Load per unit length $w := 5 \frac{lbf}{ft}$

Half distance to support (a=x) $a := 50ft$

Allowable Sag $S := 5ft$

Guess for equation $c := 32ft$

Given

$$S = c\cdot\left(\cosh\left(\frac{a}{c}\right) - 1\right)$$

$c_1 := \text{Find}(c)$

Caternary parameter $c_1 = 250.829\ ft$

$$s := c_1\cdot\left(\sinh\left(\frac{a}{c_1}\right)\right)$$

Length of cable to half (L=s) $s = 50.332\ ft$

$y_d := c_1 + S$

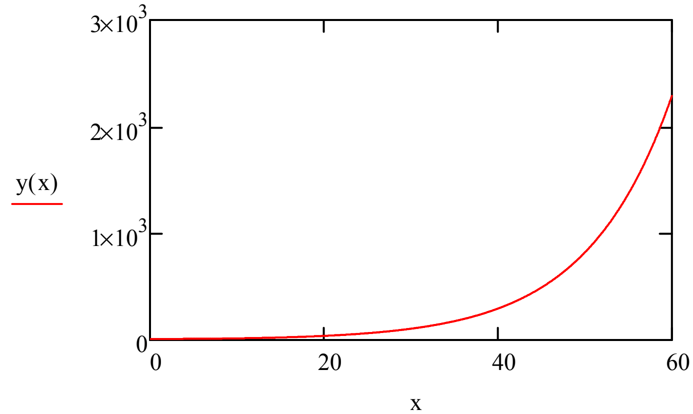
Height of post $y_d = 255.829\ ft$

Tension at point of lowest sag $H := w\cdot c_1$ $H = 1254.145\ lbf$

Distributed Load $F := w\cdot s$ $F = 251.659\ lbf$

Tension at support $T := w\cdot y_d$ $T = 1279.145\ lbf$

$$y(x) := c \cdot \cosh\left(\frac{x}{c}\right)$$



Since the load is vertical everywhere, the horizontal component of tension is constant everywhere in the cable. The tension, T_C, at any point C can be found by applying the equilibrium conditions to the cable segment BC.

$$T_{C,x} = H = \frac{wa^2}{2S} \quad 43.52$$

$$T_{C,y} = wx \quad 43.53$$

$$T_C = \sqrt{(T_{C,x})^2 + (T_{C,y})^2} = w\sqrt{\left(\frac{a^2}{2S}\right)^2 + x^2} \quad 43.54$$

The angle of the cable at any point is

$$\tan\theta = \frac{wx}{H} \quad 43.55$$

The tension and angle are maximum at the supports.

If the lowest sag point, point B, is used as the origin, the shape of the cable is

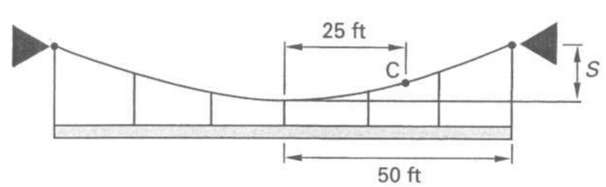
$$y(x) = \frac{wx^2}{2H} \quad 43.56$$

The approximate length of the cable from the lowest point to the support (i.e., length BD) is

$$L \approx a\left[1 + \left(\frac{2}{3}\right)\left(\frac{S}{a}\right)^2 - \left(\frac{2}{5}\right)\left(\frac{S}{a}\right)^4\right] \quad 43.57$$

Example 43.12

A pedestrian foot bridge has two suspension cables and a flexible floor weighing 28 lbf/ft. The span of the bridge is 100 ft. When the bridge is empty, the tension at point C is 1500 lbf. Assuming a parabolic shape, what is the maximum cable sag, S?



Solution

Since there are two cables, the floor weight per suspension cable is

$$w = \frac{28\ \frac{\text{lbf}}{\text{ft}}}{2} = 14\ \text{lbf/ft}$$

From Eq. 43.54,

$$T_C = w\sqrt{\left(\frac{a^2}{2S}\right)^2 + x^2}$$

$$1500\ \text{ft} = 14\ \frac{\text{lbf}}{\text{ft}}\sqrt{\left[\frac{(50\ \text{ft})^2}{2S}\right]^2 + (25\ \text{ft})^2}$$

$$S = 12\ \text{ft}$$

41. CABLES CARRYING DISTRIBUTED LOADS

An idealized tension cable with a distributed load is similar to a linkage made up of a very large number of axial members. The cable is an axial member in the sense that the internal tension acts tangentially to the cable everywhere.

Since the load is vertical everywhere, the horizontal component of cable tension is constant along the cable. The cable is horizontal at the point of lowest sag. There is no vertical tension component, and the cable tension is minimum. By similar reasoning, the cable tension is maximum at the supports.

Figure 43.19 illustrates a general cable with a distributed load. The shape of the cable will depend on the relative distribution of the load. A free-body diagram of segment BC is also shown.

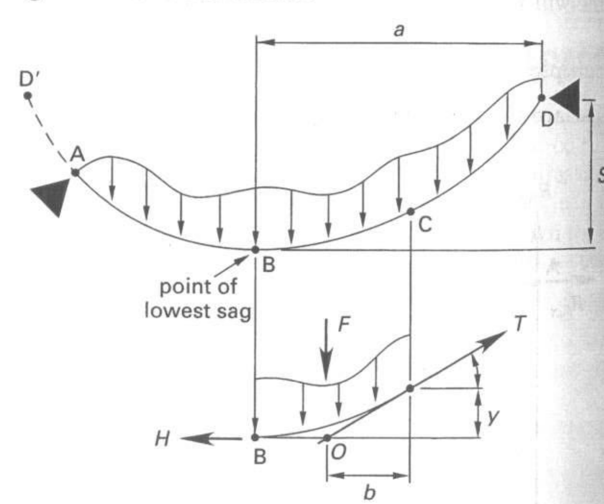


Figure 43.19 *Cable with Distributed Load*

F is the resultant of the distributed load on segment BC, T is the cable tension at point C, and H is the tension at the point of lowest sag (i.e., the point of minimum tension). Since segment BC taken as a free body is a three-force member, the three forces (H, F, and T) must be concurrent to be in equilibrium. The horizontal component of tension can be found by taking moments about point C.

$$\sum M_C = Fb - Hy = 0 \quad 43.58$$

$$H = \frac{Fb}{y} \quad 43.59$$

Also, $\tan\theta = y/b$. Therefore,

$$H = \frac{F}{\tan\theta} \quad 43.60$$

The basic equilibrium conditions can be applied to the free-body cable segment BC to determine the tension in the cable at point C.

$$\sum F_x = T_C\cos\theta - H = 0 \quad 43.61$$

$$\sum F_y = T_C\sin\theta - F = 0 \quad 43.62$$

The resultant tension at point C is

$$T_C = \sqrt{H^2 + F^2} \quad 43.63$$

42. CATENARY CABLES

If the distributed load is constant along the length of the cable, as it is with a loose cable loaded by its own weight, the cable will have the shape of a *catenary*. A vertical axis catenary has a shape determined by Eq. 43.64, where c is a constant and *cosh* is the *hyperbolic cosine*. The quantity x/c is in radians.[20]

$$y(x) = c\cosh\left(\frac{x}{c}\right) \quad 43.64$$

Referring to Fig. 43.20, the vertical distance, y, to any point C on the catenary is measured from a reference plane located a distance c below the point of greatest sag, point B. The distance c is known as the *parameter of the catenary*. Although the value of c establishes the location of the x-axis, the value of c does not correspond to any physical distance, nor is the reference plane the ground level.

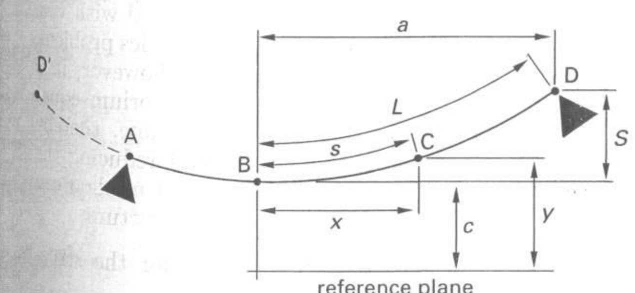


Figure 43.20 *Catenary Cable*

In order to define the cable shape and determine cable tensions, it is necessary to have enough information to calculate c. For example, if a and S are known,

[20] In order to use Eqs. 43.64 through 43.67, you must reset your calculator from degrees to radians.

Eq. 43.67 can be solved by trial and error for c.[21] Once c is known, the cable geometry and forces are determined by the remaining equations.

For any point C, the equations most useful in determining the shape of the catenary are

$$y = \sqrt{s^2 + c^2} = c\left(\cosh\left(\frac{x}{c}\right)\right) \quad 43.65$$

$$s = c\left(\sinh\left(\frac{x}{c}\right)\right) \quad 43.66$$

$$\text{sag} = S = y_D - c = c\left(\cosh\left(\frac{a}{c}\right) - 1\right) \quad 43.67$$

$$\tan\theta = \frac{s}{c} \quad 43.68$$

The equations most useful in determining the cable tensions are

$$H = wc \quad 43.69$$

$$F = ws \quad 43.70$$

$$T = wy \quad 43.71$$

$$\tan\theta = \frac{ws}{H} \quad 43.72$$

$$\cos\theta = \frac{H}{T} \quad 43.73$$

Example 43.13

A cable 100 m long is loaded by its own weight. The maximum sag is 25 m and the supports are on the same level. What is the distance between the supports?

Solution

Since the two supports are on the same level, the cable length, L, between the point of maximum sag and support D is half of the total length.

$$L = \frac{100\ \text{m}}{2} = 50\ \text{m}$$

Writing Eqs. 43.65 and 43.67 for point D (with S = 25 m),

$$y_D = c + S = \sqrt{L^2 + c^2}$$

$$c + 25\ \text{m} = \sqrt{(50\ \text{m})^2 + c^2}$$

$$c = 37.5\ \text{m}$$

Substituting a for x and $L = 50$ for s in Eq. 43.66,

$$s = c\left(\sinh\left(\frac{x}{c}\right)\right)$$

$$50 = (37.5)\left(\sinh\left(\frac{a}{37.5}\right)\right)$$

$$a = 41.2\ \text{m}$$

The distance between supports is

$$2a = (2)(41.2\ \text{m}) = 82.4\ \text{m}$$

[21] Because obtaining the solution may require trial and error, it will be advantageous to assume a parabolic shape if the cable is taut. (See Ftn. 19.) The error will generally be small.