JEE MAIN 2021

ONLINE
24th February
1st Shift

PHYSICS

SECTION-A (MULTIPLE CHOICE QUESTIONS)

1. Four identical particles of equal masses 1 kg made to move along the circumference of a circle of radius 1 m under the action of their own mutual gravitational attraction. The speed of each particle will be

(a) $\sqrt{\frac{G}{2}(2\sqrt{2}-1)}$ (b) $\sqrt{G(1+2\sqrt{2})}$

(c) $\frac{\sqrt{(1+2\sqrt{2})G}}{2}$ (d) $\sqrt{\frac{G}{2}(1+2\sqrt{2})}$

2. If an emitter current is changed by 4 mA, the collector current changes by 3.5 mA. The value of β will be

(a) 3.5 (b) 0.5 (c) 0.875 (d) 7

3. The workdone by a gas molecule in an isolated system is given by $W = \alpha\beta^2 e^{-\frac{x^2}{\alpha kT}}$, where x is the displacement, k is the Boltzmann constant and T is the temperature. α and β are constants. Then the dimensions of β will be

(a) $[MLT^{-2}]$ (b) $[M^0LT^0]$

(c) $[ML^2T^{-2}]$ (d) $[M^2LT^2]$

4. Each side of a box made of metal sheet in cubic shape is 'a' at room temperature 'T', the coefficient of linear expansion of the metal sheet is 'α'. The metal sheet is heated uniformly, by a small temperature ΔT, so that its new temperature is $T + \Delta T$. Calculate the increase in the volume of the metal box.

(a) $3a^3\alpha\Delta T$ (b) $\frac{4}{3}\pi a^3\alpha\Delta T$

(c) $4a^3\alpha\Delta T$ (d) $4\pi a^3\alpha\Delta T$

5. Two stars of masses m and $2m$ at a distance d rotate about their common centre of mass in free space. The period of revolution is

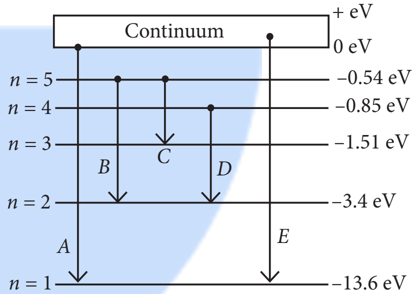
(a) $\frac{1}{2\pi}\sqrt{\frac{3Gm}{d^3}}$ (b) $2\pi\sqrt{\frac{d^3}{3Gm}}$

(c) $2\pi\sqrt{\frac{3Gm}{d^3}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{d^3}{3Gm}}$

6. Consider two satellites S_1 and S_2 with periods of revolution 1 hr and 8 hr respectively revolving around a planet in circular orbits. The ratio of angular velocity of satellite S_1 to the angular velocity of satellite S_2 is

(a) 2 : 1 (b) 1 : 4 (c) 8 : 1 (d) 1 : 8

7. In the given figure, the energy levels of hydrogen atom have been shown along with some transitions marked A, B, C, D and E. The transitions A, B and C respectively represent



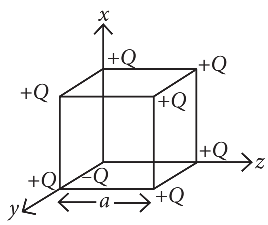
(a) The series limit of Lyman series, third member of Balmer series and second member of Paschen series.

(b) The first member of the Lyman series, third member of Balmer series and second member of Paschen series.

(c) The ionization potential of hydrogen, second member of Balmer series and third member of Paschen series.

(d) The series limit of Lyman series, second member of Balmer series and second member of Paschen series.

8. A cube of side 'a' has point charges $+Q$ located at each of its vertices except at the origin where the charge is $-Q$. The electric field at the centre of cube is



(a) $\frac{-2Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{x}+\hat{y}+\hat{z})$

(b) $\frac{2Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{x}+\hat{y}+\hat{z})$

(c) $\frac{Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{x}+\hat{y}+\hat{z})$

(d) $\frac{-Q}{3\sqrt{3}\pi\varepsilon_0 a^2}(\hat{x}+\hat{y}+\hat{z})$

9. Match List-I with List-II.

List - I		List - II	
(A)	Isothermal	(i)	Pressure constant
(B)	Isochoric	(ii)	Temperature constant
(C)	Adiabatic	(iii)	Volume constant
(D)	Isobaric	(iv)	Heat content is constant

Choose the correct answer from the options given below.

(a) (A)-(i), (B)-(iii), (C)-(ii), (D)-(iv)
(b) (A)-(iii), (B)-(ii), (C)-(i), (D)-(iv)
(c) (A)-(ii), (B)-(iii), (C)-(iv), (D)-(i)
(d) (A)-(ii), (B)-(iv), (C)-(iii), (D)-(i)

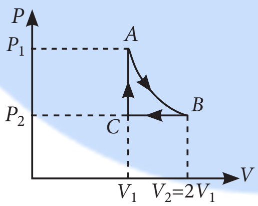
10. n mole of a perfect gas undergoes a cyclic process $ABCA$ (see figure) consisting of the following processes.

$A \to B$: Isothermal expansion at temperature T so that the volume is doubled from V_1 to $V_2 = 2V_1$ and pressure changes from P_1 to P_2.

$B \to C$: Isobaric compression at pressure P_2 to initial volume V_1.

$C \to A$: Isochoric change leading to change of pressure from P_2 to P_1.

Total workdone in the complete cycle $ABCA$ is



(a) $nRT\left(\ln 2 - \frac{1}{2}\right)$
(b) 0
(c) $nRT\ln 2$
(d) $nRT\left(\ln 2 + \frac{1}{2}\right)$

11. Moment of inertia (M.I.) of four bodies, having same mass and radius, are reported as

I_1 = M.I. of thin circular ring about its diameter,
I_2 = M.I. of circular disc about an axis perpendicular to disc and going through the centre
I_3 = M.I. of solid cylinder about its axis and
I_4 = M.I. of solid sphere about its diameter.
Then

(a) $I_1 = I_2 = I_3 < I_4$
(b) $I_1 + I_2 = I_3 + \frac{5}{2}I_4$
(c) $I_1 + I_3 < I_2 + I_4$
(d) $I_1 = I_2 = I_3 > I_4$

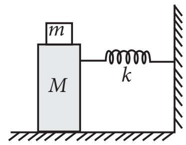
12. If Y, K and η are values of Young's modulus, bulk modulus and modulus of rigidity of any material respectively. Choose the correct relation for these parameters.

(a) $\eta = \frac{3YK}{9K+Y}\text{ N m}^{-2}$
(b) $Y = \frac{9K\eta}{2\eta+3K}\text{ N m}^{-2}$
(c) $K = \frac{Y\eta}{9\eta-3Y}\text{ N m}^{-2}$
(d) $Y = \frac{9K\eta}{3K-\eta}\text{ N m}^{-2}$

13. In a Youngs double slit experiment, the width of one of the slit is three times the other slit. The amplitude of the light coming from a slit is proportional to the slit-width. Find the ratio of the maximum to the minimum intensity in the interference pattern.

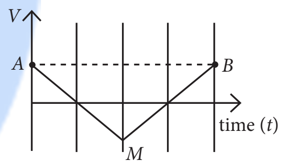
(a) 1 : 4 (b) 3 : 1 (c) 4 : 1 (d) 2 : 1

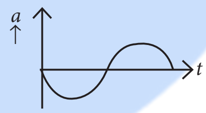
14. In the given figure, a mass M is attached to a horizontal spring which is fixed on one side to a rigid support. The spring constant of the spring is k. The mass oscillates on a frictionless surface with time period T and amplitude A. When the mass is in equilibrium position, as shown in the figure, another mass m is gently fixed upon it. The new amplitude of oscillation will be

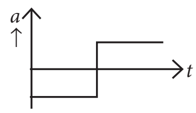


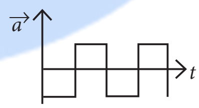
(a) $A\sqrt{\frac{M+m}{M}}$
(b) $A\sqrt{\frac{M-m}{M}}$
(c) $A\sqrt{\frac{M}{M-m}}$
(d) $A\sqrt{\frac{M}{M+m}}$

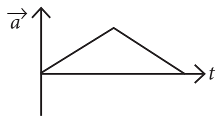
15. If the velocity-time graph has the shape AMB, what would be the shape of the corresponding acceleration-time graph?



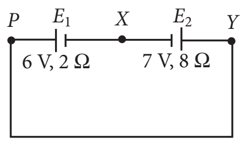
(a)


(b)


(c)


(d)


16. A cell E_1 of emf 6 V and internal resistance 2 Ω is connected with another cell E_2 of emf 4 V and internal resistance 8 Ω (as shown in the figure). The potential difference across points X and Y is



(a) 10.0 V (b) 5.6 V
(c) 3.6 V (d) 2.0 V

17. Two equal capacitors are first connected in series and then in parallel. The ratio of the equivalent capacities in the two cases will be

(a) 1 : 4 (b) 2 : 1 (c) 4 : 1 (d) 1 : 2

18. Given below are two statements :

Statement I : Two photons having equal linear momenta have equal wavelengths.

Statement II : If the wavelength of photon is decreased, then the momentum and energy of a photon will also decrease.

In the light of the above statements, choose the correct answer from the options given below.

(a) Statement I is false but Statement II is true.
(b) Both Statement I and Statement II are true.
(c) Statement I is true but Statement II is false.
(d) Both Statement I and Statement II are false.

19. The focal length f is related to the radius of curvature r of the spherical convex mirror by

(a) $f = r$ (b) $f = +\frac{1}{2}r$ (c) $f = -r$ (d) $f = -\frac{1}{2}r$

20. A current through a wire depends on time as
$i = \alpha_0 t + \beta t^{2t}$
where $\alpha_0 = 20$ A s^{-1} and $\beta = 8$ A s^{-2}. Find the charge crossed through a section of the wire in 15 s.

(a) 2250 C (b) 11250 C
(c) 21000 C (d) 260 C

SECTION-B (NUMERICAL VALUE TYPE)

Attempt any 5 questions out of 10.

21. A hydraulic press can lift 100 kg when a mass 'm' is placed on the smaller piston. It can lift ________ kg when the diameter of the larger piston is increased by 4 times and that of the smaller piston is decreased by 4 times keeping the same mass 'm' on the smaller piston.

22. An unpolarized light beam is incident on the polarizer of a polarization experiment and the intensity of light beam emerging from the analyzer is measured as 100 Lumen. Now, if the analyzer is rotated around the horizontal axis (direction of light) by 30° in clockwise direction, the intensity of emerging light will be ________ Lumen.

23. A common transistor radio set requires 12 V (D.C.) for its operation. The D.C. source is constructed by using a transformer and a rectifier circuit, which are operated at 220 V (A.C.) on standard domestic A.C. supply. The number of turns of secondary coil are 24, then the number of turns of primary are ________.

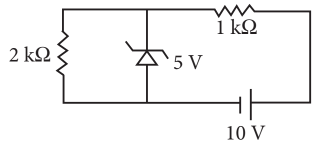
24. An inclined plane is bent in such a way that the vertical cross-section is given by $y = \frac{x^2}{4}$ where y is in vertical and x in horizontal direction. If the upper surface of this curved plane is rough with coefficient of friction $\mu = 0.5$, the maximum height in cm at which a stationary block will not slip downward is ____ cm.

