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Ground temperature history in central Europe from borehole temperature data

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SUMMARY

Ground temperature histories (GTH) are inferred from temperature measurements in several boreholes of south-eastern Germany and western Bohemia (Czech Republic). The GTHs that can be extracted from these boreholes, ranging in depth between 150 and 700 m, cover the past 250 yr. Both data sets were inverted separately and yield consistent GTHs. They were also inverted jointly to yield a regional GTH of the past 250 yr for this part of Europe. The results indicate two main episodes in the mean ground temperature with (1) a cooling period from 1750–1800 to 1930–1950, followed by (2) short colder and warmer periods until now. The same trends are found in the meteorological records at four nearby weather stations (Bayreuth, Jena, München and Praha), but the meteorological record in Berlin is clearly distinct. The GTH for this part of Europe is also markedly different from one obtained in central France. These differences are consistent with the spatial variability of climatic trends.

Key words: crystalline rock, inversion of temperature logs, recent climate, shallow boreholes, transient heat flow.

1 INTRODUCTION

It has long been recognized that past variations of the Earth's surface temperature are recorded as perturbations to the steady-state temperature conditions of the subsurface (Lane 1923). Transient surface temperature perturbations attenuate exponentially as they propagate downwards. They penetrate to a depth that depends on the frequency of the temperature signal: a few metres for the annual cycle, and about 80 m for a 100 yr cycle. Subsurface temperature is commonly measured in boreholes to determine heat flow density (HFD). Because the deglaciation and the Holocene climatic history have caused large transient perturbations of the geothermal gradients, corrections have been routinely applied to eliminate these effects and determine the steady-state HFD (e.g. Benfield 1939; Bullard 1939; Birch 1948; Jessop 1971). These authors assume a post-glacial temperature history in order to calculate corrections for the temperature gradient and the HFD. Beck (1977) pointed out that, because HFD is usually determined from measurements in shallow boreholes (<500 m), the Holocene variations in surface temperature have a larger effect on the gradient than the deglaciation, and that the temperature variations of the last millennium have the largest impact on the measured temperature gradients. The inverse problem, of determining the past history of ground surface temperature from borehole temperature measurements, was first formulated by Hotchkiss & Ingersoll (1934). Beck & Judge (1969) and Beck (1982) have interpreted the shallow perturbations of the temperature profiles measured in two boreholes in southern Ontario (Canada) in terms of recent warming of the region. Čermák (1971) inferred the variations in ground surface temperature near Kapuskasing, in northern Ontario, from borehole temperature data. Vasseur *et al.* (1983) were the first to apply inversion techniques to the extraction of past climate from borehole temperature data. They determined the Holocene ground temperature history from measurements obtained in a borehole in the Limousin (central France). Interest in borehole temperature measurements was further aroused by the study of Lachenbruch & Marshall (1986) who inferred a 1–2 K warming in Alaska over the past 100 yr. They suggested that this warming might be the first sign of an enhanced greenhouse effect predicted by global circulation models. This interpretation is consistent with the analysis of worldwide temperature records that show strong warming in the Arctic (Hansen & Lebedeff 1987). In the past few years, several studies have been concerned with the inversion of ground temperature history from borehole data (e.g. Nielsen & Beck 1989; Mareschal & Beltrami 1992; Shen & Beck

1991; Wang 1992). Borehole temperature data from different parts of the world have also been interpreted by several authors (see the collection of papers edited by Lewis 1992). Studies in Canada show that eastern Canada has experienced a 1–2 K warming over the past 150 yr (Beltrami, Jessop & Mareschal 1992) and a cold period prior to this warming (Wang & Lewis 1992). The effect of deforestation on ground temperature history was demonstrated by studies in Cuba (Čermák, Bodri & Šafanda 1992) and Zaire (Sebagenzi, Vasseur & Louis 1992). In Europe, a study of two boreholes in France shows that the ground temperature history there has recorded three major episodes in the past 1000 yr: the climatic optimum of the middle ages, the little ice ages, and the recent warming trend (Mareschal & Vasseur 1992). A study in the Bohemian mountains yields an apparent stepwise warming 480 yr ago that may be related to deforestation (Šafanda & Kubik 1992). These studies aim at obtaining information complementary to meteorological records and other proxy data analyses. If successful, the analyses of borehole temperature data could provide records of ground temperature variations that cover most of the continents and extend further back in time than meteorological records. The purpose of the present paper is to infer recent ground temperature trends in central Europe from temperature measurements in many boreholes in south-eastern Germany and in neighbouring Bohemia (Table 1). The German boreholes were logged in a programme that was preliminary to the drilling of the KTB pilot hole. The deeper of these boreholes extend to 350 m, but most of the boreholes are no deeper than 300 m. The Bohemian boreholes are exploration boreholes ranging in depth between 250 and 700 m. The two sets of data were inverted separately. Because the results of the inversion of both sets are similar and suggest that borehole temperature data in both regions have recorded the same ground temperature history, both sets were inverted jointly to yield an average ground temperature history for this part of Europe. The inferred ground temperature history compares well with records from nearby meteorological stations that have monitored the weather for more than 200 yr.

2 THEORETICAL FRAMEWORK

For a horizontally layered conductive earth, the equilibrium subsurface temperature $T_c(z)$ is given by

$$T_c(z) = T_0 + q_0 R(z) - M(z), \tag{1}$$

where T_0 is the (reference) equilibrium surface temperature, q_0 is the reference heat flow density, $M(z)$ is a term due to heat production, and $R(z)$ is the thermal resistance to depth z:

$$R(z) = \int_0^z \frac{1}{k(z')} dz' \tag{2a}$$

$$M(z) = \int_0^z \frac{dz'}{k(z')} \int_0^{z'} A(z'')\, dz'', \tag{2b}$$

where $k(z)$ is the thermal conductivity and $A(z)$ is the heat production. Both $A(z)$ and $k(z)$ can be either measured on core samples or estimated from the lithology.

Many factors can affect the shape of the temperature profile, such as departure from 1-D assumptions (horizontal variations in conductivity, horizontal changes in surface temperature, effect of topography, etc.), non-conductive transport of heat, or departure from thermal equilibrium conditions caused by transient variations in surface temperature. If all other factors affecting the shape of the temperature profile can be ruled out, the perturbations of a temperature profile can be interpreted in terms of variations in ground surface temperature. In that case, the subsurface temperature is the superposition of the equilibrium temperature, $T_e(z)$, with the transient perturbation $T(z, t)$. The inversion problem consists of determining the equilibrium heat flow and the varying past surface boundary condition from the temperature profile and from thermal conductivity and heat production measurements (e.g. Vasseur *et al.* 1983; Shen & Beck 1983; Nielsen & Beck 1989; Wang 1992; Shen & Beck 1991). The method used in the present study is based on singular value decomposition (Lanczos 1961; Jackson 1972), and is described by Mareschal & Beltrami (1992).

