Market responses to a VIX impulse

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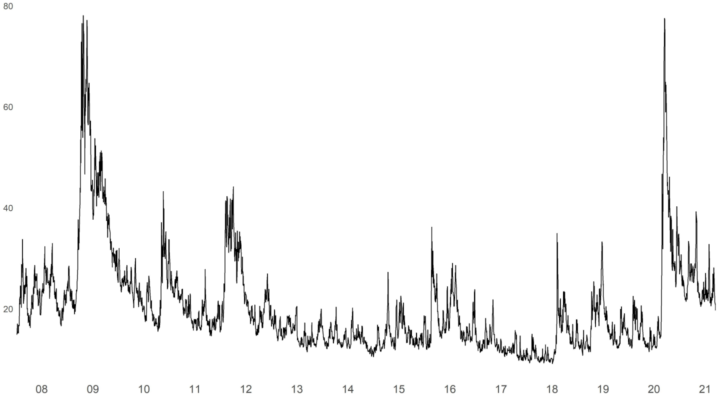
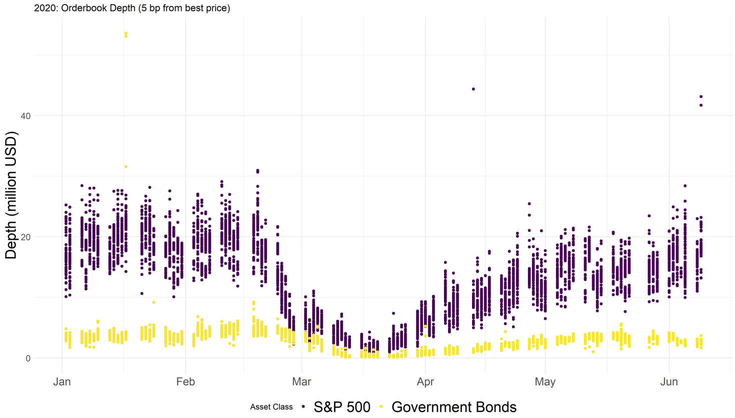
How does a VIX impulse ripple through financial markets?

- VIX - the **fear index**: Closely monitored by investors, regulators, media
- measures the risk neutral expectation of future realized variance



Worrisome liquidity dynamics?



- Vivid examples: GFC and COVID-19 market turmoils in 2020 (Duffie, 2020)
- Liquidity evaporates entirely during periods of major market turmoil (Brunnermeier, 2009; Pedersen, 2009)
- Liquidity trends concern regulators: BIS and IOSCO (2012); SEC (2016)

How does a VIX impulse ripple through financial markets?

VIX impulse

Risk-neutral measure

- higher perceived crash-risk probability
- marginal investor more risk averse
- expected returns adjust or rebalancing (Caballero and Krishnamurthy, 2008)

Flight to safety

Expected realized variance

- News hit the market
- Adverse selection concerning for liquidity providers (Campbell et al., 1993)
- Illiquidity (Nagel, 2012; Drechsler et al., 2020)

Evaporating liquidity

This paper: Nature of VIX impulse matters

$\Rightarrow$ How does an VRP or ERV shock ripple through financial markets?

- Decomposition of VIX changes into two canonical components: variance risk premium (VRP) and expected realized volatility (ERV)
- 2007-2021 sample of *all* Nasdaq trading messages for two ETFs: equity and government bonds
- Impulse response analysis

Main finding: ERV and VRP trigger different responses

- *VRP* shock leads to a flight to safety
- *ERV* shock yields fragile market liquidity and active *buying*(!) of equities

A model to reconcile the responses

- Formal analysis of the two types of shocks in the model yields equilibrium results that are consistent with the empirical findings
- Extensions of Grossman and Stiglitz (1980): Some agents experience an increase in their risk aversion (VRP channel) or private news hits the market (ERV channel)

