k^2 - 7 > 0$ is valid for every x in R, is

(a) 0 (b) 2 (c) 3 (d) 4

72. The value of the integral

$$\int \frac{\sin\theta \cdot \sin 2\theta(\sin^6\theta+\sin^4\theta+\sin^2\theta)\sqrt{2\sin^4\theta+3\sin^2\theta+6}}{1-\cos 2\theta}\,d\theta$$

is (where c is a constant of integration)

(a) $\frac{1}{18}[9-2\sin^6\theta-3\sin^4\theta-6\sin^2\theta]^{3/2}+c$

(b) $\frac{1}{18}[11-18\cos^2\theta+9\cos^4\theta-2\cos^6\theta]^{3/2}+c$

(c) $\frac{1}{18}[9-2\cos^6\theta-3\cos^4\theta-6\cos^2\theta]^{3/2}+c$

(d) $\frac{1}{18}[11-18\sin^2\theta+9\sin^4\theta-2\sin^6\theta]^{3/2}+c$

73. The image of the point (3, 5) in the line $x - y + 1 = 0$, lies on

(a) $(x - 4)^2 + (y + 2)^2 = 16$ (b) $(x - 2)^2 + (y - 2)^2 = 12$

(c) $(x - 4)^2 + (y - 4)^2 = 8$ (d) $(x - 2)^2 + (y - 4)^2 = 4$

74. If Rolle's theorem holds for the function $f(x) = x^3 - ax^2 + bx - 4$, $x \in [1, 2]$ with $f'\left(\frac{4}{3}\right)=0$, then ordered pair (a, b) is equal to

(a) (5, –8) (b) (5, 8) (c) (–5, 8) (d) (–5, –8)

75. Let α be the angle between the lines whose direction cosines satisfy the equations $l + m - n = 0$ and $l^2 + m^2 - n^2 = 0$. Then the value of $\sin^4\alpha + \cos^4\alpha$ is

(a) 3/4 (b) 5/8 (c) 3/8 (d) 1/2

76. The statement $A \to (B \to A)$ is equivalent to

(a) $A \to (A \wedge B)$ (b) $A \to (A \to B)$

(c) $A \to (A \vee B)$ (d) $A \to (A \leftrightarrow B)$

77. If a curve passes through the origin and the slope of the tangent to it at any point (x, y) is $\frac{x^2-4x+y+8}{x-2}$, then this curve also passes through the point

(a) (5, 5) (b) (4, 4) (c) (4, 5) (d) (5, 4)

78. $\lim\limits_{n\to\infty}\left(1+\frac{1+\frac{1}{2}+.....+\frac{1}{n}}{n^2}\right)^n$ is equal to

(a) $1/e$ (b) 1/2 (c) 1 (d) 0

79. The coefficients a, b and c of the quadratic equation, $ax^2 + bx + c = 0$ are obtained by throwing a dice three times. The probability that this equation has equal roots is

(a) $\frac{5}{216}$ (b) $\frac{1}{72}$ (c) $\frac{1}{36}$ (d) $\frac{1}{54}$

80. The value of $\int_{-1}^{1} x^2 e^{[x^3]}\,dx$, where $[t]$ denotes the greatest integer $\le t$, is

(a) $\frac{1}{3e}$ (b) $\frac{e+1}{3e}$ (c) $\frac{e-1}{3e}$ (d) $\frac{e+1}{3}$

SECTION-B (NUMERICAL VALUE TYPE)

Attempt any 5 questions out of 10.

81. If $A=\begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix}$ and $(I_2 + A)(I_2 - A)^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then $13(a^2 + b^2)$ is equal to ________.

82. Let $f(x)$ be a polynomial of degree 6 in x, in which the coefficient of x^6 is unity and it has extrema at $x = -1$ and $x = 1$. If $\lim\limits_{x\to 0}\frac{f(x)}{x^3}=1$, then $5 \cdot f(2)$ is equal to ________.

83. Let $A=\begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$, where x, y and z are real numbers such that $x + y + z > 0$ and $xyz = 2$. If $A^2 = I_3$, then the value of $x^3 + y^3 + z^3$ is ________.

84. The locus of the point of intersection of the lines $(\sqrt{3})kx+ky-4\sqrt{3}=0$ and $\sqrt{3}x-y-4(\sqrt{3})k=0$ is a conic, whose eccentricity is ________.

85. The total number of numbers, lying between 100 and 1000 that can be formed with the digits 1, 2, 3, 4, 5, if the repetition of digits is not allowed and numbers are divisible by either 3 or 5, is ________.

86. If the system of equations

$kx + y + 2z = 1$, $3x - y - 2z = 2$, $-2x - 2y - 4z = 3$

has infinitely many solutions, then k is equal to ________.

87. The graphs of sine and cosine functions, intersect each other at a number of points and between two consecutive points of intersection, the two graphs enclose the same area A. Then A^4 is equal to ________.

88. Let $A_1, A_2, A_3, ...$ be squares such that for each $n \ge 1$, the length of the side of A_n equals the length of diagonal of A_{n+1}. If the length of A_1 is 12 cm, then the smallest value of n for which area of A_n is less than one, is ________.

89. Let $\vec{a}=\hat{i}+2\hat{j}-\hat{k}$, $\vec{b}=\hat{i}-\hat{j}$ and $\vec{c}=\hat{i}-\hat{j}-\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r}\times\vec{a}=\vec{c}\times\vec{a}$ and $\vec{r}\cdot\vec{b}=0$, then $\vec{r}\cdot\vec{a}$ is equal to ________.

90. The number of points, at which the function $f(x) = |2x + 1| - 3|x + 2| + |x^2 + x - 2|$, $x \in R$ is not differentiable, is ________.

HINTS & EXPLANATIONS

1. **(a) :** $h \to [ML^2T^{-1}]$, $E \to [ML^2T^{-2}]$, $V \to [M^1L^2T^{-2}C^{-1}]$
$P \to [MLT^{-1}]$

$\therefore$ (A) $\to$ (ii), (B) $\to$ (iii), (C) $\to$ (iv), (D) $\to$ (i)

2. **(c) :**

	Charge	**Mass**
Proton	e	m
α-particle	$2e$	$4m$
Deuteron	e	$2m$

speed, $v = P/m$

$$v_1 = \frac{P}{m},\ v_2 = \frac{P}{2m},\ v_3 = \frac{P}{4m}$$

$\therefore$ $v_1 : v_2 : v_3 = 4 : 2 : 1$

$$F = qvB = \frac{qPB}{m}$$

$$F_1 = \frac{qPB}{m},\ F_2 = \frac{q \times P \times B}{2m},\ F_3 = \frac{2qPB}{4m}$$

$\therefore$ $F_1 : F_2 : F_3 = 4 : 2 : 2 = 2 : 1 : 1$

3. **(d) :** $T_{(1/2)}(x) = \frac{1}{2} T_{(1/2)}(y)$

$\lambda_x = 2\lambda_y$

Now, $t = 3T_y$

By the equation of radioactivity,

$N_x = N_1 e^{-\lambda x \cdot 3T_y}$ ... (i)

$N_y = N_2 e^{-\lambda y \cdot 3T_y}$ ... (ii)

$N_x = N_y$ (from question)

$N_1 e^{-\lambda x \cdot 3T_y} = N_2 e^{-\lambda y \cdot 3T_y}$

$$N_1 e^{-2\lambda_y \cdot \frac{3\times 0.693}{\lambda_y}} = N_2 e^{-\lambda_y \cdot \frac{3\times 0.693}{\lambda_y}}$$

$N_1 e^{-6\ln 2} = N_2 e^{-3\ln 2}$

$$\therefore \quad \frac{N_1}{N_2} = e^{3\ln 2} = e^{\ln 2^3} = 8 = 8 : 1$$

4. **(d) :** $\frac{I_1}{I_2} = 2x$

$$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{2x}+1}{\sqrt{2x}-1}\right)^2$$

$$\frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{(\sqrt{2x}+1)^2 - (\sqrt{2x}-1)^2}{(\sqrt{2x}+1)^2 + (\sqrt{2x}-1)^2}$$

$$= \frac{(2\sqrt{2x})(2)}{2x+1+2x+1} = \frac{4\sqrt{2x}}{2(2x+1)} = \frac{2\sqrt{2x}}{2x+1}$$

5. **(d) :** If positive terminal is at x, it means D_2 is in reverse bias, and no current flows through it.

(Voltage drop in forward bias is 0.7 V)

$$\therefore \quad I = \frac{5 - 0.7}{10} = 0.43 \text{ A}$$

6. **(a) :** Escape velocity of planet does not depend on mass of object.

