

C, Newton's method

$$H_f(x,y) = \begin{pmatrix} f_{xx}(x,y) & f_{xy}(x,y) \\ f_{yx}(x,y) & f_{yy}(x,y) \end{pmatrix} = \begin{pmatrix} 3x^2+2 & -2 \\ -2 & 2 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} - \left(H_f(x_0, y_0) \right)^{-1} \nabla f(x_0, y_0)$$

$$= \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \boxed{\begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}^{-1}} \cdot \begin{pmatrix} -4 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \frac{1}{4-4} \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} \begin{pmatrix} -4 \\ 4 \end{pmatrix}$$

