Companion software for "Volker Ziemann, *Hands-on Accelerator physics using MATLAB, CRCPress, 2019*" (https://www.crcpress.com/9781138589940)

Quadrupole (Section 4.3.3)

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Important: this example requires the PDE toolbox!

In this example we analyze a quadrupole magnet. We immediately realize, that it has a four-fold symmetry, such that we can restrict the simulation volume to one quadrant and later specify von-Neumann boundary conditions on the symmetry planes.

Let's start with the definition of the iron yoke, which has a more complicated geometry thatn the dipoles we considered previously. In particular, the pole-face that has a hyperbolic shape; we approximate it by five straight segments.

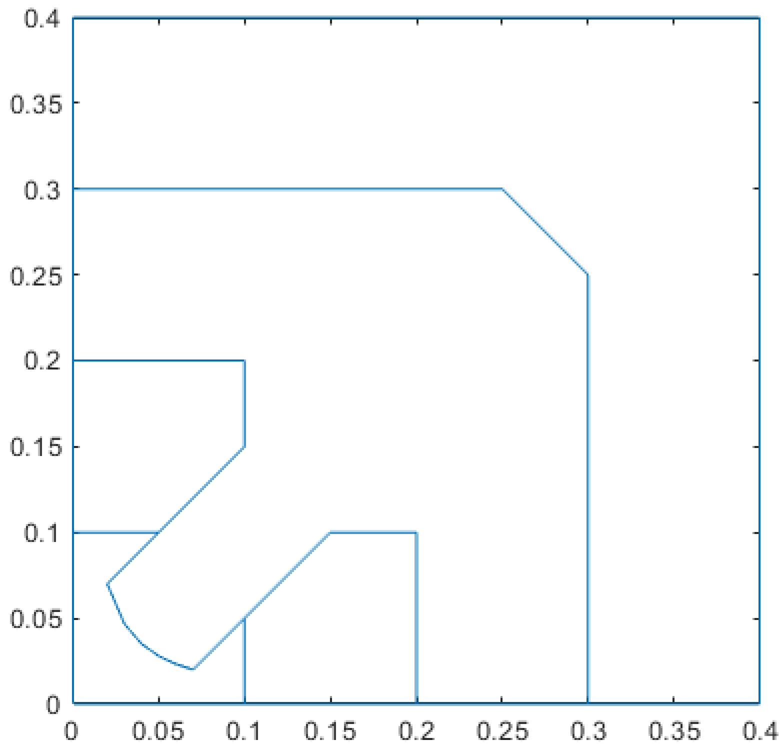
```
clear all; close all
yoke=[2;16; ...
  0; 0.25;0.3;0.3;0.2;0.2;0.15;0.07;0.06;0.05;0.04;0.03;0.02;0.10;0.10;0;
  0.3;0.3;0.25;0;0;0.1;0.1;0.02;0.0233;0.028;0.035;0.0467;0.07;0.15;0.2;0.2];
```

Also the coils have a more complicated shape. They are deifned as a five-sided polygon. Note that we define the enclosing volumen, the World, as five-sided polygon, with the origin expluded. Excluding the origin is necessary, because the there the von-Neumann boundary conditions (normal derivatives to the surface) of the left-hand and the bottom boundary would clash.

```
C1=[2;5;0.1;0.2;0.2;0.15;0.1;0;0;0.1;0.1;0.05;zeros(22,1)];
C2=[2;5;0;0.1;0.1;0.05;0;0.2;0.2;0.15;0.1;0.1;zeros(22,1)];
World=[2;5;  0.001;0.4;0.4;0;0;  0;0;0.4;0.4;0.001;  zeros(22,1)];
```

