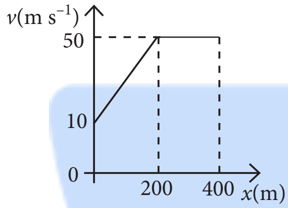
JEE MAIN 2021

ONLINE
16th March
1st Shift

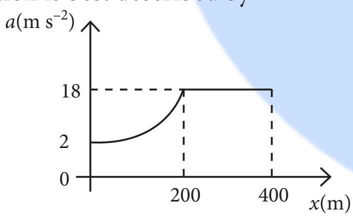
PHYSICS

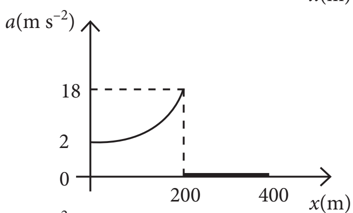
SECTION-A (MULTIPLE CHOICE QUESTIONS)

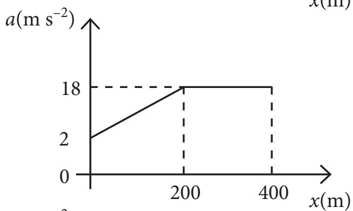
1. The velocity-displacement graph describing the motion of a bicycle is shown in the figure.

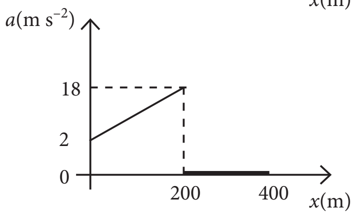


The acceleration-displacement graph of the bicycle's motion is best described by

(a)


(b)


(c)


(d)


2. For an electromagnetic wave travelling in free space, the relation between average energy densities due to electric (U_e) and magnetic (U_m) fields is

(a) $U_e < U_m$ (b) $U_e \neq U_m$
(c) $U_e > U_m$ (d) $U_e = U_m$

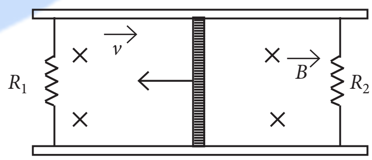
3. The volume V of an enclosure contains a mixture of three gases, 16 g of oxygen, 28 g of nitrogen and 44 g of carbon dioxide at absolute temperature T. Consider R as universal gas constant. The pressure of the mixture of gases is

(a) $\frac{4RT}{V}$ (b) $\frac{3RT}{V}$
(c) $\frac{88RT}{V}$ (d) $\frac{5}{2}\frac{RT}{V}$

4. One main scale division of a vernier callipers is 'a' cm and n^{th} division of the vernier scale coincide with $(n-1)^{th}$ division of the main scale. The least count of the callipers in mm is

(a) $\left(\frac{n-1}{10n}\right)a$ (b) $\frac{10na}{(n-1)}$
(c) $\frac{10a}{n}$ (d) $\frac{10a}{(n-1)}$

5. A conducting bar of length L is free to slide on two parallel conducting rails as shown in the figure.



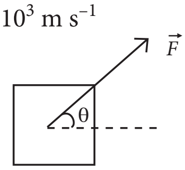
Two resistors R_1 and R_2 are connected across the ends of the rails. There is a uniform magnetic field $\vec{B}$ pointing into the page. An external agent pulls the bar to the left at a constant speed v.
The correct statement about the directions of induced currents I_1 and I_2 flowing through R_1 and R_2 respectively is

(a) both I_1 and I_2 are in clockwise direction
(b) both I_1 and I_2 are in anticlockwise direction
(c) I_1 is in clockwise direction and I_2 is in anticlockwise direction
(d) I_1 is in anticlockwise direction and I_2 is in clockwise direction.

6. The maximum and minimum distances of a comet from the Sun are 1.6×10^{12} m and 8.0×10^{10} m respectively . If the speed of the comet at the nearest point is 6×10^4 m s^{-1}, the speed at the farthest point is

(a) 3.0×10^3 m s^{-1} (b) 4.5×10^3 m s^{-1}
(c) 1.5×10^3 m s^{-1} (d) 6.0×10^3 m s^{-1}

7. A block of mass m slides along a floor while a force of magnitude F is applied to it at an angle θ as shown in figure. The coefficient of kinetic



friction is μ_K. Then, the block's acceleration 'a' is given by (g is acceleration due to gravity)

(a) $\frac{F}{m}\cos\theta-\mu_K\left(g+\frac{F}{m}\sin\theta\right)$

(b) $\frac{F}{m}\cos\theta-\mu_K\left(g-\frac{F}{m}\sin\theta\right)$

(c) $-\frac{F}{m}\cos\theta-\mu_K\left(g-\frac{F}{m}\sin\theta\right)$

(d) $\frac{F}{m}\cos\theta+\mu_K\left(g-\frac{F}{m}\sin\theta\right)$

8. A block of 200 g mass moves with a uniform speed in a horizontal circular groove, with vertical side walls of radius 20 cm. If the block takes 40 s to complete one round, the normal force by the side walls of the groove is

(a) 9.859×10^{-4} N (b) 6.28×10^{-3} N
(c) 9.859×10^{-2} N (d) 0.0314 N

9. A conducting wire of length 'l', area of cross-section A and electric resistivity ρ is connected between the terminals of a battery. A potential difference V is developed between its ends, causing an electric current. If the length of the wire of the same material is doubled and the area of cross-section is halved, the resultant current would be

(a) $\frac{1}{4}\frac{VA}{\rho l}$ (b) $4\frac{VA}{\rho l}$

(c) $\frac{1}{4}\frac{\rho l}{VA}$ (d) $\frac{3}{4}\frac{VA}{\rho l}$

10. A plane electromagnetic wave of frequency 500 MHz is travelling in vacuum along y-direction. At a particular point in space and time, $\vec{B}=8.0\times10^{-8}\,\hat{z}$T. The value of electric field at this point is
(speed of light = 3×10^8 m s^{-1})
$\hat{x}, \hat{y}, \hat{z}$ are unit vectors along x, y and z directions.

(a) $-2.6\,\hat{y}$ Vm^{-1} (b) $24\hat{x}$ Vm^{-1}
(c) $2.6\hat{x}$ Vm^{-1} (d) $-24\hat{x}$ Vm^{-1}

11. A bar magnet of length 14 cm is placed in the magnetic meridian with its north pole pointing towards the geographic north pole. A neutral point is obtained at a distance of 18 cm from the center of the magnet. If B_H = 0.4 G, the magnetic moment of the magnet is (1 G = 10^{-4} T)

(a) 28.80 J T^{-1} (b) 2.880 J T^{-1}
(c) 2.880×10^3 J T^{-1} (d) 2.880×10^2 J T^{-1}

12. Time period of a simple pendulum is T inside a lift when the lift is stationary. If the lift moves upwards with an acceleration $\frac{g}{2}$, the time period of pendulum will be

(a) $\sqrt{3}T$ (b) $\frac{T}{\sqrt{3}}$ (c) $\sqrt{\frac{3}{2}}T$ (d) $\sqrt{\frac{2}{3}}T$

13. In thermodynamics, heat and work are

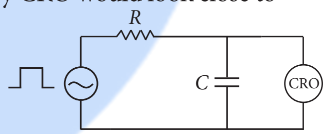
(a) extensive thermodynamic state variables
(b) point functions
(c) path functions
(d) intensive thermodynamic state variables.

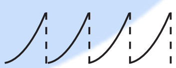
14. The pressure acting on a submarine is 3×10^5 Pa at a certain depth. If the depth is doubled, the percentage increase in the pressure acting on the submarine would be
(Assume that atmospheric pressure is 1×10^5 Pa, density of water is 10^3 kg m^{-3}, g = 10 m s^{-2})

(a) $\frac{3}{200}\%$ (b) $\frac{200}{3}\%$

(c) $\frac{200}{5}\%$ (d) $\frac{5}{200}\%$

15. An RC circuit as shown in the figure is driven by a AC source generating a square wave. The output wave pattern monitored by CRO would look close to



(a)  (b) 

(c)  (d) 

16. For changing the capacitance of a given parallel plate capacitor, a dielectric material of dielectric constant K is used, which has the same area as the plates of the capacitor. The thickness of the dielectric slab is $\frac{3}{4}d$, where 'd' is the separation between the plates of parallel plate capacitor. The new capacitance (C') in terms of original capacitance (C_0) is given by the following relation.

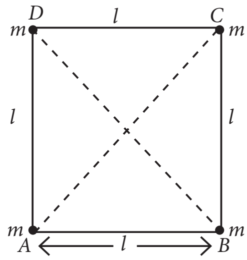
(a) $C'=\frac{3+K}{4K}C_0$ (b) $C'=\frac{4}{3+K}C_0$

(c) $C'=\frac{4+K}{3}C_0$ (d) $C'=\frac{4K}{K+3}C_0$

17. A 25 m long antenna is mounted on an antenna tower. The height of the antenna tower is 75 m. The wavelength (in meter) of the signal transmitted by this antenna would be

(a) 100 (b) 300 (c) 200 (d) 400

18. For equal masses, m each are placed at the corners of a square of length (l) as shown in the figure. The moment of inertia of the system about an axis passing through A and parallel to DB would be



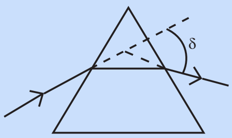
(a) $2\ ml^2$ (b) $\sqrt{3}\ ml^2$ (c) ml^2 (d) $3\ ml^2$

19. The stopping potential in the context of photoelectric effect depends on the following property of incident electromagnetic radiation

(a) phase (b) frequency

(c) intensity (d) amplitude.

20. The angle of deviation through a prism is minimum when



(A) incidentrayandemergentray are symmetric to the prism

(B) the refracted ray inside the prism becomes parallel to its base

(C) angle of incidence is equal to that of the angle of emergence

(D) when angle of emergence is double the angle of incidence.

(a) Only statements (A) and (B) are true.

(b) Statements (B) and (C) are true.

(c) Statements (A), (B) and (C) are true.

(d) Only statement (D) is true.

SECTION-B (NUMERICAL VALUE TYPE)

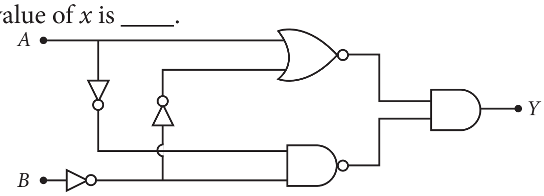
Attempt any 5 questions out of 10.

21. A fringe width of 6 mm was produced for two slits separated by 1 mm apart. The screen is placed 10 m away. The wavelength of light used is 'x' nm. The value of 'x' to the nearest integer is _______.

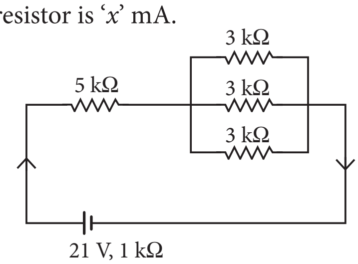
22. The first three spectral lines of H-atom in the Balmer series are given λ_1, λ_2, λ_3 considering the Bohr atomic model, the wavelengths of first and third spectral lines $\left(\frac{\lambda_1}{\lambda_3}\right)$ are related by a factor of approximately 'x' $\times 10^{-1}$.

The value of x, to the nearest integer, is ______.

23. In the logic circuit shown in the figure, if input A and B are 0 to 1 respectively, the output at Y would be 'x'. The value of x is _____.

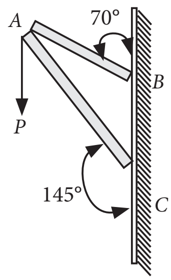


24. In the figure given, the electric current flowing through the 5 kΩ resistor is 'x' mA.



The value of x to nearest integer is _____.

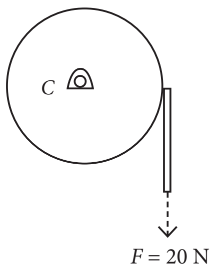
25. Consider a frame that is made up of two thin massless rods AB and AC as shown in the figure. A vertical force $\vec{P}$ of magnitude 100 N is applied at point A of the frame.



Suppose the force is $\vec{P}$ resolved parallel to the arms AB and AC of the frame. The magnitude of the resolved component along the arm AC is x N. The value of x, to the nearest integer, is ______.

[Given : sin(35°) = 0.573, cos(35°) = 0.819, sin(110°) = 0.939, cos(110°) = −0.342]

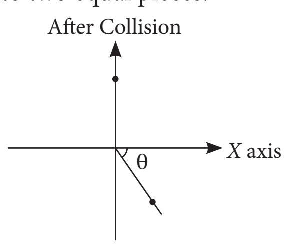
26. Consider a 20 kg uniform circular disk of radius 0.2 m. It is pin supported at its center and is at rest initially. The disk is acted upon by a constant force $F = 20$ N through a massless string wrapped around its periphery as shown in figure.



Suppose the disk makes n number of revolutions to attain an angular speed of 50 rad s^{-1}. The value of n, to the nearest integer, is _____.

[Given : In one complete revolution, the disk rotates by 6.28 rad]

27. A ball of mass 10 kg moving with a velocity $10\sqrt{3}$ m s^{-1} along X-axis, hits another ball of mass 20 kg which is at rest. After collision, the first ball comes to rest and the second one disintegrates into two equal pieces. One of the pieces starts moving along Y-axis at a speed of 10 m s^{-1}. The second piece starts moving at a speed of 20 m s^{-1} at an angle θ (degree) with respect to the X-axis.

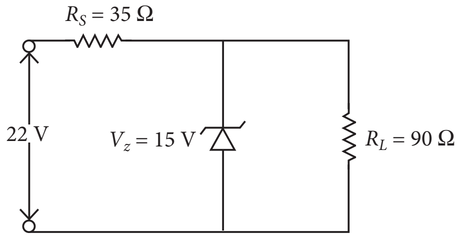


The configuration of pieces after collision is shown in the figure. The value of θ to the nearest integer is _________.

28. The resistance $R = \frac{V}{I}$, where V = (50 ± 2) V and I = (20 ± 0.2)A. The percentage error in R is 'x' %. The value of 'x' to the nearest integer is _______.

29. A sinusoidal voltage of peak value 250 V is applied to a series *LCR* circuit, in which $R = 8\ \Omega$, $L = 24$ mH and $C = 60\ \mu F$. The value of power dissipated at resonant condition is 'x' kW. The value of x to the nearest integer is ______.

30. The value of power dissipated across the zener diode ($V_z = 15$ V) connected in the circuit as shown in the figure is $x \times 10^{-1}$ watt.



The value of x, to the nearest integer, is _______.

CHEMISTRY

SECTION-A (MULTIPLE CHOICE QUESTIONS)

31. Which among the following pairs of vitamins is stored in our body relatively for longer duration?
(a) Vitamin A and vitamin D
(b) Thiamine and ascorbic acid
(c) Thiamine and vitamin A
(d) Ascorbic acid and vitamin D

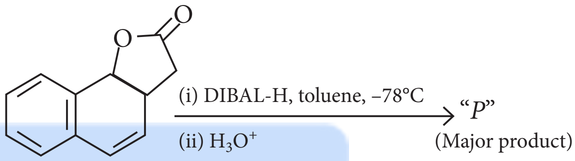
32. Given below are two statements :
Statement I : Both $CaCl_2 \cdot 6H_2O$ and $MgCl_2 \cdot 8H_2O$ undergo dehydration on heating.
Statement II : BeO is amphoteric whereas the oxides of other elements in the same group are acidic.
In the light of the above statements, choose the correct answer from the options given below :
(a) Statement I is true but statement II is false.
(b) Both statement I and statement II are true.
(c) Statement I is false but statement II is true.
(d) Both statement I and statement II are false.

33. **Assertion (A) :** Enol form of acetone [CH_3COCH_3] exists in < 0.1% quantity. However, the enol form of acetylacetone [$CH_3COCH_2OCCH_3$] exists in approximately 15% quantity.
Reason (R) : Enol form of acetylacetone is stabilized by intramolecular hydrogen bonding, which is not possible in enol form of acetone.
Choose the correct statement.
(a) A is true but R is false.
(b) Both A and R are true but R is not the correct explanation of A.
(c) Both A and R are true and R is the correct explanation of A.
(d) A is false but R is true.

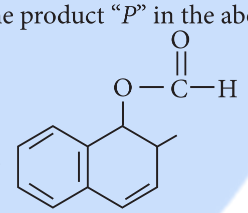
34. A group 15 element, which is a metal and forms a hydride with strongest reducing power among group 15 hydrides. The element is
(a) Bi (b) Sb (c) P (d) As

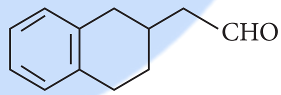
35. Given below are two statement : one is labelled as Assertion A and the other is labelled as Reason R.
Assertion (A) : Size of Bk^{3+} ion is less than Np^{3+} ion.
Reason (R) : The above is a consequence of the lanthanoid contraction.
In the light of the above statements, choose the correct answer from the options given below.
(a) Both A and R are true but R is not the correct explanation of A.
(b) A is false but R is true.
(c) Both A and R are true and R is the correct explanation of A.
(d) A is true but R is false.

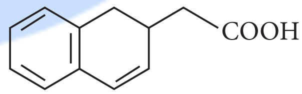
36.

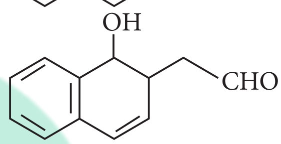


The product "*P*" in the above reaction is

(a) 

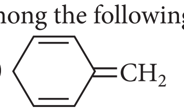
(b) 

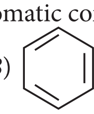
(c) 

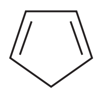
(d) 

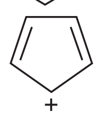
37. The functions of antihistamine are
(a) antiallergic and antidepressant
(b) antacid and antiallergic
(c) analgesic and antacid
(d) antiallergic and analgesic.

38. Among the following, the aromatic compounds are

(*A*) 

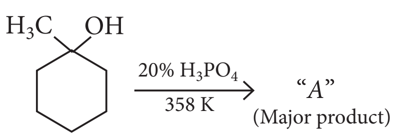
(*B*) 

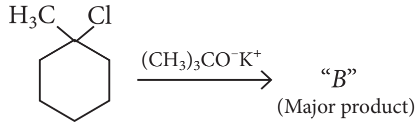
(*C*) 

(*D*) 

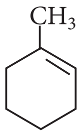
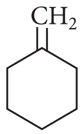
Choose the correct answer from the following options.
(a) (*B*) and (*C*) only (b) (*A*) and (*B*) only
(c) (*A*), (*B*) and (*C*) only (d) (*B*), (*C*) and (*D*) only

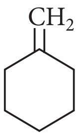
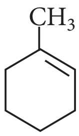
39.

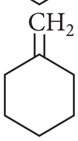
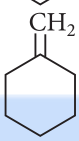


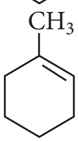
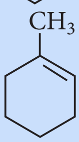


The products "*A*" and "*B*" formed in above reactions are

(a) *A*–  ; *B* – 

(b) *A* –  ; *B* – 

(c) *A* –  ; *B* – 

(d) *A* –  ; *B* – 

40. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion (A) : The H — O — H bond angle in water molecule is 104.5°.

Reason (R) : The lone pair-lone pair repulsion of electrons is higher than the bond pair-bond pair repulsion.

In the light of the above statements, choose the correct answer from the options given below.

(a) Both A and R are true, and R is the correct explanation of A.
(b) A is false but R is true.
(c) A is true but R is false.
(d) Both A and R are true, but R is not the correct explanation of A.

41. The type of pollution that gets increased during the day time and in the presence of O_3 is

(a) reducing smog (b) global warming
(c) oxidising smog (d) acid rain.

42. Match List – I with List – II.

	List - I Name of oxo acid		List - II Oxidation state of 'P'
(A)	Hypophosphorous acid	(i)	+5
(B)	Orthophosphoric acid	(ii)	+4
(C)	Hypophosphoric acid	(iii)	+3
(D)	Orthophosphorous acid	(iv)	+2
		(v)	+1

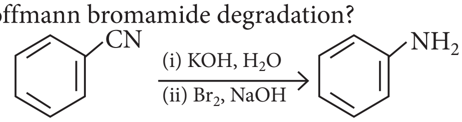
Choose the correct answer from the options given below.

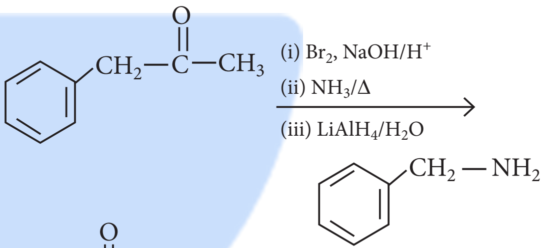
(a) (A) – (iv), (B) – (v), (C) – (ii), (D) – (iii)
(b) (A) – (iv), (B) – (i), (C) – (ii), (D) – (iii)
(c) (A) – (v), (B) – (iv), (C) – (ii), (D) – (iii)
(d) (A) – (v), (B) – (i), (C) – (ii), (D) – (iii)

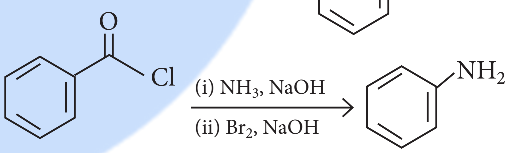
43. Which of the following is Lindlar catalyst?

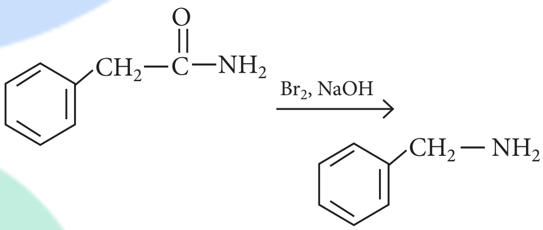
(a) Cold dilute solution of $KMnO_4$
(b) Partially deactivated palladised charcoal
(c) Zinc chloride and HCl
(d) Sodium and liquid NH_3

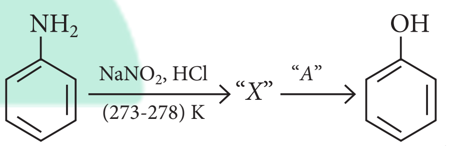
44. Which of the following reaction does not involve Hoffmann bromamide degradation?

(a) 

(b) 

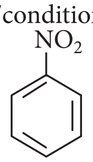
(c) 

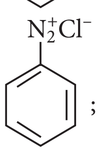
(d) 

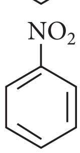
45. 

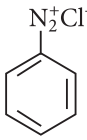
Major Product

In the above chemical reaction, intermediate "*X*" and reagent/condition "*A*" are

(a) *X* –  ; *A* – $H_2O/NaOH$

(b) *X* –  ; *A* – $H_2O/NaOH$

(c) *X* –  ; *A* – H_2O/Δ

(d) X – 

; A – H_2O/Δ

46. In chromatography technique, the purification of compound is independent of
(a) mobility or flow of solvent system
(b) solubility of the compound
(c) physical state of the pure compound
(d) length of the column or TLC plate.

47. Match List – I with List – II

List – I Industrial process		List – II Application	
(A)	Haber's process	(i)	HNO_3 synthesis
(B)	Ostwald's process	(ii)	Aluminium extraction
(C)	Contact process	(iii)	NH_3 synthesis
(D)	Hall-Heroult process	(iv)	H_2SO_4 synthesis

Choose the correct answer from the options given below.
(a) (A) – (iv), (B) – (i), (C) – (ii), (D) – (iii)
(b) (A) – (iii), (B) – (i), (C) – (iv), (D) – (ii)
(c) (A) – (iii), (B) – (iv), (C) – (i), (D) – (ii)
(d) (A) – (ii), (B) – (iii), (C) – (iv), (D) – (i)

48. Given below are two statements :
Statement I : The $E°$ value for Ce^{4+}/Ce^{3+} is +1.74 V.
Statement II : Ce is more stable in Ce^{4+} state than Ce^{3+} state.
In the light of the above statements, choose the most appropriate answer from the options given below :
(a) Both statement I and statement II are incorrect.
(b) Statement I is incorrect but statement II is correct.
(c) Statement I is correct but statement II is incorrect.
(d) Both statement I and statement II are correct.

49. The process that involves the removal of sulphur from the ores is
(a) leaching (b) smelting
(c) refining (d) roasting.

50. Given below are two statement :
Statement I : H_2O_2 can act as both oxidising and reducing agent in basic medium.
Statement II : In the hydrogen economy, the energy is transmitted in the form of dihydrogen.
In the light of the above statements choose the correct answer from the options given below :
(a) Statement I is false but statement II is true.
(b) Statement I is true but statement II is false.
(c) Both Statement I and statement II are true.
(d) Both statement I and statement II are false.

SECTION-B (NUMERICAL VALUE TYPE)

Attempt any 5 questions out of 10.