7. **(a) :** $f_m = 2$ kHz

$f_c = 1$ MHz $= 1000$ kHz

Bandwidth $= 2f_m = 4$ kHz

$\therefore$ Side frequencies will be $= f_c + f_m$

$= (1000 \pm 2)$ kHz

$= 998$ kHz and 1002 kHz

8. **(d) :** When the key is opened, the current is

$15 = I_{rms} \times R$

$15 = I_{rms} \times 60 \ \Rightarrow\ I_{rms} = 0.25$ A

$$\text{And, } 20 = I_{rms} \times X_L \ \Rightarrow\ X_L = \frac{20}{0.25} = 80\ \Omega$$

$X_L = \omega L$

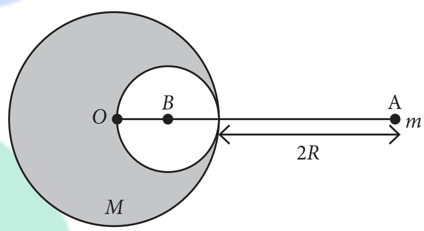
$80 = 100 \times L \ \Rightarrow\ L = 0.8$ H

$$\text{or } 10 = I_{rms} \times X_C \ \Rightarrow\ X_C = \frac{10}{0.25} = 40\ \Omega$$

$$X_C = \frac{1}{\omega C} \ \Rightarrow\ C = \frac{1}{100 \times 40} = 250 \times 10^{-6} \text{ F}$$

9. **(d) :** Let initial mass of sphere is 'M'.

$$F_1 = \frac{GMm}{9R^2} \quad \text{... (i)}$$



$$\text{Mass of cavity, } M' = \frac{M}{\frac{4}{3}\pi R^3} \cdot \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 \quad \text{or} \quad M' = \frac{M}{8}$$

$$F_2 = \frac{GMm}{9R^2} - \frac{G(M/8)m}{(5R/2)^2}$$

$$F_2 = \frac{GMm}{9R^2} - \frac{GMm}{50R^2} \quad \text{or} \quad F_2 = \frac{41GMm}{50 \times 9R^2} \quad \text{...(ii)}$$

Using (i) and (ii), we get

$$\therefore \quad \frac{F_1}{F_2} = \frac{GMm \times 50 \times 9R^2}{9R^2 \times 41 \times GMm} = 50 : 41$$

10. **(c) :** Thermal stress is there, if the rod is fixed between two supports. The rod will expand due to increase in temperature.

11. **(b) :** $h_A = 600$ km, $h_B = 1600$ km

$R = 6400$ km, $M = 6 \times 10^{24}$ kg

$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

$$T_A = 2\pi\sqrt{\frac{(6400+600)^3 \times 10^9}{6.67\times10^{-11} \times 6\times10^{24}}} = 5813.9 \text{ s}$$

$$T_B = 2\pi\sqrt{\frac{(6400+1600)^3 \times 10^9}{6.67\times10^{-11} \times 6\times10^{24}}} = 7103.2 \text{ s}$$

$\therefore \quad T_B - T_A = 1.289 \times 10^3\text{s} \simeq 1.33 \times 10^3\text{s}$

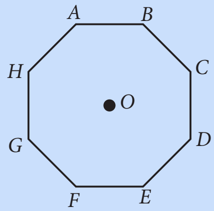
12. (b) : $\overrightarrow{OA} = \vec{a}$

$\overrightarrow{AO} = 2\hat{i} + 3\hat{j} - 4\hat{k} = \vec{a}$

$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} + \overrightarrow{AG} + \overrightarrow{AH}$

$= (\vec{b}-\vec{a}) + (\vec{c}-\vec{a}) + (\vec{d}-\vec{a}) + (\vec{e}-\vec{a}) + (\vec{f}-\vec{a}) + (\vec{g}-\vec{a}) + (\vec{h}-\vec{a})$

$= \vec{b} + \vec{c} + \vec{d} + \vec{e} + \vec{f} + \vec{g} + \vec{h} - 7\vec{a}$ ... (i)



Here, $\dfrac{\vec{a}+\vec{b}+\vec{c}+\vec{d}+\vec{e}+\vec{f}+\vec{g}+\vec{h}}{8} = 0$

$\vec{a}+\vec{b}+\vec{c}+\vec{d}+\vec{e}+\vec{f}+\vec{g}+\vec{h} = 0$

$\vec{b}+\vec{c}+\vec{d}+\vec{e}+\vec{f}+\vec{g}+\vec{h} = -\vec{a}$

On putting value of a in eqn (i), we get

$= \ -\vec{a} - 7\vec{a} = -8\vec{a}$

$= -8\,\overrightarrow{OA} = 8\overrightarrow{AO} = 8(2\hat{i} + 3\hat{j} - 4\hat{k})$

$= 16\hat{i} + 24\hat{j} - 32\hat{k}$

13. (b) : Least count = 0.01 mm

Positive zero error = 0.08 mm

Reading = 1 mm + 72 × 0.01 = 1.72 mm

True reading = 1.72 – zero error = 1.72 – 0.08 = 1.64 mm

$\therefore$ Radius $= \dfrac{1.64}{2} = 0.82$ mm

14. (a) : Diameter = 0.06 m

$f = 504$ Hz, $v = 336$ m/s

Wavelength, $\lambda = \dfrac{v}{f} = \dfrac{336}{504} = 66.66$ cm

For first resonance,

$\lambda/4 = l + e = l + 0.3d$

$\therefore \quad \dfrac{66.66}{4} = l + 0.3\times6 \Rightarrow l = 14.86$ cm

15. (b) : $C_P = \dfrac{7}{2}R,\ C_V = \dfrac{5}{2}R$

$dU = nC_V\Delta T = n\times\dfrac{5}{2}R\Delta T$ ... (i)

$dQ = nC_P\Delta T = n\times\dfrac{7}{2}R\Delta T$ ... (ii)

$dW = nR\Delta T$ ... (iii)

From equation (i), (ii) and (iii), we have

$$dU : dQ : dW = n\frac{5}{2}R\Delta T : n\frac{7}{2}R\Delta T : nR\Delta T = 5 : 7 : 2$$

16. (d) : de Broglie wavelength, $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mK}}$

$\lambda = \dfrac{h}{\sqrt{2mqV}}$

$\lambda \propto \dfrac{1}{\sqrt{mvq}}$

$\dfrac{\lambda_P}{\lambda_\alpha} = \sqrt{\dfrac{m_\alpha q_\alpha}{m_P q_P}} = \sqrt{\dfrac{4m_P\cdot 2e}{m_P\times e}} = 2\sqrt{2} = 2.8$

17. (a) : $T = 2$ s, $L = 2$ m

Time period for simple pendulum,

$T = 2\pi\sqrt{\dfrac{L}{g}} \Rightarrow T^2 = 4\pi^2\dfrac{L}{g}$

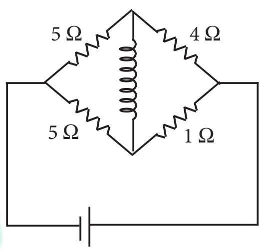
$\Rightarrow \ g = \dfrac{4\pi^2 L}{T^2} = \dfrac{4\pi^2\times2}{2\times2} = 2\pi^2$ m/s^2

18. (d) : The circuit is shown in figure.

At $t = 0$, current through inductor is zero ($L = 0$).

Hence, $R_{eq} = \dfrac{6\times9}{15} = \dfrac{18}{5}$

$I = \dfrac{E}{18/5} = \dfrac{5E}{18}$

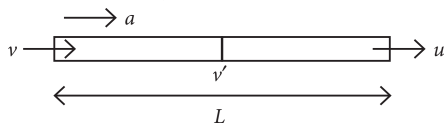


At $t = \infty$, inductor becomes a simple wire

Hence, $R_{eq} = \dfrac{5}{2} + \dfrac{4\times1}{4+1} = \dfrac{5}{2} + \dfrac{4}{5} = \dfrac{33}{10}$

$\therefore \quad I = \dfrac{E}{33/10} = \dfrac{10E}{33}$

19. (d) : Let the velocity at midpoint is v'.



