

Orbit correction in a ring (Section 8.5.1)

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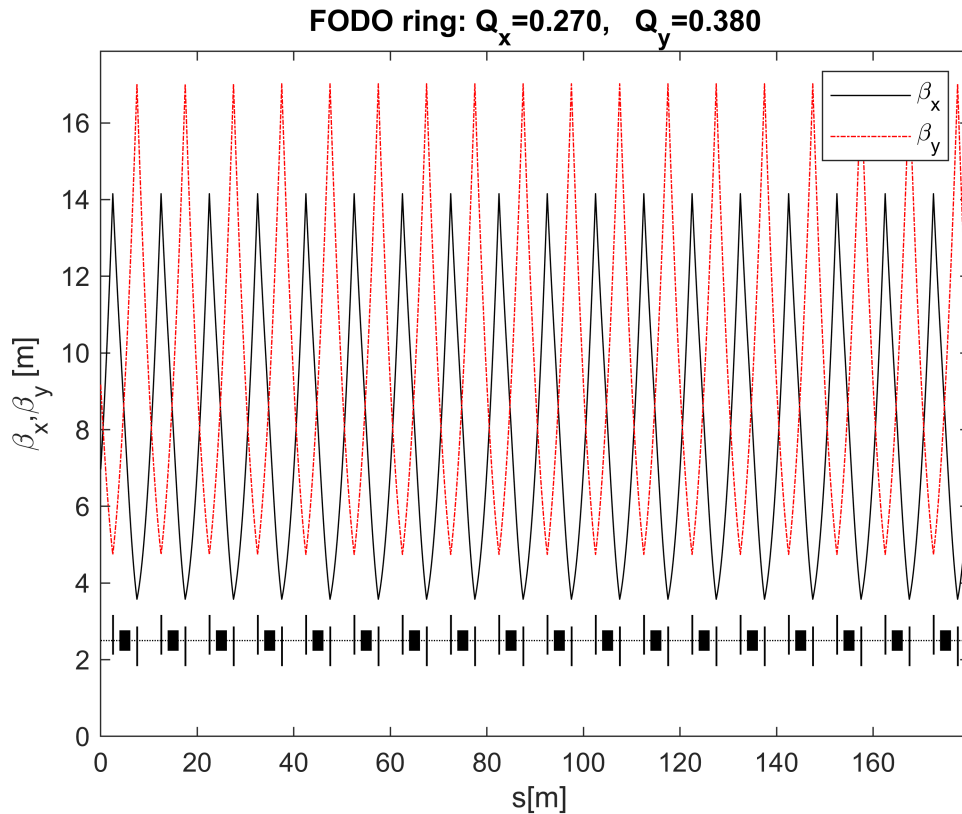
In this example, we illustrate correcting the horizontal orbit in a ring made of eighteen FODO cells with one corrector magnet and one beam position monitor (BPM) in each cell.

We add the support functions for the 4D beam optics calculations and define the FODO cell that has a phase advance of close to 90° in the horizontal plane and 60° in the vertical plane. Then the focal lengths of the thin-lens quadrupoles were slightly changed to give overall tunes of 4.27 and 3.38 for the full ring with 18 equal cells. Note also that we introduced two new element codes: 200 to indicate a corrector magnet and 300 to indicate a BPM.

```
clear
addpath ./4D
fodo=[
    1,    25,    0.1,    0;
    2,     1,    0,    4.357;    % QF
    200,  1,  0,    0;    % Corrector
    300,  1,  0,    0;    % BPM
    1,    15,    0.1,    0;
    4,    20,    0.1,    1;    % Dipole
    1,    15,    0.1,    0;
    2,     1,    0,   -4.4542;    % QD
    1,    25,    0.1,    0];
beamline= repmat(fodo,18,1);
```

In order to verify that the beta functions and the tunes are the desired ones we calculate the transfer matrices R_{acc} and from those the tunes, the periodic sigma matrix with the beta functions σ_0 , and then plot the beta functions.

```
[Racc,spos,nmat,nlines]=calcmat(beamline); Rend=Racc(:,:,end);
Q=tunes(Rend);
sigma0=periodic_beammatrix(Rend,1,1);
plot_betas(beamline,sigma0);
legend('AutoUpdate','off') % cosmetic, prevent extra entry in legend
drawmag(beamline,2,1)
title(['FODO ring: Q_x=',num2str(Q(1),'%5.3f'),' ', 'Q_y=',num2str(Q(2),'%5.3f')])
```



