JEE MAIN 2021

ONLINE
26th February
1st Shift

PHYSICS

SECTION-A (MULTIPLE CHOICE QUESTIONS)

1. Four identical solid spheres each of mass 'm' and radius 'a' are placed with their centres on the four corners of a square of side 'b'. The moment of inertia of the system about one side of square where the axis of rotation is parallel to the plane of the square is

(a) $\frac{8}{5}ma^2 + mb^2$ (b) $\frac{4}{5}ma^2 + 2mb^2$

(c) $\frac{4}{5}ma^2$ (d) $\frac{8}{5}ma^2 + 2mb^2$

2. If λ_1 and λ_2 are the wavelengths of the third member of Lyman and first member of the Paschen series respectively, then the value of $\lambda_1 : \lambda_2$ is

(a) 7 : 135 (b) 7 : 108 (c) 1 : 3 (d) 1 : 9

3. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : An electron microscope can achieve better resolving power than an optical microscope.

Reason R : The de Broglie's wavelength of the electrons emitted from an electron gun is much less than wavelength of visible light.

In the light of the above statements, choose the correct answer from the options given below:

(a) A is false but R is true.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) Both A and R are true and R is the correct explanation of A.

4. A particle is moving with uniform speed along the circumference of a circle of radius R under the action of a central fictitious force F which is inversely proportional to R^3. Its time period of revolution will be given by

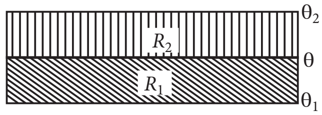
(a) $T \propto R^{4/3}$ (b) $T \propto R^2$

(c) $T \propto R^{5/2}$ (d) $T \propto R^{3/2}$

5. LED is constructed from Ga-As-P semiconducting material. The energy gap of this LED is 1.9 eV. Calculate the wavelength of light emitted and its colour.
[$h = 6.63 \times 10^{-34}$ J s and $c = 3 \times 10^8$ m s^{-1}]

(a) 1046 nm and red colour
(b) 654 nm and red colour
(c) 1046 nm and blue colour
(d) 654 nm and orange colour

6. The temperature θ at the junction of two insulating sheets, having thermal resistances R_1 and R_2 as well as top and bottom temperatures θ_1 and θ_2 (as shown in figure) is given by



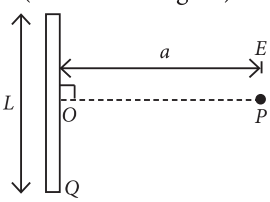
(a) $\frac{\theta_1 R_1 + \theta_2 R_2}{R_1 + R_2}$ (b) $\frac{\theta_2 R_2 - \theta_1 R_1}{R_2 - R_1}$

(c) $\frac{\theta_1 R_2 - \theta_2 R_1}{R_2 - R_1}$ (d) $\frac{\theta_1 R_2 + \theta_2 R_1}{R_1 + R_2}$

7. The normal density of a material is ρ and its bulk modulus of elasticity is K. The magnitude of increase in density of material, when a pressure P is applied uniformly on all sides, will be

(a) $\frac{PK}{\rho}$ (b) $\frac{\rho P}{K}$ (c) $\frac{K}{\rho P}$ (d) $\frac{\rho K}{P}$

8. Find the electric field at point P (as shown in figure) on the perpendicular bisector of a uniformly charged thin wire of length L carrying a charge Q. The distance of the point P from the centre of the rod is $a = \frac{\sqrt{3}}{2}L$.



(a) $\frac{Q}{3\pi\varepsilon_0 L^2}$ (b) $\frac{\sqrt{3}Q}{4\pi\varepsilon_0 L^2}$

(c) $\frac{Q}{4\pi\varepsilon_0 L^2}$ (d) $\frac{Q}{2\sqrt{3}\pi\varepsilon_0 L^2}$

9. A planet revolving in elliptical orbit has

(A) a constant velocity of revolution.
(B) has the least velocity when it is nearest to the sun.
(C) its areal velocity is directly proportional to its velocity.
(D) areal velocity is inversely proportional to its velocity.
(E) to follow a trajectory such that the areal velocity is constant.

Choose the correct answer from the options given below:

(a) C only (b) A only (c) E only (d) D only

10. Consider the combination of 2 capacitors C_1 and C_2, with $C_2 > C_1$, when connected in parallel, the equivalent capacitance is $\frac{15}{4}$ times the equivalent capacitance of the same connected in series. Calculate the ratio of capacitors, $\frac{C_2}{C_1}$.

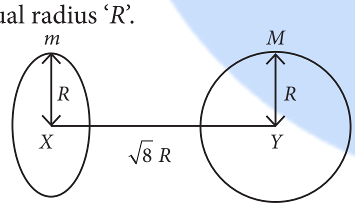
(a) $\frac{29}{15}$ (b) $\frac{15}{11}$ (c) $\frac{111}{80}$ (d) $\frac{15}{4}$

11. In a typical combustion engine the work done by a gas molecule is given by $W = \alpha^2 \beta e^{\frac{-\beta x^2}{kT}}$, where x is the displacement, k is the Boltzmann constant and T is the temperature. If α and β are constants, dimensions of α will be

(a) $[M^2LT^{-2}]$ (b) $[MLT^{-2}]$

(c) $[MLT^{-1}]$ (d) $[M^0LT^0]$

12. Find the gravitational force of attraction between the ring and sphere as shown in the diagram, where the plane of the ring is perpendicular to the line joining the centres. If $\sqrt{8}R$ is the distance between the centres of a ring (of mass 'm') and a sphere (mass 'M') where both have equal radius 'R'.



(a) $\frac{1}{3\sqrt{8}} \cdot \frac{GMm}{R^2}$ (b) $\frac{2\sqrt{2}}{3} \cdot \frac{GMm}{R^2}$

(c) $\frac{\sqrt{8}}{27} \cdot \frac{GmM}{R^2}$ (d) $\frac{\sqrt{8}}{9} \cdot \frac{GmM}{R}$

13. An alternating current is given by the equation $i = i_1 \sin\omega t + i_2 \cos\omega t$. The rms current will be

(a) $\frac{1}{2}(i_1^2 + i_2^2)^{1/2}$ (b) $\frac{1}{\sqrt{2}}(i_1^2 + i_2^2)^{1/2}$

(c) $\frac{1}{\sqrt{2}}(i_1 + i_2)^2$ (d) $\frac{1}{\sqrt{2}}(i_1 + i_2)$

14. A large number of water drops, each of radius r, combine to have a drop of radius R. If the surface tension is T and mechanical equivalent of heat is J, the rise in heat energy per unit volume will be

(a) $\frac{3T}{rJ}$ (b) $\frac{2T}{rJ}$

(c) $\frac{3T}{J}\left(\frac{1}{r} - \frac{1}{R}\right)$ (d) $\frac{2T}{J}\left(\frac{1}{r} - \frac{1}{R}\right)$

15. Assume that a tunnel is dug along a chord of the earth, at a perpendicular distance $(R/2)$ from the earth's centre, where 'R' is the radius of the earth. The wall of the tunnel is frictionless. If a particle is released in this tunnel, it will execute a simple harmonic motion with a time period

(a) $\frac{1}{2\pi}\sqrt{\frac{g}{R}}$ (b) $2\pi\sqrt{\frac{R}{g}}$ (c) $\frac{g}{2\pi R}$ (d) $\frac{2\pi R}{g}$

16. If two similar springs each of spring constant K_1 are joined in series, the new spring constant and time period would be changed by a factor

(a) $\frac{1}{2}, \sqrt{2}$ (b) $\frac{1}{4}, 2\sqrt{2}$ (c) $\frac{1}{2}, 2\sqrt{2}$ (d) $\frac{1}{4}, \sqrt{2}$

17. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Body P having mass M moving with speed u has head-on collision elastically with another body Q having mass m initially at rest. If $m << M$, body Q will have a maximum speed equal to $2u$ after collision.

Reason R : During elastic collision, the momentum and kinetic energy are both conserved.

In the light of the above statements, choose the most appropriate answer from the options given below:

(a) Both A and R are true but R is not the correct explanation of A.

(b) A is correct but R is not correct.

(c) Both A and R are correct and R is the correct explanation of A.

(d) A is not correct but R is correct.

18. A short straight object of height 100 cm lies before the central axis of a spherical mirror whose focal length has absolute value $|f| = 40$ cm. The image of object produced by the mirror is of height 25 cm and has the same orientation of the object. One may conclude from the information:

(a) Image is virtual, opposite side of convex mirror.

(b) Image is real, same side of concave mirror.

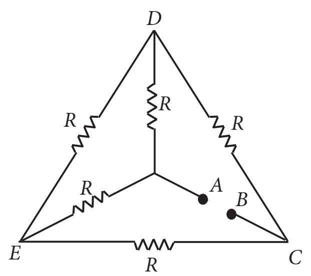
(c) Image is virtual, opposite side of concave mirror.

(d) Image is real, same side of convex mirror.

19. In a Young's double slit experiment two slits are separated by 2 mm and the screen is placed one meter away. When a light of wavelength 500 nm is used, the fringe separation will be

(a) 0.75 mm (b) 0.25 mm (c) 1 mm (d) 0.50 mm

20. Five equal resistances are connected in a network as shown in figure. The net resistance between the points A and B is



(a) R (b) $2R$ (c) $R/2$ (d) $3R/2$

SECTION-B (NUMERICAL VALUE TYPE)

Attempt any 5 questions out of 10.

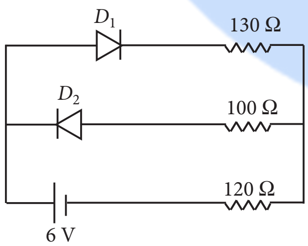
21. In an electrical circuit, a battery is connected to pass 20 C of charge through it in a certain given time. The potential difference between two plates of the battery is maintained at 15 V. The work done by the battery is ________ J.

22. A radiation is emitted by 1000 W bulb and it generates an electric field and magnetic field at P, placed at a distance of 2 m. The efficiency of the bulb is 1.25%. The value of peak electric field at P is $x \times 10^{-1}$ V/m. Value of x is ________. (Rounded off to the nearest integer)

[Take $\varepsilon_0 = 8.85 \times 10^{-12}$ $C^2 N^{-1} m^{-2}$, $c = 3 \times 10^8$ ms^{-1}]

23. The mass per unit length of a uniform wire is 0.135 g/cm. A transverse wave of the form $y = -0.21\sin(x + 30t)$ is produced in it, where x is in meter and t is in second. Then, the expected value of tension in the wire is $x \times 10^{-2}$ N. Value of x is ________. (Round off to the nearest integer)

24. The circuit contains two diodes each with a forward resistance of 50 Ω and with infinite reverse resistance. If the battery voltage is 6 V, the current through the 120 Ω resistance is ________ mA.