Equation of motion,

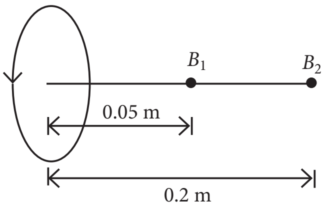
$v^2 = u^2 + 2aL$ ... (i)

$v'^2 = u^2 + aL$

From (i), $v'^2 = u^2 + \left(\dfrac{v^2-u^2}{2}\right) = \dfrac{v^2+u^2}{2}$

$\therefore \quad v' = \sqrt{\dfrac{v^2+u^2}{2}}$

20. (d) : Magnetic field due to circular coil,



$$B = \frac{\mu_0 I R^2}{2(R^2+X^2)^{3/2}}$$

As $B_1 : B_2 = 8 : 1$

So, $\dfrac{B_1}{B_2} = \dfrac{8}{1} = \left[\dfrac{(R^2+X_2^2)}{(R^2+X_1^2)}\right]^{3/2}$

$\Rightarrow \quad (2^3)^{2/3} = \dfrac{R^2+X_2^2}{R^2+X_1^2}$

$\Rightarrow \quad 4[R^2 + (0.05)^2] = [R^2 + (0.2)^2]$

$\Rightarrow \quad 4R^2 - R^2 = 0.0375$

$\Rightarrow \quad R^2 = 0.01 \quad$ or $\quad R = 0.1$ m

21. (144) $: L = 2$ H, $V = 3t$, $t = 4$ s

$V = L\dfrac{di}{dt} \quad$ or $\quad 3t = L\dfrac{di}{dt}$

$L\int di = \int 3t\, dt$

$Li = \dfrac{3t^2}{2} \quad \Rightarrow i = \dfrac{3t^2}{2L}$

Enery, $E = \dfrac{1}{2}Li^2 = \dfrac{1}{2}\times L \times \dfrac{(3t^2)^2}{4L^2} = \dfrac{9}{8L}\times t^4$

$\therefore \quad E = \dfrac{9}{8\times 2}\times(4)^4 = 144$ J

22. (3600) : Degree of freedom for monoatomic gas, $f = 3$

Using conservation of energy,

$$\frac{1}{2}mu^2 + \frac{f}{2}nRT_i = \frac{f}{2}nRT_f$$

$$\frac{3}{2}nR(T_f - T_i) = \frac{1}{2}mu^2$$

$$\frac{3}{2}\times 1\times R\Delta T = \frac{1}{2}\times 4\times(30)^2$$

$$\Delta T = \frac{4\times 900}{3R} = \frac{1200}{R}$$

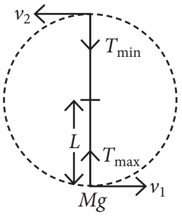
According to given statement, $\dfrac{x}{3R} = \dfrac{1200}{R} \quad \Rightarrow \quad x = 3600$

23. (5) : Let the speed of bob at lowest position be v_1 and at the highest position be v_2.

Maximum tension is at lowest position and minimum tension is at highest position.

Now, $T_{max} - mg = \dfrac{mv_1^2}{L}$

$\Rightarrow \quad T_{max} = mg + \dfrac{mv_1^2}{L} \quad$ ...(i)



$T_{min} + mg = \dfrac{mv_2^2}{L}$

$\Rightarrow \quad T_{min} = \dfrac{mv_2^2}{L} - mg \quad$ ...(ii)

From equation (i) and (ii), $\dfrac{T_{max}}{T_{min}} = \dfrac{mg + \dfrac{mv_1^2}{L}}{\dfrac{mv_2^2}{L} - mg} \quad$ ...(iii)

Also, $\dfrac{T_{max}}{T_{min}} = \dfrac{5}{1} \quad$ ...(iv)

From equation (iii) and (iv), $\dfrac{mg + \dfrac{mv_1^2}{L}}{\dfrac{mv_2^2}{L} - mg} = \dfrac{5}{1}$

$\Rightarrow \quad mg + \dfrac{mv_1^2}{L} = \left[\dfrac{mv_2^2}{L} - mg\right]\times 5$

$\Rightarrow \quad mg + \dfrac{m}{L}\left[v_2^2 + 4gL\right] = \dfrac{5mv_2^2}{L} - 5mg$

$\Rightarrow \quad 10mg = \dfrac{4mv_2^2}{L}$

$\Rightarrow \quad v_2^2 = \dfrac{10\times 10\times 1}{4} \quad (\because\ L = 1\text{ m})$

$\Rightarrow \quad v_2 = 5$ m/s

24. (50) : $T_1 = 100°C$; $T_2 = 300°C$

$\Delta T = T_2 - T_1 = 200°C$

Now, $PV = nRT$

Also, $P = kV^3$

$\therefore \quad kV^4 = nRT$

$4kV^3 dV = nRdT$

$\Rightarrow \quad PdV = \dfrac{nRdT}{4}$

Work done $= \int PdV = \int \dfrac{nRdT}{4} = \dfrac{nR}{4}\Delta T$

$W = \dfrac{200}{4}\times nR = 50nR$

25. (128) $:$ Let the radius of big drop is R

$512\times\dfrac{4}{3}\pi r^3 = \dfrac{4}{3}\pi R^3$

$R = 8r$

$V = \dfrac{Kq}{r} \quad$ ... (i); $\quad V' = \dfrac{Kq\times 512}{R} \quad$ ... (ii)

From equation (i) and (ii)

$$\frac{V}{2} = \frac{512Kq \cdot r}{R \times Kq} \Rightarrow V = \frac{2 \times 512 \times r}{8r} = 128 \text{ V}$$

26. (10) : $\lambda = 960$ m; $C = 256\ \mu$F

At resonance, $\omega_0 = \frac{1}{\sqrt{LC}}$

Squaring both sides, $\frac{4\pi^2 c^2}{\lambda^2} = \frac{1}{LC}$

$$\Rightarrow \frac{4 \times 10 \times (3 \times 10^8)^2}{(960)^2} = \frac{1}{L \times 2.56 \times 10^{-6}}$$

$\Rightarrow L = 10 \times 10^{-8}$ H

$L = 10^{-7}$ H $= 10 \times 10^{-8}$ H

27. (15) : $u = 20$ cm or 10 cm, $m = 1$

Magnification, $m = \frac{f}{f+u}$ $\quad (\because m_1 = -m_2)$

So, $\frac{f}{f-10} = \frac{-f}{f-20} \Rightarrow f - 20 = -f + 10$

$2f = 30 \Rightarrow f = 15$ cm

28. (1) : $U = \frac{\alpha}{r^{10}} - \frac{\beta}{r^5} - 3$

For equilibrium, $\frac{dU}{dr} = 0$

$\alpha(10)r^{-11} - \beta(5)r^{-6} = 0$

$r^5 = \frac{10\alpha}{5\beta} = \frac{2\alpha}{\beta}$

$$r = \left(\frac{2\alpha}{\beta}\right)^{1/5} = \left(\frac{2\alpha}{\beta}\right)^{a/b}$$

$\therefore \frac{a}{b} = \frac{1}{5} \Rightarrow a = 1, b = 5$

29. (1) : $E_1 \propto 380$ cm

$E_2 \propto 760$ cm

$\frac{E_1}{E_2} = \frac{380}{760} = \frac{1}{2} = \frac{a}{b}$

$a = 1$

30. (1) : $\vec{E} = \frac{3}{5}E_0\hat{i} + \frac{4}{5}E_0\hat{j}$, $A' = 0.2 \text{ m}^2$ $(y - z$ plane)

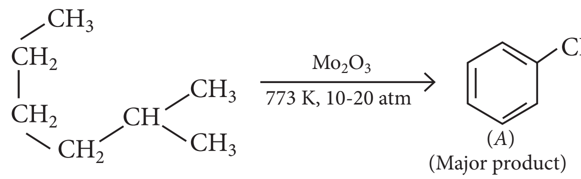
$A'' = 0.3 \text{ m}^2$ $(x - y$ plane)

$$\phi_1 = \vec{E} \cdot \vec{A}' = \left(\frac{3}{5}E_0\hat{i} + \frac{4}{5}E_0\hat{j}\right) \cdot (0.2\hat{i}) = \frac{0.6}{5}E_0 \quad \text{... (i)}$$

$$\phi_2 = \vec{E} \cdot \vec{A}'' = \left(\frac{3}{5}E_0\hat{i} + \frac{4}{5}E_0\hat{j}\right) \cdot (0.3\hat{j}) = \frac{1.2}{5}E_0 \quad \text{... (ii)}$$

$$\frac{a}{b} = \frac{\phi_1}{\phi_2} = \frac{0.6}{1.2} = \frac{1}{2} \Rightarrow a = 1$$

31. (b) :



32. (d) : $HCN \rightleftharpoons H^+ + CN^-$

$$K_a = \frac{[H^+][CN^-]}{[HCN]} \Rightarrow \frac{[HCN]}{[CN^-]} = \frac{[H^+]}{[K_a]}$$

Given, $K_a = 6.2 \times 10^{-10}$, pH = 3.0

$\Rightarrow [H^+] = 10^{-3}$

$$\therefore \frac{[HCN]}{[CN^-]} = \frac{1 \times 10^{-3}}{6.2 \times 10^{-10}} = 1.6 \times 10^6$$

Solubility equilibrium, $AgCN_{(s)} \rightleftharpoons \underset{x}{Ag^+} + \underset{x}{CN^-}$

But CN^- will react with H^+ ions

	CN^-	+ H^+	$\rightleftharpoons$	HCN
Initial conc.	x	1×10^{-3}		0
At Equilibrium	$x - y$	1×10^{-3}		y

(Concentration of H^+ remains same due to buffer action)

Thus, $[CN^-]_{Total} = x = [CN^-]_{eqm.} + [HCN]_{eqm.}$

Since, $[HCN]_{eqm.} = 1.6 \times 10^6\ [CN^-]_{eqm.}$ hence $[CN^-]_{eqm.}$ can be neglected.

