Module 3

CHAPTER 10

Basics of Antenna

10.1 INTRODUCTION

The Maxwell's equations describe the relation between various electric and magnetic field quantities. The wave propagation can be explained with the help of these equations. These equations are

Faraday's law : $\nabla \times \bar{E} = -\dot{\bar{B}}$ ...(10.1.1(a))

Ampere's law : $\nabla \times \bar{H} = \bar{J} + \dot{\bar{D}}$ ...(10.1.1(b))

Gauss law (electric) : $\nabla \cdot \bar{D} = \rho_v$ ...(10.1.1(c))

Gauss law (magnetic) : $\nabla \cdot \bar{B} = 0$ ...(10.1.1(d))

Along with these four equations there is one more equation added with this set is nothing but continuity equation which is

Continuity equation : $\nabla \cdot \bar{J} = -\dot{\rho}_v$ ...(10.1.1(e))

But this equation can be derived from Equation (10.1.1(b) and (c)), and hence contains no additional information. It simply says when charge density is changing with respect to it produces $\bar{J}$.

10.1.1 Significance of First Two Maxwell's Equations

The generation of electromagnetic waves can be explained with the help of first two equations. These equations signifies.

i) Equation (10.1.1(a)) says that the source of electric field E is the changing magnetic field $(-\dot{\bar{B}})$.

ii) Equation (10.1.1(b)) says that the source of magnetic field H is the changing electric field $(\dot{\bar{D}})$.

But the source of electric and magnetic fields are; electric field is produced by charge (ρ_v) and magnetic field is produced by current (J).

It means ultimate sources of an electromagnetic field are the current J and charge ρ_v. But from continuity equation J and ρ_v are always related. Thus we conclude that

It is possible to derive the time varying electromagnetic field from the knowledge of the current density J alone.

10.2 GENERATION OF A WAVE USING MAXWELL'S EQUATIONS

Now in this section we will see how Maxwell's equations can be used to explain the phenomenon of generation of wave.

Consider a loop of wire through which time varying current is flowing. The conduction current causes a circulating magnetic field to be generated around the loop as shown in Fig. 10.2.1(a).

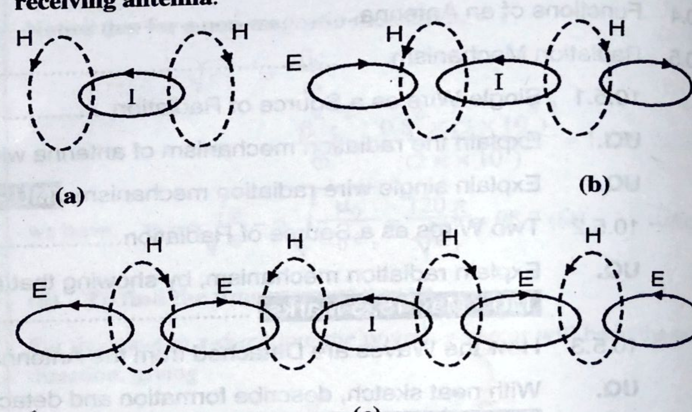
According to Maxwell's equations, the changing magnetic field creates a circulating, or curling, electric field that encircle the magnetic field lines as shown in Fig. 10.2.1(b).

Again according to Maxwell's equations, this changing electric field will produce the changing magnetic field as shown in Fig. 10.2.1(c). This phenomenon repeats resulting in continual growth or spreading of electromagnetic field into all space surrounding the current loop.

Here we say, the time varying current in the loop produces electromagnetic waves which travels away from the source.

In other words, the loop is acting like transmitting device which we call as **transmitting antenna.**

Instead if the loop is used to receive the wave it is called as **receiving antenna.**



aw(3.1)**Fig. 10.2.1 : The generation of an electromagnetic wave from a current loop**

This brief discussion is used just to show how Maxwell's equations are important in antenna analysis. The detailed study is done in the next chapters.

10.3 DEFINITION OF AN ANTENNA

There are many ways an antenna is defined :

i) According to Webster's Dictionary; it is a metallic device (as a rod or wire) for radiating or receiving radio waves.

ii) It is a structure associated with the region of transition between a guided wave and a free space wave, or vice versa.

iii) Antenna is a transition device, or transducer, between a guided wave and a free-space wave, or vice versa.

iv) Antenna is a device which interfaces a circuit and space.

v) An antenna is a sensor of electromagnetic waves.

vi) An antenna is a impedance matching device.

10.4 FUNCTIONS OF AN ANTENNA

Different functions an antenna has to performed are as follows :

i) In the transmitting mode it is used to radiate EM waves.

ii) In the receiving mode it is used as a sensor of EM waves.

iii) An antenna has to function like transducer which converts electrical energy into EM energy at the transmitter side. And it converts EM energy into electrical energy at the receiving side.

iv) For maximum power radiated in the space, the condition for maximum power transfer must be satisfied. The role of antenna is very important here. It serves as an impedance matching device, which matches the transmitter and free-space impedance on the transmitter side. And it matches free space and receiver impedance on the receiver side.

v) The antenna must also serve as a directional device means it should radiate more in the desired directions and suppresses in the unwanted directions.

10.5 RADIATION MECHANISM

- Let us study some basic sources of radiation, which are
  i) Single wire source ii) Two wires source

10.5.1 Single Wire as a Source of Radiation

UQ. Explain the radiation mechanism of antenna with single wire system. **(MU - May 16, 5 Marks)**

UQ. Explain single wire radiation mechanism. **(MU - Q. 1(c), Dec. 19, 5 Marks)**

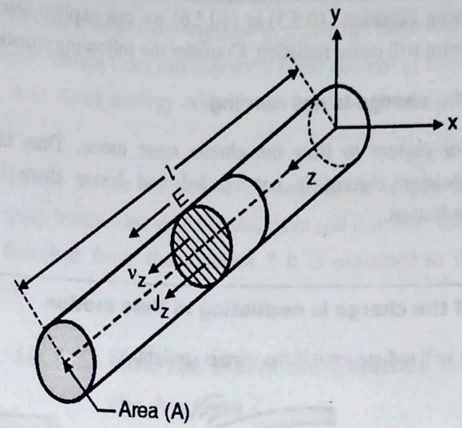
Consider a single wire of cross sectional area A and length L. When the current flows through wire, the current density (A/m^2) can be expressed as

$$J_z = \rho_v v_z \Rightarrow (C/m^3 \times m/s) \quad ...(10.5.1)$$

where ρ_v = volume charge density (C/m^3)

v_z = velocity of charge Q within volume V in z-direction.

Here the total cross sectional area of the wire is carrying the current.



aw(3.4)**Fig. 10.5.1 : Current carrying wire**

But when the wire is an ideal conductor, the current will have tendency to flow over the surface, resulting in surface charge density ρ_s (C/m^2). The surface current density J_s (A/m) is now given by

$$J_s = \rho_s v_z \Rightarrow (C/m^2 \times m/s) \quad ...(10.5.2)$$

Now if the wire is thin (ideally zero radius), then instead of current density, we will express current in z-direction as

$$I_z = \rho_l v_z \Rightarrow (C/m \times m/s)$$

where ρ_l = line charge density (C/m) ...(10.5.3)

For the time varying current Equation (10.5.3) can be written by differentiating both sides as

$$\frac{dI_z}{dt} = \rho_l \frac{dv_z}{dt} = \rho_l a_z \quad ...(10.5.4)$$

where a_z is the acceleration in z direction = dv_z / dt.

For the wire of length L, multiplying by it with Equation (10.5.4)

$$L \frac{dI_z}{dt} = L \rho_l a_z \quad ...(10.5.5)$$

It can also be expressed irrespective of any direction as

$$\dot{I} L = Q \dot{v} \quad ...(10.5.6)$$

where $Q = l \rho_l$ is the total charge and dot over the quantities is having usual meaning of differentiation with respect to time.

This is the **fundamental relation of electromagnetic radiation.** It says that

To create radiation, there must be a time varying current or an acceleration (or deceleration) of charge.

- For steady state harmonic variation, we usually focus on current.
- For transient or pulse, we focus on charge.

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Now using Equation (10.5.5) or (10.5.6) we can explain when the single wire will cause radiation. Consider the following situations.

1. If a charge is not moving

For current to flow the charge must move. Thus in this situation current is not created and hence there is no **radiation.**

2. If charge is moving with a uniform velocity

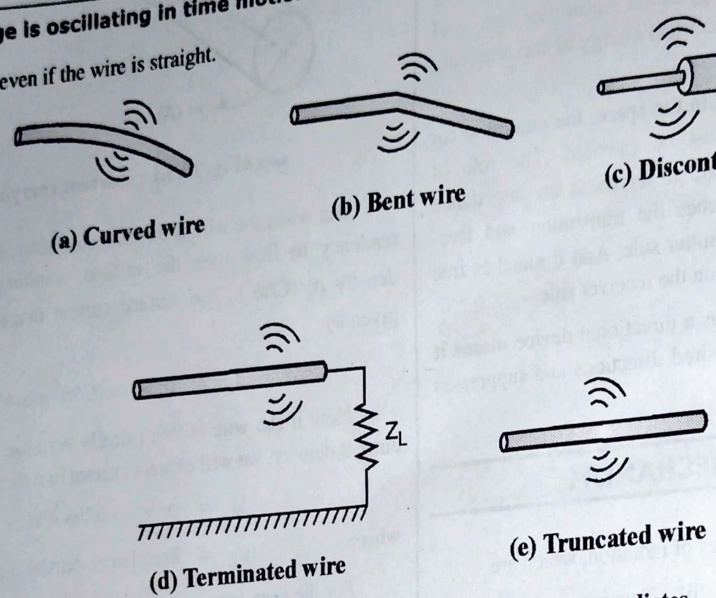
Whether radiation will take place or not depend on

a) When the wire is straight and infinite in extent there is no radiation.

b) There is radiation if the wire is curved, bent, discontinuous, terminated or truncated as shown in Fig. 10.5.2.

3. If the charge is oscillating in time motion

It will radiate even if the wire is straight.



aw(3.5)**Fig. 10.5.2 : Situations showing a single wire radiates**

10.5.2 Two Wires as a Source of Radiation

> **UQ.** Explain radiation mechanism, by showing that a parallel wire can act as source of radiation.
> **MU - May 15, 5 Marks**

In the previous section we studied under what circumstances a single wire radiates a wave. Now we will see radiation due to two wires. To understand this you should know the properties of electric and magnetic fields.

Properties of electric field

1) Electric field starts on positive charges and terminate on negative charges.
2) If negative charge is not present then electric field start on positive charge and end at infinity.
3) If positive charge is not available then electric field start at infinity and end on a negative charge.
4) If no charge is available then field lines form closed loops neither starting or ending on any charge.

Property of magnetic field

Magnetic field lines always form closed loops enclosing current carrying conductors.

Consider now a two wire transmission line with one end connected to a voltage source while other end is connected to an antenna.

When the voltage applied is sinusoidal, some parts of the transmission line will have positive cycles and on some parts negative cycles appear.

This voltage will produce electric field between the conductors. At the voltage peak points the electric field is maximum and is shown as crowded lines at those points in the Fig. 10.5.4.

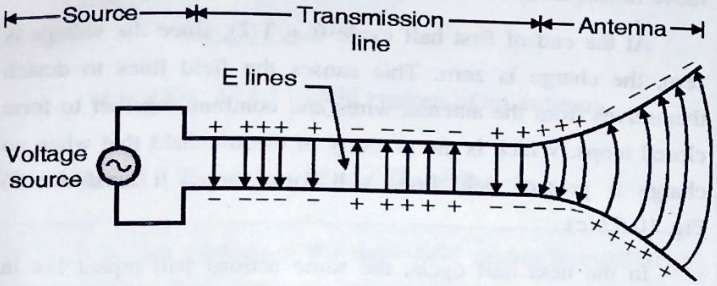
Note that on the transmission line part the E lines are perpendicular to conductor surface. Here these lines appear as vertical lines since wires are horizontal. But the far end of the transmission line where antenna is connected these lines bend so that lines will be perpendicular to the flared section of the antenna.

These lines are shown upward or downward depending upon the polarity of the voltage at those points.

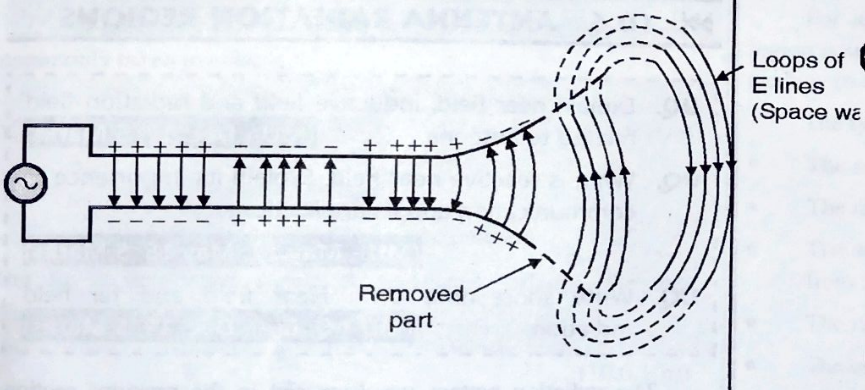
The effect of these electric field lines is to produce electric lines of force which are tangent to the electric field.