25. The maximum and minimum amplitude of an amplitude modulated wave is 16 V and 8 V respectively. The modulation index for this amplitude modulated wave is $x \times 10^{-2}$. The value of x is ________.

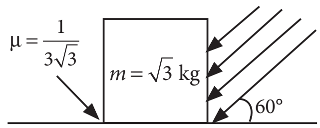
26. A person standing on a spring balance inside a stationary lift measures 60 kg. The weight of that person if the lift descends with uniform downward acceleration of 1.8 m/s^2 will be ________ N. [g = 10 m/s^2]

27. A boy pushes a box of mass 2 kg with a force $\vec{F} = (20\hat{i} + 10\hat{j})$ N on a frictionless surface. If the box was initially at rest, then ________ m is displacement along the x-axis after 10 s.

28. As shown in figure, a block of mass $\sqrt{3}$ kg is kept on a horizontal rough surface of coefficient of friction $\frac{1}{3\sqrt{3}}$.

The critical force to be applied on the vertical surface as shown at an angle 60° with horizontal such that it does not move, will be $3x$. The value of x will be

[g = 10 m/s^2, $\sin 60° = \frac{\sqrt{3}}{2}$; $\cos 60° = \frac{1}{2}$]



29. In a series LCR circuit, the quality factor is measured as 100. If the inductance is increased by two fold and resistance is decreased by two fold, then the quality factor after this change will be ________.

30. A container is divided into two chambers by a partition. The volume of first chamber is 4.5 litre and second chamber is 5.5 litre. The first chamber contain 3.0 moles of gas at pressure 2.0 atm and second chamber contains 4.0 moles of gas at pressure 3.0 atm. After the partition is removed and the mixture attains equilibrium, then, the common equilibrium pressure existing in the mixture is $x \times 10^{-1}$ atm. Value of x is ________.

CHEMISTRY

SECTION-A (MULTIPLE CHOICE QUESTIONS)

31. The orbital having two radial as well as two angular nodes is

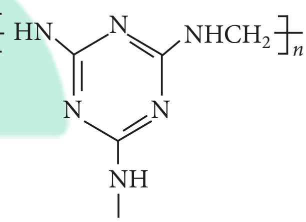
(a) $4f$ (b) $4d$ (c) $5d$ (d) $3p$

32. The structure of neoprene is

(a) $\left[CH_2CH = CH - CH_2 - CH_2 - \underset{\underset{CN}{|}}{CH} \right]_n$

(b) $\left[CH_2 - \underset{\underset{CN}{|}}{CH} \right]_n$

(c) $\left[CH_2 - \overset{\overset{Cl}{|}}{C} = CH - CH_2 \right]_n$

(d) 

33. Find A, B and C in the following reactions.

$NH_3 + A + CO_2 \rightarrow (NH_4)_2CO_3$

$(NH_4)_2CO_3 + H_2O + B \rightarrow NH_4HCO_3$

$NH_4HCO_3 + NaCl \rightarrow NH_4Cl + C$

(a) $A - H_2O$; $B - O_2$; $C - Na_2CO_3$

(b) $A - H_2O$; $B - O_2$; $C - NaHCO_3$

(c) $A - O_2$; $B - CO_2$; $C - Na_2CO_3$

(d) $A - H_2O$; $B - CO_2$; $C - NaHCO_3$

34. Compound A used as a strong oxidizing agent is amphoteric in nature. It is the part of lead storage batteries. Compound A is

(a) PbO_2 (b) $PbSO_4$ (c) Pb_3O_4 (d) PbO

35. Match List-I with List-II.

List-I Electronic configuration of elements		List-II $\Delta_i H$ in kJ mol^{-1}	
(A)	$1s^2 2s^2$	(i)	801
(B)	$1s^2 2s^2 2p^4$	(ii)	899
(C)	$1s^2 2s^2 2p^3$	(iii)	1314
(D)	$1s^2 2s^2 2p^1$	(iv)	1402

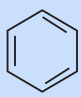
Choose the most appropriate answer from the options given below.
(a) (A) → (i), (B) → (iii), (C) → (iv), (D) → (ii)
(b) (A) → (i), (B) → (iv), (C) → (iii), (D) → (ii)
(c) (A) → (ii), (B) → (iii), (C) → (iv), (D) → (i)
(d) (A) → (iv), (B) → (i), (C) → (ii), (D) → (iii)

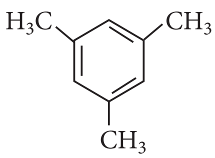
36. For the given reaction

$$\underset{\underset{CH_3}{|}}{CH}{=}CH{-}Br \xrightarrow[\text{(ii) Red hot iron tube, 873 K}]{\text{(i) } NaNH_2} \underset{\text{(Major product)}}{(A)}$$

What is 'A' ?

(a) $\underset{\underset{CH_3}{|}}{CH}{=}CH{-}NH_2$

(b) 

(c) 

(d) $CH_3CH_2CH_2NH_2$

37. Match List-I with List-II.

List-I (Ore)		List-II (Element present)	
(A)	Kernite	(i)	Tin
(B)	Cassiterite	(ii)	Boron
(C)	Calamine	(iii)	Fluorine
(D)	Cryolite	(iv)	Zinc

Choose the most appropriate answer from the options given below.
(a) (A) → (i), (B) → (iii), (C) → (iv), (D) → (ii)
(b) (A) → (iii), (B) → (i), (C) → (ii), (D) → (iv)
(c) (A) → (ii), (B) → (i), (C) → (iv), (D) → (iii)
(d) (A) → (ii), (B) → (iv), (C) → (i), (D) → (iii)

38. Which of the following vitamin is helpful in delaying the blood clotting?
(a) Vitamin K (b) Vitamin B
(c) Vitamin C (d) Vitamin E

39. Given below are two statements :
Statement I : *o*-Nitrophenol is steam volatile due to intramolecular hydrogen bonding.
Statement II : *o*-Nitrophenol has high melting due to hydrogen bonding.
In the light of the above statements, choose the most appropriate answer from the options given below.
(a) Statement I is true but statement II is false.
(b) Statement I is false but statement II is true.
(c) Both statement I and statement II are false.
(d) Both statement I and statement II are true.

40. On treating a compound with warm dil. H_2SO_4, gas X is evolved which turns $K_2Cr_2O_7$ paper acidified with dil. H_2SO_4 to a green compound Y. X and Y respectively are
(a) $X = SO_2$, $Y = Cr_2(SO_4)_3$
(b) $X = SO_2$, $Y = Cr_2O_3$
(c) $X = SO_3$, $Y = Cr_2(SO_4)_3$
(d) $X = SO_3$, $Y = Cr_2O_3$

41. Given below are two statements : One is labelled as Assertion A and the other is labelled as Reason R.
Assertion A : Dipole-dipole interactions are the only non-covalent interactions, resulting in hydrogen bond formation.
Reason R : Fluorine is the most electronegative element and hydrogen bonds in HF are symmetrical.
In the light of the above statement choose the most appropriate answer from the options given below.
(a) A is false but R is true.
(b) Both A and R are true but R is not the correct explanation of A.
(c) Both A and R are true and R is the correct explanation of A.
(d) A is true but R is false.

42. Given below are two statements :
Statement I : A mixture of chloroform and aniline can be separated by simple distillation.
Statement II : When separating aniline from a mixture of aniline and water by steam distillation, aniline boils below its boiling point.
In the light of the above statements, choose the most appropriate answer from the options given below.
(a) Statement I is true but statement II is false.
(b) Both statement I and statement II are true.
(c) Both statement I and statement II are false.
(d) Statement I is false but statement II is true.

43. The presence of ozone in troposphere
(a) protects us from greenhouse effect
(b) generates photochemical amog
(c) protects us from the X-ray radiation
(d) protects us from the UV radiation.

44. Statements about heavy water are given below.
A. Heavy water is used in exchange reactions for the study of reaction mechanisms.
B. Heavy water is prepared by exhaustive electrolysis of water.
C. Heavy water has higher boiling point than ordinary water.
D. Viscosity of H_2O is greater than D_2O.

Choose the most appropriate answer from the options given below.

(a) A and C only (b) A and D only
(c) A and B only (d) A, B and C only

45. Which one of the following lanthanoids does not form MO_2? [M is lanthanoid metal.]

(a) Dy (b) Nd (c) Pr (d) Yb

46. $\underset{(C_4H_8Cl_2)}{A} \xrightarrow[373\text{ K}]{\text{Hydrolysis}} \underset{(C_4H_8O)}{B}$

B reacts with hydroxylamine but does not give Tollen's test. Identify A and B.

(a) 1, 1-Dichlorobutane and butanal
(b) 2, 2-Dichlorobutane and butanal
(c) 2, 2-Dichlorobutane and butan-2-one
(d) 1, 1-Dichlorobutane and 2-butanone

47. An amine on reaction with benzenesulphonyl chloride produces a compound insoluble in alkaline solution. This amine can be prepared by ammonolysis of ethyl chloride. The correct structure of amine is

(a) $CH_3CH_2CH_2\overset{\overset{\large H}{|}}{N}-CH_2CH_3$

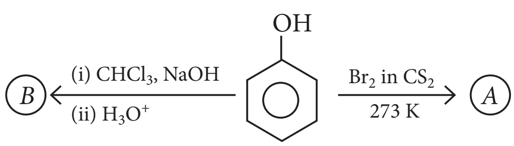
(b) 

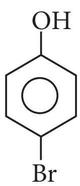
(c) $CH_3CH_2CH_2NHCH_3$ (d) $CH_3CH_2NH_2$

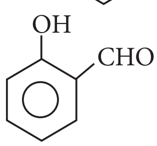
48. Which of the following is a false statement?

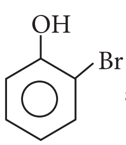
(a) Kjeldahl's method is used for the estimation of nitrogen in an organic compound.
(b) Phosphoric acid produced on oxidation of phosphorus present in an organic compound is precipitated as $Mg_2P_2O_7$ by adding magnesia mixture.
(c) Carius tube is used in the estimation of sulphur in an organic compound.
(d) Carius method is used for the estimation of nitrogen in an organic compound.

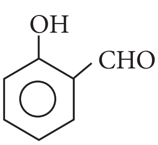
49. Identify the major products A and B respectively in the following reactions of phenol.

