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A CLIMATIC CORRECTION ON TEMPERATURE GRADIENTS USING SURFACE-TEMPERATURE SERIES OF VARIOUS PERIODS

CHR. CLAUSER

Institut für Angewandte Geophysik, Petrologie und Lagerstättenforschung, Technische Universität Berlin, Ackerstr. 71–76, D-1000 Berlin 65 (Federal Republic of Germany)

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ABSTRACT

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The instationary heat diffusion equation is being solved for a horizontally stratified medium with a varying, non periodic surface-temperature as time-dependent boundary condition.

A solution is sought by means of the Laplace transform, because most non-periodic variations, as a rule, cannot be described analytically. In the Laplace-transform domain the solution of the problem is found as the product of the transforms of the surface-temperature variation and the subsurface-response function. The temperature–depth profile for any instant after the onset of the surface-temperature variation can be found by discrete inverse Fourier-transformation of this product series (since the subsurface-response function is a stable filter and the time series of the surface-temperature variation is finite).

The results are being demonstrated using surface-temperature series of different periods. Depending on the length and resolution of the specific data-series, the corrections cover the whole spectrum from short-term, over middle-term, to long-term corrections.

The same algorithm can be used to compute the corrections for the annual temperature wave as well as for the ice ages. The results show that between these two periods there are short- and middle-term periodicities with relevant amplitudes, whose influence should not be neglected without consideration.

If only air temperature series are available, a method is proposed to determine a correction using the mean surface temperature and long term air temperature means.

INTRODUCTION

The need for climatic corrections on measured temperature data from vertical profiles has long been recognized. Before determining the terrestrial heat flow density by multiplying the thermal conductivity with the temperature gradient, all downward travelling temperature disturbances have to be eliminated from the temperature data. These disturbances are being caused by temperature variations at the earth's surface. They follow a diffusion process and are damped and phase-shifted as they propagate into the subsurface.

Up to now, corrections have been computed for two types of temperature variations: harmonic and step functions. For both of them there are analytical solutions to the diffusion problem. The actual surface-temperature variation can be approximated by a series of harmonic or step functions, and the final solution, i.e. the noise field, is then gained by the superposition of all the individual solutions.

Harmonic functions are mainly used in correcting for the annual temperature wave, whereas step functions are more customary for longer periods, as in paleoclimatic corrections.

There is an amazing number of continous temperature records available today that go back 300 years for air temperatures and over 100 years for surface temperatures respectively. Therefore the idea appeared attractive to use these records directly, without approximation for a temperature correction. It also seemed worthwhile to investigate the influence of temperature variations whose periods lie somewhere between one year and some hundred years. Especially temperature profiles from shallow boreholes might well be disturbed by middle-term fluctuations.

Therefore an algorithm was sought that would compute the noise field in a horizontally stratified substratum due to an arbitrary surface-temperature variation.

The basic idea was to seek a solution of the instationary heat conduction equation with time-dependent boundary condition in the Laplace-transform domain. The solution is obtained by multiplying the transform of the surface-temperature variation, the "input", with an underground response function. This response function, which describes the downward propagation of the surface-temperature variation into the substratum and contains its physical parameters, is a stable filter. Moreover, since the temperature records are finite, it is possible to use a numerical inverse Fourier-transformation to obtain the solution in the time domain. This solution gives the noise field at any depth for any time after the onset of the temperature variation.

SOME THEORY

In a horizontally stratified medium whose N layers are homogeneous and isotropic, the temperature $u(z, t)$ as a function of depth z (positive downward) and time t must satisfy the diffusion equation:

$$\partial u_n(z, t)/\partial t = a_n \cdot \partial^2 u_n(z, t)/\partial z^2 \qquad \begin{matrix} n = 1, 2, \ldots, N \\ h_{n-1} \leqslant z \leqslant h_n \end{matrix} \tag{1}$$

where h_n is the depth of a layer-interface and a_n the thermal diffusivity of the layer.