25. A ball with a speed of 9 m s^{-1} collides with another identical ball at rest. After the collision, the direction of each ball makes an angle of 30° with the original direction. The ratio of velocities of the balls after collision is $x : y$, where x is ________.

26. The coefficient of static friction between a wooden block of mass 0.5 kg and a vertical rough wall is 0.2. The magnitude of horizontal force that should be applied on the block to keep it adhere to the wall will be ________ N. [$g = 10$ m s^{-2}]

27. A resonance circuit having inductance and resistance 2×10^{-4} H and 6.28 Ω respectively oscillates at 10 MHz frequency. The value of quality factor of this resonator is ________. [$\pi = 3.14$]

28. In connection with the circuit drawn below, the value of current flowing through 2 kΩ resistor is ________ $\times 10^{-4}$ A.



29. An audio signal $v_m = 20 \sin 2\pi(1500t)$ amplitude modulates a carrier $v_c = 80 \sin 2\pi(100{,}000t)$. The value of percent modulation is ________.

30. An electromagnetic wave of frequency 5 GHz, is travelling in a medium whose relative electric permittivity and relative magnetic permeability both are 2. Its velocity in this medium is ________ $\times 10^7$ m s^{-1}

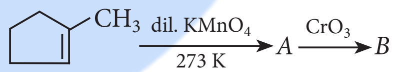
CHEMISTRY

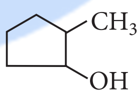
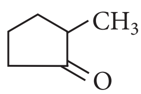
SECTION-A (MULTIPLE CHOICE QUESTIONS)

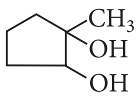
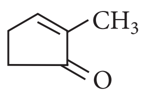
31. Out of the following, which type of interaction is responsible for the stabilisation of α-helix structure of proteins?

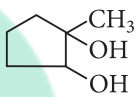
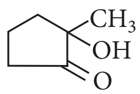
(a) Ionic bonding (b) van der Waals' forces
(c) Hydrogen bonding (d) Covalent bonding

32. Identify products A and B.



(a) A:  B: 

(b) A:  B: 

(c) A:  B: 

(d) A: $OHC-CH_2CH_2CH_2-\overset{\overset{O}{||}}{C}-CH_3$

B: $HOOC-CH_2CH_2CH_2-\overset{\overset{O}{||}}{C}-CH_3$

33. Which of the following are isostructural pairs?

A. SO_4^{2-} and CrO_4^{2-} B. $SiCl_4$ and $TiCl_4$
C. NH_3 and NO_3^- D. BCl_3 and $BrCl_3$

(a) C and D only (b) B and C only
(c) A and C only (d) A and B only

34. Given below are two statements.

Statement I : Colourless cupric metaborate is reduced to cuprous metaborate in a luminous flame.

Statement II : Cuprous metaborate is obtained by heating boric anhydride and copper sulphate in a non-luminous flame.

In the light of the above statements, choose the most appropriate answer from the options given below.

(a) Statement I is false but statement II is true.
(b) Both statement I and statement II are true.
(c) Both statement I and statement II are false.
(d) Statement I is true but statement II is false.

35. Match List-I with List-II.

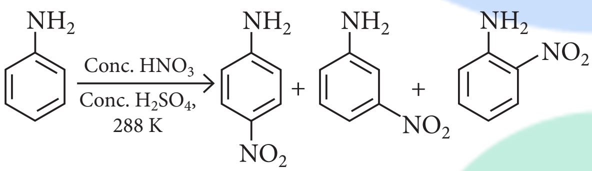
	List-I (Monomer Unit)		List-II (Polymer)
(A)	Caprolactum	(i)	Natural rubber
(B)	2-Chloro-1,3-butadiene	(ii)	Buna-N
(C)	Isoprene	(iii)	Nylon 6
(D)	Acrylonitrile	(iv)	Neoprene

Choose the correct answer from the options given below.
(a) (A)-(iv), (B)-(iii), (C)-(ii), (D)-(i)
(b) (A)-(ii), (B)-(i), (C)-(iv), (D)-(iii)
(c) (A)-(i), (B)-(ii), (C)-(iii), (D)-(iv)
(d) (A)-(iii), (B)-(iv), (C)-(i), (D)-(ii)

36. Which of the following ore is concentrated using group 1 cyanide salt?
(a) Sphalerite (b) Malachite
(c) Calamine (d) Siderite

37. The major components in "Gun Metal" are
(a) Cu, Zn and Ni (b) Cu, Ni and Fe
(c) Cu, Sn and Zn (d) Al, Cu, Mg and Mn.

38. In the following reaction the reason why *meta*-nitro product also formed is

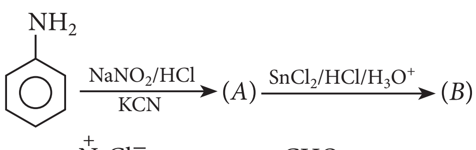


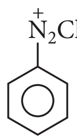
[A] 51% [B] 47% [C] 2%

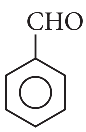
(a) $—NH_2$ group is highly *meta*-directive
(b) low temperature
(c) $—NO_2$ substitution always takes place at *meta*-position
(d) formation of anilinium ion.

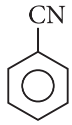
39. Consider the elements Mg, Al, S, P and Si, the correct increasing order of their first ionization enthalpy is
(a) Al < Mg < S < Si < P (b) Mg < Al < Si < S < P
(c) Al < Mg < Si < S < P (d) Mg < Al < Si < P < S

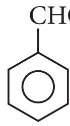
40. '*A*' and '*B*' in the following reactions are :

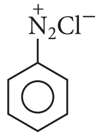


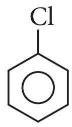
(a) (*A*): 

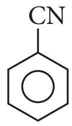
(*B*): 

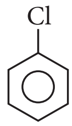
(b) (*A*): 

(*B*): 

(c) (*A*): 

(*B*): 

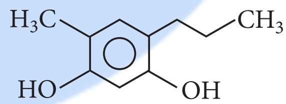
(d) (*A*): 

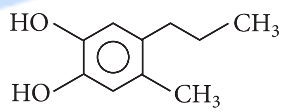
(*B*): 

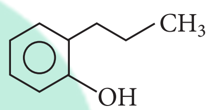
41. The electrode potential of M^{2+}/M of 3*d*-series elements shows positive value for
(a) Co (b) Zn (c) Cu (d) Fe

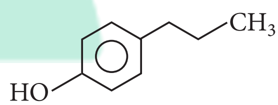
42. The gas released during anaerobic degradation of vegetation may lead to
(a) corrosion of metals
(b) acid rain
(c) ozone hole
(d) global warming and cancer.

43. Which of the following compound gives pink colour on reaction with phthalic anhydride in conc. H_2SO_4 followed by treatment with NaOH?

(a) 

(b) 

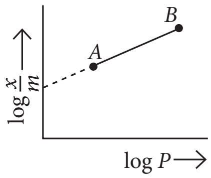
(c) 

(d) 

44. Which of the following reagent is used for the following reaction?
$CH_3CH_2CH_3 \xrightarrow{?} CH_3CH_2CHO$
(a) Copper at high temperature and pressure
(b) Molybdenum oxide
(c) Potassium permanganate
(d) Manganese acetate

45. Al_2O_3 was leached with alkali to get *X*. The solution of *X* on passing of gas *Y*, forms *Z*. *X*, *Y* and *Z* respectively are
(a) $X = Al(OH)_3$, $Y = SO_2$, $Z = Al_2O_3.xH_2O$
(b) $X = Na[Al(OH)_4]$, $Y = SO_2$, $Z = Al_2O_3$
(c) $X = Na[Al(OH)_4]$, $Y = CO_2$, $Z = Al_2O_3.xH_2O$
(d) $X = Al(OH)_3$, $Y = CO_2$, $Z = Al_2O_3$

46. In Freundlich adsorption isotherm, slope of AB line is



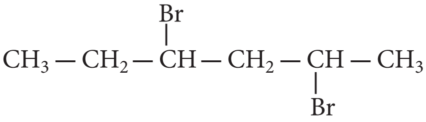
(a) $\log \frac{1}{n}$ with $(n < 1)$

(b) $\frac{1}{n}$ with $\left(\frac{1}{n} = 0 \text{ to } 1\right)$

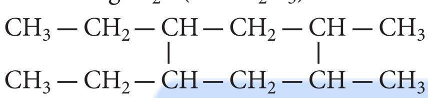
(c) $\log n$ with $(n > 1)$

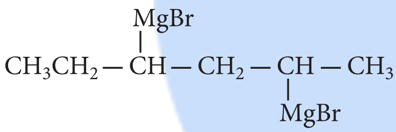
(d) n with $(n, 0.1 \text{ to } 0.5)$

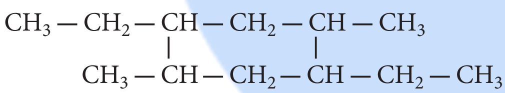
47. The product formed in the first step of the reaction of

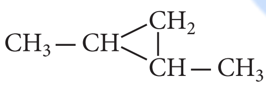


with excess $Mg/Et_2O(Et = C_2H_5)$ is

(a) 

(b) 

(c) 

(d) 

48. (A) $HOCl + H_2O_2 \rightarrow H_3O^+ + Cl^- + O_2$

(B) $I_2 + H_2O_2 + 2OH^- \rightarrow 2I^- + 2H_2O + O_2$

Choose the correct option.

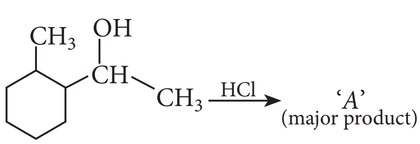
(a) H_2O_2 acts as reducing and oxidising agent respectively in equations (A) and (B).

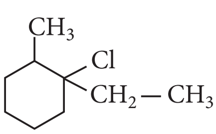
(b) H_2O_2 acts as reducing agent in equations (A) and (B).

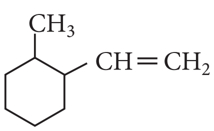
(c) H_2O_2 acts as oxidising agent in equations (A) and (B).

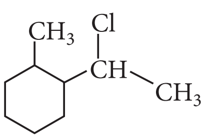
(d) H_2O_2 acts as oxidising and reducing agent respectively in equations (A) and (B).

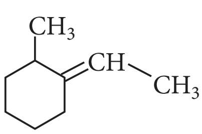
49. What is the final product (major) 'A' in the given reaction?



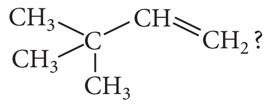
(a) 

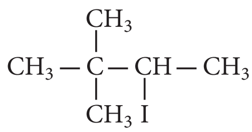
(b) 

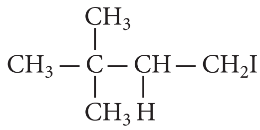
(c) 

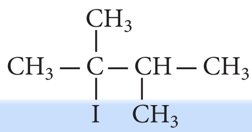
(d) 

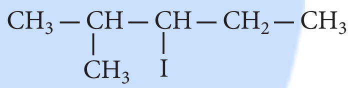
50. What is the major product formed by HI on reaction with



(a) 

(b) 

(c) 

(d) 

SECTION-B (NUMERICAL VALUE TYPE)

Attempt any 5 questions out of 10.