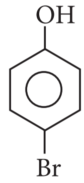


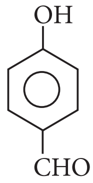
(a) 

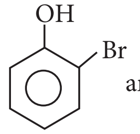
and 

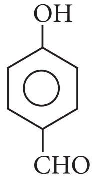
(b) 

and 

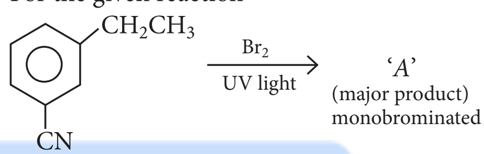
(c) 

and 

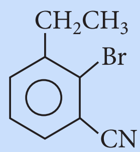
(d) 

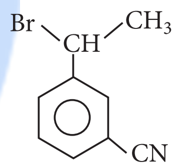
and 

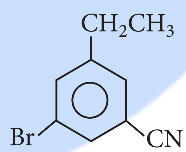
50. For the given reaction

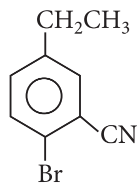


What is 'A'?

(a) 

(b) 

(c) 

(d) 

SECTION-B (NUMERICAL VALUE TYPE)

Attempt any 5 questions out of 10.

51. For a chemical reaction, $A + B \rightleftharpoons C + D$ ($\Delta_r H^\circ = 80$ kJ mol^{-1}) the entropy change $\Delta_r S^\circ$ depends on the temperature T (in K) as $\Delta_r S^\circ = 2T$ (J K^{-1} mol^{-1}). Minimum temperature at which it will become spontaneous is ________ K. (Integer answer)

52. 224 mL $SO_{2(g)}$ at 298 K and 1 atm is passed through 100 mL of 0.1 M NaOH solution. The non-volatile solute produced is dissolved in 36 g of water. The lowering of vapour pressure of solution (assuming the solution is dilute) [$P^\circ_{(H_2O)} = 24$ mm of Hg] is $x \times 10^{-2}$ mm of Hg, the value of x is ________. (Integer answer)

53. An exothermic reaction $X \rightarrow Y$ has an activation energy 30 kJ mol^{-1}. If energy change ΔE during the reaction is –20 kJ, then the activation energy for the reverse reaction in kJ is ________. (Integer answer)

54. A homogeneous ideal gaseous reaction $AB_{2(g)} \rightleftharpoons A_{(g)} + 2B_{(g)}$ is carried out in a 25 litre flask at 27°C. The initial amount of AB_2 was 1 mole and the equilibrium pressure was 1.9 atm. The value of K_p is $x \times 10^{-2}$. The value of x is ________. (Integer answer) [$R = 0.08206$ dm^3 atm K^{-1} mol^{-1}]

55. The number of significant figures in 50000.020×10^{-3} is ________.

56. A certain gas obeys $P(V_m - b) = RT$. The value of $\left(\frac{\partial Z}{\partial P}\right)_T$ is $\frac{xb}{RT}$. The value of x is ________. (Integer answer) (Z : compressibility factor)

57. Consider the following reaction
$MnO_4^- + 8H^+ + 5e^- \longrightarrow Mn^{2+} + 4H_2O$, $E° = 1.51$ V.
The quantity of electricity required in Faraday to reduce five moles of MnO_4^- is ________. (Integer answer)

58. 3.12 g of oxygen is a adsorbed on 1.2 g of platinum metal. The volume of oxygen adsorbed per gram of the adsorbent at 1 atm and 300 K in L is ________.
$[R = 0.0821 \text{ L atm K}^{-1} \text{ mol}^{-1}]$

59. Dichromate ion is treated with base, the oxidation number of Cr in the product formed is ________.

60. Number of bridging CO ligands in $[Mn_2(CO)_{10}]$ is ________.

MATHEMATICS

SECTION-A (MULTIPLE CHOICE QUESTIONS)

61. Let A be a symmetric matrix of order 2 with integer entries. If the sum of the diagonal elements of A^2 is 1, then the possible number of such matrices is
(a) 6 (b) 4 (c) 12 (d) 1

62. The value of $\int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^x} dx$ is
(a) 2π (b) $\pi/2$ (c) $\pi/4$ (d) 4π

63. The value of $\sum_{n=1}^{100} \int_{n-1}^{n} e^{x-[x]} dx$, where $[x]$ is the greatest integer $\le x$, is
(a) $100(e - 1)$ (b) $100(1 - e)$
(c) $100(1 + e)$ (d) $100e$

64. The value of $\lim_{h \to 0} 2\left\{\frac{\sqrt{3}\sin\left(\frac{\pi}{6}+h\right)-\cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}h(\sqrt{3}\cos h-\sin h)}\right\}$ is
(a) 3/4 (b) 4/3 (c) 2/3 (d) $2/\sqrt{3}$

65. The rate of growth of bacteria in a culture is proportional to the number of bacteria present and the bacteria count is 1000 at initial time $t = 0$. The number of bacteria is increased by 20% in 2 hours. If the population of bacteria is 2000 after $\frac{k}{\log_e\left(\frac{6}{5}\right)}$ hours, then $\left(\frac{k}{\log_e 2}\right)^2$ is
(a) 16 (b) 8 (c) 2 (d) 4

66. If $\frac{\sin^{-1}x}{a} = \frac{\cos^{-1}x}{b} = \frac{\tan^{-1}y}{c}$; $0 < x < 1$, then the value of $\cos\left(\frac{\pi c}{a+b}\right)$ is
(a) $\frac{1-y^2}{y\sqrt{y}}$ (b) $\frac{1-y^2}{1+y^2}$ (c) $1 - y^2$ (d) $\frac{1-y^2}{2y}$

67. If (1, 5, 35), (7, 5, 5), (1, λ, 7) and (2λ, 1, 2) are coplanar, then the sum of all possible values of λ is
(a) $-\frac{39}{5}$ (b) $-\frac{44}{5}$ (c) $\frac{44}{5}$ (d) $\frac{39}{5}$

68. A fair coin is tossed a fixed number of times. If the probability of getting 7 heads is equal to probability of getting 9 heads, then the probability of getting 2 heads is
(a) $\frac{15}{2^8}$ (b) $\frac{15}{2^{13}}$ (c) $\frac{15}{2^{14}}$ (d) $\frac{15}{2^{12}}$

69. If $\vec{a}$ and $\vec{b}$ are perpendicular, then $\vec{a}\times(\vec{a}\times(\vec{a}\times(\vec{a}\times\vec{b})))$ is equal to
(a) $\vec{a}\times\vec{b}$ (b) $|\vec{a}|^4\,\vec{b}$ (c) $\frac{1}{2}|\vec{a}|^4\,\vec{b}$ (d) $\vec{0}$

70. Let f be any function defined on R and let it satisfy the condition : $|f(x) - f(y)| \le |(x - y)^2|$, $\forall$ $(x, y) \in R$
If $f(0) = 1$, then
(a) $f(x) < 0$, $\forall\ x \in R$ (b) $f(x) = 0$, $\forall\ x \in R$
(c) $f(x)$ can take any value in R
(d) $f(x) > 0$, $\forall\ x \in R$

71. In an increasing geometric series, the sum of the second and the sixth term is $\frac{25}{2}$ and the product of the third and fifth term is 25. Then, the sum of 4^{th}, 6^{th} and 8^{th} terms is equal to
(a) 30 (b) 26 (c) 32 (d) 35

72. The sum of the infinite series
$1+\frac{2}{3}+\frac{7}{3^2}+\frac{12}{3^3}+\frac{17}{3^4}+\frac{22}{3^5}+\ldots..$ is equal to
(a) $\frac{15}{4}$ (b) $\frac{13}{4}$ (c) $\frac{11}{4}$ (d) $\frac{9}{4}$

73. The intersection of three lines $x - y = 0$, $x + 2y = 3$ and $2x + y = 6$ is a
(a) None of these (b) Equilateral triangle
(c) Isosceles triangle (d) Right angled triangle

74. The maximum value of the term independent of t in the expansion of $\left(tx^{1/5}+\frac{(1-x)^{1/10}}{t}\right)^{10}$, where $x \in (0, 1)$ is
(a) $\frac{2 \cdot 10!}{3(5!)^2}$ (b) $\frac{10!}{\sqrt{3}(5!)^2}$
(c) $\frac{2 \cdot 10!}{3\sqrt{3}(5!)^2}$ (d) $\frac{10!}{3(5!)^2}$

75. Consider the three planes
$P_1 : 3x + 15y + 21z = 9$, $P_2 : x - 3y - z = 5$ and
$P_3 : 2x + 10y + 14z = 5$
Then, which one of the following is true?
(a) P_2 and P_3 are parallel.
(b) P_1, P_2 and P_3 all are parallel.
(c) P_1 and P_3 are parallel.
(d) P_1 and P_2 are parallel.

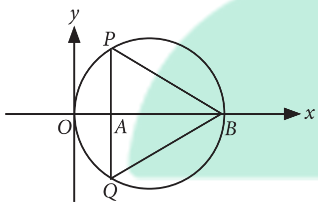
76. Let $R = \{(P, Q) \mid P$ and Q are at the same distance from the origin$\}$ be a relation, then the equivalence class of $(1, -1)$ is the set
(a) $S = \{(x, y) \mid x^2 + y^2 = 1\}$
(b) $S = \{(x, y) \mid x^2 + y^2 = 2\}$
(c) $S = \{(x, y) \mid x^2 + y^2 = 4\}$
(d) $S = \{(x, y) \mid x^2 + y^2 = \sqrt{2}\}$

77. The maximum slope of the curve $y = \frac{1}{2}x^4 - 5x^3 + 18x^2 - 19x$ occurs at the point
(a) $(2, 9)$ (b) $\left(3, \frac{21}{2}\right)$ (c) $(0, 0)$ (d) $(2, 2)$

78. The number of seven digit integers with sum of the digits equal to 10 and formed by using the digits 1, 2 and 3 only is
(a) 82 (b) 42 (c) 35 (d) 77

79. The value of $\begin{vmatrix} (a+1)(a+2) & a+2 & 1 \\ (a+2)(a+3) & a+3 & 1 \\ (a+3)(a+4) & a+4 & 1 \end{vmatrix}$ is
(a) $(a + 2)(a + 3)(a + 4)$ (b) 0
(c) $(a + 1)(a + 2)(a + 3)$ (d) -2

80. In the circle given below, let $OA = 1$ unit, $OB = 13$ unit and $PQ \perp OB$. Then, the area of the triangle PQB (in square units) is



(a) $26\sqrt{3}$ (b) $24\sqrt{2}$ (c) $24\sqrt{3}$ (d) $26\sqrt{2}$

SECTION-B (NUMERICAL VALUE TYPE)

Attempt any 5 questions out of 10.

81. The difference between degree and order of a differential equation that represents the family of curves given by $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right)$, $a > 0$ is ________.

82. Let $m, n \in N$ and $gcd\ (2, n) = 1$. If
$$30\binom{30}{0} + 29\binom{30}{1} + \ldots + 2\binom{30}{28} + 1\binom{30}{29} = n \cdot 2^m, \text{ then}$$
$n + m$ is equal to ________. $\left(\text{Here, } \binom{n}{k} = {}^nC_k\right)$

83. If $y = y(x)$ is the solution of the equation
$e^{\sin y} \cos y \frac{dy}{dx} + e^{\sin y} \cos x = \cos x$, $y(0) = 0$, then
$1 + y\left(\frac{\pi}{6}\right) + \frac{\sqrt{3}}{2} y\left(\frac{\pi}{3}\right) + \frac{1}{\sqrt{2}} y\left(\frac{\pi}{4}\right)$ is equal to ________.

84. Let $(\lambda, 2, 1)$ be a point on the plane which passes through the point $(4, -2, 2)$. If the plane is perpendicular to the line joining the points $(-2, -21, 29)$ and $(-1, -16, 23)$, then $\left(\frac{\lambda}{11}\right)^2 - \frac{4\lambda}{11} - 4$ is equal to ________.

85. The number of solutions of the equation $\log_4(x - 1) = \log_2(x - 3)$ is ________.

86. The number of integral values of k for which the equation $3\sin x + 4\cos x = k + 1$ has a solution, $k \in R$ is ________.

87. If $\sqrt{3}(\cos^2 x) = (\sqrt{3} - 1)\cos x + 1$, the number of solutions of the given equation when $x \in \left[0, \frac{\pi}{2}\right]$ is ________.

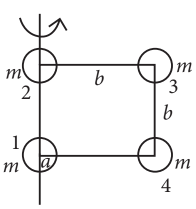
88. The sum of 162^{th} power of the roots of the equation $x^3 - 2x^2 + 2x - 1 = 0$ is ________.

89. The area bounded by the lines $y = ||x - 1| - 2|$ is ________.

90. The value of the integral $\int_0^{\pi} |\sin 2x|\, dx$ is ________.

HINTS & EXPLANATIONS

1. **(d) :** Moment of inertia of solid sphere about an axis passing through centre $= \frac{2}{5}ma^2$



$I_1 = \frac{2}{5}ma^2 = I_2$

By use of parallel axis theorem

$I_3 = \frac{2}{5}ma^2 + mb^2 = I_4$

Moment of inertia about one side

$I = I_1 + I_2 + I_3 + I_4 = 2I_1 + 2I_3$

$\Rightarrow I = 2 \times \frac{2}{5}ma^2 + 2 \times \frac{2}{5}ma^2 + 2mb^2 \Rightarrow I = \frac{8ma^2}{5} + 2mb^2$

2. **(a) :** For third line of Lyman series

$$\frac{1}{\lambda_1} = R\left[\frac{1}{1^2} - \frac{1}{4^2}\right] = \frac{15}{16} \quad \ldots\text{(i)}$$

For first line of Paschen series

$$\frac{1}{\lambda_2} = R\left[\frac{1}{3^2} - \frac{1}{4^2}\right] = \frac{7}{144}R \quad \ldots\text{(ii)}$$

From equation (i) and (ii), we get

$$\frac{\lambda_1}{\lambda_2}=\frac{7\times16}{144\times15}=\frac{7}{135}$$

3. (d) : The resolving power of electron microscope is more than optical microscope. The de Broglie wavelength of the electrons (≈ 0.1 Å)emitted from the electron gun is much less than the wavelength of visible light (380nm to 700 nm).

4. (b) : Centripetal force, $F=\dfrac{mv^2}{R}$ ...(i)

According to question, $F=\dfrac{K}{R^3}$ ...(ii)

Where K is a proportional constant.
From equation (i) and (ii),

$$\Rightarrow \frac{mv^2}{R}=\frac{K}{R^3}\Rightarrow v=\sqrt{\frac{K}{m}\cdot\frac{1}{R}}$$

$$\therefore\ T=\frac{2\pi R}{v}=\frac{2\pi R}{\sqrt{K}}R\sqrt{m}=2\pi\sqrt{\frac{m}{K}}R^2 \Rightarrow T\propto R^2$$

5. (b) : Energy gap, E = 1.9 eV

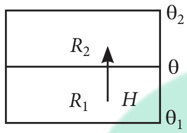
As $E=\dfrac{hc}{\lambda} \Rightarrow \lambda=\dfrac{6.63\times10^{-34}\times3\times10^8}{1.9\times1.6\times10^{-19}}$

$\Rightarrow \lambda\approx 654$ nm

Wavelength of red colour range from 620 – 750 nm.

6. (d) : As the two sheets are in series, so rate of flow of heat is same from both sheets.

$$\therefore\ H=\frac{\theta_1-\theta}{R_1}=\frac{\theta-\theta_2}{R_2}$$



$$\Rightarrow \theta_1R_2-\theta R_2=\theta R_1-\theta_2R_1$$

$$\Rightarrow \frac{\theta_1R_2+\theta_2R_1}{R_1+R_2}=\theta$$

7. (b) : Bulk Modulus, $K=\dfrac{-PV}{\Delta V}$ ...(i)

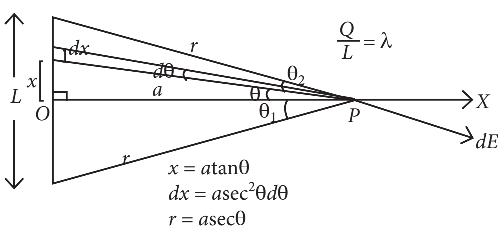
As mass is constant, so

$\rho V=\text{constant} \Rightarrow V=\dfrac{1}{\rho}$

$\dfrac{\Delta V}{V}=-\dfrac{\Delta\rho}{\rho}$ put in (i)

$K=\dfrac{P\rho}{\Delta\rho} \Rightarrow \Delta\rho=\dfrac{P\rho}{K}$

8. (d) : $a=\dfrac{\sqrt{3}}{2}L$, Let us assume an elementary portion of length dx at distance x from O.



Electric field at point P due to this elementary portion is

$$dE=\frac{K\lambda d\theta}{r^2}=\frac{K\lambda dx}{r^2}$$

$$dE=\frac{K\lambda\cdot a\sec^2\theta d\theta}{a^2\sec^2\theta}$$

$$dE_x=dE\cos\theta\, d\theta=\frac{K\lambda}{a}\cos\theta\, d\theta$$

$dE_y=dE\sin\theta$ (They balances each other)

$$\therefore\ E=\int dE_x=\frac{K\lambda}{a}\int_{-\theta_1}^{\theta_2}\cos\theta\, d\theta=\frac{K\lambda}{a}[\sin\theta]_{-\theta_1}^{\theta_2}$$

$$\Rightarrow E=\frac{K\lambda}{a}[\sin\theta_2+\sin\theta_1]=\frac{K\lambda}{a}\left[\frac{L}{2r}+\frac{L}{2r}\right]$$

$$\Rightarrow E=\frac{K\lambda\cdot2L}{a\cdot2r}=\frac{K\lambda L}{a\cdot r}=\frac{KQ}{a\cdot\sqrt{\frac{L^2}{4}+\frac{3L^2}{4}}}$$

$$E=\frac{KQ\times2}{\sqrt{3}L^2}=\frac{1}{4\pi\varepsilon_0}\cdot\frac{2Q}{\sqrt{3}L^2} \Rightarrow E=\frac{Q}{2\sqrt{3}\pi\varepsilon_0L^2}$$

9. (c) : According to Kepler's second law, areal velocity of planet is constant.

10. (*) : When connected in parallel, $C_{eq}=C_1+C_2$

When connected in series, $C_{eq}=\dfrac{C_1C_2}{C_1+C_2}$

Here, $C_1+C_2=\dfrac{15}{4}\dfrac{C_1C_2}{C_1+C_2}$

$$\Rightarrow \frac{(C_1+C_2)^2}{C_1C_2}=\frac{15}{4}\Rightarrow\frac{C_1^2+C_2^2+2C_1C_2}{C_1C_2}=\frac{15}{4}$$

$$\Rightarrow \frac{C_1}{C_2}+\frac{C_2}{C_1}=\frac{15}{4}-2$$

Let $\dfrac{C_2}{C_1}=x \Rightarrow \dfrac{1}{x}+x=\dfrac{7}{4}$

$\Rightarrow 4x^2-7x+4=0$

$D=b^2-4ac=49-64<0$

No solution exist.

11. (d) : $W=\alpha^2\beta e^{-\frac{\beta x^2}{KT}}$

As, $\dfrac{\beta x^2}{KT}=\text{dimensionless} \Rightarrow [\beta]=\dfrac{[KT]}{[x^2]}=\dfrac{ML^2T^{-2}}{L^2}$

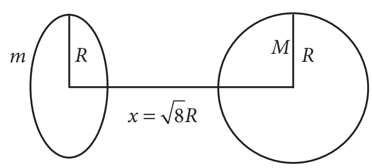
Dimensions of $(\alpha^2\beta)$ = Dimension of work

$\Rightarrow \alpha^2\times[MT^{-2}]=[ML^2T^{-2}]$

$\Rightarrow \alpha=[L]$

12. (c) : Gravitational field due to ring at a distance x is

$$E=G\cdot\frac{mx}{(R^2+x^2)^{3/2}}$$



Force on sphere,
$F=ME$

$$F=\frac{MGmx}{(R^2+x^2)^{3/2}}$$

$$\Rightarrow\quad F=\frac{GMm\cdot\sqrt{8}R}{(R^2+8R^2)^{3/2}}$$

$$\Rightarrow\quad F=\frac{GMm\cdot\sqrt{8}R}{9^{3/2}\cdot R^3}=\frac{GMm\sqrt{8}}{3^3\cdot R^2}$$

$$\Rightarrow\quad F=\frac{\sqrt{8}}{27}\cdot\frac{GMm}{R^2}$$

13. (b) : Here current, $i=i_1\sin\omega t+i_2\cos\omega t$

$$\Rightarrow\quad i=\sqrt{i_1^2+i_2^2}\left[\frac{i_2\cos\omega t}{\sqrt{i_1^2+i_2^2}}+\frac{i_1\sin\omega t}{\sqrt{i_1^2+i_2^2}}\right]$$

$$\Rightarrow\quad i=\sqrt{i_1^2+i_2^2}\,[\sin\alpha\cos\omega t+\cos\alpha\sin\omega t]$$

$$\Rightarrow\quad i=\sqrt{i_1^2+i_2^2}\,\sin(\omega t+\alpha)=i_0\sin(\omega t+\alpha)$$

$$\Rightarrow\quad i_{rms}=\frac{i_0}{\sqrt{2}}=\sqrt{\frac{i_1^2+i_2^2}{2}}$$

14. (c) : Let the number of water drops $= N$

Volume remains constant

$$\therefore\quad \frac{4}{3}\pi r^3\cdot N=\frac{4}{3}\pi R^3\Rightarrow\quad R^3=Nr^3$$

