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JEE MAIN 2021

ONLINE
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2nd Shift

PHYSICS

SECTION-A (MULTIPLE CHOICE QUESTIONS)

1. An electron of mass m_e and a proton of mass $m_p = 1836\ m_e$ are moving with the same speed. The ratio of their de Brogl ie wavelength $\dfrac{\lambda_{\text{electron}}}{\lambda_{\text{proton}}}$ will be

(a) 1 (b) $\dfrac{1}{1836}$

(c) 918 (d) 1836

2. For extrinsic semiconductors, when doping level is increased

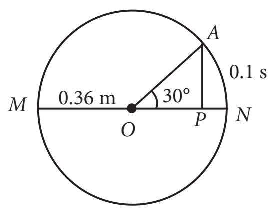
(a) Fermi-level of *p*-type semiconductors will go downward and Fermi-level of *n*-type semiconductor will go upward.

(b) Fermi-level of *p*-type semiconductor will go upward and Fermi-level of *n*-type semiconductors will go downward.

(c) Fermi-level of both *p*-type and *n*-type semiconductors will go upward for $T > T_F$ K and downward for $T < T_F$ K, where T_F is Fermi temperature.

(d) Fermi-level of *p* and *n*-type semiconductors will not be affected.

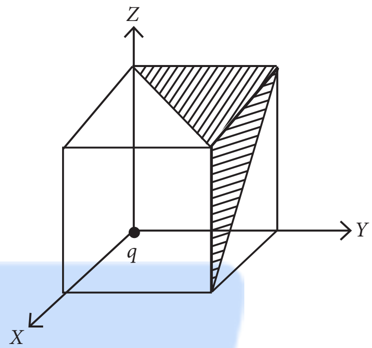
3. The point A moves with a uniform speed along the circumference of a circle of radius 0.36 m and covers 30° in 0.1 s. The perpendicular projection 'P' from 'A' on the diameter MN represents the simple harmonic motion of 'P'. The restoration forces per unit mass when P touches M will be



(a) 9.87 N (b) 0.49 N

(c) 100 N (d) 50 N

4. A charge 'q' is placed at one corner of a cube as shown in figure. The flux of electrostatic field $\vec{E}$ through the shaded area is



(a) $\dfrac{q}{8\varepsilon_0}$ (b) $\dfrac{q}{48\varepsilon_0}$ (c) $\dfrac{q}{4\varepsilon_0}$ (d) $\dfrac{q}{24\varepsilon_0}$

5. If e is the electronic charge, c is the speed of light in free space and h is Planck's constant, the quantity $\dfrac{1}{4\pi\varepsilon_0}\dfrac{|e|^2}{hc}$ has dimensions of

(a) $[\mathrm{M\,L\,T^{-1}}]$ (b) $[\mathrm{L\,C^{-1}}]$

(c) $[\mathrm{M\,L\,T^{0}}]$ (d) $[\mathrm{M^0\,L^0\,T^0}]$

6. The stopping potential for electrons emitted from a photosensitive surface illuminated by light of wavelength 491 nm is 0.710 V. When the incident wavelength is changed to a new value, the stopping potential is 1.43 V. The new wavelength is

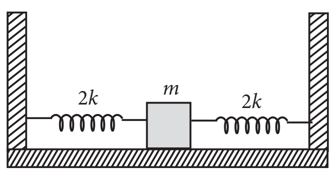
(a) 382 nm (b) 309 nm (c) 329 nm (d) 400 nm

7. The wavelength of the photon emitted by a hydrogen atom when an electron makes a transition from $n = 2$ to $n = 1$ state is

(a) 913.3 nm (b) 121.8 nm

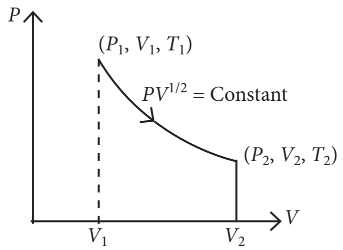
(c) 490.7 nm (d) 194.8 nm

8. Two identical springs of spring constant '$2k$' are attached to a block of mass m and to fixed support (see figure). When the mass is displaced from equilibrium position on either side, it executes simple harmonic motion. The time period of oscillations of this system is



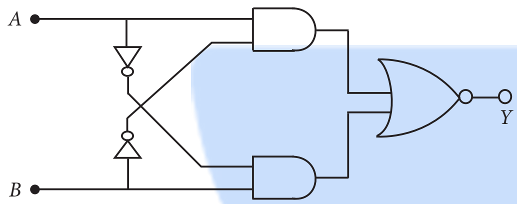
(a) $\pi\sqrt{\dfrac{m}{2k}}$ (b) $\pi\sqrt{\dfrac{m}{k}}$ (c) $2\pi\sqrt{\dfrac{m}{k}}$ (d) $2\pi\sqrt{\dfrac{m}{2k}}$

9. Thermodynamic process is shown below on a P-V diagram for one mole of an ideal gas. If $V_2 = 2V_1$ then the ratio of temperature T_2/T_1 is



(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) 2 (d) $\sqrt{2}$

10. The truth table for the following logic circuit is



(a)

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

(b)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

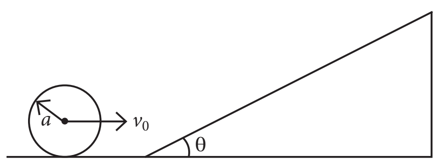
(c)

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

(d)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

11. A sphere of radius 'a' and mass 'm' rolls along a horizontal plane with constant speed v_0. It encounters an inclined plane at angle θ and climbs upward. Assuming that it rolls without slipping, how far up the sphere will travel?



(a) $\frac{2}{5}\frac{v_0^2}{g\sin\theta}$ (b) $\frac{v_0^2}{2g\sin\theta}$

(c) $\frac{v_0^2}{5g\sin\theta}$ (d) $\frac{10v_0^2}{7g\sin\theta}$

12. An electron with kinetic energy K_1 enters between parallel plates of a capacitor at an angle 'α' with the plates. It leaves the plates at angle 'β' with kinetic energy K_2. Then the ratio of kinetic energies $K_1 : K_2$ will be

(a) $\frac{\sin^2\beta}{\cos^2\alpha}$ (b) $\frac{\cos\beta}{\cos\alpha}$

(c) $\frac{\cos\beta}{\sin\alpha}$ (d) $\frac{\cos^2\beta}{\cos^2\alpha}$

13. In a ferromagnetic material, below the curie temperature, a domain is defined as

(a) a macroscopic region with consecutive magnetic dipoles oriented in opposite direction.

(b) a macroscopic region with randomly oriented magneic dipoles.

(c) a macroscopic region with saturation magnetization.

(d) a macroscopic region with zero magnetization.

14. Consider the diffraction pattern obtained from the sunlight incident on a pinhole of diameter 0.1 μm. If the diameter of the pinhole is slightly increased, it will affect the diffraction pattern such that

(a) its size decreases, and intensity increases

(b) its size increases, and intensity decreases

(c) its size decreases, and intensity decreases

(d) its size increases, and intensity increases

15. $Y = A\sin(\omega t + \phi_0)$ is the time-displacement equation of a SHM. At $t = 0$, the displacement of the particle is $Y = \frac{A}{2}$ and it is moving along negative x-direction. Then the initial phase angle ϕ_0 will be

(a) $\pi/3$ (b) $2\pi/3$ (c) $5\pi/6$ (d) $\pi/6$

16. Match List I with List II.

List I		List II	
(A)	Rectifier	(i)	Used either for stepping up or stepping down the a.c. voltage
(B)	Stabilizer	(ii)	Used to convert a.c. voltage into d.c. voltage
(C)	Transformer	(iii)	Used to remove any ripple in the rectified output voltage
(D)	Filter	(iv)	Used for constant output voltage even when the input voltage or load current change

Choose the correct answer from the options given below :

(a) (A) – (ii), (B) – (iv), (C) – (iv), (D) – (iii)

(b) (A) – (iii), (B) – (iv), (C) – (i), (D) – (ii)

(c) (A) – (ii), (B) – (i), (C) – (iii), (D) – (iv)

(d) (A) – (ii), (B) – (iv), (C) – (i), (D) – (iii)

17. An LCR circuit contains resistance of 110 Ω and a supply of 220 V at 300 rad/s angular frequency. If only capacitance is removed from the circuit, current lags behind the voltage by 45°. If on the other hand, only inductor is removed the current leads by 45° with the applied voltage. The rms current flowing in the circuit will be
(a) 1.5 A (b) 2 A (c) 2.5 A (d) 1 A

18. A stone is dropped from the top of a building. When it crosses a point 5 m below the top, another stone starts to fall from a point 25 m below the top. Both stones reach the bottom of building simultaneously. The height of the building is
(a) 50 m (b) 35 m
(c) 45 m (d) 25 m

19. If a message signal of frequency f_m is amplitude modulated with a carrier signal of frequency f_c and radiated through an antenna, the wavelength of the corresponding signal in air is
(a) $\frac{c}{f_c - f_m}$ (b) $\frac{c}{f_c + f_m}$
(c) $\frac{c}{f_m}$ (d) $\frac{c}{f_c}$

20. Given below are two statements.
Statement I : In a diatomic molecule, the rotational energy at a given temperature obeys Maxwell's distribution
Statement II : In a diatomic molecule, the rotational energy at a given temperature equals the translational kinetic energy for each molecule.
In the light of the above statements, choose the correct answer from the options given below.
(a) Statement I is true but statement II is false.
(b) Statement I is false but statement II is true.
(c) Both statement I and Statement II are true.
(d) Both statement I and Statement II are false.

SECTION-B (NUMERICAL VALUE TYPE)

Attempt any 5 questions out of 10.

21. Two small spheres each of mass 10 mg are suspended from a point by threads 0.5 m long. They are equally charged and repel each other to a distance of 0.20 m. The charge on each of the sphere is $\frac{a}{21} \times 10^{-8}$ C. The value of 'a' will be ______. [Given $g = 10$ m s^{-2}]

22. The initial velocity v_i required to project a body vertically upward from the surface of the earth to reach a height of $10R$, where R is the radius of the earth, may be described in terms of escape velocity v_e such that $v_i = \sqrt{\frac{x}{y}} \times v_e$. The value of x will be ______.

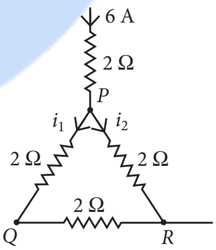
23. The wavelength of an X-ray beam is 10Å. The mass of a fictitious particle having the same energy as that of the X-ray photons is $\frac{x}{3}h$ kg. The value of x is ______.
[h = Planck's constant)

24. Two identical conducting spheres with negligible volume have 2.1 nC and –0.1 nC charges, respectively. They are brought into contact and then separated by a distance of 0.5 m. The electrostatic force acting between the spheres is ______ $\times 10^{-9}$ N. $\left[\text{Given : } 4\pi\varepsilon_0 = \frac{1}{9\times10^9} \text{ SI unit}\right]$

25. If $\vec{P} \times \vec{Q} = \vec{Q} \times \vec{P}$, the angle between $\vec{P}$ and $\vec{Q}$ is $\theta (0° < \theta < 360°)$. The value of '$\theta$' will be ______°.

26. The percentage increase in the speed of transverse waves produced in a stretched string if the tension is increased by 4%, will be ______%.