These lines of force act on the free electrons in the wire and force them to be displaced.

The movement of the charges create a current that in turn creates a magnetic field which gives rise to magnetic lines of force which are tangent to magnetic fields. This is how electromagnetic field is generated.



(a) Electric field generation on the transmission line



(b) Space wave formation

aw(3.6)**Fig. 10.5.3 : Showing field generation and space wave formation**

We can summarize the above discussion as :

When the source is ON, the electric and magnetic field is generated which forms electromagnetic waves.

These waves are guided along transmission line, hence called guided waves. These waves then enter the antenna thus current and charges associated with it also enters antenna.

If we remove part of the antenna structure (Fig. 10.5.3(b)), space waves can be formed. As charges are not available there over the removed part, open ends of the electric field lines are connected to form closed loops.

Electric charges are required to excite the fields but are not needed to sustain them and may exist in the absence of the charges. This is in direct analogy with waver waves explained in section 10.5.1.

In this section we studied the mechanism of generation of free space waves. One doubt is still there and it is **how the waves are detached from the antenna ?** It is explained in the next section.

10.5.3 How the Waves are Detached from the Antenna ?

UQ. With neat sketch, describe formation and detachment of electric field lines for short dipole. **(MU - Q. 2(a), Dec. 19, 10 Marks)**

In the previous sections we shown the antenna structure as a flared transmission line. If the flaring is done to the maximum extent so that two wires of the antenna stand along the straight line, the resultant structure is called as dipole. It is shown in Fig. 10.5.4.

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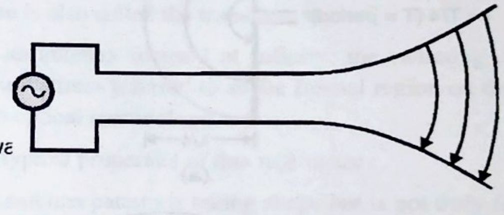
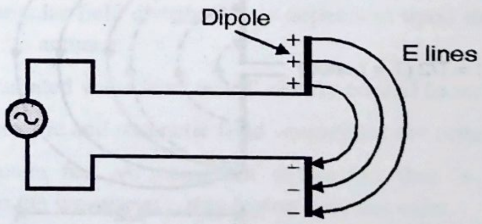


(a) Original antenna structure



(b) Dipole antenna

aw(3.7)**Fig. 10.5.4 : Dipole formation**

Fig. 10.5.4(b) also shows the generation of electric field lines when the source is ON.

Now consider different instants of time at the interval of T/4 i.e. quarter cycle. Also just for simplicity the transmission line and generator are not shown in Fig. 10.5.5. We will just focus on the field lines at the antenna structure.

Remember the terms, duration of cycle (T), wavelength (λ) which are shown in the Fig. 10.5.5.

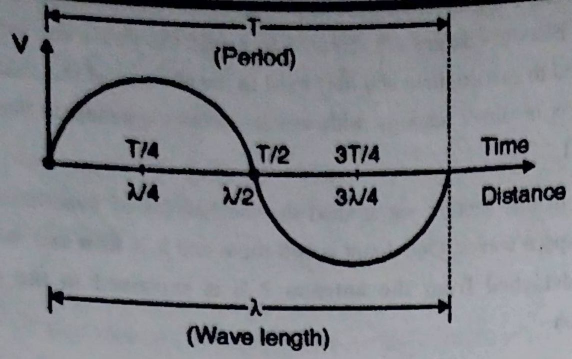


Fig. 10.5.5 : Basic parameters of a sinusoidal voltage waveform

Now we come to the point. In the first quarter cycle of the generator voltage when the voltage starts increasing, the charge on the antenna wires also increases. Consider the upper section of the antenna is charged positively and lower section is having negative charge. Due to this charge field lines in downward direction are generated and starts expanding away from the antenna.

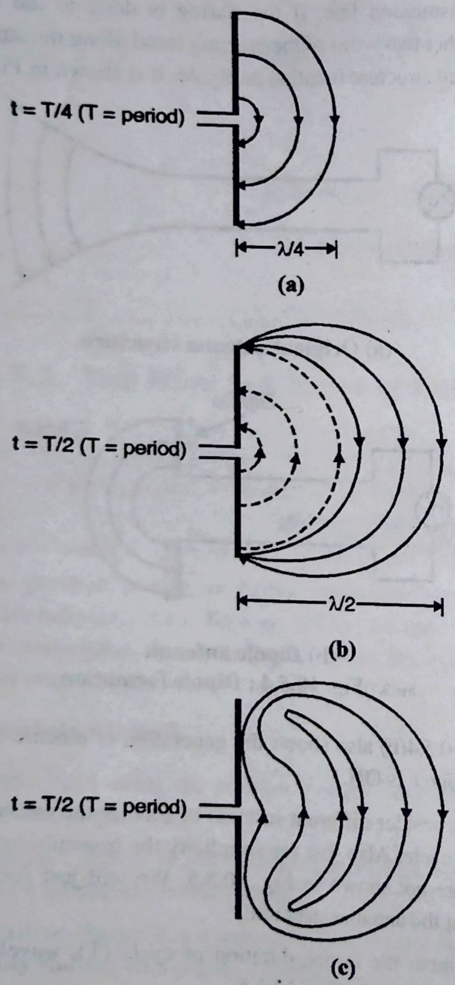


Fig. 10.5.6 : Detachment of fields from antenna

At the end of first quarter cycle (at t = T/4), when the voltage peak is reached then the charge is also maximum, the field is travelled a distance λ/4. Just for simplicity only three lines are shown in Fig. 10.5.6(a).

During the next quarter cycle when the voltage is decreasing the charge on the wires also starts decreasing. Or in other words opposite charge starts increasing. This will make upper wire of the antenna more and more negative while lower wire is becoming positive. Now the new field lines (shown as dash lines) will change the direction going in upward direction. The original field lines will move further away from the antenna. It is shown in Fig. 10.5.6(b).

At the end of first half cycle (t = T/2), since the voltage is zero, the charge is zero. This causes the field lines to detach themselves from the antenna wires and combine together to form closed loops. Which is the property of electric field that when no charge is present field lines will form loops. It is shown in Fig. 10.5.6(c).

In the next half cycle, the same actions will repeat but in opposite direction. After that the process is repeated continuously.

10.6 ANTENNA RADIATION REGIONS

UQ. Explain near field, inductive field and radiation field related to antenna. **(MU - Dec. 17, 5 Marks)**

UQ. What is reactive near field. Explain its importance in communication and its applications. **(MU - May 17, May 18, 5 Marks)**

UQ. Write short note on : Near field and far field radiation **(MU - Q. 6(e), Dec. 19, 5 Marks)**

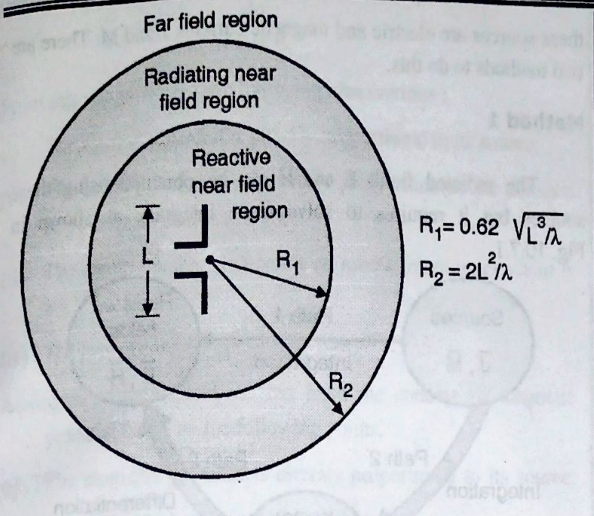
The radiation pattern we discussed in the previous section with the help of field surrounding the antenna. But it is interesting to see that whether we get same pattern at all distances from the antenna. The answer is no. There are three regions formed where the characteristics of the radiated wave are different. So we divide the space surrounding an antenna into three distinct regions :

a) Reactive near-field region,

b) Radiating near field (Fresnel) region,

c) Far field (Fraunhofer) region.

These regions are so designated to identify the field structure in each. The boundaries for these regions are not firm. These regions are now discussed one by one.



aw (4.14)**Fig. 10.6.1 : Field regions of an antenna**

10.6.1 Reactive Near Field Region

> **It is that portion of the near-field region immediately surrounding the antenna where in the reactive field predominates.**

For most of the antennas, the outer boundary of this region is commonly taken to exist at

$$R < 0.62\sqrt{\frac{L^3}{\lambda}} \quad ...(10.6.1(a))$$

Where, λ is the wave length and

L is the large dimension of the antenna.

But for a very short dipole or equivalent radiator, the outer boundary is,

$$R < \frac{\lambda}{2\pi} \quad ...(10.6.1(b))$$

Practically what is the significance of this distance ?

In general, **objects within this region will result in coupling with the antenna and distortion of the ultimate far-field antenna pattern.**

Analysis of a near field coupling can be deceiving. For example, with a 40 GHz antenna

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{40\times10^9} = 7.5\times10^{-3} \text{ m.}$$

which is less than a centimetre, making near field boundary on the order of a couple of millimetres. Thus it is unlikely that anything will be close enough to this 40 GHz antenna to cause near-field coupling.

Now consider a 4 MHz ($\lambda = c/f = 75$ m) antenna.

For this antenna

$$R < \frac{\lambda}{2\pi} = 12 \text{ m}$$

Thus any large conductor (relative to wavelength) within this distance will couple with the antenna and "detune" it. The result can be an altered resonant frequency, radiation resistance (defined later), and/or radiation pattern.

It is important to note that the radiation pattern discussed previously is not at all developed in this region.

10.6.2 Radiating Near-Field (Fresnel) Region

> **It is that region of the field of an antenna between the reactive near field region and the far field region.**

For this region, the distance from the antenna R is

$$0.62\sqrt{\frac{L^3}{\lambda}} \le R < \frac{2L^2}{\lambda}$$

where L is the largest dimension of the antenna. ...(10.6.2)

This region is also called the transition region.

For an antenna focused at infinity, the radiating near field region is sometimes referred to as the fresnel region on the basis of analogy to optical terminology.

The typical properties of this region are :

- The antenna pattern is taking shape but is not truly formed.
- The radiation field predominates the reactive field.
- The angular field distribution is dependent upon the distance from the antenna.
- The radiated wave front is still clearly curved (nonplaner).
- The electric and magnetic field vectors are not orthogonal.

In the antenna has as maximum dimension that is not large compared to the wavelength, this region may not exist.

10.6.3 Far-Field (Fraunhofer) Region

> **It is that region of the field of an antenna where the angular field distribution is essentially independent of the distance from the antenna.**

The radiation pattern studied in the previous section is the pattern in this region. It is defined as the region for which distance from the antenna,

$$R > \frac{2L^2}{\lambda} \quad ...(10.6.3)$$

In this region

- The wavefront becomes approximately planer.

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1 ghz = 1000 mhz (10^3) 1 khz = 1000 h (10^3)
1 mhz = 1000 khz (10^3)

- The radiation pattern is completely formed and does not vary with distance.
- The electric and magnetic field vectors are orthogonal to each other.
- The angular field distribution is independent of the radial distance.

For an antenna focused at infinity, the far field region is sometimes referred to as the **Fraunhofer region** based on analogy to optical terminology.

For electrically small antennas ($L << \lambda$) the far field boundary defined by the preceding equation may actually fall within the near-field region. In such cases, the far field boundary should be taken as equal to the near-field boundary rather than within it.

Table 10.6.1

Summary of antenna regions
There are three regions, i) Reactive near field region $R < 0.62\sqrt{\frac{L^3}{\lambda}}$ - For most of the antennas $R < \frac{\lambda}{2\pi}$ - For short dipole ii) Radiating near - field (Fresnel region) $0.62\sqrt{L^3/\lambda} < R < 2L^2/\lambda$ iii) For field (Fraunhofer) region $R > \frac{2L^2}{\lambda}$

Ex. 10.6.1 : Calculate the minimum distance required to measure the field pattern of an antenna of diameter 2 m at a frequency of 3 GHz.

c = speed of light 3×10^8 m/s
f = frequency here f = 3ghz

Soln. :

Convert 3ghz to hz

Given : $L = 2$ m, $f = 3$ GHz $\rightarrow \lambda = \frac{c}{f} = \frac{3\times10^8}{3\times10^9} = 0.1$ m

To measure the radiation pattern, it should be completely formed. The minimum distance for this is Equation (10.6.3).

