① 2021 Broyden's method

$$x^0 = (x_0, y_0) = (1,2)^T$$

$$f(x,y) = \frac{1}{4}x^4 + (x-y-2)^2$$

$$\nabla f(x,y) = \begin{pmatrix} x^3 + 2(x-y-2) \\ -2(x-y-2) \end{pmatrix} \qquad Hf(x,y) = \begin{pmatrix} 3x^2+2 & -2 \\ -2 & 2 \end{pmatrix}$$

$$x^0 = (1,2)^T$$

$$\nabla f(1,2) = \begin{pmatrix} -5 \\ 6 \end{pmatrix}$$

$$(A^0)^{-1} = (Hf(1,2))^{-1} = \begin{pmatrix} 5 & -2 \\ -2 & 2 \end{pmatrix}^{-1} = \frac{1}{10-4}\begin{pmatrix} 2 & 2 \\ 2 & 5 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{5}{6} \end{pmatrix}$$

$$x^1 = x^0 - (A^0)^{-1}\nabla f(x^0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{5}{6} \end{pmatrix}\begin{pmatrix} -5 \\ 6 \end{pmatrix} = \begin{pmatrix} \frac{2}{3} \\ -\frac{4}{3} \end{pmatrix}$$

$$x^1 = \begin{pmatrix} \frac{2}{3} \\ -\frac{4}{3} \end{pmatrix} \qquad g^1 = \begin{pmatrix} \frac{8}{27} \\ 0 \end{pmatrix} - \begin{pmatrix} -5 \\ 6 \end{pmatrix} =$$

$$d^1 = \begin{pmatrix} \frac{2}{3} \\ -\frac{4}{3} \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \end{pmatrix} =$$

$$\nabla f\left(\frac{2}{3}, -\frac{4}{3}\right) = \begin{pmatrix} \frac{8}{27} \\ 0 \end{pmatrix}$$

$$(A^1)^{-1} = (A^0)^{-1} - \frac{((A^0)^{-1}g^1 - d^1)\, d_1^T (A^0)^{-1}}{d_1^T (A^0)^{-1} g_1}$$

$$= \begin{pmatrix} 0.3444 & 0.3596 \\ 0.3444 & 0.8596 \end{pmatrix}$$

$$x^2 = \begin{pmatrix} \frac{2}{3} \\ -\frac{4}{3} \end{pmatrix} - \begin{pmatrix} 0.344 & 0.359 \\ 0.344 & 0.8596 \end{pmatrix}\begin{pmatrix} \frac{8}{27} \\ 0 \end{pmatrix} = \begin{pmatrix} 0.5646 \\ -1.4354 \end{pmatrix}$$

$x^2 =$

$\nabla f(x^2) =$

$(A^2)^{-1} =$