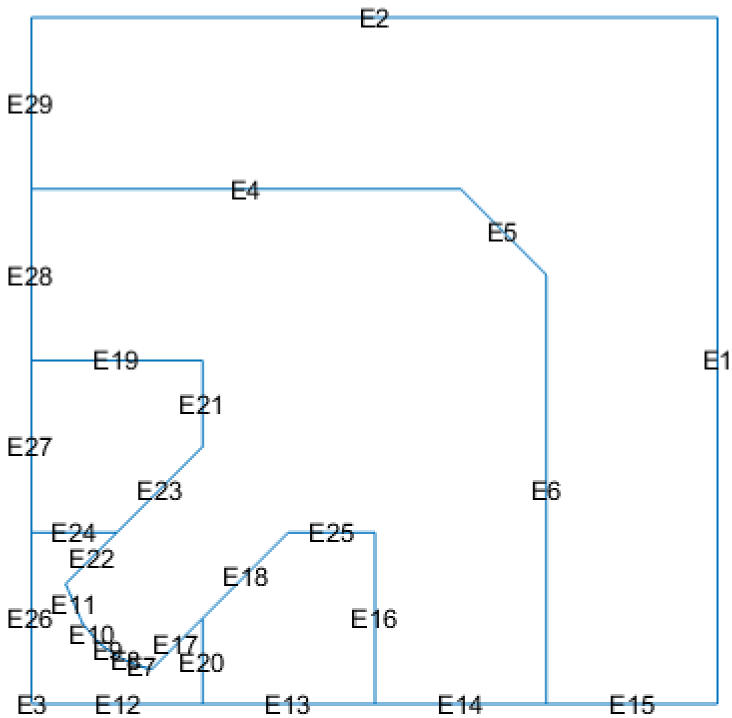
Now we can define geometry g, create the `model` with one degree of freedom u $= A_z$, include the geometry in the model with geometryFromEdges(), and plot it.

```
gd=[World,yoke,C1,C2];  % assemble geometry
ns=char('World','yoke','C1','C2')';  % names of the regions
sf='World+yoke+C1+C2';
g=decsg(gd,sf,ns);
model=createpde(1);
geometryFromEdges(model,g);
pdegplot(model)
```



After plotting the geometry once again, this time with `EdgeLabels` displayed, we can use this information to specify boundary conditions. We set the potential `u` = A_z on the boundaries `E1, E2,` and `E3` (the small edge at the origin) of the `World` to zero. On the left-hand boundaries E26 to E29 and the bottom E12 to E15 we apply von-Neumann conditions, which will cause the model to obey the symmetries of the full magnet.

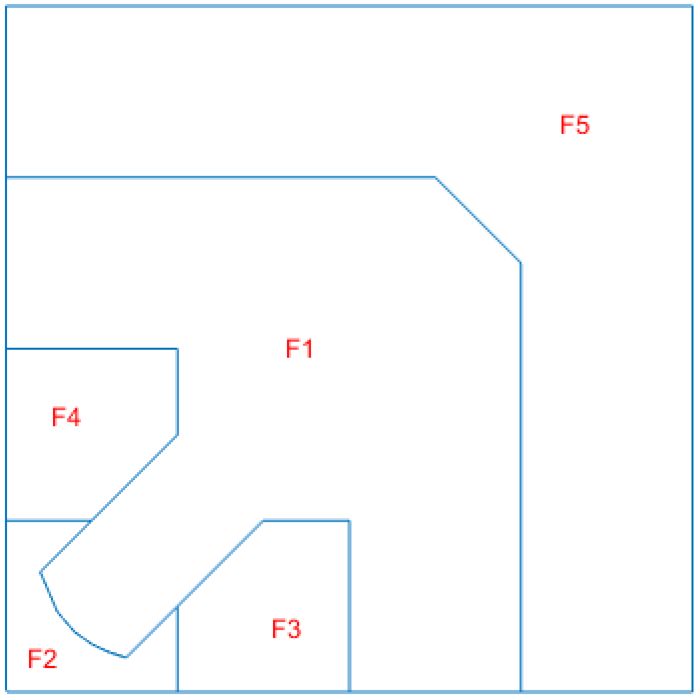
```
figure
pdegplot(model,'EdgeLabels','on'); axis off; axis equal
```




```
applyBoundaryCondition(model,'Edge',[1:3],'u',0);
applyBoundaryCondition(model,'Edge',[12:15,26:29],'q',0,'g',0);
```

In the next step we plot the model with SubDomainLabels shown and use that information to specify the material properties. The current density used is $J_z = 10^6$ A/m^2 and $\mu_r = 5000$.