Table 1. Location, elevation h, depth z_{max}, and logging year of all the boreholes used in this study.

borehole	label	latitude N	longitude E	h, m	z_{max}, m	logging year
Wülfersreuth	1	50° 3.5'	11°45.6'	720	200	1986
Weissenstadt	2	50° 5.3'	11°54.4'	635	220	1986
Röthenbach	3	50° 3.3'	12° 9.0'	536	140	1986
Neusorg	4	49°57.5'	11°59.8'	636	301	1986
Falkenberg NB3, PB7	5	49°51.6'	12°12.0'	507	301, 500	1984
Püllersreuth	6	49°47.1'	12° 8.1'	548	301	1986
Remmelberg	7	49°40.3	12°16.1'	570	140	1986
Griesbach	8	49°51.9'	12°30.0'	720	301	1986
Zadni Chodov	A	49°53.0'	12°39.0	507	250	1975
Vitkov	B	49°49.0'	12°39.8'	513	653	1972
Pernolec	C	49°46.3'	12°40.7'	485	265	1987
Lom u Stříbra	D	49°44.0'	12°53.0'	480	330	1977
Bor u Tachova	E	49°24.0'	12°47.0'	473	430	1975
Holubov	F	48°53.7	14°19.1'	511	690	1972

A first approximation to the temperature perturbation is obtained as a solution of the 1-D diffusion equation for a conductive half-space with initial and boundary conditions (Carslaw & Jaeger 1959):

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial z^2}, \tag{3}$$

where κ is the thermal diffusivity of the rock, z is the depth (positive downwards), and t is the time. The use of the 1-D equation is valid if long-term surface temperature changes have a wavelength much larger than the depth to which they penetrate (usually less than 1 km). Thermal conductivity can vary with depth, but, unless there are large conductivity contrasts, this variation produces only a small disturbance of the transient perturbation which was neglected in this analysis.

The present temperature perturbation, $T(z, t=0)$, in a semi-infinite solid with past surface temperature $T(z=0, t)$, where t is time before the present, is given by (e.g. Vasseur *et al.* 1983)

$$T(z, t=0) = \frac{z}{2\sqrt{\pi\kappa}} \int_0^\infty T(z=0, \tau)\tau^{-3/2} \times \exp(-z^2/4\kappa\tau)\, d\tau. \tag{4}$$

This expression can be integrated for various functions describing the surface temperature history. Integration of eq. (4), for an instantaneous change of the surface ΔT at time t before the present, yields

$$T(z, t=0) = \Delta T \operatorname{erfc}\left(\frac{z}{2\sqrt{\kappa t}}\right), \tag{5}$$

where erfc is the complementary error function. If the surface temperature can be approximated by a function varying in K steps at times t_k before present, i.e. if $T(z=0, t) = T_k$ for $t_{k-1} < t < t_k$ $(k = 1, \ldots, K)$, the temperature perturbation is given by

$$T(z, t=0) = \sum_{k=1}^{K} T_k \left[\operatorname{erfc}\left(\frac{z}{2\sqrt{\kappa t_k}}\right) - \operatorname{erfc}\left(\frac{z}{2\sqrt{\kappa t_{k-1}}}\right)\right]. \tag{6}$$

The inverse problem can be formally written as a system of linear equations:

$$\theta_j = A_{jk} X_k, \tag{7a}$$

where θ_j is the measured temperature at depth z_j, corrected for heat production if necessary:

$$\theta_j = \theta(z_j) - M(z_j). \tag{7b}$$

X_k are the $K+2$ unknown parameters $(T_0, q_0, T_1, \ldots, T_K)$, and the matrix A_{jk} contains 1 in the first column, the thermal resistances to depth z_j, $R(z_j)$, in the second column, and in columns 3 to $K+2$ the elements obtained by calculating the differences between error functions at times t_k and t_{k-1} for depth z_j, as given in eq. (6).

This yields a system of N linear equations which must be solved for the $M = K+2$ unknown parameters. In general, this system is both under- and overdetermined. It is also unstable. Such systems of equations are commonly solved by singular value decomposition (SVD) (Lanczos 1961; Jackson 1972; Menke 1989). This approach is theoretically attractive because SVD permits the control of the level of noise that can be tolerated in the inversion. This improvement in the stability of the solution is at the expense of resolution. The SVD involves the decomposition of the $(N \times M)$ matrix $\mathbf{A}$ as follows:

$$\mathbf{A} = \mathbf{U}\Lambda\mathbf{V}^{\mathrm{T}} \tag{8}$$

where $\mathbf{U}$ is an $(N \times N)$ orthonormal matrix in data space, $\mathbf{V}$ is an $(M \times M)$ orthonormal matrix in parameter space and Λ is an $(N \times M)$ diagonal matrix [the only non-zero elements are the L 'singular values' λ_l $(l = 1, \ldots, L)$ on the diagonal, $L \leq \min(N, M)$]. The matrix $\mathbf{U}$ is formed from N orthonormal column vectors spanning the data space, and $\mathbf{V}$ is formed from M orthonormal column vectors spanning parameter space. If $L < M$, the system of equations is underdetermined; if $L < N$, it is overdetermined. When the system of equations $\mathbf{Ax} = \mathbf{b}$ is mixed-determined, i.e. both under- and overdetermined, it has a generalized solution given by

$$\mathbf{x} = \mathbf{V}\Lambda^{-1}\mathbf{U}^{\mathrm{T}}\mathbf{b}, \tag{9}$$

where the $(M \times N)$ matrix Λ^{-1} is a diagonal matrix with the L elements $1/\lambda_l$ on the diagonal (for $\lambda_l \neq 0$) completed with zeros. The instability of the inversion procedure results from the existence of very small singular values, whose inverses have a larger impact on the solution; noise or errors in the data are thus amplified by the very small eigenvalues. In practice, this problem is eliminated in the inversion by retaining only the $P \leq L$ singular values larger than a given cut-off value. The elimination of the smaller singular values reduces the resolution. The resolving power of a set of data to identify the parameters of the model can be evaluated by the model resolution matrix

$$\mathbf{R} = \mathbf{V}_P\mathbf{V}_P^{\mathrm{T}}, \tag{10}$$

where $\mathbf{V}_P$ is the matrix containing the P vectors in parameter space that correspond to singular values larger than the cut-off. The remaining $M-P$ columns are completed with zeros. The data resolution matrix is defined as

$$\mathbf{Q} = \mathbf{U}_P\mathbf{U}_P^{\mathrm{T}}, \tag{11}$$

where $\mathbf{U}_P$ is the matrix containing the P vectors in data space that correspond to singular values larger than the cut-off. The remaining $N-P$ columns are completed with zeros.

Given the vector $\mathbf{m}$ of known model parameters, the parameters that would be obtained by inversion are

$$\mathbf{m}^{\mathrm{est}} = \mathbf{Rm}. \tag{12}$$

The model resolution matrix thus determines the adequacy of a set of data for the identification of the parameters of the model. If the system is underdetermined $(P \leq L < M)$, the solution is not unique: any linear combination of the column vectors of $\mathbf{V}$ corresponding to the singular values that are zero or have been neglected can be added to the solution, since these vectors are mapped on zero in parameter space. If the system is overdetermined $(P \leq L < N)$, the generalized inverse is equivalent to the least-square inverse (Lanczos 1961). For inversion of surface temperature from borehole temperature measurements, the system is in general mixed-determined, i.e. both under- and overdetermined.

Because the correlation distance for meteorological trends is larger than 500 km (Hansen & Lebedeff 1987), boreholes from the same region may have been affected by the same surface temperature trends. This implies that surface conditions are identical for all the boreholes. If this is indeed the case, the subsurface temperature perturbations should be consistent between boreholes. If it is assumed that the errors and noise in the data are incoherent, stacking temperature perturbations from different boreholes should improve the signal-to-noise ratio. The stacking concerns only the temperature perturbation and not the measured temperature, because surface temperature and HFD are different between sites. In practice, the stacking is done by inverting simultaneously temperature data from all the boreholes in the same region. For I boreholes, the unknown parameters are the I surface temperatures, the I HFD values and the K parameters of the ground temperature history. The data are all the temperature measurements from the boreholes. The construction of the matrix is straightforward. If N_i is the number of temperature measurements at borehole i, the matrix will have $N_1 + N_2 + \cdots N_I$ rows and $K + 2I$ columns. The first N_1 elements of the first column will be 1 and all the others will be zero; the following N_2 elements of the second column will contain 1 again and all the other elements will be zero, and so on. The following I columns are obtained by replacing these values of 1 by the thermal resistances to depths $z_j^{(i)}$ (depth z_j in borehole i). Finally the last K rows are obtained by subtracting the differences between error functions for every depth and every borehole $z_j^{(i)}$ at times t_k and t_{k-1}:

$$\begin{bmatrix} \boldsymbol{\theta}^{(1)} \\ \boldsymbol{\theta}^{(2)} \\ \cdots \\ \boldsymbol{\theta}^{(I)} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \cdots & 0 & \mathbf{R}^{(1)} & 0 & \cdots & 0 & \mathbf{A}^{(1)} \\ 0 & 1 & \cdots & 0 & 0 & \mathbf{R}^{(2)} & \cdots & 0 & \mathbf{A}^{(2)} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 & 0 & 0 & \cdots & \mathbf{R}^{(I)} & \mathbf{A}^{(I)} \end{bmatrix} \times \begin{bmatrix} T_0^{(1)} \\ T_0^{(2)} \\ \cdots \\ T_0^{(I)} \\ q_0^{(1)} \\ q_0^{(2)} \\ \cdots \\ q_0^{(I)} \\ \mathbf{T} \end{bmatrix} \tag{13}$$

where $\boldsymbol{\theta}^{(i)}$ denote the vectors of temperature data for borehole i, $\mathbf{R}^{(i)}$ denote the vectors containing thermal resistance to each depth in borehole i, $\mathbf{A}^{(i)}$ is the matrix formed by taking the difference between error functions at times t_k and t_{k-1} for each depth of borehole i. The unknown parameters are the I reference surface temperatures $T_0^{(i)}$, the I reference heat flow densities $q_0^{(i)}$, and the K parameters of the ground temperature history in vector $\mathbf{T}$. The procedure assumes that all the boreholes contain the same information: i.e. the profiles must cover the same depth range and have the same resolution. Such a procedure was applied successfully in eastern Canada, where many boreholes have recorded consistent signals (Beltrami & Mareschal 1992).

3 DATA USED IN THIS STUDY

The 15 boreholes used in this study come from the western part of the Bohemian Massif, a highly metamorphosed crystalline complex of Variscan age. They include nine boreholes in the Oberpfalz region of Germany and six boreholes from western Bohemia (Czech Republic). Their location is shown in Fig. 1, and they are listed with label, geographic coordinates, elevation and depth in Table 1. Most of the boreholes in Germany were drilled in different geological units during the pre-drilling site survey of the German Continental Deep Drilling Programme (KTB) (Burkhardt *et al.* 1989). The boreholes in Bohemia were drilled for mining exploration. Although most of these boreholes are within a radius of 50 km, the sites in Germany are separated from those in Bohemia by a series of mountain ranges, rising several hundreds of metres above their foreland up to maximum elevations of approximately 800 m in the north of the study data (Oberpfälzer Wald and Česky Les), and 1300 m in the south (Šumava), respectively. Table 2 gives the mean values for thermal conductivity k, thermal diffusivity κ, and heat production rate A for the boreholes, where measured data are available. Fig. 2 shows the individual temperature profiles. The German boreholes that had been continuously logged were resampled for this study at about 3 m intervals. The Bohemian boreholes were measured at 10 m intervals. Conductivity and heat production data for the Oberpfalz boreholes are from Burkhardt *et al.* (1989), except for the conductivity of the Falkenberg boreholes, which was measured by Rodemann (1981). Conductivity and heat production are reported by Čermák (1975, 1981). These data are given in columns 3 and 6 of Table 2. Additional k and κ measurements, performed by the Niedersachsen Geological Survey in Hannover (NLfB–GGA), are given in columns 4 and 5 of Table 2. The Czech data were kindly provided for this study by Drs V. Čermák and J. Šafanda (Geophysical Institute, Czech Academy of Sciences, Praha).

4 RESULTS

The first attempts to determine the ground temperature history by inversion of the temperature logs from shallow boreholes near the KTB drill site yielded quite inconsistent results. This is believed to be due to small hydraulic (and other) disturbances in these logs that do not seem to affect heat flow density determination by averaging interpretation techniques such as Bullard plots (see Burkhardt *et al.* 1989). The inversion results, in contrast, turn out to be quite sensitive to noise such as these small hydraulic signals, because the inversion procedure attempts to fit any deviation from a linear temperature gradient with a transient ground temperature history. The temperature perturbations in the borehole are therefore the superposition of a real climatic signal with noise due to a variety of other sources, such as conductivity heterogeneity or anisotropy causing lateral refraction of heat, water flow, etc. Because these boreholes were drilled in different geological

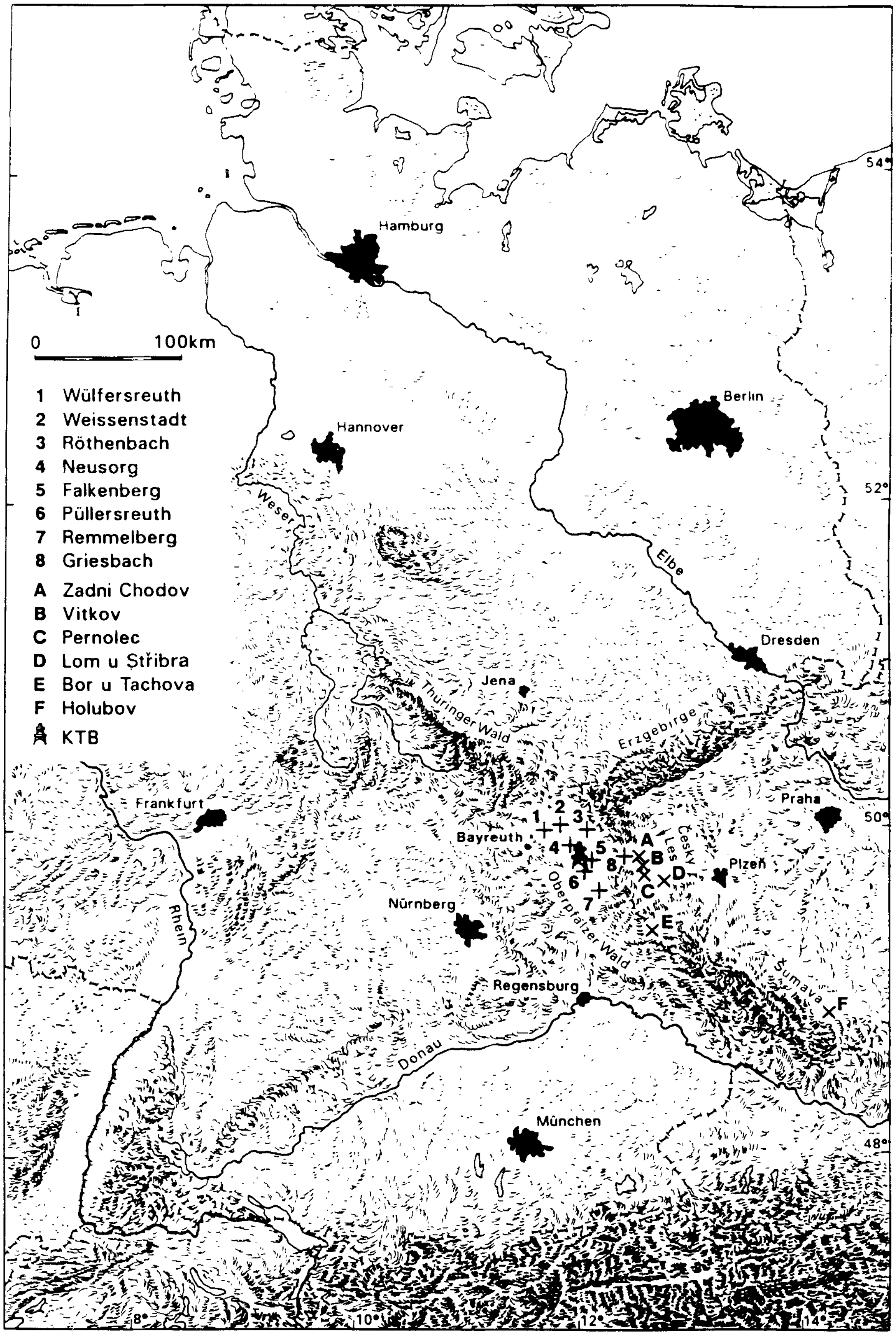

Figure 1. Location of boreholes and weather stations used in this study, plotted on a map of central Europe, showing major topographic features. Labels identifying boreholes correspond to those in Tables 1 and 2. Weather stations used are those in Bayreuth, Berlin, Jena, München and Praha.

Table 2. Name, label (in Fig. 1) and mean values for thermal conductivity k, thermal diffusivity κ and heat production rate A of the 15 boreholes used in this study. The number of individual measurements is given in parentheses (see text for details). Values for the location of Falkenberg are averages from two neighbouring boreholes. An additional diffusivity value of $\kappa = 0.926 \times 10^{-6}\,\mathrm{m^2\,s^{-1}}$ is available from another shallow hole in the area, Poppenreuth, which, however, had to be excluded from the inversion due to hydraulic disturbances. The mean diffusivity determined for the available eight locations in the Oberpfalz was determined to be $0.84 \pm 0.06 \times 10^{-6}\,\mathrm{m^2\,s^{-1}}$. Thermal capacity k/κ was determined from measurements on identical core samples. k therefore does not correspond to the average value given in column 3. No thermal diffusivity and only some heat production data are available for the Bohemian boreholes.

borehole	label	k, W m⁻¹ K⁻¹	κ, 10^{-6} m² s⁻¹	k/κ GJ m⁻³ K⁻¹	A, μW m⁻³
Wülfersreuth	1	3.60 (12)	0.891 (1)	3.30	2.33 (9)
Weissenstadt	2	3.42 (16)	0.900 (1)	3.54	9.80 (16)
Röthenbach	3	2.64 (15)	0.820 (1)	3.54	3.46 (20)
Neusorg	4	3.77 (24)	0.778 (1)	3.73	2.69 (17)
Falkenberg NB3, PB7	5	3.45 (20)	-	3.21	6.74 (15)
Püllersreuth	6	3.02 (42)	0.785 (1)	2.91	1.84 (17)
Remmelberg	7	2.73 (14)	0.767 (1)	3.66	0.19 (20)
Griesbach	8	3.43 (74)	0.832 (1)	3.56	2.10 (20)
Zadni Chodov	A	2.34 (7)	-	-	1.26 (?)
Vitkov	B	2.40 (39)	-	-	3.14 (41)
Pernolec	C	2.40 *⁾	-	-	-
Lom u Stříbra	D	4.09 (12)	-	-	-
Bor u Tachova	E	2.71 (11)	-	-	-
Holubov	F	2.89 (16)	-	-	0.63 (30)

*⁾ Because of lack of data, thermal conductivity in Pernolec is assumed to be the same as in nearby Vitkov.

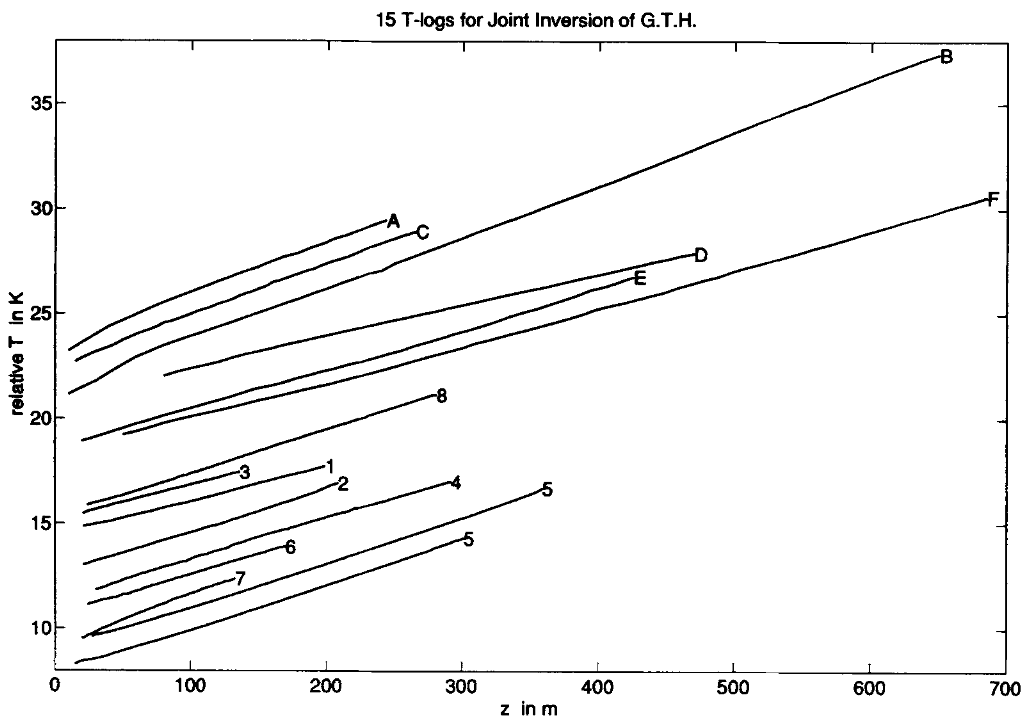

Figure 2. Temperature profiles of all boreholes used in this study. Logs are offset for better viewing. Labels for boreholes correspond to those in Fig. 1 and Tables 1 and 2. At location 5, two boreholes are within 300 m of each other.

units and because the metamorphic crystalline rocks in this region are faulted and folded, it is very unlikely that geological disturbances will be correlated. If the number of boreholes is sufficiently large over the area studied, and the noise is not correlated between the individual boreholes, stacking of the temperature logs will improve the signal-to-noise ratio. If these boreholes have experienced similar surface temperature histories, the joint inversion will extract only the signal that is common to all of them, attenuating the impact of small individual disturbances. Table 3 lists the reference heat flow density q_0 for each borehole obtained by inversion.

