Einfaches Samplen

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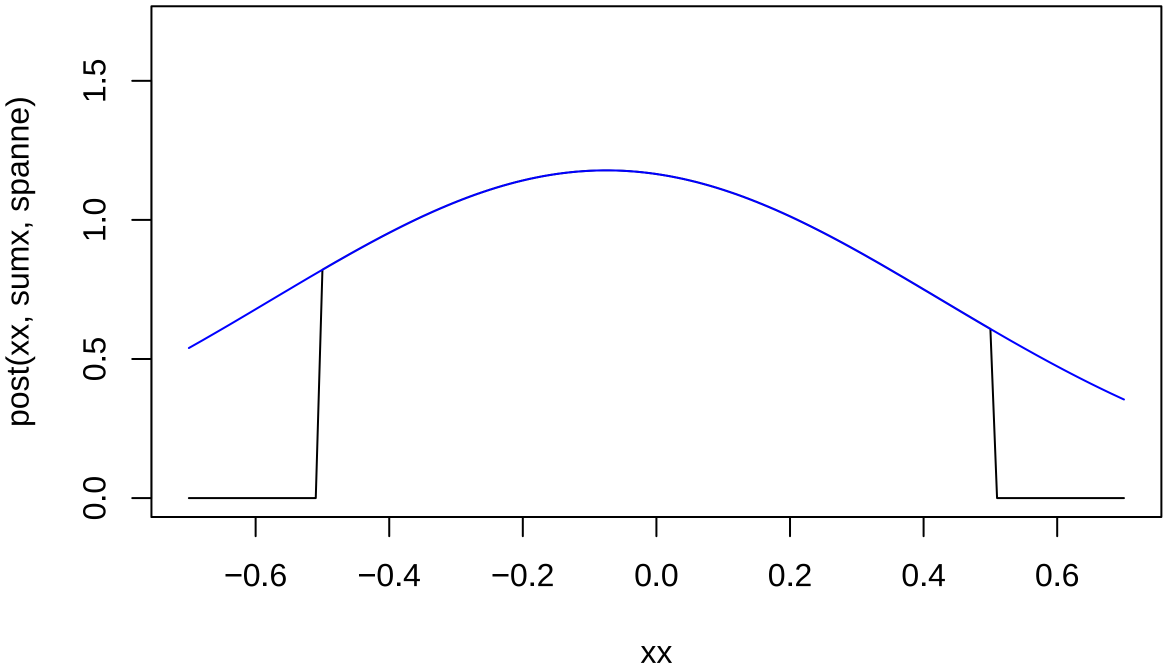
Einfaches Sampling

Sei $x_i \sim N(\mu, \tau^{-1})$ mit $\tau = 1$ bekannt. Sei $\mu \sim N_{[-1,1]}(0,1)$ und $x = 1, 0.2, -1.5$. Samplen Sie aus der Posteriori mit Acception-Rejection-Methode.

Posteriori: $\mu|x \sim N_{[-1,1]}(\sum x_i/4, 1/4)$

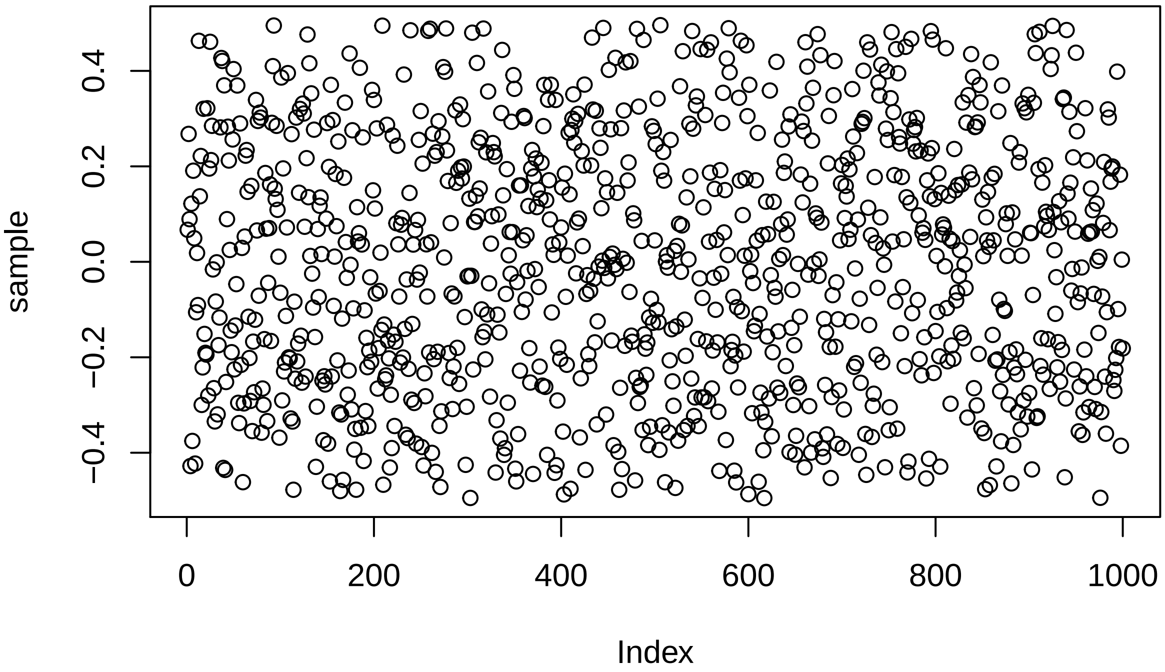
```r
x=c(1,0.2,-1.5)
n=3
sumx <- sum(x)
unten <- pnorm(-.5, sumx/4, sd=1/2)
oben <- pnorm(.5, sumx/4, sd=1/2)
spanne <- oben-unten
xx <- seq(-.7,.7, by=0.01)
post<-function(x,sumx,spanne)
{
  y<-dnorm(x, sumx/4, 1/2)
  y <- y/spanne
  y[x< -.5]<-0
  y[x>.5]<-0
  return(y)
}
plot(xx, post(xx, sumx, spanne), type="l", ylim=c(0,1.7))

factor <- post(-.5, sumx, spanne)/dnorm(-.5,sumx/4,1/2)

lines(xx, factor*dnorm(xx,sumx/4,1/2), col="blue")
```



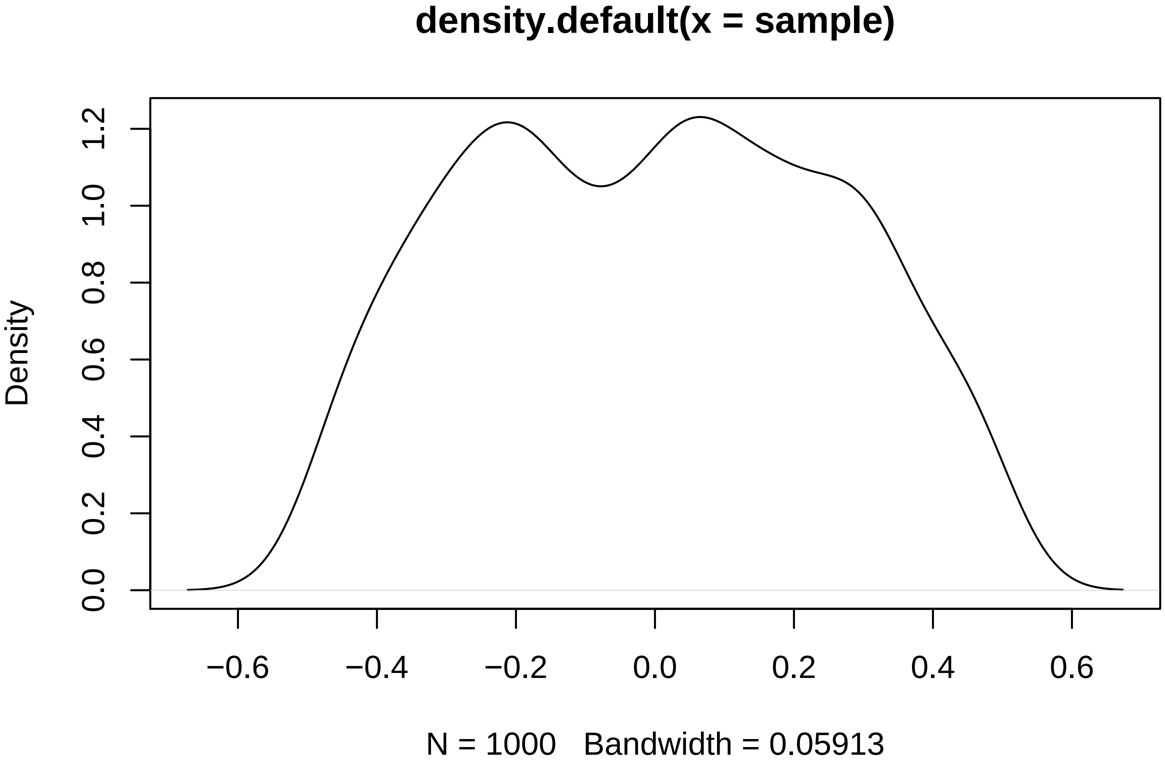
```r
sample <- c()
N<-1000
while(N!=0)
{
  cat(".")
  prop <- rnorm(N, sumx/4, 1/2)
  alpha <- post(prop, sumx, spanne)/dnorm(prop, sumx/4, 1/2)/factor
  u <- runif(N)
  sample <- c(sample, prop[alpha>u])
  N <- 1000-length(sample)
}
```

```
## ......
```

