Kaye and Higgins 1926 *Proc. Roy. Soc.* A, **113**, 335.
Koenigsberger and Mühlberg 1911 *N. Jb. Min. Geol. Paläont.* Beilage **31**, 107.
Krige, L. J. 1939 *Proc. Roy. Soc.* A, **173**, 450.
Peirce, B. O. 1903 *Proc. Amer. Acad. Arts Sci.* **38**, 651.
Poole, H. H. 1914 *Phil. Mag.* **27**, 58.
Powell and Griffiths 1937 *Proc. Roy. Soc.* A, **163**, 189.
Sherlock, R. L. 1921 *Mem. Geol. Surv. U.K.* (spec. Publ. Min. Res.), **18**, 45.
Sverdrup 1935 *Geogr. Ann.* **17**, 53.

Borehole temperatures in the Transvaal and Orange Free State*

By L. J. Krige

(*Communicated by B. F. J. Schonland, F.R.S.—Received* 15 *July* 1939)

During the past few years numerous deep boreholes have been sunk in the search for the gold-bearing reefs of the Witwatersrand system, beneath thick covers of younger formations, in the southern Transvaal and adjoining parts of the Orange Free State. The opportunity for temperature determinations in these boreholes is being utilized by the Geological Survey of South Africa, but a considerable time will elapse before the investigations are completed and the results become available. It therefore seems desirable that the publication of Dr E. C. Bullard's conductivity determinations of representative rock samples from the two deepest boreholes in which the temperatures have been measured should be accompanied by an account of these investigations. The two boreholes, on the farms Gerhardminnebron No. 4 near Potchefstroom and Jacoba No. 878 near Bothaville, were selected for the conductivity determinations on account of their depth and also because of the exceptionally small amount of change in the stratigraphic columns. For the sake of comparison three other boreholes are included in the description.

In this paper the term "geothermic step" is used to indicate the increase in vertical depth corresponding to a rise in temperature of 1° C.

* Published by permission of the Minister of Mines.

Diamond drills with an external diameter of $2\frac{5}{16}$ in. (5·9 cm.) were used in all the boreholes, but near the surface the width was greater. Except in the Driefontein hole, where it is wider, the upper portion of each is now lined with $2\frac{1}{2}$ in. (6 cm.) casing.

All the holes investigated were filled with water till near the top. It is therefore necessary to consider whether the temperature of the water is not affected by convection currents. The thermal expansion of water increases with rising temperature, while the compressibility remains approximately constant. At 20° the pressure necessary to counterbalance the expansion caused by a rise of 1° is 5 atm., at 30° it is 7 atm., and from 40 to 50° it is 11 atm., corresponding to columns of water 170, 240 and 370 ft. (50, 70 and 110 m.) high, respectively. Where the geothermic step is greater than these heights the density of the water increases downward. This is the case in all the holes examined down to about 1000 m. (3000 ft.), but at greater depths, with temperatures above 30°, the geothermic step is sometimes smaller than the height of the water column necessary to counteract the expansion, so that there is a downward decrease in density. Contrary to what at first sight might be expected an increase of density downwards is not a sufficient condition for stability. In the absence of viscosity the critical gradient above which convection occurs is (Jeffreys 1929, p. 138). $g\alpha T/c_p$ where g is the acceleration due to gravity, T is the absolute temperature, α is the coefficient of expansion and c_p is the specific heat at constant pressure. If viscosity is present Hales (1937) has shown that the critical gradient is increased by $C\nu\kappa/g\alpha a^4$ where ν is the kinematical viscocity, κ is the thermometric conductivity, a is the radius of the tube, and C is a numerical constant which has the value 216 for a tube whose length is great compared to its diameter. For water in a tube with a diameter of $2\frac{5}{16}$ in. the constants are:

Temp.	25	50	75	100° C	
α	2·6	4·5	6·0	7·0	$\times 10^{-4}$ ° C^{-1}
κ	1·49	1·56	1·64	1·71	$\times 10^{-3}$ cm.²/sec.
ν	8·9	5·6	3·9	2·9	$\times 10^{-3}$ cm.²/sec.
c_p	4·2	4·3	4·4	4·6	$\times 10^{7}$ ergs/° C g.
$C\nu\kappa/g\alpha$	11·2	4·3	2·3	1·56	$\times 10^{-3}$ ° C/cm.³
$C\nu\kappa/g\alpha a^4$	15·1	5·8	3·2	2·1	° C/km.
$g\alpha T/c_p$	0·2	0·3	0·5	0·6	,,
Critical gradient	15·3	6·1	3·7	2·7	,,

From these data it is concluded that the Driefontein and Doornkloof bores are stable at all depths at which temperatures have been taken, Gerhardminnebron is stable down to about 7000 ft. and unstable below, and that Jacoba and Doornhoutrivier are stable down to about 3000 ft. and unstable below. The Reef Nigel bore discussed in the accompanying paper by Bullard is stable, and the Dubbeldevlei one unstable. In spite of the resulting unstable equilibrium, the smallness and the regularity of the changes in the geothermic step, in the same rock, show that convection currents are not disturbing the temperatures. Indeed, van Orstrand (1924*a*) has not found evidence of convection in boreholes when the diameter is less than 8 in. (20 cm.), which is more than three times as much as it is in the cases under consideration.

In order that the true rock temperatures might be acquired by the water the determinations were not begun until at least two months after drilling had stopped. On only one occasion, at the bottom of the Gerhardminnebron hole, this was not possible, and the value of the results, obtained after 16 days, is therefore reduced.

The temperatures were recorded by maximum mercury thermometers, six of which were enclosed in a strong brass tube, with an internal diameter of $\frac{3}{4}$ in. (19 mm.) and an external diameter of $1\frac{1}{4}$ in. (31 mm.). Pipe-joint composition was used to make the screw plugs water-tight. The container was lowered by means of a 20-gauge piano wire and a hoisting apparatus made according to van Orstrand's specifications (1924*b*). At greater depths than about 5000 ft. (1500 m.) the apparatus began to give way under the hoisting strain. Both the winding drum and the frame were therefore strengthened sufficiently to bear the extra tension of the wire.

