PHYS 414 Final Project

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This is the report of the Final Project of PHYS 414 by Volkan Işık, id :69018. The codes of the project can be found in github : https://github.com/volkanisk/PHYS414.git .

NEWTON

Part -a

Lane–Emden Equation

$$\frac{dm}{dr} = 4\pi r^2 \rho, \tag{1}$$

$$\frac{1}{\rho}\frac{dP}{dr} = -\frac{Gm}{r^2}, \tag{2}$$

Getting derivative of P with r again:

$$\frac{d}{dr}\left(\frac{1}{\rho}\frac{dP}{dr}\right) = \frac{2Gm}{r^3} - \frac{Gdm}{r^2 dr} \tag{3}$$

$$\frac{d}{dr}\left(\frac{1}{\rho}\frac{dP}{dr}\right) = \frac{2dP}{\rho r dr} - 4\pi G\rho \tag{4}$$

Multiplying with r^2 and colleting P on the left:

$$r^2\frac{d}{dr}\left(\frac{1dP}{\rho dr}\right) + \frac{2rdP}{\rho dr} = \frac{d}{dr}\left(\frac{r^2 dP}{\rho dr}\right) = -4\pi G r^2 \rho \tag{5}$$

Using $P = \rho_c^{1+\frac{1}{n}}\theta^n$ and $\rho = \rho_c\theta^n$, and dividing with r^2:

$$\frac{d}{r^2 dr}\left(r^2 K \rho_c^{\frac{1}{n}} (n+1)\frac{d\theta}{dr}\right) = -4\pi G \rho_c \theta^n \tag{6}$$

Gathering constants as $r = \alpha\xi$, where:

$$\alpha^2 = (n+1)K\rho_c^{\frac{1}{n}-1}/4\pi G \tag{7}$$

We have finally the Lane-Emden equation:

$$\frac{1}{\xi^2}\frac{d}{d\xi}\left(\xi^2\frac{d\theta}{d\xi}\right) + \theta^n = 0 \tag{8}$$

Here we have the Mathematica calculation:

Mass as Function of Radius

$$R = \alpha\xi_1 = \left[\frac{K}{G}\frac{n+1}{4*pi}\right]^{\frac{1}{2}}\rho_c^{\frac{1-n}{2n}}\xi_1 \tag{9}$$

$$M = \int_0^R 4\pi r^2 \rho\, dr \;= 4\pi\alpha^3\rho_c \int_0^{\xi_1} \xi^2\theta^n\, d\xi \tag{10}$$

$$M = 4\pi\alpha^3\rho_c \int_0^{\xi_1} -\frac{d}{d\xi}\left(\xi^2\frac{d\theta}{d\xi}\right) d\xi \tag{11}$$

$$M = 4\pi\left[\frac{K}{G}\frac{n+1}{4\pi}\right]^{\frac{3}{2}}\rho^{\frac{3-n}{2n}}\left[-\xi^2\frac{d\theta}{d\xi}\right]_{\xi=\xi_1} \tag{12}$$

Combining these two equations, we have:

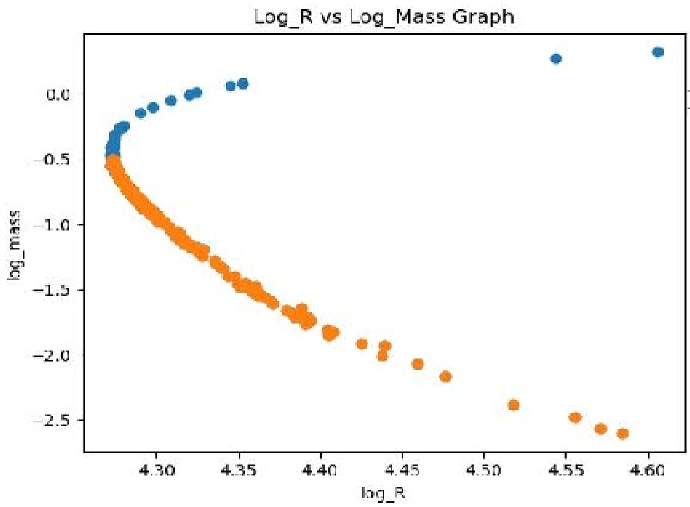
$$R^{\frac{3-n}{n}} M^{\frac{n-1}{n}} = \frac{K}{GN_n} \tag{13}$$

where

$$N_n = \frac{(4\pi)^{\frac{1}{n}}}{n+1}\left(\left[-\xi^2\frac{d\theta}{d\xi}\right]_{\xi=\xi_1}\right)^{\frac{1-n}{n}}\xi^{\frac{n-3}{n}} \tag{14}$$

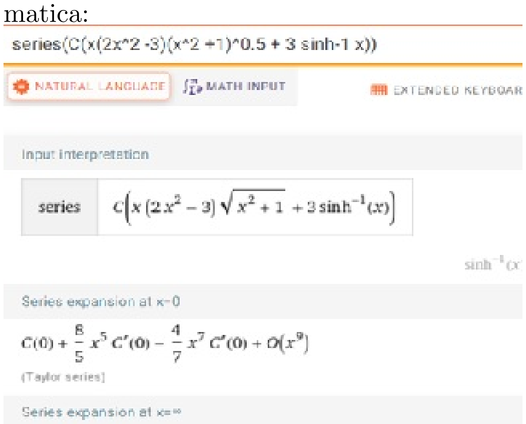
Part -b

Here is the logarith of radius vs logarith of mass graph:



Part -c

To get the leading part in expansion, I used Mathematica:



From there, derivation of the K_* and n_* is in below:

$$P = \frac{8C}{5}x^5 = K_*\rho^{1+\frac{1}{n_*}} \tag{15}$$

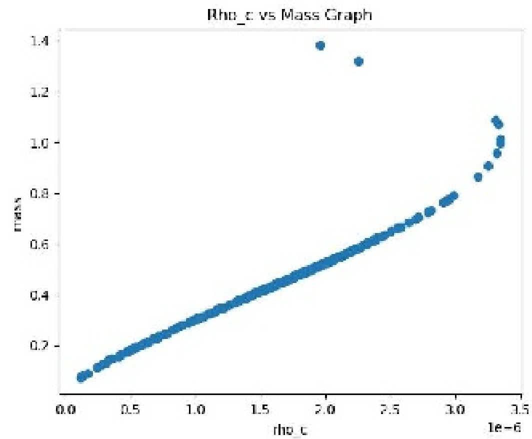
$$x = (\frac{q}{D})^{\frac{1}{q}} K_* = \frac{8C}{5D^{\frac{5}{q}}} \tag{16}$$

$$\rho^{\frac{5}{q}} = \rho^{1+\frac{1}{n_*}} \frac{1}{n_*} = \frac{5-q}{q} \tag{17}$$

Using python I found the n_*, q, and K_* values:

$$n_* = 1.2, q = 3, K_* = 8.047 * 10^{-6} \tag{18}$$

I also calculated the ρ_c vs M graph:



EINSTEIN

Part - a

In Part -a, I calculated the integrals. For the correct R value, I kept the R that gives 0 integral for pressure. I used equations (13) and (15).

Here is the M-R graph with changing ρ_c: