JEE MAIN 2021

ONLINE
24th February
2nd Shift

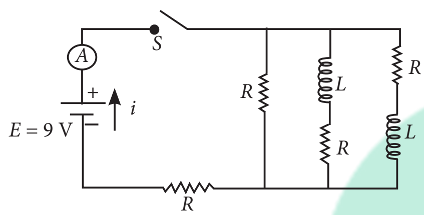
PHYSICS

SECTION-A (MULTIPLE CHOICE QUESTIONS)

1. The de Broglie wavelength of a proton and α-particle are equal. The ratio of their velocities is
(a) 4 : 3 (b) 4 : 2 (c) 4 : 1 (d) 1 : 4

2. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of '$L$' is 1.0 m from meter scale having a minimum division of 1 mm and time of one complete oscillation is 1.95 s measured from stopwatch of 0.01 s resolution. The percentage error in the determination of 'g' will be
(a) 1.03% (b) 1.30% (c) 1.13% (d) 1.33%

3. Figure shows a circuit that contains four identical resistors with resistance R = 2.0 Ω, two identical inductors with inductance L = 2.0 mH and an ideal battery with emf E = 9 V. The current 'i' just after the switch 'S' is closed will be



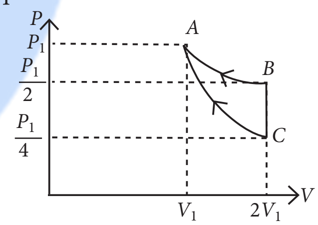
(a) 3.37 A (b) 3.0 A (c) 2.25 A (d) 9 A

4. An X-ray tube is operated at 1.24 million volt. The shortest wavelength of the produced photon will be
(a) 10^{-1} nm (b) 10^{-4} nm (c) 10^{-2} nm (d) 10^{-3} nm

5. When a particle executes SHM, nature of graphical representation of velocity as a function of displacement is
(a) elliptical (b) circular
(c) straight line (d) parabolic

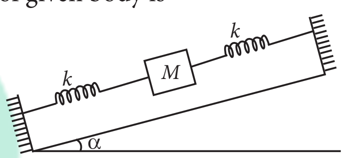
6. Given below are two statements :
Statement I : PN junction diodes can be used to function as transistor, simply by connecting two diodes, back to back, which acts as the base terminal.
Statement II : In the study of transistor, the amplification factor β indicates ratio of the collector current to the base current.
In the light of the above statements, choose the correct answer from the options given below.
(a) Statement I is true but Statement II is false.
(b) Statement I is false but Statement II is true.
(c) Both Statement I and Statement II are false.
(d) Both Statement I and Statement II are true.

7. If one mole of an ideal gas at (P_1, V_1) is allowed to expand reversibly and isothermally (A to B) its pressure is reduced to one-half of the original pressure (see figure). This is followed by a constant volume cooling till its pressure is reduced to one-fourth of the initial value ($B \to C$). Then it is restored to its initial state by a reversible adiabatic compression (C to A). The net workdone by the gas is equal to



(a) 0
(b) $-\frac{RT}{2(\gamma-1)}$
(c) $RT\ln 2$
(d) $RT\left(\ln 2 - \frac{1}{2(\gamma-1)}\right)$

8. In the given figure, a body of mass M is held between two massless springs, on a smooth inclined plane. The free ends of the springs are attached to firm supports. If each spring has spring constant k, the frequency of oscillation of given body is

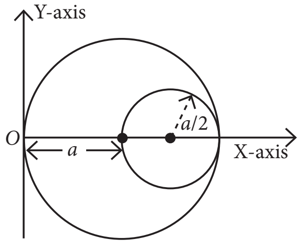


(a) $\frac{1}{2\pi}\sqrt{\frac{k}{2M}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{k}{Mg\sin\alpha}}$
(c) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{2k}{Mg\sin\alpha}}$

9. According to Bohr atom model, in which of the following transitions will the frequency be maximum?
(a) $n = 4$ to $n = 3$ (b) $n = 3$ to $n = 2$
(c) $n = 2$ to $n = 1$ (d) $n = 5$ to $n = 4$

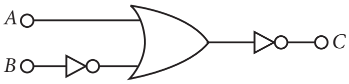
10. If the source of light used in a Young's double slit experiment is changed from red to violet
(a) the intensity of minima will increase.
(b) the central bright fringe will become a dark fringe.
(c) the fringes will become brighter.
(d) consecutive fringe lines will come closer.

11. A circular hole of radius $\left(\frac{a}{2}\right)$ is cut out of a circular disc of radius 'a' as shown in figure. The centroid of the remaining circular portion with respect to point 'O' will be

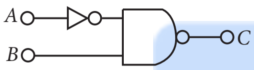


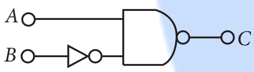
(a) $\frac{1}{6}a$ (b) $\frac{10}{11}a$ (c) $\frac{2}{3}a$ (d) $\frac{5}{6}a$

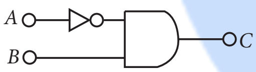
12.

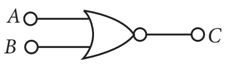


The logic circuit shown above is equivalent to

(a)


(b)


(c)


(d)


13. On the basis of kinetic theory of gases, the gas exerts pressure because its molecules
(a) suffer change in momentum when impinge on the walls of container.
(b) continuously stick to the walls of container.
(c) continuously lose their energy till it reaches wall.
(d) are attracted by the walls of container.

14. A soft ferromagnetic material is placed in an external magnetic field. The magnetic domains
(a) decrease in size and changes orientation.
(b) increase in size but no change in orientation.
(c) may increase or decrease in size and change its orientation.
(d) have no relation with external magnetic field.

15. A body weighs 49 N on a spring balance at the north pole. What will be its weight recorded on the same weighing machine, if it is shifted to the equator?
[Use $g = \frac{GM}{R^2} = 9.8\ \text{m s}^{-2}$ and radius of earth, $R = 6400$ km]
(a) 49.83 N (b) 48.83 N
(c) 49.17 N (d) 49 N

16. Two electrons each are fixed at a distance '$2d$'. A third charge proton placed at the midpoint is displaced slightly by a distance $x(x << d)$ perpendicular to the line joining the two fixed charges. Proton will execute simple harmonic motion having angular frequency (m = mass of charged particle)

(a) $\left(\frac{\pi\varepsilon_0 md^3}{2q^2}\right)^{1/2}$ (b) $\left(\frac{2q^2}{\pi\varepsilon_0 md^3}\right)^{1/2}$

(c) $\left(\frac{q^2}{2\pi\varepsilon_0 md^3}\right)^{1/2}$ (d) $\left(\frac{2\pi\varepsilon_0 md^3}{q^2}\right)^{1/2}$

17. Which of the following equations represents a travelling wave?
(a) $y = Ae^x\cos(\omega t - \theta)$ (b) $y = A\sin(15x - 2t)$
(c) $y = A\sin x \cos\omega t$ (d) $y = Ae^{-x^2}(vt + \theta)$

18. Match List-I with List-II.

List-I		List-II	
(A)	Source of microwave frequency	(i)	Radioactive decay of nucleus
(B)	Source of infrared frequency	(ii)	Magnetron
(C)	Source of Gamma Rays	(iii)	Inner shell electrons
(D)	Source of X-rays	(iv)	Vibration of atoms and molecules
		(v)	LASER
		(vi)	*RC* circuit

Choose the correct answer from the options given below
(a) (A)-(vi), (B)-(v), (C)-(i), (D)-(iv)
(b) (A)-(ii), (B)-(iv), (C)-(vi), (D)-(iii)
(c) (A)-(ii), (B)-(iv), (C)-(i), (D)-(iii)
(d) (A)-(vi), (B)-(iv), (C)-(i), (D)-(v)

19. Zener breakdown occurs in a p-n junction having p and n both
(a) heavily doped and have narrow depletion layer.
(b) heavily doped and have wide depletion layer.
(c) lightly doped and have wide depletion layer.
(d) lightly doped and have narrow depletion layer.

20. A particle is projected with velocity v_0 along x-axis. A damping force is acting on the particle which is proportional to the square of the distance from the origin *i.e.*, $ma = -\alpha x^2$. The distance at which the particle stops

(a) $\left(\frac{2v_0^2}{3\alpha}\right)^{1/2}$ (b) $\left(\frac{3v_0^2}{2\alpha}\right)^{1/2}$

(c) $\left(\frac{2v_0}{3\alpha}\right)^{1/3}$ (d) $\left(\frac{3v_0^2}{2\alpha}\right)^{1/3}$

SECTION-B (NUMERICAL VALUE TYPE)

Attempt any 5 questions out of 10.

21. Two solids A and B of mass 1 kg and 2 kg respectively are moving with equal linear momentum. The ratio of their kinetic energies $(\text{K.E.})_A : (\text{K.E.})_B$ will be $\frac{A}{1}$, so the value of A will be ________.

22. A series LCR circuit is designed to resonate at an angular frequency $\omega_0 = 10^5$ rad/s. The circuit draws 16 W power from 120 V source at resonance. The value of resistance 'R' in the circuit is ________ Ω.

23. A signal of 0.1 kW is transmitted in a cable. The attenuation of cable is –5 dB per km and cable length is 20 km. The power received at receiver is 10^{-x} W. The value of x is ________.

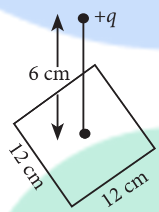
$$\left[\text{Gain in dB} = 10\log_{10}\left(\frac{P_o}{P_i}\right)\right]$$

24. A uniform metallic wire is elongated by 0.04 m when subjected to a linear force F. The elongation, if its length and diameter is doubled and subjected to the same force will be ________ cm.

25. The root mean square speed of molecules of a given mass of a gas at 27°C and 1 atmosphere pressure is 200 ms^{-1}. The root mean square speed of molecules of the gas at 127°C and 2 atmosphere pressure is $\frac{x}{\sqrt{3}}$ m s^{-1}. The value of x will be ________.

26. A uniform thin bar of mass 6 kg and length 2.4 meter is bent to make an equilateral hexagon. The moment of inertia about an axis passing through the centre of mass and perpendicular to the plane of hexagon is ________ $\times 10^{-1}$ kg m^2.

27. A point charge of +12 μC is at a distance 6 cm vertically above the centre of a square of side 12 cm as shown in figure. The magnitude of the electric flux through the square will be ________ $\times 10^3$ N m^2/C.



28. An electromagnetic wave of frequency 3 GHz enters a dielectric medium of relative electric permittivity 2.25 from vacuum. The wavelength of this wave in that medium will be ________ $\times 10^{-2}$ cm.

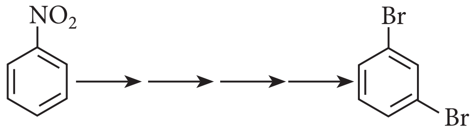
29. Two cars are approaching each other at an equal speed of 7.2 km/hr. When they see each other, both blow horns having frequency of 676 Hz. The beat frequency heard by each driver will be ________ Hz. [Velocity of sound in air is 340 m/s.]

30. A cylindrical wire of radius 0.5 mm and conductivity 5×10^7 S/m is subjected to an electric field of 10 mV/m. The expected value of current in the wire will be $x^3\pi$ mA. The value of x is ________

CHEMISTRY

SECTION - A (MULTIPLE CHOICE QUESTIONS)

31. What is the correct sequence of reagents used for converting nitrobenzene into m-dibromobenzene?



(a) $\xrightarrow{Sn/HCl} / \xrightarrow{KBr} / \xrightarrow{Br_2} / \xrightarrow{H^+}$

(b) $\xrightarrow{Br_2/Fe} / \xrightarrow{Sn/HCl} / \xrightarrow{NaNO_2/HCl} / \xrightarrow{CuBr/HBr}$

(c) $\xrightarrow{NaNO_2} / \xrightarrow{HCl} / \xrightarrow{KBr} / \xrightarrow{H^+}$

(d) $\xrightarrow{Sn/HCl} / \xrightarrow{Br_2} / \xrightarrow{NaNO_2} / \xrightarrow{NaBr}$

32. Given below are two statements :

Statement I : The value of the parameter "Biochemical Oxygen Demand (BOD)" is important for survival of aquatic life.

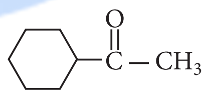
Statement II : The optimum value of BOD is 6.5 ppm.