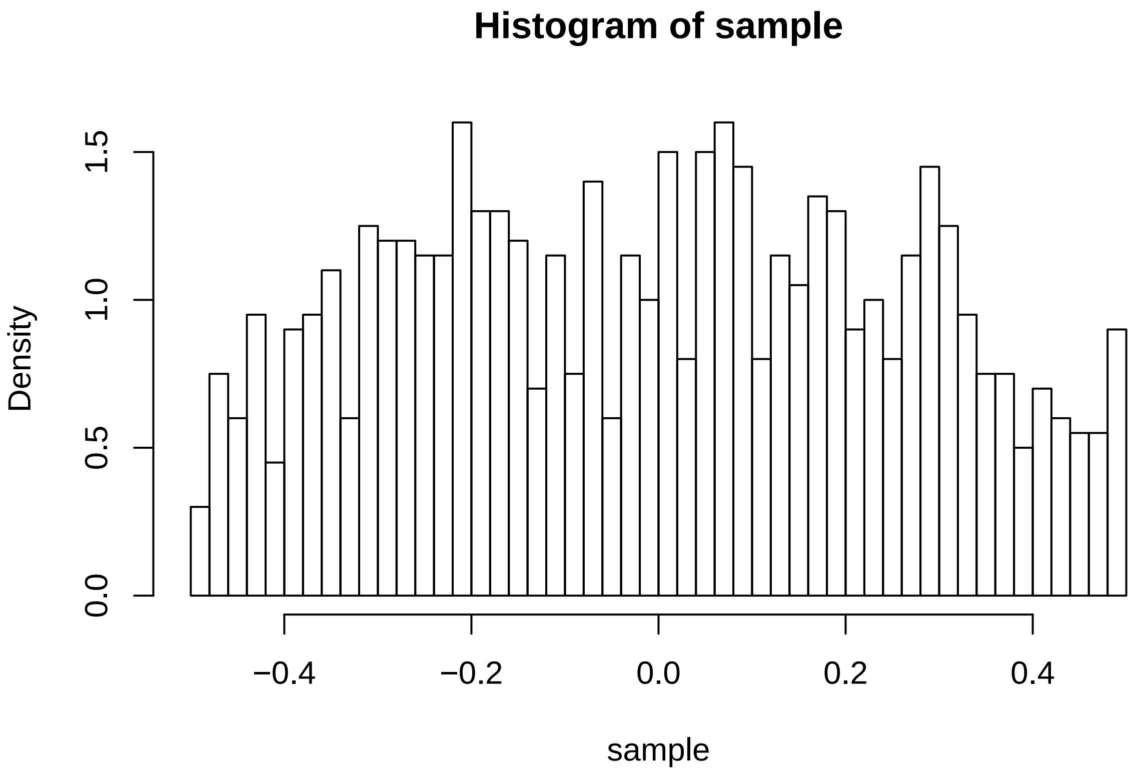
```r
plot(sample)
```



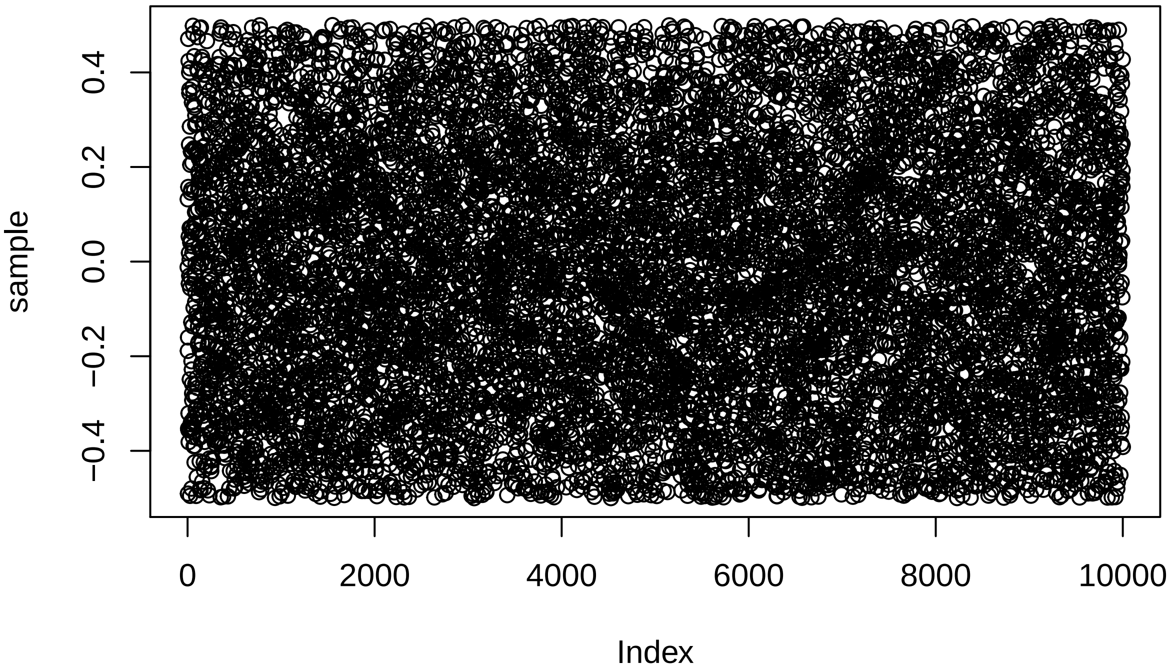
```r
plot(density(sample))
```



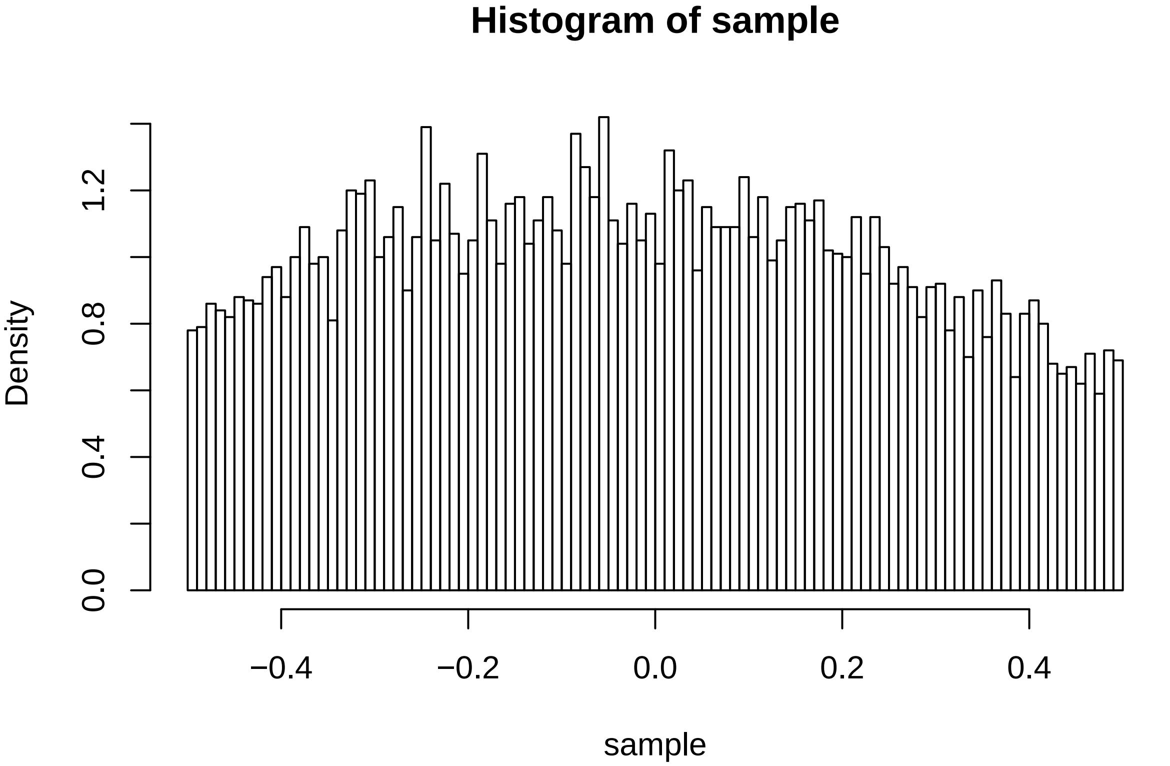
```r
hist(sample,breaks = 50, freq=FALSE)
```



```
draw.from.post<-function(N,mean,sd,unten=-.5,oben=.5)
{
  prop <- rnorm(N, mean, sd)
  while((sum(prop<unten)+sum(prop>oben))!=0)
  {
    prop[prop<unten]<-rnorm(sum(prop<unten),mean,sd)
    prop[prop>oben]<-rnorm(sum(prop>oben),mean,sd)
  }
  return(prop)
}
sample<-draw.from.post(10000,sumx/4,1/2,-.5,.5)
plot(sample)
```



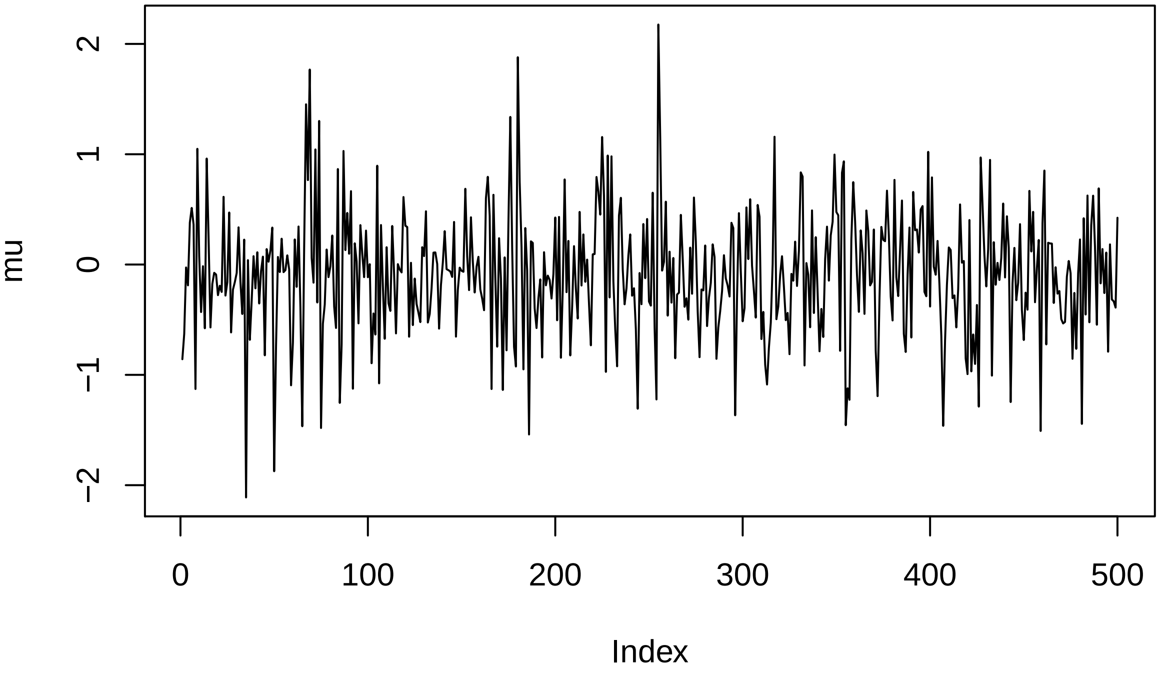
```r
hist(sample,breaks = 100, freq=FALSE)
```