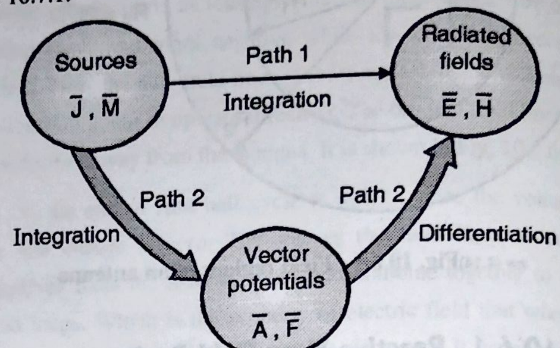
$$R = \frac{2L^2}{\lambda} = \frac{2(2)^2}{0.1} = 80 \text{ (m)}.$$

10.7 CONCEPT OF POTENTIALS

In antenna theory we are always interested in finding electric field (E) and magnetic field (H) due to sources of fields. Obviously these sources are electric and magnetic currents $\bar{J}$ and $\bar{M}$. There are two methods to do this.

Method 1

The radiated fields $\bar{E}$ and $\bar{H}$ can be obtained using these sources but it requires to solve some integrals, as shown in Fig. 10.7.1.



aw(5.1)**Fig. 10.7.1 : Finding E and H using J and M**

Method 2

These radiated fields can also be obtained using some auxiliary functions $\bar{A}$ and $\bar{F}$. As shown in Fig. 10.7.1 these fields are obtained by using a two step procedure.

Step 1 : Involves calculation of $\bar{A}$ and $\bar{F}$ from $\bar{J}$ and $\bar{M}$ by solving some integrals.

Step 2 : From $\bar{J}$ and $\bar{M}$ obtained in step 1, we can determine E and H by solving some differentiations.

As seen method 2 requires integral as well as different ions to be solved, but method 1 requires only integrals to be solved. But integrals in method 2 are much simpler as compared to in method 1.

Conclusion

In radiation problem involving finding $\bar{E}$ and $\bar{H}$ using sources $\bar{J}$ and $\bar{M}$, we introduce the auxiliary functions $\bar{A}$ and $\bar{F}$. These auxiliary functions are called as vector potentials.

10.8 CONCEPT OF VECTOR MAGNETIC POTENTIAL

The concept of vector magnetic potential can be developed by considering the analogy between the electric field and the magnetic field. Thus, as for the point charge configuration in electric field the electric potential at any point is defined as

$$dV = \frac{dq}{4\pi \in R}$$

From this equation we have following observations :

(i) The electric potential is directly proportional to its source.

(ii) The electric potential is inversely proportional to the distance of point from source (i.e. R).

(iii) The electric potential depends on media (medium constant is $\in$).

(iv) The proportionality constant is $1/4\pi$.

Analogous to these lines we can have the concept of magnetic potential based on the following points.

(i) The magnetic potential is directly proportional to its source. (i.e. differential current element $I\,d\bar{l}$).

(ii) the magnetic potential is inversely proportional to its distance of point from source. (i.e. R)

(iii) The magnetic potential depends on media (medium constant is μ)

(iv) The proportionality constant is $1/4\pi$.

Thus analogous to mathematical definition of electric potential we can write magnetic potential as

$$d\bar{A} = \frac{\mu I d\bar{l}}{4\pi r} \quad ...(10.8.1)$$

Secondly, we know that $\bar{E} = -\nabla V$, this means the space derivative of electric potential is equal to one of the electric field quantities which depends on media i.e. electric field intensity. But for the magnetic potential which is defined in above Equation (10.8.1), we can easily observe that the magnetic potential is a vector quantity (due to presence of $d\bar{l}$ which is a vector.) Thus if we want to equate the space derivative of magnetic vector potential with some magnetic quantity which is a vector then the possible space derivative is curl operation and we can write either

$$\nabla \times \bar{A} = \bar{H} \text{ or } \nabla \times \bar{A} = \bar{B}$$

But as we have accepted that the magnetic potential depends on media then its space derivative i.e. $\nabla \times \bar{A}$ must depend on media. Out of the two magnetic quantities, i.e. $\bar{B}$ and $\bar{H}$ we have studied that $\bar{H}$ is independent of media and $\bar{B}$ depend on media, thus we can define

$$\nabla \times \bar{A} = \bar{B} \quad ...(10.8.2)$$

This concept of magnetic vector potential is practically useless but many times, for deriving the expression of magnetic field intensity for complex fields the concept is widely used.

10.9 RETARDED VECTOR POTENTIAL

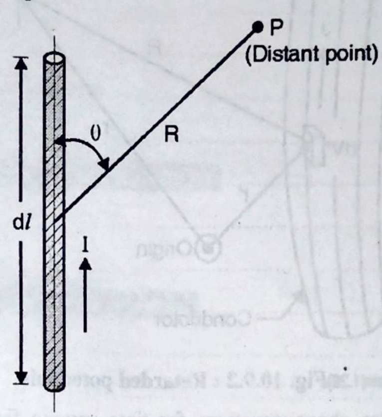
In antenna analysis usually the distance between the transmitting and receiving antennas is very large. The waves radiated by transmitting antenna requires some finite time to reach the receiving antenna.

While dealing with radiators or antennas the finite time of propagation is of great importance. If an alternating current I flowing in the short element (dl), its effect is not realized at a distant R instantaneously but only after an interval equal to the time needed for the disturbance to propagate over the distance R.

Time required = R/c

where c = velocity of a wave

For example,



aw(1.27)Fig. 10.9.1 : Differential current element

Let this time be 5 minutes. Consider the current flows through the current element at 10 am then the field is observed at point P at 10.05 or if the field is required at 10 am we should consider the current at 9.55 am.

Hence, it now becomes necessary to introduce the concept of retardation or that the effect reaching a distant point P from a given element at an instant t is due to a current value which flowed at an earlier time. Thus the instantaneous current

$$[I] = I_m \sin\omega\left(t - \frac{R}{c}\right) \quad ...(10.9.1)$$

Where, R = distance travelled,

c = velocity of propagation,

[I] = retarded current and the bracket indicates that it is retarded quantity

$\left(t - \frac{r}{c}\right)$ = retarded time.

Equation (10.9.1) implies that the disturbance at time t at the distance R (point P) from the element is caused by a retarded current [I] that occurred at an earlier time $\left(t - \frac{R}{c}\right)$. We have,

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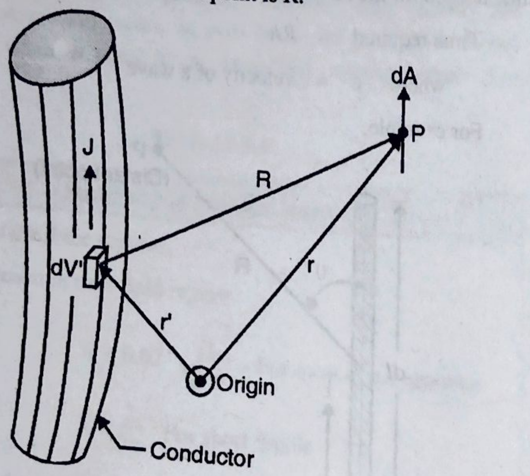
Vector magnetic potential

$$\bar{A} = \frac{\mu}{4\pi}\int_V \frac{\bar{J}}{R}\,dv \qquad ...(10.9.2)$$

Scalar electric potential

$$V = \frac{1}{4\pi\epsilon}\int_V \frac{\rho_v}{R}\,dv \qquad ...(10.9.3)$$

In the Fig. 10.9.2 source quantities are placed at a distance r' while the observation point is at a distance r from origin. The distance between source and observation point is R.



aw(1.28)**Fig. 10.9.2 : Retarded potentials**

In order to use above equations for time varying fields certain modifications are required. From the above discussion there is a finite propagation time required for the wave to travel a certain distance. Potentials from which the fields are to be derived should also display finite propagation time. Thus potential $\bar{A}$ at a distance r at time t is due to $\bar{J}$ at a distance r' at time $\left(t-\frac{R}{c}\right)$ given by,

$$\bar{A}(\bar{r}, t) = \frac{\mu}{4\pi}\int_V \frac{\bar{J}\left(\bar{r}', t-\frac{R}{c}\right)}{R}\,dv' \qquad ...(10.9.4)$$

$$V(\bar{r}, t) = \frac{1}{4\pi\epsilon}\int_V \frac{\rho_v\left(\bar{r}', t-\frac{R}{c}\right)}{R}\,dv' \qquad ...(10.9.5)$$

In above expressions a time delay $\frac{R}{c}$ sec is introduced so that now the potentials have been delayed or retarded by this amount. For this reason they are also called **retarded potentials.** Equations (10.9.4) and (10.9.5) can also be written as,

$$\bar{A} = \frac{\mu}{4\pi}\int_V \frac{[J]}{R}\,dv' \qquad ...(10.9.6)$$

$$V = \frac{1}{4\pi\epsilon}\int_V \frac{[\rho_v]}{R}\,dv' \qquad ...(10.9.7)$$

The potentials $\bar{A}$ and V are respectively called as ***retarded magnetic vector potential and retarded electric scalar potential.*** The bracketed terms [ ] are representing retarded quantities.

Chapter Ends...

□□□

Module 3

CHAPTER 11 Antenna Parameters

11.1 INTRODUCTION

In the next chapters we are going to make analysis of linear wire antennas, antenna arrays and loop antenna respectively. These chapters on analysis gives mathematical expressions for the fields. When current is passed through antenna, it radiates more in one direction and less in other directions. This is termed as radiation property of the antenna. Many times for the end user expressions for the fields are not required or these expressions does not give clear understanding of radiation properties of antenna. But using these expressions we make graphical representation of the fields. These plots give clear understanding of radiation properties.

There are some other properties of antenna and we should have knowledge of it. This will be useful when we want to select antenna for a particular application. Also these properties can be used for comparison between different antennas.

Irrespective of type of application of an antenna, all antennas possess certain basic properties which are listed below.

- Radiation pattern.
- Half-Power Beam Width (HPBW) and First Null Beam Width (FNBW).
- Beam area.
- Beam efficiency
- Directivity D or gain G
- Effective aperture A_e, and so on.

These are discussed below.

11.2 RADIATION PATTERN

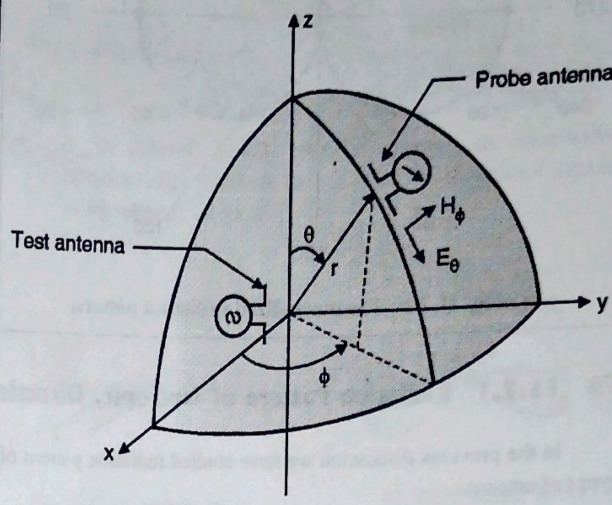
The radiation pattern is a graphical representation of the radiation (far field) properties of an antenna.

The radiation pattern of **Test antenna** can be measured by another antenna (called as **probe antenna**). In the measurement we move a probe antenna around the test antenna at a constant distance from it, and note the response as a function of angular coordinates. This is as shown in Fig. 11.2.1.

In the measurement, the probe antenna is usually maintained in a given orientation. For example, consider an ideal dipole along z-axis at the origin as shown in Fig. 11.2.1. To measure the E_θ component of the radiated field, we would choose a probe antenna that responds to this field. Another ideal dipole can also be used as probe antenna. As this probe is moved over the spherical surface its output (terminal voltage) varies and is recorded. Remember that we are moving probe antenna over spherical surface i.e. r is kept constant during measurement.

Radiation pattern can also be obtained by using mathematical expressions of the field. Consider elementary dipole antenna of length l. The magnitude of radiation for such an antenna is

$$E_\theta = \left(\frac{60 \pi I l}{r \lambda}\right) \sin \theta = E_m \sin \theta \qquad ...(11.2.1)$$



aw(4.1)**Fig. 11.2.1 : Radiated field measurement**

At a fixed distance (r) form antenna and a particular frequency (λ = constant), the bracketed term in the above expression is constant, and is replaced by E_m. (Don't bother about how the expression has arrived. This is done in next chapter).

The maximum field is obtained when $\theta = 90°$, then

$$E_{\theta\,max} = E_m \qquad ...(11.2.2)$$

The normalized field is obtained by dividing field by its maximum value. Thus

$$E_{normalized} = E_n = \frac{E_\theta}{E_{\theta\,max}} = \sin \theta \qquad ...(11.2.3)$$

and

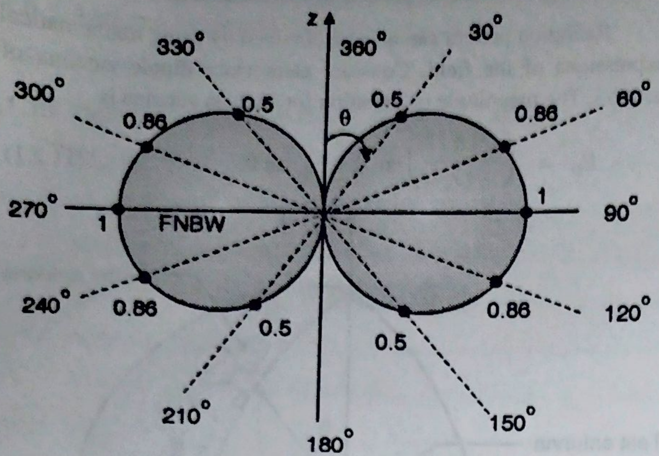
$$|E_n| = |\sin \theta| \qquad ...(11.2.4)$$

To plot the radiation pattern we calculate $|E_n|$ at different values of θ. The modulus is taken because we are plotting magnitude plot.