In the light of the above statements, choose the most appropriate answer from the options given below.

(a) Both statement I and statement II are false.
(b) Both statement I and statement II are true.
(c) Statement I is false but statement II is true.
(d) Statement I is true but statement II is false.

33. Which one of the following carbonyl compounds cannot be prepared by addition of water on an alkyne in the presence of $HgSO_4$ and H_2SO_4?

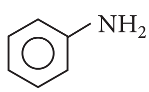
(a) $CH_3-\overset{\overset{\large O}{\|}}{C}-CH_2CH_3$ (b) $CH_3-CH_2-\overset{\overset{\large O}{\|}}{C}-H$

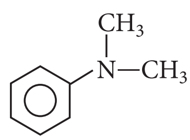
(c)  (d) $CH_3-\overset{\overset{\large O}{\|}}{C}-H$

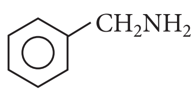
34. The calculated magnetic moments (spin only value) for species $[FeCl_4]^{2-}$, $[Co(C_2O_4)_3]^{3-}$ and MnO_4^{2-} respectively are

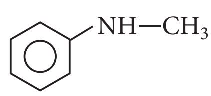
(a) 4.90, 0 and 1.73 BM (b) 5.82, 0 and 0 BM
(c) 4.90, 0 and 2.83 BM (d) 5.92, 4.90 and 0 BM.

35. The diazonium salt of which of the following compounds will form a coloured dye on reaction with β-naphthol in NaOH?

(a) 

(b) 

(c) 

(d) 

36. Match List-I and List-II.

List-I		List-II	
(A)	Valium	(i)	Antifertility drug
(B)	Morphine	(ii)	Pernicious anaemia
(C)	Norethindrone	(iii)	Analgesic
(D)	Vitamin B_{12}	(iv)	Tranquilizer

(a) (A)-(iv), (B)-(iii), (C)-(ii), (D)-(i)
(b) (A)-(ii), (B)-(iv), (C)-(iii), (D)-(i)
(c) (A)-(i), (B)-(iii), (C)-(iv), (D)-(ii)
(d) (A)-(iv), (B)-(iii), (C)-(i), (D)-(ii)

37. Match List-I and List-II.

List-I		List-II	
(A)	$R-\overset{\overset{O}{\parallel}}{C}-Cl \longrightarrow R-CHO$	(i)	$Br_2/NaOH$
(B)	$R-CH_2-COOH \longrightarrow R-\underset{\underset{Cl}{\vert}}{CH}-COOH$	(ii)	$H_2/Pd-BaSO_4$
(C)	$R-\overset{\overset{O}{\parallel}}{C}-NH_2 \longrightarrow R-NH_2$	(iii)	Zn(Hg)/Conc. HCl
(D)	$R-\overset{\overset{O}{\parallel}}{C}-CH_3 \longrightarrow R-CH_2-CH_3$	(iv)	Cl_2/Red P, H_2O

Choose the correct answer from the options given below.
(a) (A)-(iii), (B)-(iv), (C)-(i), (D)-(ii)
(b) (A)-(iii), (B)-(i), (C)-(iv), (D)-(ii)
(c) (A)-(ii), (B)-(iv), (C)-(i), (D)-(iii)
(d) (A)-(ii), (B)-(i), (C)-(iv), (D)-(iii)

38. The correct shape and I—I—I bond angles respectively in I_3^- ion are
(a) linear; 180°
(b) T-shaped; 180° and 90°
(c) trigonal planar; 120°
(d) distorted trigonal planar; 135° and 90°

39. According to Bohr's atomic theory
(A) Kinetic energy of electron is $\propto \frac{Z^2}{n^2}$.
(B) The product of velocity (v) of electron and principal quantum number (n), 'vn' $\propto Z^2$.
(C) Frequency of revolution of electron in an orbit is $\propto \frac{Z^3}{n^3}$.
(D) Coulombic force of attraction on the electron is $\propto \frac{Z^3}{n^4}$.
Choose the most appropriate answer from the options given below.
(a) (A) and (D) only
(b) (A) only
(c) (C) only
(d) (A), (C) and (D) only

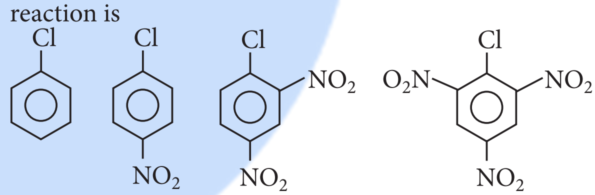
40. Match List-I and List-II.

List-I (Metal)		List-II (Ores)	
(A)	Aluminium	(i)	Siderite
(B)	Iron	(ii)	Calamine
(C)	Copper	(iii)	Kaolinite
(D)	Zinc	(iv)	Malachite

Choose the correct answer from the options given below.
(a) (A)-(iv), (B)-(iii), (C)-(ii), (D)-(i)
(b) (A)-(iii), (B)-(i), (C)-(iv), (D)-(ii)
(c) (A)-(ii), (B)-(iv), (C)-(i), (D)-(iii)
(d) (A)-(i), (B)-(ii), (C)-(iii), (D)-(iv)

41. Most suitable salt which can be used for efficient clotting of blood will be
(a) $FeCl_3$
(b) $NaHCO_3$
(c) $FeSO_4$
(d) $Mg(HCO_3)_2$

42. The correct order of the following compounds showing increasing tendency towards nucleophilic substitution reaction is

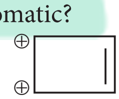
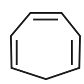
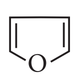
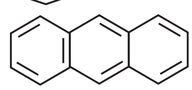


(i) (ii) (iii) (iv)

(a) (iv) < (iii) < (ii) < (i)
(b) (iv) < (i) < (ii) < (iii)
(c) (i) < (ii) < (iii) < (iv)
(d) (iv) < (i) < (iii) < (ii)

43. The correct set from the following in which both pairs are in correct order of melting point is
(a) LiCl > LiF; MgO > NaCl
(b) LiF > LiCl; NaCl > MgO
(c) LiF > LiCl; MgO > NaCl
(d) LiCl > LiF; NaCl > MgO

44. Which one of the following compounds is non-aromatic?

(a) 
(b) 
(c) 
(d) 

45. Given below are two statements : One is labelled as Assertion (A) and the other is labelled as Reason (R).
Assertion (A) : Hydrogen is the most abundant element in the universe, but it is not the most abundant gas in the troposphere.
Reason (R) : Hydrogen is the lightest element.
In the light of the above statements, choose the correct answer from the options given below.
(a) Both (A) and (R) are true and (R) is the correct explanation of (A).
(b) (A) is false but (R) is true.

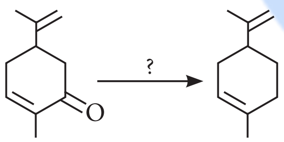
(c) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(d) (A) is true but (R) is false.

46. What is the correct order of the following elements with respect to their density?
(a) Cr < Fe < Co < Cu < Zn
(b) Zn < Cr < Fe < Co < Cu
(c) Cr < Zn < Co < Cu < Fe
(d) Zn < Cu < Co < Fe < Cr

47. Match List-I and List-II.

List-I (Salt)		List-II (Flame colour wavelength)	
(A)	LiCl	(i)	455.5 nm
(B)	NaCl	(ii)	670.8 nm
(C)	RbCl	(iii)	780.0 nm
(D)	CsCl	(iv)	589.2 nm

Choose the correct answer from the options given below.
(a) (A)-(i), (B)-(iv), (C)-(ii), (D)-(iii)
(b) (A)-(ii), (B)-(iv), (C)-(iii), (D)-(i)
(c) (A)-(ii), (B)-(i), (C)-(iv), (D)-(iii)
(d) (A)-(iv), (B)-(ii), (C)-(iii), (D)-(i)

48. 

Which of the following reagent is suitable for the preparation of the product in the above reaction?
(a) Ni/H_2 (b) $NaBH_4$
(c) Red P + Cl_2
(d) $NH_2-NH_2/C_2H_5\overset{-}{O}\overset{+}{Na}$

49. The incorrect statement among the following is
(a) red colour of ruby is due to the presence of Co^{3+}
(b) RuO_4 is an oxidizing agent
(c) Cr_2O_3 is an amphoteric oxide
(d) $VOSO_4$ is a reducing agent.

50. In polymer Buna-S, 'S' stands for
(a) strength (b) sulphonation
(c) styrene (d) sulphur.

SECTION - B (NUMERICAL VALUE TYPE)

Attempt any 5 questions out of 10.

51. The volume occupied by 4.75 g of acetylene gas at 50°C and 740 mmHg pressure is ________ L. (Rounded off to the nearest integer)
[Given $R = 0.0826$ L atm K^{-1} mol^{-1}]

52. The solubility product of PbI_2 is 8.0×10^{-9}. The solubility of lead iodide in 0.1 molar solution of lead nitrate is $x \times 10^{-6}$ mol/L. The value of x is ________. (Rounded off to the nearest integer)
[Given $\sqrt{2} = 1.41$]

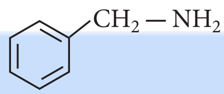
53. The magnitude of the change in oxidising power of the MnO_4^-/Mn^{2+} couple is $x \times 10^{-4}$ V, if the H^+ concentration is decreased from 1 M to 10^{-4} M at 25°C. (Assume concentration of MnO_4^- and Mn^{2+} to be same on change in H^+ concentration). The value of x is ________. (Rounded off to the nearest integer)

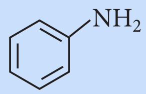
$$\left[\text{Given}: \frac{2.303RT}{F} = 0.059\right]$$

54. The total number of amines among the following which can be synthesized by Gabriel synthesis is ________.

(A) $(CH_3)_2CH-CH_2-NH_2$

(B) $CH_3CH_2NH_2$

(C) 

(D) 

55. Among the following allotropic forms of sulphur, the number of allotropic forms, which will show paramagnetism is ________.
(A) α-sulphur (B) β-sulphur
(C) S_2-form

56. Assuming ideal behaviour, the magnitude of log K for the following reaction at 25°C is $x \times 10^{-1}$. The value of x is ________. (Integer answer)

$$3HC\equiv CH_{(g)} \rightleftharpoons C_6H_{6(l)}$$

[Given : $\Delta_f G^\circ$ ($HC\equiv CH$) = -2.04×10^5 J mol^{-1};
$\Delta_f G^\circ$ (C_6H_6) = -1.24×10^5 J mol^{-1}; $R = 8.314$ J K^{-1} mol^{-1}]

57. C_6H_6 freezes at 5.5°C. The temperature at which a solution of 10 g of C_4H_{10} in 200 g of C_6H_6 freeze is ________ °C. (The molal freezing point depression constant of C_6H_6 is 5.12°C/m.)

58. Sucrose hydrolyses in acid solution into glucose and fructose following first order rate law with a half-life of 3.33 h at 25°C. After 9 h, the fraction of sucrose remaining is f. The value of $\log_{10}\left(\frac{1}{f}\right)$ is ________ $\times 10^{-2}$.
(Rounded off to the nearest integer)
[Assume : ln 10 = 2.303, ln 2 = 0.693]

59. The formula of a gaseous hydrocarbon which requires 6 times of its own volume of O_2 for complete oxidation and produces 4 times its own volume of CO_2 is C_xH_y. The value of y is ________.

60. 1.86 g of aniline completely reacts to form acetanilide. 10% of the product is lost during purification. Amount of acetanilide obtained after purification (in g) is ________ $\times 10^{-2}$.

MATHEMATICS

SECTION - A (MULTIPLE CHOICE QUESTIONS)

61. If a curve $y = f(x)$ passes through the point (1, 2) and satisfies $x\frac{dy}{dx}+y=bx^4$, then for what value of b, $\int_1^2 f(x)dx=\frac{62}{5}$?