At the surface the temperature has to be equal to the surface-temperature variation $f(t)$, and in infinity it must vanish:

$$u_1(0, t) = f(t) \qquad t > 0 \tag{2}$$

$$\lim_{z \to \infty} u_n(z, t) = 0 \tag{3}$$

Before the onset of the surface-temperature variation the subsurface temperature is assumed to be zero throughout (any other constant temperature would do as well):

$$u_n(z,t)=0, \qquad n=1,2,\ldots,N; \qquad t\leqslant 0$$

At the layer-interfaces the temperature and the heat flow density:

$$q_n(z,t)=\lambda_n\cdot\partial u_n(z,t)/\partial z$$

have to be continuous (λ: thermal conductivity):

$$u_n(h_n,t)=u_{n+1}(h_n,t) \tag{4}$$

$$q_n(h_n,t)=q_{n+1}(h_n,t) \tag{5}$$

The problem being stated, all equations can now be Laplace-transformed and a solution can be given. In the Laplace-transform domain variables will be denoted by capital letters and the transform variable by S. The solution of the transformed problem, the subsurface temperature as a function of depth and complex "frequency" S is:

$$U_n(z,S)=F(S)\cdot\left[C\cdot\left\{A_n\exp\left[-z(S/a_n)^{1/2}\right]+B_n\exp\left[z(S/a_n)^{1/2}\right]\right\}\right] \tag{6}$$

Thus the solution can indeed be considered as the product of an input $F(S)$ with an underground-response function (the right-hand term).

In eq. 6 there are $2N+1$ undetermined coefficients A_n, B_n and C. The transformed boundary condition (2), however, gives C as:

$$C=1/(A_1+B_1)$$

and from boundary condition (3) it follows that $B_N=0$. This leaves $2N-1$ independent coefficients to be determined. Using the transforms of the continuity conditions (4) and (5) and inserting them into each other, one can derive recursion formulas for the coefficients A_n and B_n (Mundry, 1974; Daniels, 1978):

$$A_n=\frac{\lambda_n p_n+\lambda_{n+1}p_{n+1}}{2\lambda_n p_n}\cdot\exp[h_n(p_n-p_{n+1})]A_{n+1}$$
$$+\frac{\lambda_n p_n-\lambda_{n+1}p_{n+1}}{2\lambda_n p_n}\cdot\exp[h_n(p_n+p_{n+1})]B_{n+1} \tag{7}$$

$$B_n=\frac{\lambda_n p_n-\lambda_{n+1}p_{n+1}}{2\lambda_n p_n}\cdot\exp[-h_n(p_n+p_{n+1})]A_{n+1}$$
$$+\frac{\lambda_n p_n+\lambda_{n+1}p_{n+1}}{2\lambda_n p_n}\cdot\exp[-h_n(p_n-p_{n+1})]B_{n+1} \tag{8}$$

where $p_n=(S/a_n)^{1/2}$.

Setting $A_N=1$ (or any other constant value), all remaining coefficients can be determined from eqs. 7 and 8. The arbitrariness in the choice of the value for A_N is removed by the "scale factor" C; since the coefficients A_1 and B_1 are functions of the physical parameters of all layers, so is C. Thus the respective value of A_N is taken into account when the solution is fitted to the boundary condition (2) by the normalizing constant C. When we are numerically evaluating formulas (7) and (8), it is sometimes advantageous to choose a much higher value for A_N than one to prevent numerical under- or overflows in the exponential expressions.

THE TEMPERATURE RECORDS

For demonstration purposes, temperature records were chosen from three classes of period-ranges:

(1) monthly means of lake bottom temperatures (depth about 40 m) in Lake Kinneret from 1966–1976 (Stanhill, 1969; C. Serruya, 1971; C. Serruya et al., 1974; S. Serruya, 1975; Villinger, 1977);

(2) annual means of air temperatures in the south-German cities of Stuttgart (1791–1955) and Karlsruhe (1799–1955) as given by Bider et al. (1959)—Fig. 8a shows the location of both cities;

