# Market responses to a VIX impulse

Nikolaus Hautsch, Albert J. Menkveld, and Stefan Voigt

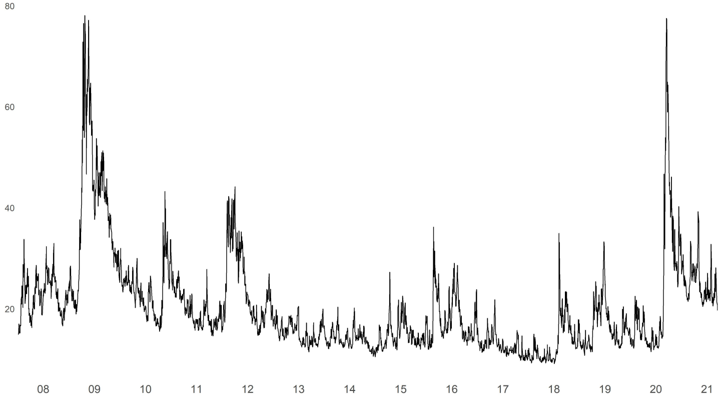
November 2022

University of Copenhagen & Danish Finance Institute

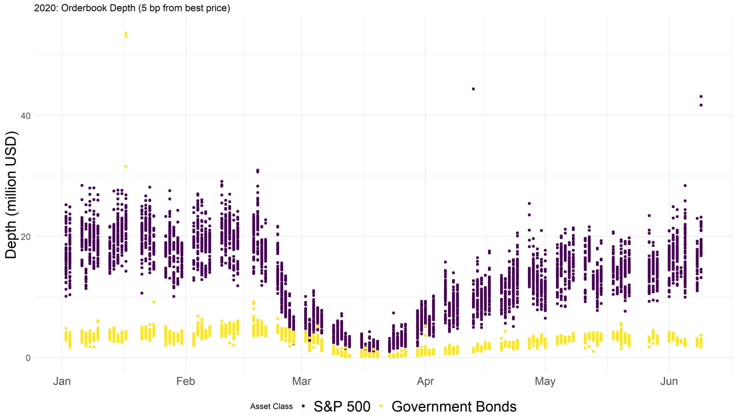
Seminar at Lancaster University 2022

# How does a VIX impulse ripple through financial markets?

- VIX - the **fear index**: Closely monitored by investors, regulators, media
- measures the risk neutral expectation of future realized variance



# Worrisome liquidity dynamics?




- Vivid examples: GFC and COVID-19 market turmoils in 2020 (Duffie, 2020)
- Liquidity evaporates entirely during periods of major market turmoil (Brunnermeier, 2009; Pedersen, 2009)
- Liquidity trends concern regulators: BIS and IOSCO (2012); SEC (2016)

# How does a VIX impulse ripple through financial markets?

VIX impulse

**Risk-neutral measure**

- higher perceived crash-risk probability
- marginal investor more risk averse
- expected returns adjust or rebalancing (Caballero and Krishnamurthy, 2008)

Flight to safety

**Expected realized variance**

- News hit the market
- Adverse selection concerning for liquidity providers (Campbell et al., 1993)
- Illiquidity (Nagel, 2012; Drechsler et al., 2020)

Evaporating liquidity

# This paper: Nature of VIX impulse matters

⇒ How does an VRP or ERV shock ripple through financial markets?

- Decomposition of VIX changes into two canonical components: variance risk premium (VRP) and expected realized volatility (ERV)
- 2007-2021 sample of *all* Nasdaq trading messages for two ETFs: equity and government bonds
- Impulse response analysis

## Main finding: ERV and VRP trigger different responses

- *VRP* shock leads to a flight to safety
- *ERV* shock yields fragile market liquidity and active *buying*(!) of equities

# A model to reconcile the responses

- Formal analysis of the two types of shocks in the model yields equilibrium results that are consistent with the empirical findings
- Extensions of Vayanos and Wang (2012): Some agents experience an increase in their risk aversion (VRP channel) or private news hits the market ("ERV channel")


- VRP channel: Unsurprisingly, investors who suddenly become more risk averse re-allocate risk to those who do not
- ERV Channel: liquidity deteriorates because of the adverse-selection cost.
- ERV Channel: *Active buying of the risky asset is an equilibrium result.* Liquidity demanders effectively bear less *posterior* risk and are the natural *holders* of the risky asset

# Decomposition of the VIX index

# The variance risk premium

- *VIX*: implied volatility of the S&P 500 index, constructed from market prices of out-of-the-money puts and calls with maturity of one month

$$IV = VRP + ERV$$

- variance risk premium: difference between *implied* variance $IV$ as the risk-neutral expectation of the future realized variance and the expected realized variance as its "physical" counterpart
- With a stochastic pricing kernel $m_{t,t'}$, VRP can be expressed as

$$\begin{aligned} VRP_t &:= E_t^{\mathbb{Q}}\left(RV_{t,t'}\right) - E_t^{\mathbb{P}}\left(RV_{t,t'}\right) \\ &= E_t^{\mathbb{P}}\left(m_{t,t'}\right)^{-1} E_t^{\mathbb{P}}\left(m_{t,t'} RV_{t,t'}\right) - E_t^{\mathbb{P}}\left(RV_{t,t'}\right) \\ &= E_t^{\mathbb{P}}\left(m_{t,t'}\right)^{-1} \left[Cov_t^{\mathbb{P}}\left(m_{t,t'}, RV_{t,t'}\right)\right]. \end{aligned}$$