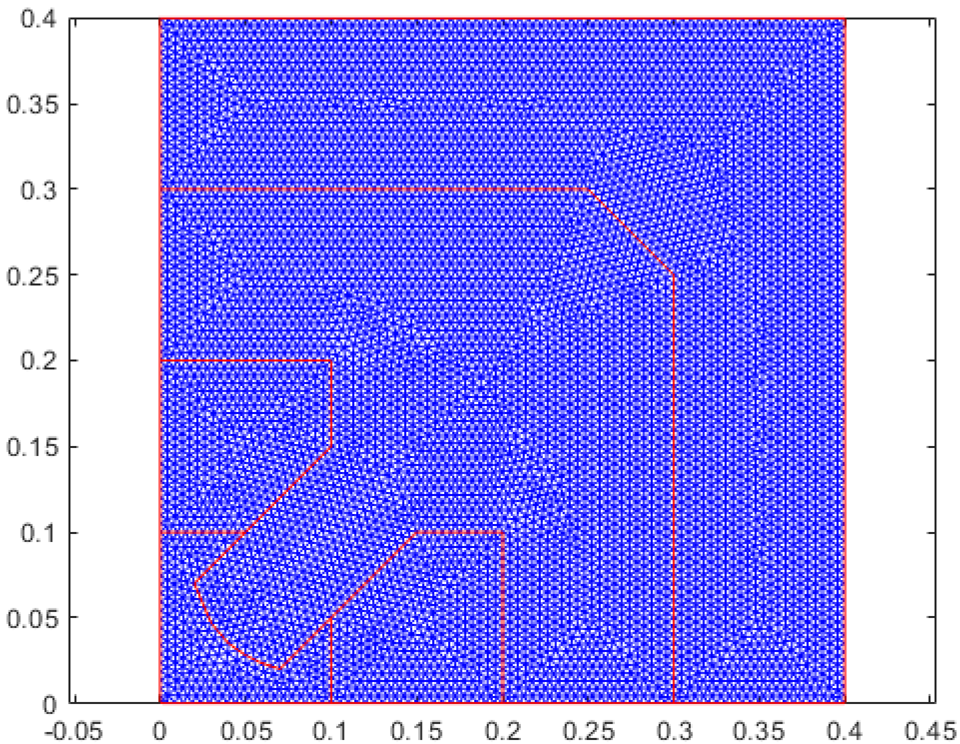
```
figure; pdegplot(model,'SubDomainLabels','on'); axis off; axis equal
```