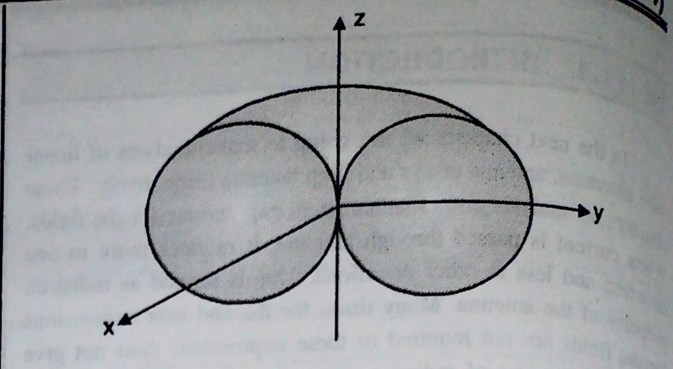
θ	0	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$\lvert E_n \rvert$	0	0.5	0.86	1	0.86	0.5	0	0.5	0.86	1	0.86	0.5	0

The plot of it as shown in Fig. 11.2.2

The advantage of making normalized pattern is the maximum normalized value can go upto one. The variation of E_n is irrespective of ϕ. Thus for all values of ϕ about z-axis plot the is same. By rotating the two dimensional pattern about the vertical axis we get 3-d pattern. The complete pattern for the ideal dipole is shown in Fig. 11.2.3 with a slice removed. It resembles a "holeless doughnut", and is often referred to as an omnidirectional pattern since it is uniform in the x-y plane.



aw(4.2)Fig. 11.2.2 : The normalized radiation pattern



aw(4.3)Fig. 11.2.3 : Three dimensional pattern (doughnut shape)

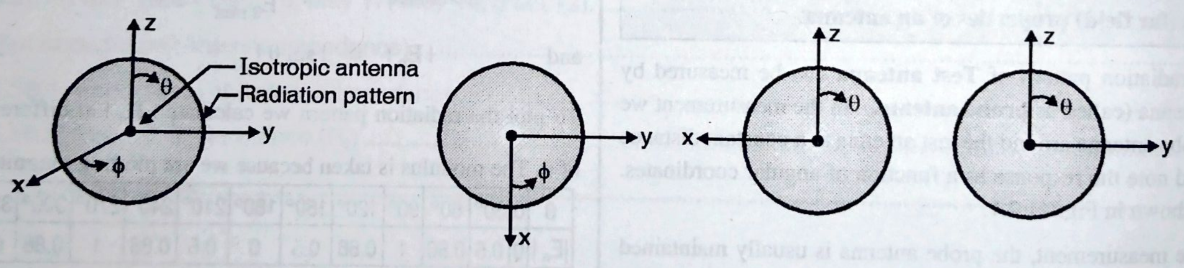
11.2.1 Radiation Pattern of Isotropic, Directional and Omnidirectional Antenna

In the previous discussion we have studied radiation pattern of Hertzian dipole. Now we will see radiation pattern of some different types of antennas.

11.2.1(A) Isotropic Antenna

UQ. What is isotropic pattern. Give one example. **(MU - Dec. 15, 2.5 Marks)**

- It is an antenna which radiates equally in all directions.
- It is a point source of radiation.
- It is not possible in practice.
- Since it radiates equally in all directions, the radiation is independent of θ and ϕ. Thus the radiation pattern of this antenna is spherical. Refer Fig. 11.2.4.
- The two dimensional plots are circles as shown in Fig. 11.2.4(b), (c), (d)
- Isotropic antenna is used as a reference antenna to compare performance of other antennas over isotropic antenna.

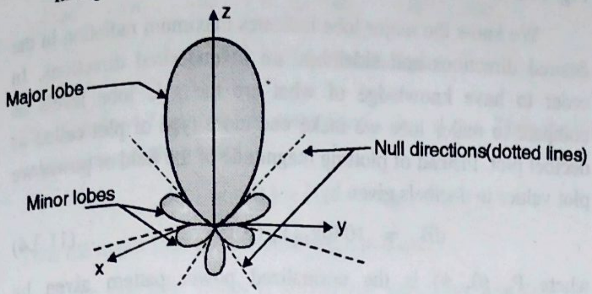


(a) 3-d pattern (b) θ = 90° plane (c) ϕ = 0° plane (d) ϕ = 90° plane

aw(4.4)Fig. 11.2.4 : The radiation pattern of a isotropic antenna

11.2.1(B) Directional Antenna

- In practice any antenna radaties more in a particular direction and less in other directions, then the antenna is called as **directional antenna.**
- The radiation pattern of a typical directional antenna is shown in Fig. 11.2.5.



aw(4.5)**Fig. 11.2.5 : The radiation pattern of directional antenna**

- In the Fig. 11.2.5 an antenna situated at origin is radiating more in z-direction and less in other directions.
- There are some directions where there is no radiation, called as null directions. These are shown as dotted lies in the Fig. 11.2.5.
- In between the two adjacent nulls, there is some radiation pattern called as lobes. These lobes are classified as :

(i) A major lobe

- Also called main beam. It is defined as,

"The radiation lobe containing the direction of maximum radiation."

- In the Fig. 11.2.5 the major lobe is pointing in the $\theta = 0$ direction.
- In some antennas, such as split beam antennas, there may exist more than one major lobe.

(ii) A minor lobe

It is any lobe except major lobe.

(iii) A side lobe

- It is a radiation lobe in any direction other than the intended lobe.
- Usually a side lobe is adjacent to the main lobe and occupies the hemisphere in the direction of the main lobe.

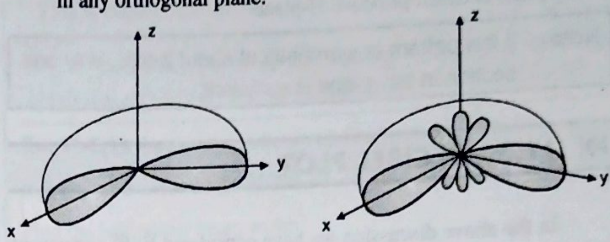
(iv) A back lobe

- It is a radiation lobe whose axis makes an angle of approximately 180° with respect to the major beam of an antenna.
- **Usually it refers to a minor lobe that occupies the hemisphere in a direction opposite to that of the major lobe.**

11.2.1(C) Omnidirectional antenna

UQ. What is Omnidirectional pattern. Give one example. **(MU - Dec. 15, 2.5 Marks)**

- It is defined as an antenna having an essentially nondirectional pattern in a given plane and directional pattern in any orthogonal plane.



(a) Without minor lobes **(b) With minor lobes**

aw(4.6)**Fig. 11.2.6 : Omnidirectional pattern**

- As shown in Fig. 11.2.6, in x-y plane it radiates equally in all directions and in a plane perpendicular to x-y plane it radiates differently in different directions.
- An omnidirectional pattern is then a special type of directional pattern.
- Some antennas such as dipoles, loops, broadside array etc. exhibit omnidirectional patterns.

11.2.2 Principal Patterns

Generally in practice to get the idea about the radiation property of antenna, only E and H plane patterns are sufficient. There is no need of 3-dimensional pattern.

The E and H plane patterns are also called principal patterns.

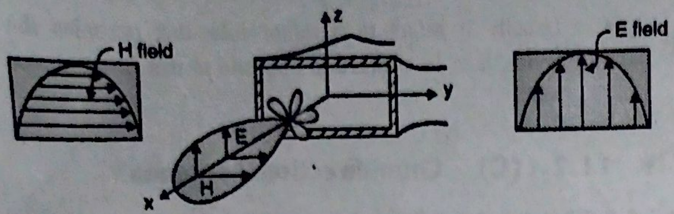
The E and H planes are defined as :

E-plane

It is a plane containing the electric field vector and the direction of maximum radiation.

H-plane

It is a plane containing magnetic field vector and the direction of maximum radiation. Consider the horn antenna with rectangular opening as shown in Fig. 11.2.7. The maximum radiation is along x axis. The electric and magnetic field of a wave radiated by horn is in z-direction and y-direction respectively as shown in Fig. 11.2.7.



aw(4.7)**Fig. 11.2.7 : The field of an horn antenna**

In the Fig. 11.2.7, the electric field and maxima are present in x-z plane (elevation plane, $\phi = 0$), so x-z plane is called principal E-plane. The magnetic field and maxima are present in x-y plane, so x-y plane is called principal H-plane.

Note : If the pattern is symmetrical about z-axis, only one section in any plane is sufficient.

11.3 DECIBEL PLOT

In the above discussion we have considered E (θ) component of a field radiated. In general the pattern can be obtained for field components and the phase components such as, $E_\theta(\theta, \phi)$, $E_\phi(\theta, \phi)$, $\delta_\theta(\theta, \phi)$ and $\delta_\phi(\theta, \phi)$ and so an. Where δ is the phase angle of the field. The term $E_\theta(\theta, \phi)$ indicates that it is a E_θ component of the field which is a function of θ and ϕ. Similarly other terms.

The plots can also be made normalized. The normalized value is obtained by dividing value by its maximum value. For example the normalized value of $E_\theta(\theta, \phi)$ is

$$E_\theta(\theta, \phi)_n = \frac{E_\theta(\theta, \phi)}{E_\theta(\theta, \phi)_{max}} \quad ...(11.3.1)$$

Similarly the normalized power plot can be made using,

$$P_n(\theta, \phi) = \frac{W(\theta, \phi)}{W(\theta, \phi)_{max}} \quad ...(11.3.2)$$

where, $W(\theta, \phi)$ = Poynting vector, obtained by

$$W(\theta, \phi) = \frac{E^2}{\eta} = \frac{\left[E_\theta^2(\theta, \phi) + E_\phi^2(\theta, \phi)\right]}{\eta} \; (W/m^2) \quad ...(11.3.3)$$

$W(\theta, \phi)_{max}$ = The maximum value of $W(\theta, \phi)$ (W/m^2)

η = Intrinsic impedance of space = $120\pi \simeq 377\ \Omega$

Note :

Throughout the subject, the letter W is reserved for power density in Watts / m^2, and letter P is for power in watts. The relation between P and W is

$$P = \int_s \overline{W} \cdot \overline{ds} \text{ (Watts)}$$

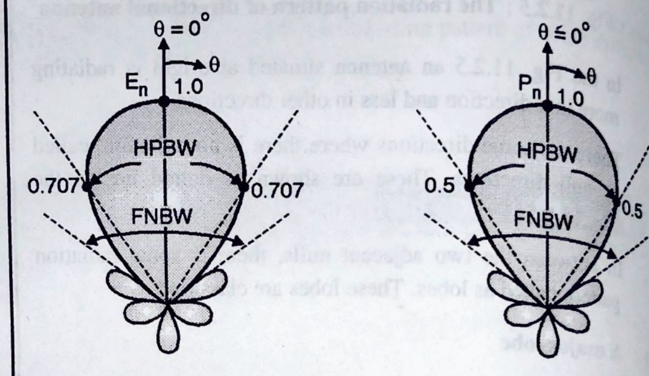
Also remember that the normalized quantity is always dimensionless.

The normalized field and power pattern is as shown in Fig. 11.3.1.

We know the major lobe indicates maximum radiation in the desired directions and sidelobes are in undesired directions. In order to have knowledge of what are the side lobe levels as compare to major lobe we make one more type of plot called as decibel plot. Instead of plotting magnitude of the field or power we plot values in decibels given by,

$$dB = 10 \log_{10} P_n(\theta, \phi) \quad ...(11.3.4)$$

where $P_n(\theta, \phi)$ is the normalized power pattern given by Equation (11.3.2).



(a) Normalized field pattern (b) Normalized power pattern

aw(4.8)**Fig. 11.3.1 : Normalized plots**

Note : It is important to recognize that the field (magnitude) pattern and power pattern are the same when plotted in decibels.

This follows directly from definitions. For field intensity in decibels,

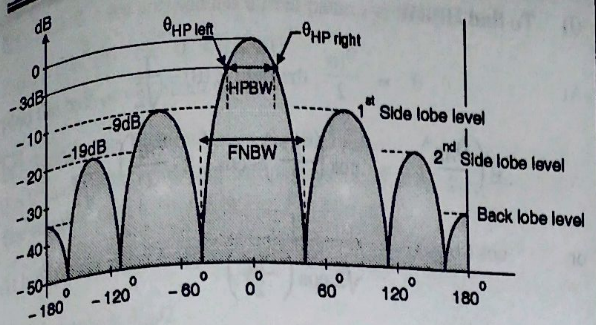
$$|E(\theta)|_{dB} = 20 \log |E(\theta)| \quad ...(11.3.5)$$

For power in decibels,

$$P(\theta)_{dB} = 10 \log P(\theta) = 10 \log |E(\theta)|^2 = 20 \log |E(\theta)| \quad ...(11.3.6)$$

and we see that $\quad P(\theta)_{dB} = |E(\theta)|_{dB}$

The decibel plot of the radiation pattern shown in Fig. 11.3.1 is shown in Fig. 11.3.2.



aw(4.9)Fig. 11.3.2 : Decibel plot

The direction of maxima has normalized value of one. Then the dB value corresponding to it is

$$dB = 10 \log P_n = 10 \log 1 = 0$$

Thus the major lobe has the peak value at 0 dB.

Since the side lobe values is normalized values are less than one, the logarithm of these values is negative giving side lobe peaks representing negative values. Using 3 dB plot we can define some very important antenna parameters.

These are :

11.3.1 Half Power Beam Width (HPBW)

HPBW defined as the angular distance between the two points on the antenna pattern main lobe that are 3 dB below the maximum gain point (which is at 0 dB).

$$HPBW = |\theta_{HP\ left} - \theta_{HP\ right}|$$

Where, $\theta_{HP\ left}$ and $\theta_{HP\ right}$ are the points to the ' left ' and ' right ' of the main beam maximum for which the power pattern has a value of one-half, field pattern has a value of $1/\sqrt{2}$ or dB plot has a value of – 3 dB. Refer Fig. 11.3.1 and 11.3.2. Notice that the HPBW can also be calculated using field pattern and power pattern as shown in Fig. 11.3.1.

11.3.2 First Null Beam Width (FNBW)

FNBW is defined as the angular distance between two points on each side of the major lobe (just adjacent) where the radiation drops to zero.

On the radiation pattern we find many null directions. But for FNBW we consider null just adjacent to the major lobe, so called first null beam width.

11.3.3 Side Lobe Level (SLL)

SLL is the ratio of the pattern value of a side lobe peak to the pattern value of the main lobe.

The largest side lobe level for the whole pattern gives the maximum (relative) side lobe level, frequently abbreviated as SLL.

In Fig. 11.3.2, the maximum side lobe level is shown 9 dB below the major lobe level.

Low side lobe level reduce the risk of undesired signal radiation or reception. Side lobe level of – 20 dB or smaller are usually not desirable in most applications.

11.3.4 Front to Back Ratio

This is ratio of the antenna gain at 0° and 180°.

It indicates how well the antenna will reject interfering signals that arrive from the rear of the antenna.

Ex. 11.3.1 : An antenna is having a field pattern given by

$E(\theta) = \cos\theta \quad$ for $0° \le \theta \le 90°$

Find the half power beam width.

☑ **Soln. :**

The given function is a cosine function, it will have maximum value at $\theta = 0$ and the value goes on decreasing as θ increases. It is zero at $\theta = 90°$. The rough sketch is shown in Fig. Ex. 11.3.1.

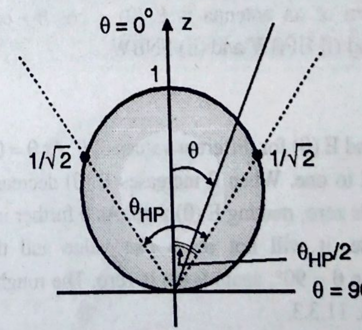
Since power, $P \propto E^2(\theta)$, when power reduce to half then $E(\theta)$ reduce to $\frac{1}{\sqrt{2}}$.

Let the value of θ at this value of $E(\theta)$ is $\theta_{HP}/2$.

$$\therefore \quad E(\theta_{HP}/2) = \cos(\theta_{HP}/2) = \frac{1}{\sqrt{2}}$$

$$\therefore \quad \theta_{HP}/2 = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$$

$$\text{or} \quad \theta_{HP} = 2 \times 45° = 90° = HPBW$$



aw(4.10)Fig. : Illustrating Ex. 11.3.1

Module 3

Ex. 11.3.2 : For an antenna E (θ) = sin θ. Find the HPBW and FNBW.

☑ **Soln. :**

The given function is a sin function. It has zero value at θ = 0° while maximum value at θ = 90°. When θ exceeds 90°, E (θ) decrease and is zero at θ = 180°. The rough sketch is shown Fig. Ex. 11.3.2.

At $\theta = 90° - \frac{\theta_{HP}}{2}$, the field drops to $\frac{1}{\sqrt{2}}$.

$$\therefore \quad E\left(90° - \frac{\theta_{HP}}{2}\right) = \sin\left(90° - \frac{\theta_{HP}}{2}\right) = \frac{1}{\sqrt{2}}$$

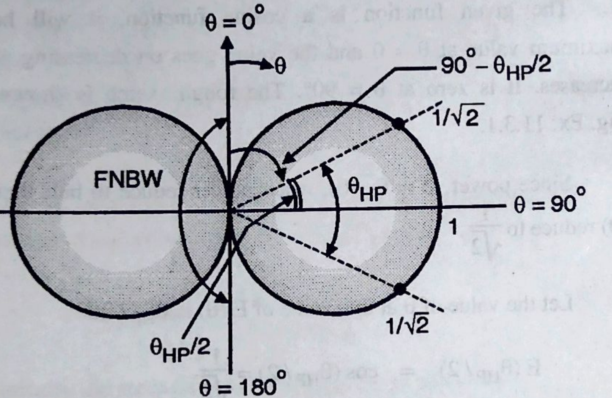
$$\therefore \quad 90° - \frac{\theta_{HP}}{2} = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$$

$$\therefore \quad \frac{\theta_{HP}}{2} = 90° - 45° = 45°$$

or $\theta_{HP} = 2 \times 45° = 90° = \text{HPBW}$

The electric field, E (θ) is zero at θ = 0° and 180°. Therefore

$$\text{FNBW} = 180° - 0° = 180°$$



aw (4.11)**Fig. : Illustrating Ex. 11.3.2**

UEx. 11.3.3 MU - May 09, 10 Marks

The field pattern of an antenna is E (θ) = cos θ · cos 2θ for 0° ≤ θ ≤ 90° Find (i) HPBW and (ii) FNBW.

☑ **Soln. :**

We can find E (θ) for different values of θ. At θ = 0°, E (θ) is maximum equal to one. When θ increases E (θ) decreases. When θ = 45°, cos 2θ is zero, making E (θ) zero. As θ further increases E (θ) will increase, it will not reach one value and then starts decreasing. When θ = 90°, again E (θ) is zero. The rough sketch is shown in Fig. Ex. 11.3.3.

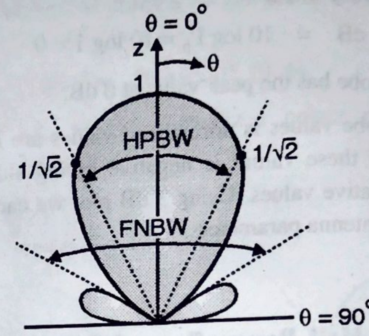
(i) To find HPBW

At $\theta = \frac{\theta_{HP}}{2}$, the field $E(\theta) = \frac{1}{\sqrt{2}}$

$$\therefore \quad E\left(\frac{\theta_{HP}}{2}\right) = \cos\left(\frac{\theta_{HP}}{2}\right) \cdot \cos\left(2 \cdot \frac{\theta_{HP}}{2}\right) = \frac{1}{\sqrt{2}}$$

$$\text{or} \quad \cos(\theta_{HP}) = \frac{1}{\sqrt{2}\cos\left(\frac{\theta_{HP}}{2}\right)}$$

$$\text{or} \quad \theta_{HP} = \cos^{-1}\left[\frac{1}{\sqrt{2}\cos\left(\frac{\theta'_{HP}}{2}\right)}\right]$$



aw (4.12)**Fig. : Illustrating Ex. 11.3.3**

We will put different values of θ'_{HP} and find θ_{HP}. When the values are equal, that is the correct value required. Let us start with $\theta'_{HP} = 0$

$\theta'_{HP} = 0$ then $\theta_{HP} = 45°$

$\theta'_{HP} = 45°$ then $\theta_{HP} = 40.06°$

$\theta'_{HP} = 40.06°$ then $\theta_{HP} = 41.17°$

$\theta'_{HP} = 41.17°$ then $\theta_{HP} = 40.95°$

$\theta'_{HP} = 40.95°$ then $\theta_{HP} = 40.99°$

$\theta'_{HP} = 40.99°$ then $\theta_{HP} = 40.98°$

Values are almost equal. Thus,

$$\theta_{HP} = 40.98° = \text{HPBW}$$

(ii) To find FNBW

At θ = 45°, cos 2θ = 0 $\quad \therefore$ E(θ) = 0

$$\therefore \quad \text{FNBW} = 2 \times 45° = 90°$$

Note : E (θ) is also zero at θ = 90° but to calculate FNBW we consider first null after major lobe, which is at 45°

Ex. 11.3.4 : An antenna has a field pattern given by,

$E(\theta) = \cos^2(\theta)$ for $0° \le \theta \le 90°$

Find the half-power beam width.

Soln. : The function given is a $\cos^2$ function. It is maximum at $\theta = 0°$ and starts decreasing with increase in θ. It is zero at $\theta = 90°$.

The rough sketch is shown in Fig. Ex. 11.3.4.

At half power point $E(\theta) = \frac{1}{\sqrt{2}} = 0.707$.

Let this angle is $\theta_{HP}/2$

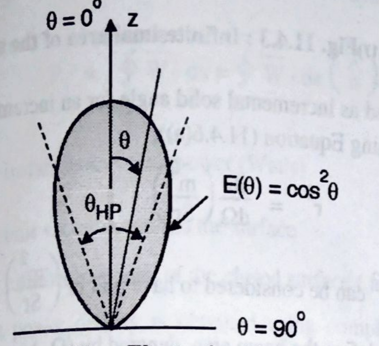
$$\therefore \quad E(\theta_{HP}/2) = \cos^2(\theta_{HP}/2) = 0.707$$

$$\text{or} \quad \cos(\theta_{HP}/2) = \sqrt{0.707}$$

$$\text{or} \quad \theta_{HP}/2 = \cos^{-1}(\sqrt{0.707}) = 32.77°$$

$$\text{or} \quad \theta_{HP} = 2(32.77°) = 65.54° = \text{HPBW}$$

or



aw (4.13)**Fig. : Illustrating Ex. 11.3.4**

11.4 BEAM AREA (Ω_A)

The beam area is also a very important antenna parameter. It is defined in terms of solid angle. So before we go for it let us understand, a plane angle and a solid angle.

11.4.1 Radian (rad)

It is defined as

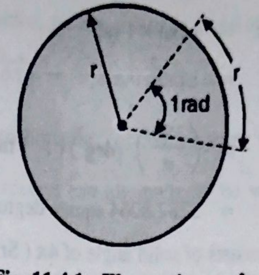
> **One radian is defined as the plane angle with its vertex at the center of a circle of radius r that is subtended by an arc whose length is r.**

This is shown in Fig. 11.4.1.

We know that the circumference of a circle with radius r is

$$\text{Circumference} = 2\pi r = 2\pi \times r \text{ (m)}.$$

This circumference can be thought of as 2π number of sections, each section is of length r. Since each r as occupies a plane angle of 1 radian, thus a circle consists of 2π radian.



aw (4.15)**Fig. 11.4.1 : Illustrating a plane angle**

Full circle consists of 2π rad. ...(11.4.1)

Now let us see what is meant by solid angle.

11.4.2 Sterdian (Sr)

The measure of a solid angle is sterdian (Sr). It is defined as :

> **One sterdian is defined as the solid angle with its vertex at the center of the sphere of radius r that is subtended by a spherical surface area of r^2.**

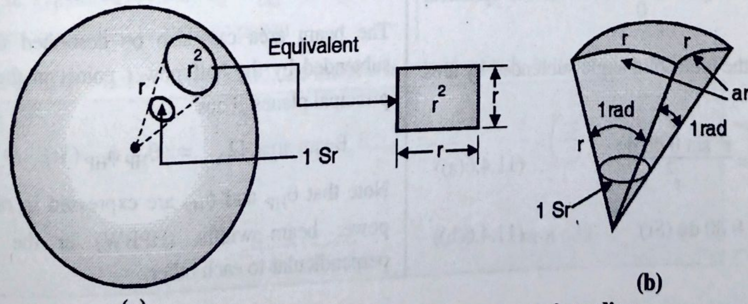
Fig. 11.4.2 below illustrates the concept of solid angle.

The area of the sphere with radius r is

$$\text{Area of sphere} = 4\pi r^2 = 4\pi \times r^2 \text{ (m}^2\text{)}$$

This area can be thought of as 4π number of sections, each section is of area r^2. Since each r^2 area occupies a solid angle of 1 sterdian, thus sphere consists of $4\pi \left(\frac{4\pi r^2}{r^2}\right)$ sterdians.