51. Gaseous cyclobutene isomerizes to butadiene in a first order process which has a 'k' value of 3.3×10^{-4} s^{-1} at 153 °C. The time in minutes it takes for the isomerization to proceed 40% to completion at this temperature is ________.
(Rounded off to the nearest integer)

52. The reaction of sulphur in alkaline medium is given below

$S_{8(s)} + a\, OH^-_{(aq)} \longrightarrow b\, S^{2-}_{(aq)} + c\, S_2O^{2-}_{3(aq)} + d\, H_2O_{(l)}$

The value of 'a' is ________. (Integer answer)

53. The stepwise formation of $[Cu(NH_3)_4]^{2+}$ is given below

$Cu^{2+} + NH_3 \overset{K_1}{\rightleftharpoons} [Cu(NH_3)]^{2+}$

$[Cu(NH_3)]^{2+} + NH_3 \overset{K_2}{\rightleftharpoons} [Cu(NH_3)_2]^{2+}$

$[Cu(NH_3)_2]^{2+} + NH_3 \overset{K_3}{\rightleftharpoons} [Cu(NH_3)_3]^{2+}$

$[Cu(NH_3)_3]^{2+} + NH_3 \overset{K_4}{\rightleftharpoons} [Cu(NH_3)_4]^{2+}$

The value of stability constants K_1, K_2, K_3 and K_4 are 10^4, 1.58×10^3, 5×10^2 and 10^2 respectively. The overall equilibrium constant for dissociation of $[Cu(NH_3)_4]^{2+}$ is $x \times 10^{-12}$. The value of x is ________. (Rounded off to the nearest integer)

54. The coordination number of an atom in a body-centered cubic structure is ________.
[Assume that the lattice is made up of atoms.]

55. At 1990 K and 1 atm pressure, there are equal number of Cl_2 molecules and Cl atoms in the reaction mixture. The value of K_P for the reaction $Cl_{2(g)} \rightleftharpoons 2Cl_{(g)}$ under the above conditions is $x \times 10^{-1}$. The value of x is ________. (Rounded off to the nearest integer)

56. 4.5 g of compound A (MW = 90) was used to make 250 mL of its aqueous solution. The molarity of the solution in M is $x \times 10^{-1}$. The value of x is ________. (Rounded off to the nearest integer)

57. When 9.45 g of $ClCH_2COOH$ is added to 500 mL of water, its freezing point drops by 0.5°C. The dissociation constant of $ClCH_2COOH$ is $x \times 10^{-3}$. The value of x is ________.
(Rounded off to the nearest integer)
$[K_{f(H_2O)} = 1.86 \text{ K kg mol}^{-1}]$

58. For the reaction $A_{(g)} \rightarrow B_{(g)}$, the value of the equilibrium constant at 300 K and 1 atm is equal to 100.0. The value of $\Delta_r G$ for the reaction at 300 K and 1 atm in J mol^{-1} is $-xR$ where x is ________.
(Rounded off to the nearest integer)
$[R = 8.31 \text{ J mol}^{-1} \text{K}^{-1}$ and $\ln 10 = 2.3]$

59. Number of amphoteric compounds among the following is ________.
(A) BeO (B) BaO
(C) $Be(OH)_2$ (D) $Sr(OH)_2$

60. A proton and a Li^{3+} nucleus are accelerated by the same potential. If λ_{Li} and λ_P denote the de Broglie wavelengths of Li^{3+} and proton respectively, then the value of $\frac{\lambda_{Li}}{\lambda_P}$ is $x \times 10^{-1}$. The value of x is ________.
(Rounded off to the nearest integer)
[Mass of Li^{3+} = 8.3 × mass of proton]

MATHEMATICS

SECTION - A (MULTIPLE CHOICE QUESTIONS)

61. A man is walking on a straight line. The arithmetic mean of the reciprocals of the intercepts of this line on the coordinate axes is $\frac{1}{4}$. Three stones A, B and C are placed at the points (1, 1), (2, 2) and (4, 4) respectively. Then which of these stones is/are on the path of the man?
(a) C only (b) A only
(c) B only (d) All the three

62. The area (in sq. units) of the part of the circle $x^2 + y^2 = 36$, which is outside the parabola $y^2 = 9x$, is
(a) $12\pi - 3\sqrt{3}$ (b) $24\pi + 3\sqrt{3}$
(c) $12\pi + 3\sqrt{3}$ (d) $24\pi - 3\sqrt{3}$

63. If $e^{(\cos^2 x + \cos^4 x + \cos^6 x + ... \infty)\log_e 2}$ satisfies the equation $t^2 - 9t + 8 = 0$, then the value of $\frac{2\sin x}{\sin x + \sqrt{3}\cos x}$ $\left(0 < x < \frac{\pi}{2}\right)$ is
(a) $\frac{3}{2}$ (b) $\frac{1}{2}$ (c) $\sqrt{3}$ (d) $2\sqrt{3}$

64. A scientific committee is to be formed from 6 Indians and 8 foreigners, which includes at least 2 Indians and double the number of foreigners as Indians. Then the number of ways, the committee can be formed, is
(a) 560 (b) 1050 (c) 1625 (d) 575

65. The population $P = P(t)$ at time 't' of a certain species follows the differential equation $\frac{dP}{dt} = 0.5P - 450$. If $P(0) = 850$, then the time at which population becomes zero is
(a) $2\log_e 18$ (b) $\log_e 9$
(c) $\frac{1}{2}\log_e 18$ (d) $\log_e 18$

66. If $\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}} dx = a\sin^{-1}\left(\frac{\sin x + \cos x}{b}\right) + c$, where c is a constant of integration, then the ordered pair (a, b) is equal to
(a) (1, –3) (b) (3, 1) (c) (1, 3) (d) (–1, 3)

67. The function $f(x) = \frac{4x^3 - 3x^2}{6} - 2\sin x + (2x - 1)\cos x$
(a) increases in $\left[\frac{1}{2}, \infty\right)$ (b) decreases in $\left(-\infty, \frac{1}{2}\right]$
(c) decreases in $\left[\frac{1}{2}, \infty\right)$ (d) increases in $\left(-\infty, \frac{1}{2}\right]$

68. The statement among the following that is a tautology is
(a) $B \rightarrow [A \wedge (A \rightarrow B)]$ (b) $[A \wedge (A \rightarrow B)] \rightarrow B$
(c) $A \vee (A \wedge B)$ (d) $A \wedge (A \vee B)$

69. $\lim_{x \to 0} \frac{\int_0^{x^2} (\sin\sqrt{t})dt}{x^3}$ is equal to
(a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{1}{15}$ (d) 0

70. If $f : R \rightarrow R$ is a function defined by $f(x) = [x - 1]\cos\left(\frac{2x - 1}{2}\right)\pi$, where $[\cdot]$ denotes the greatest integer function, then f is
(a) continuous for every real x
(b) discontinuous at all integral values of x except at $x = 1$
(c) continuous only at $x = 1$
(d) discontinuous only at $x = 1$

71. The distance of the point (1, 1, 9) from the point of intersection of the line $\frac{x - 3}{1} = \frac{y - 4}{2} = \frac{z - 5}{2}$ and the plane $x + y + z = 17$ is
(a) $\sqrt{38}$ (b) 38 (c) $2\sqrt{19}$ (d) $19\sqrt{2}$

72. The system of linear equations
$3x - 2y - kz = 10$; $2x - 4y - 2z = 6$; $x + 2y - z = 5m$
is inconsistent if
(a) $k = 3, m \neq \frac{4}{5}$ (b) $k \neq 3, m \neq \frac{4}{5}$
(c) $k = 3, m = \frac{4}{5}$ (d) $k \neq 3, m \in R$

73. Let $f : R \to R$ be defined as $f(x) = 2x - 1$ and $g : R - \{1\} \to R$ be defined as $g(x) = \frac{x - \frac{1}{2}}{x - 1}$. Then the composition function $f(g(x))$ is
(a) onto but not one-one
(b) neither one-one nor onto
(c) one-one but not onto
(d) both one-one and onto

74. Let p and q be two positive numbers such that $p + q = 2$ and $p^4 + q^4 = 272$. Then p and q are roots of the equation
(a) $x^2 - 2x + 16 = 0$ (b) $x^2 - 2x + 2 = 0$
(c) $x^2 - 2x + 136 = 0$ (d) $x^2 - 2x + 8 = 0$

75. An ordinary dice is rolled for a certain number of times. If the probability of getting an odd number 2 times is equal to the probability of getting an even number 3 times, then the probability of getting an odd number for odd number of times is
(a) $\frac{5}{16}$ (b) $\frac{3}{16}$ (c) $\frac{1}{2}$ (d) $\frac{1}{32}$

76. The equation of the plane passing through the point (1, 2, –3) and perpendicular to the planes $3x + y - 2z = 5$ and $2x - 5y - z = 7$, is
(a) $6x - 5y - 2z - 2 = 0$ (b) $6x - 5y + 2z + 10 = 0$
(c) $3x - 10y - 2z + 11 = 0$ (d) $11x + y + 17z + 38 = 0$

77. The locus of the mid-point of the line segment joining the focus of the parabola $y^2 = 4ax$ to a moving point of the parabola, is another parabola whose directrix is
(a) $x = \frac{a}{2}$ (b) $x = 0$ (c) $x = -\frac{a}{2}$ (d) $x = a$

78. The value of $-{}^{15}C_1 + 2 \cdot {}^{15}C_2 - 3 \cdot {}^{15}C_3 + \ldots\ldots - 15 \cdot {}^{15}C_{15} + {}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + \ldots\ldots + {}^{14}C_{11}$ is
(a) $2^{13} - 14$ (b) 2^{14}
(c) $2^{16} - 1$ (d) $2^{13} - 13$

79. If the tangent to the curve $y = x^3$ at the point $P(t, t^3)$ meets the curve again at Q, then the ordinate of the point which divides PQ internally in the ratio 1 : 2 is
(a) $-t^3$ (b) $-2t^3$ (c) $2t^3$ (d) 0

80. Two vertical poles are 150 m apart and the height of one is three times that of the other. If from the middle point of the line joining their feet, an observer finds the angles of elevation of their tops to be complementary, then the height of the shorter pole (in meters) is
(a) 30 (b) 25 (c) $20\sqrt{3}$ (d) $25\sqrt{3}$

SECTION - B (NUMERICAL VALUE TYPE)

Attempt any 5 questions out of 10.

81. Let B_i $(i = 1, 2, 3)$ be three independent events in a sample space. The probability that only B_1 occur is α, only B_2 occurs is β and only B_3 occurs is γ. Let p be the probability that none of the events B_i occurs and these 4 probabilities satisfy the equations $(\alpha - 2\beta)\, p = \alpha\beta$ and $(\beta - 3\gamma)\, p = 2\beta\gamma$ (All the probabilities are assumed to lie in the interval (0, 1)). Then $\frac{P(B_1)}{P(B_3)}$ is equal to ________.

82. $\lim_{n\to\infty} \tan\left\{\sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{1 + r + r^2}\right)\right\}$ is equal to ________.

83. If the least and the largest real values of α, for which the equation $z + \alpha|z - 1| + 2i = 0$ $(z \in C$ and $i = \sqrt{-1})$ has a solution, are p and q respectively, then $4(p^2 + q^2)$ is equal to ________.

84. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord of another circle 'C', whose centre is at (2, 1), then its radius is ________.

85. Let M be any 3×3 matrix with entries from the set {0, 1, 2}. The maximum number of such matrices, for which the sum of diagonal elements of $M^T M$ is seven, is ________.

86. If $\int_{-a}^{a} (|x| + |x - 2|)dx = 22, (a > 2)$ and $[x]$ denotes the greatest integer $\leq x$, then $\int_{a}^{-a} (x + [x])dx$ is equal to ________.

87. Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in R$. Suppose $Q = [q_{ij}]$ is a matrix satisfying $PQ = kI_3$ for some non-zero $k \in R$. If $q_{23} = -\frac{k}{8}$ and $|Q| = \frac{k^2}{2}$, then $\alpha^2 + k^2$ is equal to ________.

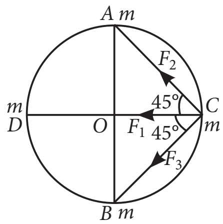
88. Let three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ be such that $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$, $\vec{a} \cdot \vec{c} = 7$ and $\vec{b}$ is perpendicular to $\vec{c}$, where $\vec{a} = -\hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{k}$, then the value of $2|\vec{a} + \vec{b} + \vec{c}|^2$ is ________.

89. Let $A = \{n \in N : n$ is a 3-digit number$\}$
$B = \{9k + 2 : k \in N\}$
and $C = \{9k + l : k \in N\}$ for some $l (0 < l < 9)$
If the sum of all the elements of the set $A \cap (B \cup C)$ is 274×400, then l is equal to ________.

90. The minimum value of α for which the equation $\frac{4}{\sin x} + \frac{1}{1 - \sin x} = \alpha$ has at least one solution in $\left(0, \frac{\pi}{2}\right)$ is ________.

HINTS & EXPLANATIONS

1. **(c) :** The force on m at C due to m at A



$$F_2 = \frac{G\cdot mm}{(\sqrt{2}R)^2} = \frac{Gm^2}{2R^2}$$

The force on m at C due to m at B

$$F_3 = \frac{G\cdot mm}{(\sqrt{2}R)^2} = \frac{Gm^2}{2R^2}$$

The force on m at C due to m at D

$$F_1 = \frac{G\cdot mm}{(2R)^2} = \frac{Gm^2}{4R^2}$$

Net force on m at C, towards the centre (O) is

$F_x = F_1 + F_2\cos45° + F_3\cos45°$

$$F_x = \frac{Gm^2}{4R^2} + \frac{Gm^2}{2R^2}\times\frac{1}{\sqrt{2}}\times 2 = \frac{Gm^2}{4R^2}(1+2\sqrt{2})$$

Net force on m at C, along vertical direction

$F_y = F_2\sin45° - F_3\sin45° = 0 \qquad (\because\ F_2 = F_3)$

$$\therefore\ F_{net} = F_x = \frac{mv^2}{R} \Rightarrow \frac{Gm^2}{4R^2}(1+2\sqrt{2}) = \frac{mv^3}{R}$$

$$\Rightarrow\ v = \sqrt{\frac{Gm(1+2\sqrt{2})}{4R}} = \sqrt{\frac{G\times 1(1+2\sqrt{2})}{4\times 1}}$$

$$\Rightarrow\ v = \frac{1}{2}\sqrt{G(1+2\sqrt{2})}$$

2. **(d) :** Given, $\Delta I_E = 4$ mA, $\Delta I_C = 3.5$ mA

$\because\ \Delta I_E = \Delta I_B + \Delta I_C \Rightarrow \Delta I_B = \Delta I_E - \Delta I_C$

$$\therefore\ \beta = \frac{\Delta I_C}{\Delta I_B} = \frac{3.5}{\Delta I_E - \Delta I_C} = \frac{3.5}{4-3.5} = 7$$

3. **(a) :** Work done, $W = \alpha\beta^2 e^{-\frac{x^2}{\alpha KT}}$

Since, exponent should be dimensionless.

Therefore, dimension of $\frac{x^2}{\alpha KT} = [M^0L^0T^0]$

$$\therefore\ \text{Dimension of } \alpha = \frac{L^2}{[ML^2T^{-2}]} = [M^{-1}T^2]$$

Dimension of $\alpha\beta^2$ = Dimension of W

$\Rightarrow\ [M^{-1}T^2]\beta^2 = [ML^2T^{-2}] \Rightarrow \beta = [M^1L^1T^{-2}]$

4. **(a) :** The increase in volume due to heat is

$\Delta V = \gamma V\Delta T \qquad$ Also, $\gamma = 3\alpha$

$\Rightarrow\ \Delta V = 3a^3\alpha\Delta T \qquad (\because V = a^3)$

5. **(b) :** Let the centre of mass is at a distance x from the centre of m.

So, $mx = 2m(d-x) \Rightarrow x = 2d - 2x \Rightarrow x = 2d/3$

The gravitational force between the stars is balanced by the centripetal force

$$\text{So, } \frac{Gm(2m)}{d^2} = (2m)(d-x)\omega^2$$

$$\Rightarrow \frac{2G\cdot m^2}{d^2} = 2m\left(d-\frac{2d}{3}\right)\cdot\left(\frac{2\pi}{T}\right)^2 \Rightarrow \frac{Gm}{d^2} = \frac{d}{3}\cdot\frac{4\pi^2}{T^2}$$

$$\Rightarrow\ T = \sqrt{4\pi^2\frac{d^3}{3Gm}} = 2\pi\sqrt{\frac{d^3}{3Gm}}$$

6. **(c) :** Given, $T_1 = 1$ hr, $T_2 = 8$ hr

$$\text{As, } T = \frac{2\pi}{\omega} \Rightarrow \frac{T_1}{T_2} = \frac{\omega_2}{\omega_1} \Rightarrow \frac{\omega_1}{\omega_2} = \frac{T_2}{T_1} = \frac{8}{1} = 8:1$$

7. **(a) :** For Lyman series, $n_2 = 1$, $n_1 = 2, 3, 4 .....\infty$

For Balmer series, $n_2 = 2$, $n_1 = 3, 4, 5, ...\ \infty$

For Paschen series, $n_2 = 3$, $n_1 = 4, 5, 6, ...\ \infty$

So $A \to (n_2 = 1, n_1 = \infty)$ Lyman series

$B \to (n_2 = 2, n_1 = 5)$ Third line of Balmer series

$C \to (n_2 = 3, n_1 = 5)$ Second line of Paschen series

$D \to (n_2 = 2, n_1 = 4)$ Second line of Balmer series

$E \to (n_2 = 1, n_1 = \infty)$ Lyman series

8. **(a) :** Due to the symmetry, the net electric field at the center of cube is due to the charge at origin and diagonally opposite to it.

$$\text{So, } \vec{E} = \frac{-2KQ\hat{r}}{r^2} \Rightarrow \vec{E} = \frac{-2KQ}{\left(\frac{a\sqrt{3}}{2}\right)^2}\left(\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}\right)$$

$$\Rightarrow\ \vec{E} = \frac{-8KQ}{3\sqrt{3}a^2}(\hat{i}+\hat{j}+\hat{k})$$

$$\Rightarrow\ \vec{E} = -\frac{8KQ}{3\sqrt{3}\times 4\pi\varepsilon_0 a^2}(\hat{i}+\hat{j}+\hat{k})$$

$$\Rightarrow\ \vec{E} = -\frac{2Q}{3\sqrt{3}\times \pi\varepsilon_0 a^2}(\hat{i}+\hat{j}+\hat{k})$$

9. **(c) :** (a) Isothermal – Constant temperature

(b) Isochoric – Volume constant

(c) Adiabatic – Heat content is constant

(d) Isobaric – Pressure constant

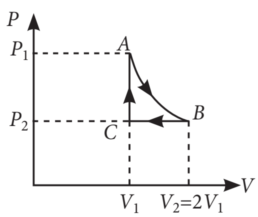
10. **(a) :** AB : Isothermal expansion

Workdone in isothermal expansion

$$W_1 = nRT\ln\left(\frac{V_2}{V_1}\right)$$

$$\Rightarrow\ W_1 = nRT\ln\left(\frac{2V_1}{V_1}\right)$$

$$= nRT\ln 2$$



BC : Isobaric compression

Workdone in isobaric compression

$$W_2 = P\cdot\Delta V = nR\Delta T$$

As, $\frac{P_2\cdot 2V_1}{T_1} = \frac{P_2\cdot V_1}{T_1} \Rightarrow T_1' = \frac{T_1}{2} = \frac{T}{2}$

$$W_2 = nR\left(\frac{T}{2} - T\right) = \frac{-nRT}{2}$$

CA : Isochoric process

Workdone in isochoric process $W_3 = P(\Delta V) = 0$

Net work done in complete cycle

$$W = W_1 + W_2 + W_3 = nRT\ln 2 - \frac{nRT}{2}$$

$$\Rightarrow W = nRT\left(\ln 2 - \frac{1}{2}\right)$$

11. (d) : Moment of inertia of ring about diameter

$$I_1 = \frac{1}{2}MR^2 = 0.5MR^2$$

Moment of inertia of disc about its axis perpendicular to its plane,

$$I_2 = \frac{1}{2}MR^2 = 0.5MR^2$$

Moment of inertia of solid cylinder about its axis

$$I_3 = \frac{1}{2}MR^2 = 0.5MR^2$$

Moment of inertia of sphere about diameter

$$I_4 = \frac{2}{5}MR^2 = 0.4MR^2$$

So, $I_1 = I_2 = I_3 > I_4$

12. (c) : The relation between Y, σ and K is

$$Y = 3K(1 - 2\sigma)$$

$$\Rightarrow \sigma = \frac{1}{2}\left(1 - \frac{Y}{3K}\right) \quad \text{...(i)}$$

The relation between σ, η and Y is

$$Y = 2\eta(1 + \sigma)$$

$$\Rightarrow \sigma = \frac{Y}{2\eta} - 1 \quad \text{...(ii)}$$

From (i) and (ii)

$$\Rightarrow \frac{1}{2}\left(1 - \frac{Y}{3K}\right) = \frac{Y}{2\eta} - 1 \Rightarrow \frac{1}{2} - \frac{Y}{6K} = \frac{Y}{2\eta} - 1$$

$$\Rightarrow 1 + \frac{1}{2} - \frac{Y}{2\eta} = \frac{Y}{6K} \Rightarrow \frac{3}{2} - \frac{Y}{2\eta} = \frac{Y}{6K}$$

$$\Rightarrow \frac{Y}{6K} = \frac{3\eta - Y}{2\eta} \Rightarrow K = \frac{Y\eta}{9\eta - 3Y}$$

13. (c) : $\omega_2 = 3\omega_1$

As, amplitude is proportional to slit width.

So, $A_2 = 3A_1$

Intensity $I \propto A^2$

$\sqrt{I_1} \propto A_1$, $\sqrt{I_2} \propto A_2$

$$\Rightarrow I_{max} \propto (A_1 + A_2)^2 \Rightarrow I_{min} \propto (A_1 - A_2)^2$$

So, $\frac{I_{max}}{I_{min}} = \left(\frac{A_1 + A_2}{A_1 - A_2}\right)^2 = \left(\frac{A_1 + 3A_1}{A_1 - 3A_1}\right)^2 = 4:1$

14. (d) : The angular frequency of oscillations (when M)

$$\omega = \sqrt{\frac{K}{M}}$$

The angular frequency of oscillation (when $M + m$)

$$\omega' = \sqrt{\frac{K}{M + m}}$$

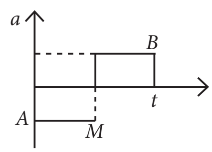
Using conservation of linear momentum

$MA\omega = (m + M)A'\omega'$

$$\Rightarrow MA\sqrt{\frac{K}{M}} = (m + M)\sqrt{\frac{K}{M + m}}A'$$

$$\Rightarrow A' = A\sqrt{\frac{M}{M + m}}$$

15. (b) : The slope of velocity time graph is acceleration



In part AM, acceleration is constant and negative.