(3) annual mean air temperatures from 10,000 B.C. until 1956 as inferred by

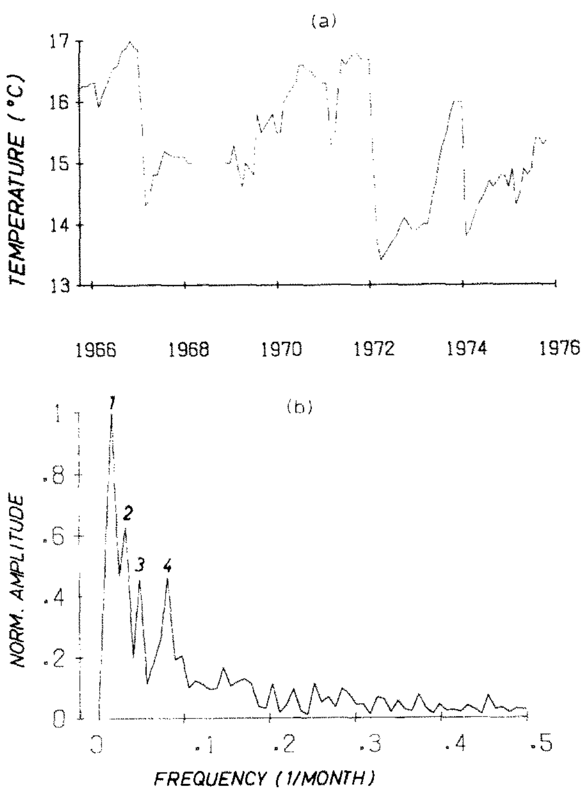

Fig. 1 a. Monthly means of bottom temperatures in the central part of Lake Kinneret in Israel (depth ≈ 40 m). b. Amplitude spectrum of the time series in Fig. 1a with its mean value of 15.32°C subtracted; see Table I for amplitudes and periods of the numbered peaks.

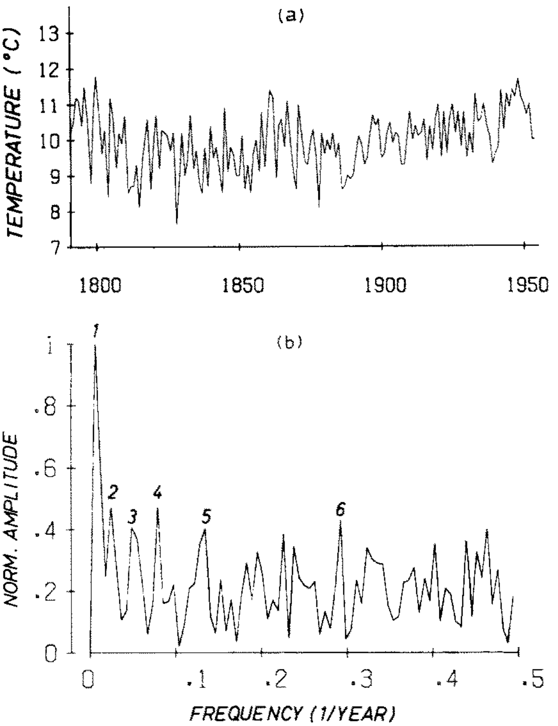

Fig. 2 a. Annual mean air temperatures at Stuttgart (Germany) from 1791 to 1955. b. Amplitude spectrum of the time series in Fig. 2a with its mean value of 9.95°C subtracted; see Table I for amplitudes and periods of the numbered peaks.

Lamb et al. (1966) for central Europe from various temperature indicators.

Figures 1a, 2a, 3a and 4a show the temperature records, while Figs. 1b, 2b, 3b and 4b show the amplitude spectra of these records with their mean temperatures subtracted (this gives zero amplitude for zero frequency). The periods and amplitudes of the numbered peaks are given in Table I as well as the mean temperatures of the respective records.

A first inspection of Fig. 1a shows a rather irregular annual temperature pattern, an impression which is confirmed by Fig. 1b, where the annual temperature wave's peak (No. *4*) is not dominating.

Figures 2a and 3a show that the annual mean temperatures at Stuttgart and Karlsruhe are not simply scattered around a long-term mean; instead, there seems to be a curved trend in Stuttgart with a minimum around the year 1810, and a linear one in Karlsruhe.

