(1)

2021 Lec. 8

$$f(x,y) = \frac{1}{4}x^4 + (x-y-2)^2$$

$x^0 = (x_0, y_0) = (0,0)$ $\qquad f(0,0) = 4$

a., successive halving of step size

$$\nabla f(x,y) = \begin{pmatrix} x^3 + 2(x-y-2) \\ -2(x-y-2) \end{pmatrix} \qquad \nabla f(0,0) = \begin{pmatrix} -4 \\ 4 \end{pmatrix}$$

β	$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} - \beta \nabla f \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$	$f(x_1, y_1)$
1	$\begin{pmatrix} 0 \\ 0 \end{pmatrix} - 1 \cdot \begin{pmatrix} -4 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \end{pmatrix}$	$64 + 36 = 100 > 4$ worse!
$\frac{1}{2}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} -4 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$	$4 + 4 = 8 > 4$
$\frac{1}{4}$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix} - \frac{1}{4}\begin{pmatrix} -4 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$	$\frac{1}{4} + 0 < 4$ ✓

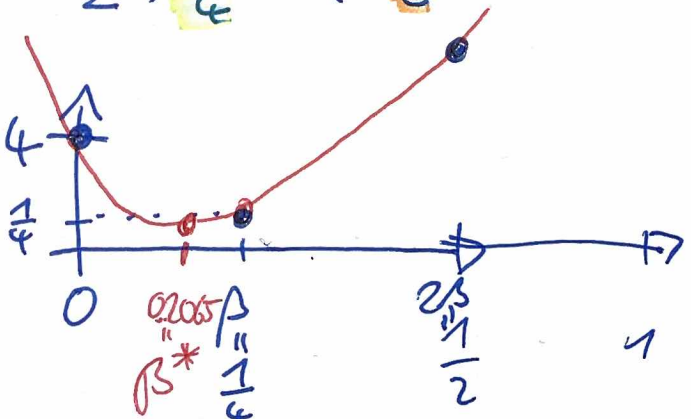
$$(x_1, y_1) = (1, -1)$$

b., ... Subsequent parabola fitting

$$\beta^* = \frac{\beta}{2} \cdot \frac{3f(x^0) - 4f(x^0 - \beta\nabla f(x^0)) + f(x^0 - 2\beta\nabla f(x^0))}{f(x^0) - 2f(x^0 - \beta\nabla f(x^0)) + f(x^0 - 2\beta\nabla f(x^0))}$$

$$= \frac{\frac{1}{4}}{2} \cdot \frac{3 \cdot 4 - 4 \cdot \frac{1}{4} + 8}{4 - 2 \cdot \frac{1}{4} + 8}$$

$$= 0.2065$$



$$\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \beta^* \begin{pmatrix} -4 \\ 4 \end{pmatrix} = \begin{pmatrix} 0.826\ldots \\ -0.826\ldots \end{pmatrix}$$

$$f(0.826\ldots, -0.826\ldots) = 0.237 < \frac{1}{4}$$

improves $\Rightarrow (x_1, y_1) = (0.826\ldots, -0.826\ldots)$