Companion software for "Volker Ziemann, *Hands-on Accelerator physics using MATLAB, CRCPress, 2019*" (https://www.crcpress.com/9781138589940)

Integrating the Sacherer equations (Section 12.1)

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The dynamics of the beam size of a beam with emittances ε_x and ε_y in the presence of space-charge can be described by Equations 12.4, often attributed to Sacherer. Here we convert these two second-order differential equations to four first-order equations and use ode45() to numerically integrate the beamsizes σ_x and σ_y through ten FODO cells with a 60^o phase advance per cell in both planes.

In this example we first define the emittaces `epsx` and `epsy`, as well as the perveance `Kperv` for a 25 MeV proton beam with a current of 10 mA. For simplicity, we make these variables available as `global` to the called functions. Here we also define the initial Twiss parameters, which are those of a zero-current 60^o FODO lattice.

```
clear all; close all
global epsx epsy Kperv                % needed in sachfun.m
epsx=1e-6; epsy=1e-6; Kperv=5e-8;     % parameters
betax=10.19292; alphax=-1.238978;     % initial values
betay=10.19292; alphay=1.238978;
```

Then we assign the initial valuesx0() for the integration. Note that $x_0(2) = dx_0(1)/ds$ and likewise $x_0(4) = dx_0(3)/ds$.

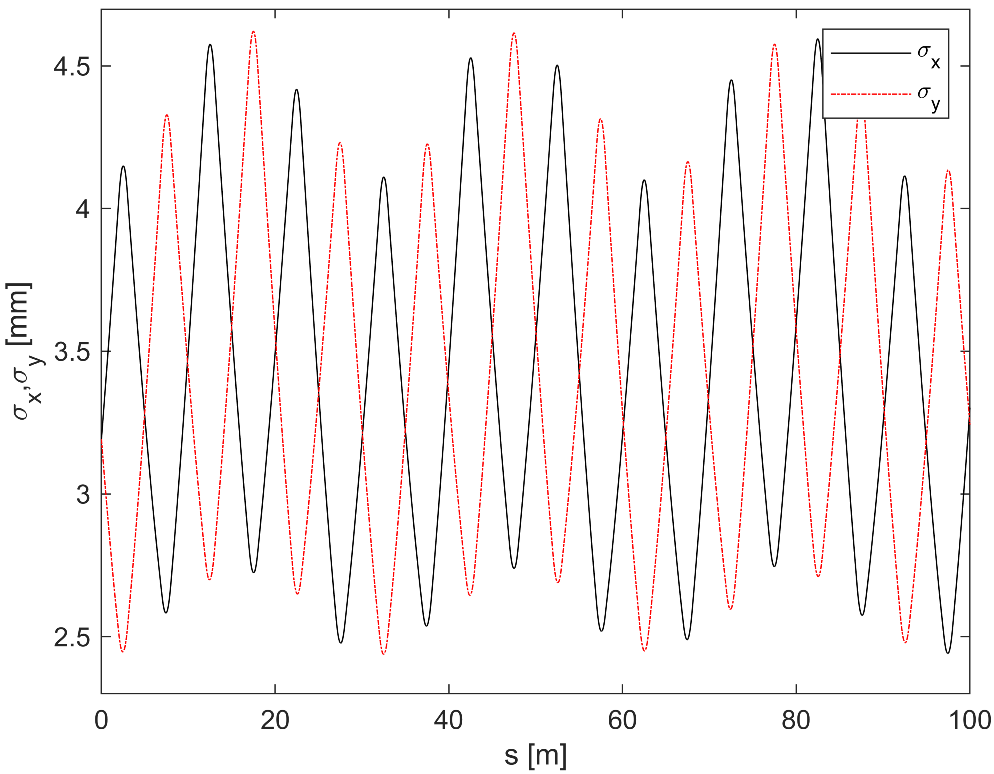
```
x0=zeros(4,1);                        % initial sigmas
x0(1)=sqrt(epsx*betax);               % sigmax
x0(2)=-alphax*sqrt(epsx/betax);       % sigmax'
x0(3)=sqrt(epsy*betay);               % sigmay
x0(4)=-alphay*sqrt(epsy/betay);       % sigmay'
```

Now we call the built-in integration routine `ode45()` with the right-hand side of the differential equation encoded in the function `sachfun()`, defined below, and initial values `x0`. Note also that we specify a small stepsize, which is necessary to catch the transitions from no-field outside to full field inside the quadrupoles.

```
[s,x]=ode45(@sachfun,[0,100],x0,odeset('MaxStep',1e-2));
```

Once ode45() completes the values of x=($\sigma_x, \sigma'_x, \sigma_y,$ and σ'_y) at position s are available. We immediately use them to plot σ_x and σ_y along our ten FODO cells.

```
plot(s,1e3*x(:,1),'k',s,1e3*x(:,3),'r-.')
ylim([2.3,4.7])
xlabel('s [m]'); ylabel('\sigma_x,\sigma_y [mm]')
legend('\sigma_x','\sigma_y')
```



And now it's time to explore different currents, encode by the value of K or different emittances.

Appendix

The function `sachfun()` returns the right-hand side of the two second-order ordinary differential equations from Equation 12.4, converted to a system of four first-order differential equations. Here we use the variables `x(1)=sigx`, `x(2)=sigx'`, `x(3)=sigy`, and `x(4)=sigy'`. The input of `sachfun()` are the position s and the vector x(). Moreover, the focusing from the quadrupoles is encapsulated in the function `k1()`, defined below. Note that the emittances ε_x and ε_y, as well as the perveance $K$are passed to the function as globals.

```
function dxds=sachfun(s,x)
global epsx epsy Kperv
dxds=zeros(4,1);
dxds(1)=x(2);
dxds(2)=-k1(s).*x(1)+epsx^2./x(1).^3+Kperv./(x(1)+x(3));
dxds(3)=x(4);
dxds(4)=k1(s).*x(3)+epsy^2./x(3).^3+Kperv./(x(1)+x(3));
end
```

The focusing strength at position s in a 60-degree FODO lattice is returned by the following function `k1()`.

```
% lattice focusing function
function out=k1(s)
kk1=0.29979*0.716792;  % quadrupole strength for 60-degree FODO
out=zeros(length(s),1);
out(mod(s,10)>=2 & mod(s,10)<=3)=kk1;
```

```
out(mod(s,10)>=7 & mod(s,10)<=8)=-kk1;
end
```