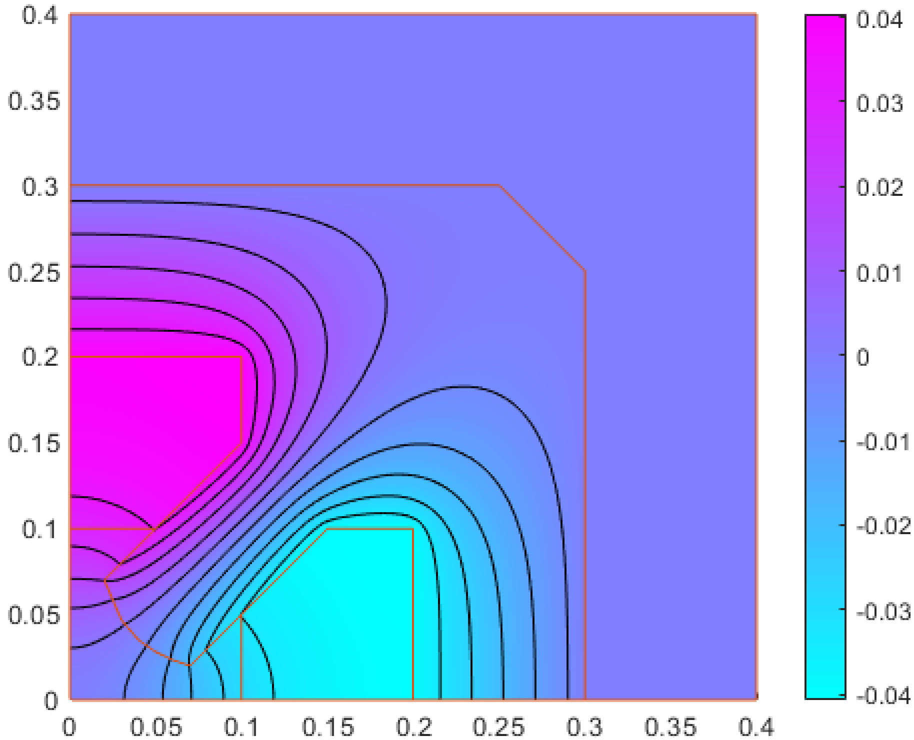

```
specifyCoefficients(model,'m',0,'d',0,'c',1/5000,'a',0,'f',0,'Face',1);
specifyCoefficients(model,'m',0,'d',0,'c',1,'a',0,'f',0,'Face',[2,5]);
specifyCoefficients(model,'m',0,'d',0,'c',1,'a',0,'f',-1.2566,'Face',3);
specifyCoefficients(model,'m',0,'d',0,'c',1,'a',0,'f',1.2566,'Face',4);
```

Now we generate the mesh with a small mesh size of 5 mm and display it before solving the model and plotting u, which is available as the `NodalSolution` in the structure `result`.

```
generateMesh(model,'Hmax',0.005);
figure;
pdemesh(model);
```

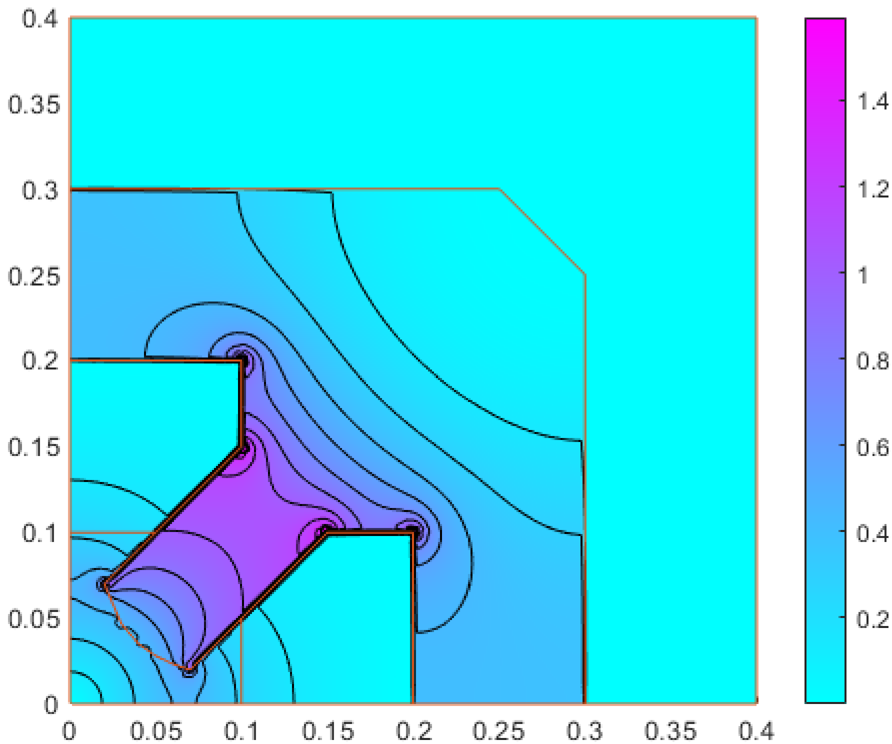



```
result=solvepde(model);
figure;
pdeplot(model,'xydata',result.NodalSolution,'contour','on');
hold on; pdegplot(model)
```