- VRP channel: Unsurprisingly, investors who suddenly become more risk averse re-allocate risk to those who do not
- ERV channel: liquidity deteriorates because of the adverse-selection cost.
- ERV channel: *Active buying of the risky asset is an equilibrium result.* Liquidity demanders effectively bear less *posterior* risk and are the natural *holders* of the risky asset

Decomposition of the VIX index

The variance risk premium

- *VIX*: implied volatility of the S&P 500 index, constructed from market prices of out-of-the-money puts and calls with maturity of one month

$$IV = VRP + ERV$$

- variance risk premium: difference between *implied* variance *IV* as the risk-neutral expectation of the future realized variance and the expected realized variance as its "physical" counterpart
- With a stochastic pricing kernel $m_{t,t'}$, VRP can be expressed as

$$\begin{aligned} VRP_t &:= E_t^{\mathbb{Q}}\left(RV_{t,t'}\right) - E_t^{\mathbb{P}}\left(RV_{t,t'}\right) \\ &= E_t^{\mathbb{P}}\left(m_{t,t'}\right)^{-1} E_t^{\mathbb{P}}\left(m_{t,t'} RV_{t,t'}\right) - E_t^{\mathbb{P}}\left(RV_{t,t'}\right) \\ &= E_t^{\mathbb{P}}\left(m_{t,t'}\right)^{-1} \left[Cov_t^{\mathbb{P}}\left(m_{t,t'}, RV_{t,t'}\right)\right]. \end{aligned}$$

- VRP is 0 if realized variance is orthogonal to the discount factor

Interpretation of VRP impulse

- With power utility and coefficient of relative risk aversion α, VRP is (Bakshi and Madan, 2006)

$$VRP_t \approx -\alpha E_t^{\mathbb{P}}\left(RV_{t,t'}\right)^{3/2} \times \rho + \frac{\alpha^2}{2} E_t^{\mathbb{P}}\left(RV_{t,t'}\right)^2 \times (\kappa - 3)$$

where ρ is the skewness and κ is the kurtosis of returns under $\mathbb{P}$.

How should we think about an VRP impulse?

- VRP increases with **risk aversion** (Campbell and Cochrane, 1999; Bekaert et al., 2013)
- VRP increases if market returns are more left-skewed: **crash risk events become more likely** (Bollerslev et al., 2015)
- VRP $\neq$ ERV: e.g., central bank communication mainly impacts risk premia rather than expected realized volatility (Bernanke and Kuttner, 2005; Drechsler et al., 2018)

High-frequency ERV estimation

A quick roadmap

- We observe $IV_{t,\tau}$, the implied volatility at day t, timestamp τ
- Aim: Decompose $IV_{t,\tau} = ERV_{t,\tau} + VRP_{t,\tau}$

$\Rightarrow$ We need $\widehat{ERV}_{t,\tau}$, an estimate of the conditional expected realized variance of the S&P 500 index over the next month

- The residual $\widehat{VRP}_{t,\tau} = IV_{t,\tau} - \widehat{ERV}_{t,\tau}$ captures the variance risk premium

High-frequency ERV estimation

- Notation: 5-minute returns $r_{t,\tau} = \log(p_{t,\tau}) - \log(p_{t,\tau-1})$ for $\tau > 0$, where $p_{t,\tau}$ is the S&P 500 index value on the day t at timestamp τ. $r_{t,0} = \log(p_{t,0}) - \log(p_{t-1,78})$ is the overnight return
- Realized variance of the S&P 500 index, computed from closure on day $t-1$ to the end of the τ-th 5 minute interval at day t:

$$\widetilde{RV}_{t,\tau} := \sum_{k=0}^{\tau} r_{t,k}^2$$

- HAR Model (Corsi, 2009) for the log one-month-ahead $RV_{t,\tau}^{(22)}$

$$\log\left(RV_{t,\tau}^{(22)}\right) = c_\tau + \beta_\tau \log\left(RV_{t-22,0}^{(21)} + \widetilde{RV}_{t,\tau}\right) + \gamma_\tau \log\left(RV_{t-5,0}^{(4)} + \widetilde{RV}_{t,\tau}\right) + \delta_\tau \log\left(\widetilde{RV}_{t,\tau}\right) + \varepsilon_{t,\tau}$$

