Statics

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Learning Objectives

- Differentiate between scalar and vector quantities and give examples.
- Describe coplanar forces and their analysis in two dimensions.
- Use Cartesian coordinates to calculate distances and angles.
- Apply the Sine and Cosine Rules to solve triangles and find resultants.
- Draw and interpret space and vector diagrams to represent forces.
- Determine the resultant of multiple forces and identify equilibrium conditions.
- Analyze sling systems and calculate tensions using equilibrium principles.

1 Scalar and Vector Quantities

In mathematics and physics, a **scalar** is a quantity with only magnitude (size), whereas a **vector** has both magnitude and direction. Examples of scalar quantities include numbers, mass, speed, temperature, volume, and time. In contrast, examples of vector quantities include velocity, acceleration, and forces like weight and friction.

A vector is represented as an arrow, where the length denotes its magnitude, and the arrowhead indicates its direction. Vector diagrams are useful for visually conveying information about vector relationships, allowing us to analyze effects like the combination of forces or the movement of objects in two or three dimensions. In such diagrams, vectors are often drawn to scale and placed with respect to an origin or another reference point.

2 Coplanar Forces

Coplanar forces are forces that act within the same plane. This means they lie along a single, flat surface, and their lines of action do not extend out of that plane. Coplanar forces are often analyzed together in physics and engineering because their combined effects can be resolved within two dimensions.

2.1 Cartesian Coordinates

In a two-dimensional Cartesian coordinate (Figure 1) system, a point is represented as (x, y), where:

- x is the horizontal distance from the origin ($the\ x-coordinate$).
- y is the vertical distance from the origin ($the\ y-coordinate$).

The distance r of a point (x, y) from the origin can be calculated using:

$$r = \sqrt{x^2 + y^2} \tag{1}$$

The angle θ between the positive $(x) - axis$ and the line joining the origin to the point can be found as:

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) \tag{2}$$

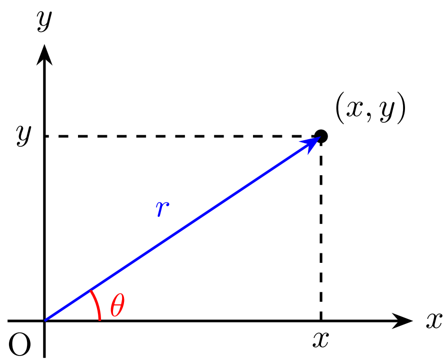

Figure 1: Cartesian Coordinates

2.2 Cosine Rule

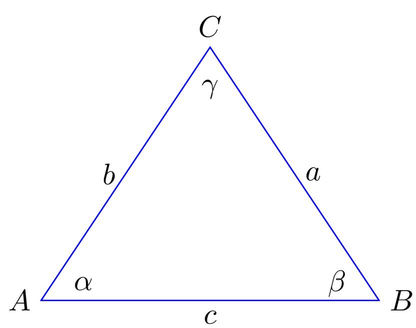

Figure 2: Rules of Cosine and Sine

The Cosine Rule is used to relate the lengths of the sides of a triangle as shown in Figure 2 to the cosine of one of its angles:

$$c^2 = a^2 + b^2 - 2ab\cos\gamma \tag{3}$$

Where:

- a, b, c are the sides of the triangle.
- γ is the angle opposite side c.

2.3 Sine Rule

The Sine Rule relates the sides and angles of a triangle in Figure 2:

$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma} \tag{4}$$

Where:

- α, β, γ are the angles of the triangle.
- a, b, c are the sides of the triangle opposite to angles α, β, γ respectively.

Example 2.1. Calculate the resultant of the forces 0.7 kN at 147° and 1.3 kN at -71° by using the cosine and sine rules.

Given the following forces:

- Force 1, $F_1 = 0.7$ kN, $at\ 147^\circ$,
- Force 2, $F_2 = 1.3$ kN, $at - 71^\circ$.

The angle between the two forces is:

$$\theta = 147^\circ - (-71^\circ) = 147^\circ + 71^\circ = 218^\circ$$

Since $(218^\circ > 180^\circ), we\ subtract\ from\ 360^\circ$ to get the smaller angle:

$$\theta = 360^\circ - 218^\circ = 142^\circ$$

Step 1: Using the Cosine Rule to Find the Resultant Magnitude

The magnitude of the resultant R is given by the cosine rule:

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos(\theta)}.$$

Substituting the known values:

$$R = \sqrt{(0.7)^2 + (1.3)^2 + 2(0.7)(1.3)\cos(142^\circ)}.$$

We calculate $\cos 142^\circ$:

$$\cos(142^\circ) = \cos(180^\circ - 38^\circ) = -\cos(38^\circ) \approx -0.788.$$