$\therefore [HCN]_{eqm.} = x$

Also, $[CN^-] = \frac{[HCN]}{1.6 \times 10^6} = \frac{x}{1.6 \times 10^6}$

Solubility product $K_{sp} = [Ag^+]\ [CN^-]$

$$2.2 \times 10^{-16} = x \times \frac{x}{1.6 \times 10^6}$$

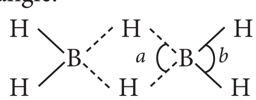
$x^2 = 1.6 \times 10^6 \times 2.2 \times 10^{-16} = 3.52 \times 10^{-10}$

$$x = \sqrt{3.52 \times 10^{-10}} = 1.88 \times 10^{-5} \approx 1.9 \times 10^{-5}$$

33. (d) : $2I^- + H_2O_2 + 2H^+ \longrightarrow I_2 + 2H_2O$

In this equation oxidation number of oxygen changes from -1 to -2 *i.e.*, reduction of H_2O_2 takes place. Therefore, it behaves as an oxidising agent and oxidises I^- (-1) to I_2 (0). In other options, oxidation state of oxygen changes from -1 to 0, that means H_2O_2 is getting oxidised, so behaving as reducing agent.

34. (a) : In B_2H_6, terminal bond angle is greater than that of bridged bond angle.



Bond angle $b > a$

Bond angle $\propto$ s-character $\propto \frac{1}{p - \text{character}}$

Therefore, terminal B — H bonds have less p-character or more s-character as compared to bridging bonds.

35. (a) : $Be_2 : \sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2\ \sigma^* 2s^2$

$$\text{Bond order} = \frac{N_b - N_a}{2} = \frac{4-4}{2} = 0$$

Bond order of Be_2 is 0. So, it does not exist.

36. (a) : Outer electronic configuration of given species :

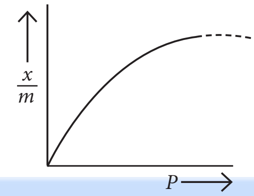
$V^{2+} = [Ar]3d^3$ $\quad$ $Cr^+ = [Ar]3d^5$
$Ni^{2+} = [Ar]3d^8$ $\quad$ $Cu^+ = [Ar]3d^{10}$
$Mn^{2+} = [Ar]3d^5$ $\quad$ $Co^+ = [Ar]3d^7 4s^1$
$Fe^{2+} = [Ar]3d^6$

So, Cr^+ and Mn^{2+} have the same outer electronic configuration.

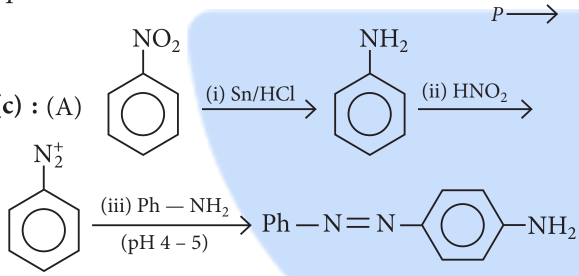
37. (c) : $\frac{x}{m} = KP^x$

At moderate pressure

$$\frac{x}{m} \propto P^{\frac{1}{n}}$$

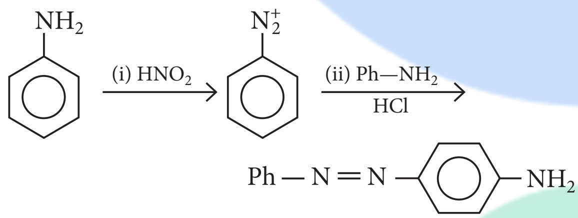


38. (c) : (A)

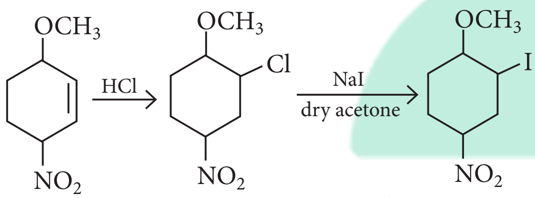


Diazonium ion

(C)



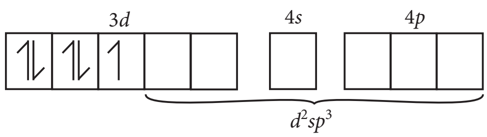
39. (a) :



(Finkelstein reaction)

40. (b) : $[Mn(CN)_6]^{4-}$:

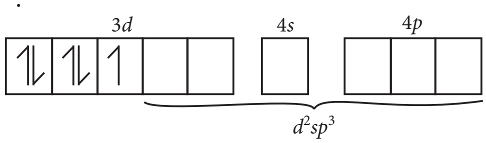
$Mn^{2+} \Rightarrow$



Due to presence of one unpaired electron, $[Mn(CN)_6]^{4-}$ is paramagnetic in nature.

$[Fe(CN)_6]^{3-}$:

$Fe^{3+} \Rightarrow$



Due to presence of one unpaired electron, $[Fe(CN)_6]^{3-}$ is also paramagnetic in nature.

41. (d) : (a) Neoprene is an addition homopolymer.
(b) Buna–N is a synthetic polymer.
(c) Buna-S is an elastomer.
(d) Synthesis of Buna-S needs nascent oxygen or free radical.

42. (d) : The graphical representation showing the variation of Gibbs energy (ΔG) with temperature (T) for the formation of oxide is known as Ellingham diagram.

43. (d) : Classical smog is reducing in nature and is known as reducing smog.
Ozone, which is an allotrope of oxygen, is an important intermediate in the formation of photochemical smog. Hydrocarbons and oxides of nitrogen contribute to the formation of photochemical smog.

44. (a) : $CH_3CH_2OH \xrightarrow{CrO_3—H_2SO_4} CH_3COOH$

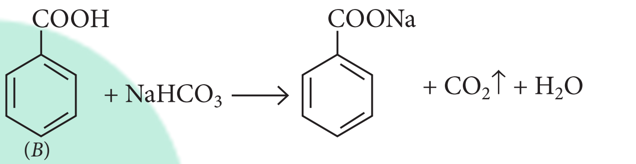
$$CH_2{=}CH_2 + O_2 \xrightarrow[H_2O]{Pd(II)/Cu(II)} CH_3CHO$$

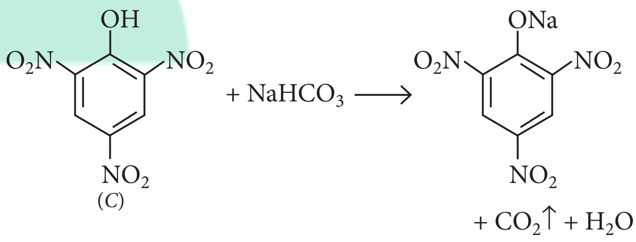
$$CH_3CH_2OH \xrightarrow[573\ K]{Cu} CH_3CHO$$

$$CH_3CN \xrightarrow[(ii)\ H_2O]{(i)\ DIBAL\text{-}H} CH_3CHO$$

$CrO_3—H_2SO_4$ is a strong oxidizing agent which further oxidises aldehyde to carboxylic acid.

45. (c) : Compounds which are more acidic than H_2CO_3, gives CO_2 gas on reaction with $NaHCO_3$.
Compound *B i.e.*, benzoic acid and compound *C i.e.*, picric acid both are more acidic than H_2CO_3. $+I$ effect of $—NH_2$ group decreases acidic nature of phenol, hence *A* is less acidic than H_2CO_3.





