of which holes the thickness of quartzite is small. At Doornkloof the range in the quartzite is from 365 to 335 ft. (111–102 m.) and the average 354 ft. (108 m.), which is also the mean value in two dykes passing through the quartzite. This agreement seems to show that the geothermic step in the quartzite is reduced by the presence of the dykes, and vice versa.

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Heat flow in South Africa

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(*Communicated by Professor B. F. J. Schonland, F.R.S.*)

1. Introduction

It has been known for many years that the increase of temperature with depth on the Witwatersrand is exceptionally slow. The normal gradient in Europe and America is about 33° C/km., whilst that in the gold mines near Johannesburg is only about 10° C/km. It has been generally supposed that the conductivities did not differ by any large factor,* and that the low-temperature gradient indicated a heat flow much less than that in other parts of the world. As there is reason to suppose (Jeffreys 1929) that most of the heat is generated by the radioactivity of a layer of granite underlying the continents, and since the principle of isostasy requires this layer to be thicker under the African plateau than elsewhere, a greater heat flow would be expected in Africa than in Europe. The conductivity data are meagre and of doubtful reliability, and it seemed desirable to

* 0·008 cal./cm. sec.° C has been quoted as typical of European sedimentary rocks, whilst Lehfeld (1916) obtained 0·0093 for the Witwatersrand quartzites.

make a systematic study of the question. The temperatures and conductivities in a number of English bores have been studied by Benfield in this laboratory, and detailed and accurate temperature measurements have been made by Dr Krige (1939) and Mr Weiss (1938) in deep boreholes in South Africa. When the author was invited to visit Johannesburg as a guest of the Bernard Price Institute of Geophysical Research he took the opportunity to measure the conductivities of specimens of rock from some of the boreholes in which temperature measurements had been made. It is the purpose of this paper to describe these measurements and to discuss the results.

As the conditions were exceptionally favourable, both for the temperature and for the conductivity measurements, the problem has been investigated in considerable detail. It was thought that, in addition to the direct interest of the results, they may be useful in indicating the disturbances to be feared in cases where no such detailed study is possible.

2. Method

The boreholes were $2\frac{5}{16}$ in. in diameter, and the cores were cylinders $1\frac{3}{8}\pm\frac{1}{32}$ in. in diameter and about 6 in. long. Each core sample was cut perpendicular to its axis with a diamond saw to give disks about 8, 4 and 1 mm. thick. These disks were ground flat with carborundum and, where suitable, polished. The flatness was tested against a surface plate and with a micrometer. There is a tendency to make specimens with convex faces. To ensure that they were flat enough they were micrometered in the centre and at six points on a 1 in. diameter circle, if the centre measurement differed by more than 0·0006 in. (15×10^{-4} cm.) from the mean of the other six, the flattening process was continued.

The quartzite cores from the Gerhardminnebron borehole had been split longitudinally and one-half assayed. Semicircular specimens were prepared from the remaining half.

The divided bar method was used to measure the conductivities. In this method the disk-like specimen is placed between two vertical brass bars of the same diameter as itself and heat is supplied at the top end by an electric heater and abstracted at the bottom by the circulation of cold water. The temperatures at a number of points in the brass bars are measured by thermojunctions, and the ratio of the temperature gradient in the brass to that in the specimen determined. The conductivity of the brass was determined by standardization with crystalline quartz and fused silica specimens. The apparatus employed was a copy of that used by

Benfield (1939), except that the bars and specimens were 1·4 in. in diameter instead of 1 in. A similar apparatus with semicircular bars was made for the semicircular specimens. The advantages of this method have been discussed by Benfield.

As the hard dolomites and quartzites whose conductivities were to be determined were much easier objects for measurement than Benfield's shales and sandstones, a rather more thorough investigation of the possible sources of error could be made. First, it was established that the temperature gradient in the bar above the specimen was equal to that in the bar below to within less than 1 %, and that heat loss sideways was therefore negligible. Next, measurements were made on several specimens at a number of different heat flows. A linear relation was found between the temperature difference across the specimen and the temperature gradient in the bar; and from the departures of the individual observations from a straight line the standard error of one temperature observation was found to be 0·009° C. The best lines through the observed points did not go exactly through the origin, that is to say, when there was no temperature difference across the specimen there appeared to be a small temperature gradient in the brass bars. From the mean of all the results from thirteen specimens this intercept was found to be $0{\cdot}009 \pm 0{\cdot}0018°$ C. Its cause has not been ascertained, but the heat flow used in the main experiments was large enough to reduce its effect on the individual measurements to about 1 %. This error will be largely eliminated in the final results, as it will affect the standardization observations in much the same way as the measurements on the rock specimens. A heat flow of about 0·6 cal./sec. was normally used. This gives 1° C temperature difference between two thermojunctions 4 cm. apart in the brass bars, and 5° C across a specimen 8 mm. thick and of conductivity 0·01 cal./cm. sec. ° C the measurements on a single specimen can be made to about 1 %.

Motor oil was at first used to improve the thermal contact between the bars and the specimens, but later a mixture of two-thirds water and one-third glycerine was used.* Weighing before and after the measurements showed that the mixture was not absorbed by the specimens. From the mean of the measurements on nineteen rocks using oil films the combined thermal resistance of the two films was found to be 0·65° C. sec./cal.; from twenty-seven rocks with glycerine it was 0·35° C sec./cal. There is

* Water has a thermal conductivity of 0·0015 and glycerine of 0·00068. It is not convenient to use water alone as it evaporates. The mixture does not evaporate, spreads better than pure glycerine, and has a much higher conductivity than oil (about 0·0010 compared to 0·00037).

therefore a considerable reduction in the temperature drop at the interfaces on using the glycerine mixture, and it is advantageous to use it when the rocks are sufficiently impervious not to absorb it. These resistances correspond to thicknesses of about 15×10^{-4} cm. for the liquid films; this agrees with the thickness determined by collecting and weighing the liquid. With the standardizing specimens the temperature drops at the interfaces were about 4 times smaller than with the rock specimens: this is due to their greater flatness.

In order to have a check on the measurements of each rock, three specimens of different thickness were measured. The conductivity and the temperature drop at the contacts were found from the results obtained with the thickest and thinnest specimens and the result to be expected with the middle specimen calculated. The distribution of the differences between the observed and calculated results (expressed as thermal resistances) was

Obs. − calc.	−0·45	−0·35	−0·25	−0·15	−0·05	+0·05	+0·15	+0·25	+0·35	+0·45
No.		0	0	7	7	15	8	5	2	1

In addition there are six specimens giving differences between −0·84 and +1·12. The latter are clearly affected by some exceptional source of error, the measurements on some of them have been repeated and practically the same result obtained; the cause of the discrepancy is therefore in the specimens rather than in the measurements, and is probably due to one of the three specimens having a conductivity very different from that of the other two prepared from the same sample or having an internal crack.

The mean of the above distribution omitting the six outlying values is $0{\cdot}019 \pm 0{\cdot}013°$ C sec./cal. and does not differ significantly from zero; the standard deviation is 0·090. Since the resistance of the thicker of the three specimens was usually about 8° C sec./cal., this result shows that the curvature of the thermal resistance-temperature relation is considerably less than 1 %; that is, that the measured conductivity is independent of the thickness of the specimen and that the method of allowing for imperfect thermal contact is satisfactory. The standard deviation of 0·090° C sec./cal. corresponds to a 2·2 % error in the measured temperatures, the difference between this and the 1 % deduced from the measurements of the individual specimens is to be ascribed to small differences in the conductivity and flatness of the three specimens made from each rock sample. The discrepancies between the observed resistance of the middle specimen and that calculated from the other two were found not to depend on the nature of the rock, the thickness of the specimen, or on whether oil or glycerine was used. From them the standard errors of the conductivity determinations in table 1 were calculated to be

Dolomite	2·9 %
Quartzite	3·7 %
Lava	1·9 %
Granite	2·2 %

The high value for the quartzites is due to the specimens being too thin for a rock of such high conductivity. These errors do not include that due to uncertainties in the standardization of the instrument. The standardization was relative to crystalline quartz with the heat flowing perpendicular to the axis of symmetry. Its conductivity has been assumed to be 0·0147 cal./cm.° C sec.* at 25° C. Check measurements were made with fused silica; this was found to have a conductivity of 0·00307 at 25° C. The conductivity of the brass bars was $0{\cdot}252 \pm 0{\cdot}002$, where the standard error does not include the uncertainty in the adopted conductivity of quartz (see Benfield (1939) for a discussion of this).

