Companion software for "Volker Ziemann, *Hands-on Accelerator physics using MATLAB, CRCPress, 2019*" (https://www.crcpress.com/9781138589940)

Dynamic aperture (Section 11.2)

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In this example we follow particles in two transverse planes and iterate Equation 11.4 in order to find the dynamic aperture, shown in Figure 11.2, in cases where the non-linearity, a single sextupole, is well centered in the beam pipe and when it is misaligned.

We determine the dynamic aperture by testing whether the particle survives `NN=1000` iterations. Then we select the horizontal and vertical tunes, `Qx` and `Qy`, respectively, and define the transfer matrix R, which is a simple rotation matrix in this case.

```
clear all; close all
NN=1000;    % maximum number of turns
Qx=0.31; Qy=0.28;
R1D=@(mu)[cos(mu),sin(mu);-sin(mu),cos(mu)];
R=[R1D(2*pi*Qx),zeros(2);zeros(2),R1D(2*pi*Qy)];
hold on
```

Sextupole not misaligned

First we consider the system without misalignment and probe the dynamic aperture along a line in the transverse (x, y) space defined by $x = r\cos\phi$ and $y = r\sin\phi$.

```
for phi=0:pi/100:pi   % loop over angles phi
```

We start by choosing r very large and then iteratively increase it by $dr = r/2$ depending on whether the particle with starting coordinates $x_0 = r\cos\phi$ and $y_0 = r\sin\phi$ survived NN turns or not. We repeat this procedure by halving the size of the interval `nmax=12` times.

```
  nmax=12; dr=10; r=dr;
  while (nmax)        % scan radially
     dr=0.5*dr;
     x0=r*cos(phi); y0=r*sin(phi);  % new starting coordinates
```

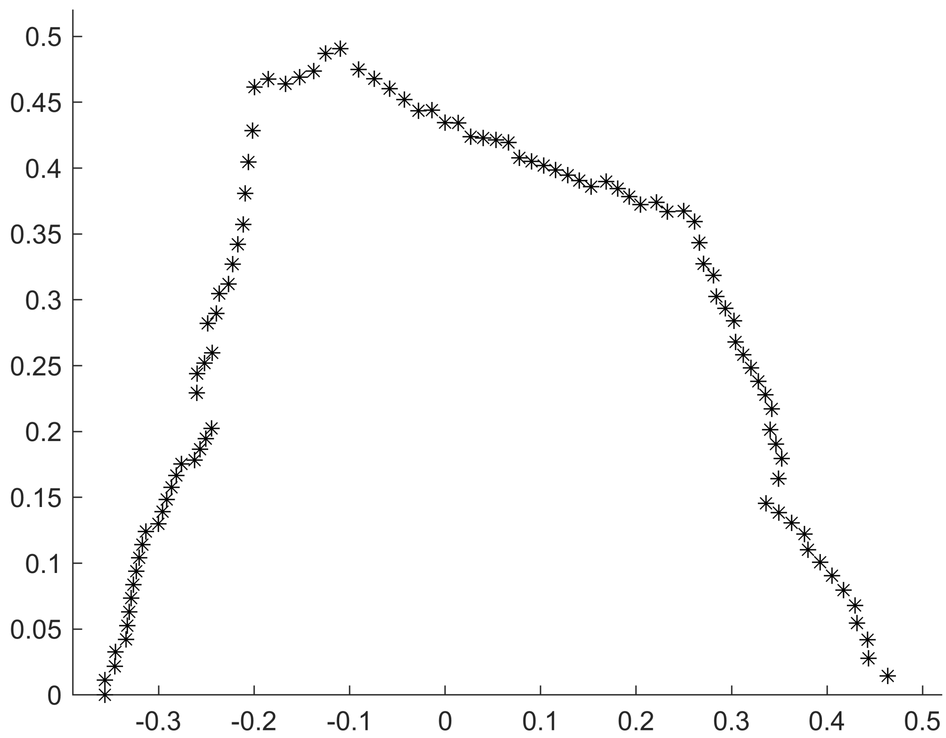
The function `survived_turns()`, defined in the Appendix, interates Equation 11.4 for a maximum on `NN` turns and returns the maximum number of turns that the particle survived.

```
     if (survived_turns(NN,R,x0,y0,0,0)==NN), r=r+dr; else, r=r-dr; end
     nmax=nmax-1;
  end
```

The value of (x_0, y_0) thus determined show the largest starting coordinates along a line, defned by the angle ϕ for which a particles survives NN turns. We show this point by a black asterisk in transverse (x, y) space.

```
  plot(x0,y0,'k*');
end
```

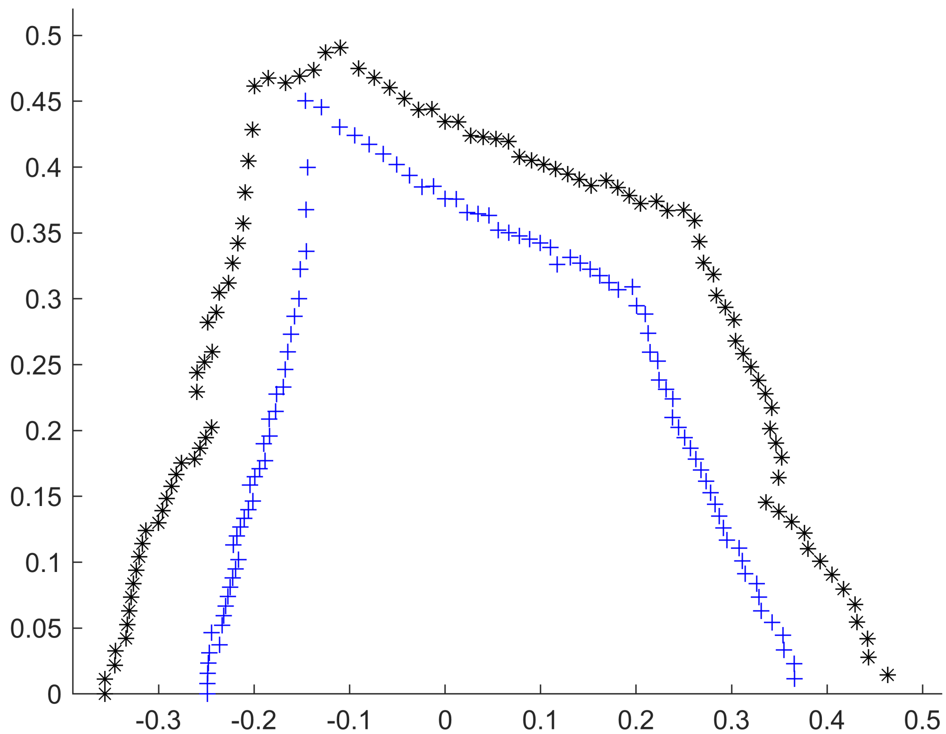
```
ylim([0,0.52]); xlim([-0.39,0.52])
```



Sextupole misaligned by dx=-0.05

Now we repat the same procedure, but horizontally displace the sextupole by `dx=-0.05`, which we pass as an argument to the function survived_turns(). In this way we find the dynamic aperture for the configuration with the misaligned sextupole, which plot as a blue plus sign. We find that the dynamic aperture, which defines the boundary between particle trajectories that are stable and those that are unstable, has become smaller.

```
for phi=0:pi/100:pi          % loop over angles phi
  nmax=12; dr=10; r=dr;
  while (nmax)               % scan radially
     dr=0.5*dr;
     x0=r*cos(phi); y0=r*sin(phi);
     if (survived_turns(NN,R,x0,y0,-0.05,0)==NN), r=r+dr; else, r=r-dr; end
     nmax=nmax-1;
  end
  plot(x0,y0,'b+');
end
```