(a) $\frac{62}{5}$ (b) $\frac{31}{5}$ (c) 5 (d) 10

62. For the system of linear equations :
$x - 2y = 1, x - y + kz = -2, ky + 4z = 6, k \in R$,
consider the following statements :
(A) The system has unique solution if $k \neq 2, k \neq -2$.
(B) The system has unique solution if $k = -2$.
(C) The system has unique solution if $k = 2$.
(D) The system has no solution if $k = 2$.
(E) The system has infinite number of solutions if $k \neq -2$.
Which of the following statements are correct?
(a) (B) and (E) only (b) (A) and (D) only
(c) (C) and (D) only (d) (A) and (E) only

63. A possible value of $\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$ is

(a) $\frac{1}{2\sqrt{2}}$ (b) $2\sqrt{2}-1$ (c) $\sqrt{7}-1$ (d) $\frac{1}{\sqrt{7}}$

64. Let $f(x)$ be a differentiable function defined on [0, 2] such that $f'(x) = f'(2 - x)$ for all $x \in (0, 2)$, $f(0) = 1$ and $f(2) = e^2$. Then the value of $\int_0^2 f(x)\,dx$ is

(a) $2(1 + e^2)$ (b) $1 + e^2$
(c) $2(1 - e^2)$ (d) $1 - e^2$

65. The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 20 seconds at the speed of 432 km/hour, the angle of elevation changes to 30°. If the jet plane is flying at a constant height, then its height is

(a) $3600\sqrt{3}$ m (b) $2400\sqrt{3}$ m
(c) $1200\sqrt{3}$ m (d) $1800\sqrt{3}$ m

66. Let f be a twice differentiable function defined on R such that $f(0) = 1, f'(0) = 2$ and $f'(x) \neq 0$ for all $x \in R$. If $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix}=0$, for all $x \in R$, then the value of $f(1)$ lies in the interval

(a) (3, 6) (b) (0, 3) (c) (9, 12) (d) (6, 9)

67. Let A and B be 3×3 real matrices such that A is symmetric matrix and B is skew-symmetric matrix. Then the system of linear equations $(A^2B^2 - B^2A^2)X = O$, where X is a 3×1 column matrix of unknown variables and O is a 3×1 null matrix, has

(a) a unique solution
(b) exactly two solutions
(c) no solution
(d) infinitely many solutions

68. If P is a point on the parabola $y = x^2 + 4$ which is closest to the straight line $y = 4x - 1$, then the co-ordinates of P are

(a) (3, 13) (b) (–2, 8) (c) (2, 8) (d) (1, 5)

69. The vector equation of the plane passing through the intersection of the planes $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=1$ and $\vec{r}\cdot(\hat{i}-2\hat{j})=-2$, and the point (1, 0, 2) is

(a) $\vec{r}\cdot(\hat{i}+7\hat{j}+3\hat{k})=7$ (b) $\vec{r}\cdot(3\hat{i}+7\hat{j}+3\hat{k})=7$

(c) $\vec{r}\cdot(\hat{i}+7\hat{j}+3\hat{k})=\frac{7}{3}$ (d) $\vec{r}\cdot(\hat{i}-7\hat{j}+3\hat{k})=\frac{7}{3}$

70. If the curve $y = ax^2 + bx + c, x \in R$, passes through the point (1, 2) and the tangent line to this curve at origin is $y = x$, then the possible values of a, b, c are

(a) $a = -1, b = 1, c = 1$ (b) $a = 1, b = 0, c = 1$

(c) $a = 1, b = 1, c = 0$ (d) $a=\frac{1}{2}, b=\frac{1}{2}, c=1$

71. The probability that two randomly selected subsets of the set {1, 2, 3, 4, 5} have exactly two elements in their intersection, is

(a) $\frac{65}{2^8}$ (b) $\frac{35}{2^7}$ (c) $\frac{135}{2^9}$ (d) $\frac{65}{2^7}$

72. The negation of the statement $\sim p \wedge (p \vee q)$ is

(a) $\sim p \vee q$ (b) $p \wedge \sim q$ (c) $p \vee \sim q$ (d) $\sim p \wedge q$

73. For which of the following curves, the line $x+\sqrt{3}y=2\sqrt{3}$ is the tangent at the point $\left(\frac{3\sqrt{3}}{2},\frac{1}{2}\right)$?

(a) $x^2 + 9y^2 = 9$ (b) $2x^2 - 18y^2 = 9$

(c) $x^2 + y^2 = 7$ (d) $y^2=\frac{1}{6\sqrt{3}}x$

74. The area of the region : $R = \{(x, y) : 5x^2 \leq y \leq 2x^2 + 9\}$ is

(a) $9\sqrt{3}$ square units (b) $6\sqrt{3}$ square units
(c) $11\sqrt{3}$ square units (d) $12\sqrt{3}$ square units

75. For the statements p and q, consider the following compound statements :
(A) $(\sim q \wedge (p \rightarrow q)) \rightarrow \sim p$
(B) $((p \vee q) \wedge \sim p) \rightarrow q$
Then which of the following statements is correct?
(a) (A) and (B) both are tautologies.
(b) (A) is a tautology but not (B).
(c) (B) is a tautology but not (A).
(d) (A) and (B) both are not tautologies.

76. Let a, b, c be in arithmetic progression. Let the centroid of the triangle with vertices (a, c), $(2, b)$ and (a, b) be $\left(\frac{10}{3},\frac{7}{3}\right)$. If α, β are the roots of the equation $ax^2 + bx + 1 = 0$, then the value of $\alpha^2 + \beta^2 - \alpha\beta$ is

(a) $-\frac{71}{256}$ (b) $\frac{71}{256}$ (c) $-\frac{69}{256}$ (d) $\frac{69}{256}$

77. If $n \geq 2$ is a positive integer, then the sum of the series ${}^{n+1}C_2 + 2({}^2C_2 + {}^3C_2 + {}^4C_2 + ... + {}^nC_2)$ is

(a) $\frac{n(n-1)(2n+1)}{6}$ (b) $\frac{n(n+1)^2(n+2)}{12}$

(c) $\frac{n(n+1)(2n+1)}{6}$ (d) $\frac{n(2n+1)(3n+1)}{6}$

78. Let $f: R \to R$ be defined as

$$f(x)=\begin{cases}-55x, & \text{if } x<-5\\ 2x^3-3x^2-120x, & \text{if } -5\le x\le 4.\\ 2x^3-3x^2-36x-336, & \text{if } x>4\end{cases}$$

Let $A = \{x \in R : f \text{ is increasing}\}$. Then A is equal to

(a) $(-5, \infty)$ (b) $(-\infty, -5) \cup (-4, \infty)$

(c) $(-5, -4) \cup (4, \infty)$ (d) $(-\infty, -5) \cup (4, \infty)$

79. Let $a, b \in R$. If the mirror image of the point $P(a, 6, 9)$ with respect to the line $\frac{x-3}{7}=\frac{y-2}{5}=\frac{z-1}{-9}$ is $(20, b, -a-9)$, then $|a+b|$ is equal to

(a) 88 (b) 86 (c) 90 (d) 84

80. The value of the integral, $\int_1^3 [x^2-2x-2]dx$, where $[x]$ denotes the greatest integer less than or equal to x, is

(a) -5 (b) -4

(c) $-\sqrt{2}-\sqrt{3}+1$ (d) $-\sqrt{2}-\sqrt{3}-1$

SECTION-B (NUMERICAL VALUE TYPE)

Attempt any 5 questions out of 10.

81. Let $i=\sqrt{-1}$. If $\frac{(-1+i\sqrt{3})^{21}}{(1-i)^{24}}+\frac{(1+i\sqrt{3})^{21}}{(1+i)^{24}}=k$, and $n = [|k|]$ be the greatest integral part of $|k|$.

Then $\sum_{j=0}^{n+5}(j+5)^2-\sum_{j=0}^{n+5}(j+5)$ is equal to ________.

82. If the variance of 10 natural numbers 1, 1, 1, ..., 1, k is less than 10, then the maximum possible value of k is ________.

83. The sum of first four terms of a geometric progression (G.P.) is $\frac{65}{12}$ and the sum of their respective reciprocals is $\frac{65}{18}$. If the product of first three terms of the G.P. is 1, and the third term is α, then 2α is ________.

84. If the area of the triangle formed by the positive x-axis, the normal and the tangent to the circle $(x-2)^2 + (y-3)^2 = 25$ at the point (5, 7) is A, then $24A$ is equal to ________.

85. The students $S_1, S_2, ..., S_{10}$ are to be divided into 3 groups A, B and C such that each group has at least one student and the group C has at most 3 students. Then the total number of possibilities of forming such groups is ________.

86. The number of the real roots of the equation $(x+1)^2+|x-5|=\frac{27}{4}$ is ________.

87. If $a+\alpha=1, b+\beta=2$ and $af(x)+\alpha f\left(\frac{1}{x}\right)=bx+\frac{\beta}{x}, x\neq 0$, then the value of the expression $\frac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}}$ is ________.

88. Let a point P be such that its distance from the point (5, 0) is thrice the distance of P from the point (–5, 0). If the locus of the point P is a circle of radius r, then $4r^2$ is equal to ________.

89. For integers n and r, let $\binom{n}{r}=\begin{cases}{}^nC_r, & \text{if } n\ge r\ge 0\\ 0, & \text{otherwise}\end{cases}$

The maximum value of k for which the sum

$$\sum_{i=0}^{k}\binom{10}{i}\binom{15}{k-i}+\sum_{i=0}^{k+1}\binom{12}{i}\binom{13}{k+1-i}$$

exists, is equal to ________.

90. Let λ be an integer. If the shortest distance between the lines $x - \lambda = 2y - 1 = -2z$ and $x = y + 2\lambda = z - \lambda$ is $\frac{\sqrt{7}}{2\sqrt{2}}$, then the value of $|\lambda|$ is ________.

HINTS & EXPLANATIONS

1. **(c) :** de Broglie wavelength of a particle is given by

$$\lambda=\frac{h}{mv}$$

where, m = mass of the particle
v = speed of the particle
h = Planck's constant

$\therefore$ de Broglie wavelength of proton

$$\lambda_p=\frac{h}{m_pv_p}$$

de Broglie wavelength of α-particle

$$\lambda_\alpha=\frac{h}{m_\alpha v_\alpha}$$

$\therefore \quad \dfrac{\lambda_p}{\lambda_\alpha} = \dfrac{h}{m_p v_p} \times \dfrac{m_\alpha v_\alpha}{h} = \left(\dfrac{m_\alpha}{m_p}\right)\left(\dfrac{v_\alpha}{v_p}\right)$

$\because \quad m_\alpha = 4m_p$

$\therefore \quad \dfrac{\lambda_p}{\lambda_\alpha} = 4\left(\dfrac{v_\alpha}{v_p}\right)$

Given, $\lambda_p = \lambda_\alpha$

$\Rightarrow \quad \dfrac{v_p}{v_\alpha} = \dfrac{4}{1} = 4:1$

2. (c) : Time period of oscillation of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$\Rightarrow T^2 = 4\pi^2 \times \frac{L}{g} \Rightarrow g = \frac{4\pi^2 L}{T^2} \Rightarrow \frac{\Delta g}{g} = \frac{\Delta L}{L} + \frac{2\Delta T}{T}$$

Given, $\dfrac{\Delta L}{L} = \dfrac{1\times 10^{-3}}{1} a \quad \dfrac{\Delta T}{T} = \dfrac{0.01}{1.95} = \dfrac{1}{195}$

$$\therefore \frac{\Delta g}{g} = 10^{-3} + 2\times\left(\frac{1}{195}\right) = 11.25 \times 10^{-3}$$

$$\Rightarrow \frac{\Delta g}{g} \times 100\% = 11.25 \times 10^{-3} \times 100\%$$

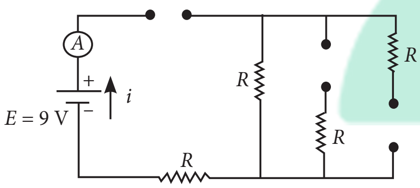
$= 1.125\% = 1.13\%$

3. (c) : Given,

$R = 2.0\ \Omega$, $L = 2.0$ mH $= 2 \times 10^{-3}$ H

$E = 9$ V

When the switch is closed at $t = 0$, the inductors will offer infinite resistance (behaves as open circuit). Therefore, the circuit can be redrawn as follows :



Current through the circuit is

$$i = \frac{E}{R_{\text{total}}} = \frac{E}{R+R} = \frac{9}{2R} = \frac{9}{2\times 2} = \frac{9}{4} = 2.25 \text{ A}$$

4. (d) : Given,

Voltage of X-ray tube $V_0 = 1.24$ million volt $= 1.24 \times 10^6$ V

Shortest wavelength produced is

$$\lambda_{\min} = \frac{hc}{eV_0} = \frac{1240}{V_0} \text{ nm} = \frac{1240}{1.24\times 10^6} \text{ nm} = 10^{-3} \text{ nm}$$

5. (a) : Velocity of particle executing SHM is given by

$$v = \omega A\sqrt{1 - \frac{x^2}{A^2}}$$

where, x = displacement

A = amplitude

ω = angular frequency

$$\Rightarrow v^2 = \omega^2 A^2\left(1 - \frac{x^2}{A^2}\right) \Rightarrow \frac{v^2}{\omega^2 A^2} + \frac{x^2}{A^2} = 1$$

This represents an ellipse.

