Tutorial 7

Statistical Computation and Analysis

Spring 2023

Outline

- Power of the statistical test

- Calculating power

- Using power to determine the sample size

- TOST

Statistical test

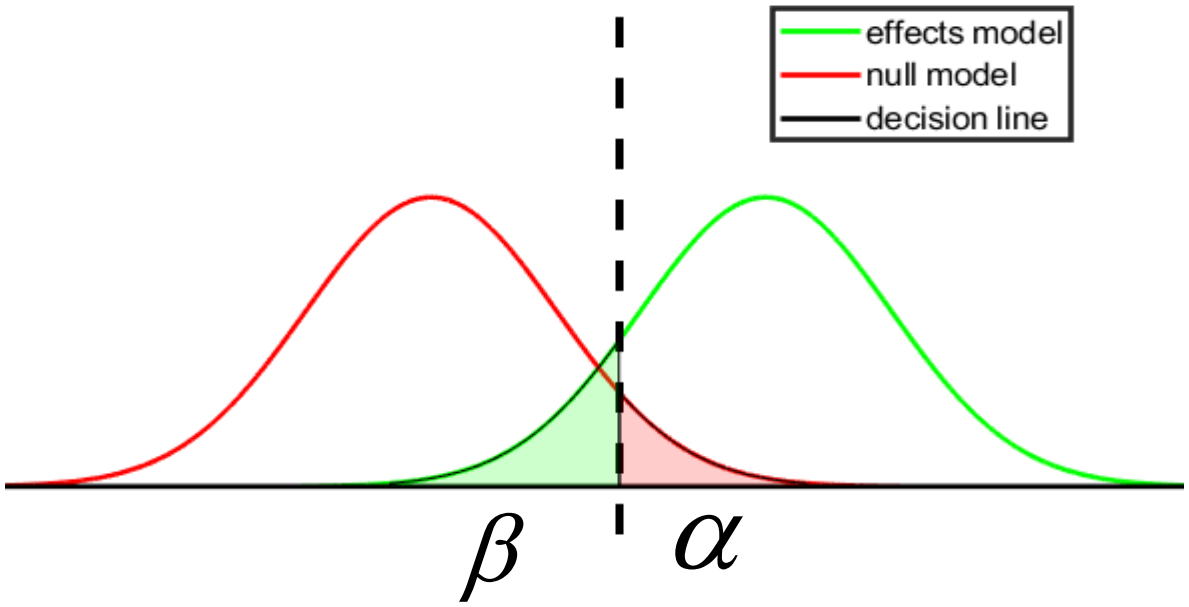
- Rejection or failing to reject the null model

- Decide on the alpha error
- Calculate the p-value
- Compare the p-value to the alpha error

If p-value is smaller than the alpha error, the null model is rejected

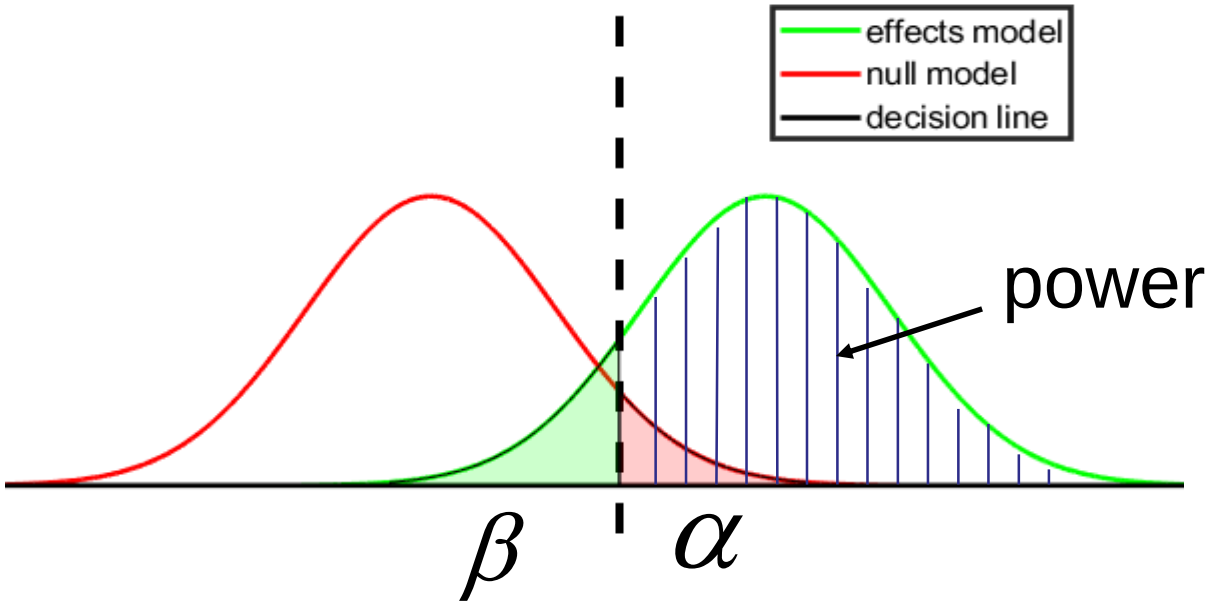
Significance and power of the statistical test