In part MB, acceleration is constant and positive.

16. (b) : Current in circuit, $I = \frac{E_1 - E_2}{R}$

$$\Rightarrow I = \frac{6 - 4}{2 + 8} = \frac{2}{10} = 0.2 \text{ A}$$

Now, applying Kirchhoff's Voltage Law

$V_x + 4 + 8 \times I - V_y = 0 \Rightarrow V_x + 4 + 1.6 - V_y = 0$

$\Rightarrow V_x - V_y = -5.6 \text{ V} \Rightarrow V_y - V_x = 5.6 \text{ V}$

17. (a) : Equivalent capacitance for series connection,

$$C_S = \frac{C \times C}{C + C} = \frac{C}{2}$$

Equivalent capacitance for parallel connection,

$C_P = C + C = 2C$

$$\therefore \frac{C_S}{C_P} = \frac{C}{2 \times 2C} = \frac{1}{4}$$

18. (c) : Linear momentum of a photon is, $P = \frac{h}{\lambda}$

Energy of photon is, $E = \frac{hc}{\lambda}$

If momentum is same, the wavelength is also same. As wavelength decreases, momentum and energy both increases.

19. (b) : The relation between focal length and radius of curvature of a mirror is

$$R = 2f \Rightarrow f = \frac{R}{2}$$

So, the focal length, $f = +\frac{r}{2}$.

20. **(b) :** Given, $i = \alpha_0 t + \beta t^2$, $\alpha_0 = 20\text{ A s}^{-1}$ and $\beta = 8\text{ A s}^{-2}$

Current, $i = \dfrac{dq}{dt}$

$$\Rightarrow \int dq = \int i dt = \int_0^{15s} (\alpha_0 t + \beta t^2) dt$$

$$\Rightarrow q = \left[\frac{\alpha_0 t^2}{2} + \frac{\beta t^3}{3}\right]_0^{15} = \left[\frac{20t^2}{2} + \frac{8t^3}{3}\right]_0^{15}$$

$$\Rightarrow q = 10(15^2 - 0) + \frac{8}{3}(15^3 - 0^3)$$

$$\Rightarrow q = 10 \times 225 + \frac{8}{3} \times 15 \times 15 \times 15 = 11250\text{ C}$$

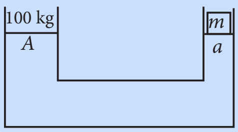
21. **(25600) :** According to Pascal's Law

$$\frac{100 \times g}{A} = \frac{mg}{a} \qquad \text{...(i)}$$

Let the mass of lift is M :

$$\frac{M' \times g}{16A} = \frac{mg}{a/16} \qquad \text{...(ii)}$$

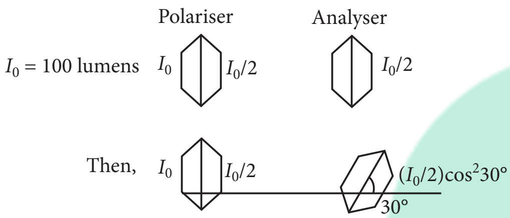
From (i) and (ii)



$$\frac{100g \times 16A}{A \times M'g} = \frac{mg\ a}{a \times mg\ 16}$$

$$\Rightarrow \frac{100 \times 16}{M'} = \frac{1}{16} \Rightarrow M' = 25600\text{ kg}$$

22. **(75) :** Consider the situation shown in figure.



Now, the energy light has intensity

$$I = \left(\frac{I_0}{2}\right)\cos^2\theta \qquad \text{(By Malus Law)}$$

$$\Rightarrow I = \frac{I_0}{2} \times \cos^2 30° = \frac{I_0}{2} \times \frac{3}{4} = \frac{3}{8} I_0$$

$$\Rightarrow I = \frac{3}{8} \times 200 = \frac{600}{8} = 75\text{ lumen}$$

23. **(440) :** As, $V_S = 12\text{ V}$, $N_S = 24$

$V_P = 220\text{ V}$

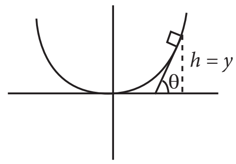
Since, $\dfrac{V_S}{V_P} = \dfrac{N_S}{N_P} \Rightarrow N_P = \dfrac{24 \times 120}{12} \Rightarrow N_P = 440$

24. **(25) :** Given, $y = \dfrac{x^2}{4}$ and $\mu = 0.5$

Let the maximum height is h.

So, the slope of tangent at height h is angle of repose.

$\therefore \quad \tan\theta = \dfrac{dy}{dx} = \dfrac{1}{4} \cdot 2x = \dfrac{x}{2}$



Slope, $m = \tan\theta$

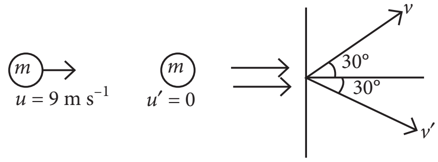
For no slipping, $\tan\theta \le \mu$

$$\Rightarrow 0.5 \ge \frac{x}{2} \Rightarrow x \le 1\text{ m}$$

So, $y = h = \dfrac{x^2}{4} \le 0.25\text{ m} \le 25\text{ cm} \Rightarrow h \le 25\text{ cm}$

$\therefore$ Maximum height, $h_{max} = 25$ cm.

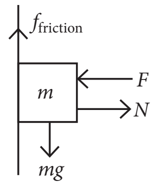
25. **(1) :** Consider collision of the two balls as shown here.



Using conservation of momentum along y axis

$$0 = m \times v\sin 30° - mv'\sin 30° \Rightarrow \frac{v}{v'} = \frac{x}{y} = \frac{1}{1} = 1$$

26. **(25) :** Given, $\mu_s = 0.2$, $m = 0.5$ kg



Let the force is F.

$F = N$ (normal force)

For no motion, friction force = μN

$\Rightarrow f = mg \Rightarrow \mu F = mg$

$$\Rightarrow F = \frac{0.5 \times 10}{0.2} = 25\text{ N}$$

27. **(2000) :** $L = 2 \times 10^{-4}$ H, $R = 6.28\ \Omega$, $f = 10$ MHz

Quality factor, $Q = \dfrac{\omega \cdot L}{R} = 2\pi f \times \dfrac{L}{R}$

$$\Rightarrow Q = \frac{2 \times 3.14 \times 10^6 \times 10 \times 2 \times 10^{-4}}{6.28} = 2000$$

The zener diode will act as voltage regulator. Therefore, voltage across 2 kΩ resistor will be constant (5 V).

28. **(25) :** So current across 2 kΩ is

$$I = \frac{5}{2 \times 1000} = 2.5 \times 10^{-3}\text{ A} \Rightarrow I = 25 \times 10^{-4}\text{ A}$$

29. **(25) :** Modulating voltage, $v_m = 20\sin 2\pi(1500t)$

Carrier voltage, $v_c = 80\sin 2\pi(100000t)$

$$\therefore \text{ Modulation index} = \frac{A_m}{A_c} \times 100\% = \frac{20}{80} \times 100 = 25\%$$

30. **(15) :** Here, $f = 5$ GHz, $\mu_r = 2$, $\varepsilon_r = 2$

Velocity in the medium, $v = \dfrac{1}{\sqrt{\mu\varepsilon}} = \dfrac{1}{\sqrt{\mu_r \varepsilon_r \varepsilon_0 \mu_0}}$

$$= \frac{1}{\sqrt{\mu_r \varepsilon_r}\sqrt{\mu_0 \varepsilon_0}}$$

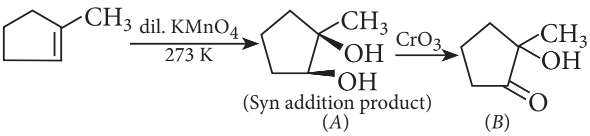
$$v = \frac{c}{\sqrt{\mu_r \varepsilon_r}} = \frac{3 \times 10^8}{\sqrt{2 \times 2}} = 1.5 \times 10^8\text{ m s}^{-1}$$

$\Rightarrow \quad v = 15 \times 10^7\text{ m s}^{-1}$

31. (c) : The α-helix structure of proteins is stabilized by hydrogen bonds between the NH and CO groups of the main chain.

32. (c) : Alkenes get converted into *cis*-diols on reaction with $KMnO_4$ solution by syn-dihydroxylation. Further CrO_3 oxidises primary alcohols to aldehydes and secondary alcohols to ketones. It is a mild oxidising agent, hence does not affect tertiary alcohols.

The reactions can be summarized as :



33. (d) : Isostructural species have same structures.

Ion	Hybridisation	Geometry
SO_4^{2-}	sp^3	Tetrahedral
CrO_4^{2-}	sp^3	Tetrahedral
$SiCl_4$	sp^3	Tetrahedral
$TiCl_4$	sd^3	Tetrahedral
NH_3	sp^3	Pyramidal
NO_3^-	sp^2	Trigonal planar
BCl_3	sp^2	Trigonal planar
$BrCl_3$	sp^3d	T-shaped

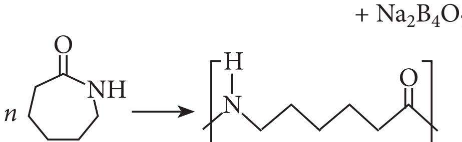
34. (c) : Cupric metaborate is formed by heating boric anhydride with $CuSO_4$ in an oxidising (non-luminous) flame.

$$B_2O_3 + CuSO_4 \xrightarrow[\text{flame}]{\text{Oxidising}} \underset{\substack{\text{(Cupric metaborate)}\\ \text{(Blue-green)}}}{Cu(BO_2)_2} + SO_3$$

Blue cupric metaborate is reduced to colourless cuprous metaborate in reducing (luminous) flame.

$$\underset{\text{(Blue)}}{2Cu(BO_2)_2} + 2NaBO_2 + C \xrightarrow[\text{flame}]{\text{Reducing}} \underset{\text{(Colourless)}}{2CuBO_2} + Na_2B_4O_7 + CO$$

35. (d) :



Caprolactum   Nylon-6 (A-iii)

$$\underset{\substack{\text{2-Chloro-1,3-butadiene}\\ \text{(Chloroprene)}}}{nCH_2=CH-\underset{|\atop Cl}{C}=CH_2} \rightarrow \underset{\substack{\text{Neoprene}\\ \text{(B-iv)}}}{\left[CH_2-CH=\underset{|\atop Cl}{C}-CH_2\right]_n}$$

$$\underset{\text{Isoprene}}{nCH_2=CH-\underset{|\atop CH_3}{C}=CH_2} \rightarrow \underset{\substack{\text{Natural rubber}\\ \text{(C-i)}}}{\left[CH_2-CH=\underset{|\atop CH_3}{C}-CH_2\right]_n}$$

$$\underset{\text{Acrylonitrile}}{nCH_2=\underset{|\atop CN}{CH}} + \underset{\text{1,3-Butadiene}}{nCH_2=CH-CH=CH_2}$$

$$\rightarrow \underset{\substack{\text{Buna-N}\\ \text{(D-ii)}}}{\left(CH_2-CH=CH-CH_2-\overset{CN\atop |}{CH}-CH_2\right)_n}$$

36. (a) :

Sphalerite	:	ZnS
Malachite	:	$CuCO_3{\cdot}Cu(OH)_2$
Calamine	:	$ZnCO_3$
Siderite	:	$FeCO_3$

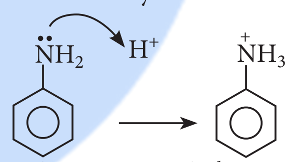
Sphalerite can be dissolved in 1st group cyanide salt *i.e.*, NaCN, KCN.