where $\varepsilon_{t,\tau}$ is zero mean white noise

High-frequency ERV estimation

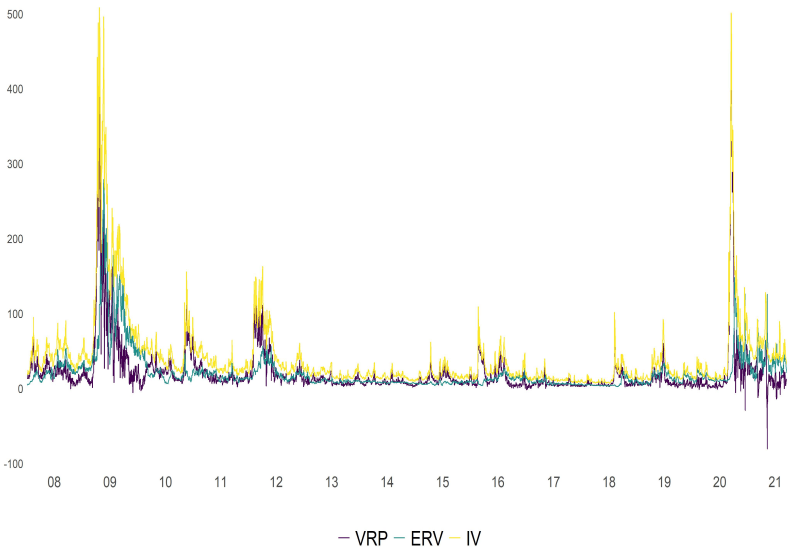
- Least squares estimation for $\tau \in \{1, \dots, 78\}$ yields estimates $\{\hat{c}_\tau, \hat{\beta}_\tau, \hat{\gamma}_\tau, \hat{\delta}_\tau\}$ and thus

$$\widehat{E}_{t,\tau}\left(RV_{t,\tau}^{(22)}\right) = \exp\left(\hat{c}_\tau + \hat{\beta}_\tau \log\left(RV_{t-22,0}^{(21)} + \widetilde{RV}_{t,\tau}\right) + \hat{\gamma}_\tau \log\left(RV_{t-5,0}^{(4)} + \widetilde{RV}_{t,\tau}\right) + \hat{\delta}_\tau \log\left(\widetilde{RV}_{t,\tau}\right)\right)$$

- Accordingly, for each day t and time stamp τ we obtain the estimated variance risk premium $\widehat{VRP}_{t,\tau}$ as

$$\widehat{VRP}_{t,\tau} = IV_{t,\tau} - \widehat{E}_{t,\tau}\left(RV_{t,\tau}^{(22)}\right)$$

Implied volatility decomposition



Market responses to a VIX impulse

Our sample

asset classes / exchange-traded funds (ETFs)

1. SPY for equities (S&P 500)
2. TLT for government bonds

Sample period: July 1st, 2007 - April 7th, 2021

- Event-time Nasdaq order book snapshots from data provider LOBSTER
- *All* Trades, Submissions, Cancellations, and Adjustments
- first 50 order book levels ($\approx$ 20 billion messages)