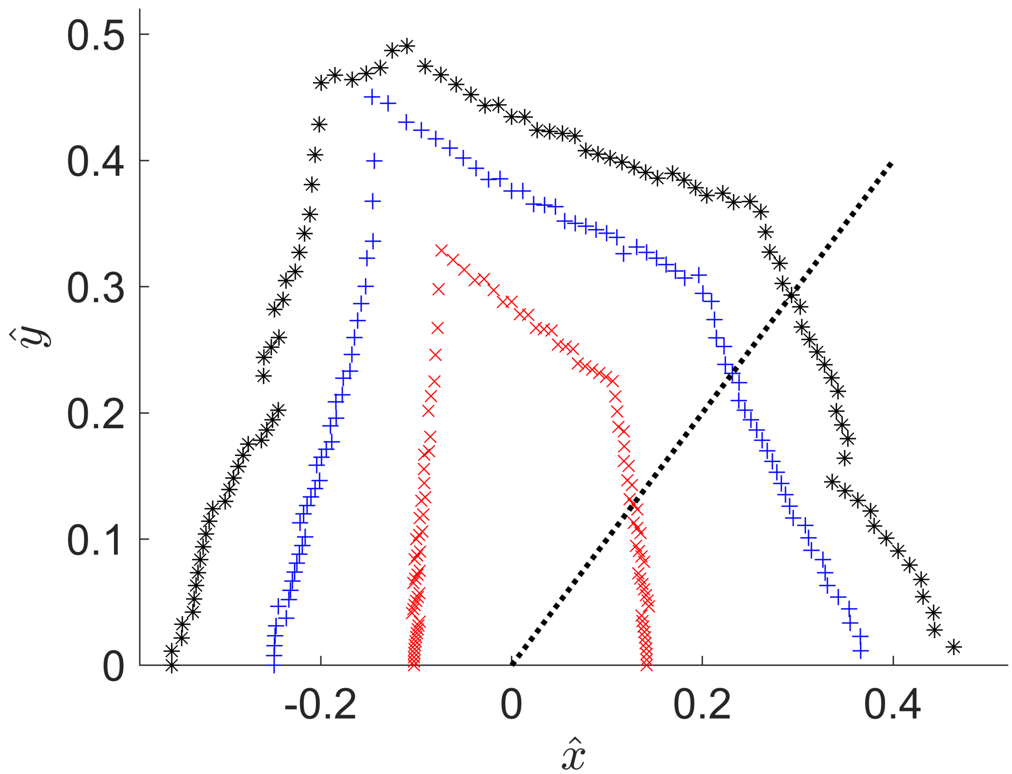
Sextupole misaligned by dx=-0.1

And in a third pass we increase the misalignement to `dx=-0.1`, repeat the procedure, and plot the dynamic aperture as red crosses. We observe that it has become even smaller.

```
for phi=0:pi/100:pi  % dx=-0.1, dy=0
  nmax=12; dr=10; r=dr;
  while (nmax)
     dr=0.5*dr;
     x0=r*cos(phi); y0=r*sin(phi);
     if (survived_turns(NN,R,x0,y0,-0.1,0)==NN), r=r+dr; else, r=r-dr; end
     nmax=nmax-1;
  end
  plot(x0,y0,'rx');
end
```

After the three dynamic apertures for `dx=0,-0.05`, and `-0.1` are displayed, we annotate the axes.

```
plot([0,0.4],[0,0.4],'k:','LineWidth',2)
xlabel('$\hat x$','interpreter','latex');
ylabel('$\hat y$','interpreter','latex');
set(gca,'FontSize',16);
```



Survived turns along $\phi = 45^o$

Now we want to determine the number of turns that a particle survives as a function of its starting coordinates located on a line with coordinates $x_0 = r\cos\phi$ and $y_0 = r\sin\phi$ where $\phi = 45^o$. The starting positions are indicated on the previous plot by the diagonal dotted line. Here we therefore define the angle `phi` and the starting values for `r` and allocate an array `turns` in which we store the survived turns for each starting coordinate. We do so for the same misalignments `dx` considered above.