27. A current of 6 A enters one corner P of an equilateral triangle PQR having 3 wires of resistance 2 Ω each and leaves by the corner R. The currents i_1 in ampere is ______.



28. A reversible heat engine converts one-fourth of the heat input into work. When the temperature of the sink is reduced by 52 K, its efficiency is doubled. The temperature in Kelvin of the source will be ______.

29. The peak electric field produced by the radiation coming from the 8 W bulb at a distance of 10 m is $\frac{x}{10}\sqrt{\frac{\mu_0 c}{\pi}} \frac{V}{m}$. The efficiency of the bulb is 10% and it is a point source. The value of x is ______.

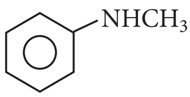
30. Two particles having masses 4 g and 16 g respectively are moving with equal kinetic energies. The ratio of the magnitude of their linear momentum is $n : 2$. The value of n will be ______.

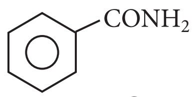
CHEMISTRY

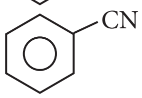
SECTION-A (MULTIPLE CHOICE QUESTIONS)

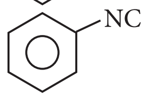
31. Which among the following species has unequal bond lengths?
(a) BF_4^- (b) XeF_4 (c) SF_4 (d) SiF_4

32. Carbylamine test is used to detect the presence of primary amino group in an organic compound. Which of the following compounds is formed when this test is performed with aniline?

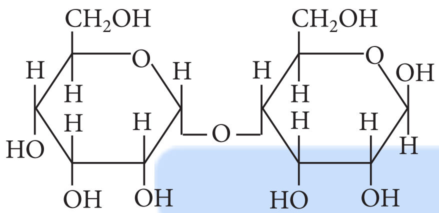
(a) 

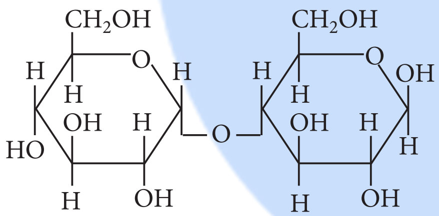
(b) 

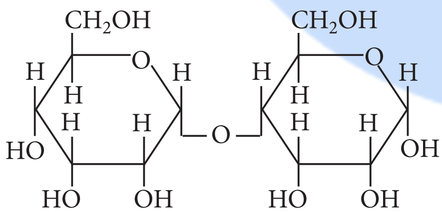
(c) 

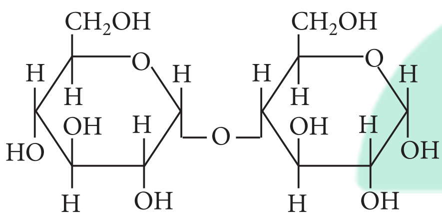
(d) 

33. Which of the following is correct structure of α-anomer of maltose?

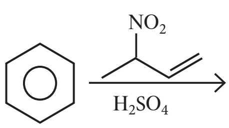
(a) 

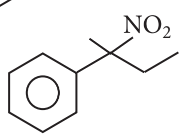
(b) 

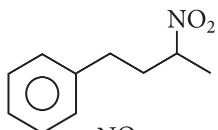
(c) 

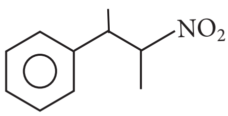
(d) 

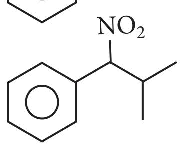
34. The major product of the following reaction is



(a) 

(b) 

(c) 

(d) 

35. The correct sequence of reagents used in the preparation of 4-bromo-2-nitroethyl benzene from benzene is

(a) HNO_3/H_2SO_4, $Br_2/AlCl_3$, $CH_3COCl/AlCl_3$, Zn-Hg/HCl

(b) $Br_2/AlBr_3$, $CH_3COCl/AlCl_3$, HNO_3/H_2SO_4, Zn/HCl

(c) $CH_3COCl/AlCl_3$, $Br_2/AlBr_3$, HNO_3/H_2SO_4, Zn/HCl

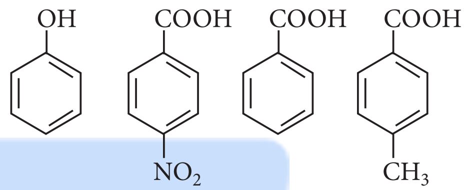
(d) $CH_3COCl/AlCl_3$, Zn-Hg/HCl, $Br_2/AlBr_3$, HNO_3/H_2SO_4

36. Water does not produce CO on reacting with

(a) CO_2 (b) C

(c) CH_4 (d) C_3H_8

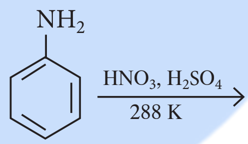
37. The correct order of acid character of the following compounds is

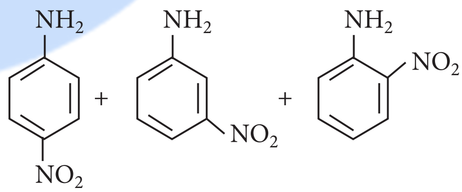


I II III IV

(a) III > II > I > IV (b) IV > III > II > I

(c) I > II > III > IV (d) II > III > IV > I

38. 



(*A*) (*B*) (*C*)

Correct statement about the given chemical reaction is

(a) $-\ddot{N}H_2$ group is *ortho* and *para* directive, so product (*B*) is not possible

(b) reaction is possible and compound (*B*) will be the major product

(c) the reaction will form sulphonated product instead of nitration

(d) reaction is possible and compound (*A*) will be major product.

39. The correct order of bond dissociation enthalpy of halogens is

(a) $Cl_2 > F_2 > Br_2 > I_2$ (b) $I_2 > Br_2 > Cl_2 > F_2$

(c) $Cl_2 > Br_2 > F_2 > I_2$ (d) $F_2 > Cl_2 > Br_2 > I_2$

40. Given below are two statements :

Statement I : The pH of rain water is normally ~5.6.

Statement II : If the pH of rain water drops below 5.6, it is called acid rain.

In the light of the above statements, choose the correct answer from the options given below.

(a) Statement I is true but statement II is false.
(b) Both statement I and statement II are false.
(c) Statement I is false but statement II is true.
(d) Both statement I and statement II are true.

41. The major components of German silver are
(a) Ge, Cu and Ag (b) Zn, Ni and Ag
(c) Cu, Zn and Ni (d) Cu, Zn and Ag.

42. In which of the following order the given complex ions are arranged correctly with respect to their decreasing spin only magnetic moment?
(i) $[FeF_6]^{3-}$ (ii) $[Co(NH_3)_6]^{3+}$
(iii) $[NiCl_4]^{2-}$ (iv) $[Cu(NH_3)_4]^{2+}$
(a) (i) > (iii) > (iv) > (ii) (b) (ii) > (iii) > (i) > (iv)
(c) (iii) > (iv) > (ii) > (i) (d) (ii) > (i) > (iii) > (iv)

43. Which of the following compounds is added to the sodium extract before addition of silver nitrate for testing of halogens?
(a) Nitric acid (b) Ammonia
(c) Hydrochloric acid (d) Sodium hydroxide

44. Which one of the following statements is false for hydrophilic sols?
(a) Their viscosity is of the order of that of H_2O.
(b) The sols cannot be easily coagulated.
(c) They do not require electrolytes for stability.
(d) These sols are reversible in nature.

45. The solubility of $Ca(OH)_2$ in water is
[Given : The solubility product of $Ca(OH)_2$ in water $= 5.5 \times 10^{-6}$]
(a) 1.77×10^{-6} (b) 1.11×10^{-6}
(c) 1.11×10^{-2} (d) 1.77×10^{-2}

46. Given below are two statements :
Statement I : The identification of Ni^{2+} is carried out by dimethyl glyoxime in the presence of NH_4OH.
Statement II : The dimethyl glyoxime is a bidentate neutral ligand.
In the light of the above statements, choose the correct answer from the options given below.
(a) Statement I is false but statement II is true.
(b) Both statement I and statement II are false.
(c) Statement I is true but statement II is false.
(d) Both statement I and statement II are true.

47. The major product of the following reaction is

$$CH_3CH_2CH{=}CH_2 \xrightarrow[\text{Rh catalyst}]{H_2/CO}$$

(a) $CH_3CH_2CH{=}CH{-}CHO$
(b) $\begin{array}{l} CH_3CH_2C{=}CH_2 \\ \quad\quad\quad\;\; | \\ \quad\quad\;\;\; CHO \end{array}$
(c) $CH_3CH_2CH_2CH_2CHO$
(d) $CH_3CH_2CH_2CHO$

48. The method used for the purification of indium is
(a) van Arkel method (b) liquation
(c) zone refining (d) vapour phase refining.

49. What is 'X' in the given reaction?

$$\begin{array}{l} CH_2OH \\ | \\ CH_2OH \end{array} + \text{oxalic acid} \xrightarrow{210°C} \underset{\text{(Major Product)}}{X}$$

(a) $\begin{array}{l} CH_2 \\ \| \\ CH_2 \end{array}$ (b) $\begin{array}{l} CH{-}OH \\ \| \\ CH_2 \end{array}$
(c) $\begin{array}{l} CHO \\ | \\ CHO \end{array}$ (d) $\begin{array}{l} CH_2OH \\ | \\ CHO \end{array}$

50. Given below are two statements :
Statement I : α and β forms of sulphur can change reversibly between themselves with slow heating or slow cooling.
Statement II : At room temperature, the stable crystalline form of sulphur is monoclinic sulphur.
In the light of the above statements, choose the correct answer from the options given below.
(a) Statement I is false but statement II is true.
(b) Both statement I and statement II are true.
(c) Statement I is true but statement II is false.
(d) Both statement I and statement II are false.

SECTION-B (NUMERICAL VALUE TYPE)

Attempt any 5 questions out of 10.

51. If a compound AB dissociates to the extent of 75% in an aqueous solution, the molality of the solution which shows a 2.5 K rise in the boiling point of the solution is ________ molal. (Rounded-off to the nearest integer)
$[K_b = 0.52 \text{ K kg mol}^{-1}]$

52. The number of compound/s given below which contain/s —COOH group is ________. (Integer Value)
(A) Sulphanilic acid (B) Picric acid
(C) Aspirin (D) Ascorbic acid

53. The rate constant of a reaction increases by five times on increase in temperature from 27°C to 52°C. The value of activation energy in kJ mol^{-1} is ________. (Rounded-off to the nearest integer) $[R = 8.314 \text{ J K}^{-1} \text{ mol}^{-1}]$

54. Among the following, number of metal/s which can be used as electrodes in the photoelectric cell is ________.
(Integer Value)
(A) Li (B) Na (C) Rb (D) Cs

55. The spin only magnetic moment of a divalent ion in aqueous solution (atomic number 29) is ________ B.M.

56. Electromagnetic radiation of wavelength 663 nm is just sufficient to ionise the atom of metal A. The ionization energy of metal A in kJ mol^{-1} is ________. (Rounded-off to the nearest integer)
$[h = 6.63 \times 10^{-34} \text{ J s}, c = 3.00 \times 10^8 \text{ m s}^{-1}, N_A = 6.02 \times 10^{23} \text{ mol}^{-1}]$

57. Consider titration of NaOH solution *versus* 1.25 M oxalic acid solution. At the end point, following burette readings were obtained.