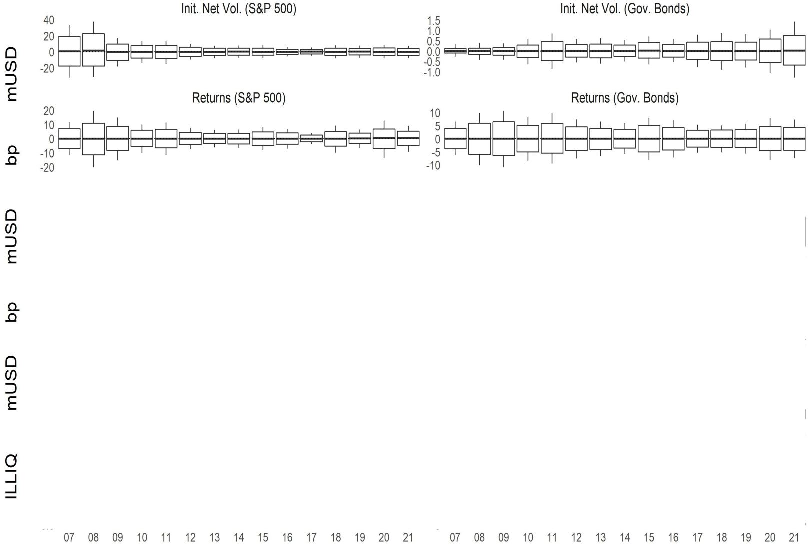
- VIX minute level data from data provider pitrading

5-minute observations: Variable definitions

- Aggregation to 5-minute sample frequency

1. *Initiator Net Volume* (in million USD) Signing based on initiating side
2. *Return* (basis points) Log difference of prevailing midquotes
3. *Trading volume* (in million USD) Cumulative trading volume
4. *Bid ask spread* (basis points) time-weighted relative bid-ask spread
5. *Depth* (million USD) time-weighted number of shares within 5 bps of best quote
6. *Amihud (2002) illiquidity measure*: $ILLIQ_{t,\tau} := \frac{\left|\log(p_{t,\tau}) - \log(p_{t,\tau-1})\right|}{V_{t,\tau}}$

Summary statistics: Box plots on an annual basis



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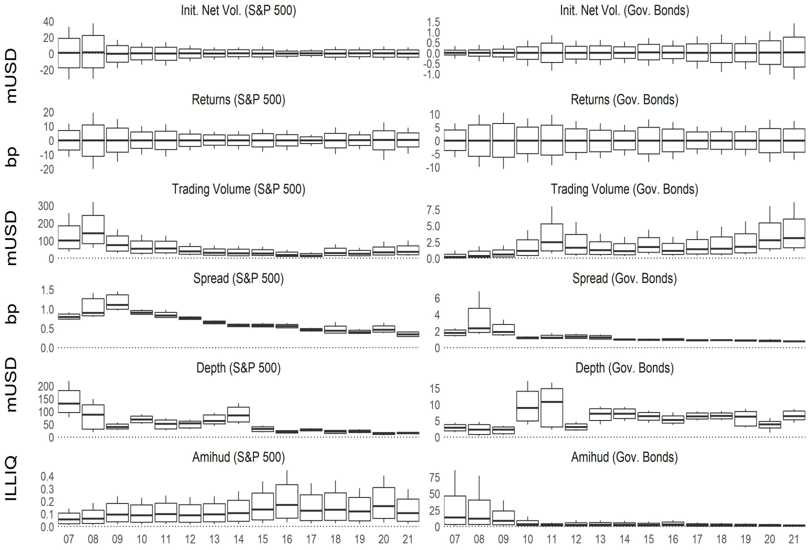


Impulse responses with local projection (Jorda, 2005)

- $\boldsymbol{y}_t = (y_{t,1}, \ldots, y_{t,N})'$: $(N \times 1)$ vector of observations at time t
- Impulse response function $\mathrm{ir}_i\,(t, h, \boldsymbol{d})$: response of $y_{t+h,i}$ at time $t+h$ on a (reduced form) shock $\boldsymbol{d}$ at time t:

$$\mathrm{ir}_i\,(t, h, \boldsymbol{d}) := E\left(y_{t+h,i} | \boldsymbol{v}_t = \boldsymbol{d}, \boldsymbol{y}_{t-1}, \ldots, \boldsymbol{y}_1\right) - E\left(y_{t+h,i} | \boldsymbol{v}_t = \boldsymbol{0}_N, \boldsymbol{y}_{t-1}, \ldots, \boldsymbol{y}_1\right)$$

  where $\boldsymbol{v}_t$ are the reduced form disturbances
- We estimate $\mathrm{ir}_i\,(t, h, \boldsymbol{d})$ from lag-augmented regressions of $y_{t+h,i}$ on $\boldsymbol{y}_t$ *and* $\boldsymbol{y}_{t-1}$ as additional control (Montiel Olea and Plagborg-Møller, 2021)

$$y_{t+h,i} = \boldsymbol{\beta}(h)' \boldsymbol{y}_t + \boldsymbol{\gamma}' \boldsymbol{y}_{t-1} + \varepsilon_t$$

- Then,

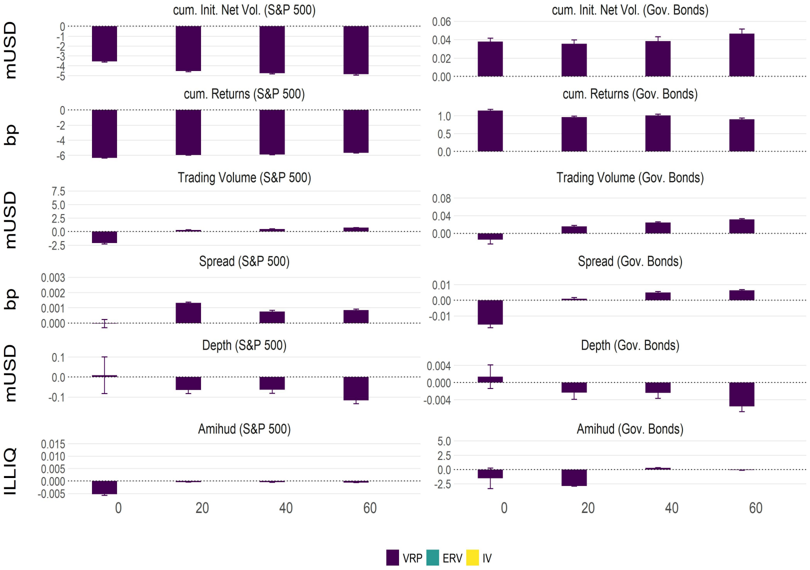
$$\mathrm{ir}_i\,(t, h, \boldsymbol{d}) = \hat{\boldsymbol{\beta}}(h)' \mathrm{d}$$

- We integrate out contemporaneous effects due to correlation using the historically observed distribution of the errors (Pesaran and Shin, 1998)

Shocking the components of the fear index

- Our system consists of the time series of order book related variables and the *VRP and ERV changes*
- We compute the responses to a (SD) shock in either VRP or ERV changes
- We report estimated cumulative impulse response functions for initiator net volume (in million USD) and return (in basis points) and impulse response functions for trading volume and depth (all in million USD), bid-ask spreads (in basis points) and the Amihud measure (ILLIQ)
- We report 95% confidence intervals with heteroscedasticity-robust standard errors

Market responses to a VIX impulse



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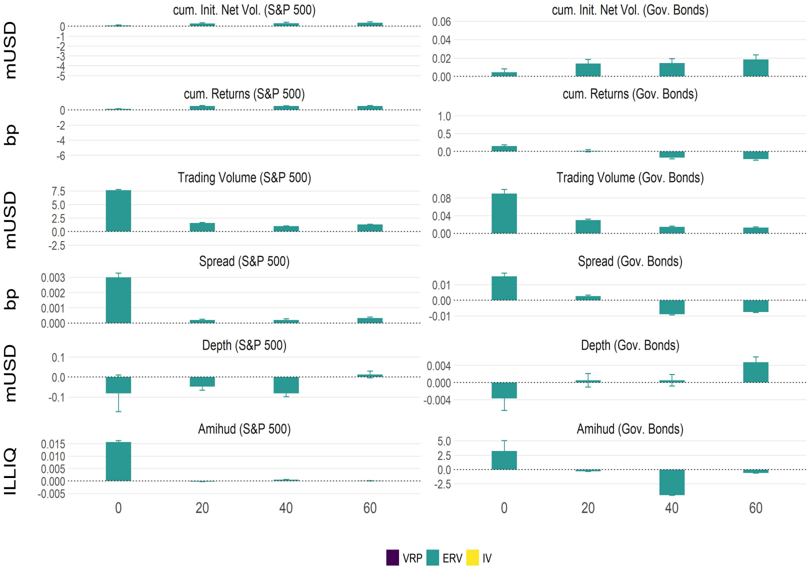