- Significance – probability of making a type I error (Alpha error), i.e. rejecting a correct null model



Significance and power of the statistical test

- Power – probability of not making Type II error (Beta error) = 1- β, i.e. probability of rejecting an incorrect null model, identifying an existing effect



Power of the statistical test

- Three things influence power
- Distance between the population to null model - real effect
- Sample size
- Significance level
- We are interested in power because:
- Low power decreases our confidence in the results of the analysis
- It helps to choose the sample size

Power of the statistical test

- Assume distribution of the population
- Estimate parameters from data
- Generate new samples
- Do statistical test on new samples
- Calculate the percentage of samples with significant effect

Example 1

We want to check if there is a connection between homework submission and final test result. We have test grades for 40 students: 20 – did their homework, 20 – did not.

Significance level - 0.05

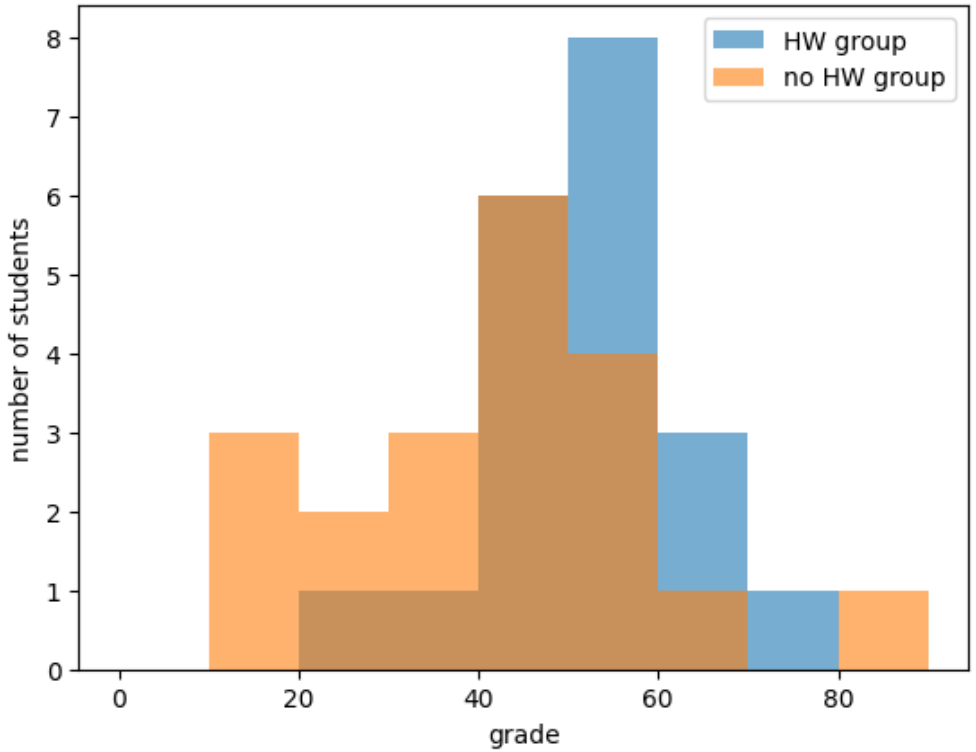
Statistic – difference of means

Methods : permutation test, t-test

P-value? Power?