51. Two salts A_2X and MX have the same value of solubility product of 4.0×10^{-12}. The ratio of their molar solubilities *i.e.*, $\frac{S(A_2X)}{S(MX)}$ = ________.
(Round off to the Nearest Integer)

52. A certain element crystallises in a *bcc* lattice of unit cell edge 27Å. If the same element under the same conditions crystallises in the *fcc* lattice, the edge length of the unit cell in Å will be ________.
(Round off to the Nearest Integer)
[Assume each lattice point has a single atom]
[Assume $\sqrt{3} = 1.73$, $\sqrt{2} = 1.41$]

53. A 6.50 molal solution of $KOH_{(aq)}$ has a density 1.89 g cm^{-3}. The molarity of the solution is ________ mol dm^{-3}. (Round off to the Nearest Integer)
[Atomic masses : K : 39.0 u; O = 16.0 u; H : 1.0 u]

54. The decomposition of formic acid on gold surface follows first order kinetics. If the rate constant at 300 K is 1.0×10^{-3} s^{-1} and the activation energy E_a = 11.488 kJ mol^{-1}, the rate constant at 200 K is ________ $\times 10^{-5}$ s^{-1}.
(Round off to the Nearest Integer)
(Given : R = 8.314 J mol^{-1} K^{-1})

55. AB_2 is 10% dissociated in water to A^{2+} and B^-. The boiling point of a 10.0 molal aqueous solution of AB_2 is ________ °C. (Round off to the Nearest Integer)
[Given : Molal elevation constant of water, K_b = 0.5 K kg mol^{-1}; boiling point of pure water = 100°C]

56. For the reaction $A_{(g)} \rightleftharpoons B_{(g)}$ at 495 K,
$\Delta_r G°$ = –9.478 kJ mol^{-1}. If we start the reaction in a closed container at 495 K with 22 millimoles of A, the amount of B in the equilibrium mixture is ________ millimoles.
(Round off to the Nearest Integer)
[R = 8.314 J mol^{-1} K^{-1}; ln10 = 2.303]

57. $2MnO_4^- + bC_2O_4^{2-} + cH^+ \longrightarrow xMn^{2+} + yCO_2 + zH_2O$
If the above equation is balanced with integer coefficients, the value of c is ________.
(Round off to the Nearest Integer)

58. When light of wavelength 248 nm falls on a metal of threshold energy 3.0 eV, the de-Broglie wavelength of emitted electrons is ________ Å.
(Round off to the Nearest Integer)
[Use : $\sqrt{3}$ = 1.73, $h = 6.63 \times 10^{-34}$ Js
$m_e = 9.1 \times 10^{-31}$ kg ; $c = 3.0 \times 10^8$ ms^{-1} ;
1 eV = 1.6×10^{-19} J]

59. Complete combustion of 750 g of an organic compound provides 420 g of CO_2 and 210 g of H_2O. The percentage composition of carbon and hydrogen in organic compound is 15.3 and ________ respectively.
(Round off to the Nearest Integer)

60. The equivalents of ethylene diamine required to replace the neutral ligands from the coordination sphere of the *trans*-complex of $CoCl_3 \cdot 4NH_3$ is ________.
(Round off to the Nearest Integer)

MATHEMATICS

SECTION-A (MULTIPLE CHOICE QUESTIONS)

61. If n is the number of irrational terms in the expansion of $\left(3^{\frac{1}{4}}+5^{\frac{1}{8}}\right)^{60}$, then $(n-1)$ is divisible by
(a) 26 (b) 7 (c) 8 (d) 30

62. Let P be a plane $lx + my + nz = 0$ containing the line, $\frac{1-x}{1}=\frac{y+4}{2}=\frac{z+2}{3}$. If plane P divides the line segment AB joining points $A(-3, -6, 1)$ and $B(2, 4, -3)$ in ratio $k : 1$, then the value of k is equal to
(a) 1.5 (b) 4 (c) 3 (d) 2

63. The locus of the midpoints of the chord of the circle, $x^2 + y^2 = 25$ which is tangent to the hyperbola, $\frac{x^2}{9}-\frac{y^2}{16}=1$ is
(a) $(x^2 + y^2)^2 - 9x^2 - 16y^2 = 0$
(b) $(x^2 + y^2)^2 - 9x^2 + 144y^2 = 0$
(c) $(x^2 + y^2)^2 - 16x^2 + 9y^2 = 0$
(d) $(x^2 + y^2)^2 - 9x^2 + 16y^2 = 0$

64. Let $A=\begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$, $i=\sqrt{-1}$. Then, the system of linear equations $A^8\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 8 \\ 64 \end{bmatrix}$ has
(a) A unique solution
(b) No solution
(c) Exactly two solutions
(d) Infinitely many solutions

65. The number of elements in the set $\{x \in R : (|x| - 3)|x + 4| = 6\}$ is equal to
(a) 1 (b) 4 (c) 3 (d) 2

66. Let a complex number z, $|z| \neq 1$, satisfy $\log_{\frac{1}{\sqrt{2}}}\left(\frac{|z|+11}{(|z|-1)^2}\right)\le 2$. Then, the largest value of $|z|$ is equal to ______.
(a) 5 (b) 7 (c) 8 (d) 6

67. If for $a > 0$, the feet of perpendiculars from the points A $(a, -2a, 3)$ and $B(0, 4, 5)$ on the plane $lx + my + nz = 0$ are points $C(0, -a, -1)$ and D respectively, then the length of line segment CD is equal to
(a) $\sqrt{31}$ (b) $\sqrt{55}$ (c) $\sqrt{41}$ (d) $\sqrt{66}$

68. Which of the following Boolean expression is a tautology?
(a) $(p \wedge q) \wedge (p \to q)$ (b) $(p \wedge q) \to (p \to q)$
(c) $(p \wedge q) \vee (p \to q)$ (d) $(p \wedge q) \vee (p \vee q)$

69. If for $x \in \left(0, \frac{\pi}{2}\right)$, $\log_{10}\sin x + \log_{10}\cos x = -1$ and $\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1)$, $n > 0$, then the value of n is equal to
(a) 9 (b) 20 (c) 16 (d) 12

70. The number of roots of the equation, $(81)^{\sin^2 x}+(81)^{\cos^2 x}=30$ in the interval $[0, \pi]$ is equal to
(a) 4 (b) 3 (c) 8 (d) 2

71. Let a vector $\alpha\hat{i}+\beta\hat{j}$ be obtained by rotating the vector $\sqrt{3}\hat{i}+\hat{j}$ by an angle 45° about the origin in counterclockwise direction in the first quadrant. Then the area of triangle having vertices (α, β), $(0, \beta)$ and $(0, 0)$ is equal to
(a) 1 (b) $2\sqrt{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{2}$

72. A pack of cards has one card missing. Two cards are drawn randomly and are found to be spades. The probability that the missing card is not a spade, is
(a) $\frac{39}{50}$ (b) $\frac{52}{867}$ (c) $\frac{3}{4}$ (d) $\frac{22}{425}$

73. Let the position vectors of two points P and Q be $3\hat{i}-\hat{j}+2\hat{k}$ and $\hat{i}+2\hat{j}-4\hat{k}$, respectively. Let R and S be two points such that the direction ratios of lines PR and QS are $(4, -1, 2)$ and $(-2, 1, -2)$, respectively. Let lines PR and QS intersect at T. If the vector $\overrightarrow{TA}$ is perpendicular to both $\overrightarrow{PR}$ and $\overrightarrow{QS}$ and the length of vector $\overrightarrow{TA}$ is $\sqrt{5}$ units, then the modulus of a position vector of A is
(a) $\sqrt{227}$ (b) $\sqrt{171}$ (c) $\sqrt{5}$ (d) $\sqrt{482}$

74. Let the functions $f : R \to R$ and $g : R \to R$ be defined as
$$f(x)=\begin{cases} x+2, & x<0 \\ x^2, & x\ge 0 \end{cases} \text{ and } g(x)=\begin{cases} x^3, & x<1 \\ 3x-2, & x\ge 1 \end{cases}$$
Then, the number of points in R where $(fog)(x)$ is NOT differentiable is equal to
(a) 3 (b) 0 (c) 2 (d) 1

75. Let $S_k=\sum_{r=1}^{k}\tan^{-1}\left(\frac{6^r}{2^{2r+1}+3^{2r+1}}\right)$.
Then $\lim_{k\to\infty} S_k$ is equal to
(a) $\cot^{-1}\left(\frac{3}{2}\right)$ (b) $\frac{\pi}{2}$
(c) $\tan^{-1}\left(\frac{3}{2}\right)$ (d) $\tan^{-1}(3)$

76. If $y = y(x)$ is the solution of the differential equation, $\frac{dy}{dx}+2y\tan x=\sin x,\ y\left(\frac{\pi}{3}\right)=0$, then the maximum value of the function $y(x)$ over R is equal to

(a) $-15/4$ (b) 8 (c) $1/2$ (d) $1/8$

77. Let $[x]$ denote greatest integer less than or equal to x. If for $n \in N$, $(1-x+x^3)^n=\sum_{j=0}^{3n}a_jx^j$, then $\sum_{j=0}^{\left[\frac{3n}{2}\right]}a_{2j}+4\sum_{j=0}^{\left[\frac{3n-1}{2}\right]}a_{2j+1}$ is equal to

(a) 2^{n-1} (b) 1 (c) n (d) 2

78. The range of $a \in R$ for which the function

$f(x) = (4a - 3)(x + \log_e 5) + 2(a - 7)\cot\left(\frac{x}{2}\right)\sin^2\left(\frac{x}{2}\right)$, $x \neq 2n\pi$, $n \in N$, has critical points, is

(a) $(-3, 1)$ (b) $\left[-\frac{4}{3}, 2\right)$

(c) $[1, \infty)$ (d) $(-\infty, -1]$

79. Consider three observations a, b, and c such that $b = a + c$. If the standard deviation of $a + 2$, $b + 2$, $c + 2$ is d, then which of the following is true?

(a) $b^2 = 3(a^2 + c^2 + d^2)$ (b) $b^2 = a^2 + c^2 + 3d^2$

(c) $b^2 = 3(a^2 + c^2) - 9d^2$ (d) $b^2 = 3(a^2 + c^2) + 9d^2$

80. If the three normals drawn to the parabola, $y^2 = 2x$ pass through the point $(a, 0)$, $a \neq 0$, then 'a' must be greater than

(a) $-1/2$ (b) 1 (c) -1 (d) $1/2$

SECTION-B (NUMERICAL VALUE TYPE)

Attempt any 5 questions out of 10.

81. If $\lim_{x\to 0}\frac{ae^x-b\cos x+ce^{-x}}{x\sin x}=2$, then $a + b + c$ is equal to ________.

82. Let $f : R \to R$ be a continuous function such that $f(x) + f(x + 1) = 2$, for all $x \in R$. If $I_1=\int_0^8 f(x)dx$ and $I_2=\int_{-1}^{3} f(x)dx$, then the value of $I_1 + 2I_2$ is equal to ________.

83. If the normal to the curve $y(x)=\int_0^x(2t^2-15t+10)dt$ at a point (a, b) is parallel to the line $x + 3y = -5$, $a > 1$, then the value of $|a + 6b|$ is equal to ________.

84. Let $P=\begin{bmatrix}-30 & 20 & 56\\ 90 & 140 & 112\\ 120 & 60 & 14\end{bmatrix}$ and $A=\begin{bmatrix}2 & 7 & \omega^2\\ -1 & -\omega & 1\\ 0 & -\omega & -\omega+1\end{bmatrix}$, where $\omega=\frac{-1+i\sqrt{3}}{2}$, and I_3 be the identity matrix of order 3. If the determinant of the matrix $(P^{-1}AP - I_3)^2$ is $\alpha\omega^2$, then the value of α is equal to ________.

85. Let $ABCD$ be a square of side of unit length. Let a circle C_1 centered at A with unit radius is drawn. Another circle C_2 which touches C_1 and the lines AD and AB are tangent to it, is also drawn. Let a tangent line from the point C to the circle C_2 meet the side AB at E. If the length of EB is $\alpha+\sqrt{3}\beta$, where α, β are integers, then $\alpha + \beta$ is equal to ________.

86. The total number of 3×3 matrices A having entries from the set $\{0, 1, 2, 3\}$ such that the sum of all the diagonal entries of AA^T is 9, is equal to ________.

87. Let z and w be two complex number such that $w=z\bar{z}-2z+2,\ \left|\frac{z+i}{z-3i}\right|=1$ and $\text{Re}(w)$ has minimum value. Then, the minimum value of $n \in N$ for which w^n is real, is equal to ________.

88. Consider an arithmetic series and a geometric series having four initial terms from the set $\{11, 8, 21, 16, 26, 32, 4\}$. If the last terms of these series are the maximum possible four digit numbers, then the number of common terms in these two series is equal to ________.