- VRP is 0 if realized variance is orthogonal to the discount factor

# Interpretation of VRP impulse

- With power utility and coefficient of relative risk aversion $\alpha$, $VRP$ is (Bakshi and Madan, 2006)

$$VRP_t \approx -\alpha E_t^{\mathbb{P}}\left(RV_{t,t'}\right)^{3/2} \times \rho + \frac{\alpha^2}{2} E_t^{\mathbb{P}}\left(RV_{t,t'}\right)^2 \times (\kappa - 3)$$

where $\rho$ is the skewness and $\kappa$ is the kurtosis of returns under $\mathbb{P}$.

## How should we think about an VRP impulse?

- VRP increases with **risk aversion** (Campbell and Cochrane, 1999; Bekaert et al., 2013)
- VRP increases if market returns are more left-skewed: **crash risk events become more likely** (Bollerslev et al., 2015)
- VRP $\neq$ ERV: e.g., central bank communication mainly impacts risk premia rather than expected realized volatility (Bernanke and Kuttner, 2005; Drechsler et al., 2018)

# High-frequency ERV estimation

## A quick roadmap

- We observe $IV_{t,\tau}$, the implied volatility at day $t$, timestamp $\tau$
- Aim: Decompose $IV_{t,\tau} = ERV_{t,\tau} + VRP_{t,\tau}$

$\Rightarrow$ We need $\widehat{ERV}_{t,\tau}$, an estimate of the conditional expected realized variance of the S&P 500 index over the next month

- The residual $\widehat{VRP}_{t,\tau} = IV_{t,\tau} - \widehat{ERV}_{t,\tau}$ captures the variance risk premium

# High-frequency ERV estimation

- Notation: 5-minute returns $r_{t,\tau} = \log(p_{t,\tau}) - \log(p_{t,\tau-1})$ for $\tau > 0$, where $p_{t,\tau}$ is the S&P 500 index value on the day $t$ at timestamp $\tau$. $r_{t,0} = \log(p_{t,0}) - \log(p_{t-1,78})$ is the overnight return
- Realized variance of the S&P 500 index, computed from closure on day $t-1$ to the end of the $\tau$-th 5 minute interval at day $t$:

$$\widetilde{RV}_{t,\tau} := \sum_{k=0}^{\tau} r_{t,k}^2$$

- HAR Model (Corsi, 2009) for the log one-month-ahead $RV_{t,\tau}^{(22)}$

$$\log\left(RV_{t,\tau}^{(22)}\right) = c_\tau + \beta_\tau \log\left(RV_{t-22,0}^{(21)} + \widetilde{RV}_{t,\tau}\right) + \gamma_\tau \log\left(RV_{t-5,0}^{(4)} + \widetilde{RV}_{t,\tau}\right) + \delta_\tau \log\left(\widetilde{RV}_{t,\tau}\right) + \varepsilon_{t,\tau}$$

where $\varepsilon_{t,\tau}$ is zero mean white noise

# High-frequency ERV estimation

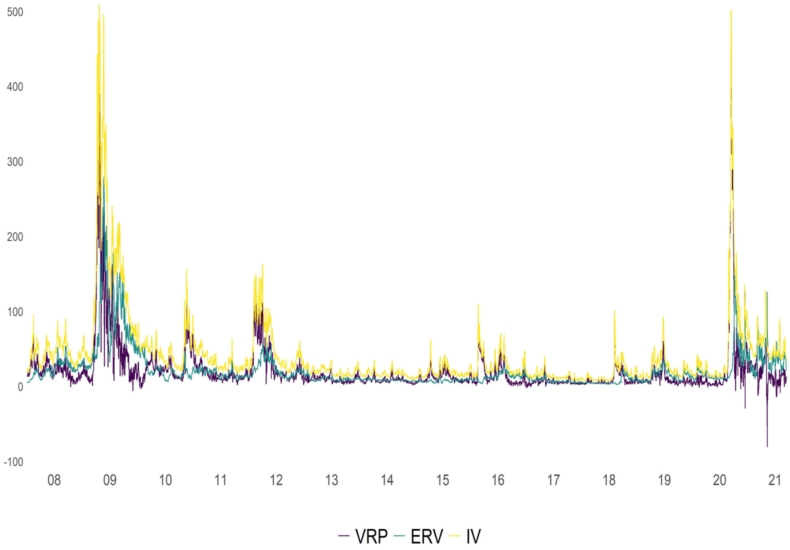
- Least squares estimation for $\tau \in \{1, \ldots, 78\}$ yields estimates $\{\hat{c}_\tau, \hat{\beta}_\tau, \hat{\gamma}_\tau, \hat{\delta}_\tau\}$ and thus

$$\widehat{E}_{t,\tau}\left(RV^{(22)}_{t,\tau}\right) = \exp\left(\hat{c}_\tau + \hat{\beta}_\tau \log\left(RV^{(21)}_{t-22,0} + \widetilde{RV}_{t,\tau}\right) + \hat{\gamma}_\tau \log\left(RV^{(4)}_{t-5,0} + \widetilde{RV}_{t,\tau}\right) + \hat{\delta}_\tau \log\left(\widetilde{RV}_{t,\tau}\right)\right)$$