Recently, the same T-logs were used to estimate vertical Péclet numbers and specific discharge rates of downward-percolating surface water (Clauser & Huenges 1993; Jobmann & Clauser 1994). This interpretation, however, was ambiguous, and the attempt to match an apparent lateral trend in the Péclet number results was only partially successful. It is expected that the extraction of a climatic signal by a joint inversion technique will allow us to discriminate more clearly between the climatic and hydrological factors that determine the curvature of the temperature logs in these shallow holes.

The inversion can by no means be entirely automated because the singular values retained in the solution are determined by the choice of an appropriate cut-off value. This selection affects the trade-off between model resolution and stability for a particular solution. In practice, inversion is first tried with a very small cut-off value, e.g. 10^{-4}–10^{-3}; usually this results in a wildly oscillating GTH, as the effect of noise is amplified. Singular values can then be eliminated one by one, by increasing the cut-off value accordingly. After some steps, these unphysical oscillations in the solution, typical for small cut-off values, will disappear. Higher cut-off values would yield solutions that are less affected by the noise. If too many singular values are eliminated, the resolution will be so low that no information can be retrieved from the inversion. It is quite clear that this procedure is intrinsically non-unique and depends on the judgement of the interpreter. The interpretation can be aided a great deal if additional information can be introduced. For the present study, results can be compared with available long-term averages of air temperature at five meteorological observatories in the area: Bayreuth, Berlin, Jena, München and Praha. For all data sets, several cut-off values were used, but a cut-off of 0.05 yielded the best results, for reasons to be discussed later.

The parameters of the model temperature history must also be selected by the interpreter, i.e. the number of time intervals and their duration. In general, it is preferable to use too many parameters for the model, because the singular value decomposition selects the linear combination of these parameters that is most strongly constrained by the data. Careful parametrization could, however, save time and, to some extent, improve the stability of solutions. Eq. (5) is useful for selecting a time interval appropriate to the depth of the boreholes available. It specifies the temperature $T(z)$ at a depth z due to a stepwise temperature rise t yr ago for a constant diffusivity. Table 4 gives the temperature perturbation as a percentage of the surface temperature jump at different times and depths. As can be seen, a model temperature history of several hundred years is reasonable for boreholes a few hundred metres deep. The inversion was performed for temperature histories with durations of 300 yr and 600 yr, with very minor differences in the results. Results will be presented for a model GTH consisting of 30 intervals, each of 10 yr.

The thermal diffusivity data were only available for the boreholes in the Oberpfalz. The diffusivity is sometimes obtained by dividing thermal conductivity by some reference value for thermal capacity. In order to test the frequently used textbook value of $2.3\ \mathrm{GJ\,m^{-3}\,K^{-1}}$ (e.g. Beck 1988), thermal conductivity and thermal diffusivity were measured

Table 3. Name, label (in Fig. 1) and reference heat flow density q_0 prior to onset of transient GTH for joint inversions of three different data sets: Oberpfalz data only (centre column), Bohemian data only (second to last column), and combined data sets from the Oberpfalz and Bohemia (last column).

borehole	label	q_0, mW m^{-2}	q_0, mW m^{-2}	q_0, mW m^{-2}
Wülfersreuth	1	66.2		66.3
Weissenstadt	2	77.0		77.2
Röthenbach	3	50.8		49.9
Neusorg	4	79.5		80.5
Falkenberg NB3, PB7	5	78.8		79.7
Püllersreuth	6	64.6		64.3
Remmelberg	7	74.9		74.0
Griesbach	8	77.2		78.0
Zadni Chodov	A		62.4	63.4
Vitkov	B		61.8	60.6
Pernolec	C		61.8	62.6
Lom u Stříbra	D		64.7	63.9
Bor u Tachova	E		54.2	54.0
Holubov	F		53.6	53.2

Table 4. Relative temperature change at depth z due to an instantaneous change ΔT at the surface, t yr ago ($\kappa = 10^{-6}\ \mathrm{m^2\,s^{-1}} = 31.6\ \mathrm{m^2\,yr^{-1}}$).

t, yrs	z, m	T/ΔT, %
50	300	0
"	200	0
"	100	8
"	50	38
100	300	0
"	200	1
"	100	21
"	50	53
300	300	3
"	200	15
"	100	47
"	50	72
1000	600	2
"	300	23

on selected cores from the Oberpfalz. The thermal capacity obtained from the ratio of measured conductivity to diffusivity was found to be uniformly greater than the textbook value, with differences ranging from 26–62 per cent (Table 2). The table shows that, in general, thermal diffusivity varies much less than thermal conductivity (less than 10 per cent from the mean value for diffusivity versus 20 per cent for conductivity). Therefore, the diffusivity was not calculated from the conductivity measured on samples from Bohemia, and the results will be presented for a diffusivity of $0.84 \times 10^{-6}\,\mathrm{m^2\,s^{-1}}$ or $26.5\,\mathrm{m^2\,yr^{-1}}$, the mean diffusivity value determined for the Oberpfalz. It can be seen from eq. (5) that changing the thermal diffusivity is equivalent to changing the time-scale, since it is the constant κt that enters in the equation. In the case of a horizontally layered earth, it is possible to include the effect of variable thermal conductivity in the inversion scheme; systematic tests have shown that it would not affect the trends of the inverted ground temperature history much (Shen *et al.* 1992).

The heat production rate measured on core material from the Oberpfalz and Bohemian boreholes varies from 0 to $10\,\mu\mathrm{W\,m^{-3}}$ (Table 2). As high values will have some effect on the curvature of individual temperature profiles, the temperature data from the Oberpfalz are corrected to remove this effect, as shown in eq. (7b). The heat production values used are those given in Table 2. The same correction was done for the Bohemian boreholes where heat production data were available. Although heat production is large in some boreholes, the correction remains small (<100 mK) compared with the temperature perturbation (500 mK).

4.1 Inversion of the Oberpfalz data set

The results of the simultaneous inversion can be compared with the data for each of the boreholes. Fig. 3 shows an example of the information obtained. The top left panel shows the measured temperature profile, the profile predicted by the model, and the reference temperature profile (i.e. the profile after elimination of the transient perturbation). The difference between the model's prediction and the data is usually too small to be detected at this scale. The top right panel shows the disagreement between the model's prediction and the data. It is small (less than 50 mK), but it is not random: it is determined by the singular vector in data space corresponding to the lowest retained eigenvalue. The smaller (5 mK) oscillations are caused by noise and random errors. The perturbation in temperature and heat flow density are shown in the bottom left and right panels of Fig. 3, respectively. These perturbations represent the difference between the equilibrium (or reference) conditions and the observed profiles. For the GTH in question, below 150 m the T and HFD perturbations become smaller than the accuracy of a measurement. Fig. 3 shows that the model provides a good fit for the data. This result is common to many interpretations of borehole temperature data. The good fit cannot only be the effect of overparametrization. With the singular cut-off selected, 23 singular values are retained. The vectors corresponding to the 18 largest singular values have a large component in the individual reference heat flow density and equilibrium surface temperature and a very small component of the GTH parameters. The model resolution matrix determined by these vectors is almost 1 on the diagonal and zero off the

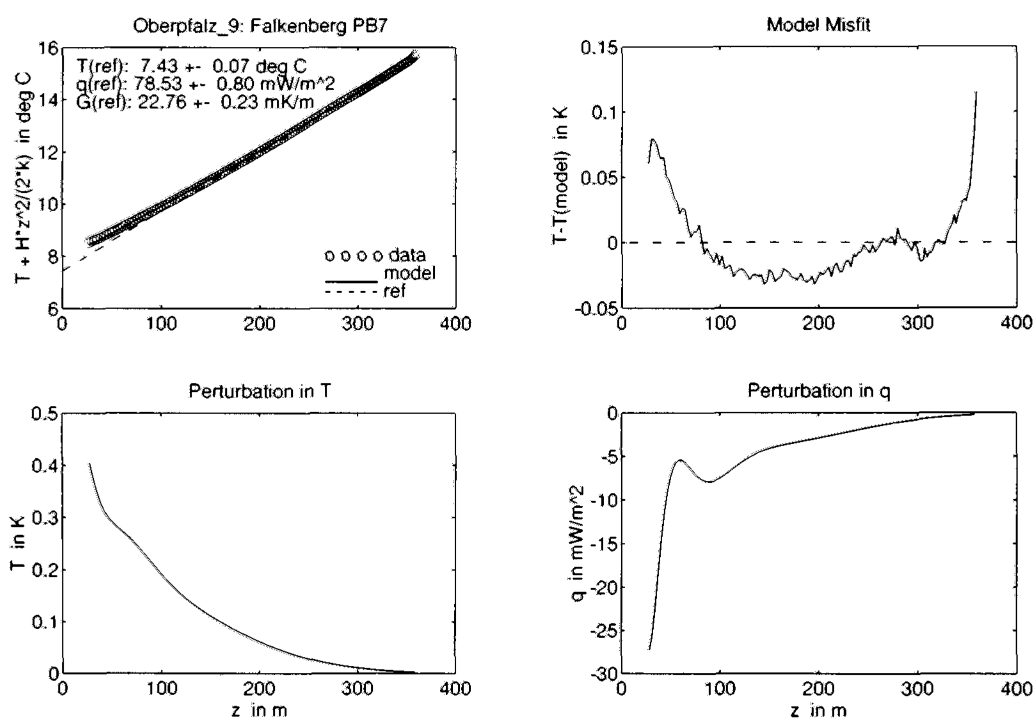

Figure 3. Model fit and perturbations of temperature and heat flow density profiles caused by transient ground temperature history for borehole Falkenberg PB7 (see Table 1 and Fig. 1). T(ref), q(ref), and G(ref) in the top left panel are reference temperature, HFD, and vertical temperature gradient, respectively.

diagonal. The next five vectors contain the components of the GTH. For these the diagonal term of the model resolution matrix is not dominant. Therefore, the data can only resolve the long periods in the GTH, and most of the short-period oscillations are lost in the analysis. Fig. 4 shows tne inverted GTH. The error envelope describes the propagation of an assumed 100 mK measurement error in the inversion process, not the absolute uncertainty of the inverted GTH. The main trends shown by this GTH are (1) cooling between 1750 and 1900, (2) warming until about 1950, and after this (3) cooling, warming and cooling again.

Tests on the inversion of synthetic, noise-free data show that a GTH will contain artificial oscillations when only a small number of singular values are retained (Mareschal & Beltrami 1992). The oscillations of the past 50 yr in our GTH might thus be due in part to the low number of singular values retained. However, comparison with meteorological records (shown later) seems to confirm these oscillations, if not their amplitude.

4.2 Inversion of the Bohemian data set

The data from Bohemia were fitted by the models as well as the Oberpfalz data set. Fig. 5 shows the reconstructed GTH. Because the boreholes are deeper than in the Oberpfalz, it was possible to retain 18 singular values. The vectors corresponding to the 12 largest singular values have a large component in the individual reference heat flow density and equilibrium surface temperature and a very small component of the GTH parameters. The next six vectors contain the components of the GTH. For these the diagonal term of the model resolution matrix is not dominant. Again, the data can therefore resolve only the long-period trends in the GTH and most of the short-period oscillations are lost in the analysis. This GTH, however, is not identical to the previous one. The main trend is a slight warming until about 1840, followed by a quick cooling up to the 1930s. Furthermore, the oscillations of the past 50 yr have a slightly lower amplitude than in Germany, and the most recent cooling (after about 1970) in the Oberpfalz GTH is now replaced by a continuation of the previous warming trend, starting in about 1960. However, as all but one of the Bohemian boreholes were logged in or prior to 1977 (Table 1), the most recent climatic history cannot be expected to be as well represented as in the Oberpfalz data set.

4.3 Inversion of the merged data sets from the Oberpfalz and Bohemia

The main difference between the Oberpfalz and Bohemian results seems to lie in the timing of the events. Bohemia shows cooling between 1840 and 1930, while cooling in the Oberpfalz seems to have taken place between 1750 and 1900. Because there is some similarity in the trends and also because the data come from locations that are very close to each other, the two data sets were combined for simultaneous inversion. The results, shown in Fig. 5, yield a temperature history that combines features from both data sets. The main trend is 0.85 K cooling between 1800 and 1920, which is about in-between the results obtained from the inversion of the Oberpfalz and Bohemian data sets. The singular value cut-off for the inversion of the combined data set was 0.05, and 36 singular values were retained. As in the previous case, six vectors contain the components of the GTH, one more than for the inversion of the Oberpfalz data set. A comparison of the error envelopes obtained for the combined data set with the individual data sets does not show significant improvement. The standard

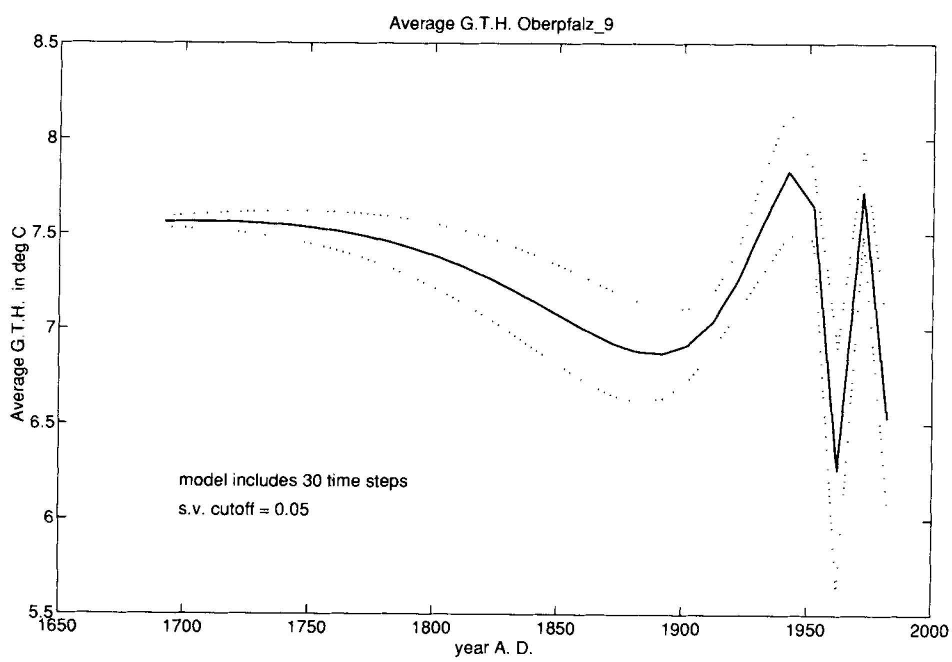

Figure 4. Ground temperature history from inversion of a data set of nine boreholes in the Oberpfalz and error envelope, showing the propagation of a 0.1 K measurement error in the inversion (see Fig. 1 for location and text for details).