NaCN reacts with ZnS to form a complex.

$$4NaCN + ZnS \longrightarrow \underset{\text{Soluble complex}}{Na_2[Zn(CN)_4]} + Na_2S$$

37. (c) : Gun metal also known as red brass is a type of bronze and consists of copper, tin and zinc.

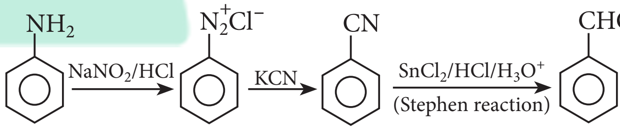
38. (d) : Due to the presence of acid in the reaction mixture $-\ddot{N}H_2$ gets converted to $-\overset{+}{N}H_3$ as $-NH_2$ contains lone pair of electron which can be easily donated.



Aniline (*o,p*-directing)   Anilinium ion (*m*-directing)

39. (c) : Ionisation enthalpy generally increases from left to right in a period due to increase in effective nuclear charge. But *I.E.* (S) < *I.E.* (P) due to the extra stability of half-filled electronic configuration of (P) and *I.E.* (Al) < *I.E.* (Mg) due to the extra stability of fully filled electronic configuration of Mg. Hence, the correct order of *I.E.* is Al < Mg < Si < S < P.

40. (b) :



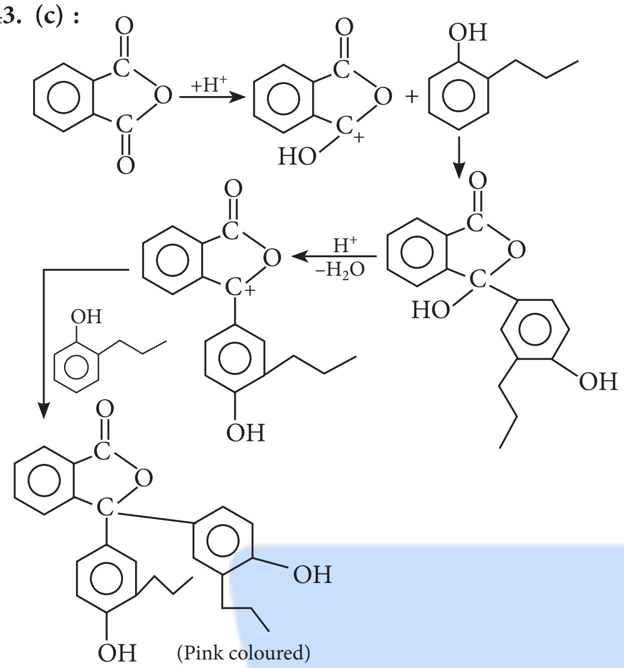
41. (c) : In 3*d* series elements, only Cu shows positive value for electrode potential of M^{2+}/M.

$Co^{2+}/Co = -0.28$ V ; $Zn^{2+}/Zn = -0.76$ V

$Cu^{2+}/Cu = +0.34$ V ; $Fe^{2+}/Fe = -0.44$ V

42. (d) : During anaerobic degradation when microorganisms break down the organic material in the absence of air (or oxygen) the gases released are methane, carbon dioxide with very small amounts of water vapour and other gases. CO_2 and CH_4 gases are responsible for global warming and cancer.

43. (c) :



44. (b) : $CH_3CH_2CH_3 \xrightarrow{Mo_2O_3} CH_3CH_2CHO$

45. (c) :

$$Al_2O_{3(s)} + 2NaOH_{(aq)} + 3H_2O_{(l)} \longrightarrow \underset{X}{2Na[Al(OH)_4]_{aq}}$$

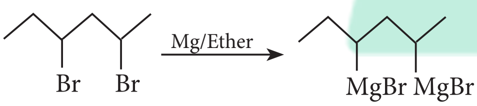
$$\underset{Z}{Al_2O_3{\cdot}xH_2O_{(s)}} + 2NaHCO_{3(aq)} \xleftarrow{\underset{Y}{CO_{2(g)}}}$$

46. (b) : According to Freundlich adsorption isotherm,

$$\frac{x}{m} = kP^{1/n} \;;\; \log\frac{x}{m} = \log k + \frac{1}{n}\log P$$

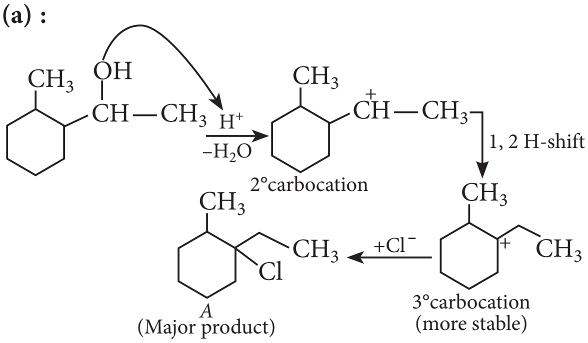
Hence, slope is $\frac{1}{n}$ with $\left(\frac{1}{n} = 0 \text{ to } 1\right)$; $0 \le \frac{1}{n} \le 1$

47. (b) :

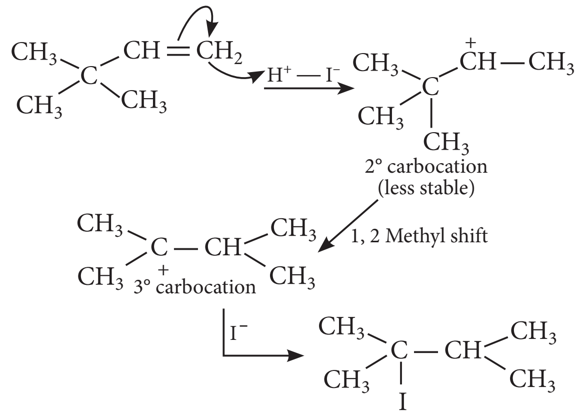


48. (b) : In reaction (A) reduction of HOCl occurs and in reaction (B) reduction of I_2 occurs hence H_2O_2 is acting as a a reducing agent in both the given equations.

49. (a) :



50. (c) :



51. (26) : $k = 3.3 \times 10^{-4}\ s^{-1}$, $x = 0.4$ $\quad (a-x) = 0.6$

$$k = \frac{2.303}{t}\log\frac{a}{a-x} \;;\; t = \frac{2.303}{3.3\times10^{-4}}\log\frac{1}{0.6} = 1554\ s$$

$$t = \frac{1554}{60} = 25.9 \text{ or } 26 \text{ minutes}$$

52. (12) : Balanced equation can be written as

$S_8 + 12OH^- \longrightarrow 4S^{2-} + 2S_2O_3^{2-} + 6H_2O$

The value of a is 12.

53. (1) : Equilibrium constant for the overall reaction

$Cu^{2+} + 4NH_3 \rightleftharpoons [Cu(NH_3)_4]^{2+}$

can be given as $K = K_1 \times K_2 \times K_3 \times K_4$

$K = 10^4 \times 1.58 \times 10^3 \times 5 \times 10^2 \times 10^2 = 7.9 \times 10^{11}$

For dissociation of $[Cu(NH_3)_4]^{2+}$,

Equilibrium constant $K' = \frac{1}{K} = \frac{1}{7.9\times10^{11}} = 1.26 \times 10^{-12}$

Hence, $x = 1$. (rounded off to the nearest integer)

54. (8) : Coordination number of an atom in a body centered cubic structure (bcc) is 8.

55. (5) : $T = 1990$ K ; $P = 1$ atm

$Cl_2 \rightleftharpoons 2Cl$

Moles of Cl_2 = Moles of Cl = x

Total moles = $2x$

$$p_{Cl_2} = \frac{x}{2x}\times 1 = \frac{1}{2} ; p_{Cl} = \frac{x}{2x}\times 1 = \frac{1}{2}$$

$$K_P = \frac{(p_{Cl})^2}{p_{Cl_2}} = \frac{\frac{1}{2}\times\frac{1}{2}}{\frac{1}{2}} = \frac{1}{2} = 0.5 = 5\times10^{-1}$$

The value of x is 5.

56. (2) : Mass of compound A = 4.5

Molar mass = 90

V = 250 mL

$$\text{Molarity} = \frac{\text{Moles of compound } A}{V(\text{mL})} \times 1000$$

$$= \frac{4.5/90}{250} \times 1000 = \frac{1000}{20\times250} = \frac{4}{20} = \frac{1}{5} = 0.2 \text{ mol/L}$$

or 2×10^{-1} M

So, the value of x is 2.

57. (35) : $\Delta T_f = 0.5°C$; $V = 500$ mL

$K_f = 1.86$ K kg mol^{-1}

Mass of compound = 9.45 g

$M = 35.5 + 24 + 3 + 32 = 94.5$

$m = \frac{9.45}{94.5} \times \frac{1000}{500} = 0.2$ [density of water ≈ 1g/mL]

$\Delta T_f = i\, K_f\, m$; $0.5 = i \times 0.2 \times 1.86$

$i = \frac{0.5}{0.2 \times 1.86} = 1.34$; $1 + \alpha = 1.34 \Rightarrow \alpha = 0.34$

At equilibrium : $\underset{C - C\alpha}{ClCH_2COOH} \rightleftharpoons \underset{C\alpha}{ClCH_2COO^-} + \underset{C\alpha}{H^+}$

$$K_a = \frac{(C\alpha)^2}{(C - C\alpha)} = \frac{C\alpha^2}{1-\alpha} = \frac{0.2 \times (0.34)^2}{(1 - 0.34)} = 35 \times 10^{-3}$$

58. (1380) : $K_{eq.} = 100$, $T = 300$ K $P = 1$ atm

$\Delta G = -2.3\, RT \log K = -2.3 \times R \times 300 \log 100$
$= -2.3 \times R \times 2 \times 300 = -1380\, R$

59. (2) : BeO — Amphoteric oxide

BaO — Basic oxide

$Be(OH)_2$ — Amphoteric hydroxide

$Sr(OH)_2$ — Basic hydroxide

60. (2) : $\lambda = \frac{h}{\sqrt{2meV}}$; $\lambda_{Li^{3+}} = \frac{h}{\sqrt{2m_{Li^{3+}}eV}}$; $\lambda_p = \frac{h}{\sqrt{2m_p eV}}$

Given, $V_{Li^{3+}} = V_p$; $m_{Li^{3+}} = 8.3m_p$

$$\frac{\lambda_{Li^{3+}}}{\lambda_p} = \frac{\sqrt{m_p(e) \times V}}{\sqrt{8.3m_p \times 3e \times V}} = \frac{1}{\sqrt{8.3 \times 3}} = \frac{1}{5} = 0.2 = 2 \times 10^{-1}$$

61. (c) : Let the line be $y = mx + c$ ...(i)

$\therefore$ x-intercept is $-\frac{c}{m}$ and y-intercept is c.

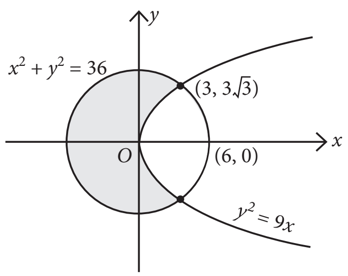
A.M. of reciprocals of the intercepts is

$$\frac{-\frac{m}{c} + \frac{1}{c}}{2} = \frac{1}{4} \Rightarrow 2(1 - m) = c$$

From (i), $y = mx + 2(1 - m)$

$\Rightarrow$ $(y - 2) - m(x - 2) = 0$, which passes through (2, 2).