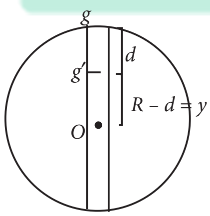
Energy released $= T\times 4\pi r^2N - T\times 4\pi R^2$

$$U=\left(T\times 4\pi r^2\cdot\frac{R^3}{r^3}-T\cdot 4\pi R^2\right)\frac{1}{J}$$

$$\therefore\quad \frac{U}{\text{Volume}}=\frac{1}{J}\,\frac{T\times 4\pi R^2\left[\frac{1}{r}R-1\right]}{\frac{4}{3}\pi R^3}$$

$$\Rightarrow\quad \frac{U}{\text{Volume}}=\frac{3T}{J}\left[\frac{1}{r}-\frac{1}{R}\right]$$

15. (b) : $g'=g\left(1-\frac{d}{R}\right)=\frac{R-d}{R}$



$$g'=-\frac{y}{R}g$$

So, $g'=\frac{-g}{R}y=-\omega^2 y$

$$\omega^2=\frac{g}{R}$$

$$T=\frac{2\pi}{\omega}=\frac{2\pi\sqrt{R}}{\sqrt{g}}$$

$$T=2\pi\sqrt{\frac{R}{g}}$$

16. (a) : When two springs are connected in series,

equivalent spring constant, $K_S=\frac{K_1K_2}{K_1+K_2}$

$$\Rightarrow\quad K_S=\frac{K_1^2}{2K_1}=\frac{K_1}{2}$$

$\therefore$ Time period,

$$T=2\pi\sqrt{\frac{m}{K_1}}\quad\text{and}\quad T'=2\pi\sqrt{\frac{m}{K_S}}=2\pi\sqrt{\frac{2m}{K_1}}$$

$$\Rightarrow\quad T'=\sqrt{2}T$$

17. (c) : $m << M$

(M) P $\longrightarrow u$ (m) Q(rest)   (Before collision)

(M) P $\longrightarrow V$ (m) Q $\longrightarrow v$ (After collision)

Using conservation of linear momentum

$Mu = MV + mv$ ...(i)

By, definition of coefficient of restitution

$e=\frac{V-v}{0-u}=1\Rightarrow V=v-u$ putting in equation (i)

$\Rightarrow\quad Mu = M(v-u) + mv$

$v=\frac{2Mu}{M+m}$ if $M>>m$; $v=2u$

18. (a) : Here, f = 40 cm, h = 100 cm, h' = 25 cm

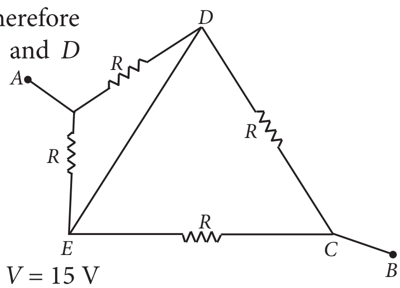
As orientation is same, the image is virtual and smaller, mirror has to be convex and image formed on opposite side.

19. (b) : Given, d = 2 mm; D = 1 m; λ = 500 nm

Fringe width, $\beta=\frac{D\lambda}{d}=\frac{1\times 500\times 10^{-9}}{2\times 10^{-3}}=2.5\times 10^{-4}$ m

$\Rightarrow\quad \beta$ = 0.25 mm

20. (a) : It is a balanced Wheatstone bridge, therefore resistance between E and D can be removed.



So,

$$\frac{1}{R_{eq}}=\frac{1}{2R}+\frac{1}{2R}=\frac{1}{R}$$

$$\Rightarrow\quad R_{AB}=R_{eq}=R$$

21. (300) : q = 20 C ; V = 15 V

Work done = qV = 20 × 15 = 300 J

22. (137) : P = 1000 W; r = 2 m; η = 1.25%

Intensity, $I=\frac{1}{2}\epsilon_0 CE_0^2$ ...(i)

$I=\frac{P}{\text{area}}\times\eta$ ...(ii)

From equation (i) and (ii),

$$\frac{1000}{4\pi(2)^2}\times\frac{1.25}{100}=\frac{1}{2}\times 8.854\times 10^{-12}\times 3\times 10^8 E_0^2$$

E_0 = 13.7 V/m

As question, $x\times 10^{-1}$ V/m = 13.7 V/m

x = 137

23. (1215) :

$$\mu=0.135\text{g/cm}=\frac{0.135\times 10^{-1}}{10^{-2}}\text{ kg/m}=0.0135\text{ kg/m}$$

$y = -0.21\sin(x + 30t)$ ...(i)

On compairing equation (i) with $y = A\sin(kx + \omega t)$, we get

$\omega = 30$, $k = 1$

$$v = \frac{\omega}{k} = 30$$

$$v = \sqrt{\frac{T}{\mu}} \Rightarrow 30 = \sqrt{\frac{T}{0.0135}}$$

$T = 12.15$ N

As per question, $T = x \times 10^{-2}$ N $= 12.15$ N

$x = 1215$

24. (20) : D_2 is in reverse bias, so no current flows through this branch.

The total resistance,

$R = 50 + 130 + 120 = 300\ \Omega$

$$i = \frac{V}{R} = \frac{6}{300} = 0.02A = 20 \text{ mA}$$

25. (33.3) : $A_{max} = 16$, $A_{min} = 8$

$$A_m = \frac{A_{max} - A_{min}}{2} = \frac{16-8}{2} = 4$$

$$A_c = \frac{A_{max} + A_{min}}{2} = \frac{16+8}{2} = 12$$

$$\mu = \frac{A_m}{A_c} = \frac{4}{12}$$

As per question, $x \times 10^{-2} = \frac{4}{12}$

$$x = \frac{400}{12} = 33.3$$

26. (492) : When lift is at rest

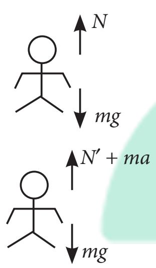
$N = mg$

$= 60 \times 10 = 600$ N

When left moves with downward acceleration

$N' = m(g - a) = 60\ (10 - 18)$

$= 492$ N



27. (500) : $a = 10$ m/s; $m = 2$ kg

$v = u + at = 0 + 10 \times 10 = 100$ m/s

Also, $\frac{1}{2}mv^2 = F \cdot S$

$$\frac{1}{2} \times 2 \times (100)^2 = 20\ S$$

$S = 500$ m

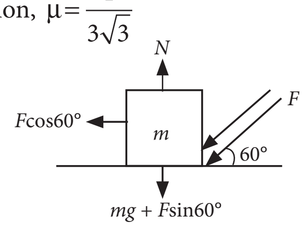
28. (3.33) : Coefficient of friction, $\mu = \frac{1}{3\sqrt{3}}$

$m = \sqrt{3}$ kg

$N = mg + F\sin 60$ ...(i)

$\mu N = F\cos 60°$ (for at rest)...(ii)

From equation (i) and (ii), we get



$$\frac{1}{3\sqrt{3}}\left(mg + F\frac{\sqrt{3}}{2}\right) = \frac{F}{2}$$

$$\frac{1}{3\sqrt{3}}\left(\sqrt{3} \times 10 + F\frac{\sqrt{3}}{2}\right) = \frac{F}{2}$$

$$\frac{10}{3} + \frac{F}{6} = \frac{F}{2}$$

$F = 10$ N

As per question, $10 = 3x$

$\Rightarrow x = 3.33$

29. (400) : Quality factor, $Q = \frac{\omega L}{R}$;

$$\frac{Q'}{Q} = \frac{L'R}{R' \cdot L} = \frac{2L}{L} \cdot \frac{R \times 2}{R} = 4$$

$Q' = 4 \times 100 = 400$

30. (25.5) : $V_1 = 4.5$ L; $V_2 = 5.5$ L

$n_1 = 3$ $\quad n_2 = 4$

$P_1 = 2$ atm $\quad P_2 = 3$ atm

According to the conservation of energy

$K_1 + K_2 = K$

$$\frac{3}{2}n_1RT_1 + \frac{3}{2}n_2RT_2 = \frac{3}{2}(n_1 + n_2)RT$$

$\therefore PV = nRT$

$$\frac{3}{2}P_1V_1 + \frac{3}{2}P_2V_2 = \frac{3}{2}P(V_1 + V_2)$$

$P = 2.55$ atm

As per question $x \times 10^{-1} = 2.55$

$x = 25.5$

31. (c) : Angular nodes = l

Radial nodes = $n - l - 1$

Orbital	Angular nodes	Radial node
$4f$	$l = 3$	$4 - 3 - 1 = 0$
$4d$	$l = 2$	$4 - 2 - 1 = 1$
$5d$	$l = 2$	$5 - 2 - 1 = 2$
$3p$	$l = 1$	$3 - 1 - 1 = 1$

32. (c) : Neoprene is a polymer of chloroprene.

$$n\underset{\text{Chloroprene}}{CH_2{=}\underset{\underset{Cl}{|}}{C}{-}CH{=}CH_2} \longrightarrow \underset{\text{Neoprene}}{\left[CH_2{-}\underset{\underset{Cl}{|}}{C}{=}CH{-}CH_2\right]_n}$$

33. (d) : $2NH_3 + H_2O + CO_2 \longrightarrow \underset{(A)}{(NH_4)_2CO_3}$

$\underset{(A)}{(NH_4)_2CO_3} + H_2O + CO_2 \longrightarrow \underset{(B)}{2NH_4HCO_3}$

$NH_4HCO_3 + NaCl \longrightarrow NH_4Cl + \underset{(C)}{NaHCO_3}$

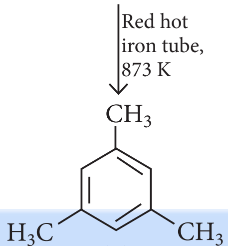
34. (a) : Due to inert pair effect, Pb^{4+} is strong oxidising agent. As given that compound A is used in lead storage batteries hence it is PbO_2.

35. (c) : As we move from left to right in a period, ionisation enthalpy increases. But half-filled and fully-filled orbitals have higher ionisation enthalpy than the next upcoming element.

Hence, the correct order of ionisation enthalpy is

	B <	Be <	O <	N
E.C. :	$1s^2 2s^2 2p^1$	$1s^2 2s^2$	$1s^2 2s^2 2p^4$	$1s^2 2s^2 2p^3$
$\Delta_i H$:	801	899	1314	1402

36. (c) : $CH_3-CH=CH-Br \xrightarrow[-HBr]{NaNH_2} CH_3-C\equiv CH$



(A)

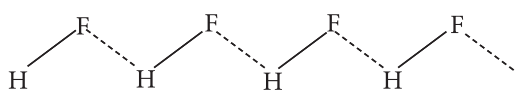
37. (c) : Kernite ($Na_2B_4O_7 \cdot 4H_2O$) (A-ii)
Cassiterite (SnO_2) (B-i)
Calamine ($ZnCO_3$) (C-iv)
Cryolite (Na_3AlF_6) (D-iii)

38. (d) : Vitamin K is a group of vitamins that the body needs for blood clotting, helping wounds to heal. On the other hand, vitamin E delays the blood clotting.