89. Let $f : (0, 2) \to R$ be defined as

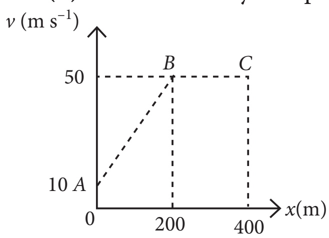
$f(x)=\log_2\left(1+\tan\left(\frac{\pi x}{4}\right)\right)$.

Then, $\lim_{n\to\infty}\frac{2}{n}\left(f\left(\frac{1}{n}\right)+f\left(\frac{2}{n}\right)+\dots+f(1)\right)$ is equal to ________.

90. Let the curve $y = y(x)$ be the solution of differential equation, $\frac{dy}{dx}=2(x+1)$. If the numerical value of area bounded by the curve $y = y(x)$ and x-axis is $\frac{4\sqrt{8}}{3}$, then the value of $y(1)$ is equal to ________.

HINTS & EXPLANATIONS

1. **(d) :** Given velocity - displacement graph



Equation of line AB

$$v = \frac{(50-10)}{(200-0)} \times (x-0) + 10 \qquad (0 \le x \le 200)$$

$$\Rightarrow \quad v = \frac{40}{200} \times x + 10 \Rightarrow \quad v = \frac{x}{5} + 10 \qquad \text{...(i)}$$

Differentiate equation (i) w.r.t. x

$$\frac{dv}{dx} = \frac{1}{5} \qquad \text{...(ii)}$$

Acceleration, $a = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = v\frac{dv}{dx}$

$$a = \left(\frac{x}{5} + 10\right) \times \frac{1}{5} \qquad \text{(Using (i) and (ii))}$$

$a = \left(\frac{x}{25} + 2\right)$ m s^{-2}, At $x = 0$, $a = 2$ m s^{-2}.

It is straight line till $x = 200$.

For $x > 200$, v = constant, $a = 0$.

Therefore, most appropriate option will be (d).

2. **(d) :** For an electromagnetic wave, the average energy density for electric and magnetic fields are equal.

$\therefore \quad U_e = U_m$

3. **(d) :** Number of moles of O_2, $n_1 = \frac{16}{32} = 0.5$ mole

Number of moles of N_2, $n_2 = \frac{28}{28} = 1$ mole

Number of moles of CO_2, $n_3 = \frac{44}{44} = 1$ mole

Total number of moles, $n = n_1 + n_2 + n_3$

$\therefore$ Now $n = 0.5 + 1 + 1 = \frac{5}{2}$ moles

Now, $PV = nRT$

$$P = \frac{(nRT)}{V} = \left(\frac{5}{2}\right)\left(\frac{RT}{V}\right)$$

4. **(c) :** One main scale division, 1 M.S.D. = a cm

n^{th} division of the vernier scale division are equal to $(n-1)^{th}$ division of main scale,

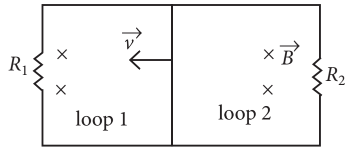
We get 1 V.S.D. $= \frac{(n-1)^{th}}{n^{th}}$ mm

The least count is given by = 1 MSD – 1 VSD

$$= \left(a - \frac{(n-1)}{(n)} \times a\right)\text{cm} \;=\frac{na-(n-1)a}{n}\text{ cm}$$

$$= \frac{a}{n}\text{ cm} = 10\frac{a}{n}\text{ mm}$$

5. **(c) :** An external agent pulls the bar, area of loop 1, decreases and that of loop 2 increases. Magnetic flux decreases in loop 1 and increases in loop 2. As a result magnetic field should be increase in loop 1 and decrease in loop 2. So the induced current I_1, should be clockwise and I_2, anticlockwise.



6. **(a) :** Here : $r_{max} = 1.6 \times 10^{12}$ m

$r_{min} = 8.0 \times 10^{10}$ m, $v_{max} = 6 \times 10^4$ m s^{-1}

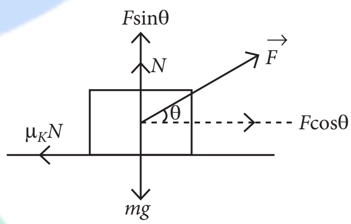
By conservation of angular momentum,

mvr = constant

$v_{min} \times r_{max} = v_{max} \times r_{min}$

$$\therefore \; v_{min} = \frac{6 \times 10^4 \times 8.0 \times 10^{10}}{1.6 \times 10^{12}} = 3.0 \times 10^3 \text{ m s}^{-1}$$

7. **(b) :** The various force acting on the block as shown in the figure.



$N + F\sin\theta = mg$ or $N = mg - F\sin\theta$

and $f = \mu_K N = \mu_K(mg - F\sin\theta)$

Also, $F\cos\theta - f = ma$ or $a = \frac{1}{m}\left[F\cos\theta - f\right]$

or $a = \frac{1}{m}[F\cos\theta - \mu_K(mg - F\sin\theta)]$

or $a = \frac{F}{m}\cos\theta - \mu_K\left(g - \frac{F}{m}\sin\theta\right)$

8. **(a) :** Given, $m = 200$ g = 0.2 kg, $T = 40$ s, $r = 20$ cm = 0.2 m

$$N = m\left(\frac{2\pi}{T}\right)^2 r = m \times \frac{4\pi^2}{T^2} \times r$$

$$= \frac{0.2 \times 4 \times (3.14)^2 \times 0.2}{(40)^2} = 9.859 \times 10^{-4}\text{ N}$$

9. **(a) :** We know that, $R = \rho\frac{l}{A}$ ...(i)

Given, new length $l' = 2l$

New area of cross section $= \frac{A}{2}$

New resistance, $R' = \rho \cdot \dfrac{2l}{A/2}$ ...(ii)

Current, $I = \dfrac{V}{\rho \cdot \dfrac{2l}{(A/2)}} = \dfrac{V}{4\dfrac{\rho l}{A}} = \dfrac{1}{4}\dfrac{VA}{\rho l}$

10. (d) : Given , frequency υ = 500 MHz = 5×10^8 Hz

$\vec{B} = 8.0 \times 10^{-8}\,\hat{z}\,T$

EM wave travelling towards + $\hat{j}$

$\vec{E} = \vec{B} \times \vec{C} \;= (8\times10^{-8}\,\hat{z})\times(3\times10^8\,\hat{y}) \;= -24\,\hat{x}\,\text{Vm}^{-1}$

11. (b) : Given, length $(2l)$ = 14 cm, l = 7 cm

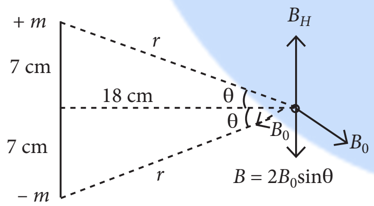
$B_H = 0.4\text{G} = 0.4 \times 10^{-4}\,\text{T}$

Magnetic field at a point on axial line, of bar magnet of length $(2l)$ and magnetic moment m is

$$\frac{2\mu_0 m}{4\pi r^2} \times \frac{7}{r} = 0.4 \times 10^{-4}$$

$$2 \times 10^{-7} \times \frac{m \times 7}{(7^2 + 18^2)^{3/2}} \times 10^4 = 0.4 \times 10^{-4}$$

$$\Rightarrow \quad m = \frac{4 \times 10^{-2} \times (373)^{3/2}}{14}$$



$$M = m \times 14\text{ cm} = m \times \frac{14}{100} = \frac{0.04 \times (373)^{3/2}}{14} \times \frac{14}{100}$$

$= 4 \times 10^{-4} \times 7203.82 = 2.88\ \text{J T}^{-1}$

12. (d) : Time period of a simple pendulum,

$$T = 2\pi\sqrt{\frac{l}{g_{eff}}}$$

When lift is stationary,

$$T = 2\pi\sqrt{\frac{l}{g}} \qquad (\because \; g_{eff} = g)$$

When lift is moving upward with an acceleration $g/2$, then effective acceleration

$$g_{eff} = g + \frac{g}{2} = \frac{3g}{2} \quad \therefore \quad T' = 2\pi\sqrt{\frac{2l}{3g}};$$

$$T' = \sqrt{\frac{2}{3}}\,T$$

13. (c) : Heat and work depends on the path taken to reach the final state from initial state.

14. (b) : The pressure acting on a submarine,

$P = P_0 + h\rho g = 3 \times 10^5$ Pa ...(i)

$h\rho g = 3 \times 10^5 - 1 \times 10^5 = 2 \times 10^5$ Pa

If depth is doubled,

$2h\rho g = 4 \times 10^5$ Pa

Now, $P' = P_0 + 4 \times 10^5$

$= 1 \times 10^5 + 4 \times 10^5 = 5 \times 10^5$ Pa ...(ii)

$$\%\text{ increase in pressure} = \frac{P' - P}{P} \times 100$$

$$= \frac{(5-3)\times 10^5}{3 \times 10^5} \times 100 \quad \text{(Using (i) and (ii))}$$

$$= 66.66\% = \frac{200}{3}\%$$

15. (b) : During the positive cycle of input the capacitor starts charging exponentially and attains maximum value. During the negative cycle it starts discharging and voltage across capacitance decreases exponentially from the maximum value. It is represented by graph (b).

16. (d) : If A be area of each plate and d is the distance between the plates, then capacitance

$$C_0 = \frac{\varepsilon_0 A}{d} \quad \text{...(i)}$$

For dielectric slab,

$$C' = \frac{\varepsilon_0 A}{\left(d - t + \dfrac{t}{K}\right)} \;\Rightarrow\; C' = \frac{\varepsilon_0 A}{\left(d - \dfrac{3d}{4} + \dfrac{3d}{4K}\right)}$$

$$\Rightarrow \quad C' = \frac{4K\varepsilon_0 A}{(3+K)d} = \frac{4KC_0}{3+K} \quad \text{(Using (i))}$$

17. (a) : Length of antenna = 25 m $= \dfrac{\lambda}{4}$

$\Rightarrow$ λ = 100 m

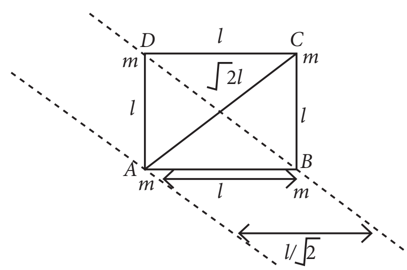
18. (d) : Moment of inertia of point mass

= mass × (perpendicular distance from axis)^2

Moment of inertia,

$$I = m\left(\frac{l}{\sqrt{2}}\right)^2 + m(l\sqrt{2})^2 + m\left(\frac{l}{\sqrt{2}}\right)^2$$

$I = 3ml^2$



19. (b) : Stopping potential charges linearly with frequency of incident radiation.

20. (c) : Incident ray and emergent ray are symmetric to the prism. Deviation is minimum in prism when $i = e$, $r_1 = r_2$ and ray inside prism is parallel to the base of the prism. Hence statements (A), (B) and (C) are true.

21. (600) : Given, $\beta = 6$ mm $= 6 \times 10^{-3}$ m

$d = 1$ mm $= 1 \times 10^{-3}$ m, $D = 10$ m

Fringe width, $\beta = \dfrac{\lambda D}{d}$

$$6 \times 10^{-3} = \frac{\lambda \times 10}{1 \times 10^{-3}}$$

$$\lambda = \frac{6 \times 10^{-3} \times 1 \times 10^{-3}}{10}$$

$\lambda = 600 \times 10^{-9}$ m $= 600$ nm

22. (15) : The wavelength of the spectral lines in the Balmer series is given by,

$$\frac{1}{\lambda} = RZ^2\left[\frac{1}{2^2} - \frac{1}{n^2}\right] \text{ where } n = 3, 4, 5, 6$$

For first line, $\dfrac{1}{\lambda_1} = RZ^2\left(\dfrac{1}{2^2} - \dfrac{1}{3^2}\right) = \dfrac{5}{36}RZ^2$ ...(i)

For third line, $\dfrac{1}{\lambda_3} = RZ^2\left(\dfrac{1}{2^2} - \dfrac{1}{5^2}\right)$

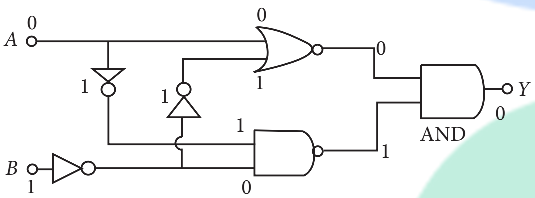
$\Rightarrow \dfrac{1}{\lambda_3} = \left(\dfrac{21}{100}\right)RZ^2$ ...(ii)

Dividing equation (ii) by (i), we get

$$\frac{\lambda_1}{\lambda_3} = \frac{21}{100} \times \frac{36}{5} = 1.512 = 15.12 \times 10^{-1}$$

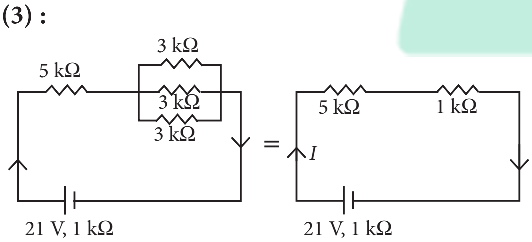
$\therefore \quad x \approx 15$

23. (0) :



The output Y of the logic circuit is zero.