The thermometers have straight stems, about 22 cm. long, with a constriction above the bulb. They are graduated in fifths of a degree C, but have two different ranges, from 0 to 60° and from 36 to 100°, respectively. The former were used below 37 or 38°, the latter for higher temperatures. As long as the mercury columns were short all the thermometers functioned satisfactorily, but when they reached a length of 8 or 10 cm. the mercury frequently ran down before the readings were taken, in spite of great care in raising and handling the instruments. In order to reduce the length of the mercury column, and the consequent danger of its running down, a third set of thermometers was acquired, graduated in tenths of a degree, from 18 to 45° C. These were used in the Jacoba borehole.

By means of a reading lens all temperatures were estimated to the second decimal. The necessary corrections were made for the errors of the instruments, as well as for the differences between the temperature registered and the temperature of the water in which they were immersed immediately before reading.

After an exposure of 3 hr. in water in a borehole the mean temperature of the six thermometers was found to be the same, to the second decimal place, as after an immersion of 10 hr. Three hours was therefore considered to be long enough for an exposure in water, while in air, in the upper portions of the holes, overnight exposures of 8 or 9 hr. were made.

1. The Jacoba borehole

This borehole, Jacoba No. 3, on the farm Jacoba No. 878, in the Bothaville District of the Orange Free State, is one of the deepest in the country. It is situated about 30 miles (50 km.) south-south-west of Klerksdorp, at an altitude of approximately 4300 ft. (1300 m.) in flat country. Except small thicknesses of Karroo sandstone and shale at the top and of Witwatersrand quartzite at the bottom of the hole, it passes through nothing but lavas of the Ventersdorp system, which are here 6900 ft. (2100 m.) thick.

The upper portion of the lava, from 157 to 1241 ft. (48–378 m.) is described as medium to dark-greenish grey and dark grey to dark or very dark, fine-grained and fine- to medium-grained crystalline lava. It includes about twice as much amygdaloidal as non-amygdaloidal rock, but the two varieties alternate in rapid succession, the maximum continuous thickness attained by each being only about 40 ft. (12 m.). Often the amygdales occur in thin scattered swarms. They usually consist of quartz and chloritic material, but sometimes of chalcedony or calcite. The chloritic amygdales are always small, the others generally large and irregular in shape. Scattered felspar phenocrysts were recorded at one horizon.

From 1277 to 2795 ft. (389–846 m.) there is felspar-quartz-porphyry, consisting of scattered or closely-packed phenocrysts of felspar and quartz, up to $\frac{1}{4}$ in. (6 mm.) across, in a dense greyish, greenish or brownish matrix, partly tuffaceous.

Below the porphyry the lava is very similar to that above it, but there are small differences. The colour is somewhat lighter, varying from light grey and light greenish grey, through greenish grey, medium grey and lead grey, to dark greenish grey, and dark slaty grey. The amygdaloidal and non-amygdaloidal varieties occur in approximately equal proportions, with a slight preponderance of the latter, which on one occasion continues for 212 ft. (65 m.). The amygdales again consist chiefly of chlorite and quartz, sometimes of chalcedony and less commonly of calcite. Those of chlorite are always small or very small, and often indistinct, while the others are mostly larger, and occasionally up to 2 in. (5 cm.) across.

The casing extends downward through the Karroo beds and weathered lava. At the time of the temperature measurements the water level in the hole was 125 ft. (38 m.) below the surface.

TABLE 1. JACOBA BOREHOLE (NO. 3), BOTHAVILLE DISTRICT

Altitude ... ... about 4300 ft. (1300 m.)
Drilled ... ... 9. iii. 37–7. i. 38
Temperatures measured ... 13. xi. 38–30. xi. 38

Observation no.	Geological section (borehole depths in feet)	Dip degrees	Borehole depth (distance drilled) ft.	Vertical depth below collar of borehole ft.	Vertical depth below collar of borehole m.	Temperature °C	Geothermic step (per °C) ft.	Geothermic step (per °C) m.
1	Overburden		50	50	15	19·50		
	32———							
	Karoo sandstone						(500 ± 90	150 ± 30)
2	113———		100	100	30	19·60 ± 0·01		
	Karoo shale							
	135———		(Water-level	125	38)		220 ± 10	69 ± 4
	Weathered lava							
3	157———		157	157	48	19·86		
		20?					345 ± 5	105 ± 2
20	Ventersdorp lava,		437	437	133	20·67 ± 0·01		
	amygdaloidal						257 ± 5	78
4	and non-		717	717	218	21·76 ± 0·01		
							255 ± 4	78
21	amygdaloidal		997	997	304	22·86		
							274 ± 3	83
	1206———							
	Sandy tuff and agglomerate							
5	1277———		1277	1276	389	23·88		
							287	87

6 and 22	Felspar-		1783	1779	542	25·63		
							272 ± 3	83
23	quartz-		2092	2086	636	26·76		
	porphyry						264 ± 4	80
7 and 24	(Ventersdorp)		2289	2281	695	27·50		
		20–25					263	80
8	2795———		2795	2776	846	29·38 ± 0·01		
		23–30					264	81
9	Ventersdorp lava,		3274	3244	989	31·15		
		30					258	79
	amygdaloidal							
10	and		3753	3713	1132	32·97		
		22–28					238	72
11	non-amygdaloidal		4232	4181	1274	34·94		
		20					237 ± 6	72 ± 2
12	4711———		4711	4653	1418	36·93 ± 0·04		
	Lava, mostly tuffaceous	15–30					238 ± 6	73 ± 2
13	5190———		5190	5130	1564	38·93		
	Ventersdorp lava,	15–30					226	69
14	amygdaloidal		5669	5607	1709	41·04 ± 0·01		
		11–21					218 ± 3	67
15	and		6148	6083	1854	43·22 ± 0·02		
		30					217 ± 3	66
16	non-amygdaloidal		6627	6560	1999	45·42		
							222	68
17	7102———		7102	7032	2143	47·55		
	Witwatersrand quartzite	9–17					380 ± 10	117 ± 2
18	and		7330	7259	2213	48·15		
	conglomerate	10–27					380 ± 30	115 ± 10
19			7383	7312	2229	48·29		
	Bottom of hole		7458					

Observation 1 was made with only three thermometers, graduated in tenths of a degree from 18 to 45°. In all other determinations down to 4181 ft. (1274 m.) the same six of these instruments were used, except one which proved unsatisfactory and was replaced by another from observation 5 onwards. In observation 12 there were three such thermometers and three others, graduated from 36 to 100° in fifths of a degree. From observation 13 downwards there were six of the latter type, two which were broken in observation 15 being replaced by two others.