- Accordingly, for each day $t$ and time stamp $\tau$ we obtain the estimated variance risk premium $VRP_{t,\tau}$ as

$$\widehat{VRP}_{t,\tau} = IV_{t,\tau} - \widehat{E}_{t,\tau}\left(RV^{(22)}_{t,\tau}\right)$$

# Implied volatility decomposition



# Market responses to a VIX impulse

# Our sample

asset classes / exchange-traded funds (ETFs)

1. SPY for equities (S&P 500)
2. TLT for government bonds

Sample period: July 1st, 2007 - April 7th, 2021

- Event-time Nasdaq order book snapshots from data provider LOBSTER
- *All* Trades, Submissions, Cancellations, and Adjustments
- first 50 order book levels ($\approx$ 20 billion messages)

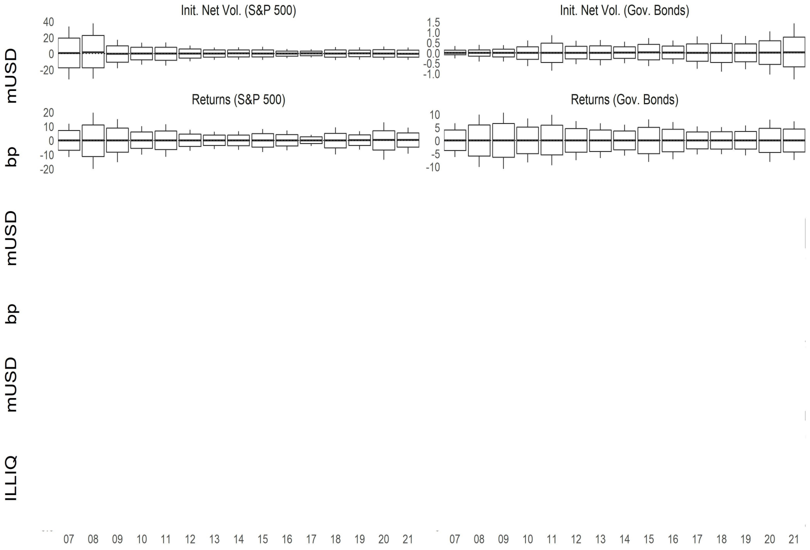
- VIX minute level data from data provider pitrading

# 5-minute observations: Variable definitions

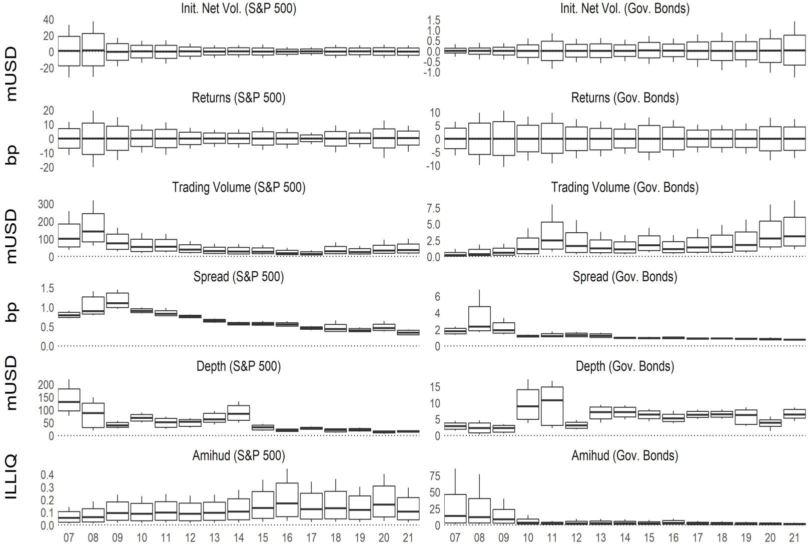
- Aggregation to 5-minute sample frequency

1. *Initiator Net Volume* (in million USD) Signing based on initiating side
2. *Return* (basis points) Log difference of prevailing midquotes
3. *Trading volume* (in million USD) Cumulative trading volume
4. *Bid ask spread* (basis points) time-weighted relative bid-ask spread
5. *Depth* (million USD) time-weighted number of shares within 5 bps of best quote
6. *Amihud (2002) illiquidity measure*: $ILLIQ_{t,\tau} := \frac{\left|\log(p_{t,\tau}) - \log(p_{t,\tau-1})\right|}{V_{t,\tau}}$