Before simulating the ring, we need to prepare arrays that contain the positions of the correctors, the BPM, and the quadrupoles. To do so we loop over all elements and segments in the beamline, increment counters for each device found; `nquad` for the quadrupoles, `nbpm` for the BPM, and `ncor` for the correctors. At the same time we record their position `ic` in the beam line; in `quadpos()` for the quadrupoles, in `corpos()` for the correctors, and in `bpmpos()` for the BPM. As a convenience, we also record the focal lengths of the found quadrupoles in `quadf()`.

```
nquad=0; quadpos=zeros(1,36); quadf=quadpos;
nbpm=0; bpmpos=zeros(1,18);
ncor=0; corpos=zeros(1,18);
ic=1;
for line=1:nlines
    for seg=1:beamline(line,2)
        ic=ic+1;
        switch beamline(line,1)
            case 2 % thin quadrupole
                nquad=nquad+1;
                quadpos(nquad)=ic;
                quadf(nquad)=beamline(line,4);
            case 200 % corrector
                ncor=ncor+1;
                corpos(ncor)=ic;
            case 300 % bpm
                nbpm=nbpm+1;
                bpmpos(nbpm)=ic;
        end
    end
end
```

```
end
```

Now that we know the positions of the correctors and the BPM, we can calculate the response matrix C_x in the horizontal plane in a nested loop over all BPM and over all correctors and for each combination calculating the 4x4 response matrix C with the function $CC()$, which is defined in the Appendix.

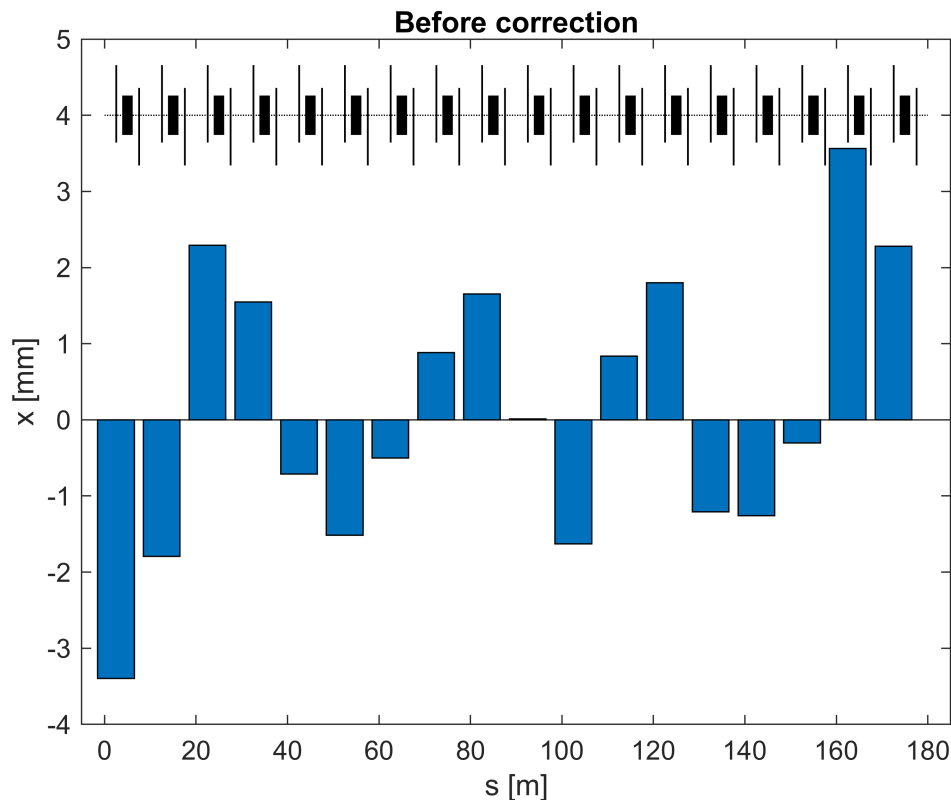
```
Cx=zeros(nbpm,ncor);
for ibpm=1:nbpm
    for icor=1:ncor
        C=CC(bpmpos(ibpm),corpos(icor),Racc);
        Cx(ibpm,icor)=C(1,2);
    end
end
```