46. (d) : $CH_3CH_2CN \xrightarrow{(1)\ H_3O^+,\ \Delta} CH_3—CH_2—\overset{\overset{\displaystyle O}{\|}}{C}—OH$

$$\downarrow (2)\ SOCl_2$$

$$\underset{\text{(Rosenmund reduction)}}{CH_3CH_2CHO} \xleftarrow[H_2]{(3)\ Pd/BaSO_4} CH_3—CH_2—\overset{\overset{\displaystyle O}{\|}}{C}—Cl$$

47. (d) : Number of radial nodes $= n - l - 1$
For $3s$-orbital $n = 3, l = 0$
$\therefore$ Number of radial nodes $= 3 - 0 - 1 = 2$

48. (b) : $C_xH_yO_z + \left(x + \frac{y}{4} - z\right)O_2 \longrightarrow xCO_2 + \frac{y}{2}H_2O$

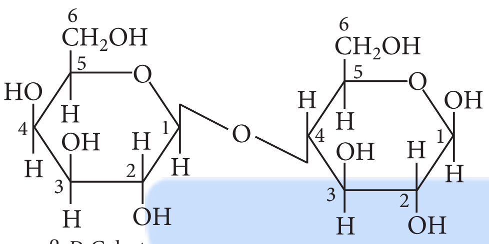
Mass of 'C' in 2.64 g of $CO_2 = \frac{12}{44} \times 2.64 = 0.72$ g

Mass of 'H' in 1.08 g of $H_2O = \frac{2}{18} \times 1.08 = 0.12$ g

Mass of 'O' in compound = 1.80 – (0.72 + 0.12) = 0.96 g

% of oxygen $= \frac{0.96}{1.80} \times 100 = 53.33\%$

49. (d) :

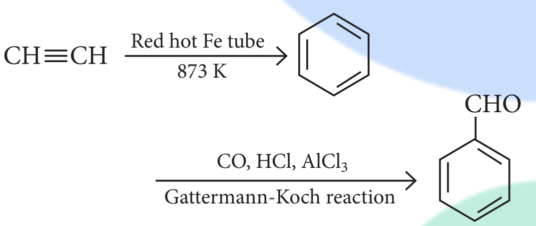


β-D-Galactose β-D-Glucose

The linkage is present between C–1 of galactose and C–4 of glucose.

50. (a) : +3 oxidation state of lanthanides is most stable and hence Ce^{4+} compounds are good oxidising agents. Eu^{2+} will act as a reducing agent so that it itself gets oxidised to a more stable +3 oxidation state.

51. (7) :



Benzaldehyde

All carbon atoms in benzaldehyde are sp^2-hybridised.

52. (1) : SF_6 does not undergo hydrolysis due to steric hindrance.

53. (173) : Balanced chemical equation is given as :

$8CrO_4^{2-} + 3S_2O_3^{2-} + 17H_2O \longrightarrow 6SO_4^{2-} + 8Cr(OH)_4^- + 2OH^-$

Applying molarity equation,

$$\frac{0.154 \times V}{8} = \frac{40 \times 0.25}{3} \Rightarrow V \approx 173 \text{ mL}$$

54. (4) : $R_f = \frac{\text{Distance travelled by compound}}{\text{Distance travelled by solvent}}$

From chromatogram, distance travelled by compound A = 2 cm and distance travelled by solvent = 5 cm

$\therefore \quad R_f = \frac{2}{5} = 4 \times 10^{-1}$

55. (4) : Let mmol of NaOH = x

and mmol of $Na_2CO_3 = y$

For first end point (phenolphthalein)

$NaOH + HCl \longrightarrow NaCl + H_2O$

$Na_2CO_3 + HCl \longrightarrow NaHCO_3 + NaCl$

$x + y = \frac{1}{10} \times 17.5$ ...(i)

For second end point (methyl orange)

$NaHCO_3 + HCl \longrightarrow NaCl + H_2O + CO_2$

mmol of $NaHCO_3$ = mmol of HCl

$y = \frac{1}{10} \times 1.5$ ...(ii)

From equation (i) and (ii), $x = 1.60$, $y = 0.15$

Weight of $Na_2CO_3 = 0.15 \times 10^{-3} \times 106 = 1.59 \times 10^{-2}$ g

Weight % of $Na_2CO_3 = \frac{1.59 \times 10^{-2}}{0.4} \times 100 = 3.975 \approx 4$

56. (375) : $A_2B_3 \longrightarrow 2A^{3+} + 3B^{2-}$

Number of ions = 2 + 3 = 5

$i = 1 + (n - 1)\alpha$

$= 1 + (5 - 1) \times 0.6 = 3.4$

$\Delta T_b = K_b \times m \times i$

$= 0.52 \times 1 \times 3.4 = 1.768°C$

$\Delta T_b = T_{b(\text{solution})} - T_{b(H_2O)}$

$1.768 = T_{b(\text{solution})} - 100$

$T_{b(\text{solution})} = 100 + 1.768 = 101.768°C$

$= 273 + 101.768 = 374.768 \text{ K} \approx 375 \text{ K}$

57. (70) : $\frac{P_1}{T_1} = \frac{P_2}{T_2}$

$$\frac{35}{(273+27)} = \frac{40}{T_2} \Rightarrow T_2 = 342.86 \text{ K}$$

$T_2 = 342.86 - 273 = 69.86\ °C \approx 70\ °C$

58. (741) : Note : The correct equation should be :

$NH_2CN_{(s)} + \frac{3}{2}O_{2(g)} \longrightarrow N_{2(g)} + CO_{2(g)} + H_2O_{(l)}$

$\Delta H = \Delta U + \Delta n_g RT$

From the given reaction, $\Delta n_g = 2 - \frac{3}{2} = 0.5$

$\therefore \quad \Delta H = -742.24 + 0.5 \times 8.314 \times 10^{-3} \times 298 = -741$ kJ

59. (526) : $\log k = \log A - \frac{E_a}{2.303RT}$ (Arrhenius equation)

$\text{Slope} = -\frac{E_a}{2.303R} = -10000$

$$\log\frac{k_2}{k_1} = \frac{E_a}{2.303R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

$$\log\frac{10^{-4}}{10^{-5}} = 10000\left(\frac{1}{500} - \frac{1}{T_2}\right)$$

$$\frac{1}{10000} = \frac{1}{500} - \frac{1}{T_2}$$

$T_2 = 526.32 \approx 526$ K

60. (5576) : $Na_{(g)} \longrightarrow Na^+_{(g)}$; $\Delta H_1 = 495.8 \text{ kJ mol}^{-1}$

$\frac{1}{2}Br_{2(l)} \longrightarrow Br^-_{(g)}$; $\Delta H_2 = -325 \text{ kJ mol}^{-1}$

$Na^+_{(g)} + Br^-_{(g)} \longrightarrow NaBr_{(s)}$; $\Delta H_3 = -728.4 \text{ kJ mol}^{-1}$

$Na_{(s)} + \frac{1}{2}Br_{2(l)} \longrightarrow NaBr_{(s)}$; $\Delta H = ?$

$\Delta H = 495.8 - 325 - 728.4 = -557.6$ kJ/mol

$= -5576 \times 10^{-1}$ kJ/mol

Note : Data is incomplete in this question. The question has been solved only with the help of given information. The value of $\Delta H_{\text{sublimation}}$ of $Na_{(s)}$, $\Delta H_{\text{vaporisation}}$ of $Br_{2(l)}$ and $\Delta H_{\text{dissociation}}$ of $Br_{2(g)}$ must be given.

61. (b) : $x = 1 + \cos^2\theta + \cos^4\theta + ... = \frac{1}{1-\cos^2\theta} = \frac{1}{\sin^2\theta}$

$$\Rightarrow \sin^2\theta = \frac{1}{x} \quad \text{.... (i)}$$

$$y = 1 + \sin^2\phi + \sin^4\phi + ... = \frac{1}{1-\sin^2\phi} = \frac{1}{\cos^2\phi}$$

$$\Rightarrow \cos^2\phi = \frac{1}{y} \quad \text{... (ii)}$$

$z = 1 + \cos^2\theta \sin^2\phi + \cos^4\theta \sin^4\phi + ...$

$$= \frac{1}{1-\cos^2\theta\sin^2\phi} = \frac{1}{1-(1-\sin^2\theta)(1-\cos^2\phi)}$$

$$\Rightarrow z = \frac{1}{1-\left(1-\frac{1}{x}\right)\left(1-\frac{1}{y}\right)} \Rightarrow z = \frac{1}{\frac{xy-(x-1)(y-1)}{xy}}$$

$$\Rightarrow z = \frac{xy}{x+y-1} \Rightarrow xy + z = z(x+y)$$

62. (d) : Given, $\sin 2\theta + \tan 2\theta > 0$

$$\Rightarrow \sin 2\theta + \frac{\sin 2\theta}{\cos 2\theta} > 0 \Rightarrow \sin 2\theta \frac{(\cos 2\theta + 1)}{\cos 2\theta} > 0$$