39. (a) : *o*-Nitrophenol has intramolecular hydrogen bonding thus, it is steam volatile. Molecular association is not possible in *o*-nitrophenol hence it has low melting point.

40. (a) : $\underset{(X)}{K_2Cr_2O_7 + H_2SO_4 + 3SO_2} \longrightarrow K_2SO_4 + \underset{\substack{(Y)\\ \text{(Green coloured)}}}{Cr_2(SO_4)_3} + H_2O$

41. (a) : Ion-dipole interactions can also result in hydrogen bonding. Fluorine is most electronegative and forms symmetrical H-bonds.



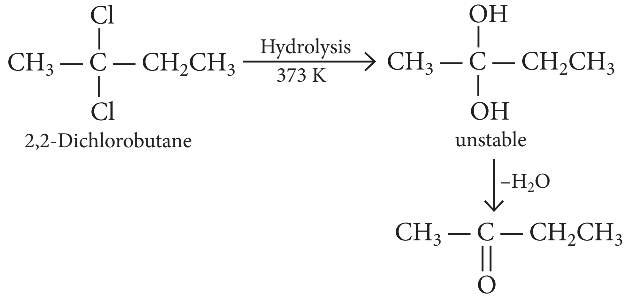
42. (b) : Chloroform (b.p. 334 K) and aniline (b.p. 457 K) have a big difference in their boiling point. Hence, the mixture can be separated by simple distillation. Statement-II is also true.

43. (b) : Ozone in stratosphere helps us to protect from UV rays. But in troposphere it generates photochemical smog.

44. (d) : Viscosity of D_2O is greater than H_2O.

45. (d) : Pr, Nd, Tb and Dy exhibit +4 state, but only in oxides MO_2. Yb^{2+}, which has f^{14} configuration does not form oxide MO_2.

46. (c) : As *B* reacts with hydroxylamine, so it is a carbonyl compound, but as it does not give Tollen's test hence it is a ketone not an aldehyde. Thus, the reactions are summarised as

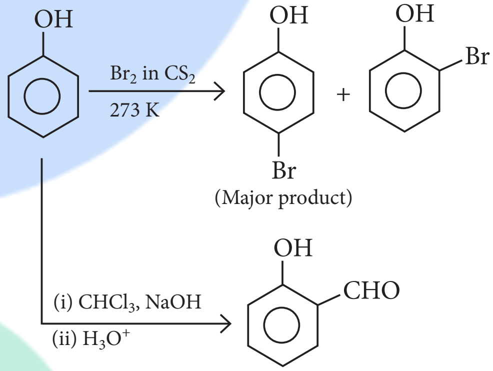


2,2-Dichlorobutane; unstable; 2-Butanone

47. (a) : As given, the product formed with benzene sulphonyl chloride is insoluble in alkaline solution hence the amine is secondary amine. Amine is prepared by ammonolysis of ethyl chloride thus it should contain CH_3CH_2NH- group. So the amine could be $CH_3CH_2CH_2NHCH_2CH_3$.

48. (d) : Carius method is used in estimation of sulphur or halogen in organic compound.

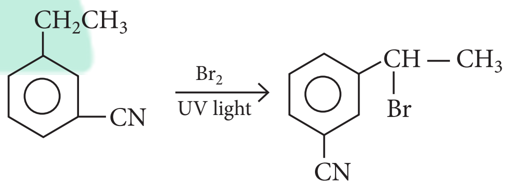
49. (a) :



(Major product)

Salicylaldehyde (Major product)

50. (b) :



51. (200) : $\Delta G° = \Delta H° - T\Delta S°$
Given, $\Delta S° = 2T$
$\Delta G° = \Delta H° - T(2T)$
Minimum temperature can be calculated when $\Delta G° = 0$
$2T^2 = \Delta H° = 80000$
$T^2 = 40000$ or $T = 200$ K

52. (18) : $SO_2 + 2NaOH \longrightarrow Na_2SO_3 + H_2O$

Number of moles of $SO_2 = \dfrac{0.224\times 1}{0.0821\times 298} = 0.0092$ mol

Number of moles of NaOH = 0.01 mol
NaOH is the limiting reagent
Hence, number of moles of $Na_2SO_3 = 0.005$ mol

Mole fraction of $Na_2SO_3 = \dfrac{0.005}{2.005} = 0.0025$

$i = 3$

Lowering in vapour pressure $= P^{\circ}_{H_2O} \times \dfrac{n_{Na_2SO_3}}{n_{H_2O}} \times i$

$= 24 \times \dfrac{0.005}{2.005} \times 3 = \dfrac{24 \times 5 \times 3}{2005} = 0.18$

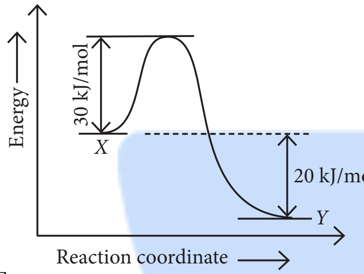
Lowering in vapour pressure $= 18 \times 10^{-2}$ mm Hg

$\therefore \quad x = 18$

53. (50) : $E_{af} = 30$ kJ/mol

$\Delta E = -20$ kJ/mol

For exothermic reaction



$\Delta E = E_{af} - E_{ab}$

$-20 = 30 - E_{ab}$

$\Rightarrow \quad E_{ab} = 50$ kJ mol^{-1}

54. (73) : $AB_{2(g)} \rightleftharpoons A_{(g)} + 2B_{(g)}$

	$AB_{2(g)}$	$A_{(g)}$	$2B_{(g)}$
Initial	1	0	0
At eqm.	$1-\alpha$	α	2α

Moles at equilibrium $= 1 + 2\alpha$

$PV = nRT$

$1.9 \times 25 = n \times 0.082 \times 300$

$n = 1.93$ or $\alpha = 0.465$

$$K_p = \frac{P_A P_B^2}{P_{AB_2}} = \frac{\left(\frac{0.465}{1.93} \times 1.9\right)\left(\frac{0.93}{1.93} \times 1.9\right)^2}{\left(\frac{0.535}{1.93} \times 1.9\right)}$$

$K_p = 0.727$ or 72.7×10^{-2}

55. (8) : It has eight significant figures.

56. (1) : $P(V_m - b) = RT$

$PV_m - Pb = RT$

$$\frac{PV_m}{RT} = 1 + \frac{Pb}{RT} \Rightarrow Z = 1 + \frac{Pb}{RT} \; ; \; \left(\frac{dZ}{dP}\right)_T = 0 + \frac{b}{RT} \times 1$$

Hence, $x = 1$.

57. (25) : For the balanced equation,

$MnO_4^- + 8H^+ + 5e^- \longrightarrow Mn^{2+} + 4H_2O$

We can say that 1 mole of MnO_4^- require 5 F electricity.

Hence, 5 moles of MnO_4^- will require 25 F of electricity.

58. (2) : Moles of oxygen $= \dfrac{3.12}{32} = 0.0975$

Volume of oxygen $= \dfrac{nRT}{P} = \dfrac{0.0975 \times 0.0821 \times 300}{1} = 2.40$ L

1.2 g of Pt metal adsorbs = 2.40 L oxygen

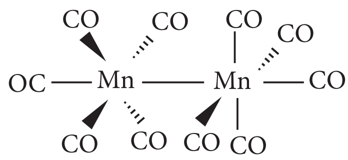
1 g of Pt metal will adsorb $= \dfrac{2.4}{1.2}$ L oxygen

= 2 L oxygen

59. (6) : $K_2Cr_2O_7 + 2KOH \longrightarrow 2K_2CrO_4 + H_2O$

or $Cr_2O_7^{2-} + 2OH^- \rightarrow 2CrO_4^{2-} + H_2O$

Oxidation state of Cr in K_2CrO_4 or CrO_4^{2-} is 6.

60. (0) : 

No bridging CO group.

61. (b) : Let $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$

$\therefore \quad A^2 = \begin{bmatrix} a^2 + b^2 & b(a+c) \\ b(a+c) & b^2 + c^2 \end{bmatrix}$

Since, $a^2 + b^2 + b^2 + c^2 = 1$

$\Rightarrow \quad a^2 + 2b^2 + c^2 = 1$, where $a, b, c \in I$

$\therefore \quad b = 0$ and $a^2 + c^2 = 1$

At $a = 0$, $c = \pm 1$

At $c = 0$, $a = \pm 1$

So, number of possible matrices is 4.

62. (c) : Let $I = \displaystyle\int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^x} dx$ ... (i)

$$= \int_{-\pi/2}^{\pi/2} \frac{\cos^2(-x)}{1+3^{(-x)}} dx \qquad \left[\because \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx\right]$$

$\therefore \quad I = \displaystyle\int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^{-x}} dx$ ... (ii)

Adding (i) and (ii), we get

$$2I = \int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^x} dx + \int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^{-x}} dx$$

$$= 2\int_0^{\pi/2} \left(\frac{\cos^2 x}{1+3^x} + \frac{3^x \cdot \cos^2 x}{1+3^x}\right) dx$$

$$\Rightarrow \quad I = \int_0^{\pi/2} \cos^2 x\,dx = \frac{1}{2}\int_0^{\pi/2} (1+\cos 2x)\,dx$$

$$= \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right]_0^{\pi/2} = \frac{1}{2}\left[\frac{\pi}{2}\right] = \frac{\pi}{4}$$