Full sphere consists of 4π sterdians. ...(11.4.2)



aw (4.16)**Fig. 11.4.2 : Illustrating the concept of sterdian**

11.4.3 Relation between Radian and Sterdian

As shown in Fig. 11.4.2(b), the area r^2 can be considered to be formed by two arcs of each length to r. From the definition of radian, each arc subtends a plane angle of 1 rad so, we can write

$$1 \text{ sterdian} = 1 \text{ Sr} = 1 \text{ rad} \times 1 \text{ rad} \quad ...(11.4.3(a))$$

$$= 1 \text{ rad}^2 \quad ...(11.4.3(b))$$

$$= \left(\frac{180}{\pi}\right)^2 (\text{deg}^2) (\because 1 \text{ rad} = 180/\pi \text{ deg})$$

$$= 3282.8064 \text{ square degrees}$$

Since full sphere consists of solid angle of 4π (Sr),

$$4\pi (\text{Sr}) = 4\pi \times 3282.8064 = 41252.96 \simeq 41253$$

square degrees = $41253^{\square}$

The square (□) symbol at the position of degree (superscript) indicates square degrees. Thus

$$\text{Solid angle in 1 sterdian} = 3282.8064^{\square} \simeq 3283^{\square}$$

$$\text{Solid angle in sphere} = 41252.96^{\square} \simeq 41253^{\square} \quad ...(11.4.4)$$

11.4.4 Incremental Area (ds) and Solid Angle (dΩ)

From the spherical coordinate system, the infinitesimal area ds on the sphere with radius r is shown in Fig. 11.4.3.

$$ds = r^2 \sin\theta \, d\theta \, d\phi \quad ...(11.4.5(a))$$

Many times in the analysis we require vector area $\bar{ds}$, given by

$$\bar{ds} = ds \, \bar{a}_n = r^2 \sin\theta \, d\theta \, d\phi \, \bar{a}_r \quad ...(11.4.5(b))$$

Here $\bar{a}_n$ is a unit vector normal to the spherical surface which is $\bar{a}_r$.

Remember for full sphere variation of angles is,

$$0 \leq \theta \leq \pi \quad \text{and} \quad 0 \leq \phi \leq 2\pi \quad ...(11.4.5(c))$$

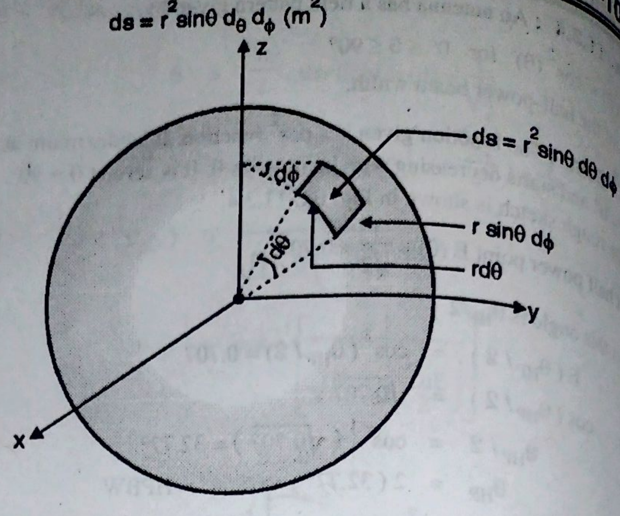
From the definition of solid angle, the area r^2 on the spherical surface

subtends an solid of 1 Sr, so the total solid angle subtended by area ds is

$$d\Omega = \frac{ds}{r^2} = \frac{r^2 \sin\theta \, d\theta \, d\phi}{r^2} \quad ...(11.4.6(a))$$

or

$$d\Omega = \sin\theta \, d\theta \, d\phi \, (\text{Sr}) \quad ...(11.4.6(b))$$



aw (4.17)Fig. 11.4.3 : Infinitesimal area of the sphere

This is called as incremental solid angle for an incremental area ds. By rearranging Equation (11.4.6(a)),

$$r^2 = \frac{ds}{d\Omega} \left(\frac{m^2}{\text{Sr}}\right) \quad ...(11.4.6(c))$$

Thus each r^2 can be considered to have unit of $\left(\frac{m^2}{\text{Sr}}\right)$.

Let us now define the beam area, denoted by (Ω_A)

Beam area (Ω_A) is the solid angle through which all of the power radiated by the antenna would stream (flow) if P (θ, ϕ) maintained its maximum value over Ω_A and was zero elsewhere.

Mathematically,

$$\Omega_A = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} P_n(\theta, \phi) \sin\theta \, d\theta \, d\phi \quad ...(11.4.7(a))$$

$$\text{i.e. Beam area, } \Omega_A = \iint_{4\pi} P_n(\theta, \phi) \, d\Omega \, (\text{Sr}) \quad ...(11.4.7(b))$$

Where $P_n(\theta, \phi)$ is normalized power.

The beam area can also be described in terms of the angles subtended by the half power points in the main lobe in the two principal planes. Thus

$$\text{Beam area, } \Omega_A \simeq \theta_{HP} \, \phi_{HP} \, (\text{Sr}) \quad ...(11.4.7(c))$$

Note that θ_{HP} and ϕ_{HP} are expressed in radians, and are the half power beam widths (HPBW) in the two principal planes perpendicular to each other.

Remember when the field is a function of only one coordinate θ or ϕ), for example $E(\theta) = \cos^2(\theta)$, here we can calculate only θ_{HP}. And since the field is not a function of ϕ, we directly write

$$\theta_{HP} = \phi_{HP} \quad ...(11.4.8)$$

11.5 RADIATION DENSITY ($\overline{W}$)

When the wave travels in the medium, the power associated with the wave travel in the direction of wave travel. The Poynting vector (W) is related to the fields (E and H) of the wave as

$$\overline{W} = \overline{E} \times \overline{H} \text{ (Watt / m}^2) \quad ...(11.5.1)$$

The term W has a unit of Watt / m^2, and is called as instantaneous power density. The power can be obtained from W using

$$P = \oint_s \overline{W} \cdot d\overline{s} = \oint_s \overline{W} \cdot ds\,(\hat{n}) \quad ...(11.5.2)$$

where P = instantaneous total power (Watts)

$\hat{n}$ = unit vector normal to the surface

ds = infinitesimal area of the closed surface (m^2)

The average power density is obtained using complex Poynting vector as

$$\overline{W}_{av} = \frac{1}{2} \text{Re}\left[\overline{E} \times \overline{H}^*\right] \quad ...(11.5.3)$$

The term RE represents the real part of, so W_{av} is real in nature.

Remember that $\overline{E}$ and $\overline{H}$ in above equation represents peak values not rms values, this is reason why 1/2 factor appear in the expression.

The power density associated with the electromagnetic fields of an antenna (we are interested in the same) in its far field region is predominately real and is normally referred to as radiation density (W_{rad}). That is

$$\overline{W}_{rad} = \overline{W}_{av} \quad ...(11.5.4)$$

By replacing $\overline{W}$ in Equation (11.5.2) by $\overline{W}_{rad}$ or $\overline{W}_{av}$, the power obtained is the power radiated or average power.

$$P_{rad} = P_{av} = \oint_s \overline{W}_{rad} \cdot d\overline{s} = \oint_s \overline{W}_{av} \cdot d\overline{s}$$

$$P_{rad} = \frac{1}{2} \oint_s \text{Re}\left[\overline{E} \times \overline{H}^*\right] \cdot d\overline{s} \quad ...(11.5.5(a))$$

In the above equation, for the antenna placed at origin,

$$d\overline{s} = r^2 \sin\theta \, d\theta \, d\phi \, \overline{a}_r \quad ...(11.5.5(b))$$

To find $\overline{W}_{av}$ from Equation (11.5.3) requires the knowledge of $\overline{E}$ and $\overline{H}$. This expression can alternatively be written only in terms of $\overline{E}$ or $\overline{H}$.

Let us assume that the antenna is at the origin of the coordinate system and the waves are travelling in radial direction. For TEM waves we know that

$$\overline{E} \times \overline{H} = E H \overline{a}_r \quad \text{and}$$

$$\frac{E}{H} = \eta \text{ (intrinsic impedance)}$$

Thus Equation (11.5.3) becomes

$$\overline{W}_{av} = \frac{1}{2} \text{Re}\left[\overline{E} \times \overline{H}^*\right] = \frac{1}{2} \text{Re}\left[E H^* \overline{a}_r\right]$$

$$= \frac{1}{2} \text{Re}\left[E \frac{E^*}{\eta} \overline{a}_r\right] = \frac{1}{2\eta} |E|^2$$

In the above step we used the fact that $E E^* = |E|^2$, which is a real.

Therefore, $$\overline{W}_{av} = \frac{|E|^2}{2\eta} \overline{a}_r = \frac{\eta}{2} |H|^2 \overline{a}_r \quad ...(11.5.6)$$

Then power radiated is

$$P_{rad} = \oint_s \frac{|E|^2}{2\eta} \overline{a}_r \cdot d\overline{s} \quad ...(11.5.7)$$

While solving numericals we may come across the integral

$$\int_0^{\pi/2} \sin^n\theta \, d\theta \quad \text{or} \quad \int_0^{\pi} \sin^n\theta \, d\theta$$

Depending upon the value of n, whether even or odd, the integral is solved quickly by following procedure.

$$\left.\begin{aligned} \int_0^{\pi/2} \sin^n\theta \, d\theta &= \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots \cdot \frac{1}{2} \times \left(\frac{\pi}{2}\right) \\ &= \frac{n-1}{n} \cdot \frac{n-3}{n-2} \cdots \cdot \frac{2}{3} \times (1) \quad : \text{For n odd} \end{aligned}\right\} \quad ...(11.5.8(a))$$

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Knowing the relation, the other integral can be solved

$$= \int_0^{\pi} \sin^n \theta \, d\theta = 2 \int_0^{\pi/2} \sin^n \theta \, d\theta \qquad ...(11.5.8(b))$$

For example,

(i) $\int_0^{\pi/2} \sin^6 \theta \, d\theta = \frac{5}{6} \times \frac{3}{4} \times \frac{1}{2} \times \left(\frac{\pi}{2}\right) = \frac{5\pi}{32}$

(ii) $\int_0^{\pi/2} \sin^5 \theta \, d\theta = \frac{4}{5} \times \frac{2}{3} \times 1 = \frac{8}{15}$

11.5.1 Beam Efficiency

The beam area or beam solid angle is the total beam area which includes major lobe and minor lobe beam areas. Thus

$$\Omega_A = \Omega_M + \Omega_m \qquad ...(11.5.9)$$

where Ω_A = total beam area ;

Ω_M = main or major lobe beam area ;

Ω_m = minor lobe beam area

The **beam efficiency** is defined as

$$\text{Beam efficiency, } \epsilon_M = \frac{\Omega_M}{\Omega_A} \text{ (dimensionless)} \qquad ...(11.5.10)$$

Instead of Ω_M in the numerator if we use Ω_m, then the ratio is called as **stray factor**. Thus

$$\text{Stray factor, } \epsilon_m = \frac{\Omega_m}{\Omega_A} \text{ (dimensionless)} \qquad ...(11.5.11)$$

Adding Equation (11.5.10) and Equation (11.5.11) and using Equation (11.5.9),

$$\epsilon_M + \epsilon_m = \frac{\Omega_M}{\Omega_A} + \frac{\Omega_m}{\Omega_A} = \frac{\Omega_M + \Omega_m}{\Omega_A} = \frac{\Omega_A}{\Omega_A} = 1$$

Ex. 11.5.1 : For isotropic antenna the radiation density is $\overline{W}_{rad} = W_0 \bar{a}_r$ (Watt / m^2), find the power radiated.

☑ Soln. :

Using Equation (11.5.5(a)), the power radiated is

$$P_{rad} = \oint_s \overline{W}_{rad} \cdot \overline{ds} = \int_0^{2\pi} \int_0^{\pi} W_0 \bar{a}_r \cdot r^2 \sin\theta \, d\theta \, d\phi \, \bar{a}_r$$

$$P_{rad} = W_0 (r^2) (-\cos\theta)_0^{\pi} (\phi)_0^{2\pi} = 4\pi r^2 W_0 \text{ (Watts)}$$

11.6 RADIATION INTENSITY (U)

Definition

The power radiated from an antenna per unit solid angle is called the radiation intensity U (Watts / Sr).

It is obtained by multiplying the radiation density (W_{rad}) by the square of the distance.

$$U = r^2 W_{rad} \qquad ...(11.6.1)$$

where U = radiation intensity (Watt / Sr) ;

W_{rad} = radiation density (Watt / m^2)

Note that from Equation (11.6.1) it seems like unit of U is

$$m^2 \times \frac{\text{Watts}}{m^2} = \text{Watts}$$

but it is not so. Equation (2.5.6(c)) says, the unit of r^2 is m^2 / Sr, so that unit of U is

$$\frac{m^2}{Sr} \times \frac{\text{Watts}}{m^2} = \frac{\text{Watts}}{Sr}$$

Also the radiation density (W_{rad}) depends on the distance from the antenna (varying inversely as the square of the distance), but the radiation intensity (U) is independent of the distance.