24. (3) :



In the circuit the resistance 3 kΩ, 3 kΩ, 3 kΩ are connected in parallel. Their effective resistance will be

$$\frac{1}{R_P} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{3}{3} \text{ or } R_P = 1\,\text{k}\Omega$$

The equivalent circuit is as shown in the figure.

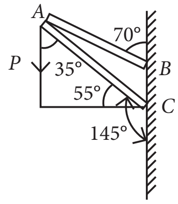
The equivalent resistance of the circuit is

$R_{eq} = 1 + 1 + 5 = 7$ kΩ

Current in the circuit, $I = \dfrac{21}{7} = 3$ mA

$\therefore \quad I = 3$ mA

25. (82) : Component along AC,



$= 100 \cos 35°$ N $= 100 \times 0.819$ N $= 81.9$ N $= 82$ N

26. (20) : Given : $m = 20$ kg, $R = 0.2$ m

$F = 20$ N, $\omega = 50$ rad s^{-1}

We know that, angular acceleration $\alpha = \dfrac{\tau}{I} = \dfrac{FR}{mR^2/2}$

$$\alpha = \frac{2F}{mR} = \frac{2 \times 20}{20 \times 0.2} = 10$$

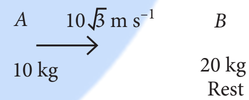
As $\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$

$(50)^2 = 0^2 + 2(10)\Delta\theta$

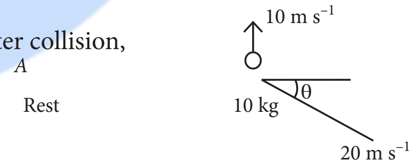
$\Delta\theta = 125$ rad

As, in one complete revolution the disk rotates by 6.28 rad, then the disk rotates by 125 rad, the number of revolution $\simeq 20$

27. (30) : Before collision,



After collision,



According to the law of conservation of linear momentum, we get

$$10 \times 10\sqrt{3} = 10 \times 20\cos\theta$$

$\cos\theta = \sqrt{3}/2 \quad \therefore \theta = 30°$

28. (5) : Here $V = (50 \pm 2)$ V and $I = (20 \pm 0.2)$ A

$R = I/V$

The percentage error in R is

$$\frac{\Delta R}{R} \times 100 = \frac{\Delta V}{V} \times 100 + \frac{\Delta I}{I} \times 100$$

$$\frac{\Delta R}{R} \times 100 = \frac{2}{50} \times 100 + \frac{0.2}{20} \times 100$$

$$\frac{\Delta R}{R} \times 100 = 4 + 1$$

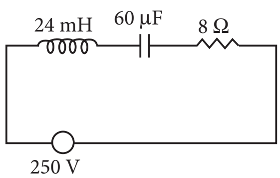
% error in $R = 5\%$

29. (4) : Given : $R = 8$ Ω, $L = 24$ mH

$C = 60$ μF

$V = 250$ V

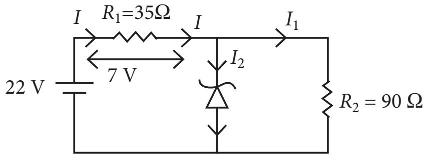
At resonance,



Power $P = \dfrac{(V_{rms})^2}{R}$

$\therefore \quad P = \dfrac{\left(\dfrac{250}{\sqrt{2}}\right)^2}{8} \quad (\because V = \sqrt{2}\, V_{rms})$

$P = 3906.25\ \text{W} = 4\ \text{kW}$

30. (5) : 

Potential difference across R_1 is $= 22\ \text{V} - 15\ \text{V} = 7\ \text{V}$

Current, $I = \dfrac{7}{35} = \dfrac{1}{5}\ \text{A}$

Current, $I_1 = \dfrac{15}{90} = \dfrac{1}{6}\ \text{A}, \quad I_2 = I - I_1 = \dfrac{1}{30}\ \text{A}$

Power across diode, $P_2 = V_2 I_2 = 15 \times \left(\dfrac{1}{30}\right) = 0.5\ \text{W}$

$\therefore \quad P = 5 \times 10^{-1}\ \text{W}$

$\therefore \quad x = 5$

31. (a) : Vitamin A and D are stored in our body for relatively longer time because these are fat soluble vitamins.

32. (d) : $CaCl_2{\cdot}6H_2O$ undergoes dehydration on heating but $MgCl_2{\cdot}8H_2O$ undergoes hydrolysis on heating.

$CaCl_2 \cdot 6H_2O \xrightarrow{\Delta} CaCl_2 + 6H_2O$

$MgCl_2 \cdot 8H_2O \xrightarrow{\Delta} MgO + 2HCl + 7H_2O$

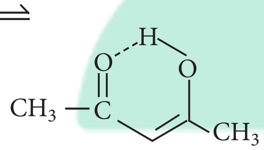
BeO is an amphoteric oxide but oxides of other elements of the same group are basic in nature.

33. (c) : $CH_3-\overset{O}{\overset{\|}{C}}-CH_3 \rightleftharpoons CH_3-\overset{OH}{\overset{|}{C}}=CH_2$

Acetone (< 0.1%)

$CH_3-\overset{O}{\overset{\|}{C}}-CH_2-\overset{O}{\overset{\|}{C}}-CH_3 \rightleftharpoons$

Acetyl acetone



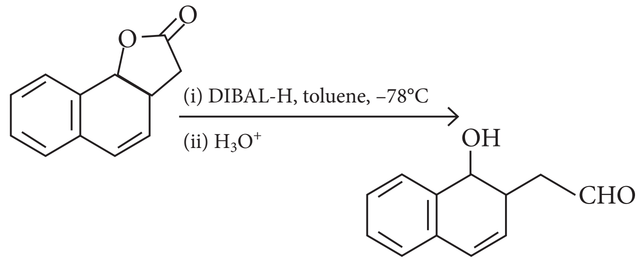
Intramolecular H-bonding
(≈15%)

Enol form of acetylacetone is stabilised by intramolecular hydrogen bonding which is not possible in enol form of acetone.

34. (a) : The stability of hydrides decreases from NH_3 to BiH_3 as their bond dissociation energy decreases, consequently their reducing power increases. Therefore, BiH_3 is the strongest reducing agent among all the hydrides of group 15.

35. (d) : Size of Bk^{3+} ion is less than Np^{3+} ion is due to actinoid contraction. Actinoid contraction is the gradual decrease in the size of M^{3+} ions across the series as atomic number increases.

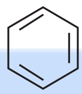
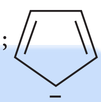
36. (d) : DIBAL-H is diisobutylaluminium hydride selectively reduces nitrites and esters to aldehydes.

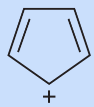


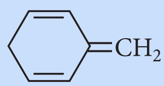
(*P*)
(Major product)

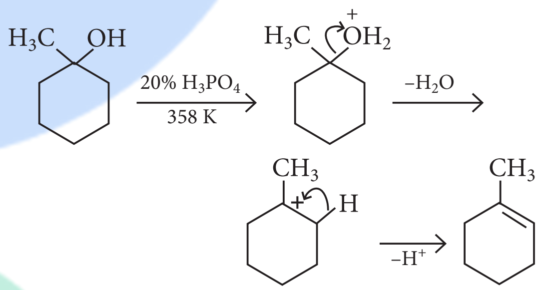
37. (b) : Antihistamine is used as an antacid and antiallergic drug.

38. (a) : A compound should be aromatic if it has planarity, complete delocalization of π-electrons in ring and follows Huckle's rule $(4n + 2)\pi$ electrons.

 $6\ \pi e^-s$ Aromatic ;  $6\ \pi e^-s$ Aromatic

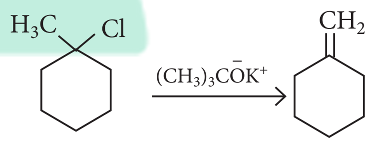
 $4\ \pi e^-s$ Antiaromatic

 Not complete delocalization
(Non-aromatic)

39. (a) : 

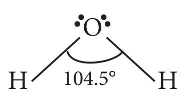
(*A*)

Alcohols gives E1 elimination with conc. H_2SO_4 and H_3PO_4 on heating.



(*B*)

t-BuO^- is a bulky base and will form Hoffman alkene *via* elimination.

40. (a) : 

Lone pair – lone pair repulsion is more than bond-pair-bond pair repulsion hence value of H — O — H angle decreases from its ideal value of 109°28′.

41. (c) : $NO_2 \longrightarrow NO + [O]$

$O + O_2 \longrightarrow O_3$

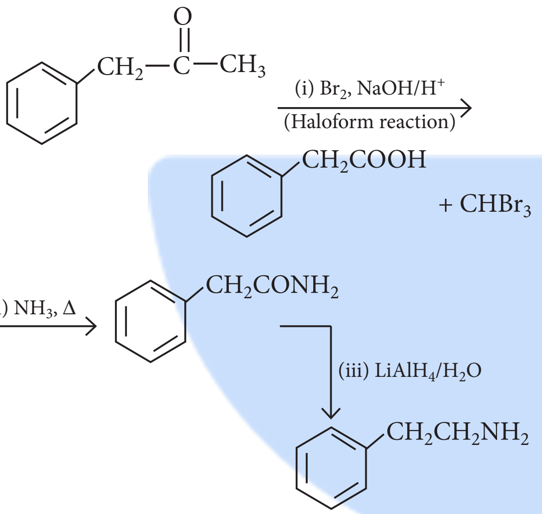
$O_3 + NO \longrightarrow NO_2 + O_2$

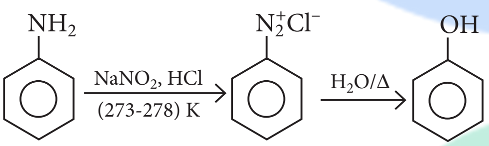
NO_2 is responsible for photochemical smog which is oxidising in nature.

42. (d) :

Acid	Formula	Oxidation state of P
Hypophosphorous acid	H_3PO_2	+1
Orthophosphoric acid	H_3PO_4	+5
Hypophosphoric acid	$H_4P_2O_6$	+4
Orthophosphorous acid	H_3PO_3	+3

43. (b) : Lindlar's catalyst is a palladium catalyst poisoned with traces of lead and quinoline.
i.e., $Pd/CaCO_3 + (CH_3COO)_2Pb$ + Quinolene.

44. (b) : 

45. (d) : 

46. (c) : In chromatography technique, purification is independent of physical state of the pure compound.

47. (b) : Haber's process $\longrightarrow NH_3$ synthesis
Ostwald's process $\longrightarrow HNO_3$ synthesis
Contact process $\longrightarrow H_2SO_4$ synthesis
Hall-Heroult process $\longrightarrow$ Aluminium extraction

48. (c) : $Ce^{4+} \xrightarrow{e^-} Ce^{3+}$; $E^\circ = +1.74$ V
Because reduction potential is positive (+ 1.74 V), therefore Ce^{4+} will reduce to Ce^{3+} ion and hence Ce^{3+} ion is more stable than Ce^{4+} ion.

49. (d) : During roasting metal sulphide is converted to metal oxide and sulphur is removed in the form of SO_2.

50. (c) : Oxidising action in basic medium,
$2Fe^{2+} + H_2O_2 \longrightarrow 2Fe^{3+} + 2OH^-$
$Mn^{2+} + H_2O_2 \longrightarrow Mn^{4+} + 2OH^-$
Reducing action in basic medium,
$I_2 + H_2O_2 + 2OH^- \longrightarrow 2I^- + 2H_2O + O_2$
$2MnO_4^- + 3H_2O_2 \longrightarrow 2MnO_2 + 3O_2 + 2H_2O + 2OH^-$
Advantage of hydrogen economy is that energy is transmitted in the form of dihydrogen and not as electric power.