Market responses to a VIX impulse

Further empirical results

- *Institutional client responses* to a VRP or ERV shocks largely mirror initiator net volume: Institutional trading responds to an VRP shock with selling of equity at large magnitudes and turns positive after an ERV shock
- Net buying of (US) equity is *not* a response of net selling of global equity. Results are qualitatively similar when including MSCI World index tracking ETF
- The results are *not* driven by estimation error in ERV (and thus in VRP): Imposing *perfect foresight* ($RV_t^{(22)}$ instead of $\hat{E}(RV_t^{(22)})$ delivers similar results
- *Sub-period analysis* for crisis periods shows that the responses magnify during crisis periods

A model to disentangle the responses

The theoretical framework in a nutshell

All traders identical

- Three periods, risk averse initially identical agents who derive (exponential) utility from consumption
- Market: a riskless and a risky asset in supply of $\bar{\theta}$ shares which pays $D \sim N(\bar{D}, \sigma^2)$ units of consumption in $t = 2$
- Just before Period 1, a fraction π of the agents learns that they will receive an endowment $z(D - \bar{D})$ of the consumption good in Period 2, with $z \sim N(0, \sigma_z^2)$ independent of D
- **Baseline insight:** Price impact from initiated volume to incentivize liquidity providers + liquidity premium already in $t = 0$

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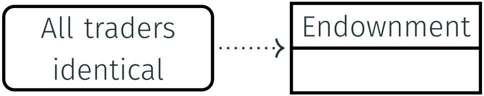

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The theoretical framework in a nutshell

All traders identical

Two extensions: Risk shocks in addition to endowment

1. *risk aversion shock (VRP channel)*: the fraction π of agents with endowment shock simultaneously experiences increase of their risk aversion
2. *cash-flow risk (ERV channel)* for the (identical) agents in $t = 0$: common knowledge that liquidity demanders learn additional information in $t = 1$ from a private signal correlated with D

The theoretical framework in a nutshell

Fear? News?

All traders identical

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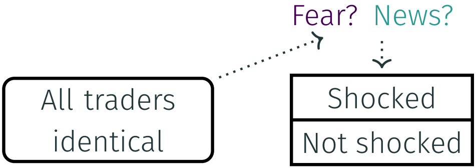

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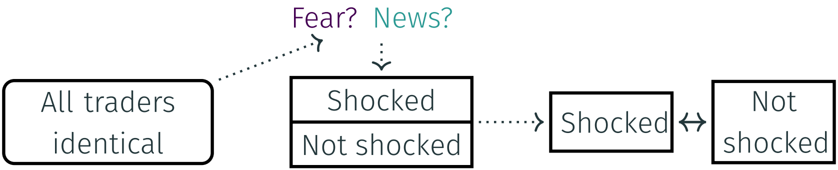

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Equilibrium outcomes from the model

- **(Initiator net volume)**. The expected equilibrium quantities for initiator net volume, defined as the signed volume of liquidity demanders are such that

$$E\left(\nu^{\text{VRP}}\right) < E(\nu) = 0 < E\left(\nu^{\text{ERV}}\right) \tag{1}$$

- **(Price impacts)**. The equilibrium quantities for price impact are such that

$$0 < \lambda = \lambda^{\text{VRP}} < \lambda^{\text{ERV}} \tag{2}$$

1. Risk aversion shocks trigger *net selling* of the risky asset and simultaneously net buying of government bonds
2. News generate *net buying* of the risky asset (resemble relative increase in risk for liquidity providers)
3. Larger price impact in response to ERV shocks than for risk aversion shocks