63. (a) : We have, $\displaystyle\sum_{n=1}^{100} \int_{n-1}^{n} e^{x-[x]}\,dx$

$$= \sum_{n=1}^{100} \int_{n-1}^{n} e^{\{x\}}\,dx \qquad [\because \; x - [x] = \{x\}]$$

$$= \sum_{n=1}^{100} \int_0^1 e^x\,dx \qquad [\because \text{ period of } \{x\} \text{ is } 1]$$

$$= 100\left[e^x\right]_0^1 = 100(e-1)$$

64. (b) : We have, $\lim_{h\to 0} 2\left\{\dfrac{\dfrac{\sqrt{3}}{2}\sin\left(\dfrac{\pi}{6}+h\right)-\dfrac{1}{2}\cos\left(\dfrac{\pi}{6}+h\right)}{\sqrt{3}h\left(\dfrac{\sqrt{3}}{2}\cos h-\dfrac{1}{2}\sin h\right)}\right\}$

$$= \lim_{h\to 0} 2\left\{\frac{\cos\frac{\pi}{6}\sin\left(\frac{\pi}{6}+h\right)-\sin\frac{\pi}{6}\cos\left(\frac{\pi}{6}+h\right)}{\sqrt{3}h\left(\cos\frac{\pi}{6}\cos h-\sin\frac{\pi}{6}\sin h\right)}\right\}$$

$$= \lim_{h\to 0} 2\left\{\frac{\sin\left(\frac{\pi}{6}+h-\frac{\pi}{6}\right)}{\sqrt{3}h\cos\left(\frac{\pi}{6}+h\right)}\right\}$$

$$= \lim_{h\to 0}\frac{2}{\sqrt{3}}\left(\frac{\sin h}{h}\right)\frac{1}{\cos\left(\frac{\pi}{6}+h\right)}$$

$$= \frac{2}{\sqrt{3}\cos\frac{\pi}{6}} = \frac{2}{\sqrt{3}\cdot\frac{\sqrt{3}}{2}} = \frac{4}{3}$$

65. (d) : Given, $\dfrac{dx}{dt} = \lambda x$

(where λ is a constant of proportionality)

Now, when $t = 0$, $x = 1000$ and

when $t = 2$, $x = 1000 + 20\%$ of $1000 = 1000 + 200 = 1200$

$$\Rightarrow \int_{1000}^{1200}\frac{dx}{x} = \lambda\int_0^2 dt \;\Rightarrow\; [\log_e x]_{1000}^{1200} = \lambda[t]_0^2$$

$$\Rightarrow \log_e\left(\frac{1200}{1000}\right) = 2\lambda \;\Rightarrow\; \lambda = \frac{1}{2}\log_e\left(\frac{6}{5}\right)$$

Also, $\displaystyle\int_{1000}^{2000}\frac{dx}{x} = \frac{1}{2}\log_e\left(\frac{6}{5}\right)\int_0^{\frac{k}{\log_e(6/5)}} dt$

$$\Rightarrow \log_e\left(\frac{2000}{1000}\right) = \frac{1}{2}\log_e\left(\frac{6}{5}\right)\cdot\frac{k}{\log_e\left(\frac{6}{5}\right)}$$

$$\Rightarrow \log_e 2 = \frac{k}{2} \;\Rightarrow\; \frac{k}{\log_e 2} = 2 \;\Rightarrow\; \left(\frac{k}{\log_e 2}\right)^2 = 4$$

66. (b) : Let $\dfrac{\sin^{-1}x}{a} = \dfrac{\cos^{-1}x}{b} = \dfrac{\tan^{-1}y}{c} = \lambda$ (say)

$\Rightarrow \sin^{-1}x = a\lambda,\ \cos^{-1}x = b\lambda,\ \tan^{-1}y = c\lambda$

$\therefore\ (a+b)\lambda = \sin^{-1}x + \cos^{-1}x = \dfrac{\pi}{2}$

$\Rightarrow \dfrac{\pi}{a+b} = 2\lambda$

Now, $\cos\left(\dfrac{\pi c}{a+b}\right) = \cos(2\lambda c) = \cos(2\tan^{-1}y)$

Let $\tan^{-1}y = \theta \Rightarrow y = \tan\theta$

$\therefore\ \cos\left(\dfrac{\pi c}{a+b}\right) = \cos 2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta} = \dfrac{1-y^2}{1+y^2}$

67. (c) : Let $A(1, 5, 35)$, $B(7, 5, 5)$, $C(1, \lambda, 7)$ and $D(2\lambda, 1, 2)$ be the given points.

For the points to be coplanar,

$$\begin{vmatrix} 6 & 0 & -30 \\ 0 & \lambda-5 & -28 \\ 2\lambda-1 & -4 & -33 \end{vmatrix} = 0$$

$\Rightarrow 6[-33(\lambda-5) - 112] - 30[-(\lambda-5)(2\lambda-1)] = 0$

$\Rightarrow -33\lambda + 53 + 5(2\lambda^2 - 11\lambda + 5) = 0$

$\Rightarrow 10\lambda^2 - 88\lambda + 78 = 0 \Rightarrow 5\lambda^2 - 44\lambda + 39 = 0$

$\therefore$ Required sum $= 44/5$

68. (b) : Let the coin be tossed n times.

Also, $P(\text{H}) = P(\text{T}) = \dfrac{1}{2}$

Since, $P(7 \text{ heads}) = P(9 \text{ heads})$

$$\Rightarrow {}^nC_7\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^{n-7} = {}^nC_9\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^{n-9}$$

$\Rightarrow {}^nC_7 = {}^nC_9 \Rightarrow n = 7 + 9 = 16$

$$\therefore\ P(2 \text{ heads}) = {}^{16}C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{14} = \frac{16\times 15}{2\times 1}\cdot\frac{1}{2^{16}} = \frac{15}{2^{13}}$$

69. (b) : Since, $\vec{a}$ and $\vec{b}$ are perpendicular.

$\therefore\ \vec{a}\cdot\vec{b} = 0$ ... (i)

Now, $\vec{a}\times(\vec{a}\times(\vec{a}\times(\vec{a}\times\vec{b})))$

$= \vec{a}\times(\vec{a}\times((\vec{a}\cdot\vec{b})\vec{a} - (\vec{a}\cdot\vec{a})\vec{b}))$

$= \vec{a}\times(\vec{a}\times(-|\vec{a}|^2\,\vec{b}))$ [using (i)]

$= -|\vec{a}|^2\,(\vec{a}\times(\vec{a}\times\vec{b})) = -|\vec{a}|^2\,(-|\vec{a}|^2\,\vec{b})$

$= |\vec{a}|^4\vec{b}$

70. (d) : We have, $|f(x) - f(y)| \le |(x-y)^2|$

$$\Rightarrow \left|\frac{f(x)-f(y)}{x-y}\right| \le |x-y|$$

$$\Rightarrow \lim_{h\to 0}\left|\frac{f(y+h)-f(y)}{h}\right| \le 0$$

$\Rightarrow |f'(y)| \le 0 \Rightarrow f'(y) = 0$

$\Rightarrow f(y) =$ constant

As $f(0) = 1$, $\therefore f(y) = 1 \Rightarrow f(x) = 1$

$\therefore\ f(x) > 0\ \forall\ x \in R$

71. (d) : Given, $T_2 + T_6 = \dfrac{25}{2} \Rightarrow ar + ar^5 = \dfrac{25}{2}$ ... (i)

Also, $T_3 \cdot T_5 = 25$

$\Rightarrow\ ar^2\,ar^4 = 25\ \Rightarrow\ a^2r^6 = 25\ \Rightarrow\ ar^3 = 5$ ... (ii)

Dividing (i) by (ii), we get

$$\frac{r+r^5}{r^3}=\frac{5}{2}\ \Rightarrow\ 2+2r^4=5r^2$$

$$\Rightarrow\ 2r^4 - 5r^2 + 2 = 0\ \Rightarrow\ r^2 = 2, \frac{1}{2}$$

$$\therefore\ r^2 = 2 \qquad \left[\text{Rejecting } r^2=\frac{1}{2}\right]$$

Now, $T_4 + T_6 + T_8 = ar^3 + ar^5 + ar^7$

$= ar^3(1 + r^2 + r^4) = 5(1 + 2 + 4) = 35$

72. (b) : Let $S=1+\frac{2}{3}+\frac{7}{3^2}+\frac{12}{3^3}+....\infty$ ... (i)

and $\frac{1}{3}S=\frac{1}{3}+\frac{2}{3^2}+\frac{7}{3^3}+\frac{12}{3^4}+...\infty$ ... (ii)

Subtracting (ii) from (i), we get

$$\frac{2}{3}S=1+\frac{1}{3}+\frac{5}{3^2}+\frac{5}{3^3}+....\infty$$

$$\Rightarrow\ \frac{2}{3}S=\frac{4}{3}+\left(\frac{5}{3^2}+\frac{5}{3^3}+....\infty\right)$$

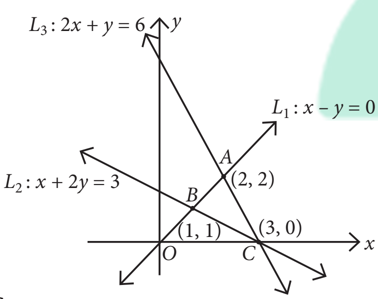
$$\Rightarrow\ \frac{2}{3}S=\frac{4}{3}+\left(\frac{\frac{5}{3^2}}{1-\frac{1}{3}}\right)=\frac{4}{3}+\frac{5}{6}=\frac{13}{6}$$

$$\therefore\ S=\frac{13}{6}\times\frac{3}{2}=\frac{13}{4}$$

73. (c) : $AB=\sqrt{(2-1)^2+(2-1)^2}=\sqrt{2}$ units,

$BC=\sqrt{(3-1)^2+(0-1)^2}=\sqrt{5}$ units and

$CA=\sqrt{(3-2)^2+(0-2)^2}=\sqrt{5}$ units



As $BC = AC$

$\therefore\ \Delta ABC$ is isosceles.

74. (c) : General term,

$$T_{r+1}={}^{10}C_r\,(tx^{1/5})^{10-r}\left(\frac{(1-x)^{1/10}}{t}\right)^r$$

Since, it is independent of t.

$\therefore\ 10 - r - r = 0\ \Rightarrow\ r = 5$

$\therefore\ T_6 = {}^{10}C_5\,x(1-x)^{1/2}$

Now, $\frac{d}{dx}(T_6)={}^{10}C_5\left[(1-x)^{1/2}-\frac{x}{2\sqrt{1-x}}\right]=0$

$\Rightarrow\ 2(1 - x) - x = 0\ \Rightarrow\ 2 - 3x = 0\ \Rightarrow\ x = 2/3$

$\therefore$ Maximum value of $T_6 = {}^{10}C_5\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^{1/2}=\frac{2\times 10!}{3\sqrt{3}\,(5!)^2}$

75. (c) : We have, $P_1 : x + 5y + 7z = 3$

$P_2 : x - 3y - z = 5$

$P_3 : x + 5y + 7z = 5/2$

So, P_1 and P_3 are parallel as direction ratios of normal are same.

76. (b) : Equivalence class of (1, –1) is a circle with centre (0, 0) and radius $\sqrt{2}$.

i.e. $x^2 + y^2 = 2$

$\therefore$ Required set is $S = \{(x, y) \mid x^2 + y^2 = 2\}$

77. (d) : We have, $y=\frac{1}{2}x^4-5x^3+18x^2-19x$

Slope $=\frac{dy}{dx}=2x^3-15x^2+36x-19$

$$\Rightarrow\ \frac{d^2y}{dx^2}=6x^2-30x+36$$

For the slope to be maximum or minimum, $\frac{d^2y}{dx^2}=0$

$\Rightarrow\ 6(x^2 - 5x + 6) = 0\ \Rightarrow\ x = 2, 3$

Now, $\frac{d^3y}{dx^3}=6(2x-5)$

At $x = 2$, $\frac{d^3y}{dx^3}=-6<0$

$\therefore$ Slope is maximum at $x = 2$.