(i) 4.5 mL (ii) 4.5 mL (iii) 4.4 mL (iv) 4.4 mL
(v) 4.4 mL
If the volume of oxalic acid taken was 10.0 mL then the molarity of the NaOH solution is _______ M. (Rounded-off to the nearest integer)

58. Five moles of an ideal gas at 293 K is expanded isothermally from an initial pressure of 2.1 MPa to 1.3 MPa against at constant external pressure 4.3 MPa. The heat transferred in this process is _______ kJ mol^{-1}. (Rounded-off to the nearest integer)
[Use $R = 8.314$ J $mol^{-1}K^{-1}$]

59. Copper reduces NO_3^- into NO and NO_2 depending upon the concentration of HNO_3 in solution. (Assuming fixed $[Cu^{2+}]$ and $P_{NO} = P_{NO_2}$), the HNO_3 concentration at which the thermodynamic tendency for reduction of NO_3^- into NO and NO_2 by copper is same is 10^x M. The value of $2x$ is _______.
(Rounded-off to the nearest integer)
[Given : $E^\circ_{Cu^{2+}/Cu} = 0.34$ V, $E^\circ_{NO_3^-/NO} = 0.96$ V,
$E^\circ_{NO_3^-/NO_2} = 0.79$ V and at 298 K, $\frac{RT}{F}(2.303) = 0.059$]

60. The unit cell of copper corresponds to a face centered cube of edge length 3.596 Å with one copper atom at each lattice point. The calculated density of copper in kg/m^3 is _______. [Molar mass of Cu : 63.54 g ; Avogadro Number = 6.022×10^{23}]

MATHEMATICS

SECTION-A (MULTIPLE CHOICE QUESTIONS)

61. Let A be a 3×3 matrix with det$(A) = 4$. Let R_i denote the i^{th} row of A. If a matrix B is obtained by performing the operation $R_2 \to 2R_2 + 5R_3$ on $2A$, then det(B) is equal to
(a) 16 (b) 80 (c) 128 (d) 64

62. The integral $\int \frac{e^{3\log_e 2x} + 5e^{2\log_e 2x}}{e^{4\log_e x} + 5e^{3\log_e x} - 7e^{2\log_e x}} dx, x > 0$, is equal to (where c is a constant of integration)
(a) $\log_e|x^2 + 5x - 7| + c$
(b) $4\log_e|x^2 + 5x - 7| + c$
(c) $\frac{1}{4}\log_e|x^2 + 5x - 7| + c$
(d) $\log_e\left|\sqrt{x^2 + 5x - 7}\right| + c$

63. The shortest distance between the line $x - y = 1$ and the curve $x^2 = 2y$ is
(a) $\frac{1}{2}$ (b) $\frac{1}{2\sqrt{2}}$ (c) $\frac{1}{\sqrt{2}}$ (d) 0

64. If $\alpha, \beta \in R$ are such that $1 - 2i$ (here $i^2 = -1$) is a root of $z^2 + \alpha z + \beta = 0$, then $(\alpha - \beta)$ is equal to
(a) -3 (b) -7 (c) 7 (d) 3

65. A hyperbola passes through the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and its transverse and conjugate axes coincide with major and minor axes of the ellipse, respectively. If the product of their eccentricities is one, then the equation of the hyperbola is
(a) $\frac{x^2}{9} - \frac{y^2}{25} = 1$ (b) $\frac{x^2}{9} - \frac{y^2}{16} = 1$
(c) $x^2 - y^2 = 9$ (d) $\frac{x^2}{9} - \frac{y^2}{4} = 1$

66. If $0 < x, y < \pi$ and $\cos x + \cos y - \cos(x + y) = 3/2$, then $\sin x + \cos y$ is equal to
(a) $\frac{1}{2}$ (b) $\frac{1+\sqrt{3}}{2}$ (c) $\frac{\sqrt{3}}{2}$ (d) $\frac{1-\sqrt{3}}{2}$

67. A plane passes through the points $A(1, 2, 3)$, $B(2, 3, 1)$ and $C(2, 4, 2)$. If O is the origin and P is $(2, -1, 1)$, then the projection of $\overrightarrow{OP}$ on this plane is of length
(a) $\sqrt{\frac{2}{7}}$ (b) $\sqrt{\frac{2}{3}}$ (c) $\sqrt{\frac{2}{11}}$ (d) $\sqrt{\frac{2}{5}}$

68. In a group of 400 people, 160 are smokers and non-vegetarian; 100 are smokers and vegetarian and the remaining 140 are non-smokers and vegetarian. Their chances of getting a particular chest disorder are 35%, 20% and 10% respectively. A person is chosen from the group at random and is found to be suffering from the chest disorder. The probability that the selected person is a smoker and non-vegetarian is
(a) $\frac{7}{45}$ (b) $\frac{14}{45}$ (c) $\frac{28}{45}$ (d) $\frac{8}{45}$

69. $\text{cosec}\left[2\cot^{-1}(5) + \cos^{-1}\left(\frac{4}{5}\right)\right]$ is equal to
(a) $\frac{56}{33}$ (b) $\frac{65}{56}$ (c) $\frac{65}{33}$ (d) $\frac{75}{56}$

70. If the curve $x^2 + 2y^2 = 2$ intersects the line $x + y = 1$ at two points P and Q, then the angle subtended by the line segment PQ at the origin is
(a) $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{3}\right)$ (b) $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{3}\right)$
(c) $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{4}\right)$ (d) $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{4}\right)$

71. The contrapositive of the statement "If you will work, you will earn money" is
(a) You will earn money, if you will not work
(b) If you will earn money, you will work
(c) If you will not earn money, you will not work
(d) To earn money, you need to work

72. A function $f(x)$ is given by $f(x)=\frac{5^x}{5^x+5}$, then the sum of the series $f\left(\frac{1}{20}\right)+f\left(\frac{2}{20}\right)+f\left(\frac{3}{20}\right)+.....+f\left(\frac{39}{20}\right)$ is equal to

(a) $\frac{19}{2}$ (b) $\frac{49}{2}$ (c) $\frac{29}{2}$ (d) $\frac{39}{2}$

73. If for the matrix, $A=\begin{bmatrix}1 & -\alpha \\ \alpha & \beta\end{bmatrix}$, $AA^T = I_2$, then the value of $\alpha^4 + \beta^4$ is

(a) 4 (b) 2 (c) 3 (d) 1

74. The minimum value of $f(x) = a^{a^x} + a^{1-a^x}$, where $a, x \in R$ and $a > 0$, is equal to

(a) $2a$ (b) $2\sqrt{a}$ (c) $a+\frac{1}{a}$ (d) $a + 1$

75. If $I_n = \int_{\pi/4}^{\pi/2} \cot^n x\, dx$, then

(a) $\frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}$ are in G.P.

(b) $I_2 + I_4, I_3 + I_5, I_4 + I_6$ are in A.P.

(c) $I_2 + I_4, (I_3 + I_5)^2, I_4 + I_6$ are in G.P.

(d) $\frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}$ are in A.P.

76. $\lim\limits_{n\to\infty}\left[\frac{1}{n}+\frac{n}{(n+1)^2}+\frac{n}{(n+2)^2}+....+\frac{n}{(2n-1)^2}\right]$ is equal to

(a) 1/2 (b) 1 (c) 1/3 (d) 1/4

77. Let A be a set of all 4-digit natural numbers whose exactly one digit is 7. Then the probability that a randomly chosen element of A leaves remainder 2 when divided by 5 is

(a) $\frac{2}{9}$ (b) $\frac{122}{297}$ (c) $\frac{97}{297}$ (d) $\frac{1}{5}$

78. Let α and β be the roots of $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10}-2a_8}{3a_9}$ is

(a) 2 (b) 1 (c) 4 (d) 3

79. Let x denote the total number of one-one functions from a set A with 3 elements to a set B with 5 elements and y denote the total number of one-one functions from the set A to the set $A \times B$. Then

(a) $y = 273x$ (b) $2y = 91x$

(c) $y = 91x$ (d) $2y = 273x$

80. The following system of linear equations

$2x + 3y + 2z = 9$

$3x + 2y + 2z = 9$

$x - y + 4z = 8$

(a) has a solution (α, β, γ) satisfying $\alpha + \beta^2 + \gamma^3 = 12$

(b) has infinitely many solutions

(c) does not have any solution

(d) has a unique solution

SECTION-B (NUMERICAL VALUE TYPE)

Attempt any 5 questions out of 10.

81. The total number of two digit numbers 'n', such that $3^n + 7^n$ is a multiple of 10, is ________.

82. A function f is defined on $[-3, 3]$ as

$$f(x)=\begin{cases}\min\{|x|, 2-x^2\}, & -2\le x\le 2\\ [|x|], & 2<|x|\le 3\end{cases}$$

where $[x]$ denotes the greatest integer $\leq x$. The number of points, where f is not differentiable in $(-3, 3)$ is ________.

83. Let $\vec{a}=\hat{i}+\alpha\hat{j}+3\hat{k}$ and $\vec{b}=3\hat{i}-\alpha\hat{j}+\hat{k}$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $8\sqrt{3}$ square units, then $\vec{a}\cdot\vec{b}$ is equal to ________.

84. If the remainder when x is divided by 4 is 3, then the remainder when $(2020 + x)^{2022}$ is divided by 8 is ________.

85. If the curves $x = y^4$ and $xy = k$ cut at right angles, then $(4k)^6$ is equal to ________.

86. A line is a common tangent to the circle $(x - 3)^2 + y^2 = 9$ and the parabola $y^2 = 4x$. If the two points of contact (a, b) and (c, d) are distinct and lie in the first quadrant, then $2(a + c)$ is equal to ________.

87. If $\lim\limits_{x\to 0}\frac{ax-(e^{4x}-1)}{ax(e^{4x}-1)}$ exists and is equal to b, then the value of $a - 2b$ is ________.

88. If the curve, $y = y(x)$ represented by the solution of the differential equation $(2xy^2 - y)dx + xdy = 0$, passes through the intersection of the lines, $2x - 3y = 1$ and $3x + 2y = 8$, then $|y(1)|$ is equal to ________.

89. The value of $\int_{-2}^{2}|3x^2-3x-6|\,dx$ is ________.

90. A line 'l' passing through origin is perpendicular to the lines $l_1:\vec{r}=(3+t)\hat{i}+(-1+2t)\hat{j}+(4+2t)\hat{k}$

$l_2:\vec{r}=(3+2s)\hat{i}+(3+2s)\hat{j}+(2+s)\hat{k}$

If the co-ordinates of the point in the first octant on 'l_2' at a distance of $\sqrt{17}$ from the point of intersection of 'l' and 'l_1' are (a, b, c), then $18(a + b + c)$ is equal to ________.

HINTS & EXPLANATIONS

1. **(d) :** Given, $m_p = 1836\, m_e$

de-Broglie wavelength, $\lambda = \frac{h}{mv}$

Ratio of de-Broglie wavelengths

$$\frac{\lambda_e}{\lambda_p} = \frac{m_p v_p}{m_e v_e} \quad (\text{As } v_p = v_e)$$

$$\therefore \quad \frac{\lambda_e}{\lambda_p} = \frac{1836\, m_e}{m_e} = 1836$$

2. **(a) :** In n-type semiconductor, pentavalent impurity is added which donates a free electron and in p-type semiconductor, a trivalent impurity is added which creates hole in the valence band. So, the fermi energy level of p-type semiconductors will go downward and for the n-type semiconductor will go upward.

3. **(a) :** $r = 0.36$ m

It covers 30° in 0.1 sec

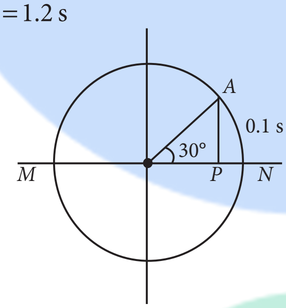
So, it covers $360°$ in $\frac{0.1}{30°} \times 360° = 1.2$ s

$T = 1.2$ s

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{1.2} = \frac{2\pi \times 10}{12} = \frac{5\pi}{3}$$

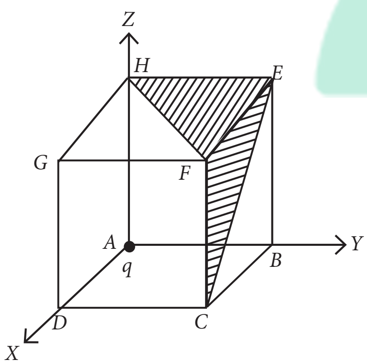
At M, $F = m\omega^2 A = m\omega^2 r$

$$\frac{F}{M} = \left(\frac{5\pi}{3}\right)^2 \times 0.36 = 9.87 \text{ N}$$



4. **(d) :** The electric flux through the cube

$$\phi = \frac{q}{8\varepsilon_0}$$



As the surfaces $ABEH$, $ADGH$, $ABCD$ are perpendicular to the field so, flux is zero through these surfaces.

Flux through ($EFGH$) = ($DCFG$) = ($EBCF$)

$$\therefore \quad \phi' = \frac{1}{3}\left(\frac{q}{8\varepsilon_0}\right) = \frac{q}{24\varepsilon_0}$$

5. **(d) :** As $F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r^2}$ and photon energy, $E = \frac{hc}{\lambda}$

So, $\frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{hc} = \frac{Fr^2}{E\lambda} = \frac{[\text{MLT}^{-2}][\text{L}^2]}{[\text{ML}^2\text{T}^{-2}][\text{L}]} = [\text{M}^0\text{L}^0\text{T}^0]$

6. **(a) :** Given, $V_s = 0.710$ V, $\lambda = 491$ nm

$V_s' = 1.43$ V, $\lambda' = ?$

According to the Einstein's photoelectric equation,

$$\frac{hc}{\lambda} = \phi + eV_s$$

$$\frac{1240}{491} = \phi + 0.710 \quad \text{...(i)}$$

When the incident wavelength is changed

$$\frac{1240}{\lambda'} = \phi + 1.43 \quad \text{...(ii)}$$

By solving eq. (i) and (ii),

$\lambda' = 382$ nm

7. **(b) :** Given, $n_1 = 1$, $n_2 = 2$

According to Rydbeg's formula,

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = 1.097 \times 10^7 \left(\frac{1}{1} - \frac{1}{4}\right)$$

$$\frac{1}{\lambda} = 1.097 \times 10^7 \times \frac{3}{4}$$

$\lambda = 121.5 \times 10^9$ m $= 121.5$ nm

The most suitable answer is option (b).

8. **(b) :** Here, the two springs are connected in parallel combination. Therefore, the effective spring constant is given by

$k_{eff} = 2k + 2k = 4k$

Time period, $T = 2\pi\sqrt{\frac{m}{k_{eff}}}$

$$\therefore \quad T = 2\pi\sqrt{\frac{m}{4k}} = \pi\sqrt{\frac{m}{k}}$$

9. **(d) :** From P-V diagram, $PV^{\frac{1}{2}} = \text{constant}$

From ideal gas equation, $PV = nRT$

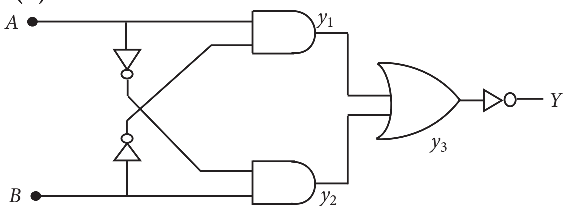
For, $n = 1$

$$P \propto \frac{T}{V}$$

$$\frac{TV^{\frac{1}{2}}}{V} = \text{constant} \quad \text{or} \quad T \propto V^{\frac{1}{2}}$$

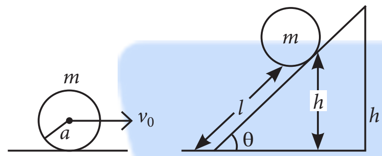
$$\text{or} \quad \frac{T_2}{T_1} = \left(\frac{V_2}{V_1}\right)^{\frac{1}{2}} \quad \text{or} \quad \frac{T_2}{T_1} = \sqrt{\frac{2V_1}{V_1}} = \sqrt{2} \qquad (\text{Given } V_2 = 2V_1)$$

10. (d) :



A	B	y_1	y_2	y_3	Y
0	0	0	0	0	1
0	1	0	1	1	0
1	0	1	0	1	0
1	1	0	0	0	1

11. (*)



Initial position Final position

Let the sphere attained height h, and travel a distance l on the inclined plane.

According to law of conservation of mechanical energy,

$$mgh = \frac{1}{2}mv_0^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}mv_0^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\omega^2 \quad \left(\because \text{ For solid sphere, } I = \frac{2}{5}mR^2\right)$$

$$= \frac{1}{2}mv_0^2 + \frac{1}{5}mv_0^2 \qquad (\because\ v = R\omega)$$

$$\text{or}\quad mgh = \frac{7}{10}mv_0^2$$

$$\text{or}\quad h = \frac{7}{10}\frac{v_0^2}{g} \qquad \text{...(i)}$$

Now, from figure, $\sin\theta = \frac{h}{l}$

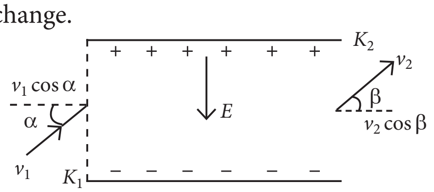
or $h = l\sin\theta$

$\therefore$ From eq (i), we have

$$l\sin\theta = \frac{7}{10}\frac{v_0^2}{g} \quad \text{or} \quad l = \frac{7}{10g}\frac{v_0^2}{\sin\theta}$$

*None of the given options is correct.

12. (d) : As the direction of electric field is downwards so there is no acceleration along x-axis, the horizontal component of velocity is constant, velocity along the plate will not change.



$v_1 \cos\alpha = v_2 \cos\beta$

Ratio of kinetic energy, $\dfrac{K_1}{K_2} = \dfrac{0.5mv_1^2}{0.5mv_2^2} = \dfrac{\cos^2\beta}{\cos^2\alpha}$

13. (c) : In ferromagnetic materials, smaller group of atoms band together into areas called domains within which all the magnetic dipoles are aligned parallel to each other. Below curie temperature, in ferromagnetic material, a domain is a macroscopic region with the saturation of magnetization.

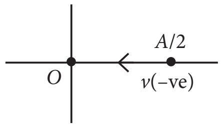
14. (a) : We know that,

Width of central maximum $= \dfrac{2\lambda D}{a}$

where a is the size of hole, λ is the wavelength of light used. When size of the hole is increased, the width of central maxima decreases. As the size of hole decreases, more and more light will pass through it, hence intensity will increase. Intensity is also increasing due to the concentration of light on a smaller area, as intensity $\propto \dfrac{1}{\text{area}}$.

15. (c) : Given, $y = A\sin(\omega t + \phi_0)$

At $t = 0$, $y = \dfrac{A}{2}$



$$\therefore \quad \frac{A}{2} = A\sin(\omega\times 0 + \phi_0)$$

$$\text{or}\quad \sin\phi_0 = \frac{1}{2}$$

$$\text{or}\quad \phi_0 = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, ...$$

Now, $v = \dfrac{dx}{dt} = A\omega\cos(\omega t + \phi_0)$

At $t = 0$, $v = A\omega\cos\phi_0$

$\therefore$ For $\phi_0 = \dfrac{\pi}{6}$, v is positive

For $\phi_0 = \dfrac{5\pi}{6}$, v is negative

$$\therefore \quad \phi_0 = \frac{5\pi}{6}$$

16. (d) : Rectifier – converts a.c. into d.c.

Stabilizer – to regulate the output

Transformer – to step up or down the a.c. voltage

Filter – to filter the a.c. component.

A→(ii), B→(iv), C→(i), D→(iii)

17. (b) : Given, $R = 110\ \Omega$, $V = 220$ V, $\omega = 300$ rad/s

$$\tan\phi = \frac{X_C}{R} = \frac{X_L}{R} \Rightarrow X_L = X_C$$

It means, the case is resonance source.

RMS current flowing in the circuit,

$$i_{rms} = \frac{V}{R} = \frac{220}{110} = 2\text{ A}$$

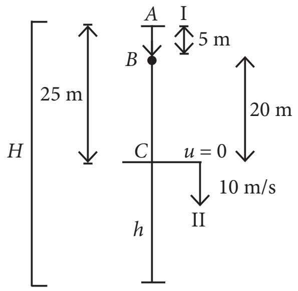
18. (c) : Let the time to meet both the stones is t

For I[st] stone,

$20^{th} = 10t$ ...(i)

For II[nd] stone,

$h = 0 + \frac{1}{2} \times gt^2$ ...(ii)



Using eq. (i) and (ii), $t = 2$ s

From (ii), $h = \frac{1}{2} \times 10 \times 2 \times 2 = 20$ m

$\because$ The height of the building, $H = 25 + h = 25 + 20 = 45$ m.

19. (d) : As carrier wave is electromagnetic wave so, wavelength of modulated signal would be same as that of the carrier signal.

$$\therefore \quad \lambda = \frac{v}{f} = \frac{c}{f_c}$$

20. (a) : Translation K.E = $= \frac{3}{2}kT$

Rotational K.E = $2\left(\frac{1}{2}kT\right) = kT$

$\therefore$ Statement I is true and Statement II is false.

21. (20) : Given, $m = 10$ mg

Let the tension is T and charge is q.