The conductivity of a plate of rock that is thinner than the grain size may differ from that of a thicker plate. For example, consider conducting particles distributed in an insulating matrix; a plate thinner than the particle diameter will conduct but a thicker plate will be an insulator. Thus, the values of the conductivity obtained with a coarsely crystalline rock consisting of a mixture of minerals of different conductivity may be slightly in error. If the thick specimen is thicker than the grain size, it may be shown that the effect is to make the measured conductivity and the temperature drop at the contacts too small. The only rocks in this investigation that are likely to be affected are the Dubbledevlei granites which have crystals of quartz, feldspar and garnet some millimetres across. These specimens are the only ones that show an apparent negative temperature drop at the interface. It is therefore possible that the measured conductivities of these specimens are 5–10 % too low. No correction has been applied.

A number of the specimens have been weighed after soaking in water and after heating at 105° C for some hours. The change in weight was always less than 1 %; the rocks are therefore very impervious and the conductivity *in situ* is not likely to differ appreciably from the measured one on account of differences in water content. A dolomite (No. 5 of table 1) was measured in its normal state with oil to make contact, and

* All conductivities in this paper are in these units, which will not be repeated every time.

the result was 0·0116; it was then remeasured using the glycerine mixture, the result being 0·0120. It was then dried for some hours at 105° C and remeasured with oil films, the result being 0·0122; it was then soaked in water overnight and gave 0·0120. These results confirm that the moisture content has no perceptible effect on the conductivity of these rocks.

The measurements were all made at about 25° C; when in the earth the temperatures of the specimens were between 20 and 70° C. The conductivity of most crystals decreases with temperature, the rate of decrease for quartz being 1 % in about 4° C. In an extreme case, therefore, the measured conductivities may be 10 % greater than those appropriate to the bottom of the holes, but the average error will be much less and no correction has been applied.

3. Results

The measured conductivities of forty-nine samples of rock are given in table 1, with the name of the borehole from which they come, the depth, and the nature of the rock. The means are harmonic means (reciprocals of means of reciprocals); these rarely differ by 0·0001 from the arithmetic means. The reasons for preferring them are given below.

It is clear from table 1 that the variation in conductivity among samples of the same kind of rock is much greater than would be accounted for by the uncertainties in the measurements on the individual rocks.* Since the standard errors given have been derived from the consistency of the results for specimens from the same piece of core this must mean that specimens taken from several hundred feet apart differ in conductivity by considerably more than the errors of measurement, and than the errors introduced by the inhomogeneities in the three specimens used in each determination. In order to see how far this variation in conductivity is correlated with a variation in composition thin sections were made of eight of the quartzite specimens and the proportions of the principal minerals determined with the integrating micrometer designed by Dr Dollar (Dollar 1937), to whom I am indebted for its loan. The proportions of quartz by volume are given in the last column of table 1, the remainder consisting mostly of finely divided material which is difficult to identify. Mr Partridge, of the Union Geological Survey, found sericite, chlorite, calcite, feldspar, rutile, epidote and tourmaline in a section made from

* The standard deviation for the Jacoba specimens is 6·1 %, for the Gerhardminnebron dolomites 7·9 %, and for the quartzites 24·7 % (the last are further subdivided below).

Table 1. Conductivities of rocks

No.	Depth ft.	Density g./cm.^{3}	Conductivity cal./cm. sec.° C	Diff. from mean $\times 10^{-4}$	Nature of rock
Jacoba Bore					
1	312	2·808	0·0073	−7	Lava
2	670	2·788	0·0081	+1	,,
3	1072	2·821	0·0080	0	,,
4	1197	2·775	0·0086	+6	,,
	Mean	2·798	0·0080 ± 0·00023		,,
5	1337	2·797	0·0078	−2	Quartz-feldspar porphyry
6	2029	2·837	0·0084	+4	,, ,,
7	2261	2·698	0·0078	−2	,, ,,
8	2477	2·728	0·0076	−4	,, ,,
9	2793	2·805	0·0086	+6	,, ,,
	Mean	2·773	0·0080 ± 0·0023		,, ,,
10	3560	2·919	0·0072	+3	Lava
11	4346	2·917	0·0068	−1	,,
12	5156	2·891	0·0074	+5	,,
13	6065	2·876	0·0069	0	,,
14	6732	2·819	0·0063	−6	,,
	Mean	2·884	0·0069 ± 0·00021		,,
Gerhardminnebron Bore					
15	1000	2·870	0·0105	−4	Dolomite
16	1519	2·827	0·0114	+5	,,
17	2032	2·841	0·0096	−13	,,
18	2500	2·846	0·0120	+11	,,
19	2995	2·844	0·0120	+11	,,
20	3546	2·843	0·0108	−1	,,
21	3991	2·855	0·0106	−3	,,
	Mean	2·847	0·0109 ± 0·00033		,,
22	*	2·711	0·0164	—	Chert 81 % cryst. quartz, rest dolomite or calcite
23	4190	3·012	0·0044	—	Black Reef shale
24	4194	2·642	0·0148	—	,, Quartzite
25	4540	2·667	0·0181	—	Witwatersrand quartzite
26	4991	2·692	0·0157	—	,, ,, 73 % quartz
27	5356	2·673	0·0192	—	,, ,, 78 % ,,
28	6024	2·711	0·0106	—	,, ,,
29	6203	2·716	0·0100	—	,, ,, (feldspathic)
30	6457	2·772	0·0066	—	Shale
31	6616	2·689	0·0089	—	Witwatersrand quartzite (feldspathic) 72 % quartz
32	6823	2·778	0·0102	—	Dyke
33	6872	2·717	0·0111	—	Witwatersrand quartzite 60 % quartz
34	7262	2·690	0·0149	—	,, ,, (chloritoid bearing)
35	7733	2·721	0·0119	—	,, ,, (chloritoid bearing)
36	7963	2·714	0·0144	—	,, ,,
37	8316	2·959	0·0078	—	Dyke
38	8524	2·672	0·0183	—	Witwatersrand quartzite (chloritoid bearing) 89 % quartz
39	8757	2·672	0·0179	—	,, ,, (chloritoid bearing)
40	8919	2·690	0·0151	—	,, ,, 75 % quartz
41	9297	2·723	0·0163	—	,, ,, (chloritoid bearing)
42	9586	2·668	0·0159	—	,, ,, (chloritoid bearing)
43	9789	2·708	0·0146	—	,, ,, (feldspathic) 71 % quartz
44	9857	2·891	0·0106	—	Dyke
45	9914	2·729	0·0109	—	Witwatersrand quartzite (feldspathic)
Dubbeldevlei Bore					
46	2687–5080	2·598	0·0068	0	Granite
47	4000–4887	2·647	0·0073	+5	,,
48	,,	2·635	0·0067	−1	,,
49	4892–4934	2·621	0·0064	−4	,,
	Mean	2·625	0·0068 ± 0·00018		,,

* As no suitable specimen of chert from Gerhardminnebron was available a specimen from 330 ft. in a bore near Lichtenberg was used.

specimen No. 31. There is a fairly well-marked positive correlation between quartz content (p %) and conductivity, the three highest conductivities having the three highest quartz contents, and the three lowest conductivities the three lowest quartz contents. The conductivity is given by

$$k = 0{\cdot}0221 - 0{\cdot}00029(p - 100).$$

Thin sections were also made of the granites Nos. 46, 47 and 49. These contained about 30 % quartz, 64 % orthoclase and 6 % mica by volume. Sections of the Gerhardminnebron dykes showed them to be much silicified and decomposed dolerites.*

In the calculation of heat flow we require the mean conductivity over some hundreds or some thousands of feet of core. If the borehole log shows long apparently homogeneous sections the best method of estimating this mean is to divide the core up according to the lithological changes and to assume that each of the measured conductivities in one of these sections occurs over an equal length. That is, the harmonic mean of the measured conductivities in an apparently homogeneous section of the borehole is taken.† The standard error of the mean conductivity is estimated from the deviations of the individual values from the mean. Straight lines are then fitted to the temperatures in each section and the heat flows calculated. If the different sections agree it is to be presumed that none of them is affected by disturbances such as climatic changes or water circulation. A better value of the heat flow H may then be obtained by combining all the observations. If T be the temperature at depth D,

$$T = T_0 + H\int_0^D \frac{dD}{k}, \tag{1}$$

where k is the thermal conductivity and T_0 is a constant. If the integral be replaced by a sum $\Sigma D/k$ over the homogeneous sections (1) gives a linear relation between the measured quantities T and $\Sigma D/k$ from which the constants T_0 and H may be found.‡ The residuals from this solution are then examined for systematic variations. In this way all the measurements of T and k are used to find H, and only one other unknown, T_0, is introduced into the solution. This method is superior to taking the mean H from the

* I am indebted to Mr Odell for help in this part of the work.

† Allowance for shale bands, dykes, and the like is easily made when their conductivity is known.

‡ If there is a surface layer of unknown conductivity it may, for convenience of computation, be assigned any arbitrary conductivity without affecting H provided we reject the temperature measurements in this layer.

solutions in each section; for we then have a different T_0 for each section, and, since the mean residual is necessarily nearly zero for each section, possible systematic variations are obscured. In fitting the straight lines (1) we have used Eddington's approximation to the least squares solution (in this method the observations are divided into three groups and the mean residual for the two outer groups made equal to each other and to half the residual of the middle group).