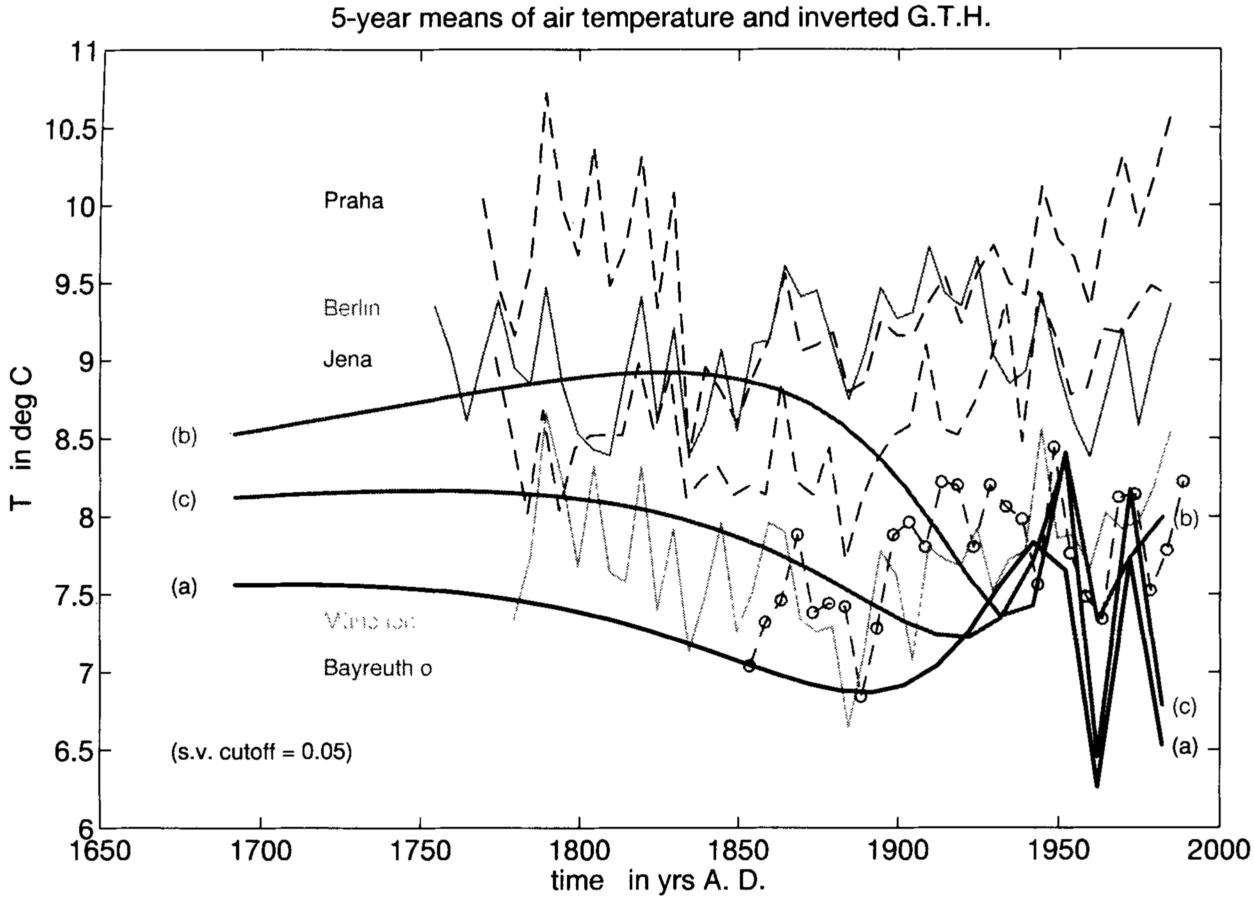

Figure 5. 5 yr means of air temperature in Praha, Berlin, Jena, München and Bayreuth (top to bottom) and inverted GTHs for the Oberpfalz (a), West Bohemia (b) and the combined data sets (c).

deviation on the oscillations of the past 50 yr (0.2–0.5 K) is fairly large, as before (see Fig. 4). Oscillations with very small amplitude would still fit the data within the error margin.

5 DISCUSSION

5.1 Comparison with meteorological data

In the present study, perturbed temperature profiles measured in boreholes are interpreted in terms of recent changes in the ground surface temperature. Comparison with a direct record of climate is useful to determine whether this interpretation is sensible and whether the inversion yields meaningful results. Fortunately, time series of measured air temperatures are available for this purpose. Air temperature has been recorded continuously in central Europe since the end of the 18th century. An excellent compilation of mean annual air temperatures up to the year 1955 for 16 meteorological observatories is given by Bider, Schüepp & von Rudloff (1959). Meteorological stations with long-term records within reasonable distance from the study area are Bayreuth (~30 km), Berlin (~300 km), Jena (~130 km), Praha (~170 km) and München (~200 km) (see Fig. 1 for locations). The record for München was provided in total by G. Tetzlaff (1992, personal communication). All the other time series were compiled from Bider *et al.* (1959) and completed up to the year 1990 with informaton provided by U. Walzer (Jena), G. Tetzlaff (Hannover), and V. Čermák (Praha). Records start in the years 1851 (Bayreuth), 1756 (Berlin), 1770 (Jena), 1775 (Praha) and 1781 (München). They are continuous, with the exception of Jena which has a 20 yr data gap during and after the Napoleonic wars from 1800 to 1820. This gap was filled with the mean value for the whole time series. The quality of these meteorological records in terms of long-term consistency was evaluated by Malcher & Schönwiese (1987). These authors conclude that München, Jena and Praha are homogeneous, while Berlin and Bayreuth are disturbed by −0.8 K in 1931 and −0.5 K in 1945, respectively. In part, this may be due to the fact that both weather stations were moved: Berlin in 1931 and Bayreuth in 1946, 1952 and 1957. In order to make these records comparable to the inversion results, 5 and 10 yr averages were calculated from the annual means using moving-window averaging. In Fig. 5, the GTHs reconstructed for the Oberpfalz, for Bohemia, and for the combined data sets (curves a, b, and c, respectively) are compared with these 5 yr averages. It is worth noting that the shorter-period peaks in all air temperature records are very well correlated over a period of more than 200 yr; however, the longer-period trends in the records from Praha, Jena, München, and Bayreuth are also correlated. These cities lie within a radius of 160 km from the Oberpfalz, but each of them is separated from the others by mountains with a difference in elevation of several hundred metres. In contrast, the long-period trends in the record from Berlin are less correlated with those from the other stations. However, Berlin is located much farther away and is closer to the Baltic Sea in the north. Thus, it may actually be in a different climatic zone from the other four cities. In addition, Berlin is a large metropolitan area where recent records are more likely to be contaminated by urbanization.

It appears that the maxima and minima of the three reconstructed GTHs fit the maxima and minima of these meteorological records reasonably well. (1) All meteorological temperature records reach a minimum between 1830 and 1890. This is well reproduced in the GTH from the Oberpfalz; that from Bohemia, which places the minimum between 1930 and 1940, fits least well. The early increase in the Bohemian GTH is perhaps best matched by the records from Berlin and Praha, while those from Bayreuth, Jena and München are more similar to the GTH from the Oberpfalz. On the whole, the earlier part of the Bohemian GTH quite clearly does not compare as well with the meteorological data as the GTH from the Oberpfalz. The combined data set yields an intermediate result. (2) The series of pronounced minima and maxima from about the year 1950 on is found in all the records, even if the timing is slightly different. (3) Immediately after 1970, Jena and München, as well as the GTH from Bohemia (Fig. 5, curve b), show a continued warming. For Praha, Berlin and the station closest to the Oberpfalz, Bayreuth, as well as the GTHs from the Oberpfalz and from the combined data sets (Fig. 5, curves a and c, respectively), the maximum around 1970 is followed by a new minimum between 1975 and 1982. After this the air temperature rises again. Because of the year in which the boreholes were logged for temperature (Table 1) and the length of the time steps used in the inversions (i.e. 10 yr), the GTHs do not resolve this very recent period.