Likewise we determine the BPM-quadrupole response matrix, which relates a transverse quadrupole displacement d_x to the change of the beam position on a BPM. As for the BPM-corrector response matrix before, we calculate 4x4 response matrix C with the function $CC()$ and copy its 12-element into the appropriate place in the matrix $C_{qx}()$. We then divide by the focal length f of the respective quadrupole, because the kick θ that a displaced quadrupole gives to the beam is $\theta = d_x/f$.

```
Cqx=zeros(nbpm,nquad);
for ibpm=1:nbpm
    for iquad=1:nquad
        C=CC(bpmpos(ibpm),quadpos(iquad),Racc);
        Cqx(ibpm,iquad)=C(1,2)/quadf(iquad);
    end
end
```

We immediately use C_{qx} to calculate effect of displaced quadrupoles on the position recorded on the BPM. We therefore randomly displace the quadrupoles by dx , sampled from a uniform distribution in the range ± 0.5 mm and loop over the BPM; for each BPM we sum the contributions from all quadrupoles to the reading of the BPM. After the loops complete, we plot $bpmx()$ in a bargraph to simulate the typical display of beam positions found in accelerator control rooms.

```
dx=1e-3*(rand(1,nquad)-0.5); % quad displacement
bpmx=zeros(1,nbpm);
for ibpm=1:nbpm
    for iquad=1:nquad
        bpmx(ibpm)=bpmx(ibpm)+Cqx(ibpm,iquad)*dx(iquad);
    end
end
figure; bar(spos(bpmpos),1e3*bpmx); drawmag(beamline,3.5,1);
xlim([-5,spos(end)+5]); xlabel('s [m]'); ylabel('x [mm]');
title('Before correction')
```



In the next step we use the BPM readings to calculate the corrector setting, based on inverting Equation 8.53, that is required to zero the BPM readings.

```
% theta=-inv(Cx)*bpmx'; % old style, less robust
theta=-Cx\bpmx'; % correction by inverting eq. 8.53
```

Next, we introduce the possibility to simulate badly calibrated correctors by giving all correctors a scale factor `corscale`. If it is less than unity, say 0.9, all corrector power supplies only produce 90% of the requested current.

```
corscale=1 % Slider for global corrector calibration error

corscale = 1
```

We can also select a single bad corrector `badcor` and give it an individual scale error by setting `badcorscale` to a value different from unity. By choosing `badcorscale=-1` we can simulate a power supply with flipped polarity.

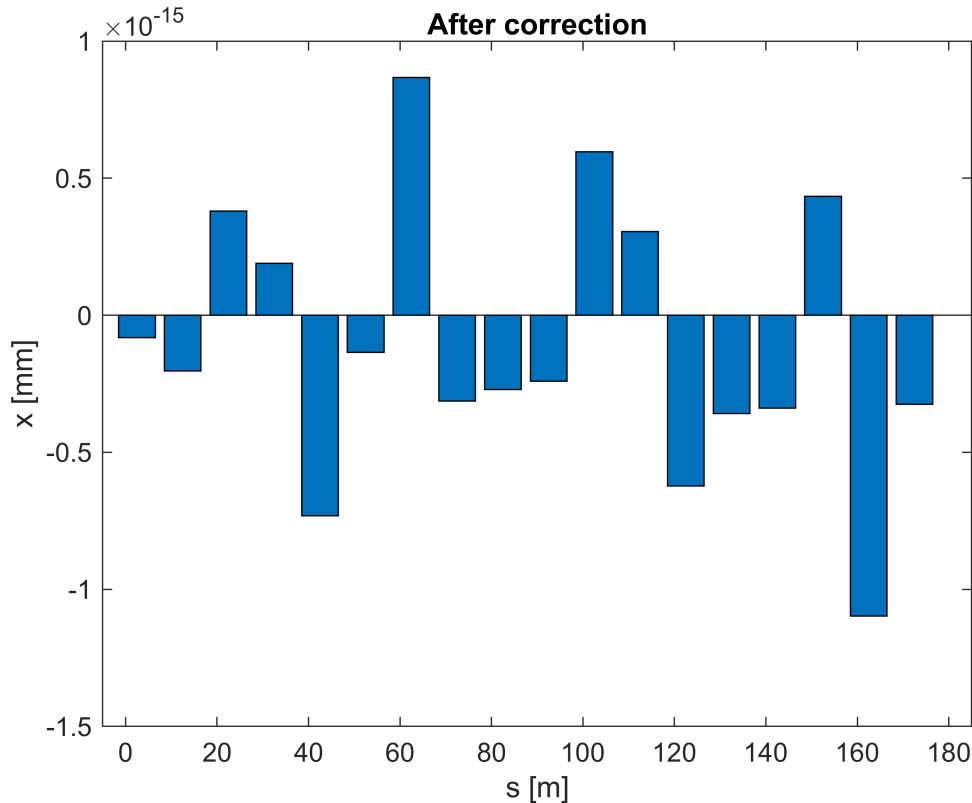
```
badcor=2; % second corrector is bad
badcorscale=1 % Slider for scale error of the bad corrector

badcorscale = 1
```

```
if badcor>0
    theta(badcor)=badcorscale*theta(badcor); % single corrector with wrong calibration
end
```