51. (50) : $\underset{}{A_2X} \rightleftharpoons \underset{S_1}{2A^+} + \underset{S_1}{X^{2-}}$

$K_{sp} = 4S_1^3 \Rightarrow S_1 = \sqrt[3]{\frac{K_{sp}}{4}} = \sqrt{\frac{4\times10^{-12}}{4}}$; $S_1 = 10^{-4}$

$MX \rightleftharpoons \underset{S_2}{M^{2+}} + \underset{S_2}{X^{2-}}$

$K_{sp} = S_2^2 \Rightarrow S_2 = \sqrt{K_{sp}} = \sqrt{4\times10^{-12}} = 2\times10^{-6}$

$S_1 = S(A_2X)$; $S_2 = S(MX)$

$$\frac{S_1}{S_2} = \frac{S(A_2X)}{S(MX)} = \frac{10^{-4}}{2\times10^{-6}} = 50$$

52. (33) : For *bcc* unit cell, $\sqrt{3}a = 4R$

$$a = \frac{4R}{\sqrt{3}} = 27 \;;\; R = \frac{27\sqrt{3}}{4}$$

For *fcc* unit cell, $\sqrt{2}a = 4R$

$$a = \frac{4R}{\sqrt{2}} = \frac{4\times27\sqrt{3}}{\sqrt{2}\times4} \;;\; a = 27\frac{\sqrt{3}}{\sqrt{2}} = 33.1 \approx 33$$

53. (9) : 1000 g solvent contains = 6.5 mole of KOH
Mass of KOH = 6.5 × 56 = 364 g KOH
Mass of solution = 1364 g

Volume of solution $= \frac{1364}{1.89}$ mL

$$\text{Molarity} = \frac{6.5\times1000}{\left(\frac{1364}{1.89}\right)} = 9\text{ M}$$

54. (10) : $\log\frac{K_2}{K_1} = \frac{E_a}{2.303R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$

$$\log\frac{(1.0\times10^{-3})}{K_1} = \frac{11.488\times1000}{2.303\times8.314}\left(\frac{1}{200} - \frac{1}{300}\right)$$

$$\log\frac{10^{-3}}{K_1} = 600\times\frac{(3-2)}{600}$$

$$\log\frac{10^{-3}}{K_1} = 1 \Rightarrow 10 = \frac{10^{-3}}{K_1}$$

$K_1 = 10^{-4}$
$\because\; x\times10^{-5} = 10^{-4} \Rightarrow x = 10$

55. (106) : $AB_2 \rightleftharpoons A^{2+} + 2B^-$

$$\alpha = \frac{i-1}{n-1} \;;\; 0.1 = \frac{i-1}{3-1} \;;\; i = 1.2$$

$\Delta T_b = i\times K_b\times m = 1.2\times0.5\times10 = 6$
$\therefore\; \Delta T_b = T_s - T_b^\circ$; $6 = T_s - 100$; $T_s = 106°C$

56. (20) : $\Delta G^\circ = -RT\ln K_{eq}$
$-9.478\times10^3 = -495\times8.314\ln K_{eq}$
$\ln K_{eq} = 2.303 = \ln10$
So, $K_{eq} = 10$

Now,	$A_{(g)}$	$\rightleftharpoons$	$B_{(g)}$
$t = 0$	22 mmol		0
$t = t_{eq}$	$22 - x$		x

$$K_{eq}=\frac{[B]}{[A]};\ 10=\frac{x}{22-x}\ \Rightarrow\ x=20$$

So, millimoles of B at equilibrium = 20

57. (16) : $2MnO_4^- + 5C_2O_4^{2-} + 16H^+ \longrightarrow 2Mn^{2+} + 10CO_2 + 8H_2O$

$b = 5, c = 16, x = 2, y = 10, z = 8.$

58. (9) : $\lambda = 248 \times 10^{-9}$ m ; $w_0 = 3 \times 1.6 \times 10^{-19}$ J

$$\frac{hc}{\lambda}=w_0+K.E.$$

$$K.E.=\frac{6.63\times10^{-34}\times3\times10^{8}}{248\times10^{-9}}-3\times1.6\times10^{-19}$$

$= 3.2 \times 10^{-19}$ J

$$p=\sqrt{2m\,K.E.}$$

$$=\sqrt{2\times9.1\times10^{-31}\times3.2\times10^{-19}}=7.63\times10^{-25}$$

$$\lambda=\frac{h}{p}=\frac{6.63\times10^{-34}}{7.63\times10^{-25}}=8.7\times10^{-10}=8.7\text{ Å}\approx 9$$

59. (3) : Liebeig's method,

$$\%\text{ of H}=\frac{2}{18}\times\frac{\text{Mass of }H_2O}{\text{Mass of compound}}\times100$$

$$=\frac{2}{18}\times\frac{210}{750}\times100=3.11\approx 3$$

60. (2) : $CoCl_3{\cdot}4NH_3$ is a *trans*-complex means octahedral geometry with one Cl^- ion out of the coordination sphere *i.e.*, $[Co(NH_3)_4Cl_2]Cl$.

Each ethylene diamine can replace two NH_3 ligands. Therefore, two ethylene diamine are required to replace all neutral monodentate (NH_3) ligands.

61. (a) : We have, general term

$$T_{r+1}={}^{60}C_r\,(3^{1/4})^{60-r}(5^{1/8})^r={}^{60}C_r\,3^{(60-r)/4}(5)^{r/8}$$

For rational terms, r should be a multiple of 8 and less than 60.

So, r can be 0, 8, 16, ..., 56 *i.e.*, 8 values

$\Rightarrow$ Number of irrational terms = 61 – 8 = 53

$\Rightarrow$ $n = 53 \Rightarrow n - 1 = 52$, which is divisible by 26.

62. (d) : The given plane is $lx + my + nz = 0$ ...(i)

and line is

$$\frac{x-1}{-1}=\frac{y-(-4)}{2}=\frac{z-(-2)}{3}\qquad ...(ii)$$

Now, plane (i) containing the line (ii), therefore

$-l + 2m + 3n = 0$ ...(iii) and $l - 4m - 2n = 0$ ...(iv)

Solving (iii) and (iv), we get $\dfrac{l}{-4+12}=\dfrac{m}{3-2}=\dfrac{n}{4-2}$

$\Rightarrow\ l : m : n = 8 : 1 : 2$

So, equation of plane is $8x + y + 2z = 0$

Now, let the plane divides the line joining A and B at point C in the ratio $k : 1$. Then,

$$\text{Coordinates of } C\equiv\left(\frac{2k-3}{k+1},\ \frac{4k-6}{k+1},\ \frac{-3k+1}{k+1}\right)$$

which satisfies the equation of plane.

$$\Rightarrow\ 8\left(\frac{2k-3}{k+1}\right)+\left(\frac{4k-6}{k+1}\right)+2\left(\frac{-3k+1}{k+1}\right)=0$$

$\Rightarrow\ 14k - 28 = 0 \Rightarrow k = 2$

63. (d) : Let (h, k) be the mid-point of the chord of circle $x^2 + y^2 = 25$ with centre (0, 0).

$\therefore$ Equation of chord is

$$hx+ky=h^2+k^2\ \Rightarrow\ y=-\frac{h}{k}x+\frac{h^2+k^2}{k}\qquad ...(i)$$

Now, (i) will be tangent to hyperbola $\dfrac{x^2}{9}-\dfrac{y^2}{16}=1$ if $c^2 = a^2m^2 - b^2$

$$\Rightarrow\ \left(\frac{h^2+k^2}{k}\right)^2=9\left(\frac{-h}{k}\right)^2-(16)\qquad \text{[Using (i)]}$$

$\Rightarrow\ (h^2 + k^2)^2 = 9h^2 - 16k^2$

$\therefore$ Required locus is$(x^2 + y^2)^2 - 9x^2 + 16y^2 = 0$

64. (b) : We have, $A=\begin{bmatrix} i & -i\\ -i & i\end{bmatrix}$

$$\Rightarrow A^2=\begin{bmatrix} i & -i\\ -i & i\end{bmatrix}\begin{bmatrix} i & -i\\ -i & i\end{bmatrix}=\begin{bmatrix} -2 & 2\\ 2 & -2\end{bmatrix}=2^1\begin{bmatrix} -1 & 1\\ 1 & -1\end{bmatrix}$$

$$\Rightarrow A^4=2^2\begin{bmatrix} -1 & 1\\ 1 & -1\end{bmatrix}\begin{bmatrix} -1 & 1\\ 1 & -1\end{bmatrix}=4\begin{bmatrix} 2 & -2\\ -2 & 2\end{bmatrix}=8\begin{bmatrix} 1 & -1\\ -1 & 1\end{bmatrix}$$

$$\Rightarrow A^8=64\begin{bmatrix} 1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix} 1 & -1\\ -1 & 1\end{bmatrix}=64\begin{bmatrix} 2 & -2\\ -2 & 2\end{bmatrix}$$

$$=128\begin{bmatrix} 1 & -1\\ -1 & 1\end{bmatrix}$$

$$\text{Now, } A^8\begin{bmatrix} x\\ y\end{bmatrix}=\begin{bmatrix} 8\\ 64\end{bmatrix}\ \Rightarrow\ 128\begin{bmatrix} 1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix} x\\ y\end{bmatrix}=\begin{bmatrix} 8\\ 64\end{bmatrix}$$

$$\Rightarrow\ x-y=\frac{1}{16}\ ...(i),\quad -x+y=\frac{1}{2}\ ...(ii)$$

Clearly, (i) and (ii) are parallel.

Thus, the system of linear equations has no solution.

65. (d) : Here, three cases arise :

Case I : When $x < -4 \Rightarrow (-x - 3)(-x - 4) = 6$

$\Rightarrow\ x^2 + 7x + 12 = 6 \Rightarrow x^2 + 7x + 6 = 0$

$\Rightarrow\ (x + 6)(x + 1) \Rightarrow x = -6$ or -1

But $x < -4$

$\therefore\ x = -6$ is the solution *i.e.*, one solution

Case II : When $-4 < x < 0$

$\Rightarrow\ (-x - 3)(x + 4) = 6 \Rightarrow x^2 + 7x + 18 = 0$

$\because\ D < 0$

$\therefore$ No solution

Case III : When $x > 0$

$\Rightarrow\ (x - 3)(x + 4) = 6 \Rightarrow x^2 + x - 18 = 0$

$$\Rightarrow\ x=\frac{-1\pm\sqrt{73}}{2}\ \Rightarrow\ x=\frac{-1+\sqrt{73}}{2}\quad(\because\ x>0)$$

$\Rightarrow$ One solution exists

$\therefore$ The elements in the given set is 2.

66. (b) : We have,

$$\log_{\frac{1}{\sqrt{2}}}\left(\frac{|z|+11}{(|z|-1)^2}\right)\le 2$$

$$\Rightarrow \frac{|z|+11}{(|z|-1)^2}\ge\left(\frac{1}{\sqrt{2}}\right)^2 \qquad \left[\because \log\frac{1}{\sqrt{2}}<0\right]$$

$\Rightarrow$ $2(|z|+11)\ge(|z|-1)^2 \Rightarrow |z|^2-4|z|-21\le 0$

$\Rightarrow$ $(|z|-7)(|z|+3)\le 0$

$\Rightarrow$ $|z|\le 7$ $\qquad (\because\ |z|$ can't be negative)

$\therefore$ Maximum value of $|z| = 7$

67. (d) : D.r.'s of line AC are

$< a-0, -2a+a, 3+1 >$ *i.e.*, $< a, -a, 4 >$

$\Rightarrow$ $l = a, m = -a, n = 4$

Also, C lies on the given plane

$\therefore$ $-am - n = 0 \Rightarrow a^2 = 4 \Rightarrow a = 2$ $\qquad (\because a > 0)$

So, equation of plane is

$2x - 2y + 4z = 0 \Rightarrow x - y + 2z = 0$

Let $D(x, y, z)$ be the foot of perpendicular from the point $B(0, 4, 5)$.

Then, D.r.'s of BD are $< 0-x, 4-y, 5-z >$

$= < -x, 4-y, 5-z >$

$\Rightarrow$ $-x = 1, 4-y=-1, 5-z=2 \Rightarrow x=-1, y=5, z=3$

$\therefore$ Coordinates of $D\equiv(-1, 5, 3)$ and that of $C\equiv(0, -2, -1)$

$\therefore$ Length of $CD=\sqrt{1^2+7^2+4^2}=\sqrt{66}$

68. (b) : $(p\wedge q)\to(p\to q)\equiv\sim(p\wedge q)\vee(\sim p\vee q)$

$\equiv(\sim p\vee\sim q)\vee(\sim p\vee q)\equiv\sim p\vee(\sim q\vee q)\equiv\sim p\vee t\equiv t$

69. (d) : We have, $\log_{10}\sin x+\log_{10}\cos x=-1$

$\Rightarrow$ $\log_{10}(\sin x\cdot\cos x)=-1 \Rightarrow \sin x\cdot\cos x=\frac{1}{10}$ $\qquad$ ...(i)

Also, $\log_{10}(\sin x+\cos x)=\frac{1}{2}(\log_{10}n-1)$

$=\frac{1}{2}(\log_{10}n-\log_{10}10)$

$$\Rightarrow 2\log_{10}(\sin x+\cos x)=\log_{10}\left(\frac{n}{10}\right)$$

$$\Rightarrow (\sin x+\cos x)^2=\frac{n}{10}$$

$$\Rightarrow 1+2\sin x\cos x=\frac{n}{10}\Rightarrow 1+\frac{2}{10}=\frac{n}{10}\ \text{(Using (i))}$$

$\Rightarrow$ $n = 12$

70. (a) : We have, $81^{\sin^2 x}+81^{\cos^2 x}=30$

$$\Rightarrow 3^{4\sin^2 x}+\frac{81}{3^{4\sin^2 x}}=30$$

$$\text{Let } 3^{4\sin^2 x}=t\Rightarrow t+\frac{81}{t}=30\Rightarrow t^2-30t+81=0$$

$$\Rightarrow t=27 \text{ or } t=3\Rightarrow 3^{4\sin^2 x}=3^3 \text{ or } 3^{4\sin^2 x}=3^1$$

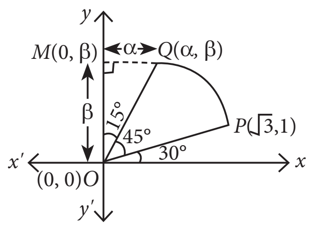
$$\Rightarrow \sin^2 x=\frac{3}{4} \text{ or } \sin^2 x=\frac{1}{4}\Rightarrow \sin x=\pm\frac{\sqrt{3}}{2} \text{ or } \pm\frac{1}{2}$$

71. (d) : Let $\overrightarrow{OP}=\sqrt{3}\,\hat{i}+\hat{j}$ and $\overrightarrow{OQ}=\alpha\hat{i}+\beta\hat{j}$

$\therefore$ $|\overrightarrow{OP}|=|\overrightarrow{OQ}|=\sqrt{3+1}=2$

In ΔOMQ, $\frac{\beta}{2}=\cos 15°$

and $\frac{\alpha}{2}=\sin 15°$ ...(i)



Area of (ΔOMQ)

$=\frac{1}{2}OM\times MQ$

$=\frac{1}{2}\alpha\beta=\frac{1}{2}(2\cos 15°)(2\sin 15°)$ $\qquad$ [Using (i)]

$=2\sin 15°\cos 15°=\sin 30°=\frac{1}{2}$

72. (a) : Let E_1 be the event that missing card is a spade and E_2 be the event that missing card is not spade.

Also, let A be the event of drawing two spades.

$\therefore$ Total probability $= P(E_1)\cdot P(A/E_1) + P(E_2)\cdot P(A/E_2)$

$$=\frac{{}^{13}C_1}{{}^{52}C_1}\times\frac{{}^{12}C_2}{{}^{51}C_2}+\frac{{}^{39}C_1}{{}^{52}C_1}\times\frac{{}^{13}C_2}{{}^{51}C_2}=\frac{1}{17}$$

Now, required probability

$$=\frac{P(E_2)\cdot P(A/E_2)}{\text{Total probability}}=\frac{39}{850}\times 17=\frac{39}{50}$$

73. (b) : Equation of line PR is

$$\frac{x-3}{4}=\frac{y+1}{-1}=\frac{z-2}{2}=\lambda\ \text{(say)}$$

Equation of line QS is

$$\frac{x-1}{-2}=\frac{y-2}{1}=\frac{z+4}{-2}=\mu\ \text{(say)}$$

Any point on line PR is $(4\lambda+3, -\lambda-1, 2\lambda+2)$ and any point on line QS is $(-2\mu+1, \mu+2, -2\mu-4)$.

Since, PR and QS intersect at T.

$\therefore$ $4\lambda+3=-2\mu+1, -\lambda-1=\mu+2, 2\lambda+2=-2\mu-4$, for some $\lambda, \mu\in R$

On solving above equations, we get $\lambda = 2, \mu = -5$

$\therefore$ Coordinates of T are $(11, -3, 6)$.

Now, as $\overrightarrow{TA}$ is $\perp$ to both $\overrightarrow{PR}$ and $\overrightarrow{QS}$.

$\therefore$ $\overrightarrow{TA}\ ||\ \overrightarrow{PR}\times\overrightarrow{QS}\Rightarrow\overrightarrow{TA}=m(\overrightarrow{PR}\times\overrightarrow{QS})$

$$\text{Now, } \overrightarrow{PR}\times\overrightarrow{QS} \text{ is parallel to}\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\4&-1&2\\-2&1&-2\end{vmatrix}$$

$$=\hat{i}(2-2)-\hat{j}(-8+4)+\hat{k}(4-2)=4\hat{j}+2\hat{k}$$

$\therefore$ $\overrightarrow{TA}=m(4\hat{j}+2\hat{k})$

Also, given that $|\overrightarrow{TA}|=\sqrt{5}\Rightarrow|m(4\hat{j}+2\hat{k})|=\sqrt{5}$

$\Rightarrow\ m^2(16+4)=5 \Rightarrow m^2=\frac{5}{20}=\frac{1}{4}\Rightarrow m=\pm\frac{1}{2}$

$\therefore\ \overrightarrow{TA}=\pm\frac{1}{2}(4\hat{j}+2\hat{k})$

$\Rightarrow$ (P.V. of A – P.V. of T) $=\pm\frac{1}{2}(4\hat{j}+2\hat{k})$

$\Rightarrow$ P.V. of $A=(11\hat{i}-3\hat{j}+6\hat{k})\pm\frac{1}{2}(4\hat{j}+2\hat{k})$

$=11\hat{i}-\hat{j}+7\hat{k}$ or $11\hat{i}-5\hat{j}+5\hat{k}$

$\therefore$ Required modulus $=\sqrt{121+1+49}=\sqrt{171}$ units

74. (d) : We have, $f(g(x))=\begin{cases} g(x)+2 & , \quad g(x)<0 \\ (g(x))^2 & , \quad g(x)\geq 0\end{cases}$

$$=\begin{cases} x^3+2 & , \quad x<0 \\ x^6 & , \quad 0\leq x<1 \\ (3x-2)^2 & , \quad x\geq 1\end{cases}$$

Now, $(fog)(x)$ is discontinuous at $x=0$.

$\therefore$ $(fog)(x)$ is non-differentiable at $x=0$.

For $x=1$, we have

$$\text{RHD}=\lim_{h\to 0}\frac{f(g(1+h))-f(g(1))}{h}$$

$$=\lim_{h\to 0}\frac{(3(1+h)-2)^2-1}{h}=6$$

$$\text{LHD}=\lim_{h\to 0}\frac{f(g(1-h))-f(g(1))}{-h}=\lim_{h\to 0}\frac{(1-h)^6-1}{-h}=6$$

$\Rightarrow$ RHD = LHD $\Rightarrow f(g(x))$ is differentiable at $x=1$.

75. (a) : We have, $S_k=\sum_{r=1}^{k}\tan^{-1}\left(\frac{6^r}{2^{2r+1}+3^{2r+1}}\right)$

$$=\sum_{r=1}^{k}\tan^{-1}\left(\frac{3^r\cdot 2^r/2^{2r+1}}{1+(3^{2r+1}/2^{2r+1})}\right)$$

$$=\sum_{r=1}^{k}\tan^{-1}\left(\frac{\frac{3^r}{2^{r+1}}}{1+\left(\frac{3}{2}\right)^r\cdot\left(\frac{3}{2}\right)^{r+1}}\right)$$

$$=\sum_{r=1}^{k}\tan^{-1}\left[\frac{\left(\frac{3}{2}\right)^{r+1}-\left(\frac{3}{2}\right)^r}{1+\left(\frac{3}{2}\right)^r\left(\frac{3}{2}\right)^{r+1}}\right]$$

$$=\tan^{-1}\frac{9}{4}-\tan^{-1}\frac{3}{2}+\tan^{-1}\left(\frac{3}{2}\right)^3-\tan^{-1}\frac{9}{4}$$

$$+\ldots\ldots+\tan^{-1}\left(\frac{3}{2}\right)^{k+1}-\tan^{-1}\left(\frac{3}{2}\right)^k$$

$$\Rightarrow S_k=\tan^{-1}\left(\frac{3}{2}\right)^{k+1}-\tan^{-1}\frac{3}{2}$$

Now, $\lim_{k\to\infty}S_k=\lim_{k\to\infty}\tan^{-1}\left(\frac{3}{2}\right)^{k+1}-\tan^{-1}\frac{3}{2}$

$$=\frac{\pi}{2}-\tan^{-1}\frac{3}{2}=\cot^{-1}\frac{3}{2}$$

76. (d) : The given differential equation is

$$\frac{dy}{dx}+2y\tan x=\sin x,$$

which is a linear differential equation

$\therefore$ I.F. $=e^{\int 2\tan x\,dx}=e^{-\log\cos^2 x}=\sec^2 x$

Now, required solution is

$$y\cdot\sec^2 x=\int\frac{\sin x}{\cos^2 x}dx=\int\tan x\cdot\sec x\,dx=\sec x+c$$

$\Rightarrow\ y=\cos x+c\cos^2 x$ ...(i)

Now, $y\left(\frac{\pi}{3}\right)=0$

$\Rightarrow\ \cos\frac{\pi}{3}+c\cos^2\frac{\pi}{3}=0\Rightarrow c=-2$ (using (i))

$\therefore\ y=\cos x-2\cos^2 x=-2\left[\left(\cos x-\frac{1}{4}\right)^2\right]+\frac{1}{8}$

$\Rightarrow\ y_{\max}=\frac{1}{8}$

77. (b) : We have,

$(1-x+x^3)^n=a_0x^0+a_1x+a_2x^2+\ldots\ldots+a_{3n}x^{3n}$ ...(i)

Putting $x=1$ in (i), we get

$a_0+a_1+a_2+\ldots\ldots+a_{3n}=1$ ...(ii)

Also, putting $x=-1$ in (i), we get

$a_0-a_1+a_2-a_3+\ldots\ldots+(-1)^{3n}\cdot a_{3n}=1$ ...(iii)

Now, adding and subtracting (ii) and (iii), we get

$$2\left\{a_0+a_2+a_4+\ldots..+a_{2\left[\frac{3n}{2}\right]}\right\}=2$$

$\Rightarrow\ \sum_{j=0}^{[3n/2]}a_{2j}=1$ ...(iv)

and $2\left\{a_1+a_3+a_5+\ldots\ldots+a_{2\left[\frac{3n-1}{2}\right]+1}\right\}=0$

$\Rightarrow\ \sum_{j=0}^{\left[\frac{3n-1}{2}\right]}a_{2j+1}=0$ ...(v)

$\therefore\ \sum_{j=0}^{[3n/2]}a_{2j}+4\sum_{j=0}^{\left[\frac{3n-1}{2}\right]}a_{2j+1}=1+4\times 0$

(Using (iv) and (v))

$=1$

78. (b) : We have,

$f(x) = (4a-3)(x+\log_e 5) + 2(a-7)\cot\left(\frac{x}{2}\right)\cdot\sin^2\left(\frac{x}{2}\right)$

$= (4a-3)(x+\log_e 5)+2(a-7)\cdot\cos\frac{x}{2}\cdot\sin\frac{x}{2}$

$= (4a-3)(x+\log_e 5) + (a-7)\sin x$

Now, $f(x)$ has critical points.

$\therefore\ f'(x) = 0$

$\Rightarrow\ (4a-3) + (a-7)\cos x = 0 \Rightarrow\ \cos x = \frac{3-4a}{a-7}$

Now, $-1 \le \cos x < 1$ $\quad (\because\ x \ne 2n\pi \Rightarrow \cos x \ne 1)$

$\Rightarrow\ -1 \le \frac{3-4a}{a-7} < 1 \Rightarrow \frac{3-4a}{a-7} \ge -1$ and $\frac{3-4a}{a-7} < 1$

$\Rightarrow\ \frac{3a+4}{a-7} \le 0$ and $\frac{5a-10}{a-7} > 0$

$\Rightarrow\ a \in \left[-\frac{4}{3}, 7\right)$ and $a \in (-\infty, 2)\cup(7, \infty)$

$\Rightarrow\ a \in \left[-\frac{4}{3}, 2\right)$

79. (c) : As we know that, standard deviation is independent of change of origin.

$\therefore$ S.D. of a, b, c is also d.

$\Rightarrow\ d^2 = \frac{a^2+b^2+c^2}{3} - \left(\frac{a+b+c}{3}\right)^2$

$\Rightarrow\ d^2 = \frac{3(a^2+b^2+c^2)-(2b)^2}{9} \Rightarrow b^2 = 3(a^2+c^2) - 9d^2$

80. (b) : We have, $y^2 = 2x$

Now, equation of normal to the parabola is

$y = mx - m - \frac{m^3}{2} = \frac{2mx - 2m - m^3}{2}$

$\because$ It passes through $(a, 0)$

$\therefore\ 2ma - 2m - m^3 = 0$

$\Rightarrow\ m^3 + 2m(1-a) = 0$ ...(i)

Let m_1, m_2, m_3 be the roots of the equation (i), then

$\Sigma m_1 = 0, \Sigma m_1 m_2 = 2(1-a), m_1 m_2 m_3 = 0$

Now, $m_1^2 + m_2^2 + m_3^2 > 0 \Rightarrow (\Sigma m_1)^2 - 2\,\Sigma m_1 m_2 > 0$

$\Rightarrow\ 0 - 2\,[2(1-a)] > 0 \Rightarrow 4(1-a) < 0 \Rightarrow a > 1$

81. (4) : We have, $\lim_{x\to 0}\frac{ae^x - b\cos x + ce^{-x}}{x\sin x} = 2$

$$\Rightarrow\ 2 = \lim_{x\to 0}\frac{a\left(1+x+\frac{x^2}{2!}+\ldots\right) - b\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-\ldots\ldots\right) + c\left(1-x+\frac{x^2}{2!}-\ldots\ldots\right)}{x^2\cdot\left(\frac{\sin x}{x}\right)}$$

$$\Rightarrow\ 2 = \lim_{x\to 0}\frac{\left[\left(\frac{a+b+c}{2}\right)x^2 + (a-c)x + (a-b+c) + \ldots\right]}{x^2}$$

For limit to exist, we have

$\frac{a+b+c}{2} = 2$, $a-c = 0$ and $a-b+c = 0$

$\Rightarrow\ a+b+c = 4$

82. (16) : We have, $f(x) + f(x+1) = 2$ ...(i)

Replacing x with $x+1$, we get

$f(x+1) + f(x+2) = 2$ ...(ii)

From (i) and (ii), we have $f(x) = f(x+2)$

$\Rightarrow\ f(x)$ is periodic with period 2.

Now, $I_1 = \int_0^8 f(x)dx = 4\int_0^2 f(x)dx$

and $I_2 = \int_{-1}^3 f(x)dx = 2\int_0^2 f(x)\,dx \Rightarrow I_1 = 2I_2$

$\therefore\ I_1 = 4\left[\int_0^1 f(x)dx + \int_1^2 f(x)dx\right]$

$= 4\left[\int_0^1 f(x)dx + \int_0^1 f(x+1)dx\right]$

$= 4\left[\int_0^1 f(x)dx + \int_0^1 (2-f(x))dx\right]$ [Using (ii)]

$= 4\left[\int_0^1 f(x)dx + \int_0^1 2dx - \int_0^1 f(x)dx\right] = 8$

$\Rightarrow\ I_1 = 8 \Rightarrow I_2 = 4$

$\therefore\ I_1 + 2I_2 = 8 + 8 = 16$

83. (406) : We have, $y(x) = \int_0^x (2t^2 - 15t + 10)dt$

$\Rightarrow\ y'(x) = 2x^2 - 15x + 10$ ...(i)

(Using Leibnitz rule)

Now, slope of normal at (a, b) $= \left.\frac{-1}{y'(x)}\right|_{(a,b)}$

Let $m_1 = \frac{-1}{2a^2 - 15a + 10}$

Now, as normal is parallel to the line $x + 3y = -5$,

having slope $(m_2) = \frac{-1}{3}$.

$\therefore\ m_1 = m_2 \Rightarrow 2a^2 - 15a + 10 = 3$

$\Rightarrow\ 2a^2 - 15a + 7 = 0 \Rightarrow (a-7)(2a-1) = 0$

$\Rightarrow\ a = 7$ or $a = \frac{1}{2}$ (Neglect) $\quad (\because\ a > 1)$

From (i), $y(x) = \frac{2x^3}{3} - \frac{15x^2}{2} + 10x + c$

At $x = 0, y = 0 \Rightarrow c = 0$

$\therefore\ y = \frac{2}{3}x^3 - \frac{15}{2}x^2 + 10x$

which passes through (a, b)

$\therefore\ b = \frac{2}{3}(7)^3 - \frac{15}{2}(7)^2 + 10(7) \Rightarrow b = \frac{-413}{6}$

Now, $|a + 6b| = |7 - 413| = 406$

84. (36) : We have, $|(P^{-1}AP - I_3)^2| = |P^{-1}AP - I_3|^2$

$\Rightarrow \alpha\omega^2 = |P^{-1}AP - P^{-1}P|^2 \qquad (\because I = P^{-1}P)$

$= |P^{-1}(AP - P)|^2 = |P^{-1}|^2 \, |AP - P|^2$

$$= |P^2|^{-1}|A - I|^2|P|^2 = \frac{1}{|P|^2}|A-I|^2|P|^2$$

$\Rightarrow \alpha\omega^2 = |A - I|^2 \qquad$ ...(i)

Now,

$$|A-I| = \begin{vmatrix} 1 & 7 & \omega^2 \\ -1 & -\omega-1 & 1 \\ 0 & -\omega & -\omega \end{vmatrix} = -6\omega \quad (\because 1+\omega+\omega^2 = 0)$$

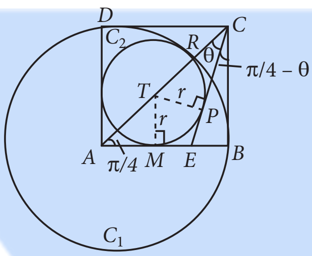
$\Rightarrow |A - I|^2 = 36\omega^2 \Rightarrow \alpha\omega^2 = 36\omega^2 \quad$ (Using (i))

$\Rightarrow \alpha = 36$

85. (1) : We have, $AR = 1$ unit (given)

$\Rightarrow AT + TR = 1 \qquad$ ...(i)

Let r be the radius of C_2.



In ΔAMT,

$$AT = \frac{r}{\sin(\pi/4)} = \sqrt{2}r$$

$\therefore$ From (i), we have

$r\sqrt{2} + r = 1$

$\Rightarrow r = \sqrt{2} - 1$

Now, $AC = \sqrt{2}$ (diagonal of square)

$\Rightarrow RC = \sqrt{2} - 1 = r \Rightarrow TC = TR + RC = r + r = 2r$

In ΔPCT, we have

$$\sin\theta = \frac{r}{TC} = \frac{r}{2r} = \frac{1}{2} \Rightarrow \theta = 30^\circ \;\therefore \angle ECB = 15^\circ$$

$$\text{Now, } \tan 15^\circ = \frac{EB}{BC} \Rightarrow 2 - \sqrt{3} = \alpha + \sqrt{3}\beta$$

$\Rightarrow \alpha = 2, \beta = -1 \therefore \alpha + \beta = 1$

86. (766) : Let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}$

$$\Rightarrow A^T = \begin{bmatrix} a_1 & a_4 & a_7 \\ a_2 & a_5 & a_8 \\ a_3 & a_6 & a_9 \end{bmatrix}$$

Sum of diagonal elements of $AA^T = 9$

$\Rightarrow a_1^2 + a_2^2 + a_3^2 + a_4^2 + a_5^2 + a_6^2 + a_7^2 + a_8^2 + a_9^2 = 9$

$\because$ All $a_i \in \{0, 1, 2, 3\} \Rightarrow a_i^2 \in \{0, 1, 4, 9\}$

Case-I : $a_i^2 = 9$, then only one $a_i = 3$ and rest will be zero.

$\therefore$ Number of matrices $= {}^9C_1 = 9$

Case II : $a_i^2 = 4, a_j^2 = 4, a_k^2 = 1$ and rest will be zero.

$\therefore$ Number of matrices $= {}^9C_2 \cdot {}^7C_1 = 252$

Case III : $a_i^2 = 4$ and five a_j's $= 1$ and rest will be zero.

$\therefore$ Number of matrices $= {}^9C_1 \cdot {}^8C_5 = 504$

Case IV : $a_i^2 = 1 \;\forall\; i$

$\therefore$ Number of matrices = 1

$\therefore$ Total number of required matrices

$= 9 + 252 + 504 + 1 = 766$

87. (4) : We have, $\left|\dfrac{z+i}{z-3i}\right| = 1 \Rightarrow |z + i| = |z - 3i|$

$\therefore$ Locus of z is the perpendicular bisector of the line segment joining $(0, -1)$ and $(0, 3)$

$\therefore$ Locus of z is $y = 1$. Let $z = x + i, x \in R$

$\Rightarrow w = (x + i)(x - i) - 2(x + i) + 2$

$= (x^2 + 1 - 2x) - 2(i - 1) = [(x - 1)^2 + 2] - 2i$

Now, Re(w) is minimum $\Rightarrow x - 1 = 0 \Rightarrow x = 1$

$\therefore w = 2(1 - i) = 2\sqrt{2}\, e^{-i\pi/4} \Rightarrow w^n = (2\sqrt{2})^n e^{-in\pi/4}$

$\therefore$ Least value of n, for which w^n is real = 4

88. (3) : Possible A.P. is 11, 16, 21, 26 .... with possible last term = 9996

Also, possible G.P. is 4, 8, 16, 32, ... with possible last term = 8192

Now, for common terms, we have

General term of A.P. = General term of G.P.

$\Rightarrow 11 + (n - 1)5 = 4(2^{n-1}) \Rightarrow 5n + 6 = 2^{n+1}$

$$\Rightarrow n = \frac{2^{n+1} - 6}{5}$$

This is only possible when, unit digit of 2^{n+1} is 6.

i.e., for $n = 3, 7, 11$. So, only 3 common terms exist.

89. (1) : Clearly,

$$\lim_{n\to\infty} \frac{2}{n}\sum_{r=1}^{n} f\left(\frac{r}{n}\right) = \lim_{n\to\infty}\frac{2}{n}\sum_{r=1}^{n}\log_2\left(1 + \tan\left(\frac{\pi r}{4n}\right)\right)$$

$$\Rightarrow I = 2\int_0^1 \log_2\left(1 + \tan\frac{\pi}{4}x\right)dx$$

$$\text{Put } \frac{\pi}{4}x = t \Rightarrow dx = \frac{4dt}{\pi}$$

$$\therefore I = \frac{8}{\pi}\int_0^{\pi/4}\log_2(1 + \tan t)dt \qquad \text{...(i)}$$

$$\Rightarrow I = \frac{8}{\pi}\int_0^{\pi/4}\log_2\left(1 + \tan\left(\frac{\pi}{4} - t\right)\right)dt$$

$$\Rightarrow I = \frac{8}{\pi}\int_0^{\pi/4}\log_2\left[1 + \frac{1 - \tan t}{1 + \tan t}\right]dt$$

$$= \frac{8}{\pi}\int_0^{\pi/4}\log_2\left(\frac{2}{1 + \tan t}\right)dt$$

$$\Rightarrow I = \frac{8}{\pi}\int_0^{\pi/4}\log_2 2\,dt - \frac{8}{\pi}\int_0^{\pi/4}\log_2(1 + \tan t)dt \qquad \text{...(ii)}$$

Adding (i) and (ii), we get

$$2I = \frac{8}{\pi}\int_0^{\pi/4}\log_2 2dt = \frac{8}{\pi}\frac{\pi}{4}$$

$(\because \log_a a = 1$, where $a > 0$ and $a \neq 1)$

$\Rightarrow I = 1$

90. **(2) :** We have, $\frac{dy}{dx} = 2(x+1) \Rightarrow y = (x+1)^2 + c$

Point of intersection with x-axis $-1 \pm \sqrt{-c} = -1 \pm m$, where $m = \sqrt{-c}$ or $c = -m^2$

Now, area bounded by the curve and x-axis

$$= 2\left|\int_{-1}^{-1+m} ((x+1)^2 + c)\,dx\right| \Rightarrow \frac{2\sqrt{8}}{3} = \left|\left[\frac{(x+1)^3}{3} + cx\right]_{-1}^{-1+m}\right|$$

$$\Rightarrow \frac{4\sqrt{2}}{3} = \left|\left[\frac{(x+1)^3 - 3m^2x}{3}\right]_{-1}^{-1+m}\right|$$

$$\Rightarrow 4\sqrt{2} = \left|[m^3 - 3m^2(-1+m)] - [0 + 3m^2]\right|$$

$$\Rightarrow 4\sqrt{2} = \left|[m^3 + 3m^2 - 3m^3 - 3m^2]\right|$$

$$\Rightarrow 2m^3 = \pm 4\sqrt{2}$$

$$\Rightarrow m = \pm\sqrt{2}$$

$$\Rightarrow c = -2$$

Thus, $y = (x+1)^2 - 2$

$\Rightarrow y(1) = 2$

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