

## Chromaticity of a FODO lattice (Section 3.4.1)

Volker Ziemann, 211124, CC-BY-SA-4.0

In this example we calculate the horizontal chromaticity of a somewhat contrived (no dipoles) ring made of 20 FODO cells.

We use the 2D software and define a FODO cell as in previous examples

```
clear
addpath ./2D
F=2.2 % Slider for the focal length of the quadrupoles
```

```
F = 2.2000
```

```
fodo=[ 1, 5, 0.2, 0;
       2, 1, 0.0, -F;
       1, 10, 0.2, 0;
       2, 1, 0.0, F;
       1, 5, 0.2, 0];
beamline=fodo;
```

First we can calculate the transfer matrices with `calcmat()`, define the matrix `Rturn`, which is the transfer matrix from start to end of the cell, and determine the phase advance of the cell  $Q$  in units of  $2\pi$  and the periodic Twiss parameters  $\alpha_0, \beta_0$ , and  $\gamma_0$ . And from those we construct the beam matrix  $\sigma_0$  which we later use to propagate the Twiss parameters through the ring.

```
[Racc,spos,nmat,nlines]=calcmat(beamline);
Rturn=Racc(:, :, end);
[Q,alpha0,beta0,gamma0]=R2beta(Rturn);
sigma0=[beta0, -alpha0; -alpha0, gamma0];
```

Next, we construct the beamline made of 20 FODO cells and calculate all transfer matrices with `calcmat()`.

```
beamline=repmat(fodo,20,1); % 20 FODO cells
[Racc,spos,nmat,nlines]=calcmat(beamline);
```

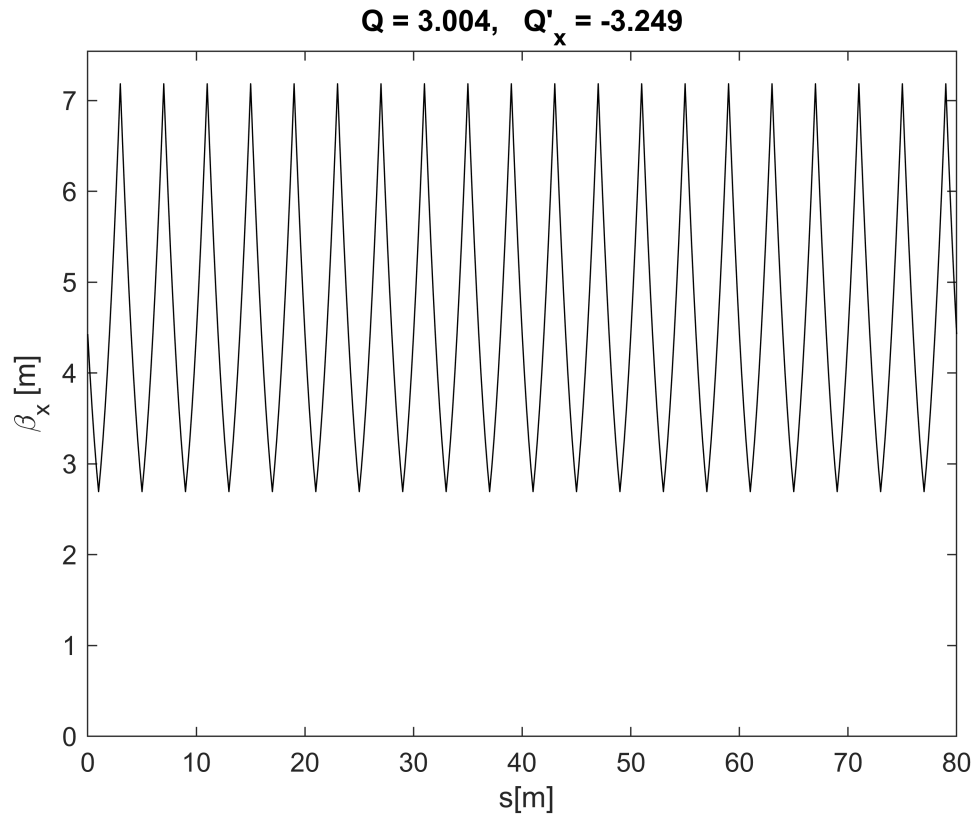
Having prepared the beam matrix  $\sigma_0$  with the Twiss parameters and all transfer matrices, we use Equation 3.43 to propagate the Twiss parameters through the ring. At each position, labeled by the loop index  $k$ , we store the value of the beta function in the previously allocated array `beta`. Once the loop completes, we plot the beta function and annotate the axes.

```
beta=zeros(nmat,1); % allocate space to store beta functions
for k=1:nmat
    sigma=Racc(:, :, k)*sigma0*Racc(:, :, k)'; % eq. 3.43
    beta(k)=sigma(1,1);
end
plot(spos,beta,'k');
```

```
xlabel(' s[m]'); ylabel('\beta_x [m]')
axis([0, spos(end), 0, 1.05*max(beta)])
```

Finally we calculate the chromaticity  $Q'_x$  from Equation 3.87. To do so we need to loop over all segments in the beamline and add  $-\beta/4\pi f$  in each quadrupole. Once the loop completes, we report the tune of the ring, which is 20 times the tune of a cell, and the chromaticity in the title bar of the plot.

```
Chomaticity=0; % initialize to zero
ic=1;
for line=1:nlines
    for seg=1:beamline(line,2)
        ic=ic+1;
        if beamline(line,1)==2 % quadrupole found
            Chomaticity=Chomaticity-beta(ic)/(4*pi*beamline(line,4)); % eq. 3.87
        end
    end
end
title(['Q = ', num2str(20*Q, '%6.3f'), ', Q' '_x = ' num2str(Chomaticity,4)])
```



And now you can play with the slider to change the focal length  $F$  of the quadrupoles so see how stronger focussing quadrupoles increase both the tune and the chromaticity.