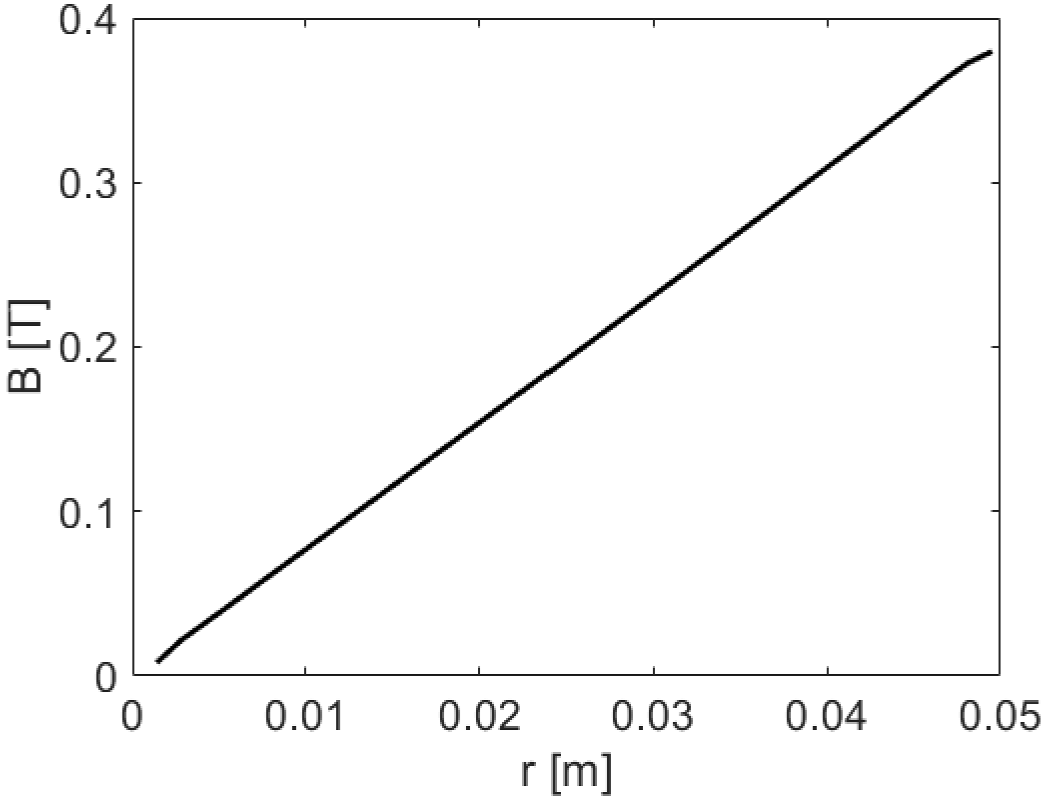
As before we calculate the extract the magnitude of the magnetic field Bn and display it with the model superimposed. We find that the maximum field in this case is 1.4 T, which is below 2 T, the typical saturation level of iron.

```
Bn=hypot(result.XGradients,result.YGradients);
figure;
pdeplot(model,'xydata',Bn,'contour','on');
hold on; pdegplot(model);
```



Since the beam dynamics depends on the gradient of a quadrupole, we determine it by evaluating Bn along a line on the diagonal from the origin (actually slightly off, because we had omitted the origin from the calculation volume) to close to the pole face. We find a nicely linear dependence.

```
x=0.001:0.001:0.035; y=x;  % Diagonal
[By,Bx]=evaluateGradient(result,x,y);  Bx=-Bx;
B=hypot(Bx,By);
figure; plot(hypot(x,y),B,'k','LineWidth',2)
xlabel('r [m]'); ylabel('B [T]');
set(gca,'FontSize',16)
```




```
dlmwrite('Qmag2.dat',[x',B],'delimiter','\t');
```

Normally the gradient is specified as $\partial B_y/\partial x$ along the horizontal axis, but we leave this as a exercise.