6. (b) : *PN* junction diodes cannot be used a transistor by simply connecting two diodes back to back. Because the base is very thin as compared to emitter and collector.

The amplification factor is given by

$$\beta = \frac{I_c}{I_b} = \frac{\text{Collector current}}{\text{Base current}}$$

Therefore, statement I is false and statement II is true.

7. (d) : Work done by the gas is given by

$W = W_{AB} + W_{BC} + W_{CA}$

It is given that, AB is isothermal process

$$\therefore W_{AB} = nRT\ln\left(\frac{V_B}{V_A}\right) = 1\times R\times T\times \ln\left(\frac{2V_1}{V_1}\right)$$

$= RT\ln(2)$

Also, the process BC is isochoric

$\therefore \quad W_{BC} = 0$

The process CA is adiabatic

$$\therefore W_{CA} = \frac{P_1V_1 - P_2V_2}{\gamma - 1} = \frac{P_CV_C - P_AV_A}{\gamma - 1}$$

$$= \frac{\frac{P_1}{4}\times 2V_1 - P_1\times V_1}{\gamma - 1} = \frac{-P_1V_1/2}{\gamma-1} = \frac{-nRT}{2(\gamma-1)} = \frac{RT}{2(1-\gamma)}$$

$$\therefore W = RT\ln(2) + 0 + \frac{RT}{2(1-\gamma)}$$

$$= RT\ln(2) - \frac{RT}{2(\gamma-1)} = RT\left(\ln(2) - \frac{1}{2(\gamma-1)}\right)$$

8. (c) : Given two springs are connected in parallel. Therefore, equivalent spring constant is given by

$k_{eq} = k_1 + k_2 = k + k = 2k$

Time period oscillation of the body is

$$T = 2\pi\sqrt{\frac{M}{k_{eq}}} = 2\pi\sqrt{\frac{M}{2k}}$$

$\therefore$ Frequency of oscillation, $f = \dfrac{1}{T} = \dfrac{1}{2\pi}\sqrt{\dfrac{2k}{M}}$

9. (c) : Energy for transition from n_1 to n_2 is

$$\Delta E = E_2 - E_1 = 13.6\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)\text{eV}$$

$\Delta E = h\upsilon \Rightarrow \Delta E \propto \upsilon$

Therefore, more is ΔE more will be frequency (υ) of transition.

It is clear that ΔE is maximum for

$$n_1 = 1 \text{ and } n_2 = 2$$

Thus frequency will be maximum for this transition.

10. (d) : The fringe width in *YDSE* is given by

$$\beta = \left(\frac{D}{d}\right)\lambda$$

Where D = distance of screen from slits.

d = separation between the slits.

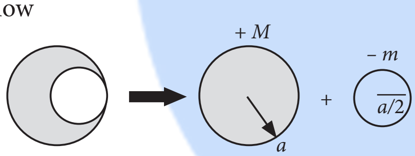
Cleary $\beta \propto \lambda$

Also, $\lambda_R > \lambda_V$

$\therefore \quad \beta_R > \beta_V$

Therefore, if light source is changed from red to violet consecutive fringe lines will come closer.

11. (d) : The given disc can be assumed as a combination of a complete disc of positive mass and a negative mass as shown below



Let us assume that mass per unit area is

$$\sigma = \frac{M}{\pi(a)^2}$$

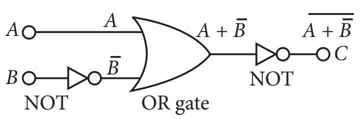
$$\therefore \quad m = \sigma \times \pi\left(\frac{a}{2}\right)^2 = \frac{\sigma\pi a^2}{4} = \frac{M}{4}$$

$\therefore$ Centre of mass of the remaining portion

$$X_{cm} = \frac{MX_1 - mX_2}{M - m} = \frac{M \times a - \frac{M}{4} \times \left(\frac{3a}{2}\right)}{M - \frac{M}{4}}$$

$$\therefore \quad X_{cm} = \frac{M\left(a - \frac{3a}{8}\right)}{M\left(1 - \frac{1}{4}\right)} = \frac{5a/8}{3/4} = \frac{5a}{6}$$

12. (c) : Given logic circuit with outputs is

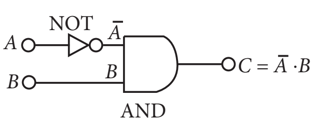


Therefore, from above circuit, net output

$$C = \overline{A + \overline{B}} = \overline{A} \cdot \overline{\overline{B}} \quad \text{(De Morgan's theorem)}$$

$$\Rightarrow \quad C = \overline{A} \cdot B$$

Now consider output of the circuit given in option (c).



The output is same, therefore this circuit is equivalent to the given circuit.

13. (a) : According to the assumption of kinetic theory of gases, the gas molecules do not exert any force either on each other or on the walls of the container. The pressure of the gas results from the force developed due to change in momentum molecules, colliding elastically with the walls.

14. (c) : Soft ferromagnetic materials can easily magnetised and demagnetised. These materials, when placed in an external magnetic field, experiences net torque which can change the orientation. Also, when the domains are aligned along the magnetic field, the size will increase and when they are aligned opposite to the field the size will decrease.

15. (b) : We know that,

Weight of a body, $W = mg$

At pole, $g_p = g$

At equator, $g_e = g - \omega^2 R < g_p$

Here, R = radius of the earth

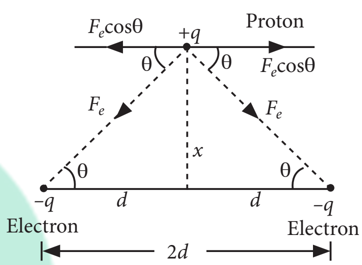
ω = angular velocity

$$\therefore \quad \frac{W_e}{W_p} = \frac{mg_e}{mg_p} = \frac{g_e}{g_p} < 1$$

$\Rightarrow \quad W_e < W_p = 49$ N

From the given option $W_e = 48.83$ N

16. (c) : Let magnitude of charge on electron and proton be q. Now, consider the arrangement of charges shown in the figure.



Let force exerted by each electron on proton is F_e.

Net force on the proton along α-direction

$F_x = F_e\cos\theta - F_e\cos\theta = 0$

Net force on the proton along Y-direction

$F_y = F_e\sin\theta + F_e\sin\theta$

$$= 2F_e \sin\theta \;= 2F_e \frac{x}{\sqrt{x^2 + d^2}}$$

$$\text{Also, } F_e = \frac{Kq^2}{\left(\sqrt{x^2 + d^2}\right)} = \frac{Kq^2}{x^2 + d^2}$$

Since, $x << \text{d}$

$\therefore \quad x^2 + d^2 \approx d^2$

$$\therefore \quad F_e = \frac{Kq^2}{d^2} \text{ and } F_y \approx 2F_e \frac{x}{d} = \frac{2Kq^2}{d^3} x$$

$$\Rightarrow \quad F_y = \frac{2}{d^3} \times \frac{1}{4\pi\varepsilon_0} q^2 x = \frac{q^2}{2\pi\varepsilon_0 d^3} x$$

As F_y and x are in opposite directions

$$\therefore \quad F_y = \frac{q^2}{2\pi\varepsilon_0 d^3}(-x) \quad \Rightarrow F_y \propto (-x)$$

Here, proportionality constant

$$K = \frac{q^2}{2\pi\varepsilon_0 d^3}$$

$$\therefore \quad \text{Net force } F = F_y = \frac{q^2}{2\pi\varepsilon_0 d^3}(-x)$$

$$\Rightarrow \quad F = -Kx$$

where constant $K = \dfrac{q^2}{2\pi\varepsilon_0 d^3}$

Also, $F \propto (-x)$

Therefore, the motion will be SHM.

Angular frequency of SHM will be

$$\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{q^2}{2\pi\varepsilon_0 m d^3}}$$

17. (b) : We know that, a travelling wave can be represented as $Y = F(x, t)$

Here the function F, should be such that Y is a linear function of x and t.

From the given options the linear function of x and t is

$Y = A\sin(15x - 2t)$

Other equations are not linear function of x and t.

18. (c) : Magnetron is the source of microwave frequency. Vibration of atoms and molecules produces infrared waves. Due to transition of inner shell electrons X-ray (K_α and K_β) are produced.

Radioactive decay of a nucleus produces α, β and γ rays.

19. (a) : In zener diode the P-type and N-type materials are heavily doped. When a high electric field is applied across the junction, the zener breakdown occurs. Also the depletion layer will be narrow.

20. (d) : For acting on the particle is

$F = -\alpha x^2$

Let the particle stops at a distance x_0.

Work done by the force, during the movement of the particle from $x = 0$ to $x = x_0$ is

$$W = \int_0^{x_0} F dx = \int_0^{x_0} -\alpha x^2 dx = \left(-\alpha \frac{x^3}{3}\right)_0^{x_0} = -\frac{\alpha x_0^3}{3}$$

From Work-Energy theorem

$\Delta W = \Delta K \quad \Rightarrow \quad W = K_f - K_i$

$$\Rightarrow \quad \frac{-\alpha x_0^3}{3} = 0 - \frac{1}{2} m v_0^2 \qquad \Rightarrow \quad v_0^2 = \frac{2\alpha x_0^3}{3m}$$

$$\Rightarrow \quad x_0^3 = \frac{3mv_0^2}{2\alpha} \quad \Rightarrow \quad x_0 = \left(\frac{3mv_0^2}{2\alpha}\right)^{\frac{1}{3}}$$

The closest option is (d).

21. (2) : Given that

Momentum of A = Momentum of B

$\Rightarrow \quad P_A = P_B$

$$\Rightarrow \quad \sqrt{2m_A K_A} = \sqrt{2m_B K_B} \qquad \left(\because K = \frac{p^2}{2m}\right)$$

$\Rightarrow \quad 2m_A K_A = 2m_B K_B$

$$\Rightarrow \quad \frac{(K.E)_A}{(K.E)_B} = \frac{m_B}{m_A} = \frac{2}{1} = \frac{A}{1}$$

$\Rightarrow \quad A = 2$

22. (900) : At resonance, $Z = R$

$$\text{Power} = P = \frac{V^2}{Z} = \frac{V^2}{R}$$

$$\Rightarrow \quad 16 = \frac{120 \times 120}{R}$$

$$\Rightarrow \quad R = \frac{12 \times 12 \times 100}{16} = 900\ \Omega$$

23. (8) : Given,

Input power = P_i = 0.1 kW

$\because$ Attenuation per km = –5 dB

$\therefore$ Attenuation in 10 km = –20 × 5 = –100 dB

According to the question

$$-100 = 10\log_{10}\left(\frac{P_o}{P_i}\right)$$

$$\Rightarrow \quad -10 = \log_{10}\left(\frac{P_o}{P_i}\right) \quad \Rightarrow \quad \frac{P_o}{P_i} = 10^{-10}$$

$\Rightarrow \quad P_o = P_i \times 10^{-10} = 0.1 \times 1000 \times 10^{-10}$

$= 100 \times 10^{-10} = 10^{-8}$ W

According to the question

$P_o = 10^{-x} = 10^{-8}$

$\Rightarrow \quad x = 8$

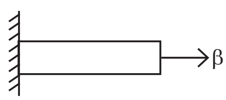
24. (2) : Consider the metallic wire on which a force F is applied

Now, we can write

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F/A}{\Delta l/l} = \frac{F}{A} \times \frac{l}{\Delta l}$$

Now,

$$\frac{F_1}{A_1} \times \frac{l_1}{\Delta l_1} = \frac{F_2}{A_2} \times \frac{l_2}{\Delta l_2}$$



$$\Rightarrow \quad \frac{\Delta l_2}{\Delta l_1} = \left(\frac{A_1}{A_2}\right) \times \frac{F_2}{F_1} \times \frac{l_2}{l_1}$$

$$= \left(\frac{r_1^2}{r_2^2}\right) \times \left(\frac{F}{F}\right) \times \left(\frac{2l_1}{l_1}\right) = \left(\frac{r_1}{2r_1}\right)^2 \times 1 \times 2 = \frac{1}{4} \times 2 = \frac{1}{2}$$

$$\Rightarrow \quad \Delta l_2 = \frac{\Delta l_1}{2} = \frac{0.04}{2} = 0.02 \text{ m} \; = 2 \text{ cm}$$

25. (400) : The rms of a gas is

$$v_{rms} = \sqrt{\frac{3RT}{M}} \Rightarrow v_{rms} \propto \sqrt{T}$$

$$\Rightarrow \frac{(v_{rms})_2}{(v_{rms})_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{273+127}{273+27}} = \sqrt{\frac{400}{300}} = \frac{2}{\sqrt{3}}$$

According to the question

$$\frac{(v_{rms})_2}{(v_{rms})_1} = \frac{2}{\sqrt{3}}$$

$$\Rightarrow (v_{rms})_2 = \frac{2}{\sqrt{3}} \times 200 = \frac{400}{\sqrt{3}} = \frac{x}{\sqrt{3}} \qquad \text{(Given)}$$

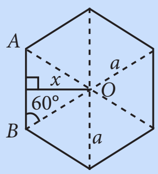
$$\Rightarrow x = 400$$

26. (8) : Let length of each side be a.