Now, $T\cos\theta = mg = 10 \times 10^{-6} \times 10 = 10^{-4}$ N ...(i)

$$T\sin\theta = f = \frac{kq^2}{(0.2)^2}$$

$$T\sin\theta = \frac{9\times10^9 \times q^2}{0.04} \quad \text{...(ii)}$$

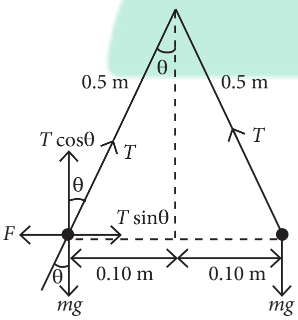
$$\tan\theta = \frac{0.1}{\sqrt{.5^2 - .1^2}} = \frac{0.1}{\sqrt{0.24}} = \frac{10^9 \times 9 \times q^2}{0.04 \times 10^{-4}}$$

$$\frac{9\times10^9 q^2}{4\times10^{-6}} = \frac{0.1}{\sqrt{0.24}}$$

$$q = \frac{2\sqrt{10}\times10^{-8}}{3\sqrt{\sqrt{24}}} = 0.95\times10^{-8}$$

So, $\frac{q}{21}\times10^{-8} = q = 0.95\times10^{-8}$

$a = 20$



22. (10) : When the body is projected vertically upwards from the Earth's surface with initial velocity, v_i, it reaches a maximum height, where R is the radius of Earth. Let M be the mass of Earth and m be the mass of the body. By the law of conservation of mechanical energy,

$U_B + K_B = U_A + K_B$

$$\frac{-GMm}{(10R+R)} + 0 = \frac{-GMm}{R} + \frac{1}{2}mv_i^2$$

$$\frac{1}{2}v_i^2 = \frac{GM}{R} - \frac{GM}{11R} = \frac{10GM}{11R}$$

or $v_i = \sqrt{\frac{20GM}{11R}}$ or $v_i = \sqrt{\frac{10}{11R}} \times v_e$ $\left(\because v_e = \sqrt{\frac{2GM}{R}}\right)$

On comparing the given value with $\sqrt{\frac{x}{y}} \cdot v_e$, we get

$x = 10$

23. (10) : Wavelength of X-ray beam, $\lambda = 10$Å

Energy, $\frac{hc}{\lambda} = mc^2 = E_1 = \frac{hc}{10\times10^{-10}}$

Energy of fictitious particle, $E_2 = mc^2 = \frac{x}{3}hc^2$

Given that, $E_1 = E_2$

$$\therefore \quad \frac{hc}{10\times10^{-10}} = \frac{x}{3}hc^2$$

$$\frac{1}{10^{-9}} = \frac{x \times 3 \times 10^8}{3} \Rightarrow x = 10$$

24. (36) : Given, $q = 2.1$ nC, $q' = -0.1$ nC, $r = 0.5$ m

When the two conducting spheres A and B are brought in contact, each sphere will attain equal charge Q,

$$Q = \frac{q+q'}{2} = \frac{2.1-0.1}{2} = 1\text{ nC}$$

When the electrostatic force acting between the spheres, is

$$F = \frac{KQ^2}{r^2} = \frac{9\times10^9 \times 1 \times 1 \times 10^{-18}}{(0.5)^2} = 36 \times 10^{-9}\text{ N}$$

25. (180) : Given, $\vec{P}\times\vec{Q} = \vec{Q}\times P$

$PQ\sin\theta = -PQ\sin\theta \Rightarrow \theta = 180°$ $(\because 0° < \theta < 360°)$

26. (2) : Speed of transverse wave, $v = \sqrt{\frac{T}{\mu}}$

$$\frac{\Delta v}{v} = \frac{1}{2}\frac{\Delta T}{T}$$

$$\frac{\Delta v}{v}\times100 = \frac{1}{2}\times4\% = 2\%$$

27. (2) : Applying Kirchhoff's second Law at P,

$i_1 + i_2 = 6$ A ...(i)

Applying Kirchhoff's Voltage law to the closed loop, $PQRP$

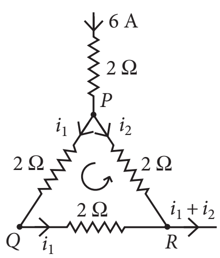
$2i_1 + 2i_1 - 2i_2 = 0$

$i_1 \times 4 = i_2 \times 2$

So, $i_2 = 2i_1$ ...(ii)

Using (i) and (ii),

$3i_1 = 6$ or $i_1 = 2$ A



28. (208) : Efficiency of an engine, $\eta = 1 - \frac{T_2}{T_1}$

where T_1 is the source temperature and T_2 is the sink temperature.

Given that, $\eta = \frac{1}{4}$

$\therefore \quad \frac{1}{4} = 1 - \frac{T_2}{T_1}$ or $\frac{T_2}{T_1} = \frac{3}{4}$

When temperature is reduced by 52 K, efficiency of the heat engine gets double. Thus, we can write

$$\text{Now, } \eta' = 2\eta = 1 - \frac{(T_2 - 52)}{T_1} \quad \text{...(ii)}$$

$$\Rightarrow \frac{1}{2} = 1 - \frac{(T_2 - 52)}{T_1}$$

$$\frac{\frac{3}{4}T_1 - 52}{T_1} = \frac{1}{2} \Rightarrow T_1 = 208 \text{ K}$$

29. **(*) :** $P = 8$ W, $d = 10$ m, $\eta = 10\%$

$$I = \frac{8 \times 10}{100 \times 4\pi(10)^2}$$

Half of intensity belongs to electric field and half of that of magnetic field.

$$\frac{I}{2} = \frac{1}{4}\varepsilon_0 E_0^2 c$$

$$\frac{8}{4\pi \times 10^2} \times \frac{1}{10} \times \frac{1}{2} = \frac{1}{4} c \times \frac{1}{\mu_0 c^2} E_0^2$$

$$E_0 = \frac{2}{10} \times \sqrt{\frac{\mu_0 c}{10\pi} \frac{V}{M}}$$

On comparing the given value with $\frac{x}{10}\sqrt{\frac{\mu_0 c}{\pi}}$, we get

$x = \frac{2}{\sqrt{10}}$.

***Note :** In the official answer key of NTA.

$x = 2$.

It is only possible when

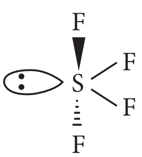
$\eta = 100\%$ or $P = 80$ W.

30. **(1) :** Kinetic energy of a particle, $K = \frac{p^2}{2m}$

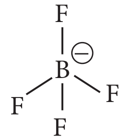
$$\frac{p_1^2}{2 \times 4} = \frac{p_2^2}{2 \times 16} \Rightarrow \frac{p_1^2}{p_2^2} = \frac{1}{4} \Rightarrow \frac{p_1}{p_2} = \frac{1}{2} \quad \text{...(i)}$$

As in question, $\frac{p_1}{p_2} = \frac{n}{2}$

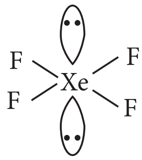
So, $\frac{n}{2} = \frac{1}{2}$ or $n = 1$ (Using (i)

31. **(c) :** SF_4 :  , See-saw

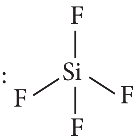
sp^3d hybridization, axial bond length is more than equatorial bond length.

BF_4^- :  , Regular tetrahedral

All bond lengths are equal.

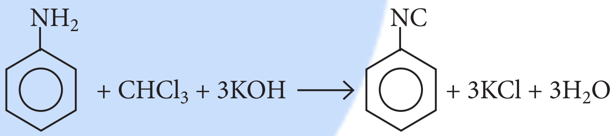
XeF_4 :  , Square planar

All bond lengths are equal.

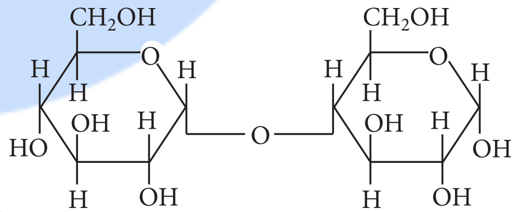
SiF_4 :  , Regular tetrahedral

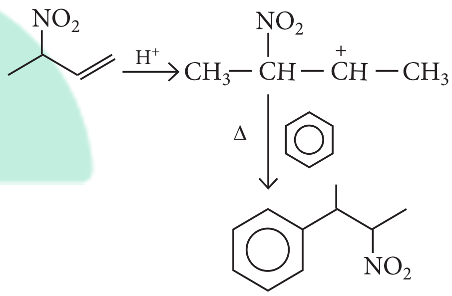
All bond lengths are equal.

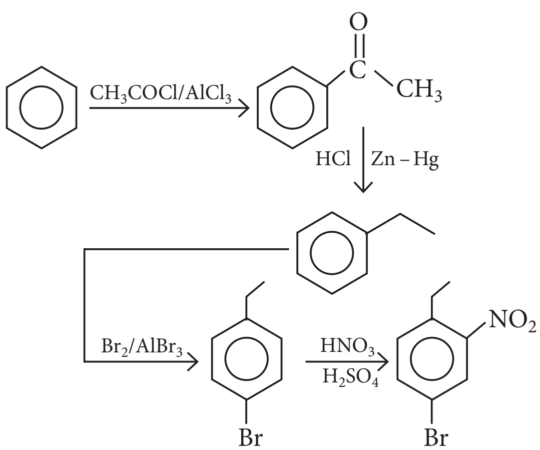
32. **(d) :** Carbylamine test is given by 1° amine. In carbylamine test, isocyanide is formed.



33. **(d) :** α-Anomer of maltose is composed of two molecules of α-D-glucose joined by C_1 — C_4 glycosidic linkage.



34. **(c) :** 

35. **(d) :** 

36. (a) : $CO_2 + H_2O \longrightarrow H_2CO_3$

$C + H_2O \longrightarrow \underbrace{H_2 + CO}_{\text{Water gas}}$

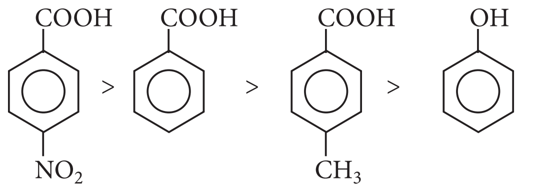
$CH_4 + H_2O \longrightarrow CO + 3H_2$

$C_3H_8 + 3H_2O \longrightarrow 3CO + 7H_2$

37. (d) : As we know, acidic strength $\propto -I$ effect or $-M$ effect

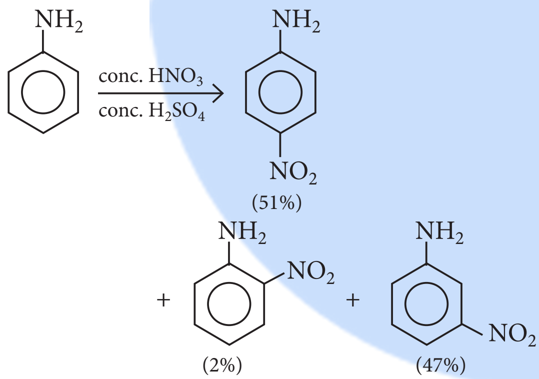
$$\propto \frac{1}{+I \text{ effect or } +M \text{ effect}}$$

Therefore, the correct acid character order is



II > III > IV > I

38. (d) :



In acidic medium, aniline is converted into anilinium ion which is *meta* directing. So, *meta* product is formed in significant amount.

39. (c) : F_2 has F — F bond, which involves repulsion of non-bonding electrons. Moreover, its size is small and hence due to high repulsion, its bond dissociation energy is very low and smaller than Cl — Cl and Br — Br bonds.

40. (d) : pH of rain water is 5.6. If pH is below 5.6, then it is called acid rain.

41. (c) : German silver is an alloy which does not have silver. Major components of German silver are Cu, Zn and Ni.

42. (a) : $[FeF_6]^{3-} \Rightarrow Fe^{3+} \Rightarrow 3d^5$

F^- is a weak field ligand. So, $3d^5$: ↑ | ↑ | ↑ | ↑ | ↑

Number of unpaired electrons = 5

$[Co(NH_3)_6]^{3+} \Rightarrow Co^{3+} \Rightarrow 3d^6$

NH_3 is a strong field ligand. So, $3d^6$: ↿⇂ | ↿⇂ | ↿⇂ | |

Number of unpaired electrons = 0

$[NiCl_4]^{2-} \Rightarrow Ni^{2+} \Rightarrow 3d^8$

Cl^- is a weak field ligand. So, $3d^8$: ↿⇂ | ↿⇂ | ↿⇂ | ↑ | ↑

Number of unpaired electrons = 2

$[Cu(NH_3)_4]^{2+} \Rightarrow Cu^{2+} \Rightarrow 3d^9$

NH_3 is a strong field ligand. So,

$3d^9$: ↿⇂ | ↿⇂ | ↿⇂ | ↿⇂ | ↑

Number of unpaired electrons = 1

Magnetic moment is directly proportional to number of unpaired electrons. So, the correct order of spin only magnetic moment is (i) > (iii) > (iv) > (ii).

43. (a) : Nitric acid is added to the sodium extract to remove CN^- and S^{2-} before the test of halides otherwise they will react with silver nitrate and hence will interfere with the test.

$NaCN + HNO_3 \longrightarrow NaNO_3 + HCN\uparrow$

$Na_2S + 2HNO_3 \longrightarrow 2NaNO_3 + H_2S\uparrow$

44. (a) : Viscosity of hydrophilic sol is higher than that of water.

45. (c) : $Ca(OH)_{2(s)} \rightleftharpoons \underset{S}{Ca^{2+}_{(aq)}} + \underset{2S}{2OH^-_{(aq)}}$

$K_{sp} = (S)\,(2S)^2$

$5.5 \times 10^{-6} = 4S^3 \Rightarrow S = 1.11 \times 10^{-2}$

46. (c)

$Ni^{2+} + dmg + NH_4OH \longrightarrow \underset{\text{Rosy red ppt.}}{[Ni(dmg)_2]}$

$CH_3 - C = N - O^-$
$\qquad\quad |$
$CH_3 - C = N - OH$

Dimethylglyoxime ion

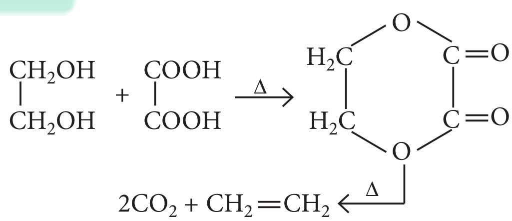
It is a negative bidentate ligand.

47. (c) : $CH_3CH_2CH{=}CH_2 + CO + H_2 \xrightarrow{\text{Rh Catalyst}}$

$\underset{\text{(Minor)}}{CH_3CH_2\underset{|}{\overset{CHO}{C}}H - CH_3} + \underset{\text{(Major)}}{CH_3CH_2CH_2CH_2CHO}$

48. (c) : Ga, In, Ge, Si etc. are refined by zone refining method.

49. (a) :



50. (c) : $\alpha\text{-sulphur} \underset{< 369\text{ K}}{\overset{> 369\text{ K}}{\rightleftharpoons}} \beta\text{-sulphur}$

At room temperature, rhombic sulphur is most stable.

51. (3) : $AB \longrightarrow A^+ + B^-$

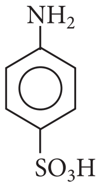
$n = 2$; $\alpha = 0.75$

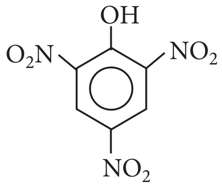
$i = 1 - \alpha + n\alpha \Rightarrow 1 - 0.75 + 2 \times 0.75 = 1.75$

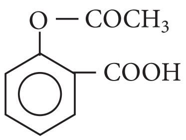
$\Delta T_b = i\,K_b\,m$

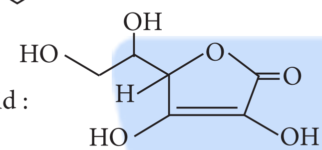
$2.5 = 1.75 \times 0.52 \times m$

$m = 2.74 \approx 3$

52. (1) : Sulphanilic acid : 

Picric acid : 

Aspirin : 

Ascorbic acid : 

Only aspirin has —COOH group.

53. (52) : $\frac{k_{52°C}}{k_{27°C}} = 5;\ T_1 = 300\text{ K};\ T_2 = 325\text{ K}$

$$\ln\frac{k_2}{k_1} = \frac{E_a}{R}\left[\frac{1}{T_1} - \frac{1}{T_2}\right]$$

or $\ln 5 = \frac{E_a}{8.314}\left[\frac{1}{300} - \frac{1}{325}\right]$

$$E_a = \frac{0.7\times 2.303\times 8.314\times 325\times 300}{25}$$

$= 52271\text{ J mol}^{-1} = 52.271\text{ kJ mol}^{-1}$ or 52 kJ mol^{-1}

54. (1) : Cs is used in photoelectric cell due to its very low ionization enthalpy.

55. (2) : Cu^{2+} : $[Ar]3d^9 \Rightarrow$ $\boxed{\upharpoonleft\downharpoonright}\boxed{\upharpoonleft\downharpoonright}\boxed{\upharpoonleft\downharpoonright}\boxed{\upharpoonleft\downharpoonright}\boxed{\uparrow}$

Number of unpaired electron = 1

Magnetic moment (μ) $= \sqrt{n(n+2)} = \sqrt{1(1+2)}$

$= 1.73$ B.M. ≈ 2 B.M.

56. (181) : *I.E* or energy $= \frac{hc}{\lambda}$

$$= \frac{6.63\times 10^{-34}\times 3\times 10^8}{663\times 10^{-9}} = 3\times 10^{-19}\ \text{J atom}^{-1}$$

I.E. per mole $= 3\times 10^{-19} \times 6.02 \times 10^{23} = 18.06 \times 10^4$ J

$= 180.6$ kJ ≈ 181 kJ

57. (6) : $(COOH)_2 + 2NaOH \longrightarrow Na_2C_2O_4 + 2H_2O$

Millimoles of $H_2C_2O_4 = 10 \times 1.25$

Volume of NaOH consumed = 4.4 mL

Equivalent of NaOH = Equivalent of $H_2C_2O_4$

$M \times 4.4 \times 1 = 1.25 \times 10 \times 2$ or $M = 5.68\text{ M} \approx 6\text{ M}$

58. (3) : $\Delta V = 0$, $n = 5$, $T = 293$ K

$P_1 = 2.1$ MPa, $P_2 = 1.3$ MPa

$P_{ext} = 4.3$ MPa

$$W = -P_{ext}(V_2 - V_1) = -P_{ext}\left(\frac{nRT}{P_2} - \frac{nRT}{P_1}\right)$$

$$W = -P_{ext}\cdot nRT\left(\frac{1}{P_2} - \frac{1}{P_1}\right)$$

$$= -4.3\times 5\times 8.314\times 293\left(\frac{1}{1.3} - \frac{1}{2.1}\right) = -15345.6\text{ J}$$

≈ -15.35 kJ

$W = -15.35$ kJ, $\Delta U = q + W$

$q = -W$ (As $\Delta U = 0$)

$= 15.35$ kJ (For 5 moles)

q per mol $= \frac{15.35}{5} = 3.07\text{ kJ} \approx 3\text{ kJ}$

59. (4) : $3Cu + 8H^+ + 2NO_3^- \longrightarrow 3Cu^{2+} + 2NO + 4H_2O$

$Cu + 4H^+ + 2NO_3^- \longrightarrow Cu^{2+} + 2NO_2 + 2H_2O$

Let $[HNO_3] = x$, so $[H^+] = [NO_3^-] = x$

$$E_1 = \left[(0.96 - 0.34) - \frac{0.059}{6}\log\frac{(P_{NO})^2[Cu^{2+}]^3}{x^{10}}\right]$$

$$= \left[0.62 - \frac{0.059}{6}\log\frac{(P_{NO})^2[Cu^{2+}]^3}{x^{10}}\right]$$

$$E_2 = \left[(0.79 - 0.34) - \frac{0.059}{2}\log\frac{(P_{NO_2})^2[Cu^{2+}]}{x^6}\right]$$

$$= \left[0.45 - \frac{0.059}{2}\log\frac{(P_{NO_2})^2[Cu^{2+}]}{x^6}\right]$$

Now, $E_1 = E_2$ or, $\left[0.62 - \frac{0.059}{6}\log\frac{(P_{NO})^2[Cu^{2+}]^3}{x^{10}}\right]$

$$= \left[0.45 - \frac{0.059}{2}\log\frac{(P_{NO_2})^2[Cu^{2+}]}{x^6}\right]$$

$$\text{or, } 0.62 - 0.45 = \frac{0.059}{6}\log\frac{(P_{NO})^2[Cu^{2+}]^3}{x^{10}} - \frac{0.059}{2}\log\frac{(P_{NO_2})^2[Cu^{2+}]}{x^6}$$

$$\text{or, } 0.17 = \frac{0.059}{6}\log\frac{(P_{NO})^2[Cu^{2+}]^3}{x^{10}} - \frac{0.059}{6}\log\frac{(P_{NO_2})^6[Cu^{2+}]^3}{x^{18}}$$

$$\text{or, } 0.17 = \frac{0.059}{6}\log\left[\frac{(P_{NO})^2[Cu^{2+}]^3}{x^{10}} \times \frac{x^{18}}{(P_{NO_2})^6[Cu^{2+}]^3}\right]$$

or, $0.17 = \frac{0.059}{6}\log\frac{x^8}{(P_{NO_2})^4}$ $\left[\because P_{NO} = P_{NO_2}\right]$

or, $0.17 = \frac{0.059}{6}\log x^8$ $\left[\text{Assuming } P_{NO_2} = 1\text{ atm}\right]$

or, $0.17 = \frac{0.059}{6} \times 8\log[HNO_3]$

or, $0.17 = \frac{0.059}{6} \times 8 \times \log[HNO_3]$

or, $\log[HNO_3] = 2.16$ or, $[HNO_3] = 10^{2.16} = 10^x$

$\Rightarrow x = 2.16$

$2x = 4.32 \approx 4$

60. (9076) : For *fcc*, $Z = 4$

$$d = \frac{Z \times M}{N_A \times a^3} = \frac{4 \times 63.54}{6.022 \times 10^{23} \times (3.596 \times 10^{-10})^3 \times 1000}$$

$$= 9076.2 \approx 9076 \text{ kg/m}^3$$

61. (d) : $|A| = 4$

$\therefore$ $|2A| = 2^3|A| = 2^3 \times 4 = 32$

$\because$ B is obtained by performing operation $R_2 \to 2R_2 + 5R_3$ on $2A$.

$\therefore$ $|B| = 2 \times 32 = 64$

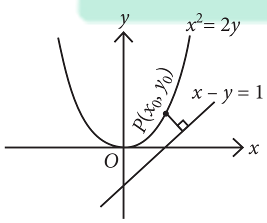
62. (b) : $\int \frac{e^{3\log_e 2x} + 5e^{2\log_e 2x}}{e^{4\log_e x} + 5e^{3\log_e x} - 7e^{2\log_e x}}dx, x > 0$

$$= \int \frac{(2x)^3 + 5(2x)^2}{x^4 + 5x^3 - 7x^2}dx = \int \frac{4x^2(2x+5)}{x^2(x^2+5x-7)}dx$$

$$= 4\int \frac{d(x^2+5x-7)}{(x^2+5x-7)} = 4\log_e|x^2+5x-7| + c$$

63. (b) : We have, $x - y = 1$ and $x^2 = 2y$

Shortest distance between curves is always along common normal.



Slope of line $= \left[\frac{dy}{dx}\right]_{P(x_0,\, y_0)} = 1$

$\Rightarrow x_0 = 1$ and $y_0 = 1/2$

So, $P \equiv \left(1, \frac{1}{2}\right)$

$\therefore$ Shortest distance from point $P(1, 1/2)$ to the line $x - y = 1$ is given by

$$\left|\frac{1 - \frac{1}{2} - 1}{\sqrt{1^2 + 1^2}}\right| = \frac{1}{2\sqrt{2}}$$

64. (b) : If one root is $1 - 2i$, then other root is $1 + 2i$.

$\alpha = -(\text{Sum of roots}) = -(1 - 2i + 1 + 2i) = -2$

$\beta = \text{Product of roots} = (1 - 2i)(1 + 2i) = 5$

$\therefore$ $\alpha - \beta = -2 - 5 = -7$

65. (b) : Given equation of ellipse is $\frac{x^2}{25} + \frac{y^2}{16} = 1$ ...(i)

$\therefore$ Eccentricity, $e_1 = \sqrt{1 - \frac{16}{25}} = \frac{3}{5}$

Coordinates of foci are (3, 0) and (–3, 0).

Now, let the equation of hyperbola be $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ ...(ii)

Eccentricity of hyperbola, $e_2 = \frac{5}{3}$ $[\because e_1e_2 = 1]$

$\because$ (ii) passes through (3, 0) $\Rightarrow \frac{9}{a^2} = 1 \Rightarrow a^2 = 9$

Now, $b^2 = a^2(e_2^2 - 1) = 9\left(\frac{25}{9} - 1\right) = 16$

$\therefore$ Equation of hyperbola is $\frac{x^2}{9} - \frac{y^2}{16} = 1$

66. (b) : We have, $\cos x + \cos y - \cos(x+y) = \frac{3}{2}$

$$\Rightarrow 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) - 2\cos^2\left(\frac{x+y}{2}\right) + 1 = \frac{3}{2}$$

$$\Rightarrow \cos^2\left(\frac{x+y}{2}\right) - \cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) + \frac{1}{4} = 0$$

$$\Rightarrow \cos^2\left(\frac{x+y}{2}\right) - \cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$+ \frac{1}{4}\cdot\cos^2\left(\frac{x-y}{2}\right) + \frac{1}{4}\sin^2\left(\frac{x-y}{2}\right) = 0$$

$$\Rightarrow \left(\cos\left(\frac{x+y}{2}\right) - \frac{1}{2}\cos\left(\frac{x-y}{2}\right)\right)^2 + \frac{1}{4}\sin^2\left(\frac{x-y}{2}\right) = 0$$

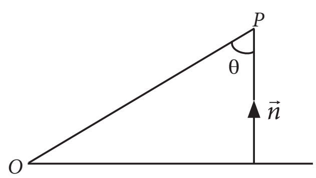
$$\Rightarrow \sin\left(\frac{x-y}{2}\right) = 0 \text{ and } \cos\left(\frac{x+y}{2}\right) = \frac{1}{2}\cos\left(\frac{x-y}{2}\right)$$

$$\Rightarrow x = y \text{ and } \cos x = \frac{1}{2} = \cos y$$

$$\therefore \sin x = \frac{\sqrt{3}}{2} \Rightarrow \sin x + \cos y = \frac{1+\sqrt{3}}{2}$$

67. (c) : We have, $A(1, 2, 3)$, $B(2, 3, 1)$ and $C(2, 4, 2)$.

$\therefore$ $\overrightarrow{AB} = \hat{i} + \hat{j} - 2\hat{k}$ and $\overrightarrow{BC} = \hat{j} + \hat{k}$



Normal to plane ABC is given by, $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -2 \\ 0 & 1 & 1 \end{vmatrix} = 3\hat{i} - \hat{j} + \hat{k}$

Also, $\overrightarrow{OP} = 2\hat{i} - \hat{j} + \hat{k} \Rightarrow |\overrightarrow{OP}| = \sqrt{4+1+1} = \sqrt{6}$

$\therefore \quad \cos\theta = \left|\frac{6+1+1}{\sqrt{11}\sqrt{6}}\right| = \frac{8}{\sqrt{66}} \quad \Rightarrow \quad \sin\theta = \sqrt{\frac{2}{66}}$

$\therefore$ Projection of $\overrightarrow{OP}$ on the plane $= |\overrightarrow{OP}|\sin\theta = \sqrt{\frac{2}{11}}$

68. (c) : Consider the following events.

A : Person chosen is a smoker and non-vegetarian.

B : Person chosen is a smoker and vegetarian.

C : Person chosen is a non-smoker and vegetarian.

E : Person chosen has a chest disorder.

$\therefore \quad P(A) = \frac{160}{400},\ P(B) = \frac{100}{400},\ P(C) = \frac{140}{400},$

and $P\left(\frac{E}{A}\right) = \frac{35}{100},\ P\left(\frac{E}{B}\right) = \frac{20}{100},\ P\left(\frac{E}{C}\right) = \frac{10}{100}$

Required probability $= P\left(\frac{A}{E}\right)$

$$= \frac{P(A)\,P\left(\frac{E}{A}\right)}{P(A)\cdot P\left(\frac{E}{A}\right) + P(B)\cdot P\left(\frac{E}{B}\right) + P(C)\cdot P\left(\frac{E}{C}\right)}$$

[Using Bayes' theorem]

$$= \frac{\frac{160}{400}\times\frac{35}{100}}{\frac{160}{400}\times\frac{35}{100} + \frac{100}{400}\times\frac{20}{100} + \frac{140}{400}\times\frac{10}{100}} = \frac{28}{45}$$

69. (b) : $\operatorname{cosec}\left[2\tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{3}{4}\right)\right]$

$$= \operatorname{cosec}\left[\tan^{-1}\left(\frac{2/5}{1-(1/5)^2}\right) + \tan^{-1}\left(\frac{3}{4}\right)\right]$$

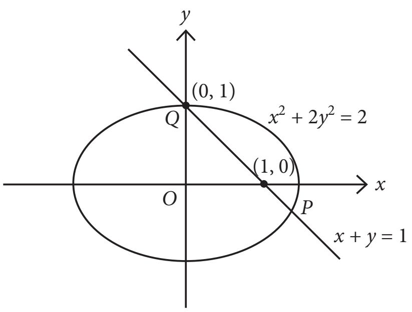
$$= \operatorname{cosec}\left[\tan^{-1}\left(\frac{5}{12}\right) + \tan^{-1}\left(\frac{3}{4}\right)\right]$$

$$= \operatorname{cosec}\left[\tan^{-1}\left(\frac{\frac{5}{12}+\frac{3}{4}}{1-\frac{5}{12}\cdot\frac{3}{4}}\right)\right]$$

$$\left[\because\ \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right),\ xy < 1\right]$$

$$= \operatorname{cosec}\left[\tan^{-1}\left(\frac{56}{33}\right)\right] = \frac{65}{56}$$

70. (d) :



Homogenising ellipse using line, we get

$x^2 + 2y^2 - 2(x+y)^2 = 0$

$\Rightarrow\ x^2 + 2y^2 - 2(x^2 + y^2 + 2xy) = 0$

$\Rightarrow\ x^2 + 4xy = 0 \quad \Rightarrow \quad x(x + 4y) = 0$

$\Rightarrow\ x = 0 \quad$ and $\quad y = -x/4$

$\therefore\ $ Angle between the lines OP and OQ is $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{4}\right)$.

71. (c) : Contrapositive of $p \to q$ is $\sim q \to \sim p$

So, contrapositive of given statement is

"If you will not earn money, you will not work".

72. (d) : $f(x) = \frac{5^x}{5^x + 5}$

$$f(2-x) = \frac{5^{2-x}}{5^{2-x}+5} = \frac{25}{25 + 5\cdot 5^x} = \frac{5}{5^x + 5}$$

$\therefore\ f(x) + f(2-x) = 1$

Now, $f\left(\frac{1}{20}\right) + f\left(\frac{2}{20}\right) + f\left(\frac{3}{20}\right) + \dots + f\left(\frac{39}{20}\right)$

$$= \left(f\left(\frac{1}{20}\right) + f\left(\frac{39}{20}\right)\right) + \left(f\left(\frac{2}{20}\right) + f\left(\frac{38}{20}\right)\right) + \dots$$

$$+ \dots + \left(f\left(\frac{19}{20}\right) + f\left(\frac{21}{20}\right)\right) + f\left(\frac{20}{20}\right)$$

$$= 1\times 19 + \frac{1}{2} = \frac{39}{2}$$

73. (d) : Given, $AA^T = I_2$

$$\Rightarrow \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}\begin{bmatrix} 1 & \alpha \\ -\alpha & \beta \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1+\alpha^2 & \alpha-\alpha\beta \\ \alpha-\alpha\beta & \alpha^2+\beta^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow\ \alpha^2 = 0$ and $\beta^2 = 1$

$\therefore\ \alpha^4 + \beta^4 = 1$

74. (b) : Given, $f(x) = a^{a^x} + a^{1-a^x}$

Since A.M. $\geq$ G.M.

$$\therefore\ \frac{a^{a^x} + \frac{a}{a^{a^x}}}{2} \geq \left(a^{a^x}\cdot\frac{a}{a^{a^x}}\right)^{1/2}$$

$$\Rightarrow\ a^{a^x} + \frac{a}{a^{a^x}} \geq 2\sqrt{a}$$

$\therefore\ $ Minimum value is $2\sqrt{a}$.

75. (d) : $I_n = \int_{\pi/4}^{\pi/2} \cot^n x\,dx = \int_{\pi/4}^{\pi/2} \cot^{n-2} x(\operatorname{cosec}^2 x - 1)dx$

$$= \left[-\frac{\cot^{n-1}x}{n-1}\right]_{\pi/4}^{\pi/2} - I_{n-2} = \frac{1}{n-1} - I_{n-2}$$

$$\Rightarrow\ I_n + I_{n-2} = \frac{1}{n-1}$$

$\therefore\ I_2+I_4=\frac{1}{3},\ I_3+I_5=\frac{1}{4}$ and $I_4+I_6=\frac{1}{5}$

$\therefore\ \frac{1}{I_2+I_4},\ \frac{1}{I_3+I_5},\ \frac{1}{I_4+I_6}$ are in A.P.

76. (a) : $\lim\limits_{n\to\infty}\left[\frac{1}{n}+\frac{n}{(n+1)^2}+\frac{n}{(n+2)^2}+\ldots+\frac{n}{(2n-1)^2}\right]$

$=\lim\limits_{n\to\infty}\sum_{r=0}^{n-1}\frac{n}{(n+r)^2}=\lim\limits_{n\to\infty}\sum_{r=0}^{n-1}\frac{n}{n^2+2nr+r^2}$

$=\lim\limits_{n\to\infty}\frac{1}{n}\sum_{r=0}^{n-1}\frac{1}{(r/n)^2+2(r/n)+1}$

$=\int_0^1\frac{dx}{(x+1)^2}=\left[\frac{-1}{(x+1)}\right]_0^1=\frac{1}{2}$

77. (c) : $n(S)=n$(7 appears on thousand's place)
+ n(7 does not appear on thousand's place)
$=9\times9\times9+8\times9\times9\times3$
$=33\times9\times9$

Now, a number leaves remainder 2 when divided by 5 only when its unit digit is 2 or 7.

$n(E)=n$(Unit digit 7 when 7 appears once)
+ n(Unit digit 2 when 7 appears once)
$=8\times9\times9+(9\times9+8\times9\times2)$
$=8\times9\times9+9\times25=9(72+25)=9\times97$

$\therefore$ Required probability $=\frac{n(E)}{n(S)}=\frac{9\times97}{33\times9\times9}=\frac{97}{297}$

78. (a) : Since α,β are the roots of equation $x^2-6x-2=0$

$\therefore\ \alpha^2-6\alpha-2=0\ \Rightarrow\ \alpha^{10}-6\alpha^9-2\alpha^8=0$ ... (i)

Similarly, $\beta^{10}-6\beta^9-2\beta^8=0$ ... (ii)

Subtracting (ii) from (i), we get

$(\alpha^{10}-\beta^{10})-6(\alpha^9-\beta^9)-2(\alpha^8-\beta^8)=0$

$\Rightarrow\ a_{10}-6a_9-2a_8=0$

$\Rightarrow\ \frac{a_{10}-2a_8}{3a_9}=2$

79. (b) : $x={}^5P_3=60$

$y={}^{15}P_3=15\times14\times13=30\times91$

$\therefore\ 2y=91x$

80. (d) : $D=\begin{vmatrix}2&3&2\\3&2&2\\1&-1&4\end{vmatrix}=2(8+2)-3(12-2)+2(-3-2)$

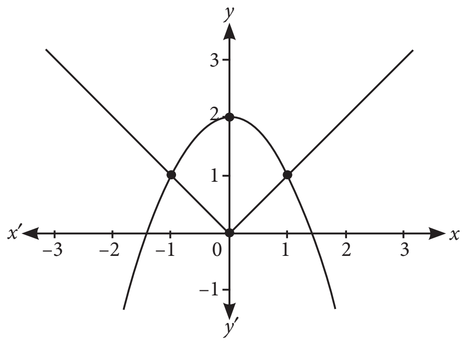
$=-20\neq0$

$\therefore$ System of linear equations has a unique solution.

81. (45) : For 3^n+7^n to be divisible by 10,
n can be any odd number.
i.e., $n=\{11, 13, \ldots, 99\}$

$\therefore$ Total number of required two digit odd numbers is 45.

82. (5) : $f(x)=\begin{cases}\min\{|x|,\ 2-x^2\}, & -2\le x\le2\\ [|x|], & 2<|x|\le3\end{cases}$



From the graph, we see that number of points of non-differentiability in $(-3, 3)=5$.

83. (2) : Given, $\vec{a}=\hat{i}+\alpha\hat{j}+3\hat{k},\ \vec{b}=3\hat{i}-\alpha\hat{j}+\hat{k}$

Now, $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&\alpha&3\\3&-\alpha&1\end{vmatrix}=\hat{i}(4\alpha)-\hat{j}(-8)+\hat{k}(-4\alpha)$

$\therefore$ Area of parallelogram, $|\vec{a}\times\vec{b}|=\sqrt{64+32\alpha^2}=8\sqrt{3}$ (Given)

$\Rightarrow\ 64+32\alpha^2=64\times3$

$\Rightarrow\ 2+\alpha^2=6\ \Rightarrow\ \alpha^2=4\ \Rightarrow\ \alpha=\pm2$

$\therefore\ \vec{a}\cdot\vec{b}=3-\alpha^2+3=2$

84. (1) : Let $x=4k+3$

$\therefore\ (2020+x)^{2022}=(2020+4k+3)^{2022}$
$=(4(505+k)+3)^{2022}=(4\lambda+3)^{2022}$
$=(16\lambda^2+24\lambda+9)^{1011}$
$=(8(2\lambda^2+3\lambda+1)+1)^{1011}=(8p+1)^{1011}$

$\therefore$ Remainder when $(2020+x)^{2022}$ is divided by 8 = 1.

85. (4) : Given curves are $x=y^4$ and $xy=k$

$\Rightarrow\ y^5=k$ ... (i)

Now, $x=y^4$

$\Rightarrow\ 1=4y^3\frac{dy}{dx}\ \Rightarrow\ \frac{dy}{dx}=\frac{1}{4y^3}=m_1$

Also, $xy=k\ \Rightarrow\ x=\frac{k}{y}$

$\Rightarrow\ 1=-\frac{k}{y^2}\cdot\frac{dy}{dx}\ \Rightarrow\ \frac{dy}{dx}=\frac{-y^2}{k}=m_2$

Since, curves cut orthogonally.

$\therefore\ m_1m_2=-1$

$\Rightarrow\ \frac{1}{4y^3}\times\left(\frac{-y^2}{k}\right)=-1\ \Rightarrow\ y=\frac{1}{4k}$

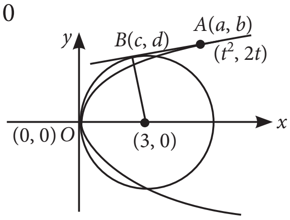
$\Rightarrow\ y^5=\frac{1}{(4k)^5}\ \Rightarrow\ \frac{1}{(4k)^5}=k$ [Using (i)]

$\Rightarrow\ (4k)^6=4$

86. (9) : Let the coordinates of point A are $(t^2, 2t)$.
Equation of tangent at point A is given by

$$yt = x + t^2 \Rightarrow x - ty + t^2 = 0$$

Centre of circle is (3, 0).



Also, radius $= \left|\frac{3-0+t^2}{\sqrt{1+t^2}}\right| = 3$

$\Rightarrow (3 + t^2)^2 = 9(1 + t^2)$

$\Rightarrow 9 + t^4 + 6t^2 = 9 + 9t^2$

$\Rightarrow t^4 - 3t^2 = 0 \Rightarrow t^2(t^2 - 3) = 0$

$\Rightarrow t = 0, -\sqrt{3}, \sqrt{3}$

So, point $A(3, 2\sqrt{3})$ is in first quadrant, where $a = 3$ and $b = 2\sqrt{3}$.

For point B which is foot of perpendicular from centre (3, 0) to the tangent $x - \sqrt{3}y + 3 = 0$.

$\therefore \frac{c-3}{1} = \frac{d-0}{-\sqrt{3}} = \frac{-(3-0+3)}{4} \Rightarrow c = \frac{3}{2}$ and $d = \frac{3\sqrt{3}}{2}$

$\therefore 2(a+c) = 2\left(3+\frac{3}{2}\right) = 9$

87. (5) : Let $L = \lim_{x\to 0} \frac{ax - (e^{4x}-1)}{ax(e^{4x}-1)}$ $\left(\frac{0}{0} \text{ form}\right)$

$= \lim_{x\to 0} \frac{ax-(e^{4x}-1)}{ax \cdot 4x}$ $\left[\text{Using } \lim_{x\to 0}\frac{e^{4x}-1}{4x} = 1\right]$

Applying L'Hospital rule, we get

$$L = \lim_{x\to 0} \frac{a - 4e^{4x}}{8ax}$$

Limit exists finitely only when $a - 4 = 0 \Rightarrow a = 4$

$\therefore L = \lim_{x\to 0} \frac{4-4e^{4x}}{32x} = \lim_{x\to 0}\frac{1-e^{4x}}{8x}$

$= \lim_{x\to 0}\frac{e^{4x}-1}{-2\times 4x} = -\frac{1}{2} \Rightarrow b = -\frac{1}{2}$

$\therefore a - 2b = 4 - 2\left(-\frac{1}{2}\right) = 5$

88. (1) : We have, $(2xy^2 - y)dx + xdy = 0$

$\Rightarrow 2xy^2dx - ydx + xdy = 0$

$\Rightarrow 2xdx = \frac{ydx - xdy}{y^2} = d\left(\frac{x}{y}\right)$

Integrating both sides, we get $x^2 = \frac{x}{y} + c$ ...(i)

We have the lines $2x - 3y = 1$ and $3x + 2y = 8$

After solving, we get $x = 2, y = 1$

Since (i) passes through point of intersection of given lines *i.e.*, (2, 1).

$\therefore 4 = \frac{2}{1} + c \Rightarrow c = 2$

$\therefore x^2 = \frac{x}{y} + 2$

Now, at $x = 1$, $y(1) = -1 \Rightarrow |y(1)| = 1$

89. (19) : We have, $\int_{-2}^{2} 3|x^2 - x - 2|\,dx$

$$= 3\left[\int_{-2}^{-1}(x^2-x-2)\,dx + \int_{-1}^{2} -(x^2-x-2)dx\right]$$

$$= 3\left[\left(\frac{x^3}{3} - \frac{x^2}{2} - 2x\right)_{-2}^{-1} - \left(\frac{x^3}{3}-\frac{x^2}{2}-2x\right)_{-1}^{2}\right]$$

$$= 3\left[\left(\frac{7}{6}+\frac{4}{6}\right) - \left(\frac{-20}{6}-\frac{7}{6}\right)\right] = 3\left(\frac{11}{6}+\frac{27}{6}\right) = 3\left[\frac{38}{6}\right] = 19$$

90. (44) : $l_1 : \vec{r} = (3+t)\hat{i} + (-1+2t)\hat{j} + (4+2t)\hat{k}$

$= (3\hat{i} - \hat{j} + 4\hat{k}) + t(\hat{i} + 2\hat{j} + 2\hat{k})$

$l_2 : \vec{r} = (3+2s)\hat{i} + (3+2s)\hat{j} + (2+s)\hat{k}$

$= (3\hat{i} + 3\hat{j} + 2\hat{k}) + s(2\hat{i} + 2\hat{j} + \hat{k})$

D.R.'s of $l_1 \equiv (1, 2, 2)$

D.R.'s of $l_2 \equiv (2, 2, 1)$

$$\therefore \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{vmatrix} = -2\hat{i} + 3\hat{j} - 2\hat{k}$$

$\therefore$ D.R.'s of l (line $\perp$ to l_1 and l_2) = (–2, 3, –2)

$\Rightarrow l : \vec{r} = -2\mu\hat{i} + 3\mu\hat{j} - 2\mu\hat{k}$

For intersection of l and l_1,

$3 + t = -2\mu$,

$-1 + 2t = 3\mu$,

$4 + 2t = -2\mu \Rightarrow t = -1$ and $\mu = -1$

$\therefore$ Point of intersection, $P \equiv (2, -3, 2)$.

Let point on l_2 be $Q\ (3 + 2s, 3 + 2s, 2 + s)$

Given, $PQ = \sqrt{17} \Rightarrow (PQ)^2 = 17$

$\Rightarrow (2s + 1)^2 + (6 + 2s)^2 + (s)^2 = 17$

$\Rightarrow 9s^2 + 28s + 20 = 0 \Rightarrow s = -2, -\frac{10}{9} \Rightarrow s = \frac{-10}{9}$

[$\because$ $s \neq -2$ as point lies in 1st octant]

$\therefore a = 3 + 2\left(-\frac{10}{9}\right) = \frac{7}{9}$, $b = 3 + 2\left(-\frac{10}{9}\right) = \frac{7}{9}$ and

$c = 2 + \left(-\frac{10}{9}\right) = \frac{8}{9}$

Hence, $18(a+b+c) = 18\left(\frac{22}{9}\right) = 44$

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