An alternative method of solution would be to divide the hole into ranges each containing one conductivity measurement and to adopt that conductivity for each range. If this method were used we should have no means of knowing how well the measured conductivities represent the means for the ranges, whereas if the ranges contain several measured values their scatter gives an indication of the uncertainty in the mean.

Borehole Jacoba No. 3. For the first 135 ft. this hole passes through surface soil, Karroo, and decomposed Ventersdorp Lava. We have no conductivity measurements in this part of the hole and will not consider it further. From 135 to 1276 ft. the rock is a greenish lava, below this there is feldspar-quartz porphyry to 2776 ft., then greenish lava again to 7032 ft. Below this come Witwatersrand quartzites to the bottom of the hole at 7387 ft. Four conductivity determinations were made on the upper lavas (table 1, Nos. 1–4). They show no systematic variation with depth, and we therefore adopt the mean $0{\cdot}0080 \pm 0{\cdot}00023$. The five specimens of feldspar-quartz porphyry also give a mean of $0{\cdot}0080 \pm 0{\cdot}00021$, and the five specimens of the lower lavas give $0{\cdot}0069 \pm 0{\cdot}00021$. The thickness of quartzites penetrated is too little to give an independent estimate of the heat flow. The straight lines through Dr Krige's temperature observations and the derived geothermal intervals and heat flows are

$$157\text{–}1276\text{ ft.}\quad T = 19{\cdot}16 + 3{\cdot}69 \pm 0{\cdot}12 \times 10^{-3}D,$$
$$271 \pm 8{\cdot}8\text{ ft./}^\circ\text{ C},\quad 12{\cdot}10 \pm 0{\cdot}39^\circ\text{ C/km.},\quad 0{\cdot}97 \times 10^{-6}\text{ cal./cm.}^2\text{ sec.}$$

$$1276\text{–}2776\text{ ft.}\quad T = 19{\cdot}11 + 3{\cdot}69 \pm 0{\cdot}06 \times 10^{-3}D,$$
$$271 \pm 4{\cdot}4\text{ ft./}^\circ\text{ C},\quad 12{\cdot}10 \pm 0{\cdot}20^\circ\text{ C/km.},\quad 0{\cdot}97 \times 10^{-6}\text{ cal./cm.}^2\text{ sec.}$$

$$2776\text{–}7032\text{ ft.}\quad T = 17{\cdot}16 + 4{\cdot}29 \times 10^{-3}D,$$
$$233\text{ ft./}^\circ\text{ C},\quad 14{\cdot}07^\circ\text{ C/km.},\quad 0{\cdot}97 \times 10^{-6}\text{ cal/cm.}^2\text{ sec.}$$

The exact agreement of the heat flows is, of course, fortuitous. The residuals from these solutions are shown in column 4 of table 2 (solution I). Those for the upper lavas and for the feldspar-quartz porphyries are quite satisfactory (standard errors 0·097 and 0·057° C). The lower lavas show a clear systematic trend from +0·31° C at the top to a minimum of

$-0{\cdot}24°$ C at 5130 ft. and then to $+0{\cdot}22°$ C at the bottom. If this range be divided into two parts we have the following solutions:

$$2776\text{–}5130 \text{ ft.} \quad T = 17{\cdot}94 + 4{\cdot}08 \times 10^{-3} D,$$
$$245 \text{ ft./° C}, \quad 13{\cdot}39° \text{ C/km.}, \quad 0{\cdot}92 \times 10^{-6} \text{ cal./cm.}^2 \text{ sec.}$$
$$5130\text{–}7032 \text{ ft.} \quad T = 15{\cdot}62 + 4{\cdot}54 \times 10^{-3} D,$$
$$220 \text{ ft./° C}, \quad 14{\cdot}89° \text{ C/km.}, \quad 1{\cdot}03 \times 10^{-6} \text{ cal./cm.}^2 \text{ sec.}$$

As the three ranges have given the same heat flow we combine them all† into one solution. Column 3 of table 2 gives $\Sigma D/k$ and column 5 the residuals from the solution (solution II),

$$T = 19{\cdot}18 + 2{\cdot}903 \times 10^{-5} \Sigma D/k, \quad H = 0{\cdot}95 \times 10^{-6} \text{ cal./cm.}^2 \text{ sec.}$$

TABLE 2. TEMPERATURES IN THE JACOBA BORE

Depth ft.	Observed temp. ° C	$\Sigma D/k$ $\times 10^2$*	Obs. − calc. ° C Solution I	Solution II
50	19·50	62	—	(+0·14)
100	19·60	125	—	(+0·06)
157	19·86	196	+0·12	+0·11
437	20·67	505	−0·10	+0·02
717	21·76	896	−0·05	−0·02
997	22·86	1246	+0·02	+0·06
1276	23·88	1595	{+0·01 / +0·06	+0·07
1779	25·63	2224	−0·04	−0·01
2086	26·76	2608	−0·05	+0·01
2281	27·50	2851	−0·03	+0·04
2776	29·38	3470	{+0·03 / +0·31	+0·13
3244	31·15	4148	+0·07	−0·07
3713	32·97	4828	−0·12	−0·23
4181	34·94	5506	−0·16	−0·22
4653	36·93	6190	−0·19	−0·22
5130	38·93	6882	−0·24	−0·23
5607	41·04	7573	−0·17	−0·11
6083	43·22	8263	−0·04	+0·05
6560	45·42	8954	+0·12	+0·25
7032	47·55	9638	+0·22	+0·39
7259	48·15	9797	—	+0·51
7312	48·29	9834	—	+0·53

* In this and the succeeding tables $\Sigma D/k$ is in ft. cm. ° C sec./cal.

† We have no measurements of the conductivity of the quartzites in this hole but adopt 0·0136, the mean from Gerhardminnebron. As there is only 200 ft. of them an accurate value is not needed.

They are satisfactory except for the systematic curvature in the lower part that has already been referred to. This might be due either to a difference in the heat flow at different depths or to a constant conductivity having been assumed over too great a range; it would be removed if a conductivity of 0·0073 were assumed from 2776 to 5130 and 0·0066 from 5130 to 7032. The means of the first three and last two observed values are 0·0071 and 0·0066; if the means were taken in this way there would be practically no anomaly. The observed variation can therefore be accounted for by changes in the conductivities that are within the permissible limits, and there is no evidence for a variation of heat flow with depth. The scatter of the measured conductivities is, however, too great for it to be said with certainty that the conductivities in the two ranges do really differ, and it seems best to accept the above solution rather than to force an agreement. The scatter of the observed points is due to errors in the temperature measurements and to changes of conductivity over short distances, the systematic trend is due to the adopted mean conductivities not representing the actual mean conductivities over long stretches of the hole. It is clear that the latter is by far the most important cause of error in the derived heat flow, and we therefore compute the standard error of H from the standard deviation of the individual conductivities (table 1); this standard deviation is 0·00046 (6 %) and gives 0·00012 for the standard error of the mean conductivity throughout the hole. The standard error of H found from this is $0{\cdot}015 \times 10^{-6}$ cal./cm.^{2} sec., and we have for the final result from this hole

$$H = 0{\cdot}95 \pm 0{\cdot}015 \times 10^{-6} \text{ cal./cm.}^2 \text{ sec.}$$

The dotted line in figure 1 shows what would be obtained with a hole having the adopted mean conductivities and a heat flow of $0{\cdot}965 \times 10^{-6}$. This shows that the standard error deduced from the scatter of the conductivities gives a reasonable estimate of the errors that would be deduced from a consideration of the differences between the observed and computed temperatures.* The residuals may give an exaggerated view of the importance of the differences between the observed and calculated temperatures; the actual temperatures and the calculated curve are therefore also shown in figure 1.

Doornhoutrivier. This borehole, which is 3·5 miles from Jacoba, is 6028 ft. deep and passes through 5661 ft. of Ventersdorp lava, into Witwatersrand quartzites. Dr Krige's temperature measurements are given in table 3. There is a stream of gas bubbles from about 1000 ft.

* The standard error cannot be computed from these by the ordinary process as they are clearly not independent.