According to the question

$$6a = 2.4 \Rightarrow a = 0.4 \text{ m}$$

Consider the triangle OAB.



$$\sin 60° = \frac{x}{a}$$

$$\Rightarrow x = a\sin 60° = \frac{\sqrt{3}}{2}$$

Moment of inertia of the hexagon about an axis passing through the centre is

$$I_0 = 6(I_D + mx^2) \qquad (m = 1 \text{ kg})$$

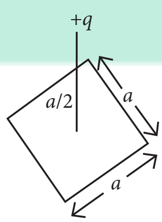
$$= 6\left[\frac{ma^2}{12} + m\left(\frac{\sqrt{3}}{2}a\right)^2\right] = 6ma^2\left[\frac{1}{12} + \frac{3}{4}\right]$$

$$= 6m \times (0.4)^2\left[\frac{1}{12} + \frac{3}{4}\right] = 6 \times 1 \times (0.4)^2\left[\frac{1+9}{12}\right]$$

$$= 16 \times 10^{-2} \times 6 \times \frac{10}{12}$$

$$= 80 \times 10^{-2} = 8 \times 10^{-1} \text{ kg m}^2$$

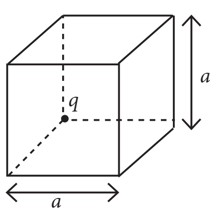
27. (226) : To find the electric flux through the square, let us assume a symmetric situation, in which the charge $+q$ is placed at the centre of a cube made by such surfaces.



From Gauss's law flux through the cube

$$\phi_c = \frac{q}{\varepsilon_0}$$

Let flux through each surface be ϕ.



$$\therefore \quad 6\phi = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \quad \phi = \frac{1}{6}\left(\frac{q}{\varepsilon_0}\right) = \frac{1}{6} \times \frac{12 \times 10^{-6}}{8.85 \times 10^{-12}}$$

$$= 225.988 \times 10^3 \text{ Nm}^2/\text{C}$$

$$= 226 \times 10^3 \text{ Nm}^2/\text{C}$$

28. (667) : Given, frequency of the EM wave

$D = 3 \text{ GHZ} = 3 \times 10^9$ Hz

Relative permittivity, $\varepsilon_r = 2.25$

Now, refractive index of the medium

$$n = \sqrt{\mu_r \varepsilon_r} = \sqrt{1 \times 2.25} = 1.5$$

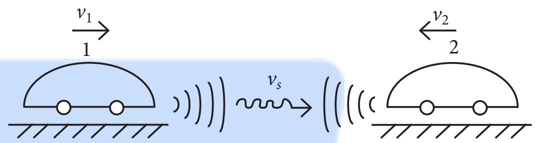
Here relative permeability, $\mu_r = 1$

$\therefore$ Wavelength of the wave in the medium,

$$\lambda_m = \frac{\lambda_{\text{vacuum}}}{n} = \frac{c/\upsilon}{1.5}$$

$$= \frac{(3 \times 10^8 / 3 \times 10^9)}{1.5} = \frac{0.1}{1.5} = \frac{1}{15} \text{ m} \approx 667 \times 10^{-2} \text{cm}$$

29. (8) : Consider the situation shown below



$$\text{Speed of each car} = \frac{7.2 \times 1000}{60 \times 60} = 2 \text{ m/s}$$

Apparent frequency heared by the driver in car-1

$$f_1 = \left(\frac{v_s + v_1}{v_s - v_2}\right) f_2$$

$$= \left(\frac{340+2}{340-2}\right) \times 676 = \frac{342}{338} \times 676 = 684 \text{ Hz}$$

The driver in car-1 will receive two frequencies one is f_1 and other the original frequency of horn of car-1, $f_0 = 676$ Hz

$\therefore$ Beat frequency $= \Delta f = f_1 - f_0 = 684 - 676 = 8$ Hz

Same beat frequency will be heared by the driver of car-2.

30. (5) : Current density is given by

$$J = \frac{I}{A} = \sigma E$$

Here, E = Electric field $= 10 \times 10^{-3}$ V/m

σ = Conductivity $= 5 \times 10^7$ S/m

$\Rightarrow \quad I = A(\sigma E)$

$= \pi r^2 (5 \times 10^7 \times 10 \times 10^{-3})$

$= \pi(0.5 \times 10^{-3})^2 (50 \times 10^4)$

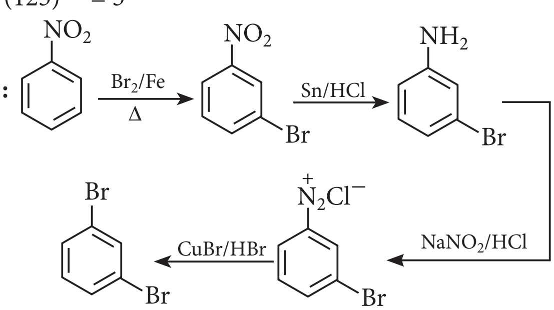
$= \pi \times 25 \times 10^{-8} \times 5 \times 10^5$

$= \pi \times 125 \times 10^{-3}$ A

$\Rightarrow \quad I = 125\pi \text{ mA} = x^3 \pi\text{m A}$

$\Rightarrow \quad x = (125)^{1/3} = 5$

31. (b) :



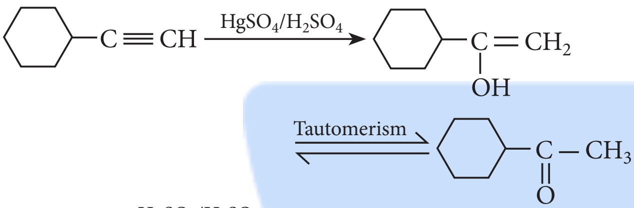
m-dibromobenzene

32. (d) : Clean water would have BOD value less than 5 ppm whereas highly polluted water could have a BOD value of 17 ppm or more.

33. (b) : $CH_3-C\equiv C-CH_3 \xrightarrow{HgSO_4/H_2SO_4}$

$$\underset{\displaystyle OH}{CH_3-\overset{|}{C}=CH-CH_3} \xrightleftharpoons{\text{Tautomerises}} CH_3-\underset{\displaystyle O}{\overset{\|}{C}}-CH_2-CH_3$$

$$CH_3-C\equiv CH \xrightarrow{HgSO_4/H_2SO_4} CH_3-\underset{\displaystyle OH}{\overset{|}{C}}=CH_2 \xrightleftharpoons{\text{Tautomerism}} CH_3-\underset{\displaystyle O}{\overset{\|}{C}}-CH_3$$



$$HC\equiv CH \xrightarrow{HgSO_4/H_2SO_4} CH_2=CH-OH \xrightleftharpoons{\text{Tautomerism}} CH_3-\underset{\displaystyle O}{\overset{\|}{C}}-H$$

Hence, CH_3CH_2CHO can not be prepared.

34. (a) : $[FeCl_4]^{2-}$ represent Fe^{2+}.
Cl^- is a weak field ligand.

$\therefore\ Fe^{2+}: [Ar]3d^6: \boxed{\uparrow\downarrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}$

Hence, 4 unpaired electrons.

$\mu = \sqrt{n(n+2)} = \sqrt{4(6)} = 4.90$

$[Co(C_2O_4)_3]^{3-}$ represents Co^{3+}

$C_2O_4^{2-}$ is a strong field ligand.

$\therefore\ Co^{3+}: [Ar]\ 3d^6: \boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\uparrow\downarrow}\boxed{\ }\boxed{\ }$

Number of unpaired electrons = 0 $\Rightarrow \mu = 0$

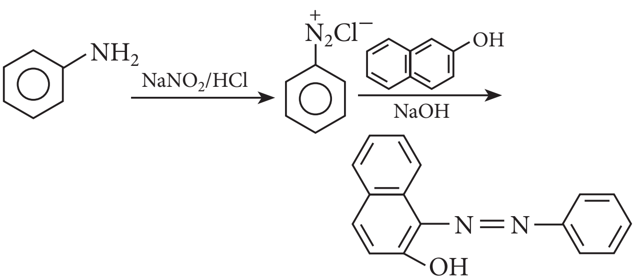
MnO_4^{2-} represents Mn^{6+}.

$Mn^{6+}: [Ar]3d^1: \boxed{\uparrow}\boxed{\ }\boxed{\ }\boxed{\ }\boxed{\ }$

Number of unpaired electrons = 1

$\Rightarrow \mu = \sqrt{1(1+2)} = \sqrt{3} = 1.73$

35. (a) : Benzene diazonium salt, when coupled with β-naphthol gives an azodye of red colour.



red coloured dye

36. (d) : Valium is used to treat anxiety hence it is a tranquilizer. (A-iv)
Morphine is an example of narcotic analgesics. (B-iii)
Norethindrone is used to prevent pregnancy *i.e.*, antifertility drug. (C-i)
Vitamin B_{12} is used to cure pernicious anaemia. (D-ii)

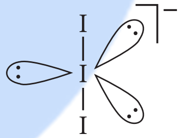
37. (c) : $R-\overset{O}{\overset{\|}{C}}-Cl \xrightarrow{H_2/Pd\text{-}BaSO_4} R-\overset{O}{\overset{\|}{C}}-H$ (A-ii)
(Rosenmund reduction)

$R-CH_2-COOH \xrightarrow{Cl_2/\text{Red P}, H_2O} R-\underset{\displaystyle Cl}{\underset{|}{CH}}-COOH$ (B-iv)
(Hell-Volhard-Zelinsky reaction)

$R-\overset{O}{\overset{\|}{C}}-NH_2 \xrightarrow{Br_2/NaOH} R-NH_2$ (C-i)
(Hoffmann bromamide degradation reaction)

$R-\overset{O}{\overset{\|}{C}}-CH_3 \xrightarrow{Zn(Hg)/\text{Conc. HCl}} RCH_2CH_3$ (D-iii)
(Clemmensen reduction)

38. (a) : I_3^- ion is linear in shape with sp^3d hybridisation and 180° bond angle.