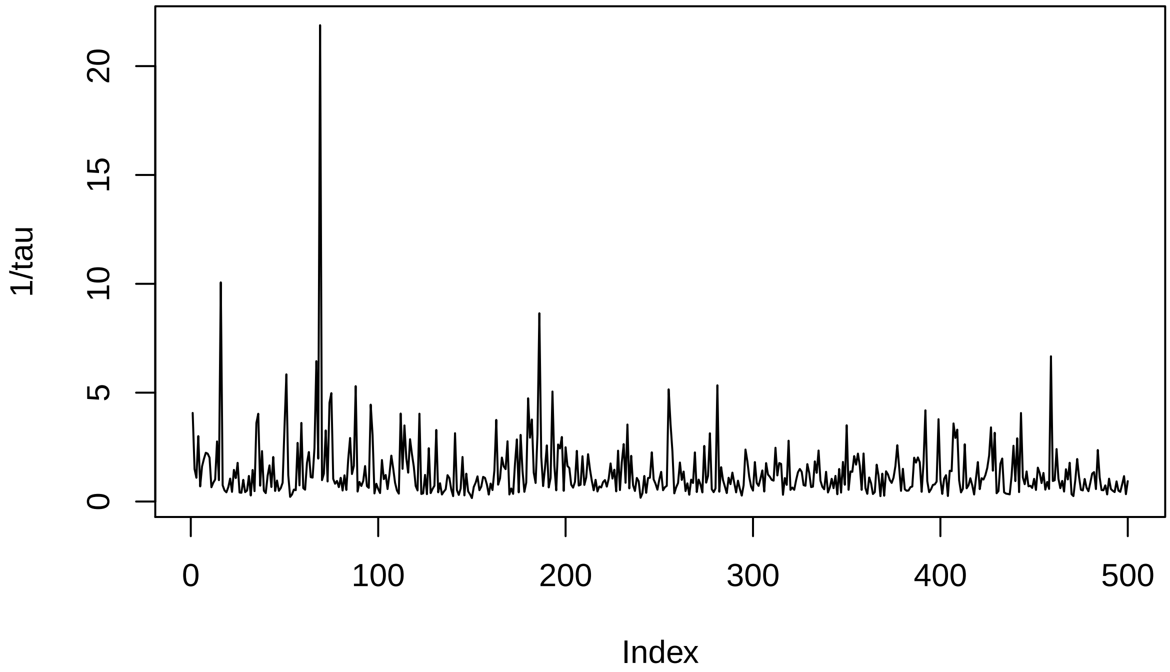
Gibbs-Sampling

Sei nun $\tau \sim Ga(1, 1/1000)$. Ziehe aus der gemeinsamen Posteriori von \$*tau* und μ mittels Gibbs-Sampling. Zuerst mal mit $\mu \sim N(0, 1)$.

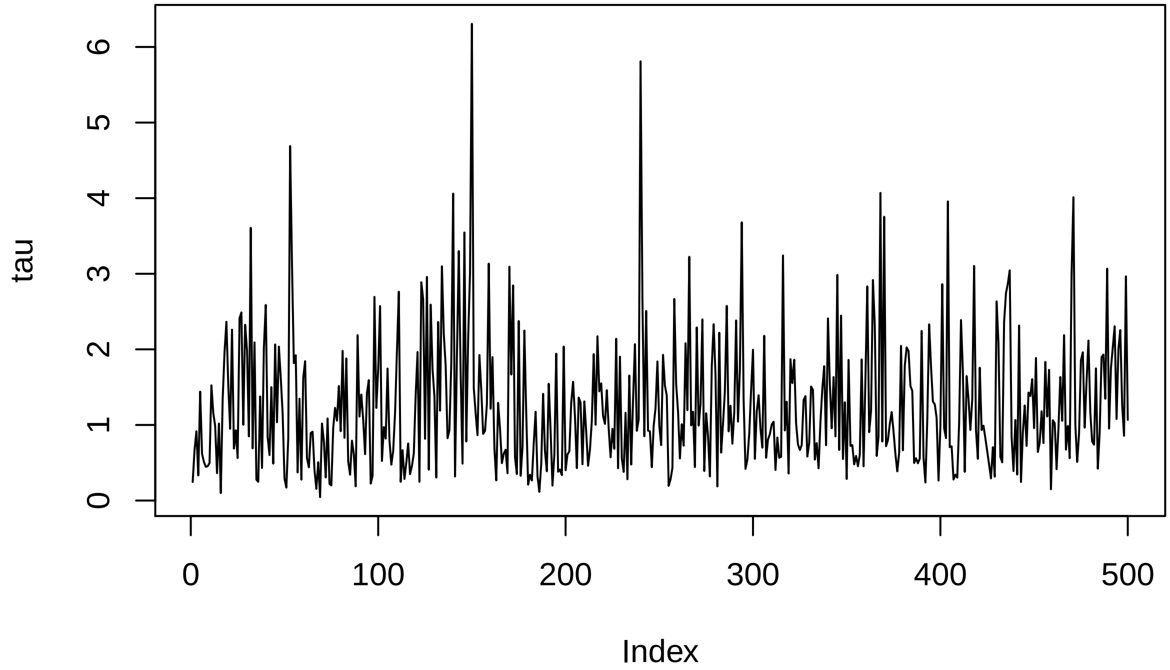
```r
I<-500

s0<-1
a<-1
b<-0.001
n<-length(x)

mu<-tau<-rep(NA,I+1)
tau[1]<-1000
mu[1]<-1

for (i in (1:I)+1)
{
  tau[i]<-rgamma(1,a+n/2,b+0.5*sum((x-mu[i-1])^2))
  m<-tau[i]*sumx
  s<-n*tau[i]+1/s0
  mu[i]<-rnorm(1,m/s,sqrt(1/s))
}

tau<-tau[-1]
mu<-mu[-1]

plot(mu,type="l")
```