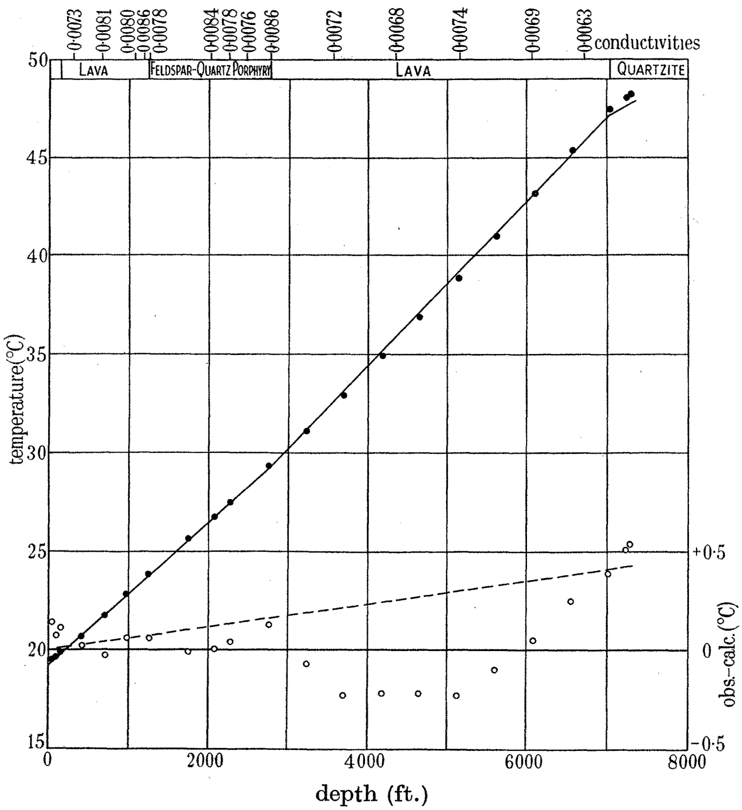

FIGURE 1. Temperatures in Jacoba bore.

● Observed temperatures (left-hand scale); —— solution II; ○ obs.-calc. (solution II) (right-hand scale); ---- calculated temperatures on increasing assumed heat flow from 0·95 to $0{\cdot}965 \times 10^{-6}$ cal./cm.^{2} sec.

TABLE 3. TEMPERATURES IN THE DOORNHOUTRIVIER BORE

Depth ft.	Temp. °C	Obs. − calc. °C
50	20·47	(+1·27)
100	20·56	(+1·17)
499	21·24	(+0·25)
998	23·27	+0·28
1496	24·98	+0·01
1995	26·87	−0·09
2494	28·80	−0·14
2991	30·79	−0·13
3490	32·74	−0·17
3989	34·72	−0·18
4547	36·96	−0·17
5104	39·42	+0·06
5657	41·89	+0·33
5990	42·76	—
6016	42·91	—

which may have disturbed the temperature distribution in the water in the higher part of the hole; the temperature measurements at 50, 100 and 499 ft. have therefore been ignored. There are no conductivity measurements on the core from this bore, but as it is so near Jacoba the conductivities found there may be used. The mean (0·0074) for the first four and the last five specimens from Jacoba was used (there is no porphyry at Doornhoutrivier). The temperature measurements give

$$T = 19{\cdot}00 + 3{\cdot}987 \times 10^{-3}D, \quad 251 \text{ ft.}/^\circ \text{C}, \\ 13{\cdot}08^\circ \text{ C/km.}, \quad 0{\cdot}97 \times 10^{-6} \text{ cal./cm.}^2 \text{ sec.}$$

The residuals show a well-marked curvature, the gradient being less in the upper part of the hole. It is probable that this is due to a decrease of conductivity with depth similar to that found at Jacoba where the top lavas gave 0·0080 and the bottom 0·0069. If we arbitrarily divide the hole into two sections above and below 3490 ft. we get

$$998\text{–}3490 \text{ ft.}, \quad T = 19{\cdot}37 + 3{\cdot}833 \times 10^{-3}D, \\ 261 \text{ ft.}/^\circ \text{C}, \quad 12{\cdot}58^\circ \text{ C/km.}, \quad 1{\cdot}01 \times 10^{-6} \text{ cal./cm.}^2 \text{ sec.}$$

$$3490\text{–}5657 \text{ ft.}, \quad T = 17{\cdot}87 + 4{\cdot}226 \times 10^{-3}D, \\ 237 \text{ ft.}/^\circ \text{C}, \quad 13{\cdot}86^\circ \text{ C/km.}, \quad 0{\cdot}96 \times 10^{-6} \text{ cal./cm.}^2 \text{ sec.}$$

A distribution of conductivity similar to that found at Jacoba thus somewhat over-corrects the curvature (the heat flow is now greater in the upper part), but the agreement is as good as can be expected when the conductivities have to be inferred from those in another hole. The last two readings are in Witwatersrand quartzites with shaly partings. A conductivity of 0·0107 for these would give a heat flow of $0{\cdot}97 \times 10^{-6}$, this is quite a reasonable value for a shaly quartzite (cf. Gerhardminnebron). We may take

$$H = 0{\cdot}97 \pm 0{\cdot}05 \times 10^{-6} \text{ cal./cm.}^2 \text{ sec.}$$

as the mean result.

Gerhardminnebron. This hole passes through 4171 ft. of dolomite and 5813 ft. of quartzites with thin shales and dykes. Except for 67 ft. of Black Reef at the top, the latter belong entirely to the upper part of the Witwatersrand system (see Krige's paper for a detailed section). The Black Reef lies directly on the eroded surface of the Witwatersrand rocks, the Ventersdorp lava being absent. The conductivity of the dolomite shows no systematic change with depth, and the mean $0{\cdot}0111 \pm 0{\cdot}00033$ has been used. The proportion of chert in each interval between the temperature measurements was determined,* and the conductivities calculated. Using these results, the best straight line through the observed temperatures is

$$T = 18{\cdot}85 + 2{\cdot}89 \times 10^{-5} \Sigma D/k, \quad H = 0{\cdot}95 \times 10^{-6} \text{ cal./cm.}^2 \text{ sec.}$$

The 99 and 346 ft. temperature measurements have not been used as the rock is weathered. The residuals, which are given in column 4 of table 4, show a systematic curvature, and from this and from the uncertainty in the mean conductivity the standard error of the heat flow may be estimated at $0{\cdot}04 \times 10^{-6}$.

The Black Reef consists of 58 ft. of quartzite of conductivity 0·0148 (table 1, No. 24) and $8\frac{1}{4}$ ft. of shale of conductivity 0·0044 (table 1, No. 23). The harmonic mean conductivity is therefore 0·0115.

The Witwatersrand quartzites show a range of conductivities from 0·0089 to 0·0181, which is much larger than the range for the Dolomite or Ventersdorp lavas. From information supplied by the African and European Investment Company, Ltd., the quartzites may be classified as "feldspathic", "non-feldspathic" or "chloritoid bearing". The harmonic means of the conductivities are

Feldspathic quartzites (4 specimens)	$0{\cdot}0107 \pm 0{\cdot}0012$
Non-feldspathic quartzites (7 specimens)	$0{\cdot}0142 \pm 0{\cdot}0012$
Chloritoid bearing (6 specimens)	$0{\cdot}0156 \pm 0{\cdot}0009$
Mean of all except feldspathic (13 specimens)	$0{\cdot}0148 \pm 0{\cdot}0008$
Mean of all (17 specimens)	0·0136

There is no evidence for a difference between the chloritoid bearing and the non-feldspathic groups but the feldspathic are definitely lower than either of the others. We have therefore treated the feldspathic quartzites separately from the rest in taking the means, but have combined the non-feldspathic and chloritoid bearing. The conductivities of the quartzites do not vary in a random way down the hole and we have adopted the following mean conductivities:

Depth	k
4237–5792	0·0175
5807–6135	0·0106
6135–6835	0·0095 (feldspathic)
6835–8299	0·0129
8299–9774	0·0167
9774–9917	0·0125 (feldspathic)

* I am very grateful to Mr P. J. Rossouw of the Union Geological Survey for carrying out this very tedious piece of work; until it had been done it was impossible to be sure that the curvature of the temperature-depth relation in the dolomite was not to be ascribed to variations in the proportion of chert. The proportion of chert varied from 0 to 11 %; the mean was 5 %.

Low values are found in the neighbourhood of the shale bands around 6500. These are presumably due to an admixture of shaly matter with the quartzites. The mean conductivities in each of the intervals between Krige's temperature measurements were estimated, using the above mean conductivities for the quartzites and the measured ones in table 1 for the shale bands and dykes.* The mean conductivity of the whole section is 0·0135, which is practically equal to the harmonic mean of the conductivities of all the quartzite specimens (0·0136). With these conductivities the temperature observations from 4236 to 9449 ft. give

$$T = 15{\cdot}07 + 3{\cdot}907 \times 10^{-5} \Sigma D/k, \quad H = 1{\cdot}28 \pm 0{\cdot}025 \times 10^{-6} \text{ cal./cm.}^2 \text{ sec.}$$

The temperatures and the residuals from this solution are shown in figure 2 and in table 4 (solution II). The calculated temperatures follow the irregularities in the observed curve remarkably well for the whole Witwatersrand section, the only systematic difference being a slight deficiency in the calculated temperature between 9000 and 10,000 ft. This is within the errors which might be caused by errors in the adopted mean conductivities. If this solution is extrapolated upwards to the Dolomite section of the hole, very large discrepancies at once arise, as is shown by the large difference in the heat flow of $0{\cdot}95 \times 10^{-6}$ found for the Dolomite and that of $1{\cdot}28 \times 10^{-6}$ for the Witwatersrand. The cause of this discrepancy is discussed below.

Driefontein. Dr Krige has made five temperature measurements in this hole between 99 and 1926 ft. From the surface to 146 ft. the rock is described as "soil, boulders, chert, and chert brecchia in broken and cavernous ground"; we therefore neglect the 99 ft. temperature reading. The other four points lie on the straight line

$$T = 18{\cdot}29 + 2{\cdot}15 \times 10^{-3} D, \quad 465 \text{ ft./}^\circ \text{ C}, \quad 7{\cdot}05^\circ \text{ C/km.}$$

with residuals $+0{\cdot}04$, $-0{\cdot}04$, $-0{\cdot}07$ and $+0{\cdot}07$; there is perhaps some tendency for the temperature gradient to increase with depth. If the conductivity of the Dolomite with its chert be taken as 0·0110 as at Gerhardminnebron, and we allow for a 40 ft. shale band, the mean conductivity is 0·0107 and the heat flow

$$H = 0{\cdot}75 \pm 0{\cdot}06 \times 10^{-6} \text{ cal./cm.}^2 \text{ sec.},$$

where the standard error has been increased to allow for the uncertainty in using the Gerhardminnebron conductivity.

* Where the record gives "quartzite with shale bands" we have taken one-third shale and two-thirds quartzite.

Doornkloof. This borehole has 90 ft. of shales and quartzites of the Pretoria series underlain by Dolomite from 90 to 4060 ft., Black Reef from 4060 to 4100 ft. and Witwatersrand quartzites and shales from 4100 to 6335 ft. It is therefore very similar to the Gerhardminnebron section except

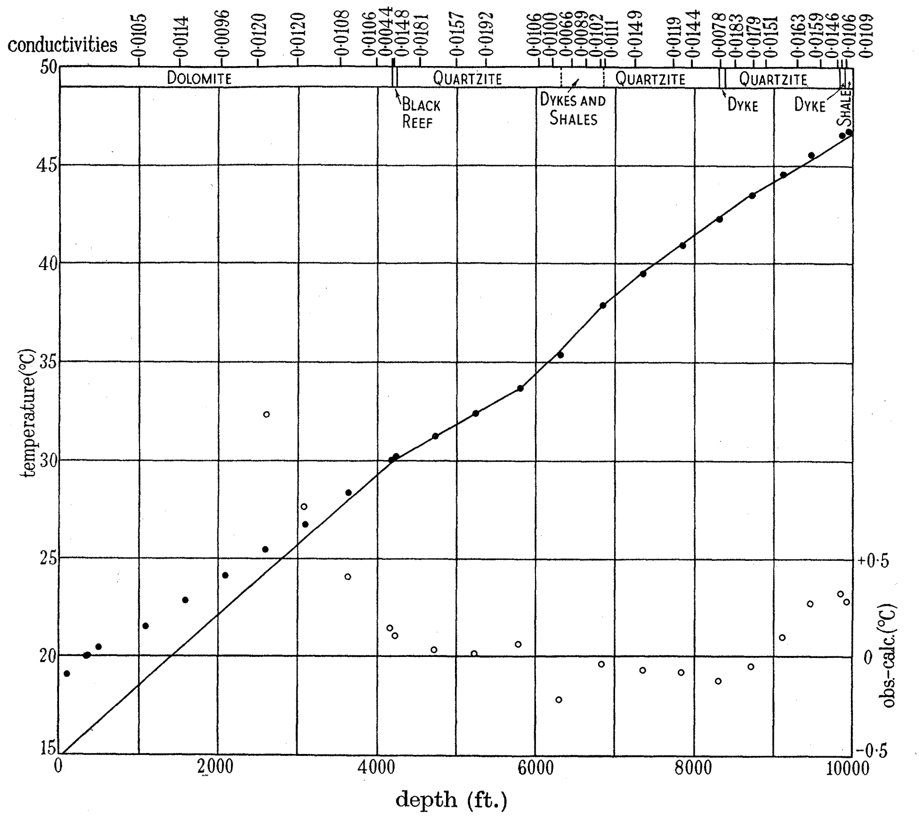

FIGURE 2. Temperatures at Gerhardminnebron.

● Observed temperatures; —— (left-hand scale), solution II;
○ (right-hand scale), observed-calculated temperatures.

for the lesser thickness of the Witwatersrand quartzites. There are considerable thicknesses of dykes (see detailed section in Krige's paper). The temperatures in the Dolomite (neglecting the top reading which is in weathered rock) give

$$T = 19{\cdot}42 + 1{\cdot}93 \times 10^{-3} D, \quad 518 \text{ ft./}^\circ\text{ C}, \quad 6{\cdot}33^\circ \text{ C/km}.$$

Taking the conductivity of Dolomite with its chert as 0·0110 as at Gerhardminnebron, the heat flow would be $0{\cdot}70 \times 10^{-6}$ cal./cm.^{2} sec., but the

TABLE 4. TEMPERATURES IN THE GERHARDMINNEBRON BORE

Depth ft.	Observed temp. ° C	$\Sigma D/k$ $\times 10^2$	Obs. − calc. ° C Solution I	Solution II
99	19·04	89	(−0·01)	(+3·62)
346	20·03	312	(+0·34)	(+3·74)
350	20·03	315	+0·33	+3·73
597	20·35	533	+0·02	+3·19
1096	21·49	977	−0·13	+2·60
1594	22·78	1427	−0·14	+2·13
2092	24·09	1876	−0·12	+1·69
2589	25·38	2329	−0·14	+1·21
3086	26·76	2783	−0·07	+0·81
3637	28·30	3293	−0·01	+0·36
4170	30·04	3787	+0·31	+0·17
4236	30·18	3845	—	+0·08
4733	31·24	4128	—	+0·04
5232	32·40	4432	—	+0·01
5791	33·68	4751	—	+0·04
6306	35·40	5257	—	−0·22
6834	37·86	5853	—	−0·08
7345	39·41	6251	—	−0·09
7837	40·91	6632	—	−0·08
8298	42·27	6990	—	−0·12
8713	43·49	7285	—	−0·05
9103	44·56	7520	—	+0·10
9449	45·54	7729	—	+0·27
9841	46·58*	7981	—	+0·32
9916	46·78*	8044	—	+0·28

* 0·36° C has been added to these two readings to make them consistent with the 9449 ft. readings (see Krige 1939, p. 467).

differences between the observed and calculated temperatures show a range of 0·99° C and clearly do not correspond to anything approaching a steady flow of heat upwards (table 5, solution I).

The lower part of the hole gives a very good straight line

$$T = 16{\cdot}17 + 2{\cdot}88 \pm 0{\cdot}026 \times 10^{-3}D, \quad 347 \text{ ft.}/^\circ\text{ C}, \quad 9{\cdot}45^\circ \text{ C/km}.$$

The standard error of a single point being only 0·054° C (table 5, solution II). If we combine this with the mean conductivity of the Witwatersrand section from Gerhardminnebron (0·0135) we get a heat flow of $1{\cdot}28 \times 10^{-6}$ cal./cm.^{2} sec. A more elaborate reduction was made allowing for the proportions of quartzites, shaly quartzites and dykes in the various sections. The result was

$$T = 15{\cdot}34 + 3{\cdot}50 \times 10^{-5}\Sigma D/k, \quad 1{\cdot}15 \times 10^{-6} \text{ cal./cm.}^2 \text{ sec.}$$

This did not fit the temperature observations as well as the simple solution (see table 5, solution III), apparently because the 423 ft. dyke from 5540 to 5963 ft. has a higher conductivity than the mean of the Gerhardminnebron dykes (0·0093). In view of the uncertainty caused by this we adopt $1{\cdot}20 \pm 0{\cdot}10 \times 10^{-6}$. If either of these solutions is extrapolated upwards to the part of the hole which traverses the Dolomite, large discrepancies appear, as at Gerhardminnebron (see table 5).