# Summary statistics: Box plots on an annual basis



# Summary statistics: Box plots on an annual basis



# Impulse responses with local projection (Jorda, 2005)

- $\boldsymbol{y}_t = (y_{t,1}, \ldots, y_{t,N})'$: $(N \times 1)$ vector of observations at time $t$
- Impulse response function $\text{ir}_i\,(t, h, \boldsymbol{d})$: response of $y_{t+h,i}$ at time $t+h$ on a (reduced form) shock $\boldsymbol{d}$ at time $t$:

$$\text{ir}_i\,(t, h, \boldsymbol{d}) := E\left(y_{t+h,i} | \boldsymbol{v}_t = \boldsymbol{d}, \boldsymbol{y}_{t-1}, \ldots, \boldsymbol{y}_1\right) - E\left(y_{t+h,i} | \boldsymbol{v}_t = \boldsymbol{0}_N, \boldsymbol{y}_{t-1}, \ldots, \boldsymbol{y}_1\right)$$

  where $\boldsymbol{v}_t$ are the reduced form disturbances
- We estimate $\text{ir}_i\,(t, h, \boldsymbol{d})$ from lag-augmented regressions of $y_{t+h,i}$ on $\boldsymbol{y}_t$ *and* $\boldsymbol{y}_{t-1}$ as additional control (Montiel Olea and Plagborg-Møller, 2021)

$$y_{t+h,i} = \boldsymbol{\beta}(h)' \boldsymbol{y}_t + \boldsymbol{\gamma}' \boldsymbol{y}_{t-1} + \varepsilon_t$$

- Then,

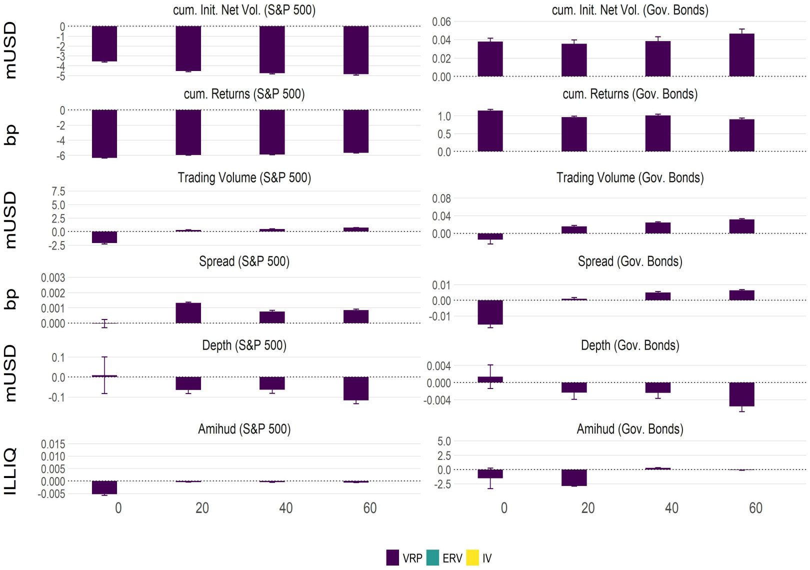
$$\text{ir}_i\,(t, h, \boldsymbol{d}) = \hat{\boldsymbol{\beta}}(h)' \mathbf{d}$$

- We integrate out contemporaneous effects due to correlation using the historically observed distribution of the errors (Pesaran and Shin, 1998)

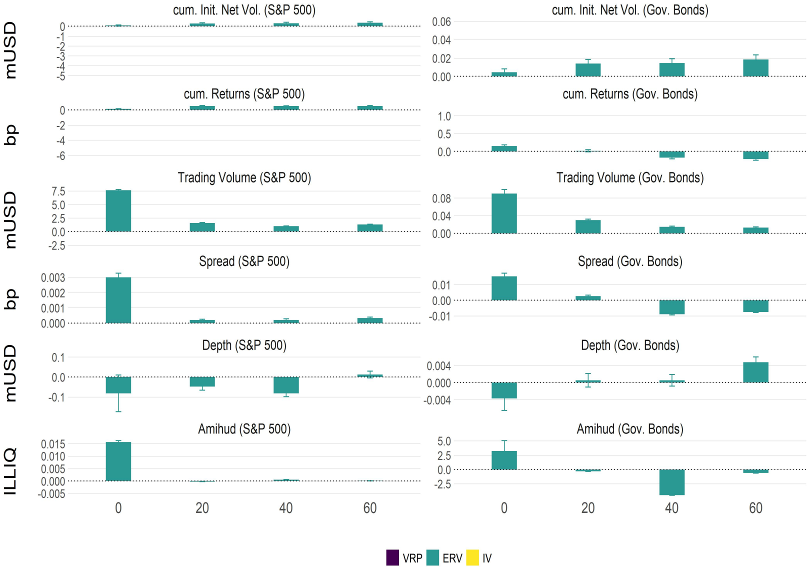
# Shocking the components of the fear index

- Our system consists of the time series of order book related variables and the *VRP and ERV changes*
- We compute the responses to a (SD) shock in either VRP or ERV changes
- We report estimated cumulative impulse response functions for initiator net volume (in million USD) and return (in basis points) and impulse response functions for trading volume and depth (all in million USD), bid-ask spreads (in basis points) and the Amihud measure (ILLIQ)
- We report 95% confidence intervals with heteroscedasticity-robust standard errors

# Market responses to a VIX impulse



# Market responses to a VIX impulse



# Market responses to a VIX impulse

# Further empirical results

- *Institutional client responses* to a VRP or ERV shocks largely mirror initiator net volume: Institutional trading responds to an VRP shock with selling of equity at large magnitudes and turns positive after an ERV shock
- Net buying of (US) equity is *not* a response of net selling of global equity. Results are qualitatively similar when including MSCI World index tracking ETF
- The results are *not* driven by estimation error in ERV (and thus in VRP): Imposing *perfect foresight* ($RV_t^{(22)}$ instead of $\hat{E}(RV_t^{(22)})$ delivers similar results
- *Sub-period analysis* for crisis periods shows that the responses magnify during crisis periods

# A model to disentangle the responses

# The theoretical framework in a nutshell

All traders identical

- Three periods, risk averse initially identical agents who derive (exponential) utility from consumption
- Market: a riskless and a risky asset in supply of $\bar{\theta}$ shares which pays $D \sim N(\bar{D}, \sigma^2)$ units of consumption in $t = 2$
- Just before Period 1, a fraction $\pi$ of the agents learns that they will receive an endowment $z(D - \bar{D})$ of the consumption good in Period 2, with $z \sim N(0, \sigma_z^2)$ independent of $D$
- **Baseline insight:** Price impact from initiated volume to incentivize liquidity providers + liquidity premium already in $t = 0$

# The theoretical framework in a nutshell

All traders identical ⋯→ Endownment

- Three periods, risk averse initially identical agents who derive (exponential) utility from consumption
- Market: a riskless and a risky asset in supply of $\bar{\theta}$ shares which pays $D \sim N(\bar{D}, \sigma^2)$ units of consumption in $t = 2$
- Just before Period 1, a fraction $\pi$ of the agents learns that they will receive an endowment $z(D - \bar{D})$ of the consumption good in Period 2, with $z \sim N(0, \sigma_z^2)$ independent of $D$
- **Baseline insight:** Price impact from initiated volume to incentivize liquidity providers + liquidity premium already in $t = 0$

# The theoretical framework in a nutshell

All traders identical ⇢ Endownment ⇢ Shocked ↔ Not shocked

- Three periods, risk averse initially identical agents who derive (exponential) utility from consumption
- Market: a riskless and a risky asset in supply of $\bar{\theta}$ shares which pays $D \sim N(\bar{D}, \sigma^2)$ units of consumption in $t = 2$
- Just before Period 1, a fraction $\pi$ of the agents learns that they will receive an endowment $z(D - \bar{D})$ of the consumption good in Period 2, with $z \sim N(0, \sigma_z^2)$ independent of $D$
- **Baseline insight:** Price impact from initiated volume to incentivize liquidity providers + liquidity premium already in $t = 0$

# The theoretical framework in a nutshell

All traders identical

Two extensions: Risk shocks in addition to endowment

1. *risk aversion shock (VRP channel)*: the fraction $\pi$ of agents with endowment shock simultaneously experiences increase of their risk aversion
2. *cash-flow risk (ERV channel)* for the (identical) agents in $t = 0$: common knowledge that liquidity demanders learn additional information in $t = 1$ from a private signal correlated with $D$

# The theoretical framework in a nutshell

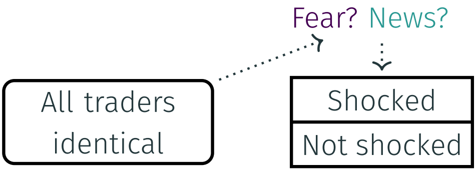
Fear? News?

All traders identical

Two extensions: Risk shocks in addition to endowment

1. *risk aversion shock (VRP channel)*: the fraction $\pi$ of agents with endowment shock simultaneously experiences increase of their risk aversion
2. *cash-flow risk (ERV channel)* for the (identical) agents in $t = 0$: common knowledge that liquidity demanders learn additional information in $t = 1$ from a private signal correlated with $D$

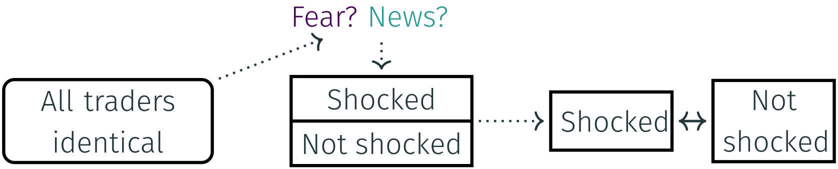
# The theoretical framework in a nutshell




Two extensions: Risk shocks in addition to endowment

1. *risk aversion shock (VRP channel)*: the fraction $\pi$ of agents with endowment shock simultaneously experiences increase of their risk aversion
2. *cash-flow risk (ERV channel)* for the (identical) agents in $t = 0$: common knowledge that liquidity demanders learn additional information in $t = 1$ from a private signal correlated with $D$