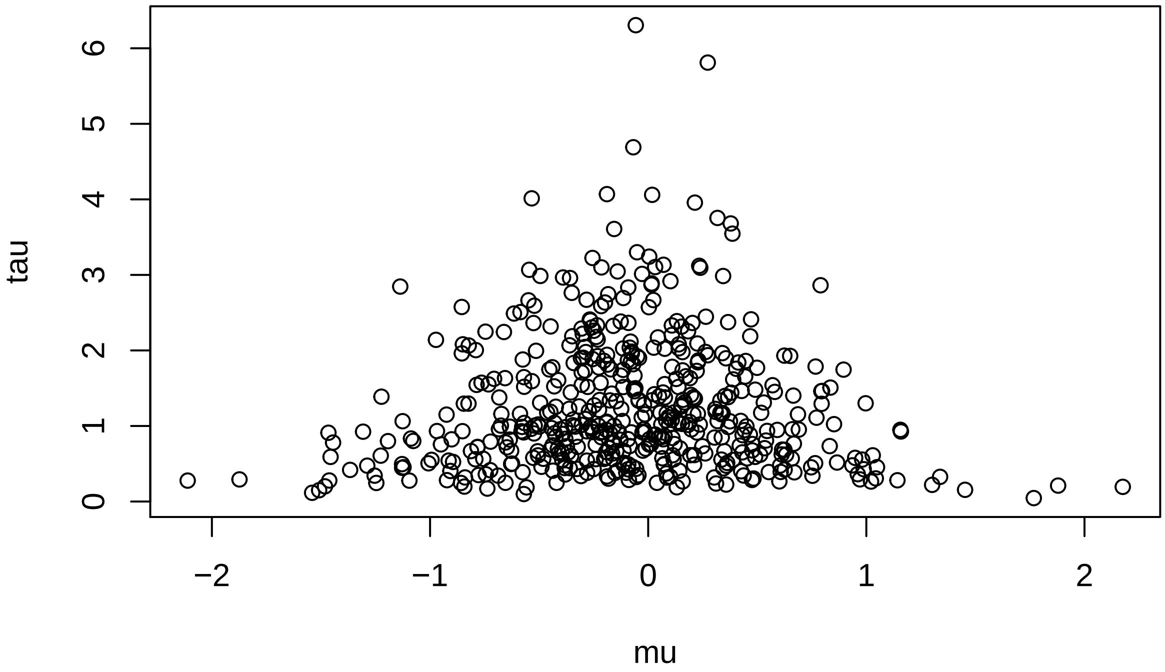
```r
plot(1/tau,type="l")
```



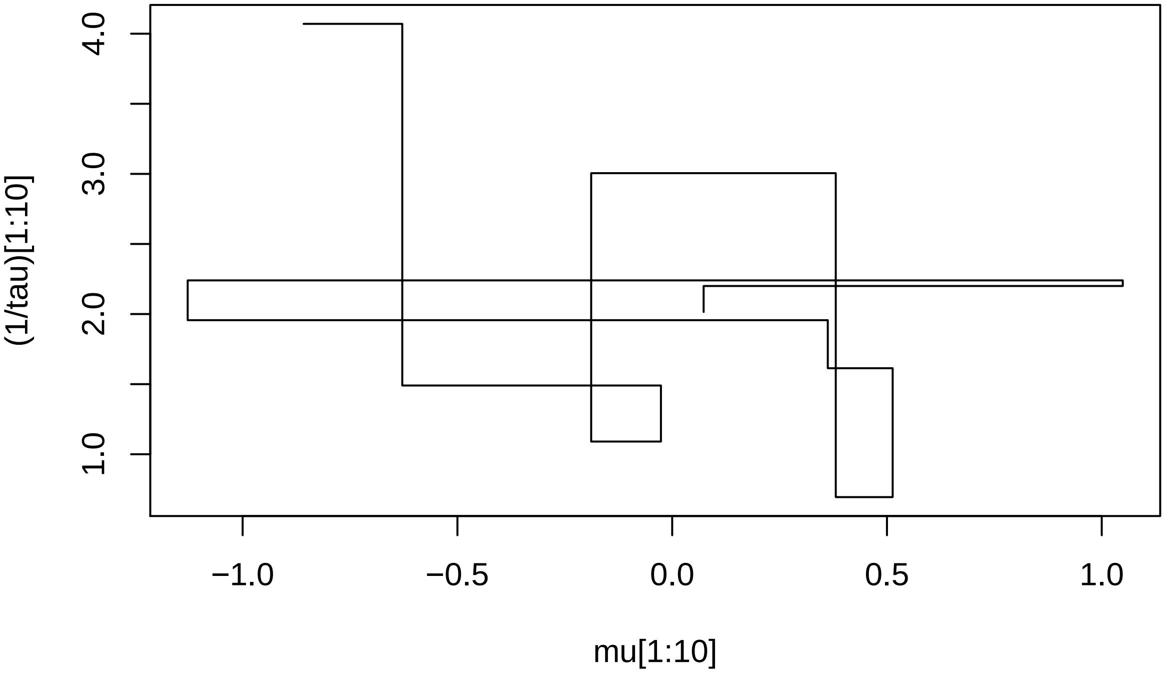
```
plot(tau,type="l")
```



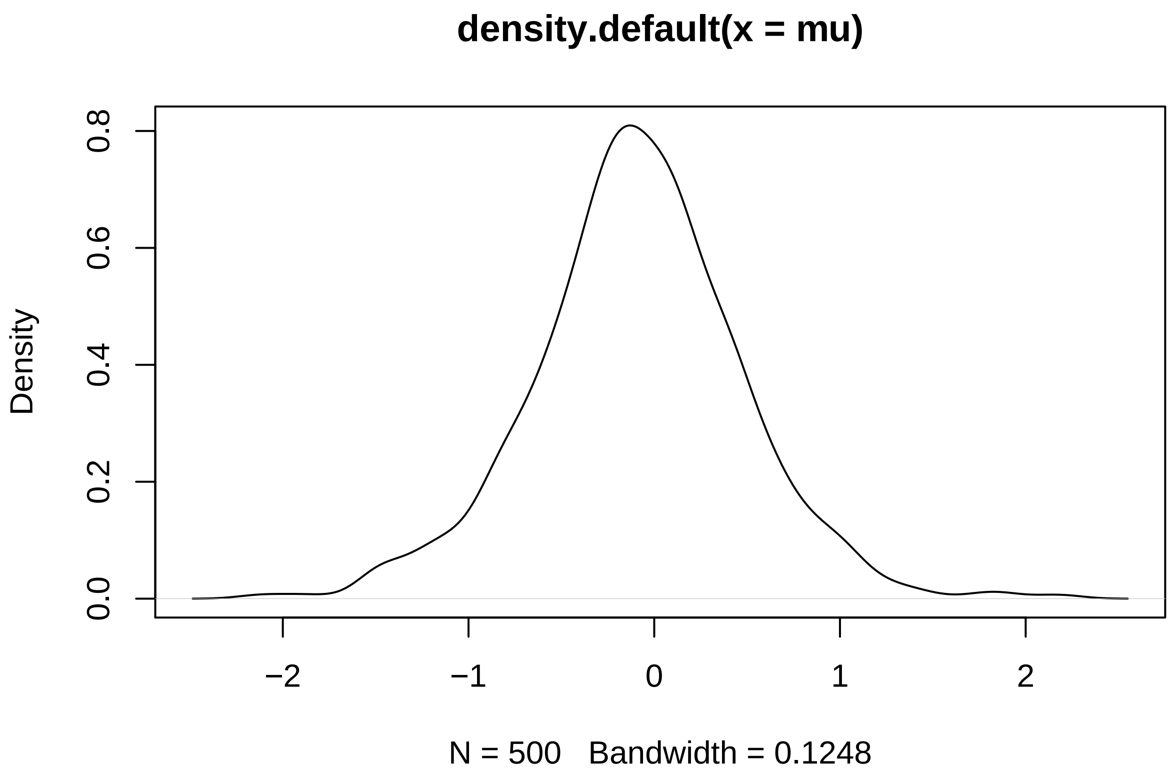
```
plot(mu,tau)
```



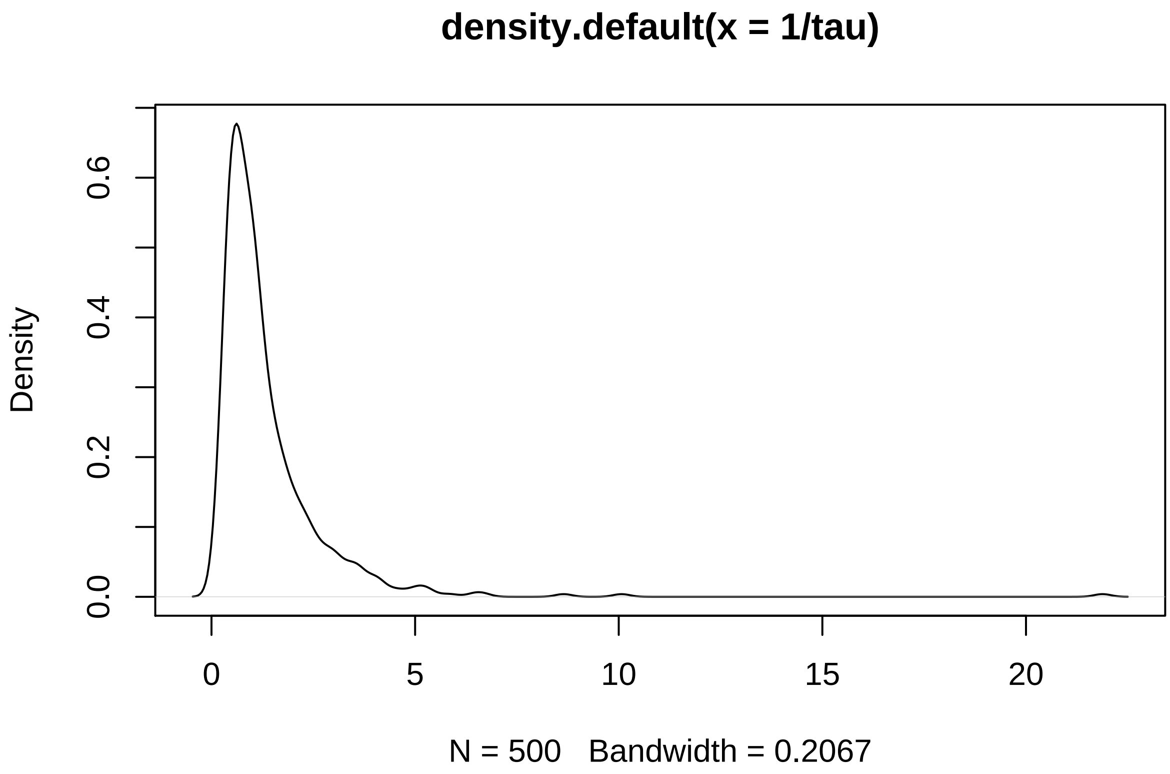
```
plot(mu[1:10],(1/tau)[1:10],type="s")
```



```
plot(density(mu))
```



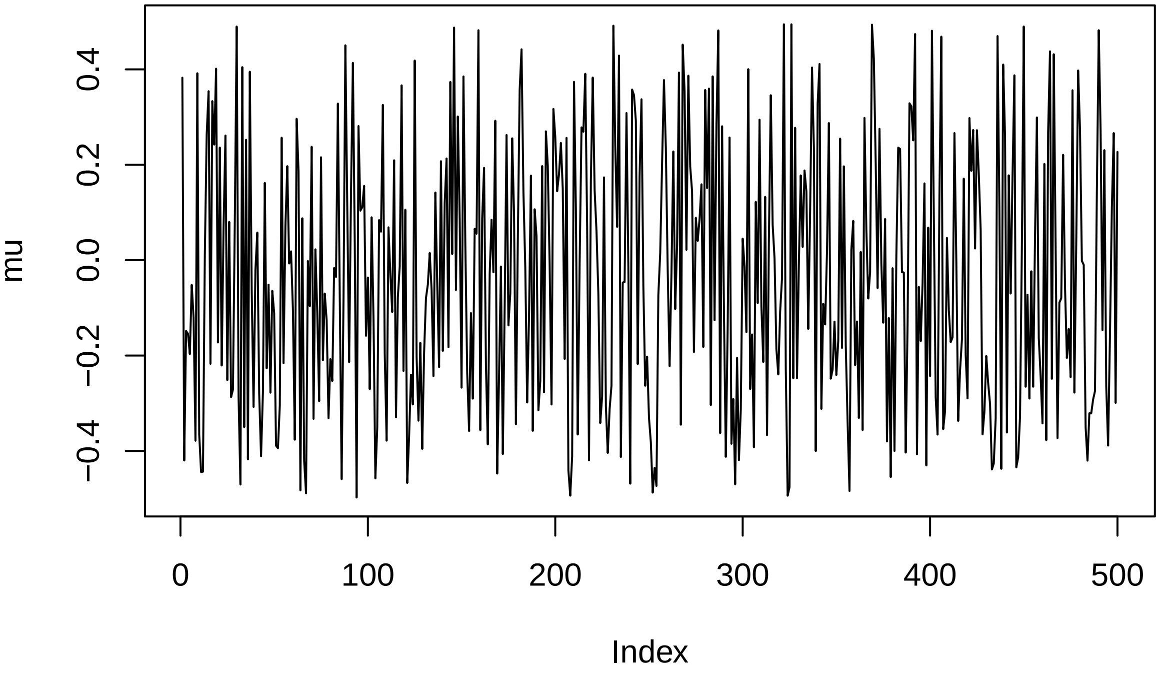
```
plot(density(1/tau))
```



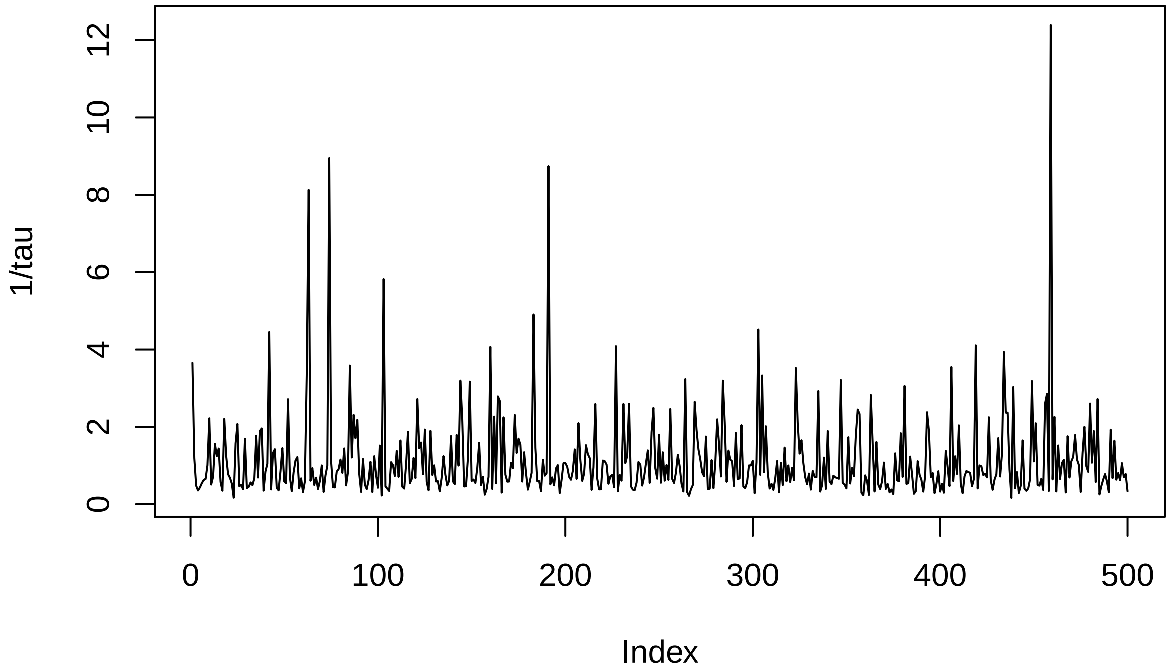
Jetzt mit beschränkter Priori:

```
I<-500
```