The agreement between the individual readings of the different observations is generally so close that the probable error is less than a unit in the second decimal place. But in observations 2, 4 and 14 it is 0·01°, while in observation 15 it is 0·02° and in observation 12, on account of the change of thermometers, it has increased to 0·04°.

In observations 8 and 20 the exposures were shorter than 3 hr., but longer than $2\frac{3}{4}$ hr., for which reason a probable error of 0·01° is assumed.

On account of a miscalculation, which was subsequently discovered and corrected, observations 6 and 7 were repeated twelve days after the original determinations. The results were identical although the times of exposure differed considerably, being 10·8 hr. for observation 7 and 3·1 hr. for observation 24. This seems to show that the temperatures are generally reliable to the second decimal place. In table 1 all the temperatures are given to the second decimal, with the addition of the probable error when it is not less than 0·01°.

There is considerable variation in the order of accuracy attained in the calculation of the geothermic step. It depends upon the precision with which the depths and the temperatures have been determined, and also upon the increments in depth and temperature in two successive sections of the borehole. In the table it is generally given to the nearest unit, but sometimes, at the top and bottom of the hole, to the nearest 5 units. The probable error is always indicated when larger than unity.

The greatest deviation of the borehole from the vertical is 12°, giving a horizontal displacement of 55 ft. or 16 m. in the geothermic step. This deviation is towards the south, in which direction the ground rises at the rate of one in 200. The effect of the displacement on the geothermic step is therefore negligible.

The first temperature determination, at the depth of 50 ft. (15 m.), is above the limit to which the annual temperature wave descends. The geothermic step in the following section of the hole is therefore a variable quantity, and the calculated result is one of several different values which occur during the year. Its large size seems to be chiefly due to this cause, as the comparatively good thermal conductivity of the sandstone must be reduced because it lies above the ground water.

In the next section the geothermic step is much smaller, corresponding to the poor conductivity of the shale, most of which is above the water table, and of the weathered lava.

From 157 to 1276 ft. (48–389 m.), in the Ventersdorp lava, the geothermic step has its highest value in the first section, with a large decrease in the second and a smaller decrease in the third, followed by an increase in the fourth section. The range is from 345 to 255 ft. (105–78 m.) and the average 278 ft. (85 m.).

In the succeeding felspar-quartz-porphyry the geothermic step is at first larger than in the preceding lava and then decreases downward more and more slowly, from 287 to 263 ft. (87–80 m.), the mean being 273 ft. (83 m.).

Below the porphyry the lava again shows a slow but almost continuous decrease in the geothermic step, from 264 to 217 ft. (81–66 m.), till near its base, where there is a slight increase. The average value is 234 ft. (71 m.). The average for all the Ventersdorp lava, from 157 to 7032 ft. (48–2143 m.), is 248 ft. (76 m.).

The average values of the geothermic step in the upper lava, quartz-felspar-porphyry and lower lava, respectively, are almost exactly proportional to the mean conductivities found in these rocks by Bullard (1939). In each of these three portions of the borehole there is also a distinct decrease in the geothermic step with increasing depth, which is not accompanied by a corresponding decrease in the conductivity determinations, except to a slight extent in the lower lava. This effect is probably partially due to a decrease in conductivity with rising temperature, and partially to another cause, possibly connected with climatic changes.

In the Witwatersrand quartzite (and conglomerate) the geothermic step is much greater than in the lavas, on account of the better conductivity.

2. Doornhoutrivier

Borehole No. 1 on the farm Doornhoutrivier No. 25, in the Bothaville District, is located 3·5 miles (5·6 km.) north of the Jacoba hole, its altitude being 50 or 100 ft. (15 or 30 m.) less. The ground is flat, sloping gently towards the Vaal river in the north. The hole entered Ventersdorp lava 25 ft. (8 m.) below the surface and passed out of it at 5665 ft. (1727 m.). The casing extends down to about 50 ft. (15 m.).

The lava varies somewhat in appearance. It is generally dense or finely crystalline, rarely medium- to coarse-grained. Amygdales are usually

TABLE 2. DOORNHOUTRIVIER BOREHOLE (NO. 1), BOTHAVILLE DISTRICT

Altitude ... ... about 4200 ft. (1300 m.)
Drilled ... ... 21. vii. 36–10. ii. 37
Temperatures measured ... 12. ii. 37 (observation 0) and 5. v. 37–8. v. 37

Observation no.	Geological section (borehole depths in ft.)	Dip degrees	Borehole depth (distance drilled) ft.	Vertical depth below collar of borehole ft.	Vertical depth below collar of borehole m.	Temperature °C	Geothermic step (per °C) ft.	Geothermic step (per °C) m.
	Overburden		Water-level	45	14			
	25							
9	Ventersdorp lava,		50	50	15	20·47 ± 0·02		
		3–35					600 ± 400	200 ± 100
4	amygdaloidal		100	100	30	20·56 ± 0·02		
		30					585 ± 50	180 ± 15
1	and		499	499	152	21·24 ± 0·03		
							245 ± 5	75 ± 2
2	non-amygdaloidal		998	998	304	23·27 ± 0·02		
							290 ± 5	89
3			1496	1496	456	24·98 ± 0·01		
	Lava + 8 % agglomerate	0–55					264 ± 3	80
5			1995	1995	608	26·87 ± 0·01		
	Lava							
		0–35					258 ± 3	79

6	———		2494	2494	760	28·80 ± 0·01		
	Amygdaloidal lava	15–40					250 ± 5	76 ± 2
7	———		2992	2991	912	30·79 ± 0·03		
	Lava + 4 % tuff + 9 % dykes	10–33					255 ± 5	78
8	———		3491	3490	1064	32·74		
	Lava	15–45					252 ± 2	77
10	———		3990	3989	1216	34·72 ± 0·01		
	Lava + 9 % quartzitic tuff*	6–45					250 ± 10	76 ± 3
11	———		4549	4547	1386	36·96 ± 0·07		
		9–34					225 ± 10	69 ± 3
12	Lava		5107	5104	1556	39·42 ± 0·03		
		0–35					224 ± 4	68
13	5665———		5664	5657	1724	41·89 ± 0·01		
	Witwatersrand quartzite†	9–24					385 ± 10	117 ± 3
14			5999	5990	1826	42·76 ± 0·01		
		10–16						
0			6025	6016	1834	42·91 ± 0·03		
	Bottom of hole		6036					

* Quartzitic tuff = tuffaceous quartzite with lava fragments and brecciated lava in quartzitic matrix.
† The quartzite contains a few scattered shaly and micaceous partings.

present, the amount of non-amygdaloidal lava being about one-third of the total, but varying from 0 to 50 % in different sections. The colour is grey in different shades, from light grey or light greenish grey to medium grey or greenish grey and dark grey. In the first 1000 ft. (300 m.) dark grey is dominant, but in the next 2000 ft. (600 m.) light shades hold sway, while in the lower portion the balance is fairly even between these two and the intermediate colours are more in evidence. Small thicknesses of agglomerate, tuff, and tuffaceous quartzite are interbedded in the lava at several horizons and two steep dykes occur close together at about 3100 ft. (940 m.).

