**EE – 598**
**Project 4a**
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**Problem 1: DTFS Markov Chains: Stationary Distributions (code: [Appendix A])**

The goal of this problem is to find the Stationary Distribution for the given two-state DTFS MC and having the following transition matrix :

$$\begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix}$$

For a two-state Markov Chain, this can be found by following this formula –

$$\left[\frac{b}{(a+b)}, \frac{a}{(a+b)}\right]$$

Therefore, the stationary distributions for the given values of **a** and **b** are as below (as obtained from the output of R code) :

1) <u>a = 1/10 , b = 1/15</u>

```
Transition Matrix is:    [ 0.9 0.06666667 ]
                         [ 0.1 0.9333333  ]

Stationary Distribution is:  [ 0.4 0.6 ]
```

2) <u>a = 0.5 , b = 0.5</u>

```
Transition Matrix is:    [ 0.5 0.5 ]
                         [ 0.5 0.5 ]

Stationary Distribution is:  [ 0.5 0.5 ]
```

3) <u>a = 1 , b = 1</u>

```
Transition Matrix is:    [ 0 1  ]
                         [ 1 0  ]

Stationary Distribution is:  [ 0.5 0.5 ]
```

4) <u>a = 0, b = 0</u>

```
Transition Matrix is:   [ 1 0  ]
                        [ 0 1  ]

Stationary Distribution is:  [ NaN NaN ]
```

The nature and the properties of the given markov chains are understood more by plotting the samples of Markov Chain which is done in the further problem.

**Problem 2: DTFS MC simulation (code: [Appendix B])**

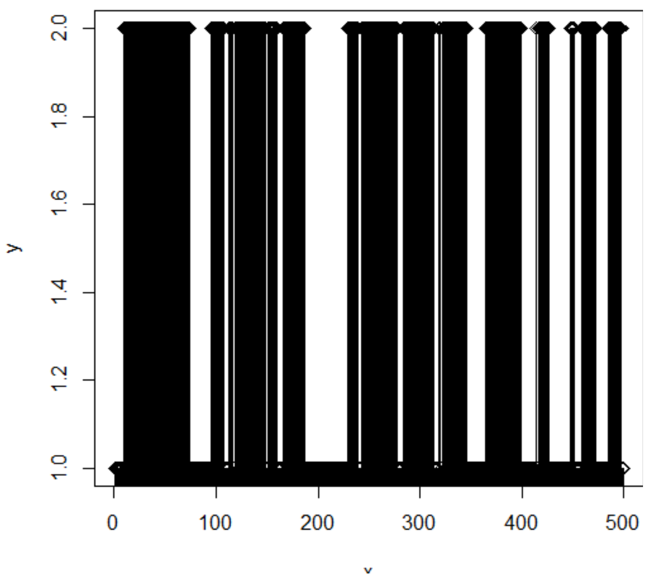
Number of samples : 500

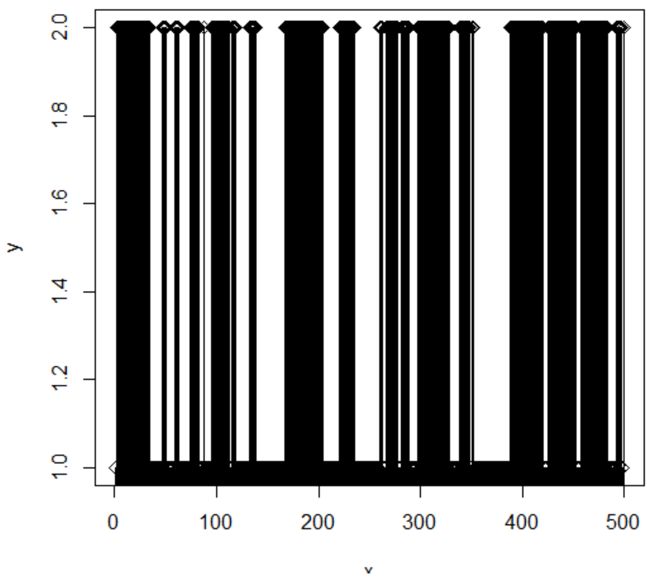
Initial Distribution : $\vec{\pi} = \begin{bmatrix} 1 & 0 \end{bmatrix}$ and $\vec{\pi} = \begin{bmatrix} 0 & 1 \end{bmatrix}$ (change of initial state)

