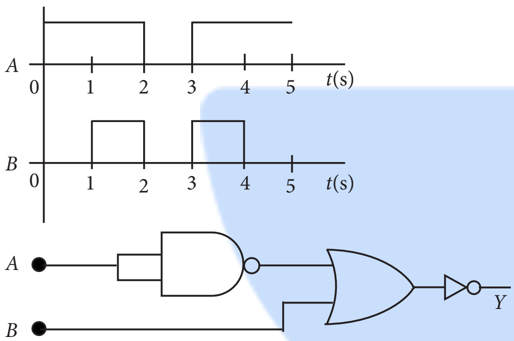
JEE MAIN 2021

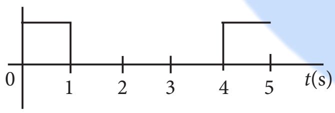
ONLINE
26th February
2nd Shift

PHYSICS

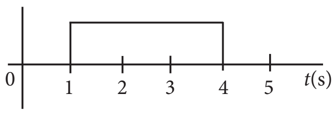
SECTION-A (MULTIPLE CHOICE QUESTIONS)

1. Draw the output signal Y in the given combination of gates.



(a) 

(b) 

(c) 

(d) 

2. A tuning fork A of unknown frequency produces 5 beats/s with a fork of known frequency 340 Hz. When fork A is filed, the beat frequency decreases to 2 beats/s. What is the frequency of fork A?
(a) 335 Hz (b) 345 Hz (c) 338 Hz (d) 342 Hz

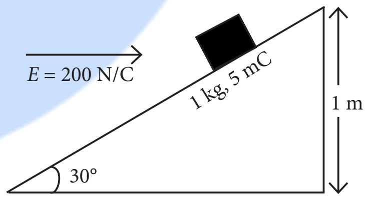
3. A scooter accelerates from rest for time t_1 at constant rate a_1 and then retards at constant rate a_2 for time t_2 and comes to rest. The correct value of $\frac{t_1}{t_2}$ will be

(a) $\frac{a_1}{a_2}$ (b) $\frac{a_1+a_2}{a_1}$ (c) $\frac{a_2}{a_1}$ (d) $\frac{a_1+a_2}{a_2}$

4. An aeroplane, with its wings spread 10 m, is flying at a speed of 180 km/h in a horizontal direction. The total intensity of earth's field at that part is 2.5×10^{-4} Wb/m^2 and the angle of dip is 60°. The emf induced between the tips of the plane wings will be _____.
(a) 88.37 mV (b) 62.50 mV
(c) 108.25 mV (d) 54.125 mV

5. An inclined plane making an angle of 30° with the horizontal is placed in a uniform horizontal electric field $200\frac{\text{N}}{\text{C}}$ as shown in figure. A body of mass 1 kg and charge 5 mC is allowed to slide down from rest at a height of 1 m. If the coefficient of friction is 0.2, find the time taken by the body to reach the bottom.

$$\left[g = 9.8\ \text{m/s}^2; \sin 30° = \frac{1}{2}; \cos 30° = \frac{\sqrt{3}}{2}\right]$$



(a) 1.3 s (b) 0.92 s (c) 0.46 s (d) 2.3 s

6. The internal energy (U), pressure (P) and volume (V) of an ideal gas are related as $U = 3PV + 4$. The gas is
(a) monoatomic only
(b) diatomic only
(c) polyatomic only
(d) either monoatomic or diatomic

7. A cord is wound round the circumference of wheel of radius r. The axis of the wheel is horizontal and the moment of inertia about it is I. A weight mg is attached to the cord at the end. The weight falls from rest. After falling through a distance 'h' the square of angular velocity of wheel will be

(a) $2gh$ (b) $\frac{2gh}{I+mr^2}$

(c) $\frac{2mgh}{I+mr^2}$ (d) $\frac{2mgh}{I+2mr^2}$

8. A particle executes S.H.M., the graph of velocity as a function of displacement is
(a) a helix (b) an ellipse
(c) a parabola (d) a circle

9. Given below are two statements:

Statement I : An electric dipole is placed at the centre of a hollow sphere. The flux of electric field through the sphere is zero but the electric field is not zero anywhere in the sphere.

Statement II : If R is the radius of a solid metallic sphere and Q be the total charge on it. The electric field at any point on the spherical surface of radius r ($< R$) is zero but the electric flux passing through this closed spherical surface of radius r is not zero.

In the light of above statements, choose the correct answer from the options given below.

(a) Both Statement I and Statement II are true.
(b) Statement I is true but Statement II is false.
(c) Both Statement I and Statement II are false.
(d) Statement I is false but Statement II is true.

10. A radioactive sample is undergoing α decay. At any time t_1, its activity is A and another time t_2, the activity is $\frac{A}{5}$. What is the average life time for the sample?

(a) $\frac{t_1 - t_2}{\ln 5}$ (b) $\frac{t_2 - t_1}{\ln 5}$
(c) $\frac{\ln(t_2 + t_1)}{2}$ (d) $\frac{\ln 5}{t_2 - t_1}$

11. A wire of 1 Ω has a length of 1 m. It is stretched till its length increases by 25 %. The percentage change in resistance to the nearest integer is

(a) 56% (b) 76% (c) 12.5% (d) 25%

12. Given below are two statements

Statement I : A second's pendulum has a time period of 1 second.

Statement II : It takes precisely one second to move between the two extreme positions.

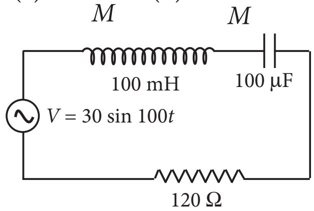
In the light of above statements, choose the correct answer from the options given below.

(a) Statement I is true but Statement II is false.
(a) Both Statement I and Statement II are true.
(c) Both Statement I and Statement II are false.
(d) Statement I is false but Statement II is true.

13. Two masses A and B, each of mass M are fixed together by a massless spring. A force acts on the mass B as shown in figure. If the mass A starts moving away from mass B with acceleration 'a' then the acceleration of mass B will be

(a) $\frac{Ma - F}{M}$ (b) $\frac{MF}{F + Ma}$ (c) $\frac{F + Ma}{M}$ (d) $\frac{F - Ma}{M}$

14. Find the peak current and resonant frequency of the following circuit (as shown in figure).



(a) 2 A and 100 Hz (b) 0.2 A and 100 Hz
(c) 2 A and 50 Hz (d) 0.2 A and 50 Hz

15. If 'C' and 'V' represent capacity and voltage respectively then what are the dimensions of λ where $C/V = \lambda$?

(a) $[M^{-2} L^{-4} I^3 T^7]$ (b) $[M^{-1} L^{-3} I^{-2} T^{-7}]$
(c) $[M^{-2} L^{-3} I^2 T^6]$ (d) $[M^{-3} L^{-4} I^3 T^7]$

16. The incident ray, reflected ray and the outward drawn normal are denoted by the unit vectors $\vec{a}, \vec{b}$ and $\vec{c}$ respectively. Then choose the correct relation for these vectors.

(a) $\vec{b} = 2\vec{a} + \vec{c}$ (b) $\vec{b} = \vec{a} - \vec{c}$
(c) $\vec{b} = \vec{a} - 2(\vec{a} \cdot \vec{c})\vec{c}$ (d) $\vec{b} = \vec{a} + 2\vec{c}$

17. The trajectory of a projectile in a vertical plane is $y = \alpha x - \beta x^2$, where α and β are constants and x and y are respectively the horizontal and vertical distance of the projectile from the point of projection. The angle of projection θ and the maximum height attained H are respectively given by

(a) $\tan^{-1}\alpha, \frac{4\alpha^2}{\beta}$ (b) $\tan^{-1}\beta, \frac{\alpha^2}{2\beta}$
(c) $\tan^{-1}\left(\frac{\beta}{\alpha}\right), \frac{\alpha^2}{\beta}$ (d) $\tan^{-1}\alpha, \frac{\alpha^2}{4\beta}$

18. The length of metallic wire is l_1 when tension in it is T_1. It is l_2 when the tension is T_2. The original length of the wire will be

(a) $\frac{T_1 l_1 - T_2 l_2}{T_2 - T_1}$ (b) $\frac{T_2 l_1 - T_1 l_2}{T_2 - T_1}$
(c) $\frac{T_2 l_1 + T_1 l_2}{T_1 + T_2}$ (d) $\frac{l_1 + l_2}{2}$

19. Given below are two statements : one is labeled as Assertion A and the other is labeled as Reason R.

Assertion R : For a simple microscope, the angular size of the object equals the angular size of the image.

Reason R : Magnification is achieved as the small object can be kept much closer to the eye than 25 cm and hence it subtends a large angle.

In the light of the above statements, choose the most appropriate answer from the options given below.

(a) Both A and R are true but R is NOT the correct explanation of A.
(b) A is true but R is false.
(c) A is false but R is true.
(d) Both A and R are true and R is the correct explanation of A.

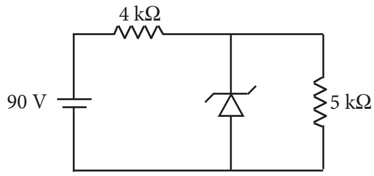
20. The recoil speed of a hydrogen atom after it emits a photon in going from $n = 5$ state to $n = 1$ state will be

(a) 4.34 m/s (b) 2.19 m/s
(c) 3.25 m/s (d) 4.17 m/s

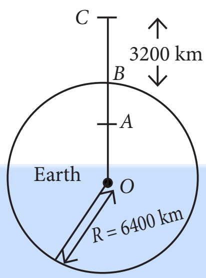
SECTION-B (NUMERICAL VALUE TYPE)

Attempt any 5 questions out of 10.

21. The zener diode has a V_z = 30 V. The current passing through the diode for the following circuit is ________ mA.



22. In the reported figure of earth the value of acceleration due to gravity is same at point A and C but it is smaller than that of its value at point B (surface of the earth). The value of $OA : AB$ will be $x : y$. The value of x is ________.



23. If the highest frequency modulating a carrier is 5 kHz, then the number of AM broadcast stations accommodated in a 90 kHz bandwidth are ________.

24. Two stream of photons, possessing energies equal to twice and ten times the work function of metal are incident on the metal surface successively. The value of ratio of maximum velocities of the photoelectrons emitted in the two respective cases is $x : y$. The value of x is ________.

25. A particle executes S.H.M. with amplitude 'a' and time period 'T'. The displacement of the particle when its speed is half of maximum speed is $\frac{\sqrt{xa}}{2}$. The value of x is ________.

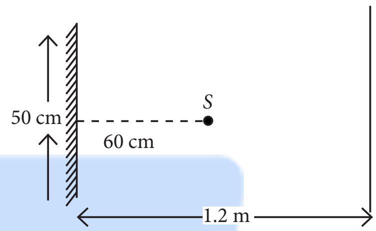
26. The volume V of a given mass of monoatomic gas changes with temperature T according to the relation $V = KT^{\frac{2}{3}}$. The workdone when temperature changes by 90 K will be xR. The value of x is ________. [R = universal gas constant]

27. Time period of a simple pendulum is T. The time taken to complete $\frac{5}{8}$ oscillations starting from mean position is $\frac{\alpha}{\beta}T$. The value of α is ________.

28. 1 mole of rigid diatomic gas performs a work of $\frac{Q}{5}$ when heat Q is supplied to it. The molar heat capacity of the gas during this transformation is $\frac{xR}{8}$. The value of x is ________. [R = universal gas constant]

29. 27 similar drops of mercury are maintained at 10 V each. All these spherical drops combine into a single big drop. The potential energy of the bigger drops is ________ times that of a smaller drop.

30. A point source of light S, placed at a distance 60 cm infront of the centre of a plane mirror of width 50 cm, hangs vertically on a wall. A man walks infront of the mirror along a line parallel to the mirror at a distance 1.2 m from it (see in the figure). The distance between the extreme points where he can see the image of the light source in the mirror is ________ cm.



CHEMISTRY

SECTION-A (MULTIPLE CHOICE QUESTIONS)

31. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).
Assertion (A) : In TlI_3, isomorphous to CsI_3, the metal is present in +1 oxidation state.
Reason (R) : Tl metal has fourteen f-electrons in its electronic configuration.
In the light of the above statements, choose the most appropriate answer from the options given below.
(a) Assertion is correct but reason is not correct.
(b) Both assertion and reason are correct but reason is not the correct explanation of assertion.
(c) Both assertion and reason are correct and reason is the correct explanation of assertion.
(d) Assertion is not correct but reason is correct.

32. Match List-I with List-II.

List-I	**List-II**
(A) Sucrose	(i) β-D-Galactose and β-D-Glucose
(B) Lactose	(ii) α-D-Glucose and β-D-Fructose
(C) Maltose	(iii) α-D-Glucose and α-D-Glucose

Choose the correct answer from the options given below.
(a) (A) → (i), (B) → (iii), (C) → (ii)
(b) (A) → (iii), (B) → (i), (C) → (ii)
(c) (A) → (iii), (B) → (ii), (C) → (i)
(d) (A) → (ii), (B) → (i), (C) → (iii)

33. The nature of charge on resulting colloidal particles when $FeCl_3$ is added to excess of hot water is
(a) sometimes positive and sometimes negative
(b) negative
(c) positive
(d) neutral.

34. Which of the following forms of hydrogen emits low energy β^- particles?

(a) Proton, H^+ (b) Deuterium, ${}^2_1H$
(c) Tritium, ${}^3_1H$ (d) Protium, ${}^1_1H$

35. The correct order of electron gain enthalpy is

(a) Te > Se > S > O (b) S > Se > Te > O
(c) O > S > Se > Te (d) S > O > Se > Te

36. Match List-I with List-II.

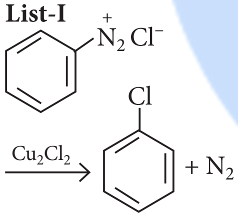
List-I	**List-II**
(A) Siderite	(i) Cu
(B) Calamine	(ii) Ca
(C) Malachite	(iii) Fe
(D) Cryolite	(iv) Al
	(v) Zn

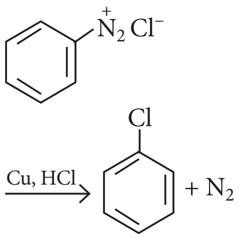
Choose the correct answer from the options given below.

(a) (A) → (i), (B) → (ii), (C) → (iii), (D) → (iv)
(b) (A) → (i), (B) → (ii), (C) → (v), (D) → (iii)
(c) (A) → (iii), (B) → (v), (C) → (i), (D) → (iv)
(d) (A) → (iii), (B) → (i), (C) → (v), (D) → (ii)

37. Match List-I with List-II.

List-I **List-II**

(A)  (i) Wurtz reaction

(B)  (ii) Sandmeyer reaction

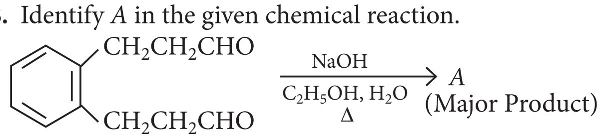
(C) $2CH_3CH_2Cl + 2Na \xrightarrow{Ether} C_2H_5-C_2H_5 + 2NaCl$ (iii) Fittig reaction

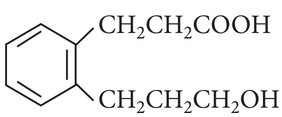
(D) $2C_6H_5Cl + 2Na \xrightarrow{Ether} C_6H_5-C_6H_5 + 2NaCl$ (iv) Gatterman reaction

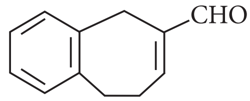
Choose the correct answer from the options given below.

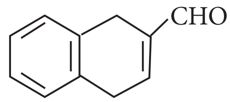
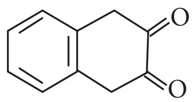
(a) (A) → (iii), (B) → (i), (C) → (iv), (D) → (ii)
(b) (A) → (iii), (B) → (iv), (C) → (i), (D) → (ii)
(c) (A) → (ii), (B) → (i), (C) → (iv), (D) → (iii)
(d) (A) → (ii), (B) → (iv), (C) → (i), (D) → (iii)

38. Identify *A* in the given chemical reaction.



(a) 

(b) 

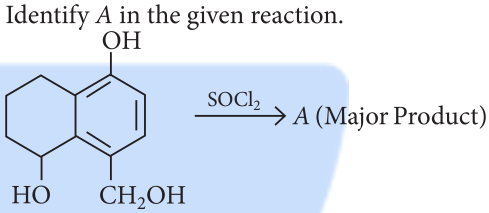
(c)  (d) 

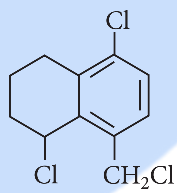
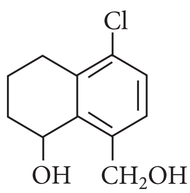
39. A. Phenyl methanamine
B. *N*, *N*-Dimethylaniline
C. *N*-Methylaniline
D. Benzenamine

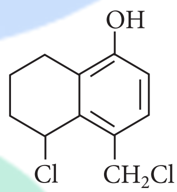
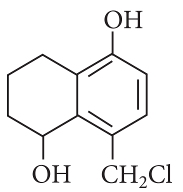
Choose the correct order of basic nature of the above amines.

(a) D > C > B > A (b) A > B > C > D
(c) D > B > C > A (d) A > C > B > D

40. Identify *A* in the given reaction.



(a)  (b) 

(c)  (d) 

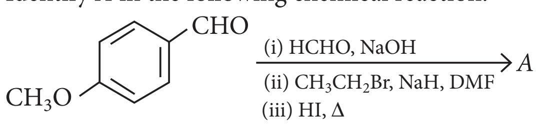
41. Match List-I with List-II.

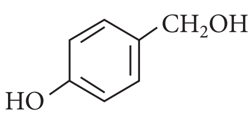
List-I	**List-II**
(A) Sodium carbonate	(i) Deacon
(B) Titanium	(ii) Castner-Kellner
(C) Chlorine	(iii) van-Arkel
(D) Sodium hydroxide	(iv) Solvay

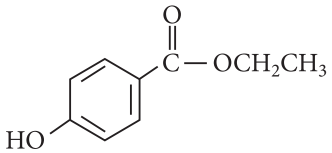
Choose the correct answer from the options given below.

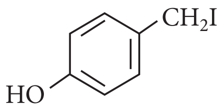
(a) (A) → (iii), (B) → (ii), (C) → (i), (D) → (iv)
(b) (A) → (iv), (B) → (i), (C) → (ii), (D) → (iii)
(c) (A) → (iv), (B) → (iii), (C) → (i), (D) → (ii)
(d) (A) → (i), (B) → (iii), (C) → (iv), (D) → (ii)

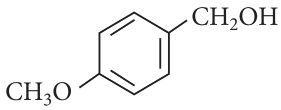
42. Identify *A* in the following chemical reaction.



(a) 

(b) 

(c) 

(d) 

43. Seliwanoff test and Xanthoproteic test are used for the identification of ______ and ______ respectively.
(a) ketoses, aldoses (b) ketoses, proteins
(c) aldoses, ketoses (d) proteins, ketoses

44. Match List-I with List-II.

List-I (Molecule)	**List-II (Bond order)**
(A) Ne_2	(i) 1
(B) N_2	(ii) 2
(C) F_2	(iii) 0
(D) O_2	(iv) 3

Choose the correct answer from the options given below.
(a) (A) → (ii), (B) → (i), (C) → (iv), (D) → (iii)
(b) (A) → (i), (B) → (ii), (C) → (iii), (D) → (iv)
(c) (A) → (iv), (B) → (iii), (C) → (ii), (D) → (i)
(d) (A) → (iii), (B) → (iv), (C) → (i), (D) → (ii)

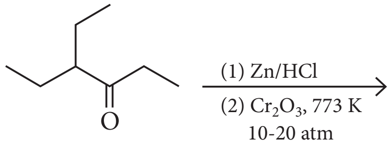
45. 2, 4-DNP test can be used to identify
(a) amine (b) aldehyde
(c) halogens (d) ether.

46. Calgon is used for water treatment. Which of the following statements is not true about calgon?
(a) Calgon contains the 2nd most abundant element by weight in the earth's crust.
(b) It is also known as Graham's salt.
(c) It is polymeric compound and is water soluble.
(d) It does not remove Ca^{2+} ion by precipitation.

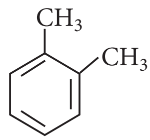
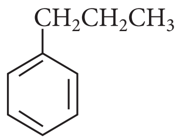
47. Ceric ammonium nitrate and $CHCl_3$/alc. KOH are used for the identification of functional groups present in ________ and ________ respectively.
(a) amine, alcohol (b) amine, phenol
(c) alcohol, amine (d) alcohol, phenol

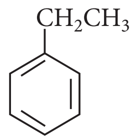
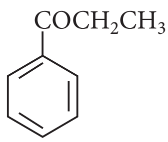
48. In $\overset{1}{C}H_2{=}\overset{2}{C}{=}\overset{3}{C}H{-}\overset{4}{C}H_3$ molecule, the hybridization of carbon 1, 2, 3 and 4 respectively, are
(a) sp^3, sp, sp^3, sp^3 (b) sp^2, sp^3, sp^2, sp^3
(c) sp^2, sp^2, sp^2, sp^3 (d) sp^2, sp, sp^2, sp^3

49. Which pair of oxides is acidic in nature?
(a) CaO, SiO_2 (b) B_2O_3, SiO_2
(c) B_2O_3, CaO (d) N_2O, BaO

50. 

Considering the above reaction, the major product among the following is

(a)  (b) 

(c)  (d) 

SECTION-B (NUMERICAL VALUE TYPE)

Attempt any 5 questions out of 10.

51. If the activation energy of a reaction is 80.9 kJ mol^{-1}, the fraction of molecules at 700 K, having enough energy to react to form products is e^{-x}. The value of x is ________. [Use $R = 8.31$ J K^{-1} mol^{-1}]

52. Emf of the following cell at 298 K in V is $x \times 10^{-2}$
$Zn|Zn^{2+}$ (0.1 M)|| Ag^+ (0.01 M)| Ag
The value of x is ________. (Rounded off to the nearest integer)
[Given: $E^\circ_{Zn^{2+}/Zn} = -0.76$ V; $E^\circ_{Ag^+/Ag} = +0.80$ V; $\frac{2.303\,RT}{F} = 0.059$]

53. The number of stereoisomers possible for $[Co(ox)_2(Br)(NH_3)]^{2-}$ is ________. [ox = oxalate]

54. The $NaNO_3$ weighed out to make 50 mL of an aqueous solution containing 70.0 mg Na^+ per mL is ________ g. (Rounded off to the nearest integer)
[Given : Atomic weight in g mol^{-1} — Na : 23; N : 14 ; O : 16]

55. When 12.2 g of benzoic acid is dissolved in 100 g of water, the freezing point of solution was found to be −0.93°C ($K_f(H_2O)$ = 1.86 K kg mol^{-1}). The number (n) of benzoic acid molecules associated (assuming 100% association) is ________.

56. The number of octahedral voids per lattice site in a lattice is ________. (Rounded off to the nearest integer)

57. A ball weighing 10 g is moving with a velocity of 90 m s^{-1}. If the uncertainty in its velocity is 5%, then the uncertainty in its position is ________ $\times 10^{-33}$ m. (Rounded off to the nearest integer)
[Given : $h = 6.63 \times 10^{-34}$ Js]

58. In mildly alkaline medium, thiosulphate ion is oxidized by MnO_4^- to "A". The oxidation state of sulphur in "A" is ________.

59. The average S — F bond energy in kJ mol^{-1} of SF_6 is ________. (Rounded off to the nearest integer)
[Given : The values of standard enthalpy of formation of $SF_{6(g)}$, $S_{(g)}$ and $F_{(g)}$ are –1100, 275 and 80 kJ mol^{-1} respectively.]

60. The pH of ammonium phosphate solution, if pK_a of phosphoric acid and pK_b of ammonium hydroxide are 5.23 and 4.75 respectively, is ________.

MATHEMATICS

SECTION-A (MULTIPLE CHOICE QUESTIONS)

61. If vectors $\vec{a}_1 = x\hat{i} - \hat{j} + \hat{k}$ and $\vec{a}_2 = \hat{i} + y\hat{j} + z\hat{k}$ are collinear, then a possible unit vector parallel to the vector $x\hat{i} + y\hat{j} + z\hat{k}$ is

(a) $\frac{1}{\sqrt{3}}(\hat{i} + \hat{j} - \hat{k})$ (b) $\frac{1}{\sqrt{2}}(-\hat{j} + \hat{k})$

(c) $\frac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$ (d) $\frac{1}{\sqrt{2}}(\hat{i} - \hat{j})$

62. Let $f(x) = \int_0^x e^t f(t)dt + e^x$ be a differentiable function for all $x \in R$. Then $f(x)$ equals

(a) $2e^{e^x} - 1$ (b) $e^{e^x} - 1$

(c) $e^{(e^x - 1)}$ (d) $2e^{(e^x - 1)} - 1$

63. Let $A = \{1, 2, 3, ..., 10\}$ and $f : A \to A$ be defined as

$$f(k) = \begin{cases} k+1 & \text{if } k \text{ is odd} \\ k & \text{if } k \text{ is even} \end{cases}$$

Then the number of possible functions $g : A \to A$ such that $gof = f$ is

(a) 5^5 (b) ${}^{10}C_5$ (c) 5! (d) 10^5

64. Let $f(x)$ be a differentiable function at $x = a$ with $f'(a) = 2$ and $f(a) = 4$. Then $\lim_{x \to a} \frac{xf(a) - af(x)}{x - a}$ equals

(a) $2a + 4$ (b) $2a - 4$ (c) $a + 4$ (d) $4 - 2a$

65. Let A_1 be the area of the region bounded by the curves $y = \sin x$, $y = \cos x$ and y-axis in the first quadrant. Also, let A_2 be the area of the region bounded by the curves $y = \sin x$, $y = \cos x$, x-axis and $x = \frac{\pi}{2}$ in the first quadrant. Then,

(a) $A_1 = A_2$ and $A_1 + A_2 = \sqrt{2}$

(b) $A_1 : A_2 = 1 : \sqrt{2}$ and $A_1 + A_2 = 1$

(c) $A_1 : A_2 = 1 : 2$ and $A_1 + A_2 = 1$

(d) $2A_1 = A_2$ and $A_1 + A_2 = 1 + \sqrt{2}$

66. If the mirror image of the point (1, 3, 5) with respect to the plane $4x - 5y + 2z = 8$ is (α, β, γ), then $5(\alpha + \beta + \gamma)$ equals

(a) 41 (b) 39 (c) 43 (d) 47

67. Let L be a line obtained from the intersection of two planes $x + 2y + z = 6$ and $y + 2z = 4$. If point $P(\alpha, \beta, \gamma)$ is the foot of perpendicular from (3, 2, 1) on L, then the value of $21(\alpha + \beta + \gamma)$ equals

(a) 142 (b) 68 (c) 102 (d) 136

68. The triangle of maximum area that can be inscribed in a given circle of radius 'r' is

(a) A right angle triangle having two of its sides of length $2r$ and r.

(b) An equilateral triangle having each of its side of length $\sqrt{3}r$.

(c) An equilateral triangle of height $\frac{2r}{3}$.

(d) An isosceles triangle with base equal to $2r$.

69. Let $A(1, 4)$ and $B(1, -5)$ be two points. Let P be a point on the circle $(x - 1)^2 + (y - 1)^2 = 1$ such that $(PA)^2 + (PB)^2$ have maximum value, then the points P, A and B lie on

(a) an ellipse (b) a straight line

(c) a hyperbola (d) a parabola

70. A natural number has prime factorization given by $n = 2^x 3^y 5^z$, where y and z are such that $y + z = 5$ and $y^{-1} + z^{-1} = \frac{5}{6}$, $y > z$. Then the number of odd divisors of n, including 1, is

(a) 11 (b) 6 (c) 12 (d) $6x$

71. If $0 < a, b < 1$, and $\tan^{-1}a + \tan^{-1}b = \frac{\pi}{4}$, then the value of $(a+b) - \left(\frac{a^2+b^2}{2}\right) + \left(\frac{a^3+b^3}{3}\right) - \left(\frac{a^4+b^4}{4}\right) + ...$ is

(a) e (b) $e^2 - 1$ (c) $\log_e 2$ (d) $\log_e\left(\frac{e}{2}\right)$

72. Let $f(x) = \sin^{-1}x$ and $g(x) = \frac{x^2 - x - 2}{2x^2 - x - 6}$. If $g(2) = \lim_{x \to 2} g(x)$, then the domain of the function fog is

(a) $(-\infty, -2] \cup [-1, \infty)$ (b) $(-\infty, -2] \cup \left[-\frac{4}{3}, \infty\right)$

(c) $(-\infty, -2] \cup \left[-\frac{3}{2}, \infty\right)$ (d) $(-\infty, -1] \cup [2, \infty)$

73. Let slope of the tangent line to a curve at any point $P(x, y)$ be given by $\frac{xy^2 + y}{x}$. If the curve intersects the line $x + 2y = 4$ at $x = -2$, then the value of y, for which the point $(3, y)$ lies on the curve, is

(a) $\frac{18}{35}$ (b) $-\frac{4}{3}$ (c) $-\frac{18}{19}$ (d) $-\frac{18}{11}$

74. Let $F_1(A, B, C) = (A \wedge \sim B) \vee [\sim C \wedge (A \vee B)] \vee \sim A$ and $F_2(A, B) = (A \vee B) \vee (B \rightarrow \sim A)$ be two logical expressions. Then

(a) F_1 and F_2 both are tautologies

(b) F_1 is not a tautology but F_2 is a tautology

(c) F_1 is a tautology but F_2 is not a tautology

(d) Both F_1 and F_2 are not tautologies

75. A seven digit number is formed using digits 3, 3, 4, 4, 4, 5, 5. The probability, that number so formed is divisible by 2, is

(a) $\frac{1}{7}$ (b) $\frac{4}{7}$ (c) $\frac{3}{7}$ (d) $\frac{6}{7}$

76. The sum of the series $\sum_{n=1}^{\infty} \frac{n^2+6n+10}{(2n+1)!}$ is equal to

(a) $\frac{41}{8}e+\frac{19}{8}e^{-1}-10$ (b) $-\frac{41}{8}e+\frac{19}{8}e^{-1}-10$

(c) $\frac{41}{8}e+\frac{19}{8}e^{-1}+10$ (d) $\frac{41}{8}e-\frac{19}{8}e^{-1}-10$

77. Let $f: R \rightarrow R$ be defined as

$$f(x)=\begin{cases} 2\sin\left(-\frac{\pi x}{2}\right), \text{ if } x<-1 \\ |ax^2+x+b|, \text{ if } -1 \le x \le 1 \\ \sin(\pi x), \text{ if } x>1 \end{cases}$$

If $f(x)$ is continuous on R, then $a + b$ equals

(a) 3 (b) –3 (c) 1 (d) –1

78. Consider the following system of equations.

$x + 2y - 3z = a$; $2x + 6y - 11z = b$; $x - 2y + 7z = c$,

where a, b and c are real constants. Then the system of equations

(a) has no solution for all a, b and c

(b) has a unique solution when $5a = 2b + c$

(c) has infinite number of solutions when $5a = 2b + c$

(d) has a unique solution for all a, b and c

79. If the locus of the mid-point of the line segment from the point (3, 2) to a point on the circle, $x^2 + y^2 = 1$ is a circle of radius r, then r is equal to

(a) 1/2 (b) 1 (c) 1/3 (d) 1/4

80. For $x > 0$, if $f(x)=\int_1^x \frac{\log_e t}{(1+t)}dt$, then $f(e)+f\left(\frac{1}{e}\right)$ is equal to

(a) 1 (b) –1 (c) 1/2 (d) 0

SECTION-B (NUMERICAL VALUE TYPE)

Attempt any 5 questions out of 10.

81. Let the normals at all the points on a given curve pass through a fixed point (a, b). If the curve passes through $(3,-3)$ and $(4,-2\sqrt{2})$, and given that $a-2\sqrt{2}b=3$, then $(a^2 + b^2 + ab)$ is equal to ________.

82. Let a be an integer such that all the real roots of the polynomial $2x^5 + 5x^4 + 10x^3 + 10x^2 + 10x + 10$ lie in the interval $(a, a + 1)$.Then, $|a|$ is equal to ________.

83. Let α and β be two real numbers such that $\alpha + \beta = 1$ and $\alpha\beta = -1$. Let $p_n = (\alpha)^n + (\beta)^n$, $p_{n-1} = 11$ and $p_{n+1} = 29$ for some integer $n \ge 1$. Then, the value of p_n^2 is ________.

84. If the matrix $A=\begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 3 & 0 & -1\end{bmatrix}$ satisfies the equation

$A^{20}+\alpha A^{19}+\beta A=\begin{bmatrix}1 & 0 & 0\\ 0 & 4 & 0\\ 0 & 0 & 1\end{bmatrix}$ for some real numbers α and β, then $\beta - \alpha$ is equal to ________.

85. If the arithmetic mean and geometric mean of the p^{th} and q^{th} terms of the sequence –16, 8, –4, 2, ... satisfy the equation $4x^2 - 9x + 5 = 0$, then $p + q$ is equal to ________.

86. Let L be a common tangent line to the curves $4x^2 + 9y^2 = 36$ and $(2x)^2 + (2y)^2 = 31$. Then the square of the slope of the line L is ________.

87. The total number of 4-digit numbers whose greatest common divisor with 18 is 3, is ________.

88. Let $X_1, X_2, \ldots, X_{18}$ be eighteen observations such that $\sum_{i=1}^{18}(X_i-\alpha) = 36$ and $\sum_{i=1}^{18}(X_i-\beta)^2=90$, where α and β are distinct real numbers. If the standard deviation of these observations is 1, then the value of $|\alpha - \beta|$ is ________.

89. If $I_{m,n}=\int_0^1 x^{m-1}(1-x)^{n-1}dx$, for $m, n \ge 1$, and

$\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx=\alpha I_{m,n}, \alpha \in R$, then α equals ________.

90. Let z be those complex numbers which satisfy

$|z+5| \le 4$ and $z(1+i)+\bar{z}(1-i) \ge -10, i=\sqrt{-1}$.

If the maximum value of $|z+1|^2$ is $\alpha+\beta\sqrt{2}$, then the value of $(\alpha + \beta)$ is ________.

HINTS & EXPLANATIONS

1. **(a) :** According to gates, by Demorgan's law,
$\overline{\overline{A}+B} = A\cdot\overline{B}$ by observation.

2. **(a) :** Initially, beat frequency = 5 Hz
So, $\upsilon_A = 340 \pm 5 = 345$ Hz or 335 Hz
After filing frequency increases slightly, so new frequency of $A > \upsilon_A$
Beat frequency = 2 Hz
New $\upsilon_A = 340 \pm 2 = 342$ Hz or 338 Hz
Hence, original frequency of A is $\upsilon_A = 335$ Hz

3. **(c) :** Initial velocity of scooter $(u) = 0$ m s^{-1}
Acceleration $(a) = a_1$ ms^{-2}
Let final velocity of scooter = v ms^{-1}
from equation of motion, $v = 0 + a_1t_1$

$$v = a_1t_1 \;\Rightarrow\; t_1 = \frac{v}{a_1} \qquad \text{... (i)}$$

Now, scooter starts to retard by a_2 m s^{-2}
Initial velocity of scooter $(u) = v$ m s^{-1}
Acceleration = $-a_2$ m s^{-2}
Final velocity of scooter = 0 m s^{-1}
Then by equation of motion, $v = u + at$

$$0 = v - a_2t_2 \;\Rightarrow\; t_2 = \frac{v}{a_2} \qquad \text{... (ii)}$$

By dividing equation (i) by (ii), we get

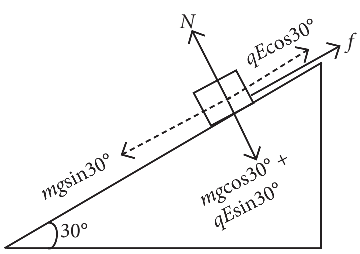
$$\therefore \quad \frac{t_1}{t_2} = \frac{a_2}{a_1}$$

4. **(c) :** The induced e.m.f. is $E = Bvl\sin\theta$
Here, $B = 2.5 \times 10^{-4}$ Wb m^{-2}
$v = 180$ km h^{-1}, $l = 10$ m

$$E = (2.5\times10^{-4}\ \text{T})\left(180\times\frac{5}{18}\ \text{ms}^{-2}\right)(10\ \text{m})\sin 60^\circ$$

$= 108.25 \times 10^{-3}$ V = 108.25 mV

5. **(a) :** The different forces on the particle are shown in figure.



From figure, $N = mg\cos30^\circ + qE\sin30^\circ$

$$N = 1\times10\times\frac{\sqrt{3}}{2} + 5\times10^{-3}\times200\times\frac{1}{2}$$

$\therefore \quad N = 9.16$ N
Acceleration is
$F = ma = mg\sin30^\circ - qE\cos30^\circ - \mu N$

$$\Rightarrow \quad a = \frac{mg\sin30^\circ - qE\cos30^\circ - \mu N}{m}$$

or $$a = \frac{1\times10\times\frac{1}{2} - 5\times10^{-3}\times200\times\frac{\sqrt{3}}{2} - 0.2\times9.16}{1}$$

$\therefore \quad a = 2.302$
Now, distance travelled in time t is

$$S = 0 + \frac{1}{2}at^2 \;\Rightarrow\; t = \sqrt{\frac{2\times S}{a}} = \sqrt{\frac{2\times2}{2.302}}$$

$\therefore \quad t = 1.31$ sec

6. **(c) :** $U = 3PV + 4$

$$\frac{nFRT}{2} = 3PV + 4$$

$$\frac{F}{2}PV = 3PV + 4 \qquad (\because PV = nRT)$$

$$F = 6 + \frac{8}{PV}$$

Since degree of freedom is more than 6, so gas is polyatomic.

7. **(c) :** The kinetic energy of rolling object is

$$K = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 \qquad \text{... (i)}$$

After falling through a distance h, the kinetic energy converted into potential energy mgh.
So, from equation (i), we can write

$$mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

Also, $v = \omega r$

$$\therefore \quad mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}m(\omega^2r^2) \;\Rightarrow\; \omega^2 = \frac{2mgh}{(I + mr^2)}$$

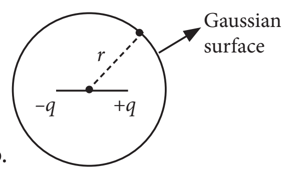
8. **(b) :** The velocity equation of a particle executing simple harmonic motion at any instant t is given by
$v^2 = \omega^2(A^2 - x^2)$

or $\dfrac{v^2}{\omega^2} = A^2 - x^2$ or $\dfrac{v^2}{\omega^2} + x^2 = A^2$

$$\therefore \quad \frac{v^2}{(\omega A)^2} + \frac{x^2}{A^2} = 1$$

This is an equation of ellipse.

9. **(b) : Statement-I :**

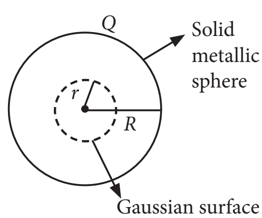


$$\oint \vec{E}\cdot d\vec{s} = \frac{q_{\text{in}}}{\varepsilon_0} = 0 = \phi$$

Flux of $\vec{E}$ through sphere is zero.
But $\oint \vec{E}\cdot d\vec{s} = 0 \Rightarrow \{\vec{E}\cdot d\vec{s} \neq 0\}$ for small section ds only.

Statement-II :
As charge encloses within gaussian surface is equal to zero

$$\phi = \oint \vec{E}\cdot d\vec{s} = 0$$



10. (b) : Let initial activity be A_0

At time t_1, $A = A_0 e^{-\lambda t_1}$ ... (i)

At time t_2, $\frac{A}{5} = A_0 e^{-\lambda t_2}$ ... (ii)

Dividing equation (i) by (ii),

$$5 = e^{\lambda(t_2 - t_1)} \quad \text{or} \quad \lambda = \frac{\ln 5}{t_2 - t_1} = \frac{1}{\tau}$$

$$\therefore \quad \tau = \frac{t_2 - t_1}{\ln 5}$$

11. (a) : As we know, $R = \frac{\rho l}{A}$

Original length, $l_0 = 1$ m

Change in length, $l_1 = \frac{1 \times 25}{100} = 0.25$ m

New length = 1.25 m

As volume of wire remains constant so

$$A_0 l_0 = A_1 l_1 \Rightarrow A_1 = \frac{l_0 A_0}{l_1}$$

If length becomes 1.25 m, area should decreases by 1.25.

$$R' = \frac{\rho(1.25)L}{A/1.25} = \frac{(1.25)^2 \rho L}{A}$$

$$\therefore \quad R' = 1.5625R \qquad \left[\because R = \frac{\rho L}{A}\right]$$

Therefore, increase in resistance = 56.25%

So, the correct option will be (a).

12. (d) : Second pendulum has a time period of 2 sec so statement 1 is false but from one extreme to other it takes only half the time period so statement 2 is true.

13. (d) : Here the two masses are not tied by inextensible string. So, acceleration in different blocks will be different.

Net force on block $A = Ma$

Remaining force on $B = F - Ma$

Acceleration on block $B = \frac{F - Ma}{M}$

14. (d) : $Z = \sqrt{(X_L - X_C)^2 + R^2}$

$X_L = \omega L = 100 \times 100 \times 10^{-3} = 10\ \Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{100 \times 100 \times 10^{-6}} = 100\ \Omega$$

$$Z = \sqrt{(10 - 100)^2 + (120)^2} = \sqrt{90^2 + 120^2} = 150\ \Omega$$

$$i = \frac{V}{Z} = \frac{30\text{ V}}{150\ \Omega} = \frac{1}{5}\text{ A} = 0.2\text{ A}$$

At resonant frequency,

$$\omega L = \frac{1}{\omega C} \Rightarrow \omega = \frac{1}{\sqrt{LC}}$$

$$2\pi\upsilon = \frac{1}{\sqrt{LC}} \Rightarrow \upsilon = \frac{1}{2\pi\sqrt{LC}}$$

$$\therefore \quad \upsilon = \frac{1}{2\pi\sqrt{100 \times 10^{-3} \times 100 \times 10^{-6}}} = 50\text{ Hz}$$

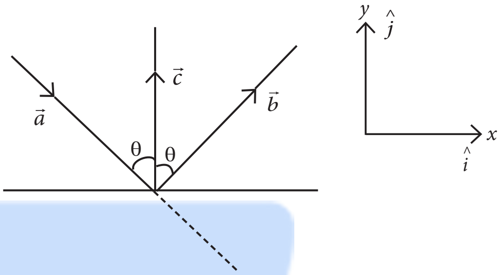
15. (a) : $\lambda = \frac{C}{V} = \frac{Q/V}{V} = \frac{Q}{V^2}$

Also, $V = \frac{\text{work done}}{\text{charge}} = \frac{W}{Q}$

So, $\lambda = \frac{Q}{(W/Q)^2} = \frac{Q^3}{W^2} = \frac{(It)^3}{(F \cdot s)^2}$

$$\lambda = \frac{[\text{I}^3\text{T}^3]}{[\text{ML}^2\text{T}^{-2}]} = [\text{M}^{-2}\text{L}^{-4}\text{T}^7\text{I}^3]$$

16. (c) :



$\vec{a} = \sin\theta\,\hat{i} - \cos\theta\,\hat{j}$ ... (i)

$\vec{b} = \sin\theta\,\hat{i} + \cos\theta\,\hat{j}$ ... (ii)

Equation (ii) – (i) gives, $\vec{b} - \vec{a} = 2\cos\theta\,\hat{j}$ ... (iii)

$\therefore \quad \vec{c} = \hat{j}$ ... (iv)

$\vec{a} \cdot \vec{c} = |a||c|\cos(180 - \theta)$ $[\because |a| = |c| = 1]$

$\vec{a} \cdot \vec{c} = -\cos\theta$

From equation (iii), $\vec{b} - \vec{a} = -2(\vec{a} \cdot \vec{c})\,\hat{j}$

$\vec{b} = \vec{a} + 2(-\vec{a} \cdot \vec{c})\,\hat{j}$

$\vec{b} = \vec{a} - 2(\vec{a} \cdot \vec{c})\,\vec{c}$ [using equation (iv)]

17. (d) : Equation of trajectory of projectile is

$$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta} \quad \text{... (i)}$$

Compare equation (i) with $y = \alpha x - \beta x^2$, we get

$\tan\theta = \alpha \Rightarrow \theta = \tan^{-1}\alpha$

$$\beta = \frac{1}{2}\frac{g}{u^2\cos^2\theta} \Rightarrow u^2 = \frac{g}{2\beta\cos^2\theta}$$

Maximum height, $H = \frac{u^2\sin^2\theta}{2g} = \frac{g}{2\beta\cos^2\theta} \cdot \frac{\sin^2\theta}{2g}$

$$\therefore \quad H = \frac{\tan^2\theta}{4\beta} = \frac{\alpha^2}{4\beta}$$

18. (b) : According to Hooke's law, $T \propto \Delta l$

$T = k\Delta l$, where k is force constant.

Let l is the original length of wire

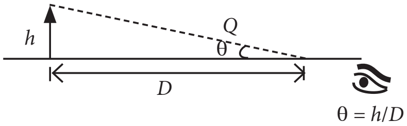
So, $T = k(l - l_0)$

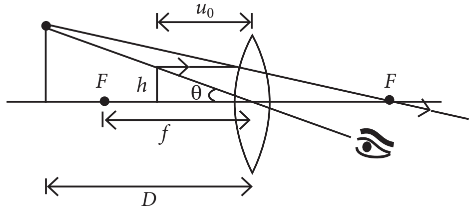
Also, $T_1 = k(l_1 - l_0)$ and $T_2 = k(l_2 - l_0)$

$$\frac{T_1}{T_2} = \frac{l_1 - l_0}{l_2 - l_0}$$

On solving, we get $l_0 = \frac{T_2 l_1 - T_1 l_2}{T_2 - T_1}$

19. (d) :





$\theta' = \dfrac{h}{u_0}$; θ′ is same for both object and image

$m = \dfrac{\theta'}{\theta} = \dfrac{D}{\mu_0}$

$u_0 < D$. Hence $m > 1$

20. (d) : For photon emitted from hydrogen atom, the wavelength is

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \quad \text{... (i)}$$

According to de-Broglie,

$$\lambda = \frac{h}{p} \Rightarrow \frac{1}{\lambda} = \frac{p}{h} \quad \text{... (ii)}$$

From equation (i) and (ii), we get

$$\frac{p}{h} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$p = Rh\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \quad \text{... (iii)}$$

According to law of conservation of linear momentum,

$p_i = p_f$

$0 = m_H v_H$

From equation (iii), $m_H v_H = Rh\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)$

$$\Rightarrow v_H = \frac{Rh}{m_H}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = \frac{1.097\times 10^7 \times 6.63\times 10^{-34}}{1.67\times 10^{-27}}\left(\frac{1}{1^2} - \frac{1}{5^2}\right)$$

$\therefore \quad v_H = 4.17 \text{ ms}^{-1}$

21. (9) : Voltage across 4 kΩ = 90 – 30 = 60 V

Total current $I = \dfrac{60}{4\times 10^3} = 15\times 10^{-3}$ A

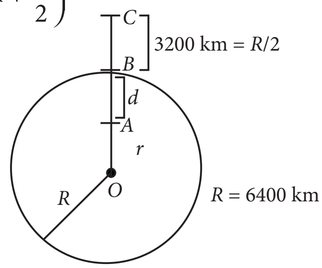
So, current in 5 kΩ, $I' = \dfrac{30}{5\times 10^3} = 6\times 10^{-3}$ A

Current in diode $I'' = I - I' = (15 - 6) \times 10^{-3} = 9$ mA

22. (4) : Let $AB = d$

Gravity at depth, $g_A = g\left(1 - \dfrac{d}{R}\right)$ ...(i)

Gravity at light, $g_C = g \times \dfrac{R^2}{\left(R + \dfrac{R}{2}\right)^2}$ ....(ii)



$gA = g_C$

from equation (i) and (ii)

$$\frac{4gR^2}{9R^2} = g\left(1 - \frac{d}{R}\right)$$

$$\frac{d}{R} = \left(1 - \frac{4}{9}\right) = \frac{5}{9}$$

$OA = OB - AB = R - \dfrac{5}{9}R = \dfrac{4R}{9}$

$AB = \dfrac{5}{9}R$

$OA : AB = 4 : 5$

So, $x : y = 4 : 5$

$\therefore \quad x = 4$

23. (9) : Highest modulating frequency = 5 kHz

Band width = 90 Hz

Band width = $n \times 2 \times$ highest modulating frequency

Here n is number of stations accommodate 90 kHz

90 kHz = $n \times 2 \times 5$ kHz

$\therefore \quad n = \dfrac{90}{10} = 9$

24. (1) : As, maximum kinetic energy = Incident photon energy – work function

$K.E._{max} = h\upsilon - \phi$

$$\frac{1}{2}mv^2 = h\upsilon - \phi \quad \text{... (i)}$$

Given, $h\upsilon_1 = 2\phi$ and $h\upsilon_2 = 10\,\phi$

Using these values in equation (i),

$$\Rightarrow \frac{1}{2}mv_1^2 = 2\phi - \phi = \phi \quad \text{... (ii)}$$

$$\Rightarrow \frac{1}{2}mv_2^2 = 10\phi - \phi = 9\phi \quad \text{... (iii)}$$

Dividing equation (i) by (ii), we get

$$\frac{v_1^2}{v_2^2} = \frac{1}{9} \Rightarrow \frac{v_1}{v_2} = \frac{1}{3}$$

So, $x : y = 1 : 3 \Rightarrow x = 1$

25. (3) : Velocity of simple harmonic motion,

$$v = \omega\sqrt{a^2 - x^2} \quad \text{... (i)}$$

The maximum velocity is at $x = 0$

$v_x = \omega a$

When speed of particle is half of maximum speed,

$$\frac{a\omega}{2} = \omega\sqrt{a^2 - x^2}$$

On squaring, $\dfrac{a^2}{4} = a^2 - x^2$

$$x^2 = \frac{3a^2}{4} \Rightarrow x = \frac{\sqrt{3}}{2}a$$

So, $\frac{\sqrt{3}}{2}a = \frac{\sqrt{xa}}{2} \Rightarrow x = 3$

26. (60) : We know that the work done is

$W = \int PdV$ ... (i)

$W = \int \frac{RT}{V} dV \quad [\because PV = nRT]$ ... (ii)

Also, $V = KT^{2/3}$

$W = \int \frac{RT}{KT^{2/3}} dV$ ... (iii)

Using $dV = \frac{2}{3}KT^{-1/3}dT$ in equation (iii)

$W = \int_{T_1}^{T_2} \frac{RT}{KT^{2/3}} \cdot \frac{2}{3}K \frac{1}{T^{1/3}} dT$

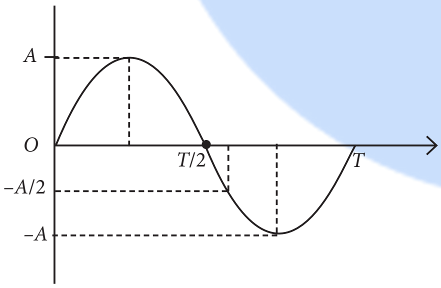
$W = \frac{2}{3}nR(T_2 - T_1)$ ... (iv)

Given, $T_2 - T_1 = 90$ K

$W = \frac{2}{3}nR \times 90 = 60R$

So, $60R = xR \Rightarrow x = 60$

27. (7) :



Total distance covered by the particle = $4A$

Divide the whole path in 8 intervals of $A/2$

So, $\frac{5}{8}$ oscillations means, it has already completed $\frac{1}{2}$ oscillations (*i.e.*, total distance = $2A$)

and is halfway to the other side. *i.e.* $A/2$

So, $\frac{A}{2} = A\sin\omega t;\ \omega = \frac{2\pi}{T}$

$\frac{A}{2} = A\sin\frac{2\pi t}{T} \Rightarrow t = \frac{T}{12}$

Total time taken = Time to complete previous half ($2A$)

+ Time taken to complete $A/2$

$= \frac{t}{2} + \frac{t}{12} = \frac{7T}{12}$

So, $\alpha\frac{T}{\beta} = \frac{7I}{12} \Rightarrow \alpha = 7$

28. (25) : From Ist law of thermodynamics,

$Q = W + \Delta U$ ... (i)

where Q is heat supplied to the gas,

W = work done by the gas

ΔU = change in internal energy

Also, $W = \frac{Q}{5}$

So, from equation (i), $\Delta U = Q - \frac{Q}{5} = \frac{4Q}{5}$ ... (ii)

Now, $\Delta U = \frac{nR\Delta T}{\gamma - 1}$, where n is number of moles.

R = universal gas constant, ΔT = change in temperature

For a diatomic gas, $\gamma = 7/5$

$\Delta U = \frac{nR\Delta T}{5/2}$ ... (iii)

From equation (ii) and (iii),

$\frac{4Q}{5} = \frac{nR\Delta T}{2/5}$

Also, $Q = nC\Delta T$

$\frac{4nC\Delta T}{5} = \frac{nR\Delta T}{2/5}$ or $C = \left(\frac{5}{4}\right)\left(\frac{5}{2}\right)R = \frac{25}{8}R$

$\therefore\ x = 25$

29. (243) : The volume of sphere is

$27 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \Rightarrow R = 3r$

where r and R are the radius of smaller and bigger sphere respectively.

$\Rightarrow R = 3r$

Capacitance, $C_1 = 4\pi\varepsilon_0 r$

Charge on each small drop, $q = C_1V_1 = 10C_1$

Total charge, $Q = 27Q = 27 \times 10C_1 = 270C_1$

Capacitance of the big drop, $C_2 = 4\pi\varepsilon_0 \cdot R$

$= 4\pi\varepsilon_0 \cdot 3r = 3C_1$

Potential of the big drop, $V_2 = \frac{Q}{C_2} = \frac{270C_1}{3C_1} - = 90$ V

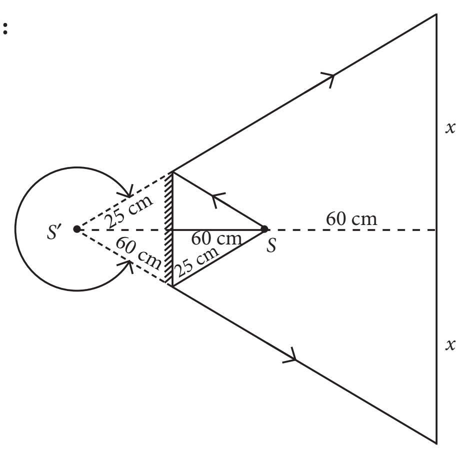
Energy of small drop, $E_1 = \frac{1}{2}C_1V_1^2 = \frac{1}{2} \times C_1 \times (10)^2$

Energy of big drop, $E_2 = \frac{1}{2}C_2V_2^2 = \frac{1}{2} \times 3C_1 \times (90)^2$

$\frac{E_2}{E_1} = \frac{3C_1}{C_1} \times \frac{(90)^2}{(10)^2} = \frac{243}{1}$

$E_2 = 243E_1$

30. (150) :



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By the symmetry, $\frac{x}{180}=\frac{25}{60}$

$x = 75$ cm

So, the man can see the image, for distance

$2x = 2 \times 75 = 150$ cm

31. (b) : $TlI_3 \Rightarrow Tl^+$ and I_3^- ; $CsI_3 \Rightarrow Cs^+$ and I_3^-

Both have same crystalline structures.

So, TlI_3 and CsI_3 are isomorphous.

Electronic configuration of Tl = [Xe] $4f^{14}\,5d^{10}\,6s^2\,6p^1$

32. (d) : Sucrose $\rightarrow$ α-*D*-Glucose + β-*D*-Fructose (A-ii)

Lactose $\rightarrow$ β-*D*-Galactose + β-*D*-Glucose (B-i)

Maltose $\rightarrow$ α-*D*-Glucose + α-*D*-Glucose (C-iii)

33. (c) : When $FeCl_3$ is added to excess of hot water, a positively charged sol of hydrated ferric oxide is formed due to adsorption of Fe^{3+} ions.

$Fe_2O_3.xH_2O/Fe^{3+}$: positively charged sol.

34. (c) : Tritium is a beta-emitting radioactive isotope of hydrogen.

35. (b) : For group 16 elements, the value of electron gain enthalpy decreases down the group from sulphur to polonium. Oxygen has low negative electron gain enthalpy due to small size.

Hence, the order of electron gain enthalpy is,

	S	>	Se	>	Te	>	O
E.A. (kJ/mol)	(–200.0)		(–195.5)		(–190.0)		(–141.4)

36. (c) : Siderite : $FeCO_3$ (A - iii)

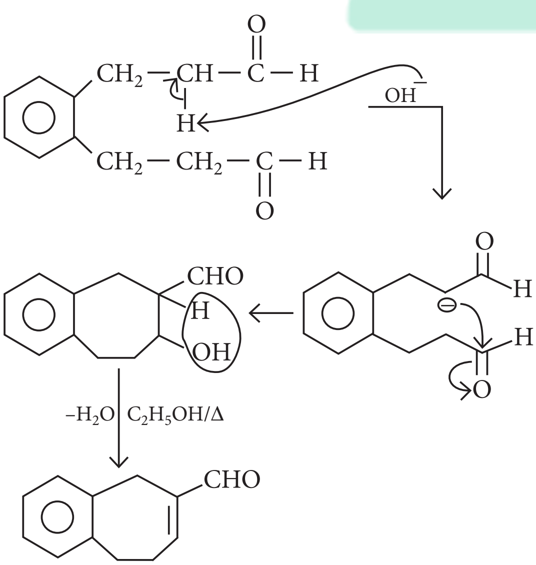
Calamine : $ZnCO_3$ (B - v)

Malachite : $CuCO_3{\cdot}Cu(OH)_2$ (C - i)

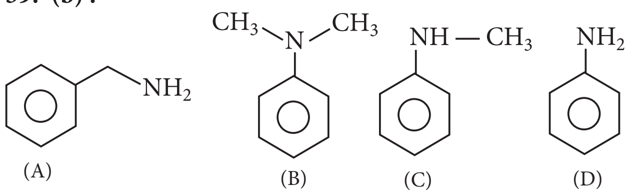
Cryolite : Na_3AlF_6 (D - iv)

37. (d)

38. (b) :

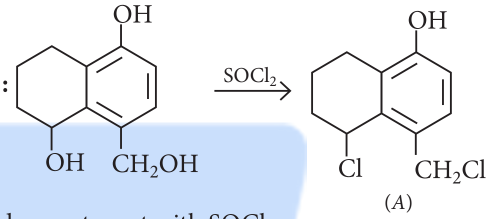


39. (b) :



Out of the given amines, phenyl methanamine (*i.e.* benzyl amine) is the most basic amine as in this delocalisation of electrons is not possible so electrons are available for donation. In aniline derivatives more the number of electron donating groups, more will be the basic character. Hence, the correct order of basicity is A > B > C > D.

40. (c) :



Phenol does not react with $SOCl_2$.

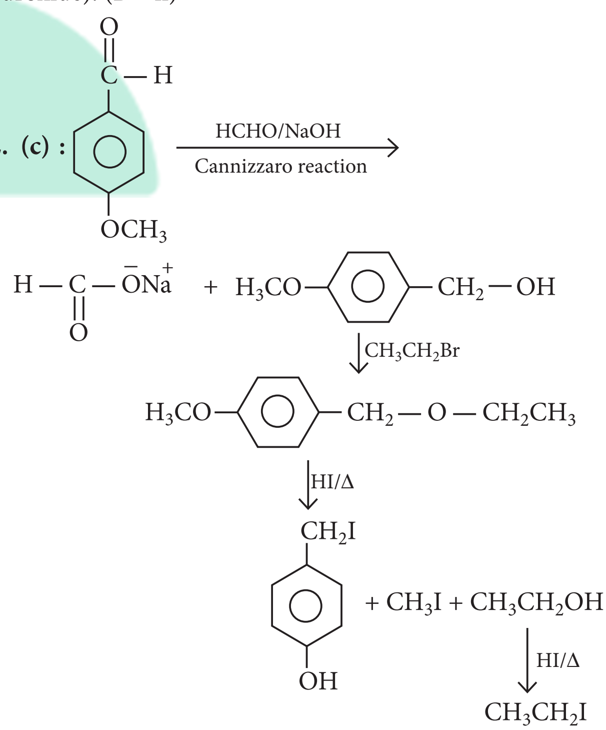
41. (c) : Sodium carbonate is prepared by the Solvay process. (A - iv)

van-Arkel method is used to prepare pure sample of titanium metal. (B - iii)

Deacon developed a process by which chlorine is produced by oxidation of gaseous HCl with O_2 in the presence of $CuCl_2$ catalyst. (C - i)

Castner-Kellner process is a method of electrolysis of an aqueous alkali chloride (sodium chloride) solution to produce the corresponding alkali hydroxide (sodium hydroxide). (D - ii)

42. (c) :



43. (b) : Seliwanoff test is used to distinguish between aldose and ketose sugars.
Xanthoproteic test is a method that can be used to detect presence of protein soluble in a solution, using conc. HNO_3.

44. (d) : $Ne_2 : \sigma 1s^2, \sigma^*1s^2, \sigma 2s^2, \sigma^*2s^2, \sigma 2p_z^2, \pi 2p_x^2 = \pi 2p_y^2, \pi^*2p_x^2 = \pi^*2p_y^2, \sigma^*2p_z^2$

$$\text{Bond order} = \frac{10-10}{2} = 0 \qquad \text{(A - iii)}$$

$N_2 : \sigma 1s^2, \sigma^*2s^2, \sigma 2s^2, \sigma^*2s^2, \pi 2p_x^2 = \pi 2p_y^2, \sigma 2p_z^2$

$$\text{Bond order} = \frac{10-4}{2} = 3 \qquad \text{(B - iv)}$$

$F_2 : \sigma 1s^2, \sigma^*1s^2, \sigma 2s^2, \sigma^*2s^2, \sigma 2p_z^2, \pi 2p_x^2 = \pi 2p_y^2, \pi^*2p_x^2 = \pi^*2p_y^2$

$$\text{Bond order} = \frac{10-8}{2} = 1 \qquad \text{(C - i)}$$

$O_2 : \sigma 1s^2, \sigma^*1s^2, \sigma 2s^2, \sigma^*2s^2, \sigma 2p_z^2, \pi 2p_x^2 = \pi 2p_y^2, \pi^*2p_x^1 = \pi^*2p_y^1$

$$\text{Bond order} = \frac{10-6}{2} = 2 \qquad \text{(D - ii)}$$

45. (b) : 2,4-DNP (2,4-dinitrophenylhydrazine) test can be used for the qualitative identification of ketone or aldehyde functional group.

46. (a) : Calgon is $Na_2[Na_4(PO_3)_6]$. Calgon does not contain Si (Si is the 2nd most abundant element by mass in earth's crust).
Calgon is also known as Graham's salt. It is found in polymeric form and is water soluble also. It removes Ca^{2+} by forming soluble complex but not by precipitation.

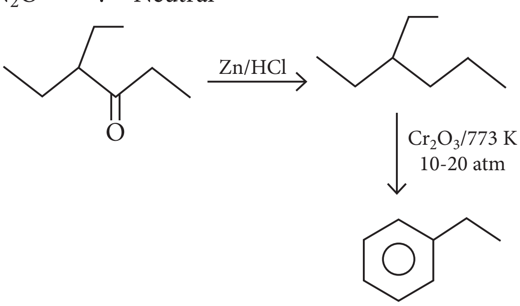
47. (c) : Alcohols on reaction with ceric ammonium nitrate forms a red or pink colour precipitate.
Amines on reaction with $CHCl_3$/alc. KOH gives foul smelling alkyl isocyanides.

48. (d) : $\underset{1}{\overset{sp^2}{CH_2}} = \underset{2}{\overset{sp}{C}} = \underset{3}{\overset{sp^2}{CH}} - \underset{4}{\overset{sp^3}{CH_3}}$

49. (b) :

CaO	:	Basic
SiO_2	:	Acidic
BaO	:	Basic
B_2O_3	:	Acidic
N_2O	:	Neutral

50. (c) :



Ethyl benzene

51. (14) : Fraction of molecules having enough energy to react $= e^{-E_a/RT}$
Comparing it with e^{-x}, we get,

$$x = \frac{E_a}{RT} = \frac{80.9\times 10^3}{8.31\times 700} = 13.9 \approx 14$$

52. (147) : $Zn_{(s)} \longrightarrow Zn^{2+}_{(aq)} + 2e^-$

$2Ag^+_{(aq)} + 2e^- \longrightarrow 2Ag_{(s)}$

$Zn_{(s)} + 2Ag^+_{(aq)} \longrightarrow Zn^{2+}_{(aq)} + 2Ag_{(s)}$

$$E_{cell} = E^\circ_{cell} - \frac{0.059}{n}\log\frac{[Zn^{2+}]}{[Ag^+]^2} \quad \text{... (i)}$$

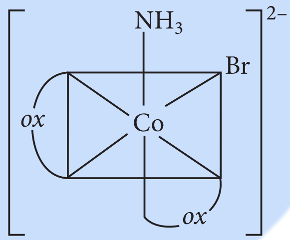
$$E^\circ_{cell} = E^\circ_{Ag^+/Ag} - E^\circ_{Zn^{2+}/Zn} = 0.80\ \text{V} - (-0.76\ \text{V}) = 1.56\ \text{V}$$

$[Zn^{2+}] = 0.1$ M, $[Ag^+] = 0.01$ M, $n = 2$
Substituting these values in eq. (i),

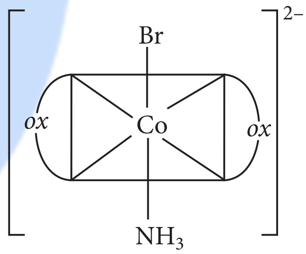
$$E_{cell} = 1.56 - \frac{0.059}{2}\log\frac{0.1}{(0.01)^2} = 1.56 - 0.0295\log 10^3$$

$$= 1.56 - 0.0295 \times 3 = 1.56 - 0.0885 = 147.15 \times 10^{-2}\ \text{V}$$

53. (3) : $[Co(ox)_2Br(NH_3)]^{2-}$ shows geometrical isomerism hence present in *cis* and *trans* forms.



(*cis*- form)
Present in *d*- and *l*- forms



(*trans*- form)

Therefore, total number of stereoisomers possible for $[Co(ox)_2(Br)(NH_3)]^{2-}$ is 3.

54. (13) : Na^+ present in 50 mL $= \dfrac{70\ \text{mg} \times 50\ \text{mL}}{1\ \text{mL}}$

$= 3500$ mg or 3.5 g

$$\text{Moles of } Na^+ = \frac{3.5}{23} = \text{moles of } NaNO_3$$

$$\text{Mass of } NaNO_3 = \frac{3.5}{23}\times 85 = 12.9 \approx 13\ \text{g}$$

55. (2) : $\Delta T_f = iK_f m$

$$m = \frac{\frac{12.2}{122}}{100}\times 1000 = 1\ \text{m}$$

$$i = \frac{\Delta T_f}{K_f\, m} = \frac{0.93}{1.86\times 1} = 0.5$$

$$i = 1 + \left(\frac{1}{n} - 1\right)\alpha$$

$$\frac{1}{2} = 1 + \left(\frac{1}{n} - 1\right)\times 1 \Rightarrow \frac{1}{n} = \frac{1}{2} \Rightarrow n = 2$$

56. (1) : Let number of lattice points $= N$
Effective octahedral voids $= N$
Thus, number of octahedral voids per lattice site is 1.

57. (1) : $m = 10$ g $= 0.01$ kg, $v = 90$ m/s

Uncertainty in velocity $= \dfrac{90\times5}{100} = 4.5$ m/s

According to Heisenberg's uncertainty principle,

$$\Delta p\cdot\Delta x = \frac{h}{4\pi} \quad \text{or} \quad m\Delta v\cdot\Delta x = \frac{h}{4\pi}$$

$$\text{or}\quad \Delta x = \frac{h}{4\pi m\Delta v} = \frac{6.63\times10^{-34}}{4\times3.14\times0.01\times4.5}$$

$$= 1.17\times10^{-33}\text{ m}$$

58. (+6) : $8MnO_4^- + 3S_2O_3^{2-} + H_2O \longrightarrow 8MnO_2 + \underset{(A)}{6SO_4^{2-}} + 2OH^-$

S has +6 oxidation state in SO_4^{2-}

59. (309) : $SF_{6(g)} \longrightarrow S_{(g)} + 6F_{(g)}$

$\Delta_r H = \Delta_f H_{(S)} + 6\times\Delta_f H_{(F)} - \Delta_f H_{(SF_6)}$

$= 275 + 6\times80 - (-1100)$

$= 275 + 480 + 1100 = 1855$ kJ/mol

S – F bond energy $= \dfrac{1855}{6} = 309.17$ kJ/mol ≈ 309 kJ/mol

60. (7) : For a salt of weak acid and weak base,

$$pH = 7 + \frac{1}{2}(pK_a - pK_b)$$

$$= 7 + \frac{1}{2}(5.23 - 4.75) = 7.24 \approx 7$$

61. (c) : Since, $\vec{a}_1$ and $\vec{a}_2$ are collinear.

$$\therefore\ \frac{x}{1} = \frac{-1}{y} = \frac{1}{z} = \lambda(\text{say})$$

Unit vector parallel to $x\hat{i} + y\hat{j} + z\hat{k}$

$$= \pm\frac{\left(\lambda\hat{i} - \frac{1}{\lambda}\hat{j} + \frac{1}{\lambda}\hat{k}\right)}{\sqrt{\lambda^2 + \frac{1}{\lambda^2} + \frac{1}{\lambda^2}}} = \pm\frac{\left(\lambda\hat{i} - \frac{1}{\lambda}\hat{j} + \frac{1}{\lambda}\hat{k}\right)}{\sqrt{\lambda^2 + \frac{2}{\lambda^2}}}$$

At $\lambda = 1$, it must be $\pm\dfrac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$

62. (d) : We have, $f(x) = \int_0^x e^t f(t)\,dt + e^x \Rightarrow f(0) = 1$

Differentiating $f(x)$ with respect to x, we get

$f'(x) = e^x f(x) + e^x$

$$\Rightarrow \int_0^x \frac{f'(x)}{f(x)+1}dx = \int_0^x e^x\,dx \Rightarrow [\ln(f(x)+1)]_0^x = \left[e^x\right]_0^x$$

$\Rightarrow \ln(f(x) + 1) - \ln(f(0) + 1) = e^x - 1$

$$\Rightarrow \ln\left(\frac{f(x)+1}{2}\right) = e^x - 1 \qquad [\because f(0) = 1]$$

$\therefore\ f(x) = 2e^{(e^x - 1)} - 1$

63. (d) : We have, $f(x) = \begin{cases} k+1, & \text{if } k \text{ is odd} \\ k, & \text{if } k \text{ is even}\end{cases}$

$\because\ g : A \to A$ such that $g(f(k)) = f(k)$

If k is even, then $g(k) = k$ ...(i)

If k is odd, then $g(k + 1) = k + 1$ ...(ii)

From (i) and (ii) we can say that

$g(k) = k$ if k is even and if k is odd, then $g(k)$ can take any value in set A.

So, number of possible functions from A to A

$= 10^5 \times 1 = 10^5$

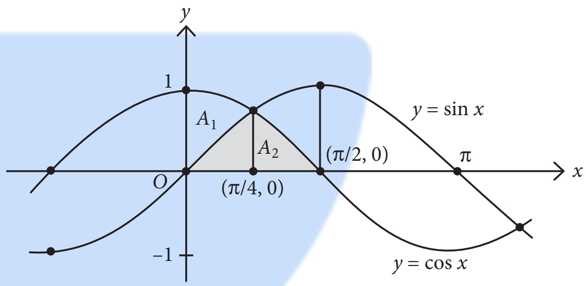
64. (d) : We have, $f'(a) = 2$ and $f(a) = 4$

$$\text{Now, } \lim_{x\to a}\frac{xf(a) - af(x)}{x-a} \qquad \left(\frac{0}{0}\text{ form}\right)$$

$$= \lim_{x\to a}\frac{f(a) - af'(x)}{1} \qquad \text{(Applying L' Hospital's rule)}$$

$= f(a) - af'(a) = 4 - 2a$

65. (b) :



$$A_1 = \int_0^{\pi/4}(\cos x - \sin x)\,dx$$

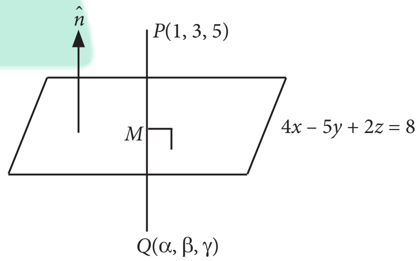
$$= (\sin x + \cos x)_0^{\pi/4} = \sqrt{2} - 1$$

$$A_2 = \int_0^{\pi/4}\sin x\,dx + \int_{\pi/4}^{\pi/2}\cos x\,dx$$

$$= (-\cos x)_0^{\pi/4} + (\sin x)_{\pi/4}^{\pi/2} = \sqrt{2}(\sqrt{2} - 1)$$

$\therefore\ A_1 : A_2 = 1 : \sqrt{2}$ and $A_1 + A_2 = \sqrt{2} - 1 + 2 - \sqrt{2} = 1$

66. (d) :



Since, point $Q(\alpha, \beta, \gamma)$ is image of point $P(1, 3, 5)$ with respect to given plane.

$\therefore$ M is mid point of P and Q that must lie on plane.

i.e., $\left(\dfrac{1+\alpha}{2}, \dfrac{3+\beta}{2}, \dfrac{5+\gamma}{2}\right)$

$$\therefore\ 4\left(\frac{1+\alpha}{2}\right) - 5\left(\frac{3+\beta}{2}\right) + 2\left(\frac{5+\gamma}{2}\right) = 8 \qquad \text{...(i)}$$

Also, PQ is perpendicular to the plane.

$\Rightarrow\ \overrightarrow{PQ} \parallel \hat{n}$

$$\therefore \quad \frac{\alpha-1}{4}=\frac{\beta-3}{-5}=\frac{\gamma-5}{2}=k \quad \text{(say)}$$

$$\therefore \quad \left.\begin{array}{l}\alpha=1+4k\\ \beta=3-5k\\ \gamma=5+2k\end{array}\right\} \qquad \text{...(ii)}$$

From (i) and (ii), we get

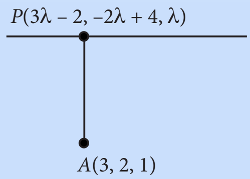
$$2(2+4k)-5\left(\frac{6-5k}{2}\right)+(10+2k)=8$$

$$\Rightarrow \quad k=\frac{2}{5}$$

From (ii), $\alpha=\frac{13}{5}$, $\beta=1$ and $\gamma=\frac{29}{5}$

$\therefore \quad 5(\alpha+\beta+\gamma)=13+5+29=47$

67. (c) : Given equations of plane are $x+2y+z=6$ and $y+2z=4 \Rightarrow 2y=8-4z$



$\therefore \quad x+8-4z+z=6$

$\Rightarrow \quad x-3z=-2$

$$\Rightarrow \quad \frac{x+2}{3}=z \quad \text{and} \quad \frac{y-4}{-2}=z$$

$\therefore$ Equation of line is

$$\frac{x+2}{3}=\frac{y-4}{-2}=\frac{z}{1}=\lambda \quad \text{(say)} \qquad \text{...(i)}$$

$\because$ AP is perpendicular to line (i).

$\therefore \quad (3\lambda-5)\cdot 3+(-2\lambda+2)(-2)+(\lambda-1)\cdot 1=0$

$$\Rightarrow \quad 9\lambda-15+4\lambda-4+\lambda-1=0 \Rightarrow 14\lambda=20 \Rightarrow \lambda=\frac{10}{7}$$

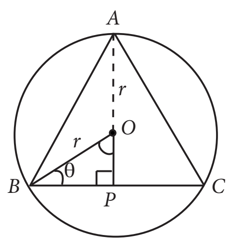
$$\therefore \quad P\equiv\left(\frac{16}{7},\frac{8}{7},\frac{10}{7}\right)$$

$$\Rightarrow \quad \alpha+\beta+\gamma=\frac{16+8+10}{7}=\frac{34}{7}$$

$\Rightarrow \quad 21(\alpha+\beta+\gamma)=102$

68. (b) : Height, $AP=r\sin\theta+r$

Base $=BC=2r\cos\theta$, where $\theta\in\left[0,\frac{\pi}{2}\right)$



$$\therefore \quad \text{Area of } \Delta ABC=\frac{1}{2}(BC)\cdot AP$$

$$=\frac{1}{2}(2r\cos\theta)\cdot(r\sin\theta+r)$$

$\therefore \quad \Delta=r^2(\cos\theta)\cdot(1+\sin\theta)$

$$\Rightarrow \quad \frac{d\Delta}{d\theta}=r^2[\cos^2\theta-\sin\theta-\sin^2\theta]$$

$$=r^2[1-\sin\theta-2\sin^2\theta]=r^2[1+\sin\theta][1-2\sin\theta]$$

Now, for maximum or minimum, $\frac{d\Delta}{d\theta}=0$

$\Rightarrow \quad (1+\sin\theta)(1-2\sin\theta)=0$

$$\Rightarrow \quad 1-2\sin\theta=0 \qquad \left[\because \theta\in\left[0,\frac{\pi}{2}\right)\right]$$

$$\Rightarrow \quad \sin\theta=\frac{1}{2} \quad \Rightarrow \quad \theta=\frac{\pi}{6}$$

$\Rightarrow$ Δ is maximum, when $\theta=\frac{\pi}{6}$.

$\therefore \quad \Delta_{max}=\frac{3\sqrt{3}}{4}r^2=$ Area of equilateral triangle with $BC=\sqrt{3}r$.

69. (b) : Let $P(1+\cos\theta, 1+\sin\theta)$ be any point on $(x-1)^2+(y-1)^2=1$.

$\therefore \quad (PA)^2+(PB)^2$

$=(\cos\theta)^2+(\sin\theta-3)^2+(\cos\theta)^2+(\sin\theta+6)^2$

$=45+6\sin\theta$, which will be maximum if $\sin\theta=1$

$\Rightarrow \quad \sin\theta=1, \cos\theta=0$

Since, $P(1,2)$, $A(1,4)$, and $B(1,-5)$ lie on line $x=1$.

$\therefore \quad P, A, B$ are collinear points.

70. (c) : Given, $y+z=5$ ...(i)

and $\frac{1}{y}+\frac{1}{z}=\frac{5}{6}$, $y>z$ ...(ii)

Solving (i) and (ii), we get

$y=3, z=2$

$\Rightarrow \quad n=2^x\cdot 3^3\cdot 5^2=(2\cdot 2\cdot 2\ ....)(3\cdot 3\cdot 3)(5\cdot 5)$

$\therefore \quad$ Number of odd divisors $=(3+1)(2+1)=4\times 3=12$

71. (c) : Given, $\tan^{-1}a+\tan^{-1}b=\frac{\pi}{4}\ \forall\ 0<a,b<1$

$$\Rightarrow \quad \frac{a+b}{1-ab}=1$$

$\Rightarrow \quad a+b=1-ab \quad \Rightarrow \quad (a+1)(b+1)=2$

$$\text{Now,} \quad \left[a-\frac{a^2}{2}+\frac{a^3}{3}-.....\right]+\left[b-\frac{b^2}{2}+\frac{b^3}{3}-....\right]$$

$=\log_e(1+a)+\log_e(1+b)$

$=\log_e[(1+a)(1+b)]=\log_e 2$

72. (b) : We have, $fog(x)=\sin^{-1}(g(x))$

For fog to be defined, $|g(x)|\le 1$

$$g(2)=\lim_{x\to 2}\frac{x^2-x-2}{2x^2-x-6}\left(\frac{0}{0}\text{ form}\right)$$

$$=\lim_{x\to 2}\left(\frac{2x-1}{4x-1}\right)=\frac{3}{7}$$

$$\text{Now, } |g(x)|\le 1 \quad \Rightarrow \quad \left|\frac{x^2-x-2}{2x^2-x-6}\right|\le 1$$

$$\Rightarrow \quad \left|\frac{(x+1)(x-2)}{(2x+3)(x-2)}\right|\le 1 \quad \Rightarrow \quad \left|\frac{x+1}{(2x+3)}\right|\le 1$$

$$\therefore \quad \frac{x+1}{2x+3}\le 1 \text{ and } \frac{x+1}{2x+3}\ge -1$$

$$\Rightarrow \quad \frac{x+1-2x-3}{2x+3}\le 0 \text{ and } \frac{x+1+2x+3}{2x+3}\ge 0$$

$$\Rightarrow \frac{x+2}{2x+3} \ge 0 \text{ and } \frac{3x+4}{2x+3} \ge 0$$

$$\Rightarrow x > \frac{-3}{2},\ x \le -2 \text{ and } x \ge \frac{-4}{3},\ x < \frac{-3}{2}$$

$$\therefore \quad x \in (-\infty, -2] \cup \left[-\frac{4}{3}, \infty\right)$$

73. (c) : Given, $\dfrac{dy}{dx} = \dfrac{xy^2 + y}{x} \Rightarrow \dfrac{xdy - ydx}{y^2} = xdx$

$$\Rightarrow -d\left(\frac{x}{y}\right) = xdx \quad \Rightarrow -\frac{x}{y} = \frac{x^2}{2} + c$$

$\because$ Curve intersects the line $x + 2y = 4$ at $x = -2$

$\Rightarrow$ Point of intersection is $(-2, 3)$.

$\therefore$ Curve passes through $(-2, 3)$.

$$\Rightarrow \frac{2}{3} = 2 + c \Rightarrow c = -\frac{4}{3} \quad \Rightarrow \frac{-x}{y} = \frac{x^2}{2} - \frac{4}{3}$$

It also passes through $(3, y)$.

$$\therefore \quad \frac{-3}{y} = \frac{19}{6} \Rightarrow y = \frac{-18}{19}$$

74. (b) : $F_1 : (A \wedge \sim B) \vee [\sim C \wedge (A \vee B)] \vee \sim A$

$F_2 : (A \vee B) \vee (B \to \sim A)$

$F_1 : \{(A \wedge \sim B) \vee \sim A\} \vee [(A \vee B) \wedge \sim C]$

$\equiv \{(A \vee \sim A) \wedge (\sim A \vee \sim B)\} \vee [(A \vee B) \wedge \sim C]$

$\equiv [t \wedge (\sim A \vee \sim B)] \vee [(A \vee B) \wedge \sim C]$

$\equiv (\sim A \vee \sim B) \vee [(A \vee B) \wedge \sim C]$

$\equiv [(\sim A \vee \sim B) \vee (A \vee B)] \wedge [(\sim A \vee \sim B) \vee \sim C]$

$\equiv t \wedge [(\sim A \vee \sim B) \vee \sim C]$

$F_1 : (\sim A \vee \sim B) \vee \sim C \ne t$ (tautology)

$F_2 : (A \vee B) \vee (\sim B \vee \sim A) = t$ (tautology)

75. (c) : Digits are 3, 3, 4, 4, 4, 5, 5

$\therefore$ Total 7 digit numbers $= \dfrac{7!}{2!\,2!\,3!}$

7 digit numbers are divisible by 2 if its unit digit is 4.

i.e. $\underbrace{\square\square\square\square\square\square}_{3,\ 3,\ 4,\ 4,\ 5,\ 5}\boxed{4}$

Now, 7 digit numbers which are divisible by 2 $= \dfrac{6!}{2!\,2!\,2!}$

$$\therefore \quad \text{Required probability} = \frac{\frac{6!}{2!\,2!\,2!}}{\frac{7!}{2!\,3!\,2!}} = \frac{3}{7}$$

76. (d) : Let $T_n = \dfrac{n^2 + 6n + 10}{(2n+1)!} = \dfrac{4n^2 + 24n + 40}{4(2n+1)!}$

$$= \frac{(2n+1)^2 + 20n + 39}{4(2n+1)!} = \frac{(2n+1)^2 + (2n+1)10 + 29}{4(2n+1)!}$$

$$= \frac{1}{4}\left[\frac{(2n+1)^2}{(2n+1)(2n)!} + \frac{(2n+1)10}{(2n+1)(2n)!} + \frac{29}{(2n+1)!}\right]$$

$$= \frac{1}{4}\left[\frac{2n+1}{(2n)!} + \frac{10}{(2n)!} + \frac{29}{(2n+1)!}\right]$$

$$= \frac{1}{4}\left[\frac{1}{(2n-1)!} + \frac{11}{(2n)!} + \frac{29}{(2n+1)!}\right]$$

Also, let $S = \displaystyle\sum_{n=1}^{\infty} \frac{n^2 + 6n + 10}{(2n+1)!} = \sum_{n=1}^{\infty} T_n = \frac{1}{4}(S_1 + S_2 + S_3)$ (say)

$$\therefore \quad S_1 = \frac{1}{1!} + \frac{1}{3!} + \frac{1}{5!} + \ldots = \frac{e - \frac{1}{e}}{2},$$

$$S_2 = 11\left[\frac{1}{2!} + \frac{1}{4!} + \frac{1}{6!} + \ldots\right] = 11\left[\frac{e + \frac{1}{e} - 2}{2}\right] \text{ and}$$

$$S_3 = 29\left[\frac{1}{3!} + \frac{1}{5!} + \frac{1}{7!} + \ldots\right] = 29\left[\frac{e - \frac{1}{e} - 2}{2}\right]$$

Now, $S = \dfrac{1}{4}[S_1 + S_2 + S_3]$

$$= \frac{1}{4}\left[\frac{e}{2} - \frac{1}{2e} + \frac{11e}{2} + \frac{11}{2e} - \frac{22}{2} + \frac{29e}{2} - \frac{29}{2e} - \frac{58}{2}\right]$$

$$= \frac{41e}{8} - \frac{19}{8e} - 10$$

77. (d) : Since, $f(x)$ is continuous on R.

$\therefore \quad f(1^-) = f(1) = f(1^+)$

$$\Rightarrow \quad |a + 1 + b| = \lim_{x \to 1} \sin(\pi x)$$

$\Rightarrow \quad |a + 1 + b| = 0 \Rightarrow a + b = -1$

78. (c) : Let $P_1 : x + 2y - 3z = a$

$P_2 : 2x + 6y - 11z = b$

$P_3 : x - 2y + 7z = c$

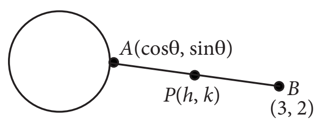
We observe that $5P_1 = 2P_2 + P_3$ if $5a = 2b + c$

$\Rightarrow$ All the planes sharing a common line of intersection.

$\therefore$ System of equations has infinite number of solutions.

79. (a) : $h = \dfrac{\cos\theta + 3}{2}$

and $k = \dfrac{\sin\theta + 2}{2}$



$$\therefore \quad \left(h - \frac{3}{2}\right)^2 + (k-1)^2 = \frac{1}{4}, \text{ which is a circle.}$$

$$\therefore \quad r = \frac{1}{2}$$

80. (c) : We have, $f(x) = \displaystyle\int_1^x \frac{\log_e t}{(1+t)}dt$

$$\therefore \quad f(e) + f\left(\frac{1}{e}\right) = \int_1^e \frac{\log_e t}{1+t}dt + \int_1^{1/e} \frac{\log_e t}{1+t}dt$$

Now, consider $I = \displaystyle\int_1^{1/e} \frac{\log_e t}{(1+t)}dt$

Put $t = \frac{1}{y} \Rightarrow dt = \frac{-1}{y^2}dy$

At $t = 1$, $y = 1$ and when $t = 1/e$, $y = e$

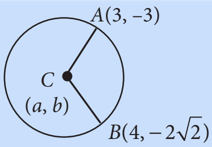
$$\therefore \quad I = \int_1^e \frac{\log_e\left(\frac{1}{y}\right)}{\left(1+\frac{1}{y}\right)} \cdot \left(-\frac{1}{y^2}\right) dy = \int_1^e \frac{\log_e y}{y(1+y)} dy$$

$$\therefore \quad f(e) + f\left(\frac{1}{e}\right) = \int_1^e \frac{\log_e t}{1+t} dt + \int_1^e \frac{\log_e t}{t(1+t)} dt$$

$$= \int_1^e \frac{\log_e t}{t} dt = \frac{1}{2}\left[(\log_e t)^2\right]_1^e = \frac{1}{2}$$

81. (9) : All normals of circle pass through a fixed point that is the centre.

$\therefore$ Radius $= CA = CB$



$\Rightarrow CA^2 = CB^2$

$\Rightarrow (a-3)^2 + (b+3)^2$

$= (a-4)^2 + (b+2\sqrt{2})^2$

$\Rightarrow -6a + 8a + 6b - 4\sqrt{2}b = 6$

$\Rightarrow a - 2\sqrt{2}b + 3b = 3$ ... (i)

Also, given that $a - 2\sqrt{2}b = 3$ ... (ii)

From (i) and (ii), we get, $a = 3$, $b = 0$

$\therefore a^2 + b^2 + ab = 9$

82. (2) : Let $f(x) = 2x^5 + 5x^4 + 10x^3 + 10x^2 + 10x + 10$

Now, $f(-2) = -34 < 0$ and $f(-1) = 3 > 0$

$\therefore f(x)$ has at least one real root in $(-2, -1)$.

Now, $f'(x) = 10x^4 + 20x^3 + 30x^2 + 20x + 10$

$$= 10x^2\left[\left(x^2 + \frac{1}{x^2}\right) + 2\left(x + \frac{1}{x}\right) + 3\right]$$

$$= 10x^2\left[\left(x + \frac{1}{x}\right)^2 + 2\left(x + \frac{1}{x}\right) + 1\right]$$

$$= 10x^2\left[\left(x + \frac{1}{x}\right) + 1\right]^2 > 0 \ \forall\ x \in R$$

$\Rightarrow f(x)$ is increasing

Hence, $f(x)$ has exactly one real root, so $|a| = 2$

83. (324) : Quadratic equation whose roots are α, β is given by $x^2 - x - 1 = 0$

$\therefore \alpha^2 - \alpha - 1 = 0 \Rightarrow \alpha^{n+1} = \alpha^n + \alpha^{n-1}$ ... (i)

and $\beta^2 - \beta - 1 = 0 \Rightarrow \beta^{n+1} = \beta^n + \beta^{n-1}$ ... (ii)

Adding (i) and (ii), we get

$(\alpha^{n+1} + \beta^{n+1}) = (\alpha^n + \beta^n) + (\alpha^{n-1} + \beta^{n-1})$

$\Rightarrow p_{n+1} = p_n + p_{n-1}$

$\Rightarrow 29 = p_n + 11 \Rightarrow p_n = 18$

$\therefore p_n^2 = 324$

84. (4) : Given, $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}$, $A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 8 & 0 \\ 3 & 0 & -1 \end{bmatrix}$ and $A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Hence, $A^{20} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^{20} & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $A^{19} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2^{19} & 0 \\ 3 & 0 & -1 \end{bmatrix}$

Now, $A^{20} + \alpha A^{19} + \beta A$

$$= \begin{bmatrix} 1+\alpha+\beta & 0 & 0 \\ 0 & 2^{20} + \alpha \cdot 2^{19} + 2\beta & 0 \\ 3\alpha + 3\beta & 0 & 1-\alpha-\beta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Therefore, $\alpha + \beta = 0$ and $2^{20} + 2^{19}\alpha - 2\alpha = 4$

$\Rightarrow \alpha = \frac{4(1-2^{18})}{2(2^{18}-1)} = -2$

and $\beta = 2$. Hence, $\beta - \alpha = 4$.

85. (10) : $4x^2 - 9x + 5 = 0 \Rightarrow (x-1)(4x-5) = 0$

$\Rightarrow x = 1, 5/4$

Given, $\frac{5}{4} = \frac{t_p + t_q}{2}$ and $1 = \sqrt{t_p t_q}$, where

$$t_p = -16\left(-\frac{1}{2}\right)^{p-1} \text{ and } t_q = -16\left(-\frac{1}{2}\right)^{q-1}$$

Now, $t_p t_q = 1 \Rightarrow 256\left(-\frac{1}{2}\right)^{p+q-2} = 1$

$\Rightarrow (-2)^{p+q-2} = (-2)^8 \Rightarrow p + q = 10$

86. (3) : Given curves are $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and $x^2 + y^2 = \frac{31}{4}$

Let slope of common tangent be m.

So, tangents of given ellipse and circle are respectively

$$y = mx \pm \sqrt{9m^2 + 4} \quad \text{and} \quad y = mx \pm \frac{\sqrt{31}}{2}\sqrt{1+m^2}$$

Hence, $9m^2 + 4 = \frac{31}{4}(1+m^2)$

$\Rightarrow 36m^2 + 16 = 31 + 31m^2 \Rightarrow 5m^2 = 15$

$\Rightarrow m^2 = 3$

87. (1000) : Let N be the required four digit number.

$\therefore$ $gcd(N, 18) = 3$

Hence, N must be an odd integer which is divisible by 3 but not by 9.

4 digit odd multiples of 3 are

1005, 1011, ..., 9999 *i.e.*, 1500 in number.

4 digit odd multiples of 9 are

1017, 1035, ..., 9999 *i.e.*, 500 in number.

$\therefore$ Required number of such $N = 1500 - 500 = 1000$

88. (4) : We have,

$$\sum_{i=1}^{18}(X_i - \alpha) = 36 \text{ and } \sum_{i=1}^{18}(X_i - \beta)^2 = 90$$

$$\Rightarrow \sum_{i=1}^{18} X_i = 18(\alpha+2) \text{ and } \sum_{i=1}^{18} X_i^2 - 2\beta \sum_{i=1}^{18} X_i + 18\beta^2 = 90$$

$$\Rightarrow \sum_{i=1}^{18} X_i^2 = 90 - 18\beta^2 + 36\beta(\alpha+2)$$

Since, $\sigma^2 = 1$ (given)

$$\Rightarrow \frac{\sum_{i=1}^{18} X_i^2}{18} - \left(\frac{\sum_{i=1}^{18} X_i}{18}\right)^2 = 1$$

$$\Rightarrow \frac{1}{18}(90 - 18\beta^2 + 36\beta(\alpha+2)) - \left(\frac{18(\alpha+2)}{18}\right)^2 = 1$$

$\Rightarrow$ $90 - 18\beta^2 + 36\beta(\alpha + 2) - 18(\alpha + 2)^2 = 18$

$\Rightarrow$ $5 - \beta^2 + 2\alpha\beta + 4\beta - \alpha^2 - 4\alpha - 4 = 1$

$\Rightarrow$ $-\alpha^2 - \beta^2 + 2\alpha\beta + 4\beta - 4\alpha = 0$

$\Rightarrow$ $(\alpha - \beta)^2 + 4(\alpha - \beta) = 0 \Rightarrow (\alpha - \beta)[(\alpha - \beta) + 4] = 0$

$\Rightarrow$ $(\alpha - \beta) = 0$ or -4

$\therefore$ $|\alpha - \beta| = 4$ [$\because$ α, β are distinct real numbers]

89. (1) : We have, $I_{m,n} = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx = I_{n,m}$

Let $x = \frac{1}{y+1} \Rightarrow dx = -\frac{1}{(y+1)^2}dy$

At $x = 0$, $y = \infty$ and when $x = 1$, $y = 0$.

$$\therefore \quad I_{m,n} = -\int_{\infty}^{0} \frac{1}{(y+1)^{m-1}} \frac{y^{n-1}}{(y+1)^{n-1}} \frac{dy}{(y+1)^2}$$

$$= \int_0^{\infty} \frac{y^{n-1}}{(1+y)^{m+n}} dy \quad \text{... (i)}$$

$$\text{Similarly, } I_{m,n} = \int_0^{\infty} \frac{y^{m-1}}{(1+y)^{m+n}} dy \quad \text{... (ii)}$$

Adding (i) and (ii), we get

$$2I_{m,n} = \int_0^{\infty} \frac{y^{m-1} + y^{n-1}}{(1+y)^{m+n}} dy$$

$$= \int_0^1 \frac{y^{m-1} + y^{n-1}}{(1+y)^{m+n}} dy + \int_1^{\infty} \frac{y^{m+1} + y^{n-1}}{(1+y)^{m+n}} dy$$

$$= \int_0^1 \frac{y^{m-1} + y^{n-1}}{(1+y)^{m+n}} dy - \int_1^0 \frac{t^{n-1} + t^{m-1}}{t^{m+n-2}} \frac{t^{m+n}}{(1+t)^{m+n}} \frac{dt}{t^2}$$

[Putting $y = \frac{1}{t}$ in 2nd integral]

$$= \int_0^1 \frac{y^{m-1} + y^{n-1}}{(1+y)^{m+n}} dy + \int_0^1 \frac{y^{m-1} + y^{n-1}}{(1+y)^{m+n}} dy$$

$$\text{Hence, } 2I_{m,n} = 2\int_0^1 \frac{y^{m-1} + y^{n-1}}{(1+y)^{m+n}} dy \Rightarrow \alpha = 1$$

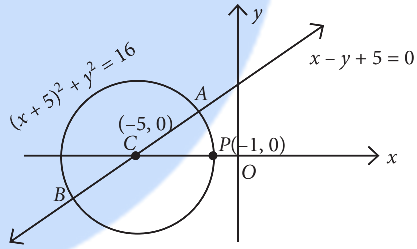
90. (48) : We have, $|z + 5| \le 4$

$\Rightarrow |x + iy + 5| \le 4 \Rightarrow \sqrt{(x+5)^2 + y^2} \le 4$

$\Rightarrow$ $(x + 5)^2 + y^2 \le 16$ ... (i)

Also, $z(1+i) + \bar{z}(1-i) \ge -10 \Rightarrow (z+\bar{z}) + i(z-\bar{z}) \ge -10$

$\Rightarrow$ $2x + i(2iy) \ge -10 \Rightarrow x - y + 5 \ge 0$ ... (ii)



$\therefore$ $|z + 1|^2 = |z - (-1)|^2$

Let $P(-1, 0)$

$\therefore$ $|z + 1|^2_{max} = PB^2$ (where B is in 3rd quadrant)

For point of intersection, solving, $(x + 5)^2 + y^2 = 16$ and $x - y + 5 = 0$, we get

$$A(2\sqrt{2} - 5,\ 2\sqrt{2}) \text{ and } B(-2\sqrt{2} - 5,\ -2\sqrt{2})$$

$$\text{Now, } PB^2 = \left(2\sqrt{2} + 4\right)^2 + (2\sqrt{2})^2$$

$\Rightarrow |z+1|^2 = 8 + 16 + 16\sqrt{2} + 8 \Rightarrow \alpha + \beta\sqrt{2} = 32 + 16\sqrt{2}$

$\Rightarrow$ $\alpha = 32$ and $\beta = 16$. Hence, $\alpha + \beta = 48$