Only three thermometers were sent down at a time, except in observation 11, where there were six. From observations 0 to 11 the same three, graduated from 0 to 60°, were used. In observation 11 there were three additional thermometers, graduated from 36 to 100°, while in the subsequent determinations these three were used alone.

One of the thermometers gave unsatisfactory readings in observations 0, 1, 2 and 4, on account of the mercury running down, and another failed in a similar way in observation 12, neither of them being replaced. In these five determinations therefore, the accepted temperatures are the mean values of only two readings, and the probable errors are equal to half the differences between the two readings. When the adopted result is the mean of three readings the probable error is assumed to be a quarter of the difference between the highest and lowest. In observation 11 the mean temperature of the three new thermometers differed by 0·10 from the mean of the three old ones, the probable error being increased by 0·05°.

Some exposures (in observations 1, 2, 5, 7 and 10) were shorter than 3 hr. by varying amounts up to 17 min. This deficiency is allowed for by adding 0·01° to the probable errors. In observation 11 the exposure lasted only 2 hr. 32 min., and 0·01° is added on this account to the probable error and also to the temperature. Except in one case, when it is negligible, the probable error is always shown in table 2.

Observation 0 was made only two days after drilling had stopped. The result does not therefore indicate the undisturbed temperature of the rock, being apparently much too high, and cannot be used to obtain the true geothermic step.

Between the depths of 50 and 100 ft. (15 and 30 m.) the temperature differences are so small and the probable error so large that the geothermic step is calculated to the nearest 100 units. Its exact value is of no great consequence, as it is subject to seasonal fluctuations in this section. In all other cases it is given to the nearest unit or 5 units, according to the size of the probable error, which is always shown in table 2, when greater than unity.

In the first three sections below 100 ft. (30 m.) the geothermic step shows large variations. These were apparently caused by a slow, but continuous, stream of gas bubbles, issuing from the water, which stood at 45 ft. (14 m.) below the surface. It is not known at what depth the gas enters the hole, but the differences in the geothermic step above and below 499 ft. (152 m.) and 998 ft. (304 m.), respectively, suggest that it happens near these depths. The water used in drilling stopped overflowing at the top and fell approximately to its present level after 1050 ft. (320 m.) was reached, but the actual depth of the leak was not found, as the core did not indicate any cavity. It is probable that the gas enters where the water flowed out of the hole, at about 1000 ft. (300 m.), and that the temperatures of the water above this point have been affected by the rising bubbles and therefore do not represent the true rock temperatures.

Between 998 ft. (304 m.) and the base of the lava there is an almost continuous slow decrease in the geothermic step, from 290 to 224 ft. (89–68 m.), the average being 250 ft. (76 m.). This figure agrees with the average value for the Ventersdorp lava in the Jacoba borehole (248 ft. = 76 m.). The effects of the small amounts of other volcanic and dyke rocks included in the lava are not apparent. Nor do the colour and the structure of the lava seem to have any marked influence.

In the Witwatersrand quartzite the geothermic step increases to 385 ft. (117 m.), which is almost identical with its value in the Jacoba borehole.

3. Gerhardminnebron

Borehole G.M.B. 1 on the farm Gerhardminnebron No. 4 in the Potchefstroom District of the Transvaal is situated 17 miles (28 km.) north-north-east of Potchefstroom in undulating country, at the altitude of 4987 ft. (1520 m.). It is by far the deepest hole in South Africa, the depth drilled being 10,718 ft. (3266 m.) and the vertical depth below the collar 9989 ft. (3045 m.). It was subsequently deflected at 9900 ft. (3017 m.), the observations (1, 2 and 3) below this depth being made immediately before the deflexion and commencing sixteen days after drilling had temporarily ceased. The other determinations were begun 72 days after the borehole was finally completed.

The standard of accuracy of the temperature measurements is not uniform. In table 3 the results are all given to the second decimal, the

TABLE 3. GERHARDMINNEBRON BOREHOLE (G.M.B. 1), POTCHEFSTROOM DISTRICT

Altitude ... ... 4987 ft. (1520 m.)
Drilled ... ... 18. xii. 35–25. i. 38, Deflexion 16. ii. 38–1. iii. 38
Temperatures measured ... 9 and 10. ii. 38 and 12. v. 38–24. v. 38

Observation no.	Geological section (borehole depths in feet)	Borehole depth (distance drilled) ft.	Vertical depth below collar of borehole ft.	Vertical depth below collar of borehole m.	Temperature ° C	Geothermic step (per ° C) ft.	Geothermic step (per ° C) m.
	Soil and boulders						
5	47	99	99	30	19·04 ± 0·01		
	Chert, clay and broken dolomite with cavities					250 ± 10	76 ± 3
25		346	346	105	20·03 ± 0·02		
		Water-level	349	106			
26		350	350	107	20·03 ± 0·03		
	440						
	Dol. + 11 % ch.					780 ± 100	235 ± 30
4		597	597	182	20·35 ± 0·01		
	Dol. + 9 % ch.					440 ± 15	133 ± 5
6		1096	1096	334	21·49 ± 0·03		
	Dol. + 4 % ch.					385 ± 15	118 ± 4
7		1595	1594	486	22·78 ± 0·01		
	Dol. + 7 % ch. ± 1 % carb. sh.					380 ± 10	116 ± 3
8		2094	2092	638	24·09 ± 0·03		
	Dol. + 6 % ch. + 2 % carb. sh.					385 ± 15	117 ± 5
9		2592	2589	789	25·38 ± 0·02		
	Dolomite					360 ± 15	110 ± 4
10		3091	3086	941	26·76 ± 0·03		
	Dol. + 1 % ch. + 2 % carb. sh.					360 ± 15	108 ± 4
11		3644	3637	1108	28·30 ± 0·02		
	Dol. + 6 % diabase					310 ± 5	95 ± 2
12	4181	4180	4170	1271	30·04 ± 0·01		
	B.Q. + 12 % sh. + 3 % calcite					470 ± 190	140 ± 60