62. (d) : Points of intersection of given curves are $(3, \pm 3\sqrt{3})$.



$$\therefore \text{ Required area} = \pi \times (6)^2 - 2\int_0^3 \sqrt{9x}\,dx - 2\int_3^6 \sqrt{36 - x^2}\,dx$$

$$= 36\pi - 6\left[\frac{x^{3/2}}{3/2}\right]_0^3 - 2\left[\frac{x}{2}\sqrt{36 - x^2} + 18\sin^{-1}\frac{x}{6}\right]_3^6$$

$$= 36\pi - 12\sqrt{3} - 2\left(9\pi - 3\pi - \frac{9\sqrt{3}}{2}\right) = \left(24\pi - 3\sqrt{3}\right) \text{ sq. units}$$

63. (b) : $e^{(\cos^2 x + \cos^4 x + \cos^6 x + ... \infty)\log_e 2}$

$$= 2^{\cos^2 x + \cos^4 x + ... \infty} = 2^{\cos^2 x\left(\frac{1}{1 - \cos^2 x}\right)} = 2^{\cot^2 x}$$

Now, $t^2 - 9t + 8 = 0 \Rightarrow t = 1, 8$

$\Rightarrow 2^{\cot^2 x} = 1, 8 \Rightarrow \cot^2 x = 0, 3$

Since, $0 < x < \frac{\pi}{2} \Rightarrow \cot x = \sqrt{3}$

$$\therefore \frac{2\sin x}{\sin x + \sqrt{3}\cos x} = \frac{2}{1 + \sqrt{3}\cot x} = \frac{2}{4} = \frac{1}{2}$$

64. (c) :

Indians	Foreigners
2	4
3	6
4	8

$\therefore$ Total number of ways

$= {}^6C_2 \times {}^8C_4 + {}^6C_3 \times {}^8C_6 + {}^6C_4 \times {}^8C_8$

$= 15 \times 70 + 20 \times 28 + 15 \times 1$

$= 1050 + 560 + 15 = 1625$

65. (a) : We have, $\frac{dP}{dt} = 0.5P - 450$

$$\Rightarrow \frac{dP(t)}{dt} = \frac{P(t) - 900}{2} \Rightarrow \int \frac{dP(t)}{P(t) - 900} = \int \frac{dt}{2}$$

$\Rightarrow \log_e |P(t) - 900| = \frac{t}{2} + c$

$\because$ $P(0) = 850$

$\therefore$ $\log_e |-50| = c \Rightarrow c = \log_e |50|$

Thus, we have $\log_e |P(t) - 900| - \log_e |50| = \frac{t}{2}$

When $P(t) = 0$, then

$$\log_e \left|\frac{900}{50}\right| = \frac{t}{2} \Rightarrow t = 2\log_e 18$$

66. (c) : Let $I = \int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}}\,dx$

$$= \int \frac{\cos x - \sin x}{\sqrt{9 - (\sin x + \cos x)^2}}\,dx$$

Put $\sin x + \cos x = t \Rightarrow (\cos x - \sin x)dx = dt$

$$\therefore I = \int \frac{dt}{\sqrt{9 - t^2}} = \sin^{-1}\frac{t}{3} + c = \sin^{-1}\left(\frac{\sin x + \cos x}{3}\right) + c$$

So, $a = 1$, $b = 3$

67. (a) : We have, $f(x) = \frac{4x^3 - 3x^2}{6} - 2\sin x + (2x - 1)\cos x$

$\Rightarrow f'(x) = (2x^2 - x) - 2\cos x + 2\cos x - \sin x(2x - 1)$
$= (2x - 1)(x - \sin x)$

For $x > 0$, $x - \sin x > 0$ and for $x < 0$, $x - \sin x < 0$

$\therefore$ For $x \in \left[\frac{1}{2}, \infty\right), f'(x) \geq 0$

and for $x \in \left(-\infty, \frac{1}{2}\right]$, sign of $f'(x)$ is not same.

So, $f(x)$ increases in $\left[\frac{1}{2}, \infty\right)$.

68. (b) : $(A \wedge (A \to B)) \to B \equiv (A \wedge (\sim A \vee B)) \to B$
$\equiv ((A \wedge \sim A) \vee (A \wedge B)) \to B \equiv (A \wedge B) \to B$
$\equiv (\sim A \vee \sim B) \vee B \equiv T$

69. (a) : $\lim\limits_{x \to 0} \dfrac{\int_0^{x^2} \sin\sqrt{t}\, dt}{x^3}$ $\left(\frac{0}{0} \text{ form}\right)$

$= \lim\limits_{x \to 0} \dfrac{(\sin x)2x}{3x^2}$ [Applying L' Hospital's Rule]

$= \lim\limits_{x \to 0} \left(\dfrac{\sin x}{x}\right) \times \dfrac{2}{3} = \dfrac{2}{3}$

70. (a) : Clearly, possible points of discontinuity are integers.
For $x = n, n \in Z$

L.H.L. $= \lim\limits_{x \to n^-} f(x) = \lim\limits_{x \to n} [x-1]\cos\left(\dfrac{2x-1}{2}\right)\pi = 0$

R.H.L. $= \lim\limits_{x \to n^+} f(x) = \lim\limits_{x \to n} [x-1]\cos\left(\dfrac{2x-1}{2}\right)\pi = 0$

Also, $f(n) = 0$

$\therefore$ L.H.L. = R.H.L. = $f(n)$

So, $f(x)$ is continuous for every real x.

71. (a) : Let $\dfrac{x-3}{1} = \dfrac{y-4}{2} = \dfrac{z-5}{2} = t$ (say)

$\therefore$ $x = 3 + t, y = 2t + 4, z = 2t + 5$, which lie on the plane $x + y + z = 17$ for some $t \in R$
$\therefore$ $3 + t + 2t + 4 + 2t + 5 = 17 \Rightarrow 5t = 5 \Rightarrow t = 1$
$\therefore$ Point of intersection is (4, 6, 7)
Hence, distance between (1, 1, 9) and (4, 6, 7)

$= \sqrt{(4-1)^2 + (6-1)^2 + (7-9)^2} = \sqrt{9 + 25 + 4} = \sqrt{38}$

72. (a) : $D = \begin{vmatrix} 3 & -2 & -k \\ 2 & -4 & -2 \\ 1 & 2 & -1 \end{vmatrix} = 0$

$\Rightarrow$ $24 - 2(0) - k(8) = 0 \Rightarrow k = 3$
At $k = 3$,

$D_1 = \begin{vmatrix} 10 & -2 & -3 \\ 6 & -4 & -2 \\ 5m & 2 & -1 \end{vmatrix}$

$= 10(8) - 2(-10m + 6) - 3(12 + 20m) = 8(4 - 5m)$
For the given system to be inconsistent, $D_1 \neq 0$

$\Rightarrow$ $8(4 - 5m) \neq 0 \Rightarrow m \neq \dfrac{4}{5}$

$\therefore$ System of linear equations is inconsistent, if $k = 3$ and $m \neq \dfrac{4}{5}$.

73. (c) : $f(g(x)) = 2g(x) - 1 = 2\left(\dfrac{2x-1}{2(x-1)}\right) - 1$

$= \dfrac{x}{x-1} = 1 + \dfrac{1}{x-1}$

Range of $f(g(x)) = R - \{1\}$
Thus, range of $f(g(x))$ is not co-domain of $f(g(x))$.
$\therefore$ $f(g(x))$ is not onto.
But $f(g(x))$ is one-one. [$\because f(g(x_1)) = f(g(x_2)) \Rightarrow x_1 = x_2$]
So, $f(g(x))$ is one-one but not onto.

74. (a) : Consider, $p^4 + q^4 = 272$
$\Rightarrow$ $(p^2 + q^2)^2 - 2p^2q^2 = 272$
$\Rightarrow$ $((p + q)^2 - 2pq)^2 - 2p^2q^2 = 272$
$\Rightarrow$ $(p + q)^4 + 4p^2q^2 - 4pq(p + q)^2 - 2p^2q^2 = 272$
$\Rightarrow$ $16 - 16pq + 2p^2q^2 = 272$ [Using $p + q = 2$]
$\Rightarrow$ $(pq)^2 - 8(pq) - 128 = 0$

$\therefore$ $pq = \dfrac{8 \pm 24}{2} = 16, -8 \Rightarrow pq = 16$

Thus, p, q are the roots of the equation $x^2 - 2x + 16 = 0$.

75. (c) : P(getting an odd number twice)
= P(getting an even number thrice)

$\Rightarrow$ ${}^nC_2\left(\dfrac{1}{2}\right)^n = {}^nC_3\left(\dfrac{1}{2}\right)^n \Rightarrow {}^nC_2 = {}^nC_3 \Rightarrow n = 2 + 3$

$\Rightarrow$ $n = 5$
Probability of getting an odd number for odd number of times $= {}^5C_1\left(\dfrac{1}{2}\right)^5 + {}^5C_3\left(\dfrac{1}{2}\right)^5 + {}^5C_5\left(\dfrac{1}{2}\right)^5 = \dfrac{1}{2^5}(5 + 10 + 1)$

$= \dfrac{16}{2^5} = \dfrac{1}{2}$

76. (d) : Normal vector of required plane is

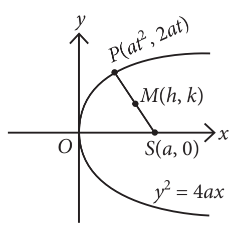
$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 2 & -5 & -1 \end{vmatrix} = -11\hat{i} - \hat{j} - 17\hat{k}$

Also, plane passes through (1, 2, –3).
So, equation of required plane is
$11(x - 1) + 1(y - 2) + 17(z + 3) = 0$
$\Rightarrow$ $11x + y + 17z + 38 = 0$

77. (b) : Here, $h = \dfrac{at^2 + a}{2}$ and $k = \dfrac{2at + 0}{2}$

$\Rightarrow$ $t^2 = \dfrac{2h - a}{a}$ and $t = \dfrac{k}{a}$

$\therefore$ $\dfrac{k^2}{a^2} = \dfrac{2h - a}{a} \Rightarrow k^2 = a(2h - a)$



Locus of (h, k) is $y^2 = a(2x - a)$

$\Rightarrow$ $y^2 = 2a\left(x - \dfrac{a}{2}\right)$, which is a parabola.