39. (a) : K.E. $\propto \dfrac{Z^2}{n^2}$ (here *A* is correct)

$v = \dfrac{KZ}{n} \Rightarrow nv \propto Z$ (here *B* is incorrect)

Frequency of revolution $= \dfrac{v_n}{2\pi r_n}$

$v_n \propto \dfrac{Z}{n}$

$r_n \propto \dfrac{n^2}{Z}$

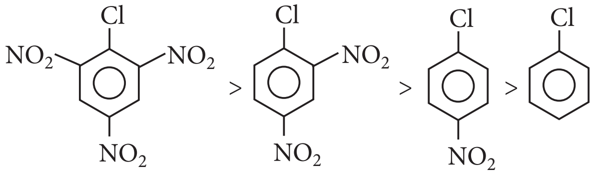
Frequency $\propto \dfrac{Z}{n(n^2/Z)} \propto \dfrac{Z^2}{n^3}$ (here *C* is incorrect)

Force $\propto \dfrac{Z}{r_n^2} \propto \dfrac{Z^3}{n^4}$ (here *D* is correct)

40. (b) : Kaolinite is $Al_2O_3(SiO_2)_2{\cdot}2H_2O$ (A-iii)
Siderite is $FeCO_3$ (B-i)
Malachite is $CuCO_3{\cdot}Cu(OH)_2$ (C-iv)
Calamine is $ZnCO_3$ (D-ii)

41. (a) : According to Hardy Schulze's rule, the cation with high charge has high coagulation power. $FeCl_3$ is an electrolyte which gives Fe^{3+} and blood contain negatively colloid. So, $FeCl_3$ coagulates blood.

42. (c) : Electron withdrawing groups such as $-NO_2$, $-CN$ etc. at *ortho* and *para* positions increase the reactivity of haloarene towards nucleophilic substitution reactions. Hence, the correct order is



(iv) (iii) (ii) (i)

43. (c) : Compound M.P.

Compound	M.P.
LiCl	605°C
LiF	848.2°C
MgO	2852°C
NaCl	801°C

Hence, correct order is LiF > LiCl ; MgO > NaCl

44. (b) : Cycloheptatriene is non-aromatic as it is a non-conjugated system.

45. (a) : Hydrogen is the most abundant element in the universe since it is the lightest element. Nitrogen is the most abundant gas in troposphere.

46. (b) : The decrease in metallic radius coupled with increase in atomic mass results in a general increase in the density of transition elements from titanium to copper.

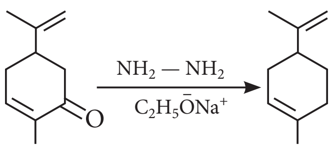
Hence, the correct order is

Zn	< Cr	< Fe	< Co	< Cu	
7.1	7.19	7.8	8.7	8.9	(Density in g/cm^3)

47. (b) :

	Li	Na	K	Rb	Cs
Colour	Crimson Red	Yellow	Violet	Red violet	Blue
λ/nm	670.8	589.2	766.5	780.0	455.5

48. (d) : This is an example of Wolff-Kishner reduction.



This reduction does not affect double-bond, only reduces $\rangle C=O$ to $\rangle CH_2$ group.

49. (a) : Ruby is the mineral also called as red corundum which has the traces of chromium element which causes the red colour in the ruby.

50. (c) : Buna-S is a polymer of styrene-butadiene. Hence, S stand for styrene.

51. (5) : $T = 50 + 273 = 323$ K

$P = 740$ mm Hg $= \dfrac{740}{760}$ atm

$R = 0.0826$ L atm K^{-1} mol^{-1}

$n = \dfrac{4.75}{26}$ (Molar mass of acetylene $(C_2H_2) = 26$ g/mol)

$= 0.1826$

$PV = nRT \Rightarrow V = \dfrac{nRT}{P}$

$$V = \frac{0.1826 \text{ mol} \times 0.0826 \text{ L atm K}^{-1}\text{ mol}^{-1} \times 323 \text{ K}}{(740/760)\text{ atm}} = 5 \text{ L}$$

52. (141) : $K_{sp} = 8.0 \times 10^{-9}$

$s = x \times 10^{-6}$ mol/L

$$\underset{}{PbI_2} \rightleftharpoons \underset{(s+0.1)}{Pb^{2+}} + \underset{(2s)}{2I^-}$$

$(s + 0.1)(2s)^2 = 8.0 \times 10^{-9}$

as $0.1 >>> s$ Hence, $s + 0.1 = 0.1$

$(0.1)\, 4s^2 = 8 \times 10^{-9}$

$$s^2 = \frac{8 \times 10^{-9}}{0.4} = 2 \times 10^{-8}$$

$s = (2 \times 10^{-8})^{1/2}$

$= 1.41 \times 10^{-4}$ mol/L or 141×10^{-6} mol/L

53. (3776) : Balanced equation is

$$MnO_4^- + 8H^+ + 5e^- \longrightarrow Mn^{2+} + 4H_2O$$

Hence, according to Nernst equation,

$$E = E^\circ - \frac{0.0591}{5}\log\frac{[Mn^{2+}]}{[MnO_4^-][H^+]^8}$$

Case-I, When $[H^+] = 1$ M

$$E_1 = E^\circ - \frac{0.0591}{5}\log\frac{[Mn^{2+}]}{[MnO_4^-]} = E^\circ - 0 = E^\circ$$

Case-II, When $H^+ = 10^{-4}$ M

$$E_2 = E^\circ - \frac{0.0591}{5}\log\frac{[Mn^{2+}]}{[MnO_4^-][10^{-4}]^8} = E^\circ - \frac{0.0591}{5} \times 32$$

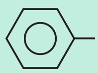
Change in electrode potential

$$E_2 - E_1 = -\frac{0.0591 \times 32}{5} = 0.3776$$

Hence x is 3776.

54. (3) : Gabriel pthalamide synthesis is used to prepare 1° aliphatic amines.

Hence,

 $-CH_2NH_2$, $\begin{matrix} CH_3 \\ CH_3 \end{matrix}\rangle CH-CH_2NH_2$ and $CH_3CH_2NH_2$

can be prepared but aniline cannot be prepared by this method because arylhalides do not undergo nucleophilic substitution with the phthalimide formed anion.

55. (1) : Sulphur exists in S_2 form in vapour phase which has two unpaired electrons while α-sulphur and β-sulphur are diamagnetic.

56. (855) : $3CH\equiv CH_{(g)} \rightleftharpoons C_6H_{6(l)}$

$\Delta G° = (\Delta G°)_P - (\Delta G°)_R$

$= (-1.24 \times 10^5)$J/mol $- (3 \times -2.04 \times 10^5)$ J/mol

$= -1.24 \times 10^5 + 6.12 \times 10^5$ J/mol

$= 4.88 \times 10^5$ J/mol

$\Delta G° = -RT\ln K$ or $-2.303RT\log K$

$4.88 \times 10^5 = -2.303 \times 8.314 \times 298 \log K$

$$\log K = -\frac{4.88 \times 10^5}{2.303 \times 8.314 \times 298}$$

$|\log K| = 85.5$ or 855×10^{-1}

57. (1) : $T_f(C_6H_6) = 5.5°C$

$C_4H_{10} = 10$ g $\quad C_6H_6 = 200$ g

$K_f = 5.12°C/m$

$\Delta T_f = iK_f m \qquad i = 1$ for C_4H_{10}

$$m = \frac{\text{Moles of solute}}{\text{Mass of solvent }(g)} \times 1000$$

$$m = \frac{10/58}{200} \times 1000 = 0.86$$

$\Delta T_f = 1 \times 5.12 \times 0.86 = 4.403$

$\Delta T_f = T_f^\circ - T'_f$

$T'_f = T_f^\circ - \Delta T_f = 5.5 - 4.40 = 1.1 \approx 1°C$

58. (81) : For first order reaction, $t_{1/2} = \dfrac{0.693}{k}$

So, $k = \dfrac{0.693}{t_{1/2}} = \dfrac{0.693}{3.33} h^{-1}$ (As $t_{1/2} = 3.33$ h)

$$k = \frac{2.303}{t} \log \frac{a}{a-x}$$

Given, $a - x = f \qquad a = 1$

$$k = \frac{2.303}{t} \log\left(\frac{1}{f}\right) \Rightarrow \log\left(\frac{1}{f}\right) = \frac{0.693}{3.33} \times \frac{9}{2.303}$$

$$\log\left(\frac{1}{f}\right) = 0.8132 = 81 \times 10^{-2}$$

59. (8) : Combustion reaction is given as,

$$C_xH_{y(g)} + \left(x + \frac{y}{4}\right)O_{2(g)} \longrightarrow xCO_{2(g)} + \frac{y}{2}H_2O_{(l)}$$

According to question,

Volume of $C_xH_y = V$

Volume of $O_2 = 6\ V = \left(x + \dfrac{y}{4}\right)$ ...(i)

Volume of $CO_2 = 4\ V = x$ ...(ii)

From (i) and (ii) we can say that $\dfrac{y}{4} = 2\ V$

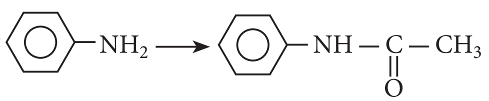
$y = 8\ V$

Ratio of $x : y = 4 : 8$

Hence, formula of C_xH_y is C_4H_8

$C_4H_{8(g)} + 6O_2 \longrightarrow 4CO_2 + 4H_2O$

60. (243) :



Moles of aniline $= \dfrac{1.86}{93} = 0.02$

Mass of acetalinilde = 0.02 moles = 0.02 × 135 g = 2.7 g

10% lost during purification *i.e.*, efficiency of reaction = 90%

Hence, obtained amount of product $= 2.7 \times \dfrac{90}{100} = 2.43$

$= 243 \times 10^{-2}$ g

61. (d) : The given differential equation is $\dfrac{dy}{dx} + \dfrac{y}{x} = bx^3$.

Here, I.F. $= e^{\int \frac{1}{x} dx} = e^{\log x} = x$

∴ Solution is given by

$$y \cdot x = \int bx^4 dx = \frac{bx^5}{5} + C \Rightarrow y = \frac{bx^4}{5} + \frac{C}{x}$$

Since $y = f(x)$ passes through (1, 2), therefore

$$2 = \frac{b}{5} + C \qquad \text{...(i)}$$

Now, $\displaystyle\int_1^2 f(x)\,dx = \int_1^2 \left(\frac{bx^4}{5} + \frac{C}{x}\right)dx = \frac{62}{5}$

$$\Rightarrow \left[\frac{bx^5}{25} + C\log x\right]_1^2 = \frac{62}{5}$$

$$\Rightarrow \frac{32b}{25} + C\log 2 - \frac{b}{25} = \frac{62}{5} \qquad \text{...(ii)}$$

Solving (i) and (ii), we get $C = 0$, $b = 10$

62. (b) : $D = \begin{vmatrix} 1 & -2 & 0 \\ 1 & -1 & k \\ 0 & k & 4 \end{vmatrix}$

$= 1(-4 - k^2) + 2(4) = 4 - k^2$

If $D \neq 0$ *i.e.*, $4 - k^2 \neq 0 \Rightarrow k^2 \neq 4$

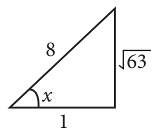
$\Rightarrow k \neq 2,\ k \neq -2$, then the system has unique solution.

At $k = 2$,

$$D_1 = \begin{vmatrix} 1 & -2 & 0 \\ -2 & -1 & 2 \\ 6 & 2 & 4 \end{vmatrix} = -48 \neq 0$$

∴ The system has no solution if $k = 2$.