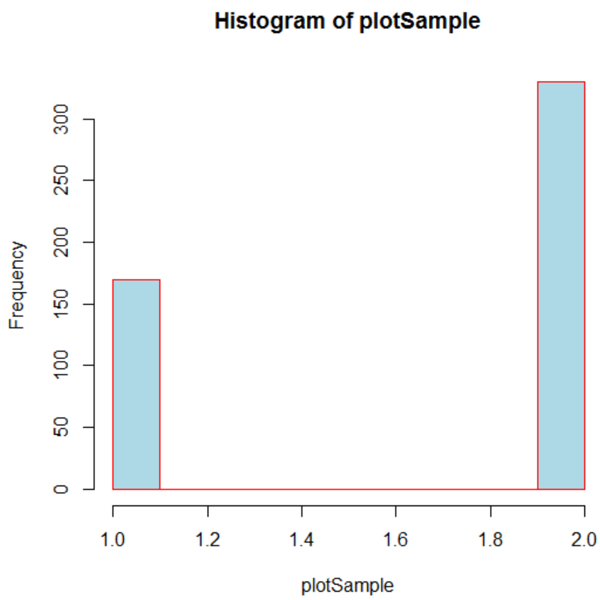
The goal is to plot the MC samples (using the first three transition matrices generated above) for 500 time steps and check for convergence using Chi-Square Goodness-of-Fit test for the last 60 time steps.

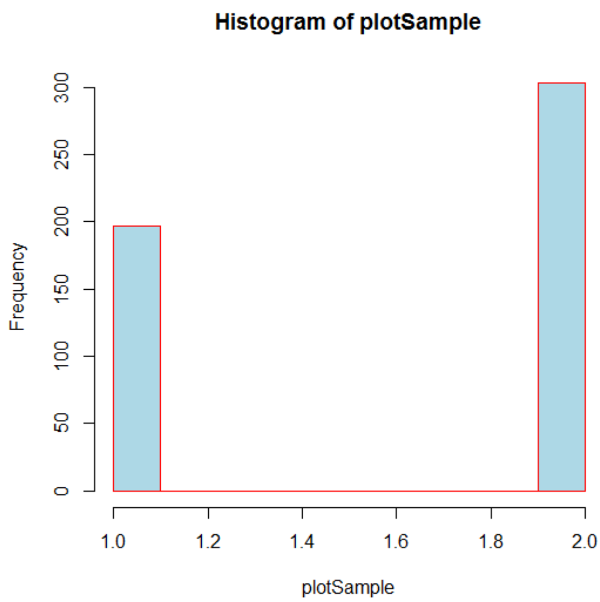
NOTE: State 0 and State 1 are represented as State 1 and State 2 respectively in all the following graphs. (**y = 1.0** represents **state 0** and **y = 2.0** represents **state 1**)

(i) **a = 1/10, b = 1/15**
(with [1 0] as initial distribution (left) and [0 1] as initial distribution (right) )

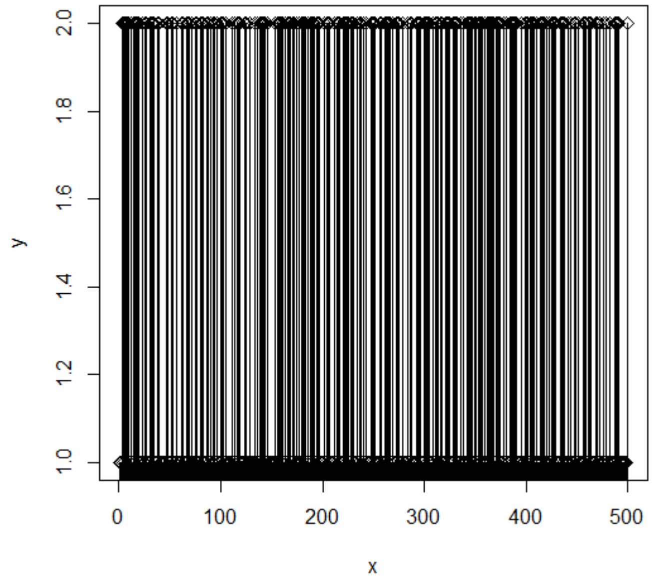


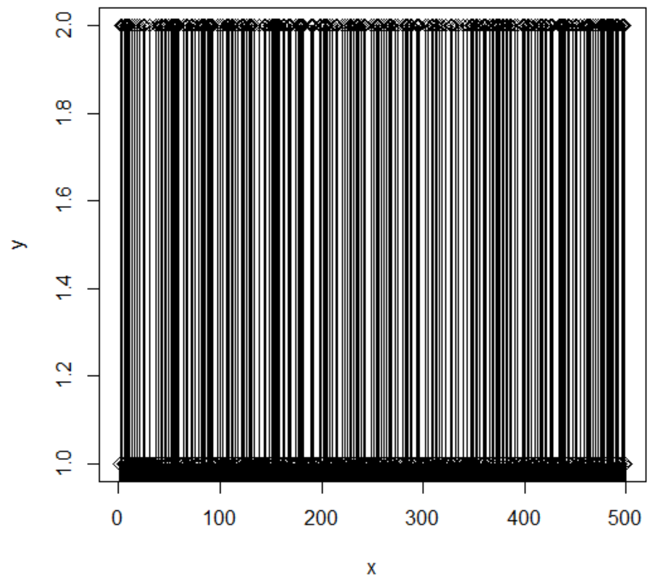



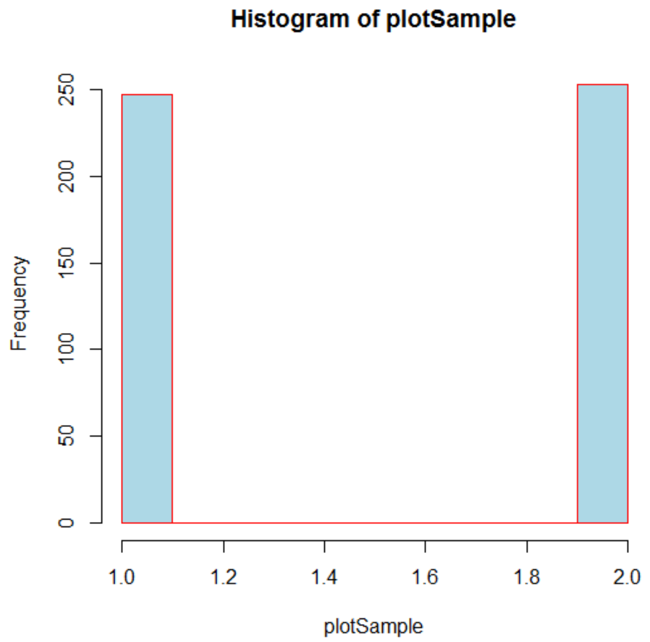



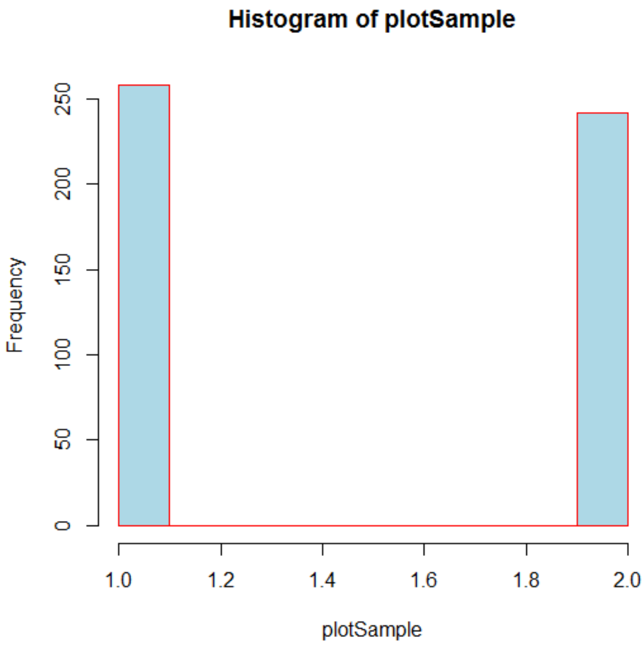



(ii) **a = 0.5 , b = 0.5**
(with [1 0] as initial distribution (left) and [0 1] as initial distribution (right) )

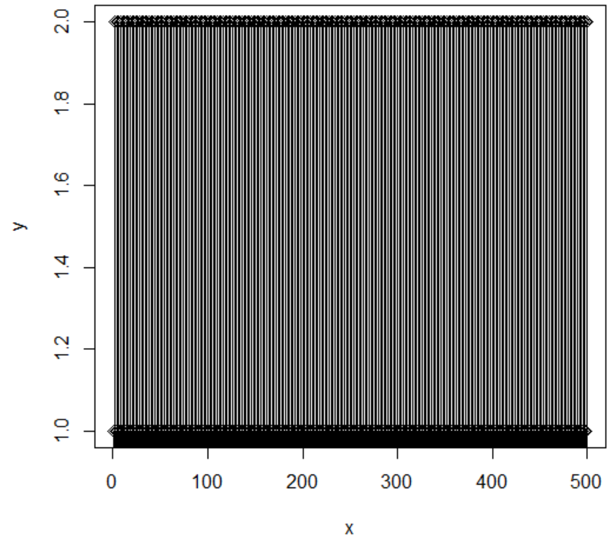


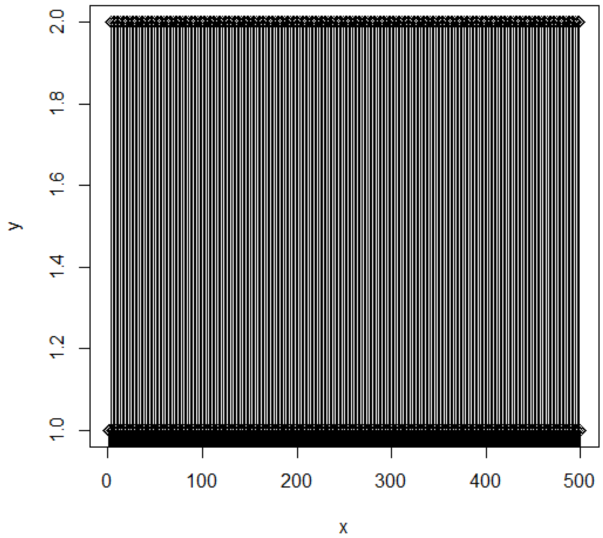



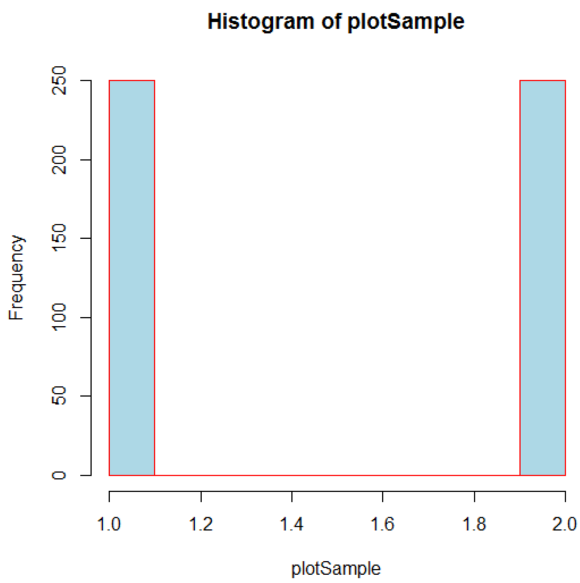



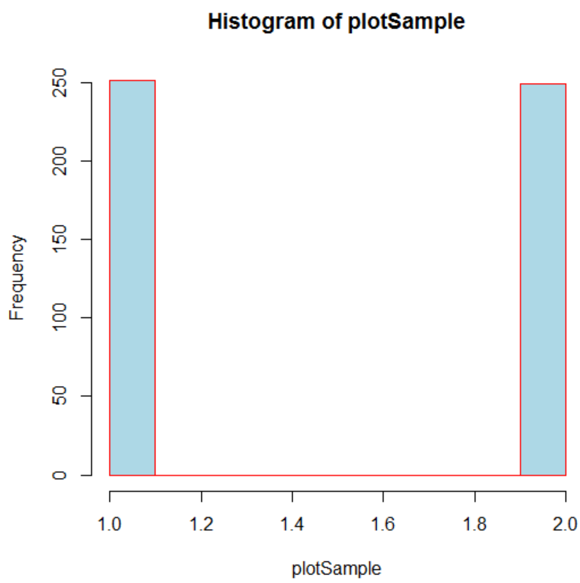



(iii) **a = 1 , b = 1**
(with [1 0] as initial distribution (left) and [0 1] as initial distribution (right) )