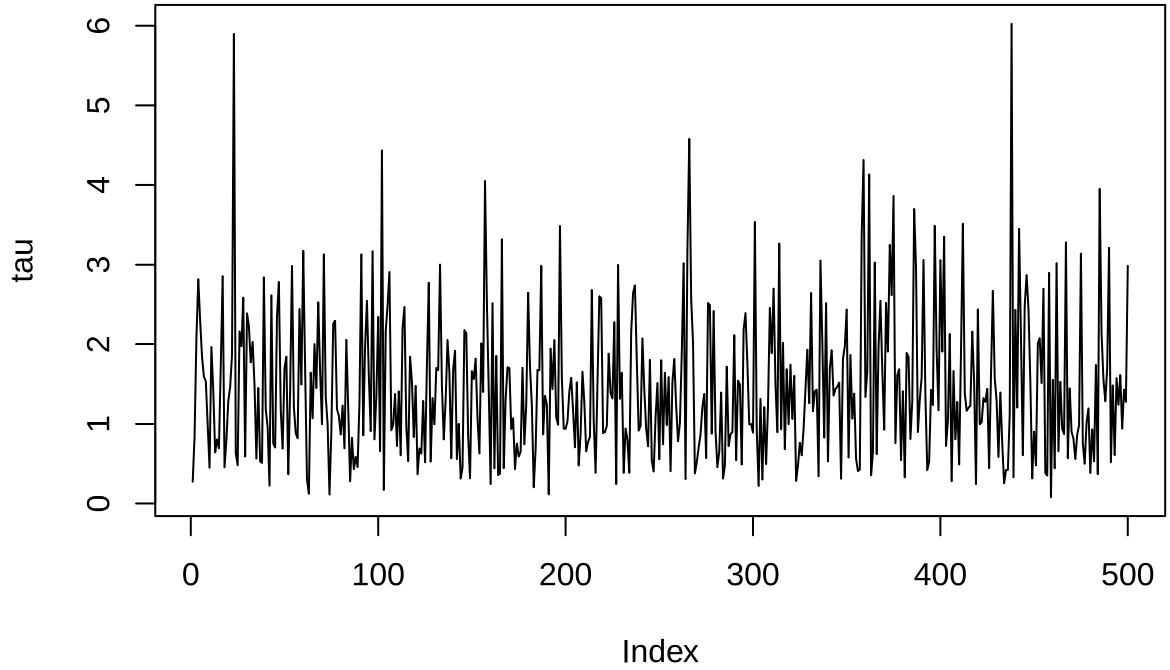
```
s0<-1
a<-1
b<-0.001
n<-length(x)

mu<-tau<-rep(NA,I+1)
tau[1]<-1000
mu[1]<-1

for (i in (1:I)+1)
{
  tau[i]<-rgamma(1,a+n/2,b+0.5*sum((x-mu[i-1])^2))
  m<-tau[i]*sumx
  s<-n*tau[i]+1/s0
  mu[i]<-draw.from.post(1,m/s,sqrt(1/s))
}

tau<-tau[-1]
mu<-mu[-1]

plot(mu,type="l")
```



