Answers to questions in

Lab 1: Filtering operations

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**Instructions**: Complete the lab according to the instructions in the notes and respond to the questions stated below. Keep the answers short and focus on what is essential. Illustrate with figures only when explicitly requested.

Good luck!

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**Question 1**: Repeat this exercise with the coordinates p and q set to (5, 9), (9, 5), (17, 9),

(17, 121), (5, 1) and (125, 1) respectively. What do you observe?

Answers:

The coordinates change the number of periods and the direction of the waves in the Fourier domain. Inputting a p value larger than 64 will result in (128-p) wave which will be negative. If either p, q are negative the wave in the spatial domain will be perpendicular to the wave produced by p, q. If both are negative it will result in the same wave in the spatial domain.

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**Question 2**: Explain how a position (p, q) in the Fourier domain will be projected as a sine wave in the spatial domain. Illustrate with a Matlab figure.

Answers: A position in the fourier domain represents a x and y frequency. A position (4, 2) means that the sine wave will have 4 periods in the x-axis and 2 periods in the y-axis of the image. The vector (4, 2) will point in the direction of the wave.

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**Question 3**: How large is the amplitude? Write down the expression derived from Equation (4) in the notes. Complement the code (variable amplitude) accordingly.

Answers:

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**Question 4**: How does the direction and length of the sine wave depend on p and q? Write down the explicit expression that can be found in the lecture notes. Complement the code (variable wavelength) accordingly.

Answers:

The sine wave will be in the direction of the vector (p, q). The length of the the sine wave will be

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**Question 5**: What happens when we pass the point in the center and either p or q exceeds half the image size? Explain and illustrate graphically with Matlab!

Answers:

When it exceeds half of the image size the resulting value will be ex. p = p - image size.

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**Question 6**: What is the purpose of the instructions following the question *What is done by these instructions?* in the code?

Answers:

Those instructions “resize” p & q if the exceed half of the image length.

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**Question 7**: Why are these Fourier spectra concentrated to the borders of the images? Can you give a mathematical interpretation? Hint: think of the frequencies in the source image and consider the resulting image as a Fourier transform applied to a 2D function. It might be easier to analyze each dimension separately!

Answers:

Because the F looks essentially like a wave in the y direction in the spatial domain, G as it is F trasnsposed looks like a wave in the x direction and H which is a linear combination of the two has frequencies in both directions but none which are diagonal.

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**Question 8**: Why is the logarithm function applied?

Answers:

The logarithm function is applied to make frequencies further from the center more apparent to make analyzing them easier.

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**Question 9**: What conclusions can be drawn regarding linearity? From your observations can you derive a mathematical expression in the general case?

Answers:

Multiplication in the spatial domain means addition in the Fourier domain.

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**Question 10**: Are there any other ways to compute the last image? Remember what multiplication in Fourier domain equals to in the spatial domain! Perform these alternative computations in practice.

Answers:

Yes, we can take the matrix multiplication of Fhat and Ghat, do an inverse Fourier transform. If we print the real part of the result, it will show the same image.

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**Question 11**: What conclusions can be drawn from comparing the results with those in the previous exercise? See how the source images have changed and analyze the effects of scaling.

Answers:

The image is stretched out along the x-axis, in the fourier domain the pattern has gone from one with quadratic(ish) pieces to a more rectangular shape, stretched in the y-direction.

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**Question 12**: What can be said about possible similarities and differences? Hint: think of the frequencies and how they are affected by the rotation.

Answers:

Because the discrete case and low resolution a rectangular shape will not get sharp edges, this creates “noise” in the fourier dimension apart from the rotation of pattern in the non-rotated image.

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**Question 13**: What information is contained in the phase and in the magnitude of the Fourier transform?

Answers:

The magnitude contains information on how bright or dark certain areas should be. The phase is a phase of the sinus waves which is important for aligning pixels in the image.

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**Question 14**: Show the impulse response and variance for the above-mentioned t-values. What are the variances of your discretized Gaussian kernel for t = 0.1, 0.3, 1.0, 10.0 and

100.0?

Answers:

For small t’s the white do in the middle of the picture “smears out” more. Higher values seem to have less effect on the image.

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**Question 15**: Are the results different from or similar to the estimated variance? How does the result correspond to the ideal continuous case? Lead: think of the relation between spatial and Fourier domains for different values of t.

Answers:

It is similar, the number supposed to be 0 are very small but the “t’s” in the matrix are around t/2 instead.

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**Question 16**: Convolve a couple of images with Gaussian functions of different variances (like t = 1.0, 4.0, 16.0, 64.0 and 256.0) and present your results. What effects can you observe?

Answers:

The higher values do not seem to have almost any effect at all on the filtered image.

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**Question 17**: What are the positive and negative effects for each type of filter? Describe what you observe and name the effects that you recognize. How do the results depend on the filter parameters? Illustrate with Matlab figure(s).

Answers:

The median filter is good at handing small amount of contrasting noise, the size is essential as a small square will miss to even out noisy pixels while a too large one will blur the image. The Gaussian is better at smoothing out the small more frequent noise, small sigma values will make the whole image greyish.

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**Question 18**: What conclusions can you draw from comparing the results of the respective methods?

Answers:

The median filter works better on the sap-noise where there are a few points deviating a lot.  
The gaussian filter is superior for the add-noise where there are a lot of pixels with more subtle magnitude devation.

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**Question 19**: What effects do you observe when subsampling the original image and the smoothed variants? Illustrate both filters with the best results found for iteration i = 4.

Answers:

With the filter the subsampled image preserves the shape of the items better. It seems that the gaussian does a better job in this regard.

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**Question 20**: What conclusions can you draw regarding the effects of smoothing when combined with subsampling? Hint: think in terms of frequencies and side effects.

Answers:

That by using filter before subsampling we can minimize the effect of “outlier frequencies” impacting the subsampled image. On the other hand smoothing too much could lead to loss of information.

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