Table 5. Temperatures in the Doornkloof bore

Depth ft.	Observed temp. °C	$\Sigma D/k$ $\times 10^2$	Obs. − calc. °C Solution I	Solution II	Solution III
100	18·66	90	(−0·95)	—	+3·00
365	19·96	329	−0·16	—	+3·47
492	20·67	443	+0·30	—	+3·78
496	20·68	447	+0·30	—	+3·77
599	20·63	540	+0·05	—	+3·40
1097	21·38	989	−0·16	—	+2·58
1595	22·22	1442	−0·28	—	+1·83
2093	23·20	1894	−0·26	—	+1·23
2592	24·14	2344	−0·28	—	+0·60
3089	25·26	2792	−0·12	—	+0·15
3588	26·50	3243	+0·16	—	−0·19
4059	27·95	3675	+0·70	+0·08	−0·25
4099	28·06	3702	—	+0·07	−0·24
4616	29·48	4051	—	+0·02	−0·04
5131	30·92	4399	—	+0·05	+0·18
5239	31·23	4515	—	−0·03	+0·09
5538	32·12	4717	—	0·00	+0·27
5961	33·31	5172	—	−0·03	−0·11
6135	33·81	5290	—	−0·03	−0·02
6207	34·07	5367	—	+0·02	−0·03
6282	34·34	5446	—	+0·08	−0·04

In both the Gerhardminnebron and the Doornkloof bores the heat flow is much less in the Dolomite than it is in the underlying quartzites. Two explanations might be suggested for this. First, one might suppose that there had been a climatic change and that the heat flow in the upper part of the hole was therefore disturbed; or secondly, one might suppose that the heat flow in the upper part of the hole had been disturbed by the circulation of water in fissures. The first explanation seems unlikely to be correct, since the change in heat flow sets in sharply at the boundary of the dolomite and the quartzite; also other holes in the same area, such as Jacoba, Doornhoutrivier and Reef Nigel show the same heat flow at all depths. Of the rocks concerned the Dolomite is obviously the most likely to be affected by circulating water; the difficulties caused by water in the sinking of shafts through it are notorious. On the farm Gerhardminnebron there is a spring delivering 13 million gallons per day (Nel, Frommurze, Willemse and Haughton 1935).

Suppose w g. of water to flow vertically every second from a depth D at which the temperature is T to the surface where it is zero. The water gives out wT cal./sec. The heat flow by conduction is kT/D cal./cm.^{2} sec., where k is the thermal conductivity of the rock. The ratio of the heat carried by the water to that conducted is therefore wD/k. If $k = 0{\cdot}01$ and $D = 1$ km. and $w = 10^{-8}$ g., this is 10 %. Thus if 10^{-8} g. of water per sec. flows up or down from every cm.^{2} of the surface, the heat flow will be disturbed by 10 %. 10^{-8} g./cm.^{2} sec. is equivalent to 0·32 g./cm.^{2} year, or 0·54 % of the rainfall at Gerhardminnebron. To account for the discrepancy 1·5 % of the rainfall would have to flow down to the bottom of the Dolomite. Since water does not, on the average, accumulate underground, it must flow down in one place and up in another. Presumably rain falls on a piece of country, sinks into fissures and flows through them to emerge at a lower place as a spring. Most of the water will flow in the upper hundred feet or so where the fissures have been enlarged by solution, but if the open fissures extend to the bottom of the Dolomite some of the water will flow down and up again precisely as an electric current, when it travels through a conductor, does not all travel by the shortest route. In a process where small flows of water percolate through numerous small fissures the water would have the same temperature as the rock at every stage of its journey. Hot springs can only be produced where the flow of water in a single channel is large enough for it to maintain a temperature greater than the surrounding rock. Moreover, most of the water flow takes place near the surface and this cool water would dilute a small amount of warm water from below.

This explanation of the low temperature gradient in the Dolomite also accounts qualitatively for the variation in temperature gradient between the nearby holes at Gerhardminnebron, Doornkloof and Driefontein, and for the apparent increase of their heat flow with depth. If, as seems very probable, this explanation is correct, the heat flows in the Dolomite at Gerhardminnebron, Doornkloof and Driefontein should be disregarded. It is unlikely that any appreciable proportion of the rainfall finds its way to great depths in the other rocks considered; this is confirmed by the constancy of the heat flow with depth. It will clearly be wise to avoid bores in dolomite or limestone in future work.

Dubbeldevlei bore. This bore, which is about 500 miles south-west of the others, is of considerable interest, as it shows gradients from 3 to 5 times those on the Witwatersrand. It passes through Karroo sandstones and shales from the surface to 2137 ft., where it enters grits which are doubtfully correlated with the Fish River series; at 2687 ft. it passes into the "Old Granite" and continues to 5080 ft. The descriptions of the strata passed through are vague and samples were only available of the granite part. The depths from which they came are not known, and we adopt the harmonic mean $0{\cdot}0068 \pm 0{\cdot}00018$. Table 6 gives the observed temperatures (Krige 1924) and their residuals from

$$T = 36{\cdot}21 + 6{\cdot}79 \pm 0{\cdot}07 \times 10^{-3}D, \quad 147\text{ ft./}^\circ\text{ C}, \quad 22{\cdot}3^\circ\text{ C/km.}$$

TABLE 6. TEMPERATURES IN THE DUBBELDEVLEI BORE

Depth ft.	Observed temp. °C	Obs. − calc. °C
2797	55·35	+0·15
3007	56·65	+0·02
3509	59·85	−0·19
3752	61·85	+0·16
4011	63·35	−0·09
4255	65·05	−0·05
4512	66·70	−0·15
4753	68·45	−0·03
4912	69·75	+0·19

There is no systematic departure from a straight line and the standard error of one observation is 0·14° C. This is partly due to errors in measuring the temperatures (which depend on one thermometer) and partly to variations in conductivity. It gives a standard error of 1 % to the temperature gradient and to H. The standard error of H from the scatter in the conductivities is 2·7 %. The heat flow is therefore

$$1{\cdot}52 \pm 0{\cdot}044 \times 10^{-6}\text{ cal./cm.}^2\text{ sec.}$$

The mean gradient in the part of the hole above the granite is about 41° C/km. This would require a conductivity of 0·0037 to give the same heat flow as the lower part. Such a conductivity is quite a reasonable value for the Karroo shales (cf. Benfield's values for Scottish coal measure shales). Anderson (1934) has also discussed this bore.

Reef-Nigel bore. Temperature measurements have been made in this hole by Weiss (1938) using a resistance thermometer. The first 100–200 ft. are in dolomite; this part will not be considered further. Below this comes

Ventersdorp lava to 1400 ft., then Witwatersrand quartzites to the bottom of the hole. Down to 3050 ft. the latter are fairly free from dykes (67 ft. or 4·1 % of dolerite between 1400 and 3050 ft.). Below this there are great thicknesses of dolerite (933 ft. or 62·2 % between 3050 and 4550 ft.). We therefore discuss the temperature measurements in three sections.

First, the twelve measurements between 300 and 1400 ft. give

$$T = 17{\cdot}72 + 4{\cdot}22 \pm 0{\cdot}052 \times 10^{-3}, \quad 237 \pm 2{\cdot}9 \text{ ft./}^\circ \text{ C}, \quad 13{\cdot}85 \pm 0{\cdot}18^\circ \text{ C/km.}$$

The differences between the observed temperatures and this expression (column 4 of table 7) show no systematic trend (six changes of sign in twelve values), and the standard error is only 0·059° C. If the mean conductivity of the lavas is assumed to be the same as at Jacoba (0·0074), a heat flow of $1{\cdot}02 \times 10^{-6}$ cal./cm.^{2} sec. is obtained.

The eighteen measurements between 1400 and 3050 ft. rejecting the reading at 1900 ft. give

$$T = 20{\cdot}29 + 2{\cdot}46 \times 10^{-3}D, \quad 406 \text{ ft./}^\circ \text{ C}, \quad 8{\cdot}07^\circ \text{ C/km.}$$

The differences between the observed temperatures and this expression show a definite curvature, the rate of increase of temperature decreasing with increasing depth, this is presumably due to variations in the conductivity of the quartzite similar to those found at Gerhardminnebron. The standard error of one observation (calculated from the residuals without allowing for the curvature) is 0·070° C. If the conductivity of the quartzites be taken as 0·0136 and of the dolerites as 0·0094 the heat flow is found to be $1{\cdot}08 \times 10^{-6}$ cal./cm.^{2} sec.

The bottom section of the hole has a large proportion of dolerite whose mean conductivity is very uncertain owing to only three specimens having been measured (Nos. 32, 37 and 44 in table 1 from Gerhardminnebron). The eighteen temperature measurements between 3050 and 4550 give

$$T = 19{\cdot}18 + 2{\cdot}81 \pm 0{\cdot}084 \times 10^{-3}D, \quad 356 \pm 10 \text{ ft./}^\circ \text{ C}, \quad 9{\cdot}22 \pm 0{\cdot}27^\circ \text{ C/km.}$$

The standard error of one observation is 0·062° C and there is no systematic trend. Assuming the conductivities of quartzite and dolerite to be 0·0136 and 0·0094, the mean conductivity is 0·0106 and the heat flow is $0{\cdot}98 \times 10^{-6}$ cal./cm.^{2} sec. The solution from all points between 300 and 4550 ft. (rejecting that at 1900 ft.) is

$$T = 17{\cdot}78 + 3{\cdot}151 \times 10^{-5} \Sigma D/k, \quad H = 1{\cdot}03 \times 10^{-6} \text{ cal./cm.}^2 \text{ sec.}$$

The standard error is 0·13° C; the residuals are given in column 5 of table 7 (solution II) and $\Sigma D/k$ in column 3.

Table 7. Temperatures in the Reef-Nigel bore

Depth ft.	Observed temp. °C	$\Sigma D/k \times 10^2$	Obs. – calc. °C Solution I	Obs. – calc. °C Solution II
100	18·12	(135)	(–0·02)	(–0·09)
200	18·32	(270)	(–0·24)	(–0·31)
300	19·10	405	+0·11	+0·04
400	19·42	540	+0·01	–0·06
500	19·76	676	–0·07	–0·15
600	20·20	811	–0·05	–0·14
700	20·70	946	+0·03	–0·06
800	21·13	1081	+0·03	–0·06
900	21·54	1216	–0·02	–0·07
1000	21·89	1351	–0·05	–0·15
1100	22·43	1486	+0·07	–0·03
1200	22·77	1621	–0·01	–0·12
1300	23·14	1756	–0·07	–0·17
1400	23·65	1880	{+0·02 –0·08	–0·05
1500	23·92	1956	–0·06	–0·02
1600	24·11	2030	–0·12	–0·07
1700	24·43	2103	–0·04	+0·02
1800	24·70	2176	–0·02	+0·06
1900	(25·59)	2252	(+0·63)	(+0·71)
2000	25·31	2329	+0·10	+0·12
2090	25·54	2395	+0·11	+0·21
2190	25·79	2469	+0·11	+0·23
2280	25·99	2535	+0·09	+0·22
2380	26·19	2608	+0·04	+0·19
2480	26·41	2682	+0·02	+0·18
2573	26·60	2750	–0·02	+0·15
2660	26·80	2814	–0·03	+0·15
2760	27·01	2887	–0·07	+0·13
2860	27·30	2961	–0·03	+0·19
2960	27·51	3043	–0·06	+0·14
3050	27·79	3114	{0·00 +0·04	+0·20
3150	27·97	3220	–0·06	+0·04
3245	28·22	3321	–0·08	–0·02
3330	28·54	3412	0·00	+0·01
3430	28·89	3518	+0·07	+0·02
3520	29·08	3614	+0·01	–0·09
3615	29·21	3700	–0·13	–0·23
3705	29·53	3766	–0·06	–0·12
3790	29·88	3829	+0·05	+0·03
3880	30·18	3897	+0·10	+0·12
3970	30·32	3993	–0·01	–0·04
4050	30·57	4078	+0·01	–0·06
4140	30·82	4174	+0·01	–0·11
4220	31·07	4259	+0·03	–0·13
4315	31·30	4341	0·00	–0·16
4390	31·42	4396	–0·10	–0·21
4460	31·74	4448	+0·03	–0·06
4550	31·99	4522	+0·02	–0·04
4630	31·91	4593	—	(–0·34)
4710	32·14	4714	—	(–0·49)

Considering that the conductivities have been derived from measurements made on samples from Gerhardminnebron and Jacoba which are 80 miles to the west, the agreement between the heat flows found from the three sections is good. The standard error of the mean is almost entirely due to the uncertainty in the assumed conductivities. Making a conservative estimate of this, the final result for this bore is

$$H = 1\cdot03 \pm 0\cdot07 \times 10^{-6} \text{ cal./cm.}^2 \text{ sec.}$$

The absence of any discontinuity in the heat flow at the lava-quartzite junction confirms the conclusion that it was the dolomite that was anomalous at Gerhardminnebron and Doornkloof.

4. Discussion

The heat flows, neglecting those in the Dolomite, are

Table 8

	cal./cm.^{2} sec.
Jacoba	$0\cdot95 \pm 0\cdot015 \times 10^{-6}$
Doornhoutrivier	$0\cdot97 \pm 0\cdot05$,,
Gerhardminnebron	$1\cdot28 \pm 0\cdot025$,,
Doornkloof	$1\cdot20 \pm 0\cdot10$,,
Reef-Nigel	$1\cdot03 \pm 0\cdot07$,,
Dubbeldevlei*	$1\cdot52 \pm 0\cdot044$,,
Mean	$1\cdot16 \pm 0\cdot09$,,

The mean heat flow of $1\cdot16 \times 10^{-6}$ cal./cm.^{2} sec. is considerably lower than those generally quoted (various authorities give values ranging from $1\cdot6$ to $3\cdot4 \times 10^{-6}$) but is not very different from Benfield's value of $0\cdot98 \pm 0\cdot12 \times 10^{-3}$ (or $1\cdot42 \times 10^{-6}$ after removing the effect of the ice age) from measurements in Great Britain. It is therefore likely that the heat flow has been considerably overestimated in the past and that the low values found by Benfield are general. Some of this heat flow must be due to the original heat of the earth, this part may be estimated from data given by Jeffreys (1929, p. 151) at $0\cdot25 \times 10^{-6}$ cal./cm.^{2} sec., leaving $0\cdot91 \times 10^{-6}$ cal./cm.^{2} sec. to be accounted for by radioactivity. The radioactivity of sedimentary and volcanic rocks is generally small. We shall consider first of all the contribution of the granite which underlies them. The mean radium content of eighteen specimens of granite from various

* Owing to the small number of specimens available this value is less reliable than the rest. The small standard error is due to the agreement of the conductivity values which may be fortuitous.

places in the Cape Province* was found by Immelman (1934) to be $2{\cdot}36 \pm 0{\cdot}16 \times 10^{-12}$ g. Ra/g. rock; this is almost identical with Jeffreys's (1936) mean of $2{\cdot}38 \pm 0{\cdot}16 \times 10^{-12}$ from specimens from all over the world. No measurements have been made on the thorium content of South African granites, but we may take Jeffreys's mean of $1{\cdot}76 \pm 0{\cdot}22 \times 10^{-5}$ g. Th/g. rock. The potassium content is about 3 %. The heat produced by the uranium equivalent to 1 g. of radium in equilibrium with its products is 0·068 cal./sec., that produced by 1 g. of thorium and its products is $6{\cdot}4 \times 10^{-9}$ cal./sec., and that produced by 1 g. of potassium is about 3×10^{-13} cal./sec. The heat produced by 1 cm.³ of granite (density 2·63 g./cm.³) is therefore $0{\cdot}74 \times 10^{-12}$ cal./sec. If H_1 cal./cm.² sec. is generated in the granitic layer and H_2 comes from below, then

$$H = H_1 + H_2 = 1{\cdot}16 \times 10^{-6},$$
$$H_1 = 0{\cdot}74 \times 10^{-12} s,$$

where s is the thickness of the granite in cm. The temperature difference between the top and bottom of the granite is

$$T = s(H + H_2)/2k,$$

with $k = 0{\cdot}0068$ (mean for Dubbeldevlei granite) we get the following values of s and T as a function of H_2/H:

H_2/H	s km.	T °C
0·0	15·7	134
0·2	12·5	128
0·4	9·4	112
0·6	6·3	86
0·8	3·1	48
1·0	0·0	0

As about 0·25 cal./cm.² sec. is contributed by the earth's original heat, H_2/H cannot be less than 0·2, and the maximum thickness of the granitic layer is 12 km. To obtain this maximum it is necessary that the layers below the granite should be entirely devoid of radioactivity, and to get any reasonable thickness a very low radioactivity must be assumed. The temperature of the base of the granite cannot exceed 130° C. This temperature is much less than that obtained by similar methods from the rather vague data hitherto available.† No reasonable adjustment of the data will give

* Unfortunately these are Nama granites and are not contemporaneous with the Old Granite that underlies the Transvaal.

† Jeffreys (1929, p. 152) gets 280° C and Anderson (1938) gets 384° C.

a substantially different result* and there is therefore no possibility of melting the granitic layer unless it is forced many kilometres below its normal position. The temperatures below the granite depend very much on the proportion of the whole radioactivity that is below the granite, and how it is distributed; they will not be further discussed here (see Bullard 1939).

The delay found by seismologists in the starting of P_n relative to P_g in near earthquakes, and the P–pP intervals in deep focus earthquakes indicate that the homogeneous material in which P_n travels (the "lower layer") has its upper surface at such a depth that a compressional wave takes about 5·4 sec. (Jeffreys 1939) to travel from it to the surface. If there were no "intermediate layer" this would correspond to 30 km. of granite (velocity 5·57 km./sec.). The heat flow measurements show that there cannot be more than 12 km. of rock with the radioactivity of surface granite, and that there must therefore be an "intermediate layer" between the bottom of the granite and the top of the lower layer. The confirmation of the existence of a layer between the granite and the lower layer is valuable, as the evidence for it has previously depended on the reality of rather dubious near earthquake pulses such as P^*. In order to make the information more precise it is desirable that measurements of the conductivity and radioactive content of specimens of the "Old Granite" from the Transvaal should be made, and that the time-distance curve for near earthquakes should be investigated there.