$\therefore$ Its directrix is $x - \frac{a}{2} = -\frac{a}{2} \Rightarrow x = 0$

78. (a) : We have

$(-{}^{15}C_1 + 2.{}^{15}C_2 - 3.{}^{15}C_3 + ... - 15.{}^{15}C_{15})$

$+ ({}^{14}C_1 + {}^{14}C_3 + ...+ {}^{14}C_{11})$

$= \sum_{r=1}^{15} (-1)^r r.{}^{15}C_r + ({}^{14}C_1 + {}^{14}C_3 + ...+ {}^{14}C_{11} + {}^{14}C_{13}) - {}^{14}C_{13}$

$= \sum_{r=1}^{15} (-1)^r r.\frac{15}{r}\, {}^{14}C_{r-1} + 2^{13} - 14$

$= 15(-{}^{14}C_0 + {}^{14}C_1 - ..... - {}^{14}C_{14}) + 2^{13} - 14$

$= 15(0) + 2^{13} - 14 = 2^{13} - 14$

79. (b) : We have, $y = x^3$ ...(i)

Slope of tangent at $P(t, t^3)$ is

$\left[\frac{dy}{dx}\right]_{(t,t^3)} = (3x^2)_{x=t} = 3t^2$

So, equation of tangent at $P(t, t^3)$ is given by

$y - t^3 = 3t^2(x - t)$ ...(ii)

From (i) and (ii), we get $x^3 - t^3 = 3t^2(x - t)$

$\Rightarrow (x - t)(x^2 + xt + t^2) = 3t^2(x - t)$

$\Rightarrow x^2 + xt + t^2 = 3t^2$ $[\because x \neq t]$

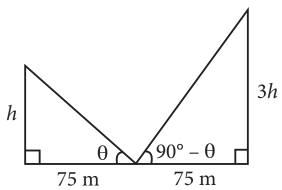
$\Rightarrow x^2 + xt - 2t^2 = 0 \Rightarrow (x - t)(x + 2t) = 0$

$\therefore x = -2t$ and $y = -8t^3$

$\Rightarrow Q \equiv (-2t, -8t^3)$

$\therefore$ Ordinate of required point is $\frac{2t^3 + (-8t^3)}{2+1} = -2t^3$

80. (d) :



Here, $\tan\theta = \frac{h}{75} = \frac{75}{3h} \Rightarrow h^2 = \frac{(75)^2}{3} \Rightarrow h = 25\sqrt{3}$ m

81. (6) : Let $P(B_1) = p_1$, $P(B_2) = p_2$ and $P(B_3) = p_3$

Given, $p_1(1 - p_2)(1 - p_3) = \alpha$ ...(i)

$p_2(1 - p_1)(1 - p_3) = \beta$ ...(ii)

$p_3(1 - p_1)(1 - p_2) = \gamma$ ...(iii)

and $(1 - p_1)(1 - p_2)(1 - p_3) = p$ ...(iv)

$\Rightarrow \frac{p_1}{1-p_1} = \frac{\alpha}{p}, \frac{p_2}{1-p_2} = \frac{\beta}{p}$ and $\frac{p_3}{1-p_3} = \frac{\gamma}{p}$

Also, $\beta = \frac{\alpha p}{\alpha + 2p} = \frac{3\gamma p}{p - 2\gamma}$

$\Rightarrow \alpha p - 2\alpha\gamma = 3\alpha\gamma + 6p\gamma \Rightarrow \alpha p - 6p\gamma = 5\alpha\gamma$

$\Rightarrow \frac{p_1}{1-p_1} - \frac{6p_3}{1-p_3} = \frac{5p_1p_3}{(1-p_1)(1-p_3)} \Rightarrow p_1 - 6p_3 = 0$

$\Rightarrow \frac{p_1}{p_3} = 6$, *i.e.*, $\frac{P(B_1)}{P(B_3)} = 6$

82. (1) : $\lim_{n\to\infty} \tan\left(\sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{1+r(r+1)}\right)\right)$

$= \lim_{n\to\infty} \tan\left(\sum_{r=1}^{n} \tan^{-1}\left(\frac{r+1-r}{1+r(r+1)}\right)\right)$

$= \tan\left(\lim_{n\to\infty} \sum_{r=1}^{n} [\tan^{-1}(r+1) - \tan^{-1}(r)]\right)$

$= \tan\left(\lim_{n\to\infty} \left(\tan^{-1}(n+1) - \tan^{-1}1\right)\right)$

$= \tan\left(\tan^{-1}(\infty) - \frac{\pi}{4}\right) = \tan\left(\frac{\pi}{2} - \frac{\pi}{4}\right) = \tan\left(\frac{\pi}{4}\right) = 1$

83. (10) : The given equation is $z + \alpha|z - 1| + 2i = 0$

Putting $z = x + iy$, we get

$x + iy + \alpha|x + iy - 1| + 2i = 0$

$\Rightarrow x + \alpha\sqrt{(x-1)^2 + y^2} + i(y+2) = 0 + 0i$

$\Rightarrow y + 2 = 0$ and $x + \alpha\sqrt{(x-1)^2 + y^2} = 0$

$\Rightarrow y = -2$ and $\alpha^2 = \frac{x^2}{x^2 - 2x + 5}$

Now, $\frac{x^2}{x^2 - 2x + 5} \in \left[0, \frac{5}{4}\right]$

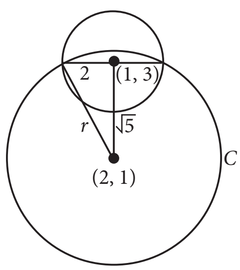
$\therefore \alpha^2 \in \left[0, \frac{5}{4}\right] \Rightarrow \alpha \in \left[-\frac{\sqrt{5}}{2}, \frac{\sqrt{5}}{2}\right]$

$\therefore p = -\frac{\sqrt{5}}{2};\ q = \frac{\sqrt{5}}{2}$

$\Rightarrow 4(p^2 + q^2) = 4\left(\frac{5}{4} + \frac{5}{4}\right) = 10$

84. (3) : The given equation of circle is $x^2 + y^2 - 2x - 6y + 6 = 0$, whose center is (1, 3) and radius is

$\sqrt{(1)^2 + (3)^2 - 6} = 2$



$\therefore$ Distance between (1, 3) and (2, 1) is $\sqrt{(2-1)^2 + (1-3)^2} = \sqrt{5}$.

$\therefore (\sqrt{5})^2 + (2)^2 = r^2 \Rightarrow r^2 = 9$

$\Rightarrow r = 3$

85. (540) : Let $M = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix} \Rightarrow M^T = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$.

Now, $M^TM = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$

Since, sum of diagonal elements of $M^TM = 7$

$\Rightarrow a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 7$

Case-I : Seven (1's) and two (0's)

So, number of possible matrices $= \dfrac{9!}{7!2!} = 36$

Case-II : One (2's), three (1's) and five (0's)

So, number of possible matrices $= \dfrac{9!}{5!3!} = 504$

$\therefore$ Total number of possible matrices = 504 + 36 = 540

86. (3) : $\int_{-a}^{0}(-2x+2)dx + \int_{0}^{2}(x+2-x)dx + \int_{2}^{a}(2x-2)dx = 22$

$\Rightarrow \left[x^2-2x\right]_0^{-a} + [2x]_0^2 + \left[x^2-2x\right]_2^a = 22$

$\Rightarrow a^2 + 2a + 4 + a^2 - 2a - (4-4) = 22$

$\Rightarrow 2a^2 = 18$

$\Rightarrow a = 3$

$\therefore \int_{3}^{-3}(x+[x])dx = -\int_{-3}^{3}[x]dx = -(-3-2-1+1+2) = 3$

87. (17) : Given $PQ = kI_3$

$\Rightarrow |PQ| = k^3|I_3|$

$\Rightarrow |P|\cdot|Q| = k^3 \Rightarrow |P|\dfrac{k^2}{2} = k^3$

$\Rightarrow |P| = 2k \neq 0 \Rightarrow P$ is an invertible matrix.

$\therefore Q = kP^{-1}I = \dfrac{k\,\text{adj}P}{|P|} = \dfrac{k}{2k}\text{adj}P \Rightarrow Q = \dfrac{\text{adj}P}{2}$

Now, $q_{23} = -\dfrac{k}{8} \Rightarrow \dfrac{-(3\alpha+4)}{2} = -\dfrac{k}{8}$ ...(i)

Also, $|P| = 2k \Rightarrow 12\alpha + 20 = 2k \Rightarrow k = 10 + 6\alpha$

$\therefore$ From (i), we get $\dfrac{-(3\alpha+4)}{2} = -\dfrac{(10+6\alpha)}{8} \Rightarrow \alpha = -1$

and $k = 10 + 6(-1) = 4$

$\therefore \alpha^2 + k^2 = (-1)^2 + 4^2 = 17$

88. (75) : Let $\vec{c} = \lambda(\vec{b}\times(\vec{a}\times\vec{b})) = \lambda((\vec{b}\cdot\vec{b})\vec{a} - (\vec{b}\cdot\vec{a})\vec{b})$

$= \lambda(5(-\hat{i}+\hat{j}+\hat{k}) + 2\hat{i}+\hat{k}) = \lambda(-3\hat{i}+5\hat{j}+6\hat{k})$

Also, $\vec{a}\cdot\vec{c} = 7 \Rightarrow 3\lambda + 5\lambda + 6\lambda = 7 \Rightarrow 14\lambda = 7$

$\Rightarrow \lambda = \dfrac{1}{2}$

Now, $\vec{a}+\vec{b}+\vec{c} = \left(\dfrac{-3}{2} - 1 + 2\right)\hat{i} + \left(\dfrac{5}{2}+1\right)\hat{j} + (3+1+1)\hat{k}$

$= -\dfrac{1}{2}\hat{i} + \dfrac{7}{2}\hat{j} + 5\hat{k}$

$\therefore 2|\vec{a}+\vec{b}+\vec{c}|^2 = 2\left(\dfrac{1}{4} + \dfrac{49}{4} + 25\right) = 2\left(\dfrac{150}{4}\right) = 75$

89. (5) : B and C will contain three digit numbers of the form $9k + 2$ and $9k + l$ respectively. We need to find sum of all 3-digit numbers in the set $B \cup C$.

Now, $S(B \cup C) = S(B) + S(C) - S(B \cap C)$,

where $S(K)$ denotes sum of 3-digit numbers of set K.

Since, 3-digit numbers in set $B = \{101, 110, \ldots\ldots, 992\}$

$\therefore S(B) = \dfrac{100}{2}(101 + 992) = 54650$

Case-I : If $l = 2$, then $B \cap C = B$

$\therefore S(B \cup C) = S(B)$, which is not possible as given sum is $274 \times 400 = 109600$

Case-II : If $l \neq 2$, then $B \cap C = \phi$

$\therefore S(B \cup C) = S(B) + S(C) = 400 \times 274$

$\Rightarrow 54650 + \sum_{k=11}^{110}(9k + l) = 109600$

$\Rightarrow 9\sum_{k=11}^{110} k + \sum_{k=11}^{110} l = 54950$

$\Rightarrow 9\left(\dfrac{100}{2}(11+110)\right) + l(100) = 54950$

$\Rightarrow 54450 + 100l = 54950 \Rightarrow l = 5$

90. (9) : Let $y = \dfrac{4}{\sin x} + \dfrac{1}{1-\sin x} = \dfrac{4 - 3\sin x}{\sin x(1-\sin x)}$

Let $\sin x = t$, where $t \in (0, 1)$

$\therefore y = \dfrac{4-3t}{t-t^2}$

Now, $\dfrac{dy}{dt} = \dfrac{(t-t^2)(-3) - (4-3t)(1-2t)}{(t-t^2)^2} = 0$

$\Rightarrow 3t^2 - 3t - (4 - 11t + 6t^2) = 0$

$\Rightarrow 3t^2 - 8t + 4 = 0 \Rightarrow (3t-2)(t-2) = 0$

$\Rightarrow t = \dfrac{2}{3}$ $[\because t \in (0,1)] \Rightarrow \sin x = \dfrac{2}{3}$

$\therefore y_{min} = \dfrac{4}{2/3} + \dfrac{1}{1-2/3} = 9$ $[\because y_{max} \to \infty]$

$\Rightarrow \alpha_{min} = 9$