Finally, we add the effect of the corrector magnets with non-zero excitation to the BPM reading and display the resulting orbit.

```
for ibpm=1:nbpm           % add corrections to bpm
    for icor=1:ncor
        bpmx(ibpm)=bpmx(ibpm)+corscale*Cx(ibpm,icor)*theta(icor);
    end
end
figure; bar(spos(bpmpos),1e3*bpmx);
xlim([-5,spos(end)+5]); xlabel('s [m]'); ylabel('x [mm]');
title('After correction')
```



Note that without any errors the orbit correction is perfect and the resulting BPM readings are zero to machine precision.

Now it's time to play with the scale errors of the corrector magnets to simulate how much badly calibrated steering magnets affect the orbit correction. You might also want to add code to simulate bad BPMs.

Appendix

CC()

The function `CC()` receives the positions of the BPM and corrector in the beam line sequence and the transfer matrices `Racc` as input and returns the 4x4 response matrix from Equation 8.28 as `out`. Inside the function, we have to separately treat the cases, where the index of the BPM is larger than that of the corrector, or vice-versa.

```
function out=CC(ipos,jpos,Racc)
```

```

Rjj=Racc(:, :, jpos)*Racc(:, :, end)*inv(Racc(:, :, jpos));
if ipos > jpos
    Rij=Racc(:, :, ipos)*inv(Racc(:, :, jpos));
else
    Rij=Racc(:, :, ipos)*Racc(:, :, end)*inv(Racc(:, :, jpos));
end
out=Rij*(inv(eye(4)-Rjj));    % eq. 8.28
end

```

calcmat()

We have amended the function `calcmat()` for this program to handle the correctors with code 200 and the BPM with code 300 to prevent many unsupported code messages. The rest of the function is unchanged.

```

function [Racc,spos,nmat,nlines]=calcmat(beamline)
ndim=size(DD(1),1);
nlines=size(beamline,1);    % number of lines in beamline
nmat=sum(beamline(:,2))+1;    % sum over repeat-count in column 2
Racc=zeros(ndim,ndim,nmat);    % matrices from start to element-end
Racc(:, :, 1)=eye(ndim);    % initialize first with unit matrix
spos=zeros(nmat,1);    % longitudinal position
ic=1;    % element counter
for line=1:nlines    % loop over input elements
    for seg=1:beamline(line,2)    % loop over repeat-count
        ic=ic+1;    % next element
        Rcurr=eye(4);    % matrix in next element
        switch beamline(line,1)
            case 1    % drift
                Rcurr=DD(beamline(line,3));
            case 2    % thin quadrupole
                Rcurr=Q(beamline(line,4));
            case 4    % sector dipole
                phi=beamline(line,4)*pi/180;    % convert to radians
                rho=beamline(line,3)/phi;
                Rcurr=SB(beamline(line,3),rho);
            case 5    % thick quadrupole
                Rcurr=QQ(beamline(line,3),beamline(line,4));
            case 20    % coordinate roll
                Rcurr=ROLL(beamline(line,4));
            case 200    % corrector
                Rcurr=eye(4);
            case 300    % BPM
                Rcurr=eye(4);
            otherwise
                disp('unsupported code')
        end
        Racc(:, :, ic)=Rcurr*Racc(:, :, ic-1);    % concatenate
        spos(ic)=spos(ic-1)+beamline(line,3);    % position of element
    end
end
end

```