Figure 4a shows basically the double-step shaped temperature history from the last intermediate period over the last glaciation up to now. The main periodicities of the amplitude-spectrum in Fig. 4b are therefore rather long-period.

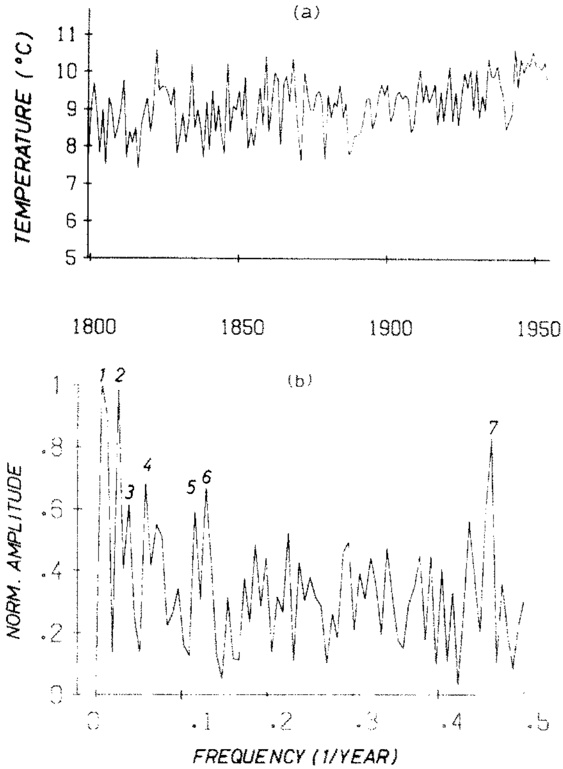

Fig. 3. a. Annual mean air temperatures at Karlsruhe (Germany) from 1799 to 1955. b. Amplitude spectrum of the time series in Fig. 3a with its mean value of 9.12°C subtracted; see Table I for amplitudes and periods of the numbered peaks.

RESULTS: THE SURFACE NOISE-GRADIENTS

The Fourier-transforms of the above temperature records were used as input to evaluate eq. 6 for three underground models. The coefficients A_n, B_n and C were determined with the recursion formulas (7) and (8) for each model. The model parameters are given in Table II. For Lake Kinneret a homogeneous halfspace model was used, while for the longer period records two 4-layer models were chosen. The physical parameters for the latter, models Neustadt 4 and Urach 4 were taken from the drillholes Landau 2 close to Karlsruhe (Sattel, 1979) and Urach (Hänel, 1980). The halfspace parameters for Lake Kinneret were synthesized from previous investigations (Ben Avraham et al., 1977).

All noise fields shown were computed for the instant when the temperature variation terminated.