13	4249	4247	4236	1291	$30{\cdot}18 \pm 0{\cdot}03$		
	Witwatersrand Quartzite					470 ± 20	143 ± 5
14		4747	4733	1443	$31{\cdot}24 \pm 0{\cdot}01$		
	Q. + 4 % sh.*					430 ± 10	131 ± 2
15		5246	5232	1595	$32{\cdot}40 \pm 0{\cdot}01$		
	Quartzite					435 ± 10	133 ± 2
16		5806	5791	1765	$33{\cdot}68 \pm 0{\cdot}01$		
	Q., 34 % felsp.					300 ± 5	91 ± 2
17		6323	6306	1922	$35{\cdot}40 \pm 0{\cdot}03$		
	Felsp. Q. + 5 % dyke + 16 % sh.*					215 ± 5	65 ± 2
18		6860	6834	2082	$37{\cdot}86 \pm 0{\cdot}04$		
	Q., 4 % felsp.					330 ± 15	101 ± 4
19		7387	7345	2239	$39{\cdot}41 \pm 0{\cdot}02$		
						330 ± 10	100 ± 3
20	Quartzite	7912	7837	2389	$40{\cdot}91 \pm 0{\cdot}03$		
						340 ± 10	103 ± 3
21		8437	8298	2529	42·27		
	Q. + 16 % dykes					340 ± 5	104
22		8937	8713	2656	43·49		
						365 ± 5	111
23	Quartzite	9435	9103	2775	44·56		
						355 ± 5	107
24		9904	9449	2880	45·54		
3		9904	9449	2880	$45{\cdot}18 \pm 0{\cdot}01$		
	Q., 17 % felsp. + 1 % dyke + 1 % quartz					375 ± 20	114 ± 5
2		10484	9841	2999	$46{\cdot}22 \pm 0{\cdot}03$		
	Felsp. Q. + 31 % dyke					375 ± 100	115 ± 30
1		10605	9916	3022	$46{\cdot}42 \pm 0{\cdot}01$		
	Bottom of hole	10718					

Abbreviations

Dol. = Dolomite.
ch. = chert.
felsp. = felspathic.
sh. = shale.

B.Q. = Quartzite of Black Reef series.
Q. = Quartzite of Witwatersrand system.
carb. sh. = carbonaceous shale.

* 4 % sh. = 12 % quartzite with shale bands, and 16 % sh. = 11 % shale + 16 % quartzite with shale bands.

reliability of this figure being indicated by the size of the probable error, which is always included unless it is smaller than 0·01°.

Observations 1, 2 and 3 were made with (the same) three thermometers, because the usual container had to be replaced by another which held only three. The agreement of the readings was satisfactory, the probable error being 0·01° or 0·02°. In all the determinations above 6500 ft. (2000 m.) the same six thermometers, graduated from 0 to 60°, were used. Of the six instruments two always gave consistent readings, except on one occasion, in observation 25. The temperatures indicated by the other four were often too low, on account of the running down of the mercury. It has therefore been decided to discard the readings of the four defective thermometers, so that the accepted temperatures are the mean values of only two readings, with half the difference as the probable error. In observation 25 five thermometers were in fairly good agreement, but the temperature recorded by the sixth was much lower. This instrument, the mercury of which had apparently run down, was one of the two which had not failed before. Its reading is therefore discarded and the mean of the other five accepted. This is justifiable, as in the following determination (observation 26) all the thermometers were in good agreement and, moreover, the mean of the two which had previously proved reliable was identical with the mean of the others.

In observations 18 and 19 there were three of the old thermometers and three others graduated from 36 to 100°. The former included the two instruments which had not failed in previous determinations, and the accepted temperatures were obtained from the mean readings of these two and of the three new thermometers. The next five observations were made with the same six thermometers, all of the new kind and including the three used in observations 18 and 19. The large probable errors in these two and in the following observation are due to the introduction of the new thermometers. Subsequently they are less than 0·01°, and the mean results are considered to be accurate to the second decimal place.

In five observations (2, 8, 10, 20 and 26) the exposure was shorter than 3 hr., although never by more than 22 min. The error due to this cause can hardly be greater than 0·01°, and this amount has therefore been added to the probable error.

The geothermic step is calculated to the nearest 5 ft. and the nearest metre, with a few exceptions, and the probable error is always shown when greater than unity.

The borehole is situated in the valley in the Gatsrand, a range of hills in which the highest points rise about 600 ft. (180 m.) above the surrounding country. It is therefore necessary to consider the influence of the topography on the geothermic gradient. Lees (1910) has investigated the effect of a single range and worked out an equation which can be applied to any range of uniform height and shape. The chief topographical features at Gerhardminnebron are two parallel ridges rising about 400 ft. (120 m.) above the intervening valley and the surrounding plain. At half its height each may be taken to be about 1600 ft. (480 m.) wide. One ridge is built up of dolomite and chert, and the other of shale and quartzite, which overlie the dolomite and dip away from the valley at an angle of about 12°. The borehole site is on the slope of the former ridge, less than 100 ft. (30 m.) above the bottom of the valley.

If the effect of the dolomite ridge on the temperatures in the borehole be reckoned at a quarter of what it is vertically below the crest, then it should cause an increase of 2 % in the geothermic step between the depths of 99 and 340 ft. (30 and 105 m.), a *decrease* of 1 % between 350 and 597 ft. (107 and 182 m.) and an increase of ½ % between 597 and 1096 ft. (182 and 334 m.). The disturbance diminishes downward and need therefore not be considered at greater depths. It is always positive except in the one case mentioned. On account of its greater distance the second ridge must have a smaller effect than the first. The combined effect of the two ridges thus appears to be less than 4, 2 and 1 %, respectively, in the first three sections of the borehole, where the probable errors in the calculated geothermic step are 4, 13 and 3 %, respectively. Except in the first section the influence of the topography may therefore be neglected. This result is in agreement with Bullard's conclusion that the correction for topography need only be applied in very mountainous country (1938, p. 362). It should be emphasized, however, that this correction is unusually small in the present case, on account of the uncommonly large geothermic step. The disturbance is proportional to the difference between the geothermic gradient and the mean thermal gradient (lapse-rate) in the air, which is regarded as constant. Hence a low geothermic gradient (large geothermic step) must have a small effect, which may even vanish or become negative in extreme cases, as in the second section of this hole.