Using Equation (11.5.6), the radiation intensity is also expressed as,

$$U = \frac{r^2}{2\eta} |E|^2 \qquad ...(11.6.2)$$

The power radiated (P_{rad}) can be related to U by

$$P_{rad} = \oint U \, d\Omega = \oint U \sin\theta \, d\theta \, d\phi \text{ (Watts)} \qquad ...(11.6.3(a))$$

Where dΩ is the incremental solid angle = sin θ dθ dϕ (Sr). The power (P_{rad}) is also obtained using Equation (11.5.5(a)) as

$$P_{rad} = \oint \overline{W}_{rad} \cdot \overline{ds} \text{ (Watts)} \qquad ...(11.6.3(b))$$

The radiation intensity is also used to express the normalized power given by

$$P_n(\theta, \phi) = \frac{U(\theta, \phi)}{U(\theta, \phi)_{max}} = \frac{W(\theta, \phi)}{W(\theta, \phi)_{max}} \qquad ...(11.6.4)$$

For more understanding of this parameter, let us solve some examples.

Ex. 11.6.1 : For isotropic radiator find the radiation intensity U_0 when the power radiated is P_{rad}.

Soln. : Using equation

$$P_{rad} = \oint U_0 \, d\Omega = \int_0^{2\pi}\int_0^{\pi} U_0 \sin\theta \, d\theta \, d\phi$$

$$= U_0 (\phi)_0^{2\pi} (-\cos\theta)_0^{\pi} = 4\pi U_0$$

So, the radiation intensity U_0 is expressed in terms of P_{rad} as

$$U_0 = \frac{P_{rad}}{4\pi} \text{ (Watts / Sr)} \quad ...(A)$$

This result is very obvious because the isotropic antenna radiates equally in all directions i.e. in 4π (Sr). Thus the radiation intensity (Watts / Sr) is simply obtained by dividing power radiated by total sterdians (4π). The result in Equation (A) is frequently required in next part of the analysis.

11.7 DIRECTIVITY

11.7.1 Definition

The directivity (D) and the antenna gain (G) are the most important parameters of an antenna. There are many ways in which the directivity is defined. The gain of the antenna we shall study in the next section.

Definition 1

The directivity of an antenna is defined as the ratio of the radiation intensity in a given direction from the antenna to the radiation intensity averaged over all directions.

This definition is used when the directivity in a given direction is required.

$$\text{Mathematically : } D = \frac{U_{\text{given direction}}}{U_{av}} = \frac{U}{U_{av}} \text{ (dimensionless)} \quad ...(11.7.1)$$

Definition 2

The average radiation intensity in above expression is simply obtained by power radiated divided by 4π, which is nothing but radiation intensity of the isotropic antenna. This fact is used to define directivity as :

The directivity of an antenna (non isotropic) is defined as the ratio of its radiation intensity (U) in a given direction over that of an isotropic source.

$$\text{Mathematically : } D = \frac{U}{U_0}$$

Using Equation (A) of Example 11.6.1, $U_0 = P_{rad} / 4\pi$

$$D = \frac{U}{U_0} = \frac{4\pi U}{P_{rad}} \quad ...(11.7.2)$$

Definition 3

Above two definitions are used when a directivity in a particular direction (given) is required. But if the direction is not specified then we calculate directivity in the direction of maxima. This gives rise to third definition of directivity. The directivity of an antenna is defined as the ratio of radiation intensity in maximum direction to the radiation intensity of isotropic source. This results in the maximum directivity, denoted by D_0.

$$\text{Mathematically : } D_{max} = D_0 = \frac{U_{max}}{U_0} = \frac{4\pi U_{max}}{P_{rad}} \quad ...(11.7.3)$$

In all the mathematical expressions, notations have the following meanings.

- D = directivity (no unit) ;
- D_0 = maximum directivity (no unit)
- U = radiation intensity (Watts / Sr) ;
- U_{max} = maximum radiation intensity (Watts / Sr)
- U_0 = radiation intensity of isotropic source (Watts / Sr)
- P_{rad} = power radiated (Watts)

Remember that P_{rad} in above expressions is obtained using the radiation intensity itself given by Equation (11.6.3(a))

$$P_{rad} = \oint U \, d\Omega = \oint U \sin\theta \, d\theta \, d\phi \quad ...(11.7.4)$$

11.7.2 Directivity from Beam Area (Ω_A)

Now we will express D in terms of beam area. For this purpose we have to assume some expression for U. Let the radiation intensity U is a function of θ and ϕ as

$$U = K_0 F(\theta, \phi) \quad \text{where } K_0 \text{ is a constant.} \quad ...(11.7.5)$$

The directivity according to Equation (11.7.3) is

$$D_0 = \frac{4\pi U_{max}}{P_{rad}} \quad ...(A)$$

So we require to calculate U_{max} and P_{rad}.

To find U_{max}

In the expression for U Equation (11.7.5), since K_0 is a constant, U is maximum when $F(\theta, \phi)$ is maximum.

$$U_{max} = K_0 F(\theta, \phi)\Big|_{max} = K_0 F_{max}(\theta, \phi) \quad ...(B)$$

To find P_{rad}

The total power radiated is obtained by putting value of U in Equation (11.7.4),

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$$P_{rad} = \oint U \sin\theta \, d\theta \, d\phi = \int_0^{2\pi}\int_0^{\pi} K_0 F(\theta,\phi) \cdot \sin\theta \, d\theta \, d\phi$$

$$= K_0 \int_0^{2\pi}\int_0^{\pi} F(\theta,\phi) \cdot \sin\theta \, d\theta \, d\phi \quad ...(C)$$

Using Equation (B) and (C) in Equation (A),

$$D_0 = \frac{4\pi K_0 F_{max}(\theta,\phi)}{K_0 \int_0^{2\pi}\int_0^{\pi} F(\theta,\phi) \cdot \sin\theta \, d\theta \, d\phi}$$

$$= 4\pi \frac{F_{max}(\theta,\phi)}{\int_0^{2\pi}\int_0^{\pi} F(\theta,\phi) \cdot \sin\theta \, d\theta \, d\phi}$$

$$D_0 = \frac{4\pi}{\left[\int_0^{2\pi}\int_0^{\pi} F(\theta,\phi) \cdot \sin\theta \, d\theta \, d\phi\right] \Big/ F_{max}(\theta,\phi)}$$

$$= \frac{4\pi}{\int_0^{2\pi}\int_0^{\pi} \frac{F(\theta,\phi)}{F_{max}(\theta,\phi)} \cdot \sin\theta \, d\theta \, d\phi}$$

Noting that $F(\theta, \phi) / F_{max}(\theta, \phi) = U / U_{max} = P_n(\theta, \phi)$

Here the definition of $P_n(\theta, \phi)$ is used. Thus

$$D_0 = \frac{4\pi}{\int_0^{2\pi}\int_0^{\pi} P_n(\theta,\phi) \cdot \sin\theta \, d\theta \, d\phi}$$

The denominator of this expression is nothing but beam area Ω_A given by,

$$D_0 = \frac{4\pi}{\Omega_A} \quad ...(11.7.6(a))$$

11.7.3 Approximate Expression for D

The above expression gives the exact value of D. But if we know half power beam widths in two principal planes (θ_{HP} and ϕ_{HP}) then approximate value of Ω_A can be obtained using equation,

$$\Omega_A \simeq \theta_{HP}\,\phi_{HP} \text{ (Sr)},$$

which gives $D_0 \simeq \dfrac{4\pi}{\theta_{HP}\,\phi_{HP}}$ ...(11.7.6(b))

Note that θ_{HP} and ϕ_{HP} are expressed in radians. If we have these two beamwidths in degrees then we should convert 4π (Sr) in numerator into degrees using equation,

$$4\pi \text{ (Sr)} = 41253^{\square} \quad \text{Thus} \quad D_0 \simeq \frac{41253^{\square}}{\theta^{\circ}_{HP}\,\phi^{\circ}_{HP}} \quad ...(11.7.6(c))$$

Many times it is desirable to express D in decibels (dB) instead of dimensionless quantities. It is given by

$$D \text{ (dB)} = 10 \log_{10} [D \text{ (dimensionless)}] \quad ...(11.7.6(d))$$

Sometimes you will find D expressed as D (dBi), which means D is calculated by comparing with isotropic (i) source.

Ex. 11.7.1 : The radiation intensity of a unidirectional antenna is $U = U_m \cos\theta \quad 0 \le \theta \le \pi/2, \quad 0 \le \phi \le 2\pi$

Find (i) exact directivity, (ii) approximate directivity.

☑ **Soln. :**

Given : $U = U_m \cos\theta$

(i) To find exact directivity

The maximum radiation intensity is,

$$U_{max} = U_m$$

The power radiated is obtained using Equation (11.7.4) as

$$P_{rad} = \int_0^{2\pi}\int_0^{\pi/2} U \, d\Omega$$

$$= \int_0^{2\pi}\int_0^{\pi/2} (U_m \cos\theta) \sin\theta \, d\theta \, d\phi$$

$$= U_m (\phi)_0^{2\pi} \int_0^{\pi/2} \frac{1}{2} \sin(2\theta) \, d\theta$$

$$= \pi U_m \left(\frac{-\cos 2\theta}{2}\right)_0^{\pi/2} = \pi U_m$$

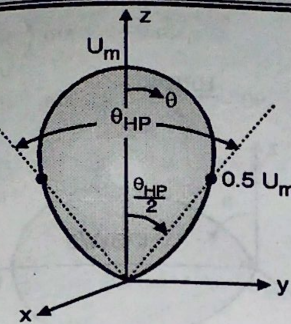
The exact directivity is obtained using Equation (11.7.3)

$$D_0 = 4\pi \frac{U_{max}}{P_{rad}} = 4\pi \frac{U_m}{\pi U_m} = 4$$

(ii) To find approximate D

The approximate value of D is obtained using half power beam widths (θ_{HP} and ϕ_{HP}). The given U is not a function of ϕ, so ϕ_{HP} is equal to θ_{HP}.

The plot of radiation intensity is shown in Fig. Ex. 11.7.1. The radiation intensity is expressed in Watts/Sr, at $\theta_{HP}/2$ the value of U drops to 0.5 U_m.



aw (4.19)**Fig. : Illustrating Ex. 11.7.1**

$$U(\theta_{HP}/2) = U_m \cos \theta_{HP}/2 = 0.5\,U_m$$

or $$\cos(\theta_{HP}/2) = 0.5$$

or $$\theta_{HP} = 2\cos^{-1}(0.5) = 120° = \phi_{HP}$$

The approximate D is given in Equation (11.7.6(c))

$$D_0 = \frac{41253^{\square}}{\theta^{\circ}_{HP}\,\phi^{\circ}_{HP}} = \frac{41253}{120° \times 120°} = 2.865$$

Note the difference in exact and approximate values. Better approximation can be obtained if the patterns have much narrower bandwidths, it is demonstrated in Example 11.7.3.

Ex. 11.7.2 : The radiation intensity of the antenna is, $U = U_m \cos^n \theta$. Prove that the directivity for a source with a unidirectional pattern can be expressed as $D(\theta) = 2(n+1)$. Find the directivity when

(i) $U = U_m \cos\theta$ (ii) $U = U_m \cos^2\theta$

☑ **Soln. :** A unidirection pattern for cosine function means

$$0 \le \theta \le \pi/2 \quad \text{and} \quad 0 \le \phi \le 2\pi$$

The maximum radiation intensity is,

$$U_{max} = U_m \cos^n \theta \Big|_{max} = U_m$$

The power radiated is,

$$P_{rad} = \int_0^{2\pi} \int_0^{\pi/2} U \, d\Omega$$

$$= \int_0^{2\pi} \int_0^{\pi/2} (U_m \cos^n \theta) \sin\theta \, d\theta \, d\phi$$

$$P_{rad} = U_m (\phi)_0^{2\pi} \int_0^{\pi/2} \cos^n \theta \sin\theta \, d\theta$$

To solve integral -

Put $\cos\theta = t$ ∴ $-\sin\theta\, d\theta = dt$ or $\sin\theta\, d\theta = -dt$

limits : $\theta \to 0$ then $t \to 1$

and $\theta \to \pi/2$ then $t \to 0$

$$\therefore \int_0^{\pi/2} \cos^n\theta \sin\theta\, d\theta = \int_1^0 t^n(-dt) = \left[\frac{t^{n+1}}{n+1}\right]_0^1 = \frac{1}{n+1}$$

$$\therefore \quad P_{rad} = (2\pi)\,U_m \left(\frac{1}{n+1}\right)$$

The directivity is given by,

$$D_0 = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi U_m}{2\pi U_m/(n+1)} = 2(n+1)$$

Hence proved.

(i) When $U = U_m \cos\theta$, here $n = 1$ ∴ $D_0 = 2(1+1) = 4$

(ii) When $U = U_m \cos^2\theta$, here $n = 2$ ∴ $D_0 = 2(2+1) = 6$

Compare the result of part (i) with the answer of Ex. 11.7.1. It is exact. So result of Ex. 11.7.2 can be used as a standard result.

For $U = U_m \cos^n \theta$, the directivity is given by

$$D_0 = 2(n+1) \quad ...(11.7.7)$$

Ex. 11.7.3 : The radiation intensity of a unidirectional antenna is $U = U_m \cos^2\theta$ $0 \le \theta \le \pi/2$, $0 \le \phi \le 2\pi$

Find (i) exact directivity,

(ii) approximate directivity.