In summary it appears that most of the prominent features in the reconstructed ground temperature history correlate well with variations in long-term air temperature records in surrounding meteorolgoical observatories.

Another way to compare the meteorological records with the inversion results is to multiply the meteorological records with the model resolution matrix **R**, given by eq. (12), to find the model parameters $\mathbf{m}^{est}$ that inversion of data would resolve. Consistent with the length of time steps in the inversions, 10 yr means of air temperature are used in this comparison. To this end, each record of 10 yr means is (1) reduced by its average temperature, and (2) padded with zeros according to the number of columns of the model resolution matrix. For all the meteorological records, Fig. 6 compares the input function **m**, the model response $\mathbf{m}^{est}$ to this forcing function, and the GTH inverted from all three data sets. Several points can be made from this comparison. (1) The only model response that begins with a minimum is that to the Berlin record. (2) Because the boreholes are not located at the site of these stations and because ground temperature is affected by more factors than air temperature alone (e.g. Beltrami & Mareschal 1991), the comparison can only be qualitative. However, the trends observed appear similar even though the amplitude of the temperature variations differ between GTH and the model responses. The reconstructed GTH for the Oberpfalz correlates best with the model response for the data sets from Praha, Jena

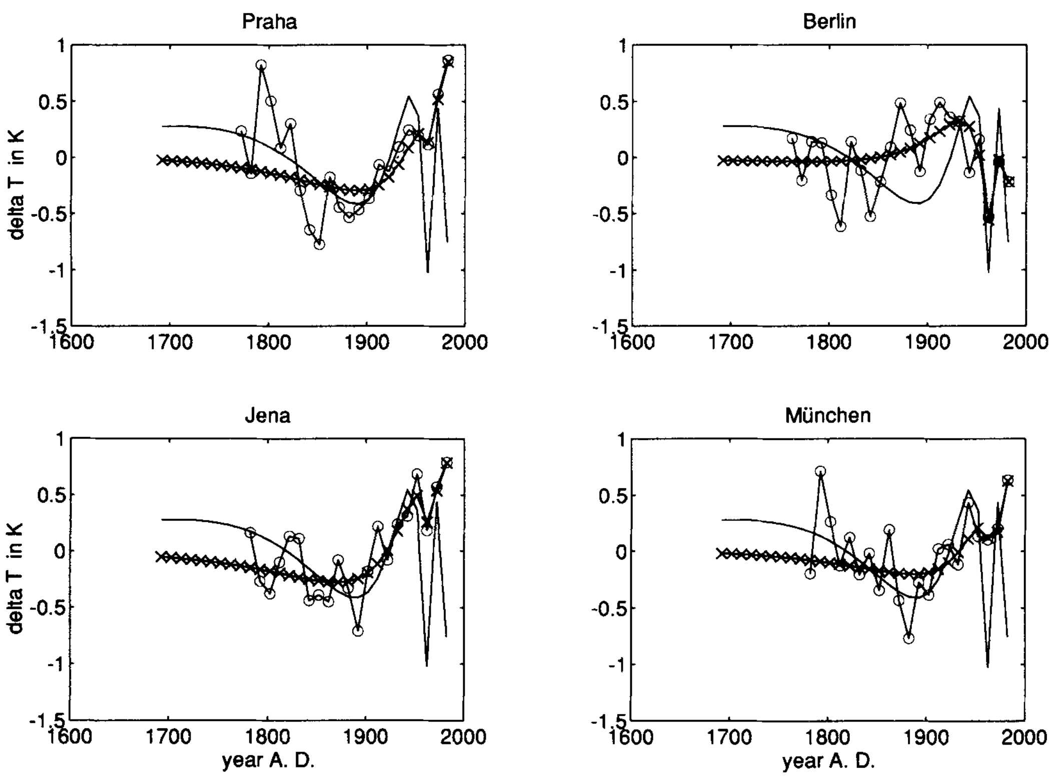

Figure 6. Model responses (×) calculated from eq. (12) for 10 yr means of air temperature (○) in Praha, Berlin, Jena and München. The GTH inverted for the Oberpfalz data set is shown by the solid line.

and München. Except for the very first 50 yr, the agreement with the Berlin data set is less satisfactory. (3) The Bohemian GTH generally correlates well with the model responses for the first 50 yr only. (4) As a consequence, the GTH from the combined data set shows a generally acceptable agreement with the model responses, but it is not quite as convincing as that from the Oberpfalz.

5.2 Comparison with GTH inferred in other studies

The climatic trends of the past 250 yr inferred by this study compare well with the meteorological records of local weather stations. However, these trends also exhibit large differences with those obtained by similar studies elsewhere. Although the two boreholes studied by Šafanda & Kubik (1992) are in Bohemia and geographically close to the region of the present investigation, comparison between the two studies is impossible because of differences in the data and in the analysis. The data used by Šafanda & Kubik (1992) come from deeper boreholes (700 and 2000 m) and thus contain a longer time record. Their method of analysis does not permit the reconstruction of a complete GTH but merely the identification of one stepwise change in ground temperature. It appears that the most important change recorded by the deep borehole is a 6 K warming at the end of the glacial period 12 000 yr ago. The shallow borehole was affected by a 1–2 K warming 480 yr ago, possibly related to deforestation. This latter event, if confirmed, is a local phenomenon that will not affect the regional GTH. Because there is no overlap between the events of Šafanda & Kubik (1992) and the regional GTH, further comparison is impossible.

Vasseur *et al.* (1983) and Mareschal & Vasseur (1992) determined a GTH for central France from the study of two boreholes in the Limousin, which were about 750 km away from the boreholes of this study. The French boreholes are deeper (700 and 1000 m) than those of the present study and yield a detailed GTH covering the last 1000 yr. A comparison of the most recent section of this GTH with the present study shows marked differences. Warming in France started 200 yr ago and lasted until 50 yr ago. It was followed by cooling and possibly very recent warming. In central Europe, the trend appears to be cooling from 200 until 90–60 yr ago, and was followed, on average, by warming. The French data suggest a cold episode, tentatively identified with the Little Ice Ages, between 400 and 200 yr ago. Unfortunately, the central European boreholes of the present study are not deep enough for this episode, if present, to be identified. The studies in eastern Canada (e.g. Beltrami & Mareschal 1992) have identified climatic trends similar to those reported for France.

Trends in meteorological records appear correlated over distances less than 500 km (Hansen & Lebedeff 1987). This is much less than the distance separating the French and the central European sites. A study of climatic trends of the past century in Europe has shown spatial variability in the climatic trends with differences between western and central Europe (Schönwiese, Rapp & Fuchs 1994). Thus the inversion of ground temperature history from borehole temperature data has been supported by evidence in meteorological records for central Europe and proxy data for France. The differences between the GTH inferred in these two studies are not inconsistent with the current understanding of climatic variability.

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