The decrease of the disturbance due to the topography with increasing depth means that in the deeper parts of the hole the isogeotherms must be very nearly horizontal. The inclination of the borehole, which is not considerable except near the bottom, could therefore hardly have introduced any appreciable errors in the calculation of the geothermic step. It is, nevertheless, indirectly responsible for another error which cannot always be neglected. The deviation from the vertical increases downwards, until it is over 50° at the bottom. As the inclination was measured every 500 ft. (150 m.), it is not known at intermediate points with sufficient accuracy to determine the exact vertical depth below the collar in the deepest parts of the hole. The error which this uncertainty introduces into the geothermic step is generally not large, being estimated at less than 3 ft. or a metre above the deflexion, but at 5 ft. or 2 m. between observations 3 and 2, and at 15 ft. or 5 m. between observations 2 and 1.

The geothermic step in the dolomite above the water table is comparatively small, as would be expected in dry and somewhat broken rocks. In the first section below the ground-water level it is abnormally large, probably as a result of the infiltration of rain water.

In the succeeding sections of the borehole the geothermic step is at first much smaller, after which it decreases more slowly, then remains approximately constant and subsequently again decreases at an increasing rate. Small irregularities are due to the presence of chert, which causes an increase, and of shale and diabase, which have the opposite effect, but these disturbances are comparatively insignificant. The large decrease, from 780 to 310 ft. (235–95 m.), and the changes in the rate at which it takes place must be due to other causes. Of these the circulation of water can be regarded as the most important, because, although there is usually a downward decrease in other rocks also, it is very much smaller than in the dolomite, which alone is suitable for circulation on a large scale. The average value below the water table is 382 ft. (116 m.).

After an unreliable value in the Black Reef series, the geothermic step in the succeeding quartzites of the Witwatersrand system is much larger than in the last dolomite section. It then again decreases with depth, but a minimum is soon reached and subsequently there is an almost continuous increase. In the first three sections, from 4236 to 5791 ft. (1291–1765 m.), where the quartzite includes only small thicknesses with shale bands, it ranges from 470 to 430 ft. (143–131 m.) and averages 445 ft. (136 m.). It drops to 300 ft. (91 m.) in the next section which contains about 34 % of felspathic quartzite. There is a further drop to 215 ft. (65 m.) in the following section, where all the quartzite is felspathic and considerable amounts of shale as well as dyke rocks* are included. From 6834 to 9449 ft. (2082–2880 m.) chloritoid is often an essential component of the quartzite, but no other rocks are present in appreciable amount, except in one section where there are two igneous bodies, thought to be dykes. They are without

* Mr R. Borchers of the African and European Investment Company states that some of the dyke rocks in this hole are very largely silicified. This accounts for the high conductivities found by Dr Bullard.

evident effect upon the geothermic step, which increases almost continuously downward, but has a comparatively small range, from 330 to 365 ft. (100–111 m.). It averages 340 ft. (104 m.), and is thus considerably less than in the higher sections.

In the first three observations near the bottom of the hole, the thermometers were lowered by means of a steel cable, with its own measuring wheel, and it is not known how closely the depths recorded agree with those of the piano wire. It is therefore possible that there is an appreciable difference between the depths of observations 3 and 24, but it could hardly be large enough to account for the whole temperature difference of 0·36°. On the other hand, the observations below the deflexion were made only 16 days after drilling had stopped, which was probably not long enough for the temperatures to return to normal, so that the results obtained cannot be accepted as the true rock temperatures. For this reason the geothermic step in the two deepest sections of the hole must be regarded as unreliable. It is, however, only slightly larger than in the preceding two sections, in agreement with the general tendency in the lower quartzites to increase slowly downward.

If there is thermal equilibrium then the geothermic step should be proportional to the mean conductivity in each section of the borehole. As the conductivity determinations in the quartzites are not sufficiently numerous to determine such mean values, it is not known whether this proportionality exists or whether the equilibrium has been disturbed.

4. Driefontein

Borehole E 3 on the farm Driefontein 118 in the Potchefstroom District, is situated 26 miles (42 km.) south-west of Krugersdorp and 17 miles (27 km.) east-north-east from the Gerhardminnebron site, at the altitude of 5123 ft. (1562 m.) in gently sloping country. It is 4129 ft. (1258 m.) deep, but the thermometers could only be lowered down to 1926 ft. (587 m.) below which depth there was an obstruction where the diameter diminishes. The top part of the hole is lined with 10 in. (25 cm.) casing. The dolomite extends down to the depth of 2563 ft. (781 m.). It contains thin beds and partings of carbonaceous shale and thin bands of chert, unevenly distributed. As the thicknesses of these rocks did not seem sufficient to cause large disturbances, the proportions were not determined.

The same three thermometers were used for each observation and the readings were always satisfactory. In two determinations the exposures were shorter than 3 hr., by 23 and 25 min., respectively, and 0·01° has therefore been added to the probable error.

The geothermic step is given to the nearest 5 ft. and 5 m. In the first section, which lies partly above the water table and includes some broken

Table 4. Driefontein borehole (E. 3), Potchefstroom district

Altitude	... ...	5123 ft. (1562 m.)
Drilled	... ...	1933–1935
Temperatures measured	...	29. iv. 37–30. iv. 37

Observation no.	Geological section (depths in feet)	Vertical depth (= borehole depth) ft.	m.	Temperature °C	Geothermic step ft.	m.
	Soil & boulders					
	67					
3	Chert in broken cavernous ground	99	30	18·72 ± 0·01		
	146	310 Water level	94		535 ± 25	165 ± 10
	Dolomitic					
1		496	151	19·46 ± 0·02		
	limestone				545 ± 35	165 ± 10
	with thin					
2		955	291	20·30 ± 0·03		
	bands of					
	chert and				465 ± 30	140 ± 10
4	carbonaceous	1434	437	21·33 ± 0·03		
					420 ± 25	130 ± 10
5	shale	1926	587	22·50 ± 0·03		
	2563					

ground, it is slightly smaller than in the next section, after which it decreases downward from 545 to 420 ft. (165–130 m.), the average below the water table being 465 ft. (143 m.). This downward decrease resembles that found at Gerhardminnebron, but the average there is much less. One obvious reason for the difference is the fact that here the observations did not extend to the base of the dolomite, where the smallest values of the geothermic step are to be expected.