Example 1

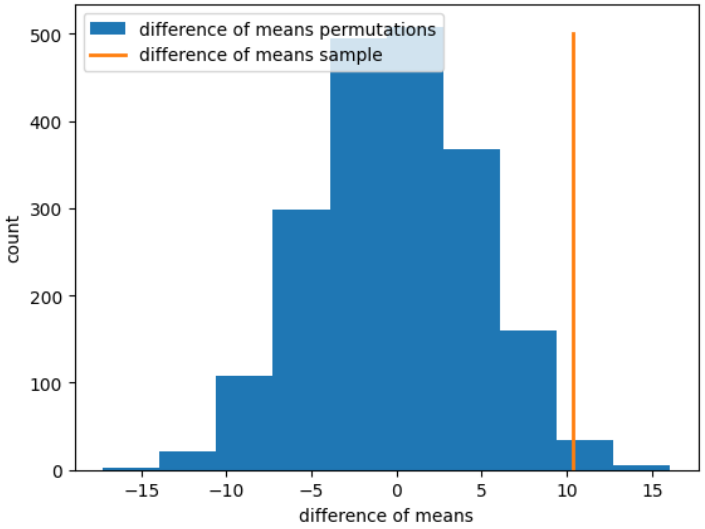
```
HW group [24 61 59 46 43 53 43 44 52 43 57 49 58 67 62 57 56
33 71 54]
no HW group [42 33 46 37 62 20 43 41 10 42 53 48 55 19 17 55
37 85 26 54]
```



Assume Normal distribution

Example 1: permutations significance and power

Permutation test:
create permuted samples find difference of means
find the p-value (portion of sample that are more extreme the the sample value)



p value = 0.01 $p < \alpha$

Find power of the permutation test:
create new samples from the suitable distribution
perform permutation test
find percentage of samples that pass the test

power of the permutations test = 70%

Example 1: t-test significance and power

T-test:
Find t-statistic
Find p-value (probability of getting the t-statistic value or more extreme under suitable t-distribution

$$\frac{\bar{x}-\mu}{s\hat{e}} \sim t_{\nu=N-1} \quad ; \quad s\hat{e} = {std(x)}/{\sqrt{N}}$$

p value = 0.0167. $p < \alpha$

Find power of the t-test:
create new samples from the distribution N(mean sample, sd sample)
perform t-test
find percentage of samples that pass the test

power of the t-test = 71%

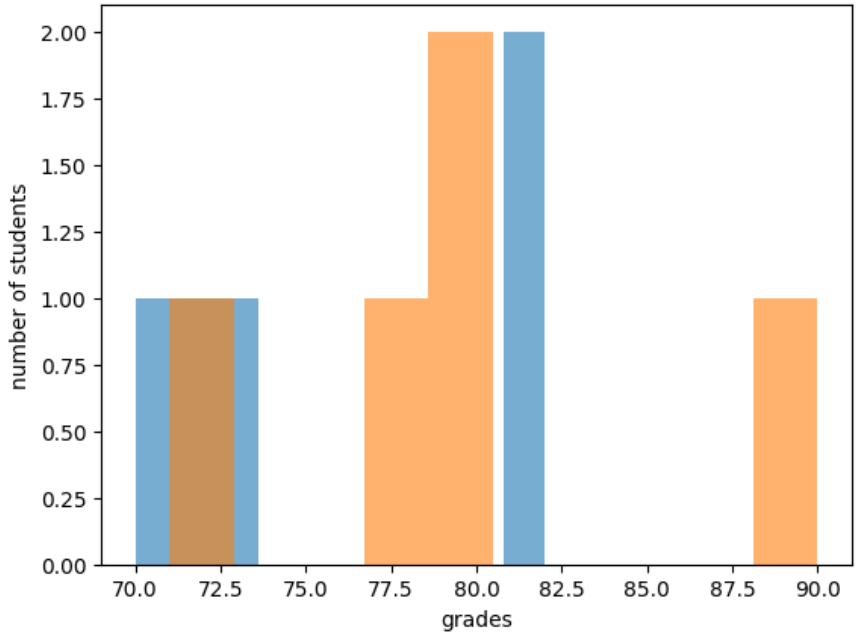
Exercise 2

We want to check if studying during the semester is more effective than studying before the exam. Before the exam, we ask 10 students about their studying method and looked at their exam grade.

Before the exam	**82**	**72**	**73**	**70**	**82**
Whole semester	**80**	**71**	**78**	**80**	**90**

What sample size should we use to show that studying during the semester is more effective with the power of 90% if we assume the sample is representative of the population?
Estimator – difference of means
Significance – 0.05
Assume normal distribution of grades

Exercise 2
t-test




```
sample mean difference
-4.0
```

Find significance using t-test

`p value = 0.1724. p >= α`

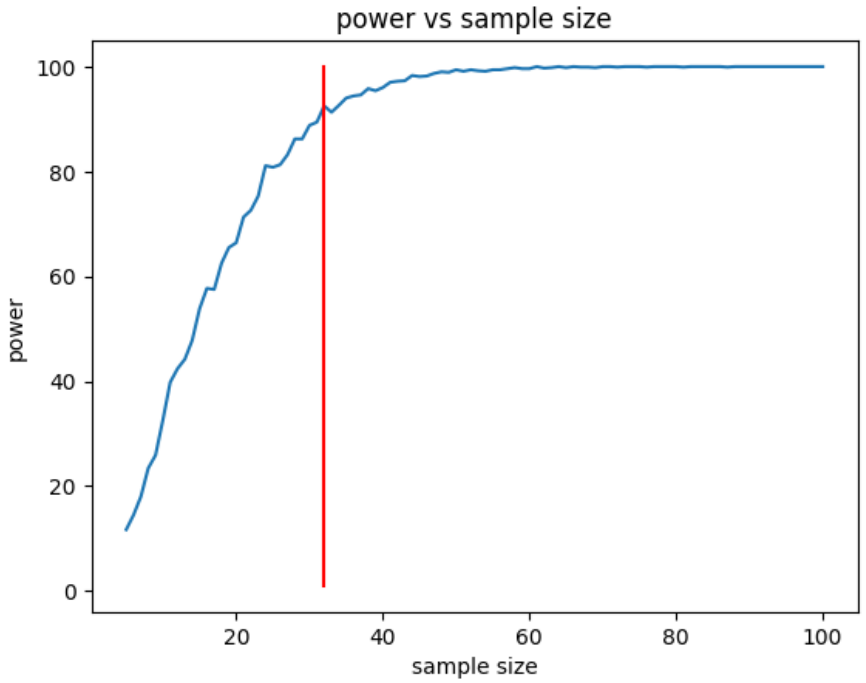
Null model not rejected

Exercise 2
t-test

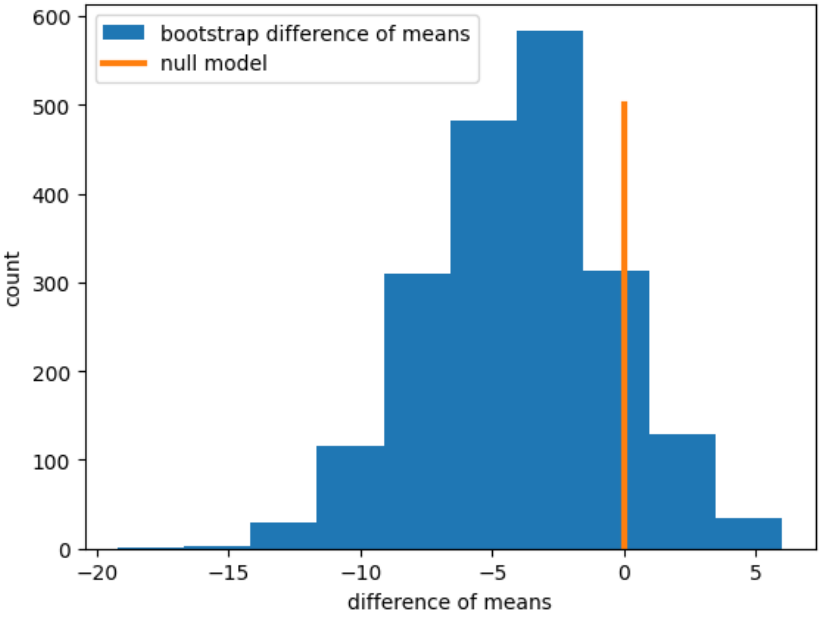
Find sample size for which t-test has the power of 90%:

Create new samples with different sizes from Normal distribution
Find the power of the t-test – perform t-test n times for each sample size
Find the minimal sample size for which t-test has the power of 90%

Sample size = 32



Exercise 2
bootstrap




```
sample mean difference
-4.0
```

Find significance using bootstrap

```
p-value = 0.126
```

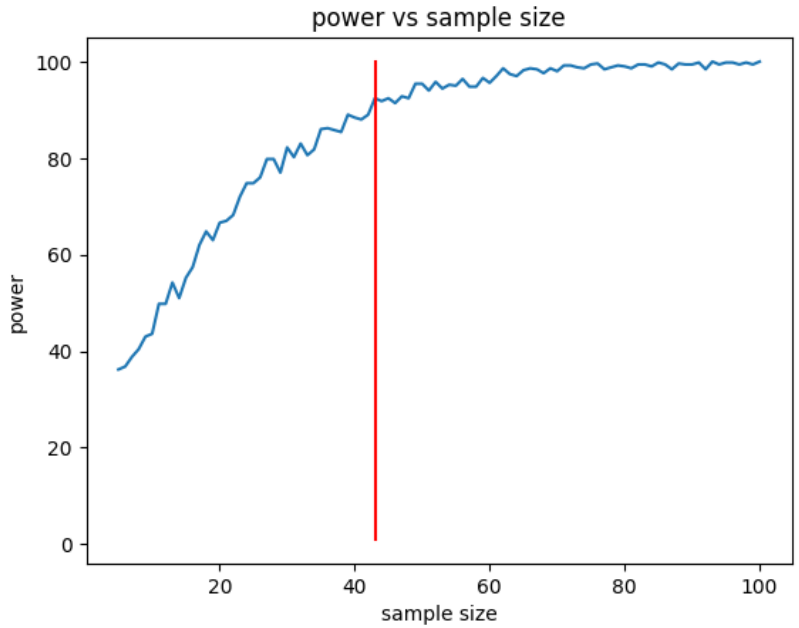
Null model not rejected

Exercise 2
bootstrap

Find sample size for which bootstrap test has the power of 90%:

Create new samples with different sizes from Normal distribution
Find the power of the bootstrap test
Find the minimal sample size for which bootstrap test has the power of 90%

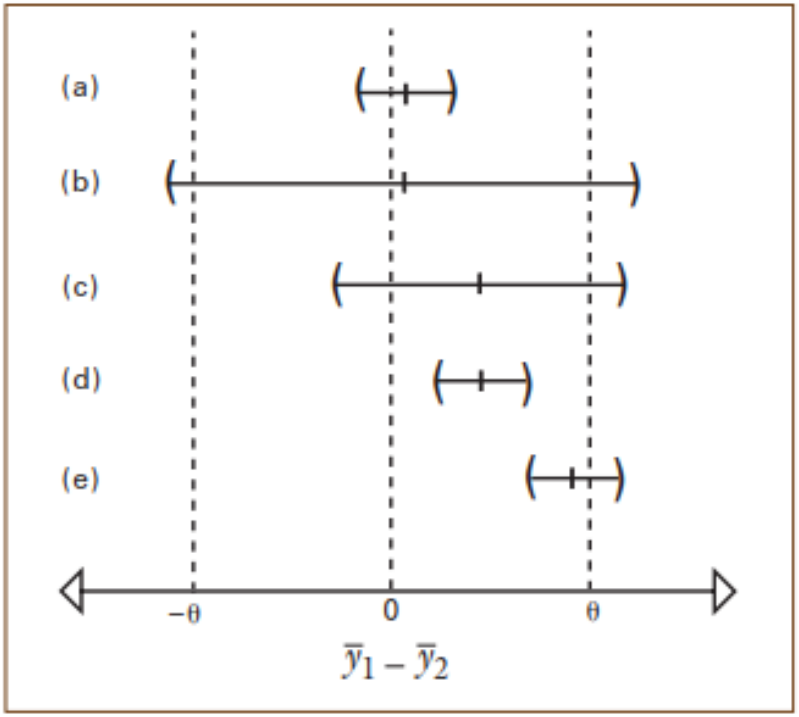
```
wanted sample size = 43
```



Equivalence testing

- ***T*wo *O*ne-*S*ided *T*** (TOST) procedure
  - One sided test with null model
    - Difference greater than $+\delta$
    - **Reject!**
  - One sided test with null model
    - Difference less than $-\delta$
    - **Reject!**

$$\underbrace{H_0 : |\mu_1 - \mu_2| > \Delta}_{\text{Inequivalent}} \quad \text{vs.} \quad \underbrace{H_A : |\mu_1 - \mu_2| < \Delta}_{\text{Equivalent}}$$



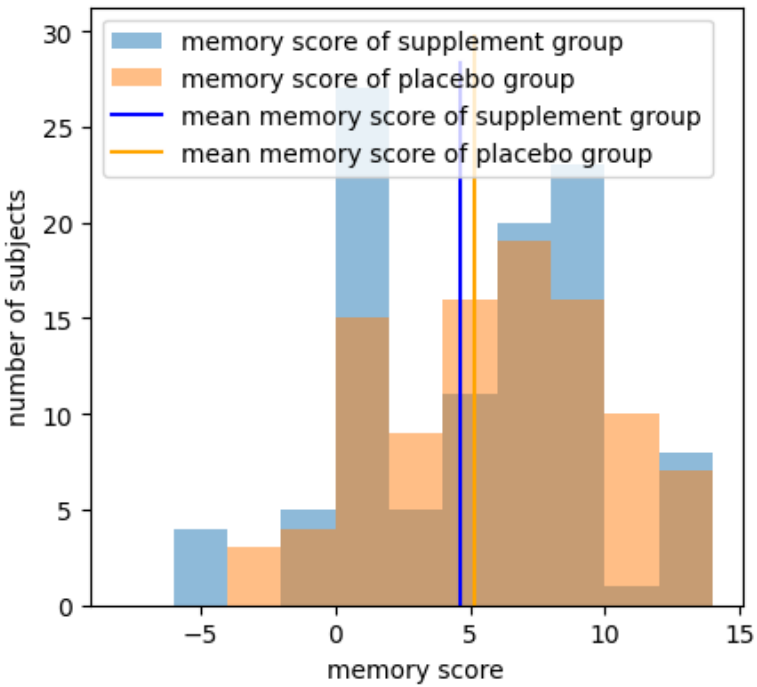
Exercise 3

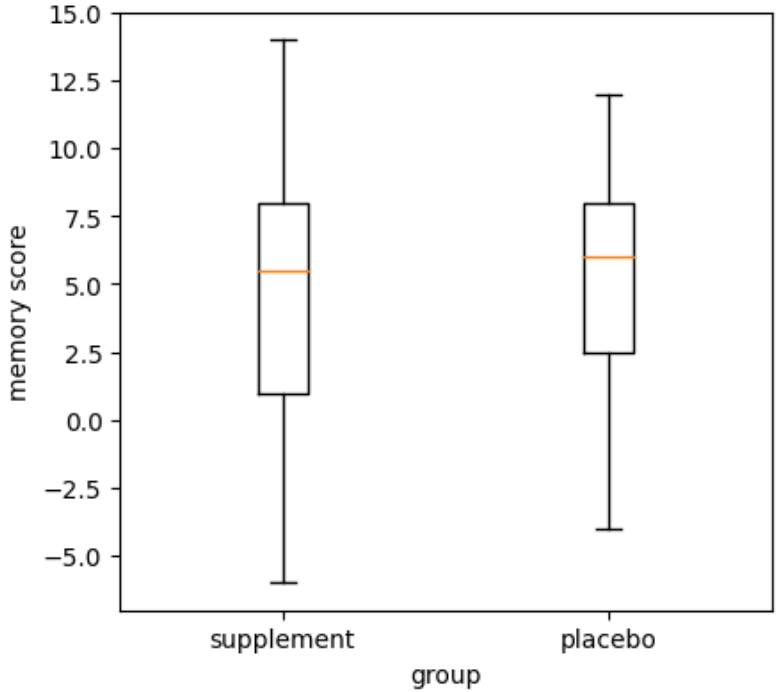
In an experiment to test the effect of ginkgo biloba on memory. 203 subjects were randomly assigned to take ginkgo biloba supplements or a placebo and took a memory test. 99 took the supplement and 104 – the placebo.

Does ginko supplement has an effect on memorization abilities?

Exercise 3

Present using histogram and boxplot





Previously, we compared the groups using bootstrap test and failed to reject the Null model

Example 3

We choose delta = 1

Perform TOST:

Find sample difference of means
Compare sample difference of means with delta using t-test
Compare sample difference of means with minus delta using t-test

```
p1 = 0.0052, p2 = 0.2374
```

We cannot say that the samples are from the same population

Example 3

Build distribution of the estimator using bootstrap and find 95% CI