☑ **Soln. :**

(i) **Exact directivity**

Using result in Equation (11.7.7), for n = 2 the exact directivity is

$$D_0 = 2(n+1) = 2 \times 3 = 6$$

(ii) **Approximate directivity**

The pattern is shown in Fig. Ex. 11.7.3.

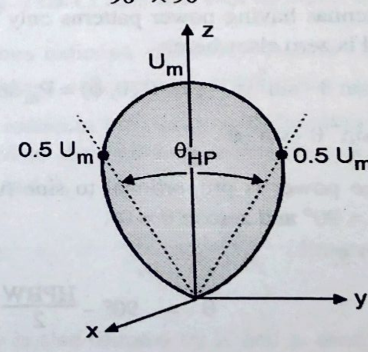
At the angle $\theta = \theta_{HP}/2$, the value of U drops to $0.5\,U_m$.

$$U(\theta_{HP}/2) = U_m \cos^2(\theta_{HP}/2) = 0.5\,U_m$$

or $$\cos^2(\theta_{HP}/2) = 0.5$$

or $$\theta_{HP} = 2\cos^{-1}(\sqrt{0.5}) = 90° = \phi_{HP}$$

Then $$D_0 = \frac{41253^{\square}}{90° \times 90°} = 5.09$$



aw (4.20)**Fig. Illustrating Ex. 11.7.3**

Let us compare now results of Ex. 11.7.1 and 11.7.3.

U =	θ°_{HP} = ϕ°_{HP} =	Exact D_0 =	Approximate D_0 =	Difference in values (%)
$U_m \cos\theta$	120°	4	2.865	28.4%
$U_m \cos^2\theta$	90°	6	5.09	15.2%

From the table we find exact and approximate values are closer when the half power beamwidth is smaller. Thus,

Better approximation is achieved when the pattern have much narrower beam widths.

Ex. 11.7.4 : Calculate the approximate directivity from the half-power beamwidths of an antenna with power pattern given by

(i) $P_n = \cos\theta$, (ii) $P_n = \cos^2\theta$, (iii) $P_n = \cos^3\theta$

In all these cases these patterns are unidirectional (z-direction) with $0° \le \theta \le 90°$ and $P_n = 0$ for $90° \le \theta \le 180°$.

Soln. : The given patterns are power patterns. First we obtain θ_{HP} and ϕ_{HP} by setting $P_n = 0.5$ at these angles.

This procedure is similar to procedure outlined in Ex. 11.7.1 and 11.7.3 with U given. Answers are same for part (i) and (ii).

Part (i) : $D \simeq 2.86$

Part (ii) : $D \simeq 5.09$

Only solution of part (iii) is given below

Part (iii) $P_n = \cos^3\theta$:

$$P_n\left(\frac{\theta_{HP}}{2}\right) = \cos^3\left(\frac{\theta_{HP}}{2}\right) = 0.5$$

$$\cos\left(\frac{\theta_{HP}}{2}\right) = (0.5)^{1/3} = 0.794$$

$$\therefore \quad \theta_{HP} = 2\cos^{-1}(0.794) = 74.93°$$

$$\therefore \quad D \simeq \frac{41253^\square}{(74.93)^2} = 7.34 \text{ (dimensionless)}$$

Ex. 11.7.5 : Calculate the approximate directivity of unidirectional antennas having power patterns only for $0 \le \theta \le \pi$, and $0 \le \phi \le \pi$, and is zero elsewhere.

(i) $P(\theta, \phi) = P_m \sin\theta \cdot \sin^2\phi$ (ii) $P(\theta, \phi) = P_m \sin\theta \cdot \sin^3\phi$

(iii) $P(\theta, \phi) = P_m \sin^2\theta \cdot \sin^3\phi$

Soln. : Since power is proportional to sine function, it has maximum along $\theta = 90°$ and zero at $\theta = 0°$.

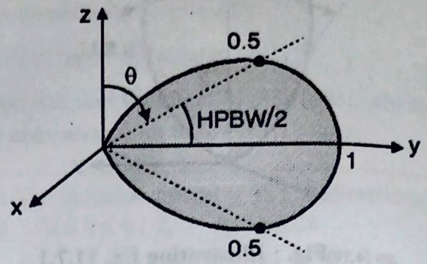
In the Fig. Ex. 11.7.5,

$$\theta = 90° - \frac{HPBW}{2}$$

For $\sin\theta$ pattern,

$$\sin\theta = \sin\left(90 - \frac{HPBW}{2}\right) = 0.5$$

$$\therefore \quad 90 - \frac{HPBW}{2} = \sin^{-1}(0.5) = 30°$$



aw (4.21) **Fig. : Illustrating Ex. 11.7.5**

$$\therefore \quad HPBW = 2(90 - 30) = 120° = \theta_{HP}$$

For $\sin^2\theta$ pattern,

$$\sin^2\theta = \sin^2\left(90 - \frac{HPBW}{2}\right) = 0.5$$

$$\therefore \quad \sin\left(90 - \frac{HPBW}{2}\right) = \sqrt{0.5} = 0.707$$

$$\therefore \quad 90 - \frac{HPBW}{2} = \sin^{-1}(0.707) = 45°$$

$$\therefore \quad HPBW = 2(90 - 45) = 90° = \theta_{HP}$$

For $\sin^3\theta$ pattern,

$$\sin^3\theta = \sin^3\left(90 - \frac{HPBW}{2}\right) = 0.5$$

or $$\sin\left(90 - \frac{HPBW}{2}\right) = \sqrt[3]{0.5} = 0.794$$

or $$90 - \frac{HPBW}{2} = \sin^{-1}(0.794) = 52.56°$$

or $$HPBW = 2(90 - 52.56) = 74.88°$$

$$\therefore \quad \theta_{HP} = 74.88°$$

Similarly for $\sin^2\phi$ and $\sin^3\phi$ functions we get ϕ_{HP} equal to 90° and 74.88° respectively.

Now we can find approximate directivities as,

(i) For $P(\theta, \phi) = P_m \sin\theta \cdot \sin^2\phi$

$$D \simeq \frac{41253}{\theta_{HP}\,\phi_{HP}} = \frac{41253}{(120)(90)} = 3.70$$

(ii) For $P(\theta, \phi) = P_m \sin\theta \cdot \sin^3\phi$

$$D \simeq \frac{41253}{\theta_{HP} \cdot \phi_{HP}} = \frac{41253}{(120)(74.88)} = 4.59$$

(iii) For $P(\theta, \phi) = P_m \sin^2\theta \cdot \sin^3\phi$

$$D \simeq \frac{41253}{\theta_{HP} \cdot \phi_{HP}} = \frac{41253}{(90)(74.88)} = 6.12$$

UEx. 11.7.6 MU - Dec. 12, 10 Marks

Normalised radiation intensity of an antenna is given by :

$U = \sin\theta \sin\phi \qquad 0 \le \theta \le \pi$ and $0 \le \phi \le \pi$

$= 0 \qquad$ elsewhere

Find exact directivity, azimuth plane HPBW and elevation plane HPBW.

Soln. :

$$U = \sin\theta \sin\phi \quad \therefore U_{max} = 1$$

$$P_{rad} = \int_0^{\pi}\int_0^{\pi} U \sin\theta \, d\theta \, d\phi$$

$$= \int_0^{\pi}\int_0^{\pi} \sin^2\theta \sin\phi \, d\theta \, d\phi$$

$$= (\phi)_0^{\pi} \int_0^{\pi} \sin^2\theta \, d\theta = \pi$$

$$\therefore D_0 = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi(1)}{\pi} = 4$$

$$(D_0)_{dB} = 10 \log(4) = 6.02 \text{ (dB)}$$

$$\text{HPBW (az.)} = 2\left[90° - \sin^{-1}(1/2)\right]$$

$$= 2(90° - 30°) = 120°$$

$$\text{HPBW (el.)} = 2\left[90° - \sin^{-1}(1/2)\right] = 120°$$

11.7.4 Properties of D

UQ. Show that the directivity of an isotropic antenna is unity. **(MU - May 15, 2 Marks)**

From the previous discussion on the directivity, some important points regarding it are :

(i) The directivity of an isotropic source is unity since the power is radiated equally well in all directions. It is independent of θ and ϕ. Thus $U = U_0$ giving

$$D = \frac{U}{U_0} = \frac{U_0}{U_0} = 1$$

$$D = 1 \;:\; \text{for isotropic source}$$

(ii) For all other sources, the maximum directivity will always be greater than 1. This is due to the fact that all antennas (except isotropic) radiates more in one particular direction, giving $U_{max} > U_0$.

$$D > 1 \;:\; \text{for non isotropic source}$$

(iii) From the expression for D [Equation 11.7.6(a)],

$$D = \frac{4\pi}{\Omega_A}$$

It is clear that, smaller the beam area, the larger the directivity D.

(iv) The beam area is approximately given by

$$\Omega_A \approx \theta_{HP} \phi_{HP}$$

Thus D is large when the half power beam width is small.

(v) The directivity provides a relative "figure-of-merit" which indicates the directional properties of antenna as compared with isotropic source.

For example if the directivity of a source is 2, it means it is two times more intense than that of isotropic radiator with the same radiated power.

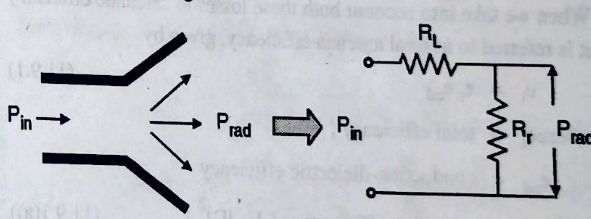
11.8 ANTENNA RADIATION EFFICIENCY (E_{CD} OR K)

While calculating directivity D we used power radiated by the antenna. When some power (P_{in}) is supplied to the antenna, all of this power is not radiated. Some power is wasted as a loss. There are two types of losses

(i) Conduction loss
(ii) Dielectric loss
} (I^2R losses)

These two losses of an antenna are very difficult to calculate and so they are actually measured. Even after measurement they are difficult to separate and they are lumped together to represent the loss resistance (R_L).

The power which is radiated is suppose to be due to radiation resistance (R_r). The radiation resistance is studied in more detail in next chapters. The antenna with P_{in} and P_{rad} and its equivalent circuit is shown in Fig. 11.8.1.



aw (4.22)**Fig. 11.8.1 : Antenna and its equivalent circuit**

The antenna radiation efficiency, denoted by e_{cd} (called as conduction-dielectric efficiency) is defined as -

Antenna radiation efficiency is defined as the ratio of the power delivered to the radiation resistance (R_r) to the power delivered to R_r and R_L.

$$\text{Mathematically : } e_{cd} = \frac{P_{rad}}{P_{in}} = \frac{R_r}{R_r + R_L} \text{ (dimensionless)} \quad ...(11.8.1)$$

This efficiency is also denoted by K and is used to relate the gain and directivity.

Module 3

For practical antenna always some losses are present, then the term $R_r + R_L > R_r$ making K less than 1.

For ideal antenna with no loss ($R_L = 0$), the efficiency K is equal to 1.

For practical antenna : K < 1

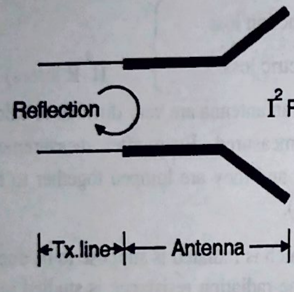
For ideal antenna : K = 1 ...(11.8.2)

11.9 ANTENNA EFFICIENCY (E_T)

The radiation efficiency (e_{cd}) just now we have seen takes into account the losses within the structure of the antenna. There is also some loss at the input terminals of the antenna. This loss is due to reflections at the input due to mismatch between the transmission line and the antenna impedance. Thus when some power is supplied to the antenna, part of it is

(i) reflected

(ii) and the remaining is lost due to $I^2 R$ effect (conduction and dielectric)



aw(4.23)Fig. 11.9.1 : To find total efficiency

When we take into account both these losses to calculate efficiency it is referred to as total antenna efficiency, given by

$$e_t = e_r e_{cd} \quad ...(11.9.1)$$

where e_t = total efficiency ;

e_{cd} = conduction-dielectric efficiency

e_r = reflection efficiency = $(1 - |\Gamma|^2)$ ...(11.9.1(a))

Γ = voltage reflection coefficient = $\dfrac{Z_{in} - Z_0}{Z_{in} + Z_0}$ ...(11.9.1(b))

Z_{in} = antenna input impedance

Z_0 = characteristic impedance of the transmission line

By introducing value of e_r in Equation (11.9.1) we get

$$e_t = e_{cd}(1 - |\Gamma|^2) \quad ...(11.9.2)$$

If the antenna is perfectly matched, the reflection coefficient is zero, giving,

$$e_t = e_{cd} \quad ...(11.9.3)$$

Ex. 11.9.1 : The antenna is supplied with a power of 10 Watts. Calculate the power radiated when the efficiency of the antenna is 90%.

Soln. