Deep Partial Functional Maps for Shape Correspondence
Project Milestone

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Motivation

The problem of finding correspondence between 3D shape is one of the most fundamental prototypical problems in geometry processing. It is widely used in domains like autonomous cars, robotics, gaming, etc. Particularly challenging subtopics are non-rigid correspondence and partial correspondence. Where non-rigid correspondence is correspondence between deformed shapes, and partial correspondence is correspondence between shapes that have missing parts. Although there are multiple papers addressing this topic recently [2],[7], [8], the partial non-rigid correspondence in context of CNNs remains under-explored and is presented this project.

Method

The method is based on deep FMNet architecture for dense shape correspondence proposed in [1] and is presented on Figure 1. This method provides a state-or-art correspondence and transfer learning results. The issue of this approach is that it does not work well for partial correspondence problems due to Laplacian eigenfunctions used to construct functional maps. Namely, they are sensitive to topological changes. One way to solve this issue is to adapt approach of partiality priors described in [2].

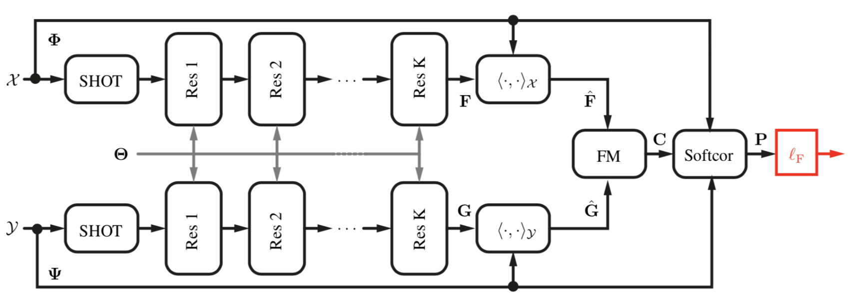

Figure 1 FMNet Architecture

In FMNet deep functional maps [4] are used to represent data points. The model structure is based on multi layered residual network with correspondence computation included directly as part of the learning procedure. The model takes point-wise descriptors (SHOT [5]) of two objects as an input and passes them through K identical residual blocks with shared weights to produce better descriptors $\mathbf{F}$ and $\mathbf{G}$. These descriptors are projected into Laplacian eigenbases resulting in spectral descriptors $\hat{\mathbf{F}}$ and $\hat{\mathbf{G}}$, which are passed as input to the functional map layer FM that produces the functional map matrix $\mathbf{C}$. The following layer calculates soft correspondence matrix $\mathbf{P}$ which is used to calculate Soft error loss. For formulas and details refer to [1].

The objective function for correspondence computation has a form of least squares:

$$min_C||\mathbf{C}\hat{\mathbf{F}} - \hat{\mathbf{G}}||_F^2 \tag{1}$$

where $\hat{\mathbf{F}} = (\langle \phi_i, f_j \rangle_{\mathcal{X}})$ and $\hat{\mathbf{G}} = (\langle \phi_i, g_j \rangle_{\mathcal{Y}})$, f_j and g_j are functions representing two shapes.

This objective has a closed closed form solution $\mathbf{C} = \hat{\mathbf{G}}\hat{\mathbf{F}}^{\dagger}$ ($\dagger$ is pseudo-inverse operation).

This formula works well with isometric shapes, however fails for non-isometric shapes as the diagonally dominant structure of $\mathbf{C}$ is broken. In [2] a new pair of bases $\hat{\phi_j}, \hat{\psi_j}$ were proposed to bring $\mathbf{C}$ back to near-diagonal structure, where

$$\hat{\phi_j} = \sum_{j=1}^{k} p_{ji}\phi_i, \qquad \hat{\psi_j} \sum_{j=1}^{k} q_{ji}\psi_i \tag{2}$$

This leads to different from 1 optimization problem:

$$\min_{Q \in S(k,r)} off(\mathbf{Q}^T \Lambda_{\mathcal{Y}} \mathbf{Q}) + \mu ||\hat{\mathbf{F}}_r - \mathbf{Q}^T \hat{\mathbf{G}}||_F^2 (3)$$

where $S(k,r)$ is the Stiefel manifold of orthogonal $k \times r$ matrices, $\hat{\mathbf{G}}_r = \mathbf{W}_r \mathbf{A}$ with $\mathbf{W}_r = (\mathbf{I}_{r\times r} \mathbf{0}_{r \times k-r})$ denotes the $r \times k$ matrix consisting of r rows of $\mathbf{A}$. $\mathbf{Q}$ is "quasi-harmonic" basis of $\{\hat{\psi}\}$. $\Lambda_{\mathcal{Y}}$ is diagonal matrix of the first k eigenvalues of $\triangle_{\mathcal{Y}}$, where $\triangle$ is the Laplacian.
The objective of our project is to blend equation (3) into FM layer of FMNet. The main difficulty in solving this problem is that it does not have closed form solution and is not easily bendable into CNN architecture. The optimization has to be done on the manifold S and, thus results in orthogonal constraint on $\mathbf{Q}$ ($\mathbf{Q^T Q} = \mathbf{I}$). Moreover, in the learning context we need to differentiate with respect to $\hat{\mathbf{F}}$ and $\hat{\mathbf{G}}$ in back propagation step. And when solution for $\mathbf{Q}$ is iterative this presents a challenge.
A conceptual gradient descent-like manifold optimization can be described as:

repeat
Compute the extrinsic gradient $\nabla f(X^{(k)})$
Projection: $\nabla_{\mathcal{M}} f(X^{(k)}) = P_{X^{(k)}}(\nabla f(X^{(k)}))$
Compute the step size $\alpha^{(k)}$ along the descent direction
Retraction: $X^{(k+1)} = R_{X^{(k)}}(-\alpha^{(k)} \nabla_{\mathcal{M}} f(X^{(k)}))$
until *convergence*;