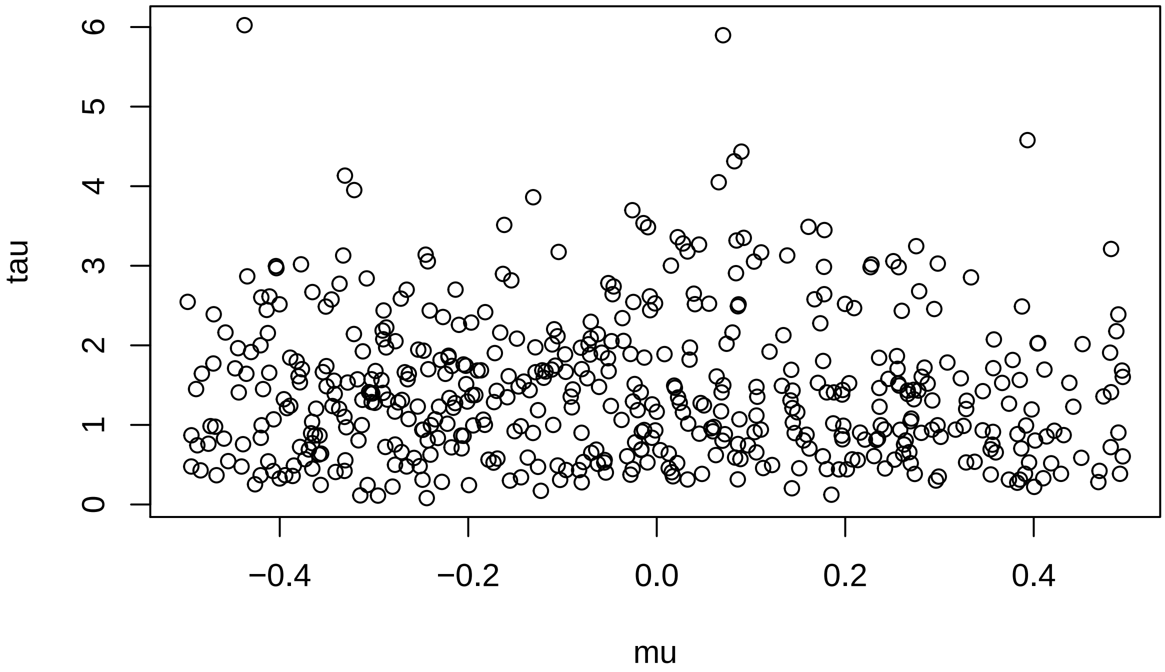
```
plot(1/tau,type="l")
```



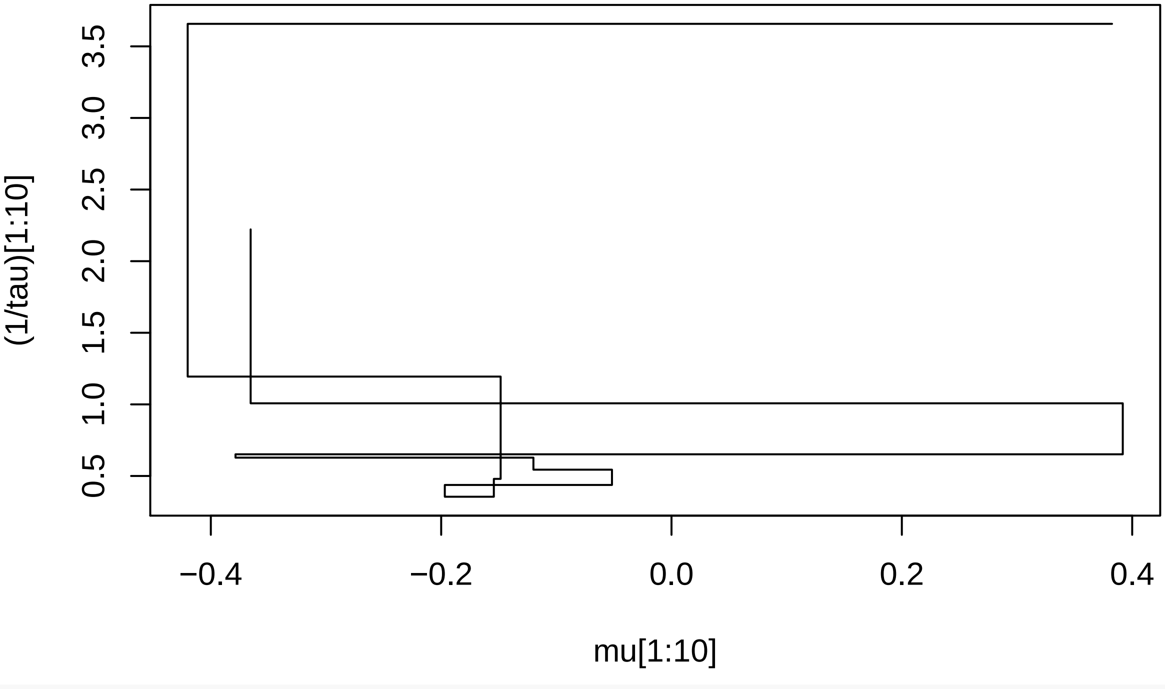
```r
plot(tau,type="l")
```



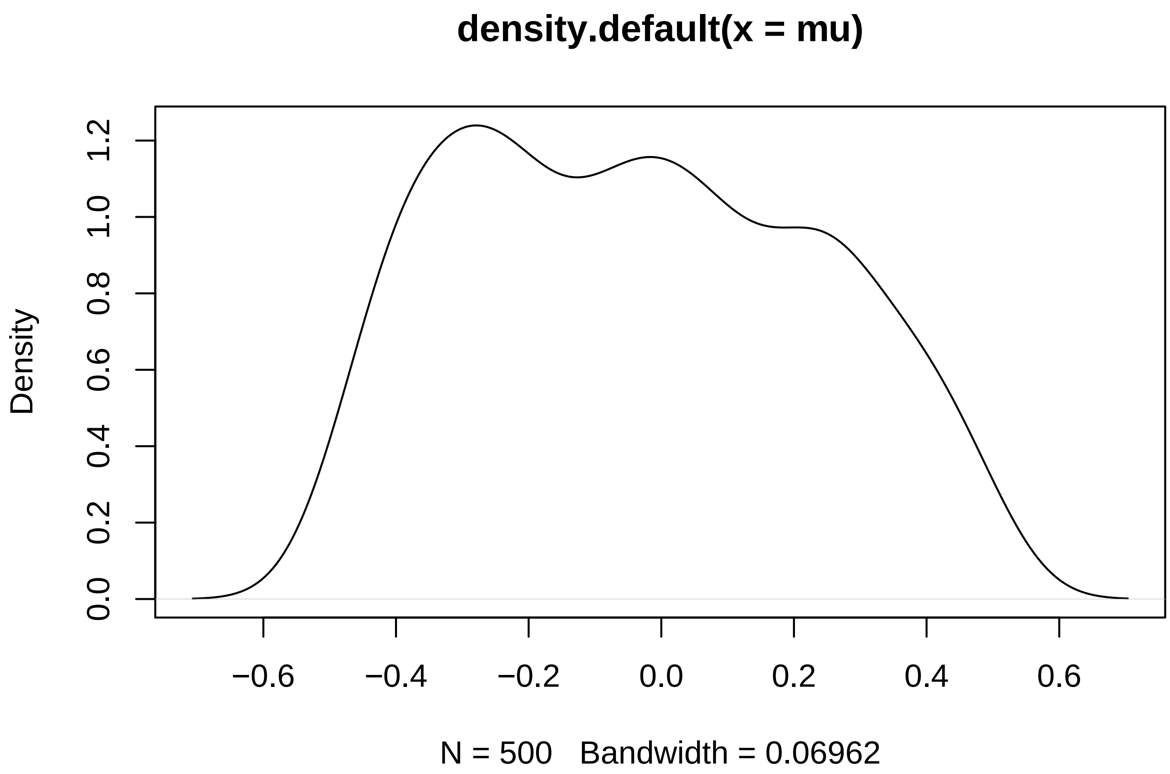
```r
plot(mu,tau)
```



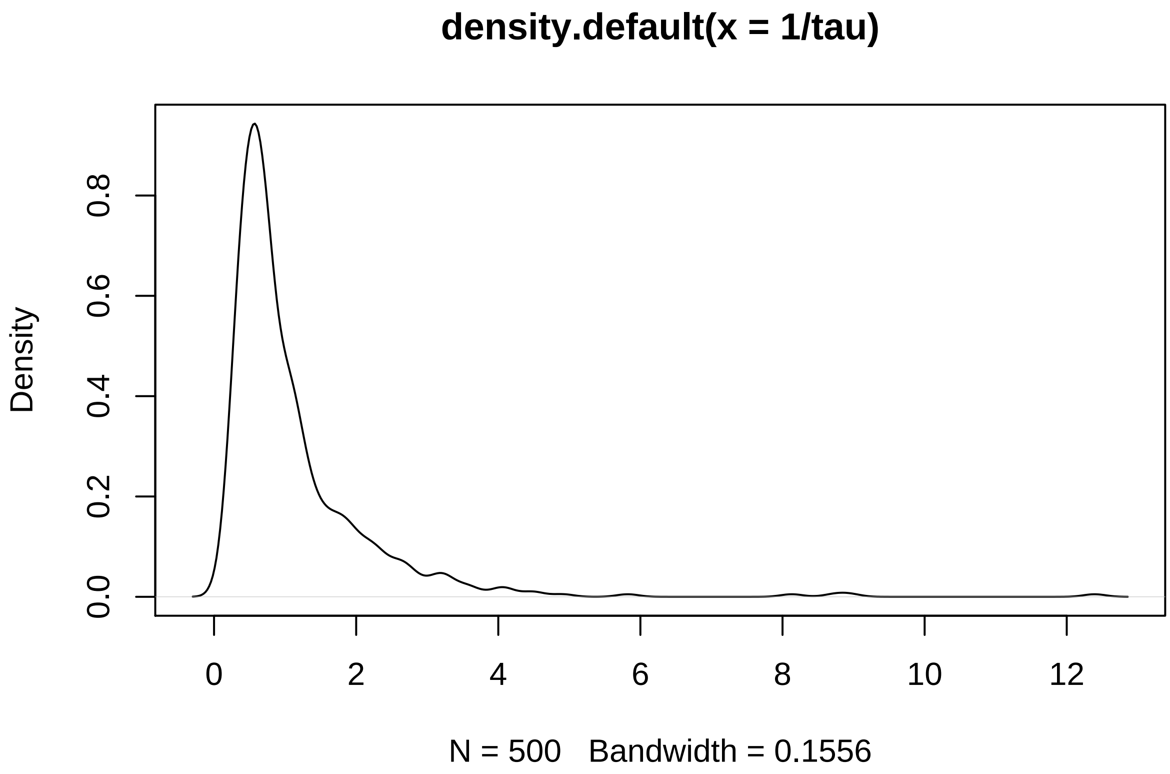
```r
plot(mu[1:10],(1/tau)[1:10],type="s")
```



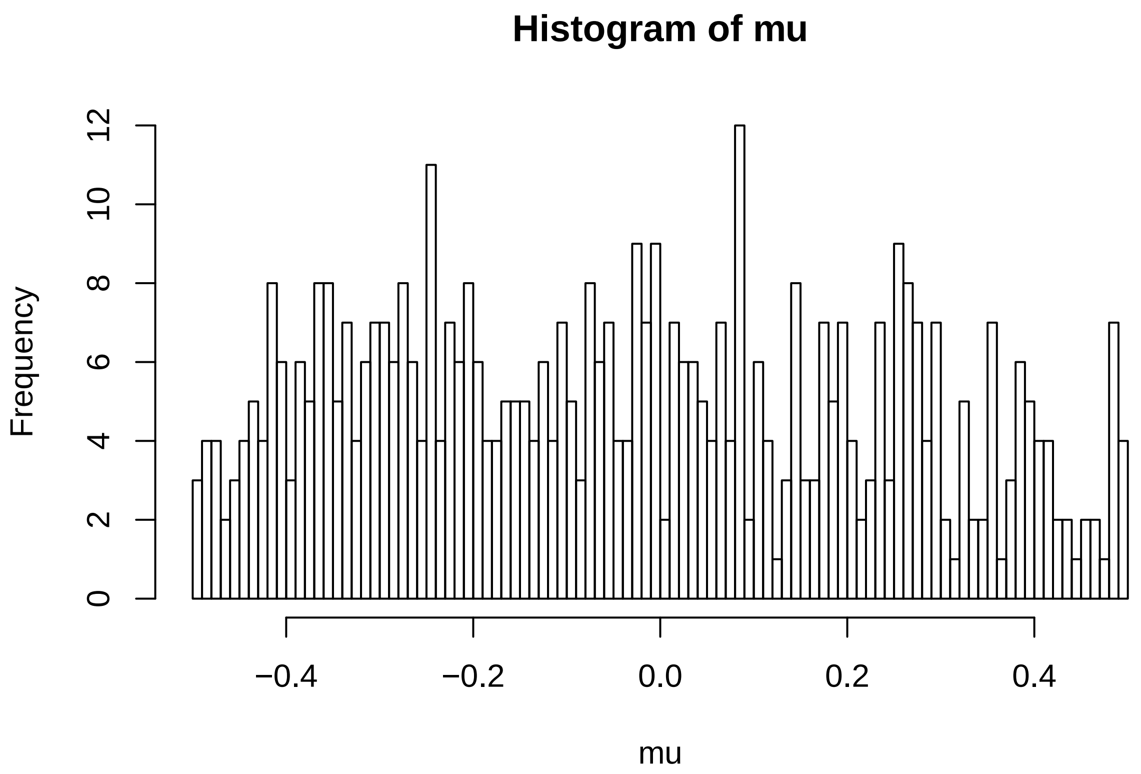
```r
plot(density(mu))
```



```r
plot(density(1/tau))
```



```r
hist(mu,breaks=100)
```



```
print(mean(mu))
```

```
## [1] -0.03427149
```

```
print(mean(tau))
```

```
## [1] 1.449831
```

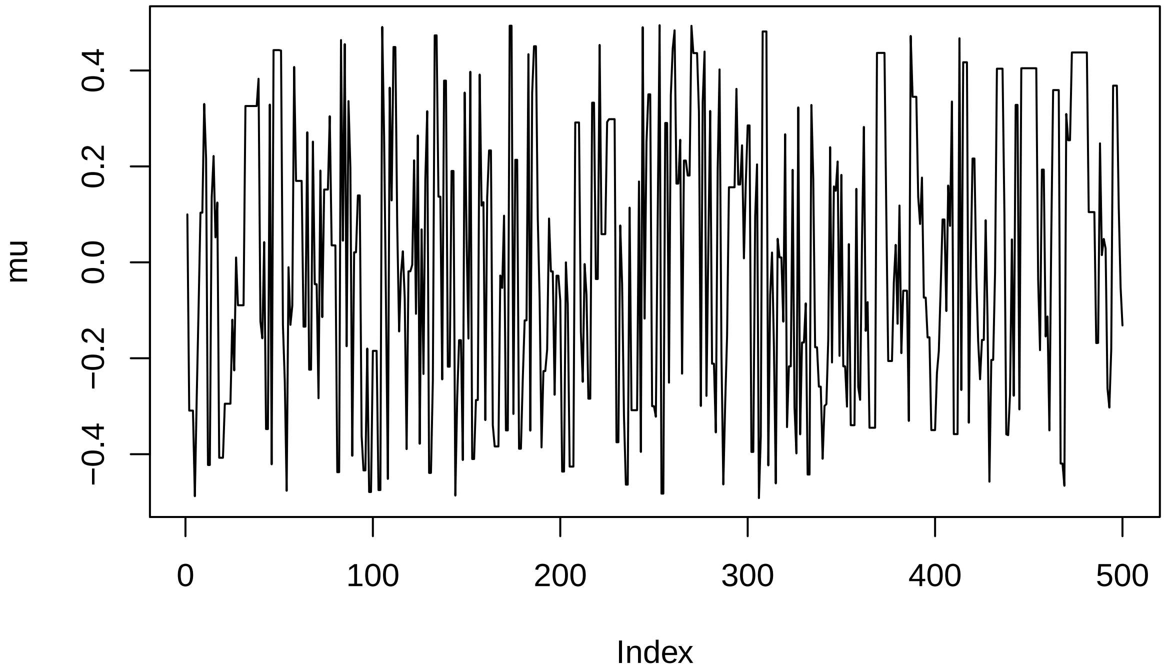
Metropolis-Hastings-Sampling

Sei nun $\tau \sim LN(0,1)$. Ziehe aus der gemeinsamen Posteriori mittels Metropolis_Hastings.