Now substitute into the formula for (R):

$$R = \sqrt{0.49 + 1.69 - 1.432}.$$

$$R = \sqrt{0.748} \approx 0.865\,\text{kN}.$$

Step 2: Using the Sine Rule to Find the Direction

The direction of the resultant α relative to F_1 is found using the sine rule:

$$\frac{\sin(\alpha)}{F_2} = \frac{\sin(\theta)}{R}.$$

We calculate $\sin 142^\circ$:

$$\sin(142^\circ) = \sin(180^\circ - 142^\circ) = \sin(38^\circ) \approx 0.616.$$

Now apply the sine rule:

$$\sin(\alpha) = \frac{1.3 \times 0.616}{0.865} \approx 0.926.$$

Thus, $\alpha \approx 67.9^\circ$.

The direction of the resultant relative to the positive x-axis is:

$$\text{Direction} = 147^\circ - 67.9^\circ = 79.1^\circ.$$

3 Space Diagrams

The space diagram (Figure 3) is an illustration of the system of forces.

4 Vector Diagrams

The vector diagram (Figure 4) is a diagram drawn to scale with the vectors joined end to end. Vector diagrams are used to analyze and combine forces to find the resultant.

5 Free Body Diagrams

The free body diagram ((Figure 5) is a diagram isolating a single object, showing all external forces and moments acting on it.

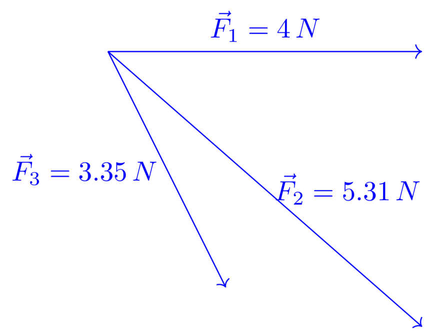

Figure 3: Space Diagram

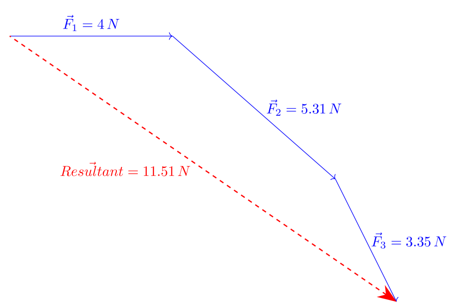

Figure 4: Vector Diagram

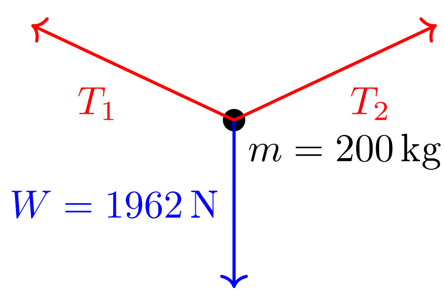

Figure 5: Free Body Diagram

6 Resultant

The resultant is a single force that represents the combined effect of two or more forces acting on an object. It has the same effect as applying all the original forces together and is found by adding the individual forces, taking both their magnitudes and directions into account. The resultant gives the overall direction and magnitude of the combined forces as shown in Figure 4.

7 Equilibrium

Equilibrium of an object occurs when all the forces acting on it are balanced, so the object remains at rest or moves at a constant speed in a straight line. In equilibrium, there is no net force or acceleration, meaning the object is in a stable state without any change in its motion.

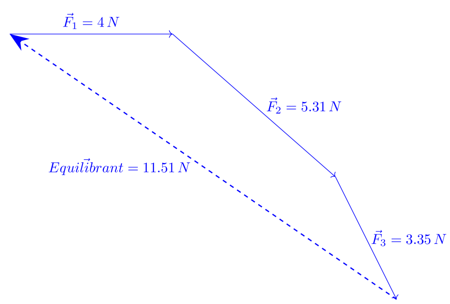

Figure 6: Equilibrant

Conditions of equilibrium:

1. Net force must be zero:

$$\sum_k \vec{F}_k = \vec{0} \tag{5}$$

2. Net torque must be zero:

$$\sum_k \vec{\tau}_k = \vec{0} \tag{6}$$

8 Slings

A sling is a device or assembly of ropes, cables, or straps used to support and lift loads. Slings play a crucial role in rigging operations, allowing objects to be lifted, lowered, or moved safely and efficiently. Slings are arranged to distribute the load evenly across their length, reducing stress points and ensuring stability.

In multi-leg slings (e.g., two-leg or four-leg), the load is shared among the sling legs, which helps stabilize and balance the load.

Example 8.1. A 200 kg mass is suspended as shown in Figure 7. Rope A is attached to a beam in two places and is passed through a ring that rest naturally at the centre of rope A. Rope B is attached to the bottom of the ring, and to the 200 kg mass.