$\Rightarrow \tan 2\theta\,(1 + \cos 2\theta) > 0$

$\Rightarrow \tan 2\theta > 0 \quad (\because \cos 2\theta + 1 > 0)$

$$\Rightarrow 2\theta \in \left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right) \cup \left(2\pi, \frac{5\pi}{2}\right) \cup \left(3\pi, \frac{7\pi}{2}\right)$$

$$\Rightarrow \theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$$

63. (b) : We have, $xyz = 2^3 \times 3$

Let $x = 2^{\alpha_1} \times 3^{\beta_1}$, $y = 2^{\alpha_2} \times 3^{\beta_2}$ and $z = 2^{\alpha_3} \times 3^{\beta_3}$

Since, $\alpha_1 + \alpha_2 + \alpha_3 = 3$

$\therefore$ Number of non-negative integral solutions

$= {}^{3+3-1}C_{3-1} = {}^5C_2 = 10$

Also, $\beta_1 + \beta_2 + \beta_3 = 1$

$\therefore$ Number of non-negative integral solutions

$= {}^{1+3-1}C_{3-1} = {}^3C_2 = 3$

Total number of positive integral solutions = 10 × 3 = 30

64. (b) : Slope of tangent, $m = 1/2$

Equation of tangent in slope form to the parabola $y^2 = 6x$ is given by

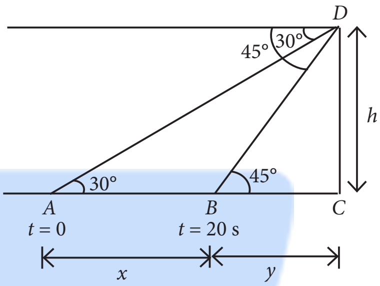
$$y = mx + \frac{3}{2m} \quad (\because 4a = 6 \Rightarrow a = 3/2)$$

$$\Rightarrow y = \frac{1}{2}x + \frac{3}{2 \times \frac{1}{2}} \Rightarrow y = \frac{x}{2} + 3$$

$\Rightarrow 2y = x + 6$

Only point (5, 4) does not lie on it.

65. (a) :



Let speed of boat is u and height of tower is h, $AB = x$ and $BC = y$.

In ΔBCD, $\frac{h}{y} = \tan 45° \Rightarrow h = y$

Also, in ΔACD, $\frac{h}{x+y} = \tan 30° \Rightarrow \frac{y}{x+y} = \frac{1}{\sqrt{3}}$

$$\Rightarrow x + y = \sqrt{3}y \Rightarrow x = (\sqrt{3}-1)y$$

Since, Speed $= \frac{\text{Distance}}{\text{Time}} \Rightarrow u = \frac{x}{20} = \frac{(\sqrt{3}-1)y}{20}$

$\therefore$ Time taken to travel from B to $C = \frac{\text{Distance}}{\text{Speed}}$

$$= \frac{y}{u} = \frac{y}{y\frac{(\sqrt{3}-1)}{20}} = \frac{20}{\sqrt{3}-1} = 10(\sqrt{3}+1) \text{ secs}$$

66. (b) : We have, $\frac{x^2}{a} + \frac{y^2}{b} = 1$ ... (i)

Differentiating (i) with respect to x, we get

$$\frac{2x}{a} + \frac{2y}{b}\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{bx}{ay} = m_1$$

Also, $\frac{x^2}{c} + \frac{y^2}{d} = 1$ ... (ii)

Differentiating (ii) with respect to x, we get

$$\frac{2x}{c} + \frac{2y}{d}\frac{dy}{dx} = 0$$

$$\Rightarrow m_2 = \frac{dy}{dx} = -\frac{dx}{cy}$$

Now, $m_1 m_2 = -1$ [Given]

$$\Rightarrow \left(-\frac{bx}{ay}\right)\left(\frac{-dx}{cy}\right) = -1 \Rightarrow bdx^2 = -acy^2 \quad \text{... (iii)}$$

Now, subtracting (ii) from (i), we get

$$\left(\frac{1}{a}-\frac{1}{c}\right)x^2+\left(\frac{1}{b}-\frac{1}{d}\right)y^2=0$$

$$\Rightarrow \left(\frac{c-a}{ac}\right)x^2+\left(\frac{d-b}{bd}\right)y^2=0$$

$$\Rightarrow \frac{(c-a)}{ac}x^2+\left(\frac{d-b}{bd}\right)\left(\frac{-bdx^2}{ac}\right)=0 \qquad \text{[Using (iii)]}$$

$\Rightarrow$ $(c-a)-(d-b)=0$ $\Rightarrow$ $c-d=a-b$

67. (a) : Required probability $=\left(\frac{2}{3}\times\frac{3}{4}\right)^3=\frac{1}{8}$

68. (d) : We have, $f(n+1)=f(n)+f(1)$

$f(2)=f(1)+f(1)=2f(1)$

$f(3)=f(2)+f(1)=3f(1)$

....

....

$f(n)=n\,f(1)$

$\therefore$ f is one-one.

Now, let $f(g(x_2))=f(g(x_1))$

$\Rightarrow$ $g(x_2)=g(x_1)$ ($\because$ f is one-one)

$\Rightarrow$ $x_1=x_2$ ($\because$ fog is one-one)

$\Rightarrow$ g is one-one

Now, $f(g(n))=g(n)\,f(1)$

$\therefore$ fog may be many-one if $g(n)$ is many-one.

69. (a) : Let $z=x+iy$ and $\bar{z}=x-iy$

$\therefore$ $(2-i)z=(2+i)\bar{z}$ $\Rightarrow$ $(2-i)(x+iy)=(2+i)(x-iy)$

$\Rightarrow$ $2x+2iy-ix+y=2x-2iy+ix+y$ $\Rightarrow$ $2y=x$ ... (i)

Also, $(2+i)z+(i-2)\bar{z}-4i=0$

$\Rightarrow$ $(2+i)(x+iy)+(i-2)(x-iy)-4i=0$

$\Rightarrow$ $2x+2iy+ix-y+ix-2x+y+2iy-4i=0$

$\Rightarrow$ $x+2y-2=0$ .... (ii)

Solving (i) and (ii), we get $x=1$, $y=1/2$

Equation of tangent is $iz+\bar{z}+1+i=0$

$\Rightarrow$ $i(x+iy)+(x-iy)+1+i=0$

$\Rightarrow$ $ix-y+x-iy+1+i=0$

$\Rightarrow$ $i(x-y+1)+(x-y+1)=0$

$\Rightarrow$ $x-y+1=0$

$$\therefore \text{ Radius, } r=\left|\frac{1-\frac{1}{2}+1}{\sqrt{2}}\right|=\frac{3}{2\sqrt{2}}$$

70. (c) : Let $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-1}{-2}=r$ (say)

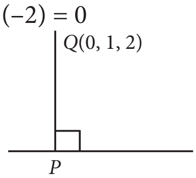
$\therefore$ $P(x,y,z)=(2r+1,3r-1,-2r+1)$

Since, $\overrightarrow{QP}\perp(2\hat{i}+3\hat{j}-2\hat{k})$.

$\therefore$ $(2r+1)\cdot 2+(3r-2)\cdot 3+(-2r-1)\cdot(-2)=0$

$\Rightarrow$ $4r+2+9r-6+4r+2=0$

$\Rightarrow$ $r=\frac{2}{17}$



So, $P\equiv\left(\frac{21}{17},\frac{-11}{17},\frac{13}{17}\right)$

$$\therefore \overrightarrow{PQ}=\frac{-21\hat{i}+28\hat{j}+21\hat{k}}{17}=\frac{7}{17}(-3\hat{i}+4\hat{j}+3\hat{k})$$

So, required line is $\frac{x}{-3}=\frac{y-1}{4}=\frac{z-2}{3}$

71. (c) : We have, $x^2-2(3k-1)x+8k^2-7>0$

Now, $D<0$

$\Rightarrow$ $4(3k-1)^2-4\times 1\times(8k^2-7)<0$

$\Rightarrow$ $9k^2-6k+1-8k^2+7<0$

$\Rightarrow$ $k^2-6k+8<0$

$\Rightarrow$ $(k-4)(k-2)<0$ $\Rightarrow$ $k\in(2,4)$

i.e., $k=3$

72. (b) : Let $I=\displaystyle\int\frac{\frac{\sin\theta\cdot\sin 2\theta(\sin^6\theta+\sin^4\theta+\sin^2\theta)}{\sqrt{2\sin^4\theta+3\sin^2\theta+6}}}{1-\cos 2\theta}\,d\theta$

$$=\int\frac{\frac{\sin\theta\cdot 2\sin\theta\cos\theta\cdot\sin^2\theta(\sin^4\theta+\sin^2\theta+1)}{(2\sin^4\theta+3\sin^2\theta+6)^{1/2}}}{2\sin^2\theta}\,d\theta$$

$$=\int\sin^2\theta\cdot\cos\theta(\sin^4\theta+\sin^2\theta+1)(2\sin^4\theta+3\sin^2\theta+6)^{1/2}\,d\theta$$

Putting $\sin\theta=t$ $\Rightarrow$ $\cos\theta\,d\theta=dt$, we get

$$I=\int t^2(t^4+t^2+1)(2t^4+3t^2+6)^{1/2}\,dt$$

$$=\int(t^5+t^3+t)\,t\,(2t^4+3t^2+6)^{1/2}\,dt$$

$$=\int(t^5+t^3+t)\,(2t^6+3t^4+6t^2)^{1/2}\,dt$$

Again put $2t^6+3t^4+6t^2=u^2$

$\Rightarrow$ $12(t^5+t^3+t)dt=2udu$

$$\therefore\ I=\int(u^2)^{1/2}\frac{2udu}{12}=\int\frac{u^2}{6}du=\frac{u^3}{18}+c$$

$$=\frac{(2t^6+3t^4+6t^2)^{3/2}}{18}+c$$

$$=\frac{1}{18}(2\sin^6\theta+3\sin^4\theta+6\sin^2\theta)^{3/2}+c$$

$$=\frac{1}{18}[2(1-\cos^2\theta)^3+3(1-\cos^2\theta)^2+6(1-\cos^2\theta)]^{3/2}+c$$

$$=\frac{1}{18}\{(1-\cos^2\theta)[2(1-\cos^2\theta)^2+3(1-\cos^2\theta)+6]\}^{3/2}+c$$

$$=\frac{1}{18}[(1-\cos^2\theta)(2\cos^4\theta-7\cos^2\theta+11)]^{3/2}+c$$

$$=\frac{1}{18}[11-18\cos^2\theta+9\cos^4\theta-2\cos^6\theta]^{3/2}+c$$

73. (d) :

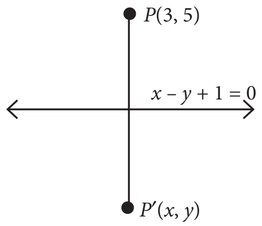


Image of point $P(3, 5)$ in the line $x - y + 1 = 0$, is given by

$$\frac{x-3}{1}=\frac{y-5}{-1}=-2\left(\frac{3-5+1}{1+1}\right)=1$$

$\Rightarrow$ $x = 4$ and $y = 4$, which lies on $(x - 2)^2 + (y - 4)^2 = 4$.

74. (b) : Since, $f(1) = f(2)$

$\Rightarrow$ $1 - a + b - 4 = 8 - 4a + 2b - 4$

$\Rightarrow$ $3a - b = 7$ ... (i)

$f(x) = x^3 - ax^2 + bx - 4$

$\Rightarrow$ $f'(x) = 3x^2 - 2ax + b$

$$\therefore\ f'\left(\frac{4}{3}\right)=3\left(\frac{4}{3}\right)^2-2a\left(\frac{4}{3}\right)+b=0 \quad \text{(given)}$$

$$\Rightarrow \frac{16}{3}-\frac{8a}{3}+b=0$$

$\Rightarrow$ $8a - 3b - 16 = 0$ ... (ii)

Solving (i) and (ii), we get $a = 5$ and $b = 8$

75. (b) : We know that $l^2 + m^2 + n^2 = 1$

And given that $l^2 + m^2 - n^2 = 0$

$$\Rightarrow\ 2n^2=1 \Rightarrow\ n=\pm\frac{1}{\sqrt{2}}$$

Also, $l + m = n \Rightarrow l^2 + m^2 + 2lm = n^2$

$\Rightarrow$ $n^2 + 2lm = n^2 \Rightarrow lm = 0$

If $l = 0$, then $m = n = \pm\frac{1}{\sqrt{2}}$

And if $m = 0$, then $l = n = \pm\frac{1}{\sqrt{2}}$

So, direction cosines of two lines are

$$\left(0,\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right) \text{ and } \left(\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}\right)$$

$$\text{So, } \cos\alpha=\frac{0+0+\frac{1}{2}}{\sqrt{1}\sqrt{1}}=\frac{1}{2} \quad\Rightarrow\quad \alpha=\frac{\pi}{3}$$

$$\therefore\ \sin^4\alpha+\cos^4\alpha=\left(\sin\frac{\pi}{3}\right)^4+\left(\cos\frac{\pi}{3}\right)^4$$

$$=\left(\frac{\sqrt{3}}{2}\right)^4+\left(\frac{1}{2}\right)^4=\frac{9}{16}+\frac{1}{16}=\frac{5}{8}$$

76. (c) : $A \to (B \to A)$

$\equiv A \to (\sim B \vee A)$

$\equiv \sim A \vee (\sim B \vee A) \equiv \sim A \vee (A \vee \sim B)$

$\equiv (\sim A \vee A) \vee \sim B \equiv T \vee \sim B \equiv T$

Also, $T \equiv T \vee B$

$\equiv (\sim A \vee A) \vee B$

$\equiv \sim A \vee (A \vee B)$

$\equiv A \to (A \vee B)$

77. (a) : Given, $y(0) = 0$

$$\text{and } \frac{dy}{dx}=\frac{x^2-4x+y+8}{x-2}=\frac{(x-2)^2+y+4}{x-2}$$

$$\Rightarrow \frac{dy}{dx}-\frac{y}{x-2}=(x-2)+\frac{4}{x-2}, \text{ which is L.D.E.}$$

$$\text{Here, I.F.}=e^{-\int\frac{1}{x-2}dx}=\frac{1}{x-2}$$

$\therefore$ Required solution is given by

$$y\cdot\frac{1}{x-2}=\int\frac{1}{x-2}\left((x-2)+\frac{4}{x-2}\right)dx$$

$$\Rightarrow \frac{y}{x-2}=x-\frac{4}{x-2}+C$$

At $x = 0, y = 0 \Rightarrow C = -2$

$\therefore$ $y = x(x - 2) - 4 - 2(x - 2)$

$\Rightarrow$ $y = x^2 - 4x$, which passes through (5, 5).

78. (c) : Given limit is of 1^∞ form.

$$\text{So, given limit } (l)=\exp\left(\lim_{n\to\infty}\frac{1+\frac{1}{2}+\frac{1}{3}+.....+\frac{1}{n}}{n}\right)$$

$$\text{Now, } 0\le 1+\frac{1}{2}+\frac{1}{3}+....+\frac{1}{n}\le 1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+....+\frac{1}{\sqrt{n}}$$

$$\le 2\sqrt{n}-1$$

So, $l = \exp(0)$ (from sandwich theorem)

$= 1$

79. (a) : Given quadratic equation is

$ax^2 + bx + c = 0$

For equal roots, $D = 0 \Rightarrow b^2 = 4ac$

Case-I : If $b = 2$, then $ac = 1$

$\therefore$ $(a, b, c) = (1, 2, 1)$

Case-II : If $b = 4$, then $ac = 4$

So, $(a, b, c) = (1, 4, 4)$ or $(4, 4, 1)$ or $(2, 4, 2)$

Case III : If $b = 6$, then $ac = 9$

So, $(a, b, c) = (3, 6, 3)$

$$\therefore\ \text{Required probability}=\frac{5}{216}$$

80. (b) : Let $I=\int_{-1}^{1}x^2e^{[x^3]}\,dx$

$$=\int_{-1}^{0}x^2e^{[x^3]}dx+\int_{0}^{1}x^2e^{[x^3]}dx \;=\int_{-1}^{0}x^2e^{-1}dx+\int_{0}^{1}x^2e^{0}dx$$

$$=\frac{1}{e}\left[\frac{x^3}{3}\right]_{-1}^{0}+\left[\frac{x^3}{3}\right]_{0}^{1}=\frac{1}{e}\times\left(0-\left(\frac{-1}{3}\right)\right)+\frac{1}{3}$$

$$=\frac{1}{3e}+\frac{1}{3}=\frac{1+e}{3e}$$

81. (13) : We have, $A=\begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix}$

and $I_2=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Now, $I_2+A=\begin{bmatrix} 1 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 1 \end{bmatrix}$

and $I_2-A=\begin{bmatrix} 1 & \tan\left(\frac{\theta}{2}\right) \\ -\tan\left(\frac{\theta}{2}\right) & 1 \end{bmatrix}$

$$(I_2-A)^{-1}=\frac{1}{\sec^2\left(\frac{\theta}{2}\right)}\begin{bmatrix} 1 & \tan\left(\frac{\theta}{2}\right) \\ -\tan\left(\frac{\theta}{2}\right) & 1 \end{bmatrix}^T$$

$$=\frac{1}{\sec^2\left(\frac{\theta}{2}\right)}\begin{bmatrix} 1 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 1 \end{bmatrix}$$

Now, $(I_2+A)(I_2-A)^{-1}$

$$=\frac{1}{\sec^2\left(\frac{\theta}{2}\right)}\begin{bmatrix} 1 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 1 \end{bmatrix}\begin{bmatrix} 1 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 1 \end{bmatrix}$$

$$=\frac{1}{\sec^2\left(\frac{\theta}{2}\right)}\begin{bmatrix} 1-\tan^2\left(\frac{\theta}{2}\right) & -2\tan\left(\frac{\theta}{2}\right) \\ 2\tan\left(\frac{\theta}{2}\right) & 1-\tan^2\left(\frac{\theta}{2}\right) \end{bmatrix}$$

$$=\frac{1+\tan^2\left(\frac{\theta}{2}\right)}{\sec^2\left(\frac{\theta}{2}\right)}\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$=\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

So, $a=\cos\theta$, $b=\sin\theta$

$\therefore\quad 13(a^2+b^2)=13(\cos^2\theta+\sin^2\theta)=13$

82. (144) : Let $f(x)=x^6+ax^5+bx^4+cx^3+dx^2+ex+f$

Since, $\lim\limits_{x\to 0}\frac{f(x)}{x^3}=1$, which is non-zero and finite.

$\therefore\quad d=e=f=0$

and $f(x)=x^3(x^3+ax^2+bx+c)$

$\Rightarrow\quad \lim\limits_{x\to 0}\frac{f(x)}{x^3}=c=1$

Now, as $f(x)=x^6+ax^5+bx^4+x^3$

$\Rightarrow\quad f'(x)=6x^5+5ax^4+4bx^3+3x^2$

Also, $f'(x)=0$ at $x=1$ and $x=-1$

Now, $f'(1)=0 \Rightarrow\quad 6+5a+4b+3=0$

$\Rightarrow\quad 5a+4b=-9$ ... (i)

and $f'(-1)=0$

$\Rightarrow\quad -6+5a-4b+3=0$

$\Rightarrow\quad 5a-4b=3$ ... (ii)

Solving (i) and (ii), we get

$$a=\frac{-3}{5}\text{ and } b=\frac{-3}{2}$$

$$\therefore\quad f(x)=x^6-\frac{3}{5}x^5-\frac{3}{2}x^4+x^3$$

Hence, $5f(2)=5\left[64-\frac{3}{5}\cdot 32-\frac{3}{2}\cdot 16+8\right]$

$=320-96-120+40=144$

83. (7) : Given, $A^2=I$

$\Rightarrow\quad AA'=I\ (\because\ A'=A)$

$\Rightarrow\quad A$ is orthogonal.

So, $x^2+y^2+z^2=1$ and $xy+yz+zx=0$

Now, $(x+y+z)^2=(x^2+y^2+z^2)+2(xy+yz+zx)$

$=1+2\,(0)$

$\Rightarrow\quad x+y+z=1\qquad [\because\ x+y+z>0]$

Hence, $x^3+y^3+z^3$

$=3xyz+(x+y+z)(x^2+y^2+z^2-xy-yz-zx)$

$=3\times 2+1\,(1-0)=7$

84. (2) : $k=\frac{4\sqrt{3}}{\sqrt{3}x+y}=\frac{\sqrt{3}x-y}{4\sqrt{3}}$

$\Rightarrow\quad 3x^2-y^2=48\quad\Rightarrow\quad\frac{x^2}{16}-\frac{y^2}{48}=1$, which is a hyperbola.

Now, $48=16(e^2-1)\quad\Rightarrow\quad 4=e^2$

$\Rightarrow\quad$ Eccentricity, $e=2$

85. (32) : Three digit numbers formed with digits 1, 2, 3, 4, 5 which are divisible by $3=4\times 3!=24$

Three digit numbers formed with digits 1, 2, 3, 4, 5 which are divisible by $5=4\times 3=12$

Three digit numbers formed with digits 1, 2, 3, 4, 5 which are divisible by $15=1\times 2\times 2!=4$

$\therefore\quad$ Required numbers $=24+12-4=32$

86. (21) : Here, $D=\begin{vmatrix} k & 1 & 2 \\ 3 & -1 & -2 \\ -2 & -2 & -4 \end{vmatrix}=0$ for $k\in R$

Also, $D_1=\begin{vmatrix} 1 & 1 & 2 \\ 2 & -1 & -2 \\ 3 & -2 & -4 \end{vmatrix}=0$

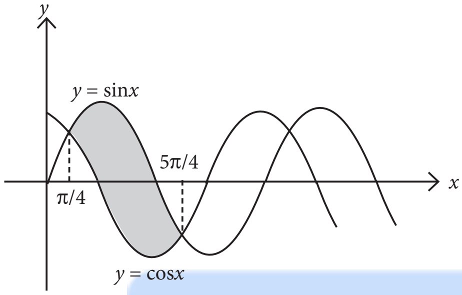
So, for the given system to have infinite many solutions, $D_2 = 0$.

$$\Rightarrow \begin{vmatrix} k & 1 & 2 \\ 3 & 2 & -2 \\ -2 & 3 & -4 \end{vmatrix} = 0$$

$\Rightarrow\ k(-8 + 6) - 1(-12 - 4) + 2(9 + 4) = 0$

$\Rightarrow\ -2k + 16 + 26 = 0 \ \Rightarrow\ 2k = 42 \ \Rightarrow\ k = 21$

87. (64) :



Area, $A = \int_{\pi/4}^{5\pi/4} (\sin x - \cos x)\, dx = \left[-\cos x - \sin x\right]_{\pi/4}^{5\pi/4}$

$$= -\left[\left(\cos\frac{5\pi}{4} + \sin\frac{5\pi}{4}\right) - \left(\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\right)\right]$$

$$= -\left[\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\right] = \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}} = 2\sqrt{2}$$

$\Rightarrow\ A^4 = (2\sqrt{2})^4 = 16 \times 4 = 64$

88. (9) : Given, length of side of $A_1 = 12$

Length of diagonal of $A_2 = 12$

$\therefore$ Length of side of $A_2 = \dfrac{12}{\sqrt{2}}$

Length of side of $A_3 = \dfrac{12}{(\sqrt{2})^2}$

Length of side of $A_n = \dfrac{12}{(\sqrt{2})^{n-1}}$

$\therefore$ Area of $A_n = \left(\dfrac{12}{(\sqrt{2})^{n-1}}\right)^2 < 1 \ \Rightarrow\ \dfrac{144}{2^{(n-1)}} < 1$

$\Rightarrow\ 2^{(n-1)} > 144 \ \Rightarrow\ n - 1 > 7$

$\Rightarrow\ n > 8$

Since, n is a natural number.

$\therefore$ Least value of n is 9.

89. (12) : Given, $\vec{r} \times \vec{a} = \vec{c} \times \vec{a} \ \Rightarrow\ (\vec{r} - \vec{c}) \times \vec{a} = \vec{0}$

$\Rightarrow\ \vec{r} - \vec{c} = \lambda\vec{a} \ \Rightarrow\ \vec{r} = \vec{c} + \lambda\vec{a}$

$\Rightarrow\ \vec{r}\cdot\vec{b} = \vec{c}\cdot\vec{b} + \lambda\vec{a}\cdot\vec{b} \ \Rightarrow\ 0 = \vec{c}\cdot\vec{b} + \lambda\vec{a}\cdot\vec{b}$

$\Rightarrow\ \lambda = \dfrac{-\vec{c}\cdot\vec{b}}{\vec{a}\cdot\vec{b}} = \dfrac{-2}{-1} = 2$,

$\therefore\ \vec{r}\cdot\vec{a} = \vec{c}\cdot\vec{a} + \lambda\vec{a}\cdot\vec{a} = 0 + 2(6) = 12$

90. (2) : $f(x) = |2x + 1| - 3|x + 2| + |x^2 + x - 2|$

$= |2x + 1| - 3|x + 2| + |x + 2||x - 1|$

$= |2x + 1| + |x + 2|(|x - 1| - 3)$

Critical points are $x = -1/2, -2$ and 1.

But $x = -2$ is making a zero twice in product so, points of non differentiability are $x = -1/2$ and $x = 1$.

$\therefore$ Number of points of non-differentiability is 2.

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