Comment : Close observation reveals that while the first two MCs change behaviour on changing the initial distribution, the third one does not. The fact that the stationary distribution of MCs two and three are the same and yet behaviour is different can be attributed to the different transition matrices, and thereby are observed in the histograms of the MC samples.

The $\chi^2$ values of the three MCs are as below:
1) With Initial distribution [1 0] –

```
(i) data:  plotSample[441:500]
  X-squared = 9.7391, df = 59
```

```
(ii) data:  plotSample[441:500]
X-squared = 9.4043, df = 59

(iii) data:  plotSample[441:500]
X-squared = 10, df = 59
```

2) With initial distribution [0  1] –

```
(i) data:  plotSample[441:500]
X-squared = 9.5806, df = 59

(ii) data:  plotSample[441:500]
X-squared = 9.5806, df = 59

(iii) data:  plotSample[441:500]
X-squared = 10, df = 59
```

Based on the values for the three MCs above, it could be said that MC-1 and MC-2 converge to unique distributions. The chi-squared value for the third MC is higher and the histograms/stem plots show constant fluctuation of the sample states visited.

## [Appendix A]

```
### Problem 1 - DTFS: Markov Chain Stationary Distributions #########

#       ------------------------------------------------------------
#            Discrete Time Finite State MC having transition matrix
#       [(1-a)  a ]
#       [b   (1-b)]
#
# A two-state MC's stationary distribution is given by
[b/(a+b),a/(a+b)]
#      -------------------------------------------------------------

statDist <- function (a,b){
 transMat <- matrix(c((1-a),a,b,(1-b)),ncol=2,byrow=TRUE)
 cat("Transition Matrix is:   ","[",transMat[1],transMat[2],"]","\n")
      cat("\t\t\t","[",transMat[3],transMat[4]," ]","\n")
 statDist <- c( (b/(a+b)), (a/(a+b)) )
      cat("Stationary Distribution is: ","[",statDist,"]","\n")
}


statDist(1/10,1/15) #(i)
statDist(1/2,1/2) #(ii)
statDist(1,1) #(iii)
statDist(0,0) #(iv)
```

**[Appendix – B]**

```
####------  Problem 2 - DTFS MC Simulation   -------################

# function to draw STEM PLOT of X vs Y (R doesn't have MATLAB like
function)
stem <- function(x,y,pch=5,linecol=1,clinecol=1,...){
if (missing(y)){
    y = x
    x = 1:length(x) }
    plot(x,y,pch=pch,...)
    for (i in 1:length(x)){
       lines(c(x[i],x[i]), c(0,y[i]),col=linecol)
    }
    lines(c(x[1]-2,x[length(x)]+2), c(0,0),col=clinecol)
}



###      ###### Simulating Markov Chains #######
# The probability transition matrix
trans  =  matrix(c(0,1,
              1,0), ncol=2, byrow=TRUE);

# Markov chain simulation function

    # The state that we're starting in
    state = 2;
    cat("Starting state:", state, "\n");

    plotSample <- rep(1,1)
    # Make twenty steps through the markov chain
    for (i in 1:499)
    {
        cat("> Dist:", paste(round(c(trans[state,]), 2)), "\n");
        newState <- sample(1:ncol(trans), 1, prob=trans[state,])
        cat("*", state, "->", newState, "\n");
        state = newState;
   plotSample <- append(plotSample, state, after=i)
    }
x <- seq(1,500,by=1)
hist(plotSample,col="lightblue",border="red")
x11();
stem(x,plotSample)

#### --- chi-square test of goodness-of fit
chisq.test(plotSample[441:500])
```