Algorithm 1: Conceptual algorithm for smooth optimization on manifold $\mathcal{M}$.

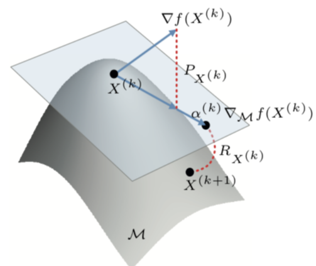

Figure 2 Conceptual algorithm for smooth optimization on manifold [3]

In the paper [2] this conceptual scheme was used to solve problem in equation (3) using Matlab manopt method. In order to blend this method into CNN it has to be fully differentiable. Recently, a state-of-art Cayley SGD algorithm was introduced by Li et al. [6]. The algorithm is based on a combination of momentum projection and the Cayley transform. It enforces orthogonality of CNN parameters, and thus can potentially be applied to our optimization objective. Remark: in Figure 3 X_k represents $\mathbf{Q}$ and $G(X_k)$ - gradient of equation (3)

Algorithm 1 Cayley SGD with Momentum

Input: learning rate l, momentum coefficient β, $\epsilon = 10^{-8}$, $q = 0.5$, $s = 2$.
Initialize X_1 as an orthonormal matrix; and $M_1 = 0$
for $k = 0$ **to** T **do**
  $M_{k+1} \leftarrow \beta M_k - \mathcal{G}(X_k)$, ▷ Update the momentum
  $\hat{W}_k \leftarrow M_{k+1} X_k^\top - \frac{1}{2} X_k (X_k^\top M_{k+1} X_k^\top)$ ▷ *Compute the auxiliary matrix*
  $W_k \leftarrow \hat{W}_k - \hat{W}_k^\top$
  $M_{k+1} \leftarrow W_k X_k$. ▷ *Project momentum onto the tangent space*
  $\alpha \leftarrow \min\{l, 2q/(\|W_k\| + \epsilon)\}$ ▷ *Select adaptive learning rate for contraction mapping*
  Initialize $Y^0 \leftarrow X + \alpha M_{k+1}$ ▷ *Iterative estimation of the Cayley Transform*
  for $i = 1$ **to** s **do**
    $Y^i \leftarrow X_k + \frac{\alpha}{2} W_k (X_k + Y^{i-1})$
  Update $X_{k+1} \leftarrow Y^s$

Figure 3 Cayley SGD with Momentum [6]

Preliminary Experiments

The first step of this project is more on the theoretical side and did not require many experiments. Namely, the main objective of the first step was to come up with optimization procedure on manifolds that is applicable to our task and can be incorporated into CNN architecture.
However, the project required adapting and running baseline model from [1] and setting up manifold optimization packages in Matlab (like manopt).
To produce resulting correspondence presented in Figure 4 I trained a baseline model on Faust Synthetic dataset for 1200 iterations. 80 shapes of 8 subjects were used for training and 20 shapes of 2 subjects for testing. This results support the statement made in [1]: the method is failing under extreme partiality and needs to be adapted.

Further Steps

The further steps include:

1. Implementation and gradient check of Cayley ADAM algorithm

2. Incorporate implemented algorithm into FMNet as a new FM layer

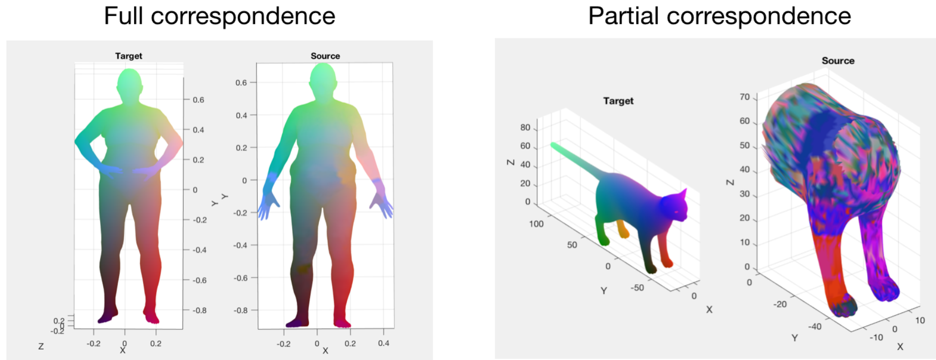

Figure 4 Baseline FMNet: correspondence between shapes

References

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