63. (d) : Let $\sin^{-1}\dfrac{\sqrt{63}}{8} = x \Rightarrow \sin x = \dfrac{\sqrt{63}}{8}$

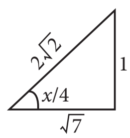


∴ $\cos x = \dfrac{1}{8}$ and $\tan x = \sqrt{63}$

Now, $2\cos^2\dfrac{x}{2} = 1 + \cos x = 1 + \dfrac{1}{8} = \dfrac{9}{8}$

$$\Rightarrow \cos\frac{x}{2} = \frac{3}{4}$$

Also, $2\cos^2\dfrac{x}{4} = 1 + \cos\dfrac{x}{2} = 1 + \dfrac{3}{4} = \dfrac{7}{4}$



$$\Rightarrow \cos\frac{x}{4} = \frac{\sqrt{7}}{2\sqrt{2}}$$

Now, $\tan\left(\dfrac{1}{4}\sin^{-1}\dfrac{\sqrt{63}}{8}\right) = \tan\left(\dfrac{x}{4}\right) = \dfrac{1}{\sqrt{7}}$

64. (b) : Given, $f'(x) = f'(2 - x)$

Integrating both sides, we get

$f(x) = -f(2 - x) + C$

At $x = 0$, $f(0) + f(2) = C$

$\Rightarrow \quad C = 1 + e^2$

$\therefore \quad f(x) + f(2-x) = 1 + e^2$

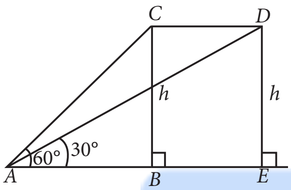
Now, let $I = \int_0^2 f(x)\,dx \Rightarrow I = \int_0^2 f(2-x)dx$

$\therefore \quad 2I = \int_0^2 [f(x) + f(2-x)]\,dx$

$\Rightarrow \quad 2I = (1+e^2)\int_0^2 dx \Rightarrow 2I = (1+e^2)(2-0)$

$\Rightarrow \quad I = 1 + e^2$

65. (c) :



Speed of jet plane $= 432 \times \dfrac{1000}{60 \times 60}$ m/sec $= 120$ m/sec

Distance CD = Speed × Time = 120 × 20 = 2400 m

In ΔACB, $\tan 60° = \dfrac{h}{AB} \Rightarrow AB = \dfrac{h}{\sqrt{3}}$

In ΔADE, $\tan 30° = \dfrac{h}{AE} \Rightarrow AE = \sqrt{3}h \Rightarrow \dfrac{h}{\sqrt{3}} + BE = \sqrt{3}h$

$\Rightarrow \quad \sqrt{3}h - \dfrac{h}{\sqrt{3}} = 2400 \quad [\because BE = CD]$

$\Rightarrow \quad 2h = 2400\sqrt{3} \Rightarrow h = 1200\sqrt{3}$ m

66. (d) : We have, $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix} = 0$

$\Rightarrow \quad f(x)f''(x) - (f'(x))^2 = 0$

$\Rightarrow \quad \dfrac{d}{dx}\left(\dfrac{f'(x)}{f(x)}\right) = 0 \quad [\because f'(x) \neq 0 \ \forall \ x \in R \Rightarrow f(x) \neq 0 \ \forall \in R]$

$\Rightarrow \quad \dfrac{f'(x)}{f(x)} = c$

At $x = 0$, $f'(0) = cf(0) \Rightarrow 2 = c$

$\therefore \quad \dfrac{f'(x)}{f(x)} = 2$

Integrating both sides, we get $\int \dfrac{f'(x)}{f(x)}dx = \int 2\,dx$

$\Rightarrow \quad \log|f(x)| = 2x + c_1$

At $x = 0$, $\log|f(0)| = 2(0) + c_1 \Rightarrow c_1 = 0$

$\therefore \quad \log|f(x)| = 2x \Rightarrow f(x) = e^{2x}$

$\therefore \quad f(1) = e^2 \in (6, 9)$

67. (d) : We have, $A^T = A$ and $B^T = -B$

Let $A^2B^2 - B^2A^2 = P$, then

$P^T = (A^2B^2 - B^2A^2)^T = (A^2B^2)^T - (B^2A^2)^T$

$= (B^2)^T(A^2)^T - (A^2)^T(B^2)^T = (B^T)^2(A^T)^2 - (A^T)^2(B^T)^2$

$= B^2A^2 - A^2B^2$

$\therefore \quad P^T = -P$

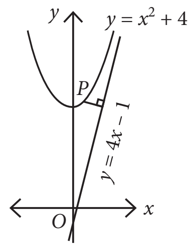
$\therefore \quad P$ is skew-symmetric matrix

$\therefore \quad |P| = 0 \quad$ [$\because$ Determinant of skew symmetic matrix of odd order is zero]

Hence, $PX = O$ have infinite solutions.

68. (c) : Let $P(x_1, y_1)$ be the required point.

$\therefore \quad y_1 = x_1^2 + 4 \quad$ ...(i)



Given, equation of line is $y = 4x - 1$

$\Rightarrow \quad y - 4x + 1 = 0$

$\therefore \quad$ Shortest distance from P

$= \left|\dfrac{y_1 - 4x_1 + 1}{\sqrt{17}}\right|$

$= \left|\dfrac{x_1^2 + 4 - 4x_1 + 1}{\sqrt{17}}\right|$ [Using (i)]

$= \left|\dfrac{(x_1-2)^2 + 1}{\sqrt{17}}\right|$, which will be minimum when $x_1 = 2$.

So, $y_1 = 2^2 + 4 = 8$

Hence, required point is (2, 8).

69. (a) : Equation of plane passing through the intersection of planes is $[\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k}) - 1] + \lambda[\vec{r}\cdot(\hat{i} - 2\hat{j}) + 2] = 0$

$\Rightarrow \quad \vec{r}\cdot[\hat{i}(1+\lambda) + \hat{j}(1-2\lambda) + \hat{k}] - 1 + 2\lambda = 0$

It passes through the point $\hat{i} + 0\hat{j} + 2\hat{k}$.

$\therefore \quad (\hat{i} + 2\hat{k})\cdot[\hat{i}(1+\lambda) + \hat{j}(1-2\lambda) + \hat{k}] - 1 + 2\lambda = 0$

$\Rightarrow \quad 1 + \lambda + 2 - 1 + 2\lambda = 0 \Rightarrow 3\lambda + 2 = 0 \Rightarrow \lambda = -\dfrac{2}{3}$

$\therefore \quad \vec{r}\cdot\left[\hat{i}\left(1 - \dfrac{2}{3}\right) + \hat{j}\left(1 + \dfrac{4}{3}\right) + \hat{k}\right] = 1 + \dfrac{4}{3}$

$\Rightarrow \quad \vec{r}\cdot[\hat{i} + 7\hat{j} + 3\hat{k}] = 7$

70. (c) : Since curve $y = ax^2 + bx + c$ is passing through (1, 2).

$\therefore \quad 2 = a + b + c \quad$ ...(i)

Also, $\dfrac{dy}{dx} = 2ax + b$

Since, $\left[\dfrac{dy}{dx}\right]_{(0,\,0)} = 1$ (given)

$\therefore \quad [2ax + b]_{(0,\,0)} = 1$

$\therefore \quad b = 1$

Also, curve passes through origin. $\therefore \quad c = 0$

Hence, $a = 1 \quad$ [From (i)]

71. (c) : Let A and B be any two subsets of given set.

Total number of possible outcomes $= 4 \times 4 \times 4 \times 4 \times 4 = 4^5$

[$\because$ Each element of given set has only four options only A, only B, Both A and B, None of A and B]

Number of favourable outcomes $= {}^5C_2 \times 3^3$

[$\because$ Each of rest three numbers has only three options only A, only B, None of A and B]

$\therefore$ Required probability $= \frac{{}^5C_2 \times 3^3}{4^5} = \frac{10 \times 27}{2^{10}} = \frac{135}{2^9}$

72. (c) : $\sim(\sim p \wedge (p \vee q))$
$\equiv \sim((\sim p \wedge p) \vee (\sim p \wedge q))$
$\equiv \sim(\sim p \wedge q)$
$\equiv p \vee \sim q$

73. (a) : Tangent to $x^2 + 9y^2 = 9$ at point $\left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)$ is
$x\left(\frac{3\sqrt{3}}{2}\right) + 9y\left(\frac{1}{2}\right) = 9$

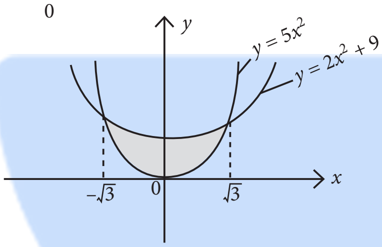
i.e., $x + \sqrt{3}y = 2\sqrt{3}$

74. (d) : Required area $= 2\int_0^{\sqrt{3}} (2x^2 + 9 - 5x^2)dx$

$= 2\left[9x - x^3\right]_0^{\sqrt{3}}$

$= 2\left[9\sqrt{3} - 3\sqrt{3}\right]$

$= 12\sqrt{3}$ sq. units



75. (a) : (A)

p	q	$\sim q$	$p \to q$	$\sim p$	$\sim q \wedge (p \to q)$	$(\sim q \wedge (p \to q)) \to \sim p$
T	T	F	T	F	F	T
T	F	T	F	F	F	T
F	T	F	T	T	F	T
F	F	T	T	T	T	T

(B)

p	q	$p \vee q$	$\sim p$	$(p \vee q) \wedge \sim p$	$((p \vee q) \wedge \sim p) \to q$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

Both (A) and (B) are tautologies.

76. (a) : Given, $\frac{a + a + 2}{3} = \frac{10}{3} \Rightarrow 2a + 2 = 10 \Rightarrow a = 4$

and $\frac{c + b + b}{3} = \frac{7}{3} \Rightarrow c + 2b = 7 \Rightarrow 2b = 7 - c$

Also, we have $2b = a + c \Rightarrow 2b = 4 + c$

$\therefore\ 7 - c = 4 + c \Rightarrow c = \frac{3}{2}$

$\Rightarrow b = \frac{7 - \frac{3}{2}}{2} = \frac{11}{4}$

$\therefore$ Given quadratic equation is $4x^2 + \frac{11}{4}x + 1 = 0$

Since α, β are roots of above equation.

$\therefore\ \alpha + \beta = -\frac{11}{16}$ and $\alpha\beta = \frac{1}{4}$

Now, $\alpha^2 + \beta^2 - \alpha\beta = (\alpha + \beta)^2 - 3\alpha\beta$

$= \left(\frac{-11}{16}\right)^2 - 3\left(\frac{1}{4}\right) = \frac{121}{256} - \frac{3}{4} = -\frac{71}{256}$

77. (c) : Clearly, ${}^{n+1}C_2 + 2({}^2C_2 + {}^3C_2 + {}^4C_2 + \ldots\ldots + {}^nC_2)$

$= {}^{n+1}C_2 + 2({}^3C_3 + {}^3C_2 + {}^4C_2 + \ldots\ldots + {}^nC_2)$

$= {}^{n+1}C_2 + 2({}^4C_3 + {}^4C_2 + {}^5C_2 + \ldots\ldots + {}^nC_2)$

$(\because\ {}^nC_{r+1} + {}^nC_r = {}^{n+1}C_{r+1})$

$= {}^{n+1}C_2 + 2({}^5C_3 + {}^5C_2 + \ldots\ldots + {}^nC_2)$

$= {}^{n+1}C_2 + 2({}^{n+1}C_3) = {}^{n+1}C_2 + {}^{n+1}C_3 + {}^{n+1}C_3$

$= {}^{n+2}C_3 + {}^{n+1}C_3$

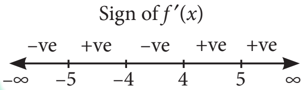
$= \frac{(n+2)!}{3!\,(n-1)!} + \frac{(n+1)!}{3!\,(n-2)!}$

$= \frac{(n+2)(n+1)(n)}{6} + \frac{(n+1)(n)(n-1)}{6}$

$= \frac{n(n+1)}{6}[2n+1]$

78. (c) : $f'(x) = \begin{cases} -55 & ; \text{ if } x < -5 \\ 6(x^2 - x - 20) & ; \text{ if } -5 < x < 4 \\ 6(x^2 - x - 6) & ; \text{ if } x > 4 \end{cases}$

$= \begin{cases} -55 & ; \text{ if } x < -5 \\ 6(x-5)(x+4) & ; \text{ if } -5 < x < 4 \\ 6(x-3)(x+2) & ; \text{ if } x > 4 \end{cases}$



$\therefore\ f(x)$ is increasing in $x \in (-5, -4) \cup (4, \infty)$

79. (a) : Here, $P \equiv (a, 6, 9)$ and $Q \equiv (20, b, -a - 9)$

$\therefore$ Mid point of PQ is $\left(\frac{a + 20}{2}, \frac{6 + b}{2}, \frac{9 - a - 9}{2}\right)$

It must lie on the line $\frac{x - 3}{7} = \frac{y - 2}{5} = \frac{z - 1}{-9}$.

i.e., $\frac{\frac{a + 20}{2} - 3}{7} = \frac{\frac{6 + b}{2} - 2}{5} = \frac{\frac{-a}{2} - 1}{-9}$

i.e., $\frac{a + 14}{14} = \frac{b + 2}{10} = \frac{a + 2}{18}$

$\therefore\ a = -56$ and $b = -32$

Hence, $|a + b| = 88$.