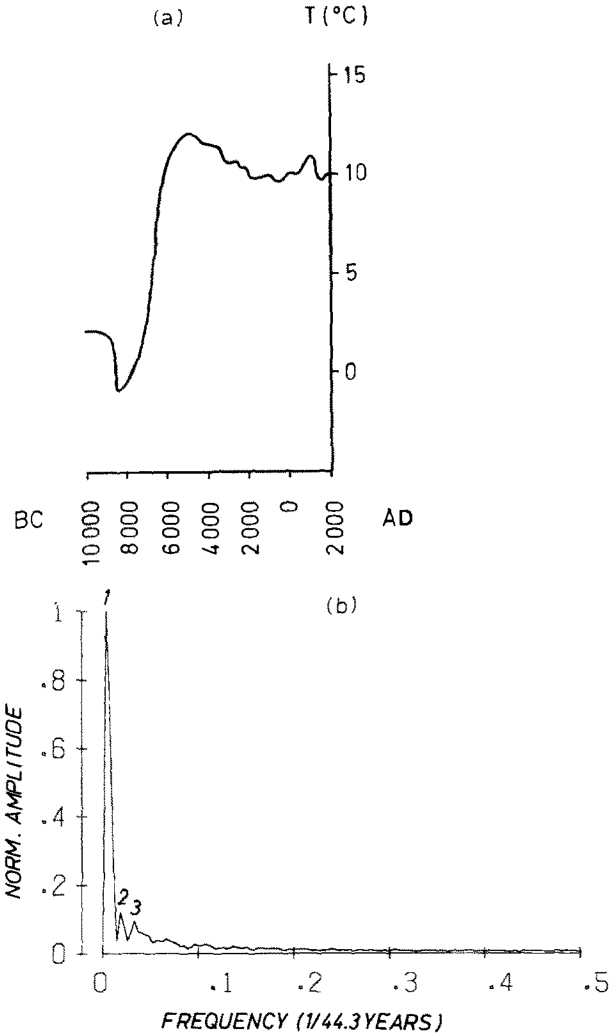

Fig. 4. a. Annual mean air temperatures for Central Europe as inferred by Lamb et al. (1966) for the last 12,000 years; redrawn after Lamb et al. (1966). b. Amplitude spectrum of the time series in Fig. 4a with its mean value of 8.02°C subtracted; see Table I for amplitudes and periods of numbered peaks.

The noise field of short period temperatures variations

The temperature- and gradient distribution caused by the temperature variation of Fig. 1a in the sediments of Lake Kinneret is shown in Fig. 5. The disturbance goes well down to about 25 m before the temperature reaches the surface temperature's mean value of 15.32°C. The disturbance is especially severe in the upper meters of the sediment that were used for geothermal investigations by Ben Avraham et al.

TABLE I

Main periods in amplitude spectra

	Peak No.	Period		Amplitude (°C)
		years	days	
Lake Kinneret	1	5.05	1854	0.79
$\bar{T}_0 = 15.32°C$	2	2.53	922	0.50
	3	1.68	615	0.36
	4	1.00	369	0.37
Stuttgart	1	164.0	59940	0.50
$\bar{T}_0 = 9.95°C$	2	41.0	14965	0.23
	3	21.5	7483	0.20
	4	12.6	4605	0.23
	5	7.5	2710	0.20
	6	3.4	1247	0.21
Karlsruhe	1	156.0	56940	0.30
$\bar{T}_0 = 9.12°C$	2	39.0	14235	0.30
	3	26.0	9490	0.19
	4	17.3	6327	0.21
	5	8.7	3136	0.18
	6	7.8	2847	0.20
	7	2.1	791	0.25
Paleotemperature	1	12000		4.68
$\bar{T}_0 = 8.02°C$	2	2401		0.56
	3	1334		0.44

TABLE II

Underground models

Model	Interface-depth	a (10^{-6} m^2 s^{-1})	λ (W m^{-1} K^{-1})
Lake Kinneret		0.20	0.75
Urach 4		1.10	2.51
	500		
		1.08	2.43
	700		
		1.12	2.51
	800		
		1.06	2.30
Neustadt 4		0.59	0.85
	493		
		0.52	0.74
	567		
		0.69	1.05
	900		
		0.88	1.77

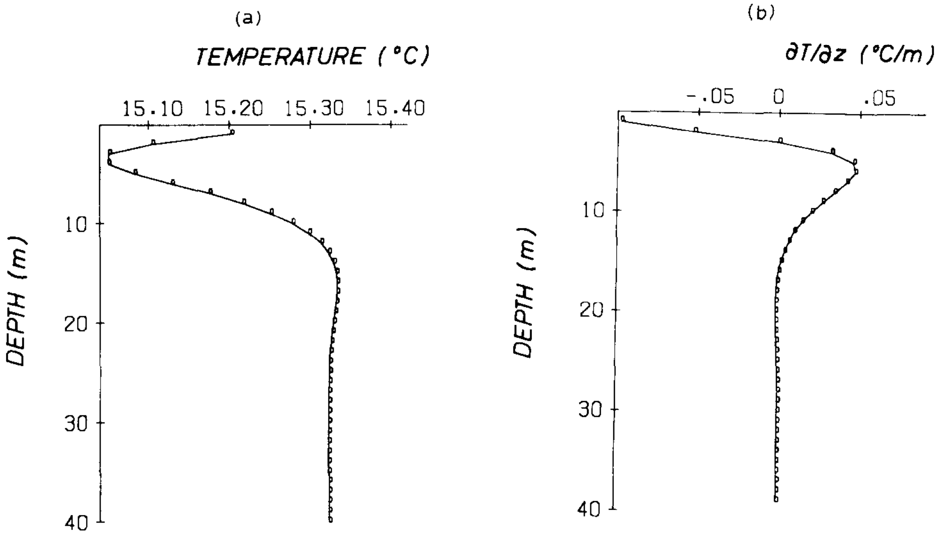

Fig. 5. Computed temperatures (a) and gradients (b) in the first 40 m of the sediment of Lake Kinneret, due to the temperature variation in Fig. 1a; see Table II for subsurface parameters.