# The theoretical framework in a nutshell




Two extensions: Risk shocks in addition to endowment

1. *risk aversion shock (VRP channel)*: the fraction $\pi$ of agents with endowment shock simultaneously experiences increase of their risk aversion
2. *cash-flow risk (ERV channel)* for the (identical) agents in $t = 0$: common knowledge that liquidity demanders learn additional information in $t = 1$ from a private signal correlated with $D$

# Equilibrium outcomes from the model

- **(Initiator net volume)**. The expected equilibrium quantities for initiator net volume, defined as the signed volume of liquidity demanders are such that

$$E\left(\nu^{\text{VRP}}\right) < E\left(\nu\right) = 0 < E\left(\nu^{\text{ERV}}\right) \tag{1}$$

- **(Price impacts)**. The equilibrium quantities for price impact are such that

$$0 < \lambda = \lambda^{\text{VRP}} < \lambda^{\text{ERV}} \tag{2}$$

1. Risk aversion shocks trigger *net selling* of the risky asset and simultaneously net buying of government bonds
2. News generate *net buying* of the risky asset (resemble relative increase in risk for liquidity providers)
3. Larger price impact in response to ERV shocks than for risk aversion shocks

# Conclusions

# Conclusions

- Responses to risk shocks depend on the *nature* of the shock
  1. Variance risk premium shocks predominantly trigger net selling of equities and net buying of government bonds
  2. Expected realized variance shocks create *net buying of risky assets* and *illiquidity*, pointing towards asymmetric information

## Implications

- Liquidity risk management: IV too broad - ERV matters!
- Central bank communication: addressing risk premia of investors (crash-risk) does not impair liquidity

# References i


Ait-Sahalia, Y., F. Matthys, E. Osambela, and R. Sircar (2021). When Uncertainty and Volatility Are Disconnected: Implications for Asset Pricing and Portfolio Performance. Working paper.

Amihud, Y. (2002). Illiquidity and Stock Returns: Cross-section and time-series Effects. *Journal of Financial Markets 5*(1), 31–56.

Angel, J. J., L. E. Harris, and C. S. Spatt (2015). Equity Trading in the 21st Century: An Update. *Quarterly Journal of Finance 5*.

Bakshi, G. and D. Madan (2006). A Theory of Volatility Spreads. *Management Science 52*(12), 1945–1956.

Beber, A., M. W. Brandt, and K. A. Kavajecz (2009). Flight-to-Quality or Flight-to-Liquidity? Evidence from the Euro-Area Bond Market. *Review of Financial Studies 22*, 925–957.

Bekaert, G. and M. Hoerova (2014). The VIX, the Variance Premium and Stock Market Volatility. *Journal of Econometrics 183*, 181–192.

Bekaert, G., M. Hoerova, and M. L. Duca (2013). Risk, Uncertainty and Monetary Policy. *Journal of Monetary Policy 60*, 771–788.

Ben-David, I., F. Franzoni, and R. Moussaw (2017). Exchange-Traded Funds. *Annual Review of Financial Economics 9*, 169–189.

Bernanke, B. S. and K. N. Kuttner (2005). What explains the stock market's reaction to federal reserve policy? *The Journal of finance 60*(3), 1221–1257.

BIS and IOSCO (2012). Principles for Financial Market Infrastructures. Manuscript, Committee on Payment and Settlement Systems and Technical Committee of the International Organization of Securities Commissions.

Bollerslev, T., J. Marrone, L. Xu, and H. Zhou (2014). Stock Return Predictability and Variance Risk Premia: Statistical Inference and International Evidence. *Journal of Financial and Quantitative Analysis 49*, 633–661.

# References ii


Bollerslev, T., N. Sizova, and G. Tauchen (2012). Volatility in Equilibrium: Asymmetries and Dynamic Dependencies. *Review of Finance 16*, 31–80.

Bollerslev, T., G. Tauchen, and H. Zhou (2009). Expected Stock Returns and Variance Risk Premia. *Review of Financial Studies 22*, 4463–4492.

Bollerslev, T., V. Todorov, and L. Xu (2015). Tail Risk Premia and Return Predictability. *Journal of Financial Economics 118*, 113–134.

Brenner, M. and Y. Izhakian (2018). Asset Pricing and Ambiguity: Empirical Evidence. *Journal of Financial Economics 130*(3), 503–531.

Brunnermeier, M. K. (2009). Deciphering the Liquidity and Credit Crunch 2007-2008. *Journal of Economic Perspectives 23*(1), 77–100.

Caballero, R. J. and A. Krishnamurthy (2008). Collective risk management in a flight to quality episode. *The Journal of Finance 63*(5), 2195–2230.

Campbell, J. Y. and J. H. Cochrane (1999). By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior. *Journal of Political Economy 107*(2), 205–251.

Campbell, J. Y., S. J. Grossman, and J. Wang (1993). Trading Volume and Serial Correlation in Stocks Returns. *The Quarterly Journal of Economics 108*, 905–939.

Corsi, F. (2009). A Simple Approximate Long-Memory Model of Realized Volatility. *Journal of Financial Econometrics 7*(2), 174–196.

Drechsler, I. (2013). Uncertainty, Time-Varying Fear, and Asset Prices. *The Journal of Finance 68*(5), 1843–1889.

Drechsler, I., A. Moreira, and A. Savov (2020, October). Liquidity and volatility. Working Paper 27959, National Bureau of Economic Research.

Drechsler, I., A. Savov, and P. Schnabl (2018). A model of monetary policy and risk premia. *The Journal of Finance 73*(1), 317–373.

Duffie, D. (2020). Still the world's safe haven? *Redesigning the US Treasury market after the COVID-19 crisis', Hutchins Center on Fiscal and Monetary Policy at Brookings, available online at https://www.brookings.edu/research/still-the-worlds-safe-haven*.

ESMA (2019). Guidelines on liquidity stress testing in UCITS and AIFs. Manuscript, European Securities and Markets Authority.

Falato, A., I. Goldstein, and A. Hortacsu (2020). Financial Fragility in the COVID-19 Crisis: The Case of Investment Funds in Corporate Bond Markets. Manuscript.

Jorda, O. (2005). Estimation and Inference of Impulse Responses by Local Projections. *American Economic Review 95*(1), 161–182.

Koop, G., M. Pesaran, and S. M. Potter (1996). Impulse Response Analysis in Nonlinear Multivariate Models. *Journal of Econometrics 74*(1), 119 – 147.

Lettau, M. and A. Madhavan (2017). Exchange-Traded Funds 101 for Economists. *Journal of Economic Perspectives 32*, 135–154.

Liu, J., J. Pan, and T. Wang (2004). An Equilibrium Model of Rare-Event Premia and Its Implication for Option Smirks. *The Review of Financial Studies 18*(1), 131–164.

Longstaff, F. A. (2004). The Flight-to-Liquidity Premium in U.S. Treasury Bond Prices. *Journal of Business 77*, 511–526.

Montiel Olea, J. L. and M. Plagborg-Møller (2021). Local Projection Inference Is Simpler and More Robust Than You Think. *Econometrica 89*(4), 1789–1823.

Nagel, S. (2012). Evaporating liquidity. *Review of Financial Studies 25*, 2005–2039.

Pedersen, L. H. (2009). When Everyone Runs for the Exit. *International Journal of Central Banking 5*, 177–199.

Pesaran, H. and Y. Shin (1998). Generalized Impulse Response Analysis in Linear Multivariate Models. *Economics Letters 58*(1), 17 – 29.

SEC (2016). Investment Company Liquidity Risk Management Programs. Manuscript, SEC Release No. 33–10233.

Vayanos, D. (2004). Flight-to-Quality or Flight-to-Liquidity? Evidence from the Euro-Area Bond Market. Manuscript, NBER Working Paper No. 10327.

Vayanos, D. and J. Wang (2012). Liquidity and Asset Returns Under Asymmetric Information and Imperfect Competition. *Review of Financial Studies 25*, 1339–1365.

# Literature (incomplete)

- *Flight to quality/liquidity* Longstaff (2004); Vayanos (2004); Beber et al. (2009)
- Ait-Sahalia et al. (2021) rationalize *asset-price dynamics* by introducing two “disconnected” stochastic processes: one that drives (realized) volatility and another that drives uncertainty (aversion) (same spirit: Liu et al. (2004), Drechsler (2013), Brenner and Izhakian (2018))
- *variance risk premium* (Bollerslev et al., 2009, 2012, 2014; Bekaert and Hoerova, 2014; Nagel, 2012)
- *Crisis periods*: GFC (Brunnermeier, 2009) and COVID-19 (Falato et al., 2020)
- *Exchange-traded funds* (Ben-David et al., 2017; Lettau and Madhavan, 2017)
- *Market liquidity trends* (Angel et al., 2015; SEC, 2016; ESMA, 2019)

# Impulse responses with local projection

## Impulse response functions (IRF)

- Denote $(N \times 1)$ vector of observations at time $t$ as $\boldsymbol{y}_t = (y_{t,1}, \ldots, y_{t,N})'$
- Impulse response function $\mathrm{ir}_i\,(t, h, \boldsymbol{d})$ measures the response of $y_{t+h,i}$ at time $t + h$ on a (reduced form) shock $\boldsymbol{d}$ at time $t$:

  $$\mathrm{ir}_i\,(t, h, \boldsymbol{d}) := E\,(y_{t+h,i}|\boldsymbol{v}_t = \boldsymbol{d}, \boldsymbol{y}_{t-1}, \ldots, \boldsymbol{y}_1) - E\,(y_{t+h,i}|\boldsymbol{v}_t = \boldsymbol{0}_N, \boldsymbol{y}_{t-1}, \ldots, \boldsymbol{y}_1)$$

  where $\boldsymbol{v}_t$ are the reduced form disturbances, $\boldsymbol{0}_N$ denotes a $(N \times 1)$ vector of zeros

## Common approach: vector autoregressive model (VAR)

- specify VAR: $\boldsymbol{y}_t = \boldsymbol{A}\boldsymbol{y}_{t-1} + \boldsymbol{u}_t$ and obtain $\mathrm{ir}_i\,(t, h, \boldsymbol{d})$ as non-linear transformation of the estimated coefficients
- $\mathrm{ir}_i\,(t, h, \boldsymbol{d}) = \boldsymbol{e}_i'\boldsymbol{\Phi}_h\boldsymbol{d}$ where $e_i$ is a $(N \times 1)$ unit vector and $\boldsymbol{\Phi}_h$ is computed recursively from the canonical $MA(\infty)$ representation
- **Problem:** High-dimensional nonlinear functions of parameter estimates $\hat{\boldsymbol{A}}$ are prone to estimation and misspecification error