At $x = 2$, $y=\frac{1}{2}(2)^4-5(2)^3+18(2)^2-19(2)$

$= 8 - 40 + 72 - 38 = 2$

$\therefore$ Required point is (2, 2).

78. (d) : Case-I : Digits can be 1, 1, 1, 1, 1, 2, 3

$\therefore$ Required number of ways $=\frac{7!}{5!}=7\times 6=42$

Case-II : Digits can be 1, 1, 1, 1, 2, 2, 2

$\therefore$ Required number of ways $=\frac{7!}{4!\,3!}=\frac{7\times 6\times 5}{3\times 2}=35$

Hence, total number of ways = 42 + 35 = 77

79. (d) : Let $\Delta=\begin{vmatrix}(a+1)(a+2) & (a+2) & 1\\ (a+2)(a+3) & (a+3) & 1\\ (a+3)(a+4) & (a+4) & 1\end{vmatrix}$

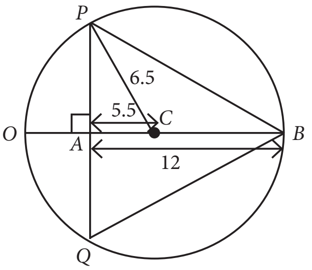
Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\Delta=\begin{vmatrix}(a+1)(a+2) & (a+2) & 1\\ 2a+4 & 1 & 0\\ 4a+10 & 2 & 0\end{vmatrix}$$

$= 2(2a + 4) - (4a + 10) = 4a + 8 - 4a - 10 = -2$

80. (c) : Given, OB = 13 units and OA = 1 unit

$\therefore\ AB$ = 12 units



Let C be the centre of circle.

Thus, $PC = \frac{13}{2} = 6.5$ units and $AC = 5.5$ units

Now, in ΔPAC, $PA^2 = PC^2 - CA^2$

$\Rightarrow \quad PA = \sqrt{(6.5)^2 - (5.5)^2} = \sqrt{42.25 - 30.25} = \sqrt{12}$ units

$\therefore \quad PQ = 2\sqrt{12}$ units

$\therefore \quad$ Area of $\Delta PQB = \frac{1}{2} \times AB \times PQ$

$= \frac{1}{2} \times 12 \times 2\sqrt{12} = 24\sqrt{3}$ sq. units

81. (2) : We have, $y^2 = a\left(x + \frac{\sqrt{a}}{2}\right)$ ... (i)

$\Rightarrow 2yy_1 = a$ ... (ii)

From (i) and (ii), we get

$$y^2 = 2yy_1\left(x + \frac{\sqrt{2yy_1}}{2}\right)$$

$\Rightarrow \quad y^2 = 2xyy_1 + \frac{(2yy_1)^{3/2}}{2} \Rightarrow y^2 - 2xyy_1 = 2^{1/2}(yy_1)^{3/2}$

Squaring both sides, we get

$(y^2 - 2xyy_1)^2 = 2y^3 y_1^3 \Rightarrow (y - 2xy_1)^2 = 2yy_1^3$

$\therefore$ Order = 1 and degree = 3

Required difference = 3 – 1 = 2

82. (45) : We have,

$$30\binom{30}{0} + 29\binom{30}{1} + \ldots\ldots\ldots + 2\binom{30}{28} + 1\binom{30}{29}$$

$$= 30\binom{30}{30} + 29\binom{30}{29} + \ldots\ldots + 2\binom{30}{2} + 1\binom{30}{1}$$

$$= \sum_{r=1}^{30} r \cdot \binom{30}{r} = \sum_{r=1}^{30} r \cdot \frac{30}{r}\binom{29}{r-1}$$

$$= 30\left[\binom{29}{0} + \binom{29}{1} + \binom{29}{2} + \ldots + \binom{29}{29}\right]$$

$= 30 \cdot 2^{29} = 15 \times 2^{30} = n \cdot 2^m$ (given)

$\therefore \quad n + m = 15 + 30 = 45$

83. (1) : Given equation is

$e^{\sin y} \cos y \frac{dy}{dx} + e^{\sin y} \cos x = \cos x$ ...(i)

Put $e^{\sin y} = t \quad \Rightarrow \quad e^{\sin y} \cos y \frac{dy}{dx} = \frac{dt}{dx}$

$\therefore$ (i) becomes, $\frac{dt}{dx} + t\cos x = \cos x$, which is a L.D.E.

Here, I.F. $= e^{\int \cos x dx} = e^{\sin x}$

$\therefore$ Solution is given by

$t \cdot e^{\sin x} = \int \cos x \cdot e^{\sin x} dx \Rightarrow e^{\sin y} e^{\sin x} = e^{\sin x} + c$

At $x = 0, y = 0 \Rightarrow c = 0$

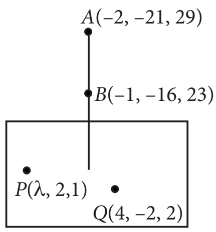
$\therefore \quad e^{\sin y} = 1 \Rightarrow \sin y = 0 \Rightarrow y = 0, \forall x$

$\therefore \quad 1 + y\left(\frac{\pi}{6}\right) + \frac{\sqrt{3}}{2} y\left(\frac{\pi}{3}\right) + \frac{1}{\sqrt{2}} y\left(\frac{\pi}{4}\right) = 1 + 0 + 0 + 0 = 1$

84. (8) : $\overrightarrow{AB} = \hat{i} + 5\hat{j} - 6\hat{k}$

$\overrightarrow{PQ} = (4 - \lambda)\hat{i} - 4\hat{j} + \hat{k}$

Since, $\overrightarrow{AB} \perp \overrightarrow{PQ}$.



$\therefore \quad \overrightarrow{AB} \cdot \overrightarrow{PQ} = 0$

$(\hat{i} + 5\hat{j} - 6\hat{k}) \cdot ((4 - \lambda)\hat{i} - 4\hat{j} + \hat{k}) = 0$

$\Rightarrow \quad 4 - \lambda - 20 - 6 = 0 \Rightarrow \lambda = -22$

$\therefore \quad \left(\frac{\lambda}{11}\right)^2 - \frac{4\lambda}{11} - 4 = \left(\frac{-22}{11}\right)^2 - 4\left(\frac{-22}{11}\right) - 4$

$= 4 + 8 - 4 = 8$

85. (1) : We have, $\log_4(x - 1) = \log_2(x - 3)$

$\Rightarrow \quad \frac{1}{2}\log_2(x - 1) = \log_2(x - 3)$

$\Rightarrow \quad \log_2(x - 1)^{1/2} = \log_2(x - 3) \Rightarrow (x - 1)^{1/2} = x - 3$

Squaring both sides, we get

$(x - 1) = x^2 + 9 - 6x$

$\Rightarrow \quad x^2 - 7x + 10 = 0 \Rightarrow x = 2, 5$

$\therefore \quad x = 5$ as $x = 2$ does not satisfy the domain of given equation

$\therefore$ Number of solution is 1.

86. (11) : We have, $3\sin x + 4\cos x = k + 1$

$\Rightarrow \quad (k + 1) \in \left[-\sqrt{3^2 + 4^2}, \sqrt{3^2 + 4^2}\right]$

$\Rightarrow \quad (k + 1) \in [-5, 5] \Rightarrow k \in [-6, 4]$

$\therefore$ Number of integral values of k is 11.

87. (1) : We have, $\sqrt{3}\cos^2 x - (\sqrt{3} - 1)\cos x - 1 = 0$

$\Rightarrow \quad \sqrt{3}\cos^2 x - \sqrt{3}\cos x + \cos x - 1 = 0$

$\Rightarrow \quad \sqrt{3}\cos x(\cos x - 1) + 1(\cos x - 1) = 0$

$\Rightarrow \quad (\cos x - 1)(\sqrt{3}\cos x + 1) = 0$

$\Rightarrow \quad \cos x = 1$ or $\cos x = -\frac{1}{\sqrt{3}}$ $\left(\text{not possible as } x \in \left[0, \frac{\pi}{2}\right]\right)$

$\therefore \quad \cos x = 1 \Rightarrow x = 0$

$\therefore$ Number of solutions is 1.

88. (3) : We have, $x^3 - 2x^2 + 2x - 1 = 0$ ....(i)

Since, $x = 1$ satisfies the given equation.

$\therefore$ $x - 1$ is a factor of (i)

$\therefore$ $(x^3 - 2x^2 + 2x - 1) = (x - 1)(x^2 - x + 1)$

$\therefore$ $x = 1, \dfrac{1 \pm i\sqrt{3}}{2}$

i.e., $x = 1, -\omega^2, -\omega$

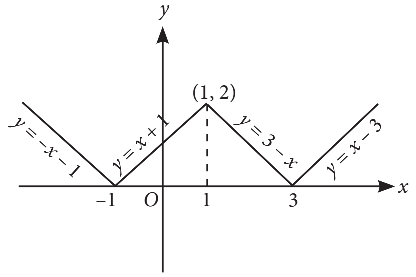
$\therefore$ Sum of 162$^{\text{th}}$ power of roots

$= (1)^{162} + (-\omega^2)^{162} + (-\omega)^{162}$

$= 1 + \omega^{324} + \omega^{162} = 1 + (\omega^3)^{108} + (\omega^3)^{54} = 1 + 1 + 1 = 3.$

89. (4) : We have, $y = ||x - 1| - 2|$

$$= \begin{cases} x-3 & , \quad x \ge 3 \\ 3-x & , \quad 1 \le x \le 3 \\ x+1 & , \quad -1 \le x \le 1 \\ -1-x & , \quad x \le -1 \end{cases}$$



$\therefore$ Required area $= \int_{-1}^{1} (x+1)\,dx + \int_{1}^{3} (3-x)\,dx$

$$= \left[\frac{x^2}{2} + x\right]_{-1}^{1} + \left[3x - \frac{x^2}{2}\right]_{1}^{3} = 0 + 2 + 6 - 4 = 4 \text{ sq. units}$$

90. (2) : Let $I = \int_0^{\pi} |\sin 2x|\,dx$

$$= 2\int_0^{\pi/2} \sin 2x\,dx$$

$$\left[\because \int_0^{2a} f(x)\,dx = 2\int_0^{a} f(x)\,dx \text{ if } f(2a-x) = f(x)\right]$$

$$= 2\left[\frac{-\cos 2x}{2}\right]_0^{\pi/2} = 2$$

❖❖❖