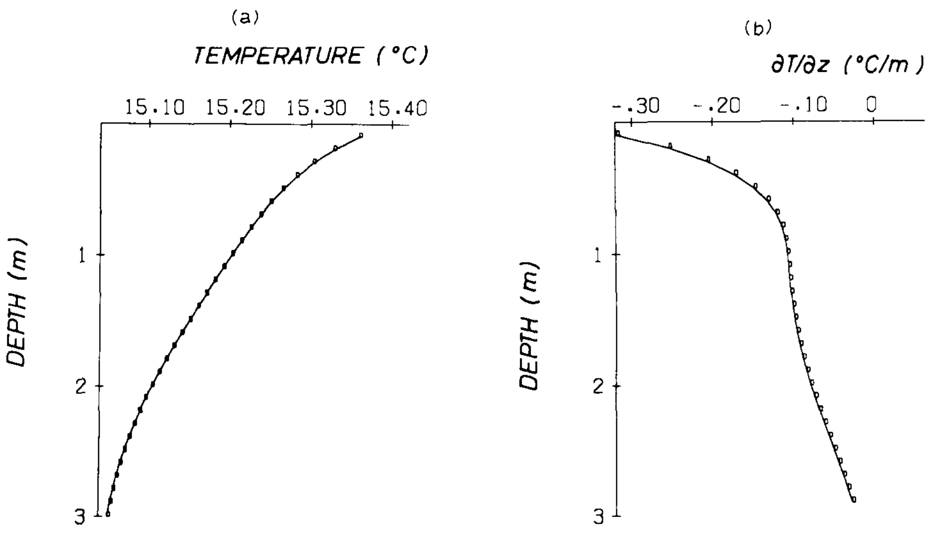

Fig. 6. Computed temperatures (a) and gradients (b) in the first 3 m of the sediment of Lake Kinneret, due to the temperature variation in Fig. 1a; see Table II for subsurface parameters.

(1977) in 1975. The temperature correction, as computed by Villinger (1977), used a 20th degree harmonic analysis of the preceeding year. A new correction on the measured temperature values of 1975, using the noise fields shown in Fig. 6, led to a mean temperature gradient that was lower by 58%. This lowered the mean value of heat flow density by 36% to 62.8 mW/m^2. This discrepancy is caused by the irregular annual temperature pattern in Lake Kinneret that is not taken into account by merely correcting for an annual temperature wave.

The noise field of intermediate temperature variations

Figure 7 shows the noise gradients in percentage of 30 K/km due to the temperature variations shown in Figs. 2a and 3a for the underground models Urach 4 and Neustadt 4 respectively. At a depth of 50 m the two noise gradients have not yet been damped below 10%; between 70 m and 110 m they vanish below 1%.

Thus, in dealing with shallow drillholes of up to 100 m depth it seems to be advisable to look into the thermal history of the last 100–200 years.

One problem should not be passed by: when using air temperatures, one has to correct for the difference to the surface temperature. For Germany, Kessels (1980) investigated this problem and derived an iso-line map for this difference. For seven meteorological stations in Germany I checked Kessels' results with somewhat different data sets and found them in good agreement with my own, shown in Table III. Figure 8a gives the location of the German Weather Service stations that were used (redrawn after: Deutsches Meteorologisches Jahrbuch, 1978).

Figure 8b illustrates how $\overline{T}_0$, the mean surface temperature was determined; daily means for the air temperature and the ground temperature in 2, 5, 10, 20, 50 and 100 cm were available on a magnetic tape for periods of 25–15 years by courtesy of the

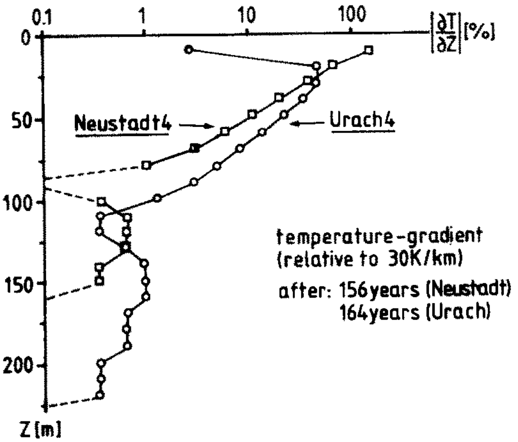

Fig. 7. Computed noise gradients in percentage of 30 K/km due to the temperature variations at Karlsruhe and Stuttgart (subsurface models Neustadt 4 and Urach 4); see Table II for model parameters.

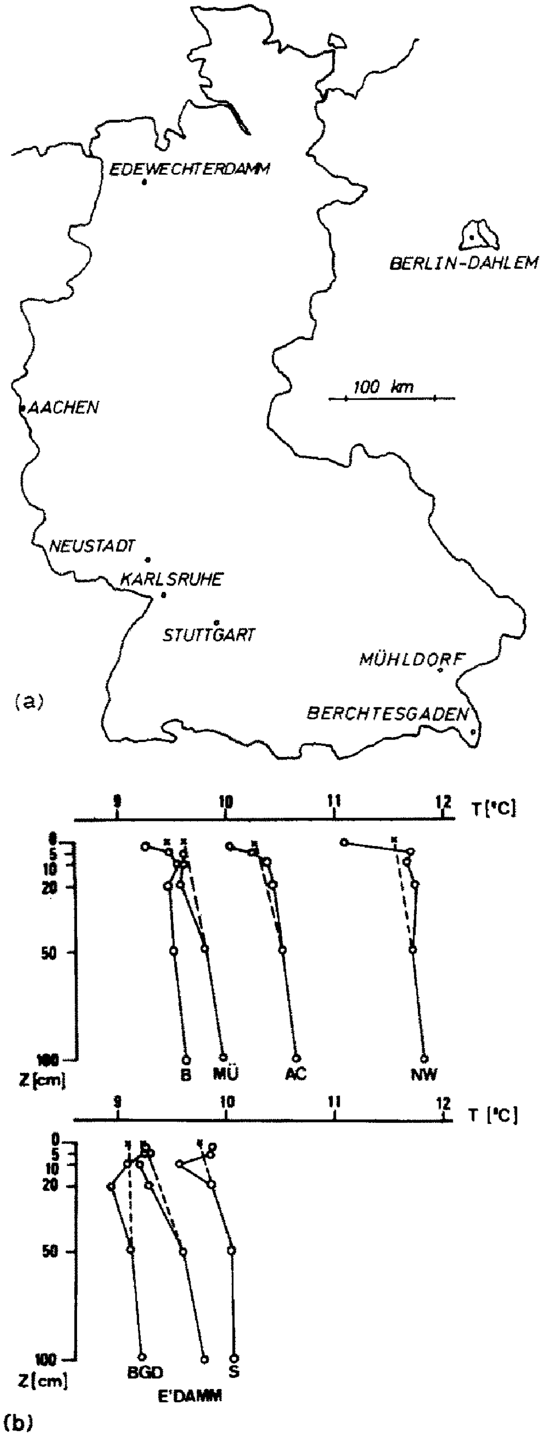

Fig. 8a. Federal Republic of Germany with locations of observation points of the German Weather Service (redrawn after Deutsches Meteorologisches Jahrbuch, 1978). b. Long term means of the soil temperature in different depths; see Table III for station identification. O = mean temperature for one depth (2, 5, 10, 20, 50, 100 cm); × = mean of the means for 2, 5, 10, and 20 cm depth, assigned to the surface ($z = 0$).