5. Doornkloof

Borehole No. 18 on the farm Doornkloof No. 155, in the Potchefstroom District, is situated 23 miles (37 km.) south-west of Krugersdorp and 10 miles (16 km.) east of the Driefontein site, at the altitude of 5448 ft.

(1660 m.), on a gentle slope at the northern foot of the Gatsrand. The effect of the topography on the isogeotherms is estimated to be approximately the same as at Gerhardminnebron, where it is not appreciable except near the top of the dolomite, and is consequently neglected.

The hole passes through 31 ft. (9 m.) of soil, 59 ft. (18 m.) of shale and quartzite of the Pretoria series, nearly 4000 ft. (1200 m.) of the Dolomite series and 40 ft. (12 m.) of Black Reef quartzite, before entering the Witwatersrand system. The Dolomite series consists of dolomitic limestone with numerous bands and thin seams of chert and, less frequently, bands and partings of carbonaceous shale. It also contains three thin bodies of dolerite. The proportions of chert and shale have not been determined, and it is therefore not possible to estimate their effect on the temperatures, which is likely to be of the same order as at Gerhardminnebron. As the thickness of dolerite does not exceed 3 % in any section it is comparatively insignificant and need not be considered. The Witwatersrand quartzites show a progressive downward decrease in grain size, from coarse and medium-grained light grey, glassy and sericitic types, with numerous small-pebble-conglomerate bands, at the top to fine-grained dark quartzite with interbedded shaly quartzite and shale at the base. Between the quartzites the borehole passed through three large thicknesses of intrusive rocks, which are thought to be dykes or parts of one dyke. The Dolomite and Black Reef dip at low angles, while the mean inclination of the Witwatersrand beds is about 35°.

The temperatures were obtained from five thermometers, of which three were used at a time. These were always the same in consecutive determinations, except when a replacement became necessary on account of a failure to register the maximum temperature. One thermometer was replaced after observation 9 and a second after it had failed successively in observations 15 and 16. The new instruments were both satisfactory. Thus three temperatures were obtained with only two thermometers, but in each case the same two also gave acceptable readings in both the preceding and following determinations. In order to obtain a connexion between the temperatures in the upper part of the hole, some of which had to be measured after the two replacements, observations 1 and 2 were repeated with the new instruments. The mean results of the earlier and later determinations agreed to the second decimal in each case.

In three observations the exposures were shorter than 3 hr. by 10 min. or less. Ample allowance is made for this discrepancy by assuming an increase of 0·01° in the probable error. Another exposure (observation 10) lasted only 2 hr. 21 min. Additions of 0·01 and 0·02° have therefore been made to the reading and to the probable error, respectively.

TABLE 5. DOORNKLOOF BOREHOLE (No. 18), POTCHEFSTROOM DISTRICT

Altitude ... ... 5448 ft. (1660 m.)
Drilled ... ... 9. i. 37–28. ii. 38
Temperatures measured ... 2. v. 38–10. v. 38

Observation no.	Geological section (borehole depths in feet)	Borehole depth (distance drilled) ft.	Vertical depth below collar of borehole ft.	Vertical depth below collar of borehole m.	Temperature °C	Geothermic step (per °C) ft.	Geothermic step (per °C) m.
	Soil						
	31 ———						
	P. sh. and Q.						
1 and 20	90 ———	100	100	30	18·66 ± 0·02		
	Leached dol.					205 ± 5	62 ± 2
24	———	365	365	111	19·96 ± 0·02		
	Dolomitic					180 ± 15	55 ± 4
23		492	492	150	20·67 ± 0·03		
	limestone	Water level	494	150		(400)	(100)
22		496	496	151	20·68 ± 0·03		
	with					(−2000)	(−600)
2 and 25		599	599	183	20·63		
	numerous					665 ± 15	201 ± 4
3		1097	1097	334	21·38 ± 0·01		
	chert bands					595 ± 25	180 ± 10
4 and 19		1596	1595	486	22·22 ± 0·02		
	and some					510 ± 15	155 ± 5
5		2095	2093	638	23·20 ± 0·01		
	bands and					530 ± 10	162 ± 4
6		2594	2592	790	24·14 ± 0·01		
	partings of					445 ± 10	136 ± 3
7		3092	3089	942	25·26 ± 0·01		
	carbonaceous					400 ± 10	123 ± 4
8 and 21		3591	3588	1094	26·50 ± 0·02		
	shale					325 ± 5	99 ± 2
9		4063	4059	1237	27·95 ± 0·01		
	4064 ———						
						360 ± 210	110 ± 60
	B.R. quartzite						
10		4103	4099	1249	28·06 ± 0·03		
	4104 ———						
						365 ± 15	111 ± 3
	W. cmg. Q.,						

11		4621	4616	1407	29·48 ± 0·01		
	partly sericitic					360 ± 5	109
12		5139	5131	1564	30·92 ± 0·01		
	5140					350 ± 25	105 ± 10
	Am. dyke						
13		5248	5239	1597	31·23 ± 0·01		
	5250					335 ± 5	102 ± 2
	Mg. quartzite						
14		5556	5538	1688	32·12		
	5558					355 ± 3	108
	Dyke						
15		5997	5961	1817	33·31		
	5999					350 ± 10	106 ± 3
	Fg. quartzite						
16		6179	6135	1870	33·81 ± 0·01		
	6181					275 ± 25	85 ± 5
	Dyke						
17		6254	6207	1892	34·07 ± 0·01		
	6257					280 ± 20	85 ± 5
	Fg. Q. (and sh.)						
18		6333	6282	1915	34·34 ± 0·01		
	6336						
	Shale						
	Bottom of hole	6388					

Abbreviations

P. sh. and Q. = shale and quartzite of Pretoria series.
Leached dol. = chert, chert-breccia and dolomite, with cavities and layers of clay, gravel and sand.
B.R. = Black Reef series.
W. cmg. Q. = Witwatersrand system. Coarse and medium-grained quartzite with numerous small-pebble-conglomerate bands.

Am. = amygdaloidal.
Mg. = medium-grained.
Fg. = fine-grained.
Fg. Q. (and sh.) = fine-grained Witwatersrand quartzite with bands of shaly quartzite and shale near base.

The temperatures are all given to the second decimal in table 5, the standard of accuracy being indicated by the probable error, which is omitted when less than 0·01°. The geothermic step is calculated to the nearest unit or 5 units, except in the third and fourth sections, where it is exceeded by the probable error and is therefore shown in brackets, to the nearest 100.

Above the ground water, which is rather low in this hole, the geothermic step is small, averaging only 195 ft. (60 m.). Its greatest numerical value occurs immediately below the water table and is negative. The cause of this inversion of temperature must be sought in the circulation of the ground water, which is likely to be most active near its surface.

In the succeeding sections there is an almost continuous decrease from 665 to 325 ft. (201–99 m.), the average below the water table being 473 ft. (144 m.). This value is much higher than the average at Gerhardminnebron, possibly as a result of more intense circulation. The rapid decrease of the geothermic step in the dolomite has been found in other boreholes, in addition to the present ones, and seems to be general.

In the first section in the Witwatersrand quartzite the geothermic step is larger than at the base of the dolomite and (apparently) about the same as in the intervening Black Reef quartzite, where the value is somewhat doubtful. It is much smaller than at the top of the Witwatersrand quartzite at Gerhardminnebron. It then remains almost constant till near the bottom of the hole, the values in two bodies of intrusive rock and in the adjoining quartzites not differing much. In a third intrusive body it shows a sharp drop, followed by no appreciable change in the succeeding quartzite, which includes some shaly quartzite and shale.

The intrusive rocks are described in the borehole report from above downward, as

(1) "a fine-grained greyish amygdaloidal dyke",

(2) "acid dolerite or basic syenite type with typical porphyritic zones highly reminiscent of the felspar-porphyry zone of the Ventersdorp lavas",

(3) "a further body of dyke".

There cannot be much doubt that the thermal conductivity of these rocks is lower than that of the quartzites, except perhaps in the deepest section, where interbedded shale is present. If they formed sills, or dykes sloping at a low angle, they should therefore have smaller geothermic steps than the quartzites. As this is not the case with the first and second intrusive bodies, it seems that they must be vertical or steeply inclined dykes, while the nature of the third is somewhat uncertain.

In vertical bodies of rock of small width the flow of heat from the earth's interior to the surface should adjust itself according to the conductivity of the adjoining rocks, so that it is less in bad than in good conductors, the isogeotherms remaining approximately parallel. In other words, if a narrow vertical dyke has a different conductivity from that of the country rock, the flow of heat in each should be proportional to its conductivity, while the temperature gradient in the dyke should be the same as in the country rock. The determination of the thermal conductivity of both the quartzites and the igneous rocks in this borehole should throw some light on the subject of heat flow in adjacent bodies of different rocks.

I wish to express my thanks to the Anglo-American Corporation, the Union Corporation, the African and European Investment Company, the New Consolidated Goldfields and to Mr Smith, the owner of the farm Jacoba, for their cordial co-operation in these investigations and for permission to publish the results.

Summary

Five deep boreholes in which temperatures were measured pass through dolomite, Ventersdorp lava, Witwatersrand quartzite, intrusive diabase and small thicknesses of other rocks. The geothermic step (increase in depth per degree C) varies over wide ranges, its mean values being highest in the dolomite and lowest in the lava. It is as a rule small above the water table, especially in the leached dolomite, and generally largest immediately below this level.

In the dolomite below the water table the geothermic step is at first very large and in one case negative, and then decreases almost continuously downward, the lowest value reached being 310 ft. (95 m.) at its base. This decrease and the negative result both seem to be due to circulation of the ground water. The average below the water-table is 382 ft. (116 m.) at Gerhardminnebron, 465 ft. (143 m.) at Driefontein and 473 ft. (144 m.) at Doornkloof.

In the Ventersdorp lava in the Jacoba borehole the geothermic step decreases from 345 ft. (105 m.) below the ground-water level to 217 ft. (66 m.) near its base, the mean being 248 ft. (76 m.), which is also the average at Doornhoutrivier below the first 1000 ft. (300 m.), where the temperatures are affected by gas.

In the Witwatersrand quartzites at Gerhardminnebron the range is from 470 to 300 ft. (143–91 m.) and the average is 366 ft. (111 m.). The value is 380 ft. (116 m.) at Jacoba and 385 ft. (117 m.) at Doornhoutrivier, in both

of which holes the thickness of quartzite is small. At Doornkloof the range in the quartzite is from 365 to 335 ft. (111–102 m.) and the average 354 ft. (108 m.), which is also the mean value in two dykes passing through the quartzite. This agreement seems to show that the geothermic step in the quartzite is reduced by the presence of the dykes, and vice versa.

References

Bullard, E. C. 1939 *Proc. Roy. Soc.* A, **173**, 474–502.
— 1938 *Mon. Not. R. Astr. Soc., Geophys. Suppl.* **4**, 360–362.
Hales, A. L. 1937 *Mon. Not. R. Astr. Soc. Geophys. Suppl.* **4**, 122–131.
Jeffreys, H. 1929 *The Earth.* Cambridge.
Lees, C. H. 1910 *Proc. Roy. Soc.* A, **83**, 339–346.
van Orstrand, C. E. 1924*a* *J. Geol.* **32**, 194–225.
— 1924*b* *Econ. Geol.* **19**, 229–248.

Heat flow in South Africa

By E. C. Bullard

Department of Geodesy and Geophysics, Cambridge

(*Communicated by Professor B. F. J. Schonland, F.R.S.*)

1. Introduction

It has been known for many years that the increase of temperature with depth on the Witwatersrand is exceptionally slow. The normal gradient in Europe and America is about 33° C/km., whilst that in the gold mines near Johannesburg is only about 10° C/km. It has been generally supposed that the conductivities did not differ by any large factor,* and that the low-temperature gradient indicated a heat flow much less than that in other parts of the world. As there is reason to suppose (Jeffreys 1929) that most of the heat is generated by the radioactivity of a layer of granite underlying the continents, and since the principle of isostasy requires this layer to be thicker under the African plateau than elsewhere, a greater heat flow would be expected in Africa than in Europe. The conductivity data are meagre and of doubtful reliability, and it seemed desirable to

* 0·008 cal./cm. sec.° C has been quoted as typical of European sedimentary rocks, whilst Lehfeld (1916) obtained 0·0093 for the Witwatersrand quartzites.