# Summary – Dynamics

Course Notes

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# 1 Summary of the Chapter: Dynamics

This document presents a concise, structured summary of the key theoretical concepts in the chapter on **Dynamics**, excluding all numerical examples and code listings.

## 1.1 Inertia

Inertia is the property of a body that resists any change in its state of motion and is directly proportional to its mass.
A net external force is required either to initiate motion from rest or to alter the velocity (magnitude or direction) of an existing velocity. The concept also appears in structural mechanics, where the resistance of a cross-section to bending is quantified by the second moment of area.

## 1.2 Momentum

Linear momentum quantifies the amount of motion possessed by a body and is defined as the product of mass and velocity. Because of its dependence on mass, objects of large mass (e.g., ships) possess considerable momentum even at moderate speeds, requiring substantial force applied over significant time to stop or redirect them.

## 1.3 Newton's Laws of Motion

1. **First Law (Law of Inertia)**
   A body remains at rest or continues in uniform rectilinear motion unless acted upon by a net external force.

$$\sum \vec{F} = 0 \quad \Longrightarrow \quad \vec{v} = \text{constant}$$

2. **Second Law (Law of Acceleration)**
   The net force acting on a body equals the rate of change of its linear momentum:

$$\vec{F} = m\vec{a}$$

3. **Third Law (Action–Reaction)**
   For every action force there exists an equal and opposite reaction force:

$$\vec{F}_{\text{action}} = -\vec{F}_{\text{reaction}}$$

## 1.4 Linear Momentum and Conservation

Linear momentum is the vector quantity

$$\vec{p} = m\vec{v}$$

In a closed system with zero net external force, the total linear momentum of the system is conserved. This principle is a direct consequence of Newton's third law in collisions and also applies to continuous flows (e.g., water jets impinging on surfaces), where the force exerted on a target equals the rate of change of momentum of the fluid.

## 1.5 Angular Momentum

Angular momentum is the moment of linear momentum about a chosen point or axis. For a rigid body rotating about a fixed axis it takes the form

$$\vec{L} = I\omega \quad \text{or} \quad L = mk^2\omega$$

where $I = mk^2$ is the moment of inertia and $k$ is the radius of gyration.

## 1.6 Radius of Gyration

The radius of gyration $k$ is the distance from the axis at which the entire mass of the body could be concentrated to produce the same moment of inertia.
- Small $k$: mass close to the axis → low rotational inertia (desirable for rapid speed changes).
- Large $k$: mass far from the axis → high rotational inertia (beneficial for energy storage in flywheels).

## 1.7 Turning Moment (Torque)

By analogy with linear motion, torque $\tau$ is related to angular acceleration $\alpha$ through

$$\tau = I\alpha = mk^2\alpha$$

This relation is derived from $\vec{F} = m\vec{a}$ and the kinematic link $a = r\alpha$.

## 1.8 Power Transmitted by Torque

Power is the rate at which work is performed in rotation. For a constant torque,

$$P = \tau\omega = \tau \cdot 2\pi n$$

where $\omega$ is angular velocity in rad/s and $n$ is rotational speed in revolutions per second. This is the fundamental relation used for shafts, propellers, and geared systems.

## 1.9 Kinetic Energy of Rotation

Rotational kinetic energy is obtained by substituting $v = \omega k$ into the linear expression, yielding

$$KE_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}mk^2\omega^2$$

This form is widely applied to flywheels, punching machines, and any system where rotational speed varies and energy is stored or released.

## 1.10 Analogy Between Linear and Rotational Motion

The chapter consistently emphasizes the parallel structure:

| Linear quantity | Rotational analogue | Relation |
|---|---|---|
| Force $F$ | Torque $\tau$ | $\tau = I\alpha$ |
| Mass $m$ | Moment of inertia $I$ | $I = mk^2$ |
| Acceleration $a$ | Angular acceleration $\alpha$ | $a = r\alpha$ |
| Momentum $p = mv$ | Angular momentum $L = I\omega$ | — |
| Kinetic energy $\frac{1}{2}mv^2$ | $\frac{1}{2}I\omega^2$ | — |
| Power $Fv$ | Power $\tau\omega$ | — |

These correspondences provide a unified framework for analysing both translational and rotational dynamics in mechanical systems.