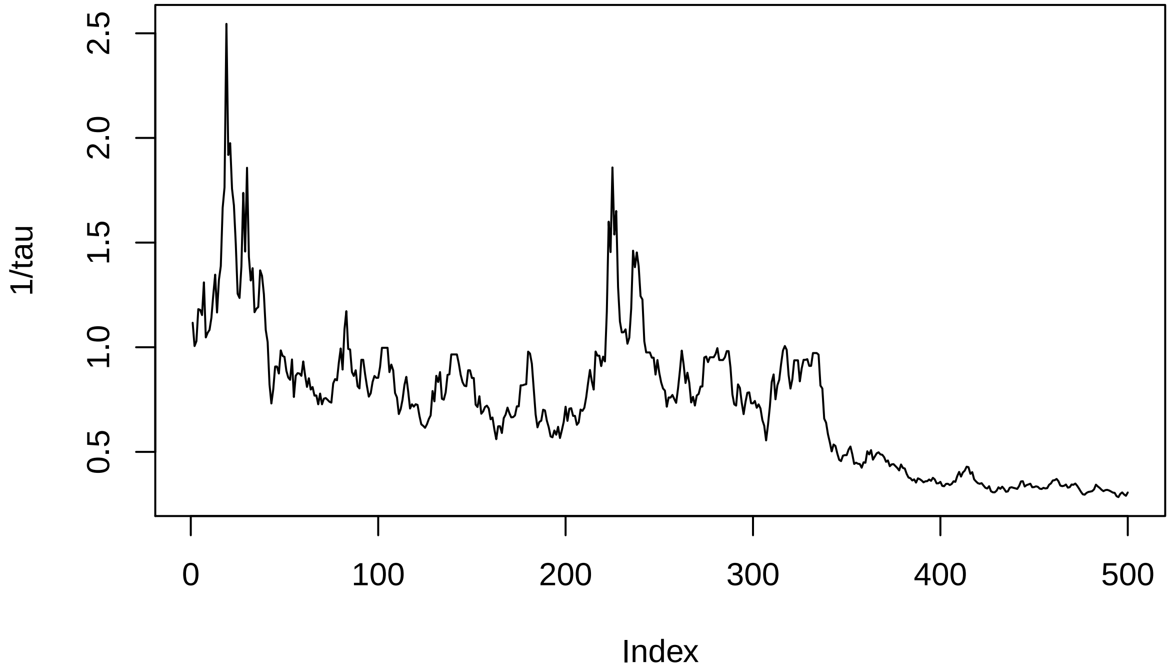
```
I<-500

s0<-1
a<-5
b<-1
n<-length(x)

mu<-tau<-rep(NA,I+1)
tau[1]<-1 # !
mu[1]<-.1 # !

log.fc.tau <- function(tau,n,sumxmu2)
{
  if (tau<0|tau>1)return(0)
  return<-n*log(tau)/2-tau*sumxmu2/2+log(dlnorm(tau))
  return(return)
}
```

```
for (i in (1:I)+1)
{
  taustern <- rnorm(1,tau[i-1],.1)
  sumxmu2<-sum((x-mu[i-1])^2)
  logalpha<-log.fc.tau(taustern,n,sumxmu2)-log.fc.tau(tau[i-1],n,sumxmu2)
  alpha<-exp(logalpha)
  if (runif(1)<alpha)
  {
    tau[i]<-taustern
  }
  else
  {
     tau[i]<-tau[i-1]
  }

  m<-tau[i]*sumx
  s<-n*tau[i]+1/s0
  mustern<-rnorm(1,m/s,sqrt(1/s))
  logalpha <- log(post(mustern,sumx,spanne))-log(post(mu[i-1],sumx,spanne))
  logalpha <- logalpha+log(dnorm(mu[i-1],m/s,sqrt(1/s)))-log(dnorm(mustern,m/s,sqrt(1/s)))
 alpha<-exp(logalpha)
 if(!is.na(alpha))if(!is.na(mustern))
   if (runif(1)<alpha)
  {
    mu[i]<-mustern
  }
  else
  {
    mu[i]<-mu[i-1]
  }
}
```

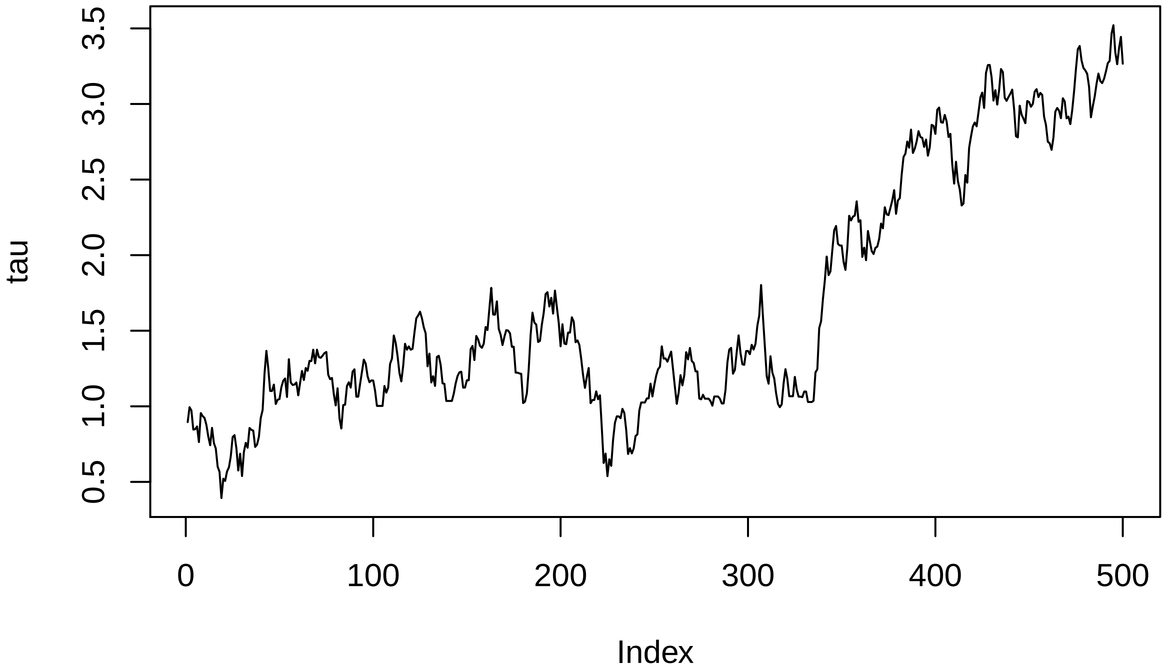
```
tau<-tau[-1]
mu<-mu[-1]

plot(mu,type="l")
```



