Short manual for *heat1dfwd*

Function *heat1dfwd* and its dependencies represent a simple GSTH forward modeling package solving the conductive heat equation

$$(\rho c)_e \frac{\partial T}{\partial t} = \frac{\partial}{dz} \cdot \left(\lambda_e \frac{\partial T}{dz} \right) + h \,,$$

with a Dirichlet BC $T=T_{surf}$ at the top, and a Neumann BC $\partial T/\partial z=Q_{bas}/\lambda$ at the bottom. The coefficients λ, ρ, and c_p are assumed to depend nonlinearly on temperature. This is realized through some functions (*kfT(T), rhof(T,P), cpfT(T,P)* for the pore fluid (water), *rhoiT(T), cpiT(T), ki=kiT(T)* for ice, and *kmT(k,T,kA,kB), cpmT(cp,T), rhomT(rho,T)* for the rock matrix. Note that the rock properties can be unit-specific. The unfrozen fluid content is given by *ftheta(Tc,Tf,w), which calculates this* parameter and its temperature derivative. The physical relations employed comefrom different sources as summarized in Mottaghy & Rath (2006) and Rath & Mottaghy (2007). The effective properties, λ_e, $(\rho c_p)_e$ are then determined by an appropriate average, i.e., an arithmetic mean for the volumetric heat capacity, and a geometric mean for the thermal conductivity.

The equation is solved by a standard finite difference method on a general grid of *nz* nodes located on cell boundaries defined by the cell sizes, *dz(1:nz-1)*. Properties (coefficients) are assign to these cells. If nz, is the number of grid nodes, *nu* ($1 \le nu \le nz-1$) is the number of physical units which are assigned to the corresponding cells by an index array *ip*. Thermal properties are assumed to be nonlinear functions of temperature, and the nonlinearity is resolved using a simple fixed point iteration.

The time integration is done with a fully implicit method with *nt* time steps of size dt(1:nt-1). The values of the time-dependent boundary condition assigned using the index array *it*, similar as the rock properties properties.

```
[T]  = …
heat1dfwd(k,kA,kB,h,r,c,p,Qb,dz,ip,dt,it,GST,T0,maxiter,tol,
freeze,out);
```

heat1dfwd is a wrapper around two other functions *heat1dns* and *heat1dnt*, which calculate a steady-state temperature profile (e.g. for initial values) and the transient temperature field , respectively. The parameters passed to this function are given below. Currently the only output is the temperature field, T(1:nz,1:nt).

heat1dfwd calls two subfunctions, *heat1dns* and *heat1dnt,* which calculate a stationary and transient solutions, the typical calls could be

```
[T0]  = …
heat1dns(k,kA,kB,h,p,Qb,GST(1),dz,ip,maxiter,tol,freeze,out);
```

Here the temperature profile T0 is calculated as equilibrium geotherm for the first value of the GSTH. Th corresponding call to the transient solution is:

```
[T] = …
heat1dnt(k,kA,kB,h,r,c,p,Qb,dz,ip,dt,it,GST,T0,maxiter,tol,
freeze,out);
```

The input parameters are given in the following table. All input parameters are necessary (currently no defaults):

Variable	Dim	Description
k, kA, kB	1:nu	Thermal conductivity (J/Km), coefficients for temperature dependence , wher here the Clauser & Huenges (1995) relation is assumed.
h	1:nu	Volumetric heat production (W/m^3)
p	1:nu	Porosity (-)
Qb	-	Basal heat flow (W/m^2)
Ts	-	Surface temperature (C)
dz	1:nz-1	Vertical grid spacing (m)
ip	1:nz-1	Index vector assigning physical units to cells
GST	1:nt	Transient upper BC (“ground surface temperature history”)
T0	1:nz	Initial values
dt	1:nt-1	
it	1:nt-1	
maxiter	-	Maximum number of nonlinear fixed point iterations (usually 2-4)
tol	-	Tolerance for nonlinear fixed point iterations (usually 1e-5)
freeze	-	Switch for activation of freeze/thaw
out	-	Switch for activation of additional output, in particular: if out=0, only the final temperature profile, T(1;nz,nt) is stored.

Remarks

Remark 1. The most useful feature in practice is the indirect assignment of properties to cells by index array *ip*. It gives the flexibility to adapt properties to homogeneous (*ip(1:nz-1)=1*) to cellwise resolved (*ip(1:nz-1)=[1:nz-1]*) models. The same advantages apply to the assignment of values to the time-dependent upper boundary conditions.

Remark 2. Due to problems with the MATLAB coder I simplified the code considerably (to the bare skeleton), removing any sparse matrix commands, use of structures, or algorithmic alternatives. However, the original toolbox contains a lot of different formulations for the physics of the problem (e.g. the averaging of phase contributions to the coefficients). Most of them are mainly important for high porosity cases, and the one included work pretty well for crystalline, low-porosity cases.

Remark 3. The algorithm used for freeze/thaw is not optimal for active layer studies, due to the smooth parametrization of the unfrozen water content.

Remark 4. In the automatically translates code the assumption of dynamic-size arrays led to problems, thus I simply put them to a fixed size – this probably should be revised.

List of matlab files:

`heat1dfwd`	- GSTH top forward modeling function
`heat1dns`	- steady state forward modeling function
`heat1dnt`	- transient forward modeling function
`kfT, kiT, kmT`	- thermal conductivity for fluid, ice, and rock matrix
`cpfT, cpiT, cpmT`	- heat capacity at const. pressure for fluid, ice, and rock matrix
`rhofT, rhoiT, rhomT`	- density for fluid, ice, and rock matrix
`ftheta`	- unfrozen water content function
`tridiag`	- tridiagonal solver (Thomas algorithm)
`c2n, n2c`	- cell-to-node and node-to cell transforms

For test case only:

`gsth_driver`	- test for forward model: glacial cycle
`set_lingst`	- set GSTH from data by linear interpolation to step functions
`set_mesh`	- generates a lin/log mesh
`wfilt,tri`	- runs a weighted average filter with a N-point triangular window

References

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