It is often only possible to make a temperature measurement at the bottom of a bore. The gradient has then to be calculated from this and from the air temperature at the surface. In such cases it is usual to assume that the intercept of a temperature depth plot (the T_0 of (1) above) is 1·0° C above the mean air temperature (Benfield 1939). The heat flow is then found from

$$H = \frac{T_1 - T_2 - 1{\cdot}0}{\Sigma D/k},$$

where T_1 is the temperature at the bottom of the hole and T_2 is the mean air temperature. It is of interest to calculate H in this manner for the bores considered in this paper, as comparison with the values obtained above will give an indication of the errors that are likely to be introduced by such a procedure. Only the Jacoba, Doornhoutrivier and Reef Nigel bores are suitable for the test, as no conductivities have been measured

* A cover of 3 km. of clays and shales and sands on the top of the granite would give an extra 100° C.

in the upper part of the Dubbeldevlei bore and the temperatures in the other bores are disturbed, probably by water circulation in the Dolomite.

The *Reseau Mondial* gives the mean air temperature at Johannesburg at a height of 1806 m. above sea level as 15·0° C. The air temperatures at the bores have been calculated from this assuming a lapse rate of 6° C/km. The data are as follows:

	Jacoba	Doornhoutrivier	Reef-Nigel
Height, m.	1311 ± 30	1300 ± 30	1565
T_1 °C	48·3	42·9	32·1
$T_2 + 1{\cdot}0°$ C	19·0	19·2	17·4
$\Sigma D/k$ cm.^{2} °C sec./cal.	3·00	2·47*	$1{\cdot}44 \times 10^7$
H cal./cm.^{2} sec.	0·98	0·96	$1{\cdot}02 \times 10^{-6}$
H (table 8) cal./cm.^{2} sec.	0·95	0·97	$1{\cdot}06 \times 10^{-6}$

* Mean $1/k$ assumed the same as at Jacoba.

The result (line 5) is never more than 4 % different from that obtained by using all the temperature data (line 6). The method is therefore reliable if there is a temperature difference of the order of 20° C between the top and bottom of the hole.

The heat flows found in the boreholes considered above vary from 0·95 to $1{\cdot}52 \times 10^{-6}$ cal./cm.^{2} sec. It is desirable to consider how much of this variation is due to differences in the rate of heat generation and how much to other causes.

The "lagging" effect of topographic irregularities on the earth's surface causes more heat to flow out under valleys than under hills. A method of calculating the amount of the disturbance has been given by Jeffreys (1937) and applied to a number of special cases by Bullard (1938). The experience gained in the calculations indicates that the effect is quite negligible for the bores considered here, and detailed calculations have not been made.

If the rocks round a borehole do not consist of horizontal homogeneous layers of indefinite extent the heat flow is disturbed, more flowing out in regions occupied by good conductors and less in those occupied by bad conductors. Since the heat is mostly generated within 10 km. of the surface it is only necessary to consider the structure within a radius of 10 or 20 km. from the bore. It is not practical to calculate the magnitude of the disturbance except in highly idealized cases, but a good idea of the amount to be expected may be obtained by experiments with a model. Figure 3 shows a section from south-east to north-west through the Gerhardminnebron borehole. This section has been prepared from the information given on the Geological Survey sheet 53 Ventersdorp, the details are somewhat hypothetical but should be close enough to the truth to give an idea of the magnitude of the disturbance. If it is assumed that the rocks shown in figure 3 extend indefinitely in a direction at right angles to the diagram the heat flow will be similar to the flow of electricity in a thin sheet of homogeneous material whose thickness is proportional to the thermal conductivity. A trough was therefore prepared and filled with salt solution in such a way that its depth was proportional to the conductivities in figure 3.* The edge of the trough representing ground level was made an

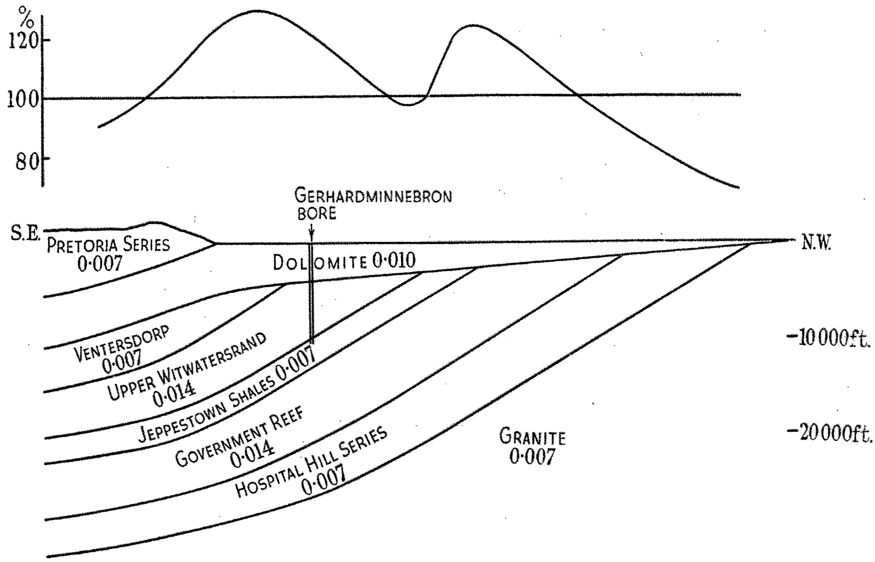

FIGURE 3. Section near Gerhardminnebron. Total length of section = 11 miles; horizontal and vertical scales equal.

equipotential and current was fed into the opposite edge through a number of electrodes connected in series with resistances large compared with that of the trough. These boundary conditions correspond to a constant temperature at the earth's surface, and the same rate of generation of heat under each unit area of the earth's surface in the thermal problem. The mean potential gradient between the surface and a distance corresponding to a depth of 10,000 ft. was then found and multiplied by the conductivity of the section. The result is shown in figure 3 as a percentage of the average for the whole section, this represents the heat flow at any point as a percentage of the average that would be obtained from measurements in

* By an oversight the model was made as if the Hospital Hill Series had a conductivity of 0·014, the effect of this will be small except at the north-west end of the figure and it was not thought worth while to do the work again.

numerous boreholes spread over the area. The disturbance at the Gerhardminnebron bore is 20 %. The uncertainties due to uncertainties in the assumed structure and the crudeness of the model are such that it would not be safe to apply this as a correction, but the curve of figure 3 does indicate that this is the order of the disturbance to be expected in a rather extreme case.

The two processes so far considered divert the heat flow from one place to another, and cause variations in the measured heat flow without corresponding variations in the production of heat. In addition to these there will be real variations in the rate of production due to variations in the thickness of granite and to variations in its radioactive content. For example, the surface of the granite is 34,000 ft. nearer the surface near the north-west end of figure 3 than it is at the south-east end. Other things being equal there will therefore be an extra heat flow at the north-west end corresponding to the heat generated in this granite (about $1{\cdot}1 \times 10^{-6}$ cal./cm.^{2} or 86 % of the measured heat flow at Gerhardminnebron). This variation is in addition to that shown in figure 3.*

The known causes of variation are therefore amply sufficient to account for the observed variation and it is hardly worth discussing them in more detail until more information is available about the radioactive contents of the various rocks and their variations from place to place.

These measurements were carried out at the Bernard Price Institute in Johannesburg, and I am indebted to Professor Schonland for inviting me to visit the Institute and for providing facilities for the work. The specimens were prepared in the Mineral Research Laboratory of the University of the Witwatersrand, and I am indebted to Professor Stanley for allowing this to be done, and to Mr Lazenger and Mr Wilson for advice on the method of doing it.

Throughout I have had the advantage of the co-operation and advice of Dr Krige of the South African Geological Survey. The accuracy of the temperature measurements, which he has allowed me to use before publication, is indicated by the consistency of the results.

Finally, I am indebted to the African and European Investment Co. Ltd., the Anglo-American Corporation of South Africa, Ltd. and the Anglo-Transvaal Consolidated Investment Company, Ltd. for providing pieces of core and boreholes logs.

* Strictly, the effects cannot be simply added and the combined effect should be determined with the model by a distribution of electrodes over the area representing the granite; it has not been thought worth while to do this.

Summary

The thermal conductivities of forty-nine rocks from bores in South Africa have been measured. The mean heat flow calculated from these and from Krige's and Weiss's temperature measurements is $1 \cdot 16 \times 10^{-6}$ cal./cm.^{2} sec. This is much lower than the values usually quoted but is near Benfield's mean of $0 \cdot 98 \times 10^{-6}$ for bores in Europe. There is no foundation for the common opinion that the heat flow is lower in South Africa than in Europe. It is likely that many of the other supposed cases of abnormal heat flow are also unfounded. The low value of the heat flow shows that there cannot be more than 12 km. of rock with the radioactivity of surface granite under South Africa. This supports the reality of the "Intermediate Layer" of seismology and leads to very low temperatures at the base of the granitic layer.

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