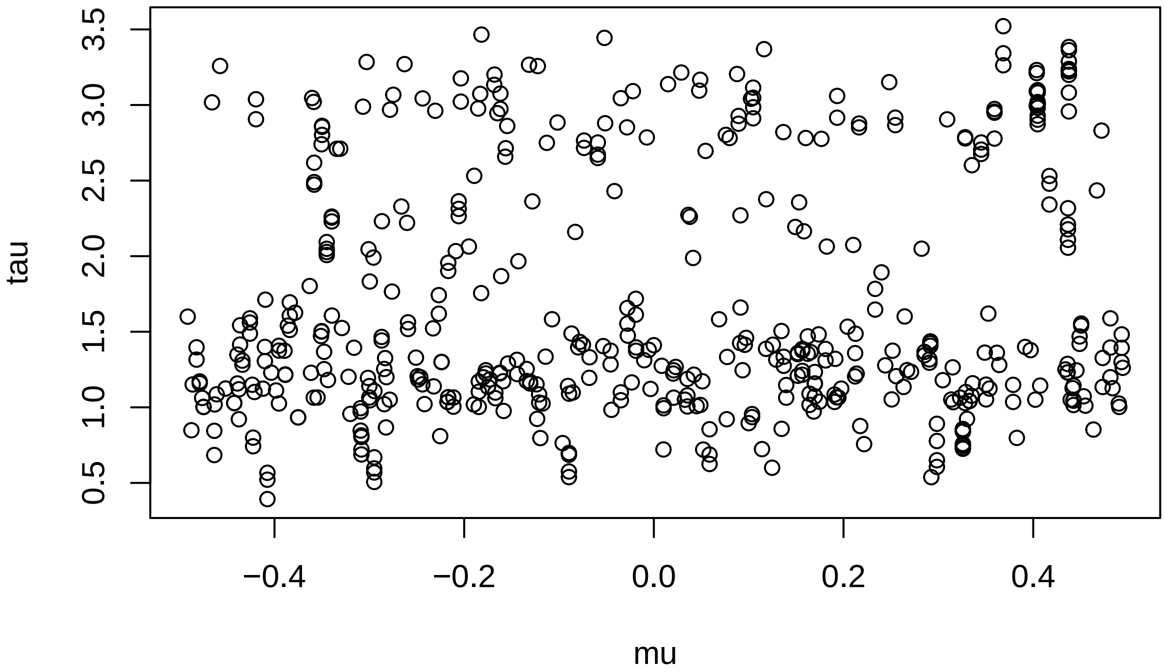
```r
plot(1/tau,type="l")
```



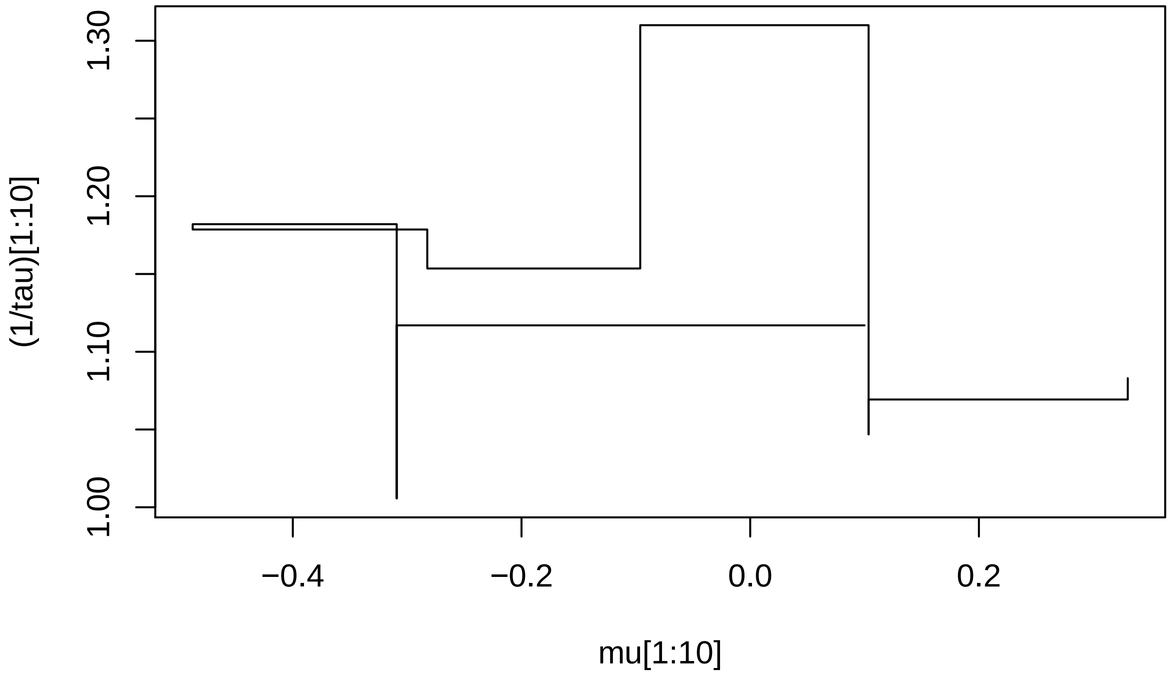
```r
plot(tau,type="l")
```



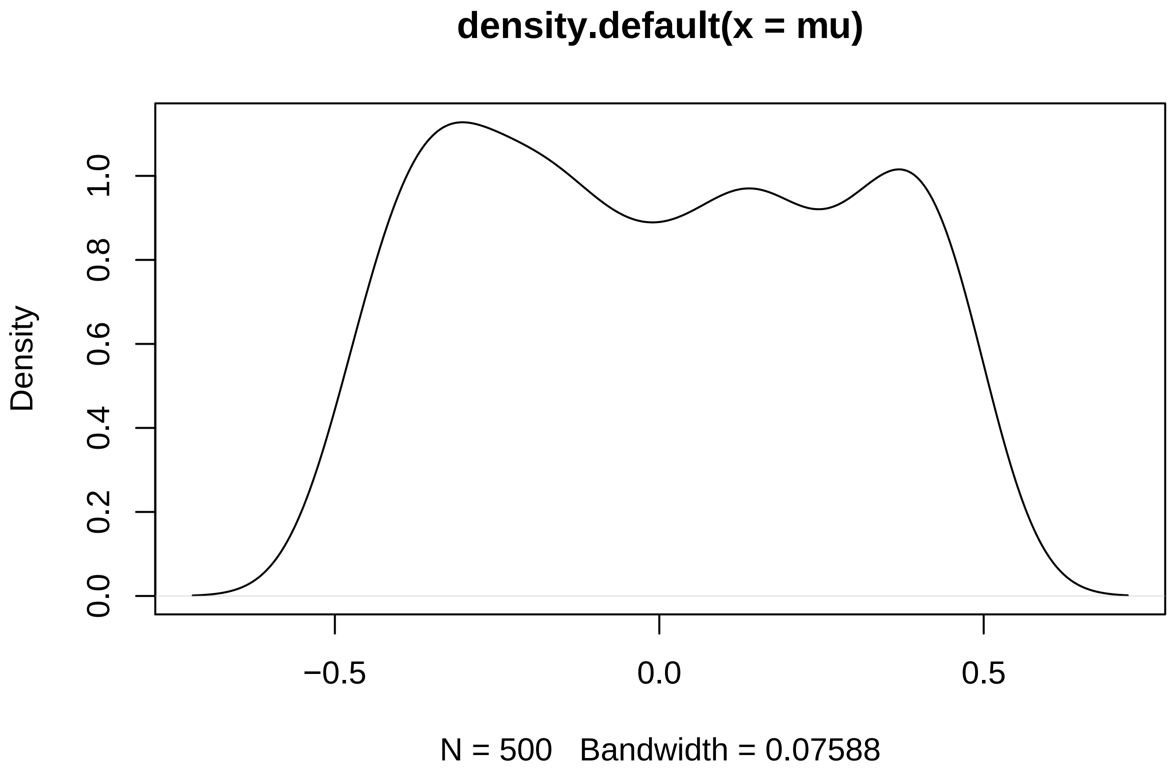
```
plot(mu,tau)
```



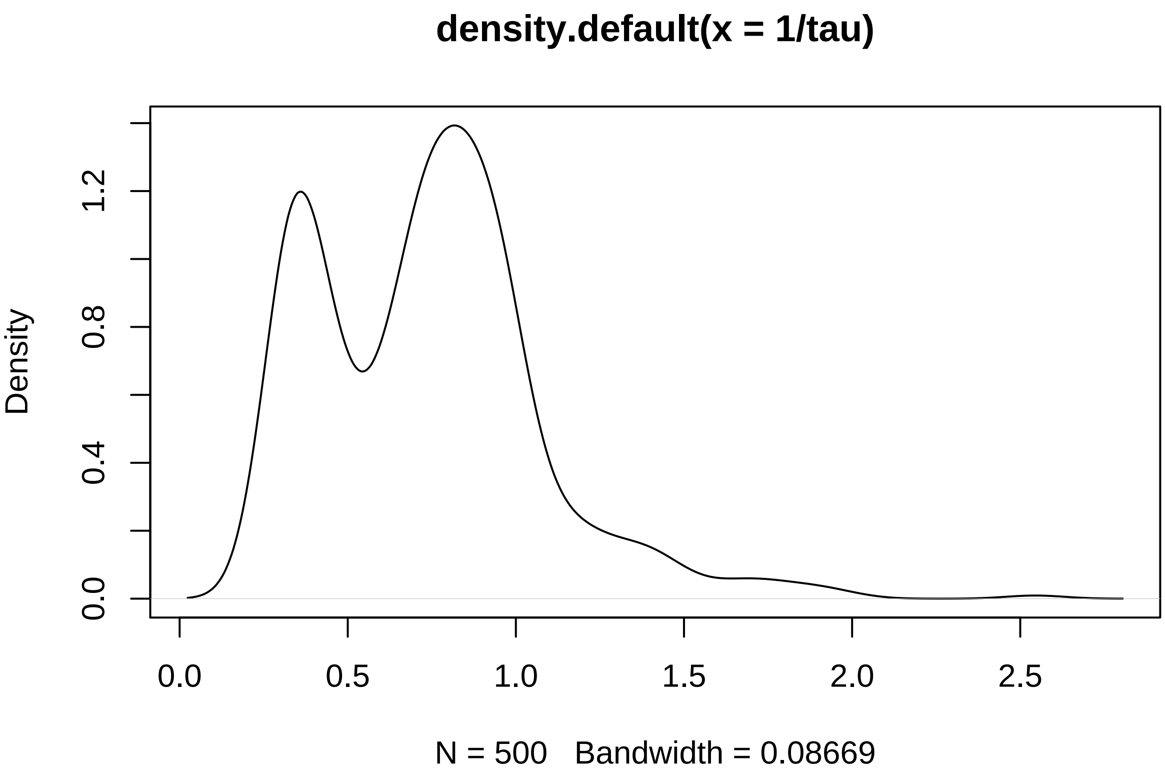
```
plot(mu[1:10],(1/tau)[1:10],type="s")
```



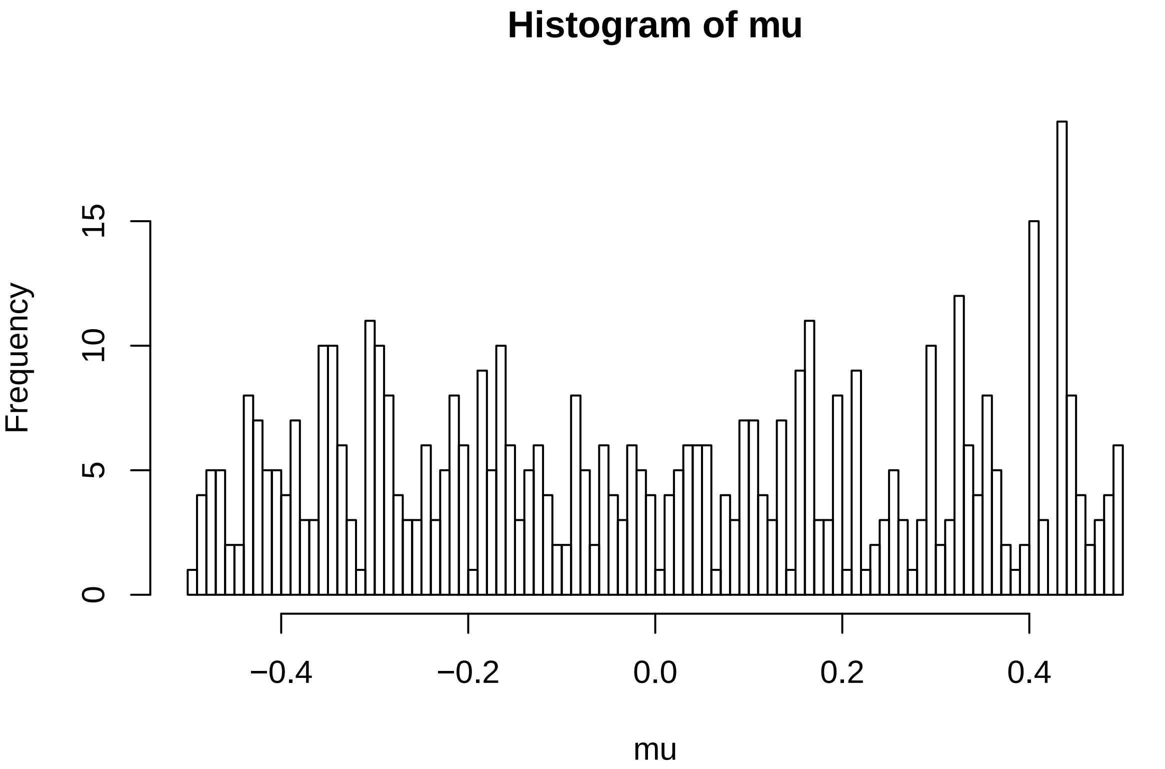
```
plot(density(mu))
```



```
plot(density(1/tau))
```



```r
hist(mu,breaks=100)
```



```r
print(mean(mu))
```

```
## [1] 0.0004174352
```

```r
print(mean(tau))
```

```
## [1] 1.676065
```