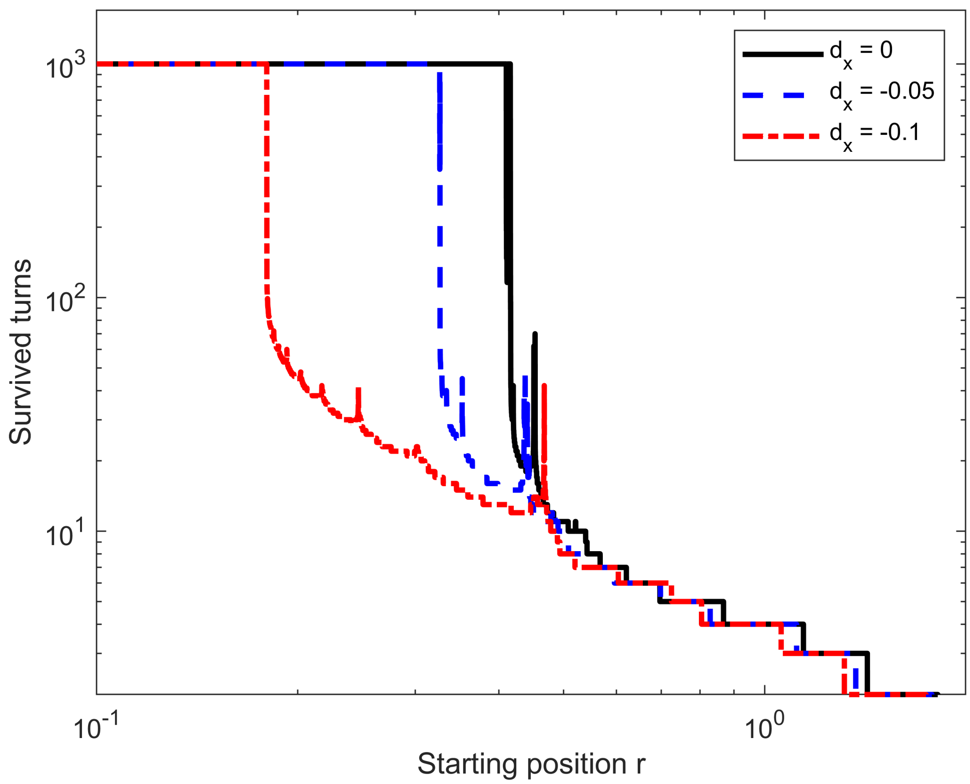
```
phi=45*pi/180;
r=2:-0.0001:0.1;
turns=zeros(3,length(r));
```

Now we loop over the radial positions r, define starting coordinates x0 and y0 and use the function survived_turns() to store the found value in the array turns.

```
for k=1:length(r)
  x0=r(k)*cos(phi); y0=r(k)*sin(phi);
  turns(1,k)=survived_turns(NN,R,x0,y0,0,0);       % dx=0
  turns(2,k)=survived_turns(NN,R,x0,y0,-0.05,0);   % dx=-0.05
  turns(3,k)=survived_turns(NN,R,x0,y0,-0.1,0);    % dx=-0.1
end
```

Once the loop completes, we open a new figure, plot the number of survived turns for the three cases on double-logarithmic scales, and annotate the axes. Note that the radial coordinate r where the respective curves start to deviate from 10^3 correspond to the intersection of the dotted black line on the previous figure with the three dynamic apertures.

```
figure(2)
loglog(r,turns(1,:),'k',r,turns(2,:),'b--',r,turns(3,:),'r-.','LineWidth',2)
axis([0.1,2,2,1700])
legend('d_x = 0','d_x = -0.05','d_x = -0.1')
xlabel('Starting position r'); ylabel('Survived turns')
```



Note how fast the survived number of turns drops once the dynamic aperture is exceeded.

Appendix

The function `survived_turns()` receives the maximum number of turns to probe `N`, the transfer matrix `R`, the starting coordinates `x0` and `y0`, as well as the misalignments `dx` and `dy` of the sextupole as input and returns the survived number of turns as `out`. Inside the function the starting coordinates and the default return value `out` are initialized . Then we calculate the sextupole kick angles `thetax` and `thetay` from Equation 11.4, but with misalignment added.

```
function out=survived_turns(N,R,x0,y0,dx,dy)
x=[x0;0;y0;0]; out=N;
for k=1:N
  thetax=(x(1)-dx)^2-(x(3)-dy)^2;  % sextupole kick, horizontal
  thetay=-2*(x(1)-dx)*(x(3)-dy);   % sextupole kick, vertical
```

In the next line we implement Equation 11.4 and update the particle coordinates by adding the kicks and transporting with the transfer matrix R.

```
  x=R*[x(1);x(2)-thetax;x(3);x(4)-thetay];
```

Finally, we test whether one coordinate exceeds a large value, here we chose 3. If that happens we set the return value `out` to the present turn number `k` and exit the function.

```
  if ((abs(x(1))>3) || (abs(x(3))>3)), out=k; return; end
end
end
```