80. (d) : We have, $\int_1^3 [x^2 - 2x - 2]dx$

$$= \int_1^3 ([(x-1)^2]-3)\,dx$$

$$= \int_0^2 [t^2]dt - 3\int_1^3 dx \qquad \begin{bmatrix} \text{Using } x-1=t \Rightarrow dx=dt \\ \text{Also, at } x=1, t=0 \\ \text{at } x=3,\ t=2 \end{bmatrix}$$

$$= \int_0^1 0dt + \int_1^{\sqrt{2}} 1dt + \int_{\sqrt{2}}^{\sqrt{3}} 2dt + \int_{\sqrt{3}}^{2} 3dt - 6$$

$$= (\sqrt{2}-1)+2\sqrt{3}-2\sqrt{2}+6-3\sqrt{3}-6$$

$$= -1-\sqrt{2}-\sqrt{3}$$

81. (310) : We know that

$$\omega = \frac{-1+\sqrt{3}i}{2} \text{ and } \omega^2 = \frac{-1-\sqrt{3}i}{2}, \omega^3 = 1$$

Also $(1+i)^2 = 1+i^2+2i = 2i$ and $(1-i)^2 = 1+i^2-2i = -2i$

$$\therefore \quad k = \frac{(-1+i\sqrt{3})^{21}}{(1-i)^{24}} + \frac{(1+i\sqrt{3})^{21}}{(1+i)^{24}}$$

$$= \frac{(2\omega)^{21}}{(-2i)^{12}} + \frac{(-2\omega^2)^{21}}{(2i)^{12}} = \frac{2^{21}\omega^{21}}{2^{12}i^{12}} - \frac{2^{21}\omega^{42}}{2^{12}i^{12}} = 0$$

$[\because\ \omega^{21} = (\omega^3)^7 = 1 \text{ and } \omega^{42} = (\omega^3)^{14} = 1]$

$\therefore \quad n = [|k|] = 0$

Now, $\displaystyle\sum_{j=0}^{n+5} (j+5)^2 - \sum_{j=0}^{n+5} (j+5) = \sum_{j=0}^{5} (j^2+25+10j-j-5)$

$$= \sum_{j=0}^{5} (j^2+9j+20) = \sum_{j=0}^{5} j^2 + 9\sum_{j=0}^{5} j + 20\sum_{j=0}^{5} 1$$

$$= \frac{5\times6\times11}{6} + 9\left(\frac{5\times6}{2}\right) + 20\times6$$

$= 55 + 135 + 120 = 310$

82. (11) : Given, $\sigma^2 = \dfrac{\Sigma x^2}{n} - \left(\dfrac{\Sigma x}{n}\right)^2 < 10$

$$\Rightarrow \quad \frac{9+k^2}{10} - \left(\frac{9+k}{10}\right)^2 < 10$$

$\Rightarrow \quad 90 + 10k^2 - 81 - k^2 - 18k < 1000$

$\Rightarrow \quad 9k^2 - 18k + 9 < 1000 \ \Rightarrow\ 9(k-1)^2 < 1000$

$$\Rightarrow \quad (k-1)^2 < \frac{1000}{9} \quad \Rightarrow \quad k < \frac{10\sqrt{10}}{3} + 1 \approx 11.54$$

$\therefore$ Maximum integral value of k is 11.

83. (3) : Let the terms are a, ar, ar^2, ar^3.

$$\therefore \quad a\left(\frac{r^4-1}{r-1}\right) = \frac{65}{12} \qquad \text{...(i)}$$

Also, $\displaystyle\frac{1}{a}\left(\frac{\frac{1}{r^4}-1}{\frac{1}{r}-1}\right) = \frac{65}{18} \Rightarrow \frac{1}{a}\left(\frac{\frac{1-r^4}{r^4}}{\frac{1-r}{r}}\right) = \frac{65}{18}$

$$\Rightarrow \quad \frac{1}{ar^3}\left(\frac{1-r^4}{1-r}\right) = \frac{65}{18} \qquad \text{...(ii)}$$

Dividing (i) by (ii), we get $a^2r^3 = \dfrac{3}{2}$ ...(iii)

Also, we have $a^3r^3 = 1$, *i.e.*, $ar = 1$

Now, from (iii), we get

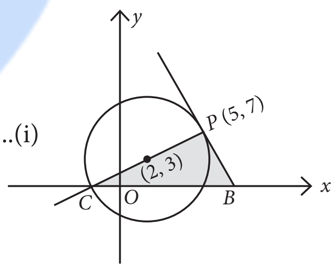
$$(a^2r^2)r = \frac{3}{2} \Rightarrow r = \frac{3}{2}$$

$$\therefore \quad a\left(\frac{3}{2}\right) = 1 \Rightarrow a = \frac{2}{3}$$

Now, third term, $ar^2 = \dfrac{2}{3}\left(\dfrac{3}{2}\right)^2 = \dfrac{3}{2} = \alpha$ (Given)

$\Rightarrow \quad 2\alpha = 3.$

84. (1225) : Equation of normal at P is



$$y-7 = \frac{7-3}{5-2}(x-5)$$

$\Rightarrow \quad 3y - 21 = 4x - 20$

$\Rightarrow \quad 4x - 3y + 1 = 0$ ...(i)

$$\therefore \quad C \equiv \left(-\frac{1}{4}, 0\right)$$

Equation of tangent at P is

$$y-7 = -\frac{3}{4}(x-5)$$

$\Rightarrow \quad 4y - 28 = -3x + 15$

$\Rightarrow \quad 3x + 4y = 43$ ...(ii)

$$\therefore \quad B \equiv \left(\frac{43}{3}, 0\right)$$

So, $BC = \sqrt{\left(\dfrac{43}{3} + \dfrac{1}{4}\right)^2} = \dfrac{175}{12}$

Hence, area of $\Delta PBC = \dfrac{1}{2} \times BC \times 7$

$$= \frac{1}{2} \times \frac{175}{12} \times 7 = \frac{1225}{24} = A \quad \text{(Given)}$$

$\therefore \quad 24A = 1225$

85. (31650) :

A	B	C
1	8	1
2	7	1
⋮	⋮	⋮
6	1	3

Number of ways to form groups

$$= ({}^{10}C_1)({}^9C_1 + {}^9C_2 + ... + {}^9C_8) + {}^{10}C_2({}^8C_1 + {}^8C_2 + ... + {}^8C_7) + {}^{10}C_3({}^7C_1 + {}^7C_2 + ... + {}^7C_6)$$

$$= 10(2^9 - 2) + 45(2^8 - 2) + 120(2^7 - 2)$$

$$= 10 \times 510 + 45 \times 254 + 120 \times 126$$

$$= 5100 + 11430 + 15120 = 31650$$

86. (2) : Case-I : When $x \le 5$

In this case, we have $(x+1)^2 - (x-5) = \frac{27}{4}$

$$\Rightarrow \quad (x+1)^2 - (x+1) - \frac{3}{4} = 0$$

$$\Rightarrow \quad x+1 = \frac{3}{2}, -\frac{1}{2} \quad \Rightarrow \quad x = \frac{1}{2}, \frac{-3}{2} \text{ which are real}$$

Case-II : When $x \ge 5$

In this case, we have, $(x+1)^2 + (x-5) = \frac{27}{4}$

$$\Rightarrow \quad (x+1)^2 + (x+1) - \frac{51}{4} = 0$$

$$\Rightarrow \quad x+1 = \frac{-1 \pm \sqrt{52}}{2}, \text{ which are not possible as } x \ge 5$$

$\therefore$ The equation has 2 real roots.

87. (2) : We have, $a + \alpha = 1$ and $b + \beta = 2$

Also, we have $af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}$ ...(i)

Interchanging x by $\frac{1}{x}$, we get

$$af\left(\frac{1}{x}\right) + \alpha f(x) = b\left(\frac{1}{x}\right) + \beta x \qquad ...(ii)$$

Adding (i) and (ii), we get

$$a\left(f(x) + f\left(\frac{1}{x}\right)\right) + \alpha\left(f\left(\frac{1}{x}\right) + f(x)\right) = b\left(x + \frac{1}{x}\right) + \beta\left(x + \frac{1}{x}\right)$$

$$\Rightarrow \quad \frac{f(x) + f\left(\frac{1}{x}\right)}{x + \frac{1}{x}} = \frac{b+\beta}{a+\alpha} = \frac{2}{1} = 2$$

88. (56.25) : Let the coordinates of point P be (h, k). Then,

$$\sqrt{(h-5)^2 + k^2} = 3\sqrt{(h+5)^2 + k^2}$$

$$\Rightarrow \quad h^2 + k^2 + 25 - 10h = 9(h^2 + k^2 + 25 + 10h)$$

$$\Rightarrow \quad 8h^2 + 8k^2 + 100h + 200 = 0 \quad \Rightarrow \quad h^2 + k^2 + \frac{25}{2}h + 25 = 0$$

Thus, the locus is $x^2 + y^2 + \frac{25}{2}x + 25 = 0$, which is a circle whose centre is $\left(-\frac{25}{4}, 0\right)$ and radius, $r = \sqrt{\left(\frac{-25}{4}\right)^2 + (0)^2 - 25}$

$$\Rightarrow \quad 4r^2 = \frac{(25)^2 - 400}{4} = \frac{225}{4} = 56.25$$

89. (24) : $\displaystyle\sum_{i=0}^{k}\binom{10}{i}\binom{15}{k-i} + \sum_{i=0}^{k+1}\binom{12}{i}\binom{13}{k+1-i}$

$$= \binom{10}{0}\binom{15}{k} + \binom{10}{1}\binom{15}{k-1} + ... + \binom{10}{k}\binom{15}{0}$$

$$+ \binom{12}{0}\binom{13}{k+1} + \binom{12}{1}\binom{13}{k} + ... + \binom{12}{k+1}\binom{13}{0}$$

$=$ Coeff. of x^k in $(1+x)^{10}(1+x)^{15}$ + Coeff. of x^{k+1} in $(1+x)^{12}(1+x)^{13}$

$=$ Coeff. of x^k in $(1+x)^{25}$ + Coeff. of x^{k+1} in $(1+x)^{25}$

$= {}^{25}C_k + {}^{25}C_{k+1}$

Now, for sum to be exists $0 \le k \le 25$ and $0 \le k+1 \le 25$.

$\Rightarrow \quad 0 \le k \le 24$

Thus, maximum value of k is 24.

90. (1) : The given lines are $\dfrac{x-\lambda}{1} = \dfrac{y-\frac{1}{2}}{\frac{1}{2}} = \dfrac{z-0}{-\frac{1}{2}}$ and $\dfrac{x-0}{1} = \dfrac{y+2\lambda}{1} = \dfrac{z-\lambda}{1}$

The shortest distance between the lines is given by

$$\left|\frac{\begin{vmatrix} 0-\lambda & -2\lambda-\frac{1}{2} & \lambda-0 \\ 1 & \frac{1}{2} & -\frac{1}{2} \\ 1 & 1 & 1 \end{vmatrix}}{\sqrt{\left(\frac{1}{2}+\frac{1}{2}\right)^2 + \left(-\frac{1}{2}-1\right)^2 + \left(1-\frac{1}{2}\right)^2}}\right| = \frac{\sqrt{7}}{2\sqrt{2}}$$

$$\Rightarrow \left|\frac{-\lambda\left(\frac{1}{2}+\frac{1}{2}\right)^2 + \left(2\lambda+\frac{1}{2}\right)\left(1+\frac{1}{2}\right) + \lambda\left(1-\frac{1}{2}\right)}{\frac{\sqrt{14}}{2}}\right| = \frac{\sqrt{7}}{2\sqrt{2}}$$

$$\Rightarrow \left|-\lambda + 3\lambda + \frac{3}{4} + \frac{\lambda}{2}\right| = \frac{7}{4} \Rightarrow \left|\frac{5\lambda}{2} + \frac{3}{4}\right| = \frac{7}{4} \Rightarrow \frac{5\lambda}{2} + \frac{3}{4} = \pm\frac{7}{4}$$

$$\Rightarrow \text{ Either } \frac{5\lambda}{2} = \frac{7}{4} - \frac{3}{4} \text{ or } \frac{5\lambda}{2} = -\frac{7}{4} - \frac{3}{4}$$

$$\Rightarrow \frac{5\lambda}{2} = 1 \text{ or } \frac{5\lambda}{2} = \frac{-10}{4}$$

$$\Rightarrow \lambda = \frac{2}{5} \text{ or } \lambda = -1 \Rightarrow \lambda = -1 \qquad [\because \lambda \text{ is an integer}]$$

$\therefore \quad |\lambda| = 1$

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