# Local Projections (Jorda, 2005)

- Simple equation-by-equation least squares estimation
- We obtain an estimate of $\mathrm{ir}_i(t, h, \boldsymbol{d})$ from baseline lag-augmented regression of $y_{t+h,i}$ on $\boldsymbol{y}_t$ *and* $\boldsymbol{y}_{t-1}$ as additional control

$$y_{t+h,i} = \boldsymbol{\beta}(h)'\boldsymbol{y}_t + \boldsymbol{\gamma}'\boldsymbol{y}_{t-1} + \varepsilon_t$$

- For a given horizon $h$, the lag-augmented local projection estimates are

$$\begin{pmatrix} \hat{\boldsymbol{\beta}}(h) \\ \hat{\boldsymbol{\gamma}} \end{pmatrix} = \left( \sum_{t=1}^{T-h} \boldsymbol{x}_t \boldsymbol{x}_t' \right)^{-1} \left( \sum_{t=1}^{T-h} \boldsymbol{x}_t y_{t+h,i} \right)$$

where $\boldsymbol{x}_t = \begin{pmatrix} \boldsymbol{y}_t, & \boldsymbol{y}_{t-1} \end{pmatrix}'$ is a $(2N \times 1)$ vector, $\hat{\boldsymbol{\beta}}(h)$ is *a consistent estimator of* $\mathrm{ir}_i(t, h, \boldsymbol{d})$ and $\hat{\boldsymbol{\gamma}}$ is a nuisance parameter

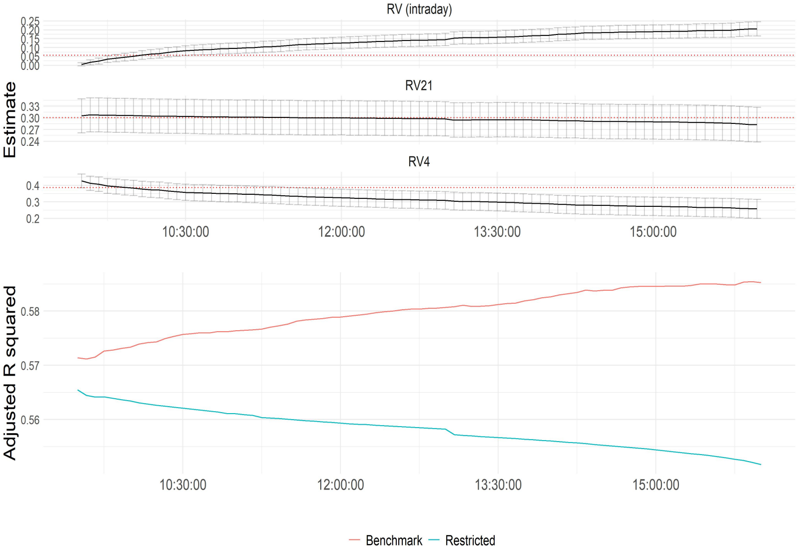
## What makes the lag-augmented estimator $\hat{\boldsymbol{\beta}}(h)$ appealing?

- Covariance of the estimator $\hat{\boldsymbol{\beta}}(h)$ does not require Newey-West inference. (Eicker-Huber-White) heteroscedasticity-robust standard errors suffice for inference (Montiel Olea and Plagborg-Møller, 2021)
- Lag augmentation handles nearly persistent series particularly well

# Choice of the shock

- Common problem: contemporaneous correlation of the errors $\varepsilon_t$
- Our (current) approach: *pseudo* generalized IRFs in spirit of Pesaran and Shin (1998) and Koop et al. (1996)
- Two steps to integrate out the effects of other shocks using the historically observed distribution of the errors
  1. estimate the parameters of a VAR($p$) regression for $\boldsymbol{y}_t$ of the standard form $\boldsymbol{y}_t = \boldsymbol{\mu} + \sum_{l=1}^{p} \tilde{\boldsymbol{A}}_l \boldsymbol{y}_{t-l} + \boldsymbol{\varepsilon}_t$ where $\tilde{\boldsymbol{A}}_l$ is a $(N \times N)$ matrix
  2. Set $\boldsymbol{d} = E(\boldsymbol{\varepsilon}_t | \varepsilon_{t,j} = \delta) = \hat{\boldsymbol{\Sigma}}_{\boldsymbol{\varepsilon}} \boldsymbol{e}_j \hat{\sigma}_{jj}^{-1} \delta$ where $\hat{\sigma}_{jj}$ is the $j$-th diagonal element of $\hat{\boldsymbol{\Sigma}}_{\boldsymbol{\varepsilon}}$ and $\delta \neq 0$ denotes the shock in element $j$

# Intraday variance risk premium estimation



# Shocking the fear gauge