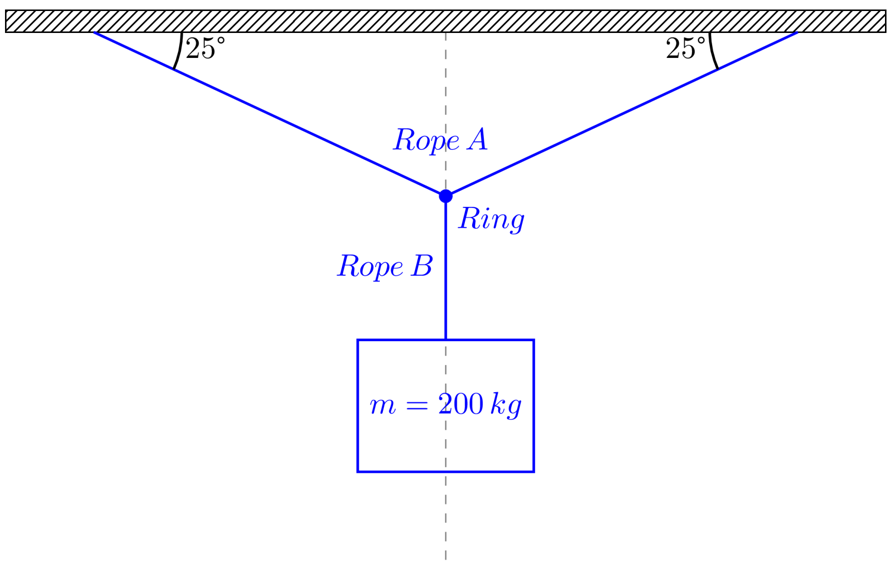

Figure 7: A 200 kg suspended mass.

a. What is the tension in rope B?
b. Draw a free body diagram of the forces exerted on the ring.
c. What is the tension in rope A?

Given Information

- Mass $m = 200\,\mathrm{kg}$
- Gravitational acceleration $g = 9.81\,\mathrm{m/s}^2$
- Angle $\theta = 25^\circ$

a. Tension in Rope B

The tension T_B in Rope B must support the entire weight of the mass. Therefore,

$$T_B = m \cdot g = 200 \times 9.81 = 1962\,\mathrm{N}$$

So, **the tension in Rope B is 1962 N**.

b. Free Body Diagram

The ring is in equilibrium, meaning the net force acting on it is zero. Here's a breakdown of the forces:

- **Tension** T_B: Acts downward, equal to the weight of the 200 kg mass.
- **Tensions** T_{A1} and T_{A2}: These are the tensions in each side of Rope A. Since the ring is at the center and the setup is symmetrical, $T_{A1} = T_{A2} = T_A$.

Since the ring is in equilibrium, the vertical components of T_{A1} and T_{A2} must balance the downward force from T_B.

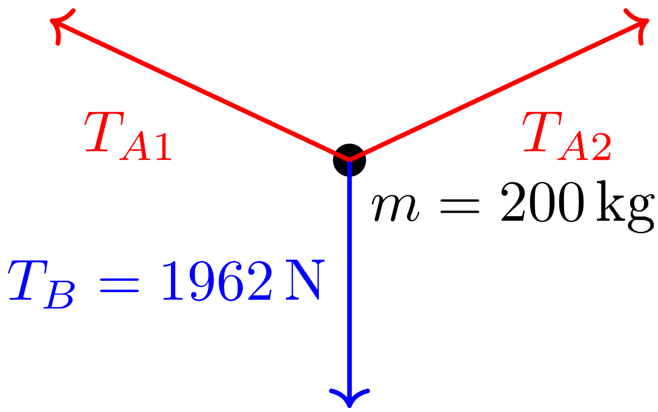

Figure 8: Free Body Diagram

c. Tension in Rope A

Since the angle between each side of Rope A and the horizontal is $\theta = 25^\circ$, we can use trigonometry to find T_A.

The equilibrium condition is:

$$2 \cdot T_A \cdot \sin(\theta) = T_B$$

Solving for T_A:

$$T_A = \frac{T_B}{2\sin(\theta)}$$

Substitute the values:

$$T_A = \frac{1962}{2 \cdot \sin(25^\circ)}$$

Calculating:

$$T_A \approx 2321.24\,\mathrm{N}$$

So, **the tension in Rope A is approximately 2321.24 N**.

9 Jib Cranes

A simple jib crane has a vertical post, a jib, and a tie. The jib is hinged at its lower end to the post, and the tie connects the top of the jib to the base of the post, forming the crane head where the tie and jib meet.

When a load is hung directly from the crane head, solving for forces involves a simple triangle of forces. In other cases, the crane may have a pulley at the head, with a rope running over it to a winch, creating a system with more than three forces.

Example 9.1. The angle between the jib and the vertical post of a jib crane is 42°, and the angle between the tie and the jib is 36°. Determine the forces in the jib and the tie when a mass of 3.822×10^3 kg is suspended from the crane head.

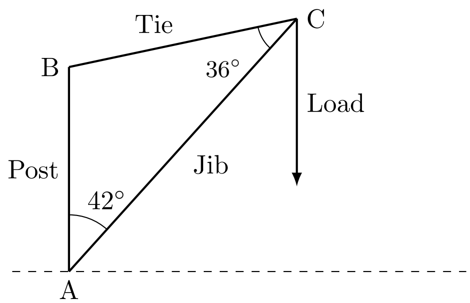

Figure 9: Jib Crane

The angle between the **jib** and the **vertical post** is 42°, and the angle between the **tie** and the **jib** is 36°.

A load of 3.822×10^3 kg (that is, $W = 37.4938$ kN) is suspended from the crane head at point A.

Find the **pull in the tie (AC)** and the **thrust in the jib (BC)**.

Let the geometry of the crane form a triangle ABC, where:

- AB is the **load** $W = 37.4938$ kN,
- $\angle B = 42^\circ$,
- $\angle C = 36^\circ$,
- hence $\angle A = 180^\circ - (42^\circ + 36^\circ) = 102^\circ$.

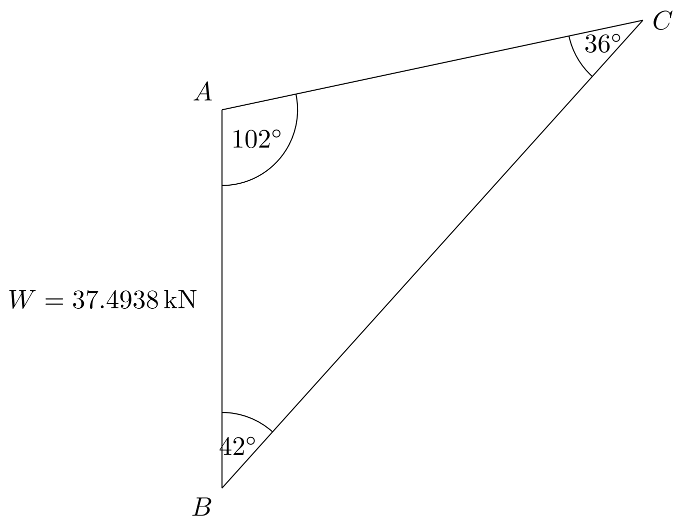

Figure 10: Jib Crane Vector Diagram

Using the **law of sines**:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C},$$

where
$a = BC$ (jib), $b = CA$ (tie), and $c = AB = W$.

Then:

$$a = c\frac{\sin A}{\sin C}, \qquad b = c\frac{\sin B}{\sin C}.$$

Substitute the given values:

$$a = 37.4938 \cdot \frac{\sin 102^\circ}{\sin 36^\circ} = 62.394 \text{ kN},$$

$$b = 37.4938 \cdot \frac{\sin 42^\circ}{\sin 36^\circ} = 42.683 \text{ kN}.$$

Final Answers

$$\boxed{\text{Pull in tie (AC)} = 42.68 \text{ kN}}$$

$$\boxed{\text{Thrust in jib (BC)} = 62.39 \text{ kN}}$$

Notes
The tie (AC) is in **tension**, while the jib (BC) is in **compression**.

Example 9.2. A vertical lifting force of 95 N is applied to a body, and simultaneously, a horizontal force of 135 N pulls on it. Determine the magnitude and direction of the resulting force.

To solve for the magnitude and direction of the resultant force, we can use vector addition.

Given:

- Vertical force, $F_v = 95\,\text{N}$
- Horizontal force, $F_h = 135\,\text{N}$

Step 1: Calculate the Magnitude of the Resultant Force

The resultant force F_r is the vector sum of the vertical and horizontal forces. Using the Pythagorean theorem:

$$F_r = \sqrt{F_v^2 + F_h^2}$$

Substituting the values:

$$F_r = \sqrt{95^2 + 135^2}$$

Calculating further:

$$F_r = \sqrt{9025 + 18225} = \sqrt{27250}$$

Thus,

$$F_r \approx 165.07\,\text{N}$$

Step 2: Determine the Direction of the Resultant Force

The direction θ of the resultant force with respect to the horizontal can be found using the tangent function:

$$\theta = \arctan\left(\frac{F_v}{F_h}\right)$$

Substituting the values:

$$\theta = \arctan\left(\frac{95}{135}\right)$$

Calculating θ:

$$\theta \approx 35.1341^\circ$$

Final Answer

The magnitude of the resultant force is approximately $165.07\,\mathrm{N}$, and its direction is 35.1341° above the horizontal.

Exercise here.

Solution here.

Index