TABLE III

Temperature difference air–ground

location	$T_{air} - T_{ground}$ (K)
Edewechterdamm (E'damm)	0.67
Berlin–Dahlem (B)	0.60
Aachen (AC)	0.71
Neustadt (NW)	1.53
Stuttgart (S)	1.17
Mühldorf (MÜ)	1.84
Berchtesgaden (BGD)	2.08

German Weather Service. Over these periods the mean values were calculated. It turns out that if one takes the mean of the mean values for 2, 5, 10 and 20 cm depth and assigns it to the surface ($z = 0$), this value almost lines up in a straight line with the values for 50 and 100 cm. This means that the gradient is almost constant, so that phase change-, circulation- and evaporation effects should be eliminated. This value was then subtracted from the long-term mean air temperature.

In order to correct the Karlsruhe and Stuttgart records this long term air–ground temperature difference was subtracted for each location: 1.60 K for Karlsruhe (model Neustadt 4), and 1.04 K for Stuttgart (model Urach 4).

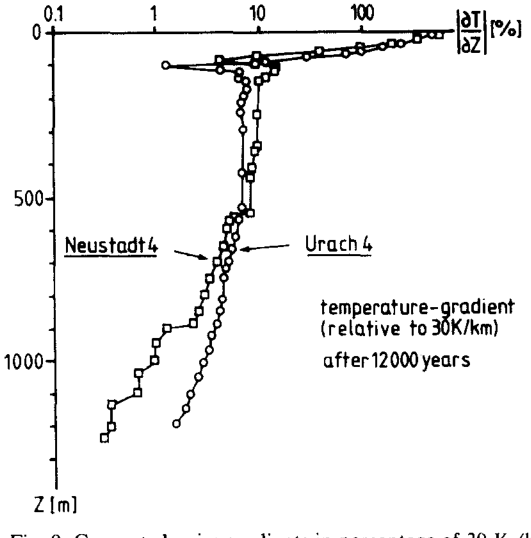

Fig. 9. Computed noise gradients in percentage of 30 K/km due to the paleoclimatic temperature change as shown in Fig. 4a (Lamb et al., 1966) for the models Neustadt 4 and Urach 4; see Table II for model parameters.

The noise field of a paleo-climatic change

The assumed air temperature history of the last 12000 years as given by Lamb et al. (1966) is to serve as an example of how an arbitrarily shaped paleo-climatic change diffuses into the subsurface. Figure 9 shows the noise gradient (again in percentage of 30 K/km) caused by the air temperature variation shown in Fig. 4a. Since the subsurface models Neustadt 4 and Urach 4 were used for calculation, 1.60 K and 1.04 K, respectively, were subtracted as before from the temperatures to correct for the air–ground temperature difference. The noise gradients do not significantly drop below 10% before 500 m depth in both models, a fact that has been recognized for quite a time.

DISCUSSION, CONCLUSIONS AND OUTLOOK

The examples in the preceeding paragraph have demonstrated the algorithm's applicability to temperature corrections of a wide period-range. Moreover, it has become clear that in certain cases it is not sufficient to restrict corrections to the annual and the paleoclimatic correction; in shallow lakes or in shallow boreholes it may be necessary to correct for the thermal history of the last couple of years up to some hundred years.

Because this algorithm is an extension of the traditional harmonic analysis approach, it should be mentioned that it also has its shortcomings. This is mainly the assumption that a given finite temperature record is indefinitely repeated in the past and in the future. This, of course, is not the case in reality; but the longer the temperature records go back into the past, the better the approximation is. One way to improve this may lie in using maximum entropy spectral analysis for the data rather than a discrete Fourier-transformation because it makes different assumptions.

On the other hand, the algorithm's advantages are its applicability to all period ranges and data sets of arbitrary length and shape (limited only by the storage-capacity of the computer) as well as its ability to compute the noise fields in a stratified medium.

The problem of determining $\overline{T}_0$ could be solved in a way outlined in this paper, at least for countries with long-term weather observations. If sufficient data are available, isoline maps could be produced for certain areas; this should certainly be possible for central Europe. $\overline{T}_0$ being determined, the air–ground temperature difference could be plotted and the wide range of air temperature records could be used for temperature corrections with a better accuracy.

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