Conclusions

Conclusions

- Responses to risk shocks depend on the *nature* of the shock
    1. Variance risk premium shocks predominantly trigger net selling of equities and net buying of government bonds
    2. Expected realized variance shocks create *net buying of risky assets* and *illiquidity*, pointing towards asymmetric information

Implications

- Liquidity risk management: IV too broad - ERV matters!
- Central bank communication: addressing risk premia of investors (crash-risk) does not impair liquidity

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Literature (incomplete)

- *Flight to quality/liquidity* Longstaff (2004); Vayanos (2004); Beber et al. (2009)
- Ait-Sahalia et al. (2021) rationalize *asset-price dynamics* by introducing two "disconnected" stochastic processes: one that drives (realized) volatility and another that drives uncertainty (aversion) (same spirit: Liu et al. (2004), Drechsler (2013), Brenner and Izhakian (2018))
- *variance risk premium* (Bollerslev et al., 2009, 2012, 2014; Bekaert and Hoerova, 2014; Nagel, 2012)
- *Crisis periods*: GFC (Brunnermeier, 2009) and COVID-19 (Falato et al., 2020)
- *Exchange-traded funds* (Ben-David et al., 2017; Lettau and Madhavan, 2017)
- *Market liquidity trends* (Angel et al., 2015; SEC, 2016; ESMA, 2019)

Impulse responses with local projection

Impulse response functions (IRF)

- Denote $(N \times 1)$ vector of observations at time t as $\boldsymbol{y}_t = (y_{t,1}, \ldots, y_{t,N})'$
- Impulse response function $\mathrm{ir}_i\,(t, h, \boldsymbol{d})$ measures the response of $y_{t+h,i}$ at time $t+h$ on a (reduced form) shock $\boldsymbol{d}$ at time t:

  $$\mathrm{ir}_i\,(t, h, \boldsymbol{d}) := E\left(y_{t+h,i} | \boldsymbol{v}_t = \boldsymbol{d}, \boldsymbol{y}_{t-1}, \ldots, \boldsymbol{y}_1\right) - E\left(y_{t+h,i} | \boldsymbol{v}_t = \boldsymbol{0}_N, \boldsymbol{y}_{t-1}, \ldots, \boldsymbol{y}_1\right)$$

  where $\boldsymbol{v}_t$ are the reduced form disturbances, $\boldsymbol{0}_N$ denotes a $(N \times 1)$ vector of zeros

Common approach: vector autoregressive model (VAR)

- specify VAR: $\boldsymbol{y}_t = \boldsymbol{A}\boldsymbol{y}_{t-1} + \boldsymbol{u}_t$ and obtain $\mathrm{ir}_i\,(t, h, \boldsymbol{d})$ as non-linear transformation of the estimated coefficients
- $\mathrm{ir}_i\,(t, h, \boldsymbol{d}) = \boldsymbol{e}_i' \boldsymbol{\Phi}_h \boldsymbol{d}$ where e_i is a $(N \times 1)$ unit vector and $\boldsymbol{\Phi}_h$ is computed recursively from the canonical $MA(\infty)$ representation
- **Problem:** High-dimensional nonlinear functions of parameter estimates $\hat{\boldsymbol{A}}$ are prone to estimation and misspecification error

Local Projections (Jorda, 2005)

- Simple equation-by-equation least squares estimation
- We obtain an estimate of $\mathrm{ir}_i\,(t, h, \boldsymbol{d})$ from baseline lag-augmented regression of $y_{t+h,i}$ on $\boldsymbol{y}_t$ *and* $\boldsymbol{y}_{t-1}$ as additional control

$$y_{t+h,i} = \boldsymbol{\beta}(h)'\boldsymbol{y}_t + \boldsymbol{\gamma}'\boldsymbol{y}_{t-1} + \varepsilon_t$$

- For a given horizon h, the lag-augmented local projection estimates are

$$\begin{pmatrix} \hat{\boldsymbol{\beta}}(h) \\ \hat{\boldsymbol{\gamma}} \end{pmatrix} = \left(\sum_{t=1}^{T-h} \boldsymbol{x}_t \boldsymbol{x}_t' \right)^{-1} \left(\sum_{t=1}^{T-h} \boldsymbol{x}_t y_{t+h,i} \right)$$

  where $\boldsymbol{x}_t = \begin{pmatrix} \boldsymbol{y}_t, & \boldsymbol{y}_{t-1} \end{pmatrix}'$ is a $(2N \times 1)$ vector, $\hat{\boldsymbol{\beta}}(h)$ is *a consistent estimator of* $\mathrm{ir}_i\,(t, h, \boldsymbol{d})$ and $\hat{\boldsymbol{\gamma}}$ is a nuisance parameter

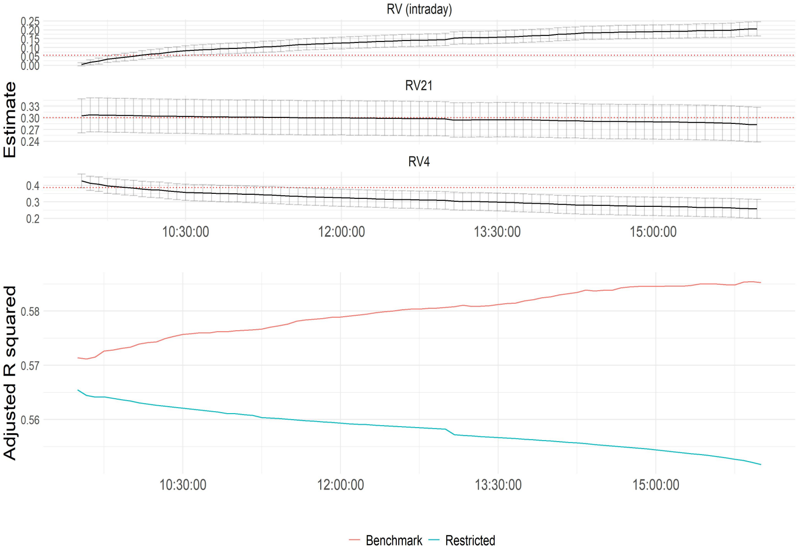
What makes the lag-augmented estimator $\hat{\boldsymbol{\beta}}(h)$ appealing?

- Covariance of the estimator $\hat{\boldsymbol{\beta}}(h)$ does not require Newey-West inference. (Eicker-Huber-White) heteroscedasticity-robust standard errors suffice for inference (Montiel Olea and Plagborg-Møller, 2021)
- Lag augmentation handles nearly persistent series particularly well

Choice of the shock

- Common problem: contemporaneous correlation of the errors ε_t
- Our (current) approach: *pseudo* generalized IRFs in spirit of Pesaran and Shin (1998) and Koop et al. (1996)
- Two steps to integrate out the effects of other shocks using the historically observed distribution of the errors
    1. estimate the parameters of a VAR(p) regression for $\boldsymbol{y}_t$ of the standard form $\boldsymbol{y}_t = \boldsymbol{\mu} + \sum_{l=1}^{p} \tilde{\boldsymbol{A}}_l \boldsymbol{y}_{t-l} + \boldsymbol{\varepsilon}_t$ where $\tilde{\boldsymbol{A}}_l$ is a $(N \times N)$ matrix
    2. Set $\boldsymbol{d} = E(\boldsymbol{\varepsilon}_t | \varepsilon_{t,j} = \delta) = \hat{\boldsymbol{\Sigma}}_{\boldsymbol{\varepsilon}} \boldsymbol{e}_j \hat{\sigma}_{jj}^{-1} \delta$ where $\hat{\sigma}_{jj}$ is the j-th diagonal element of $\hat{\boldsymbol{\Sigma}}_{\boldsymbol{\varepsilon}}$ and $\delta \neq 0$ denotes the shock in element j

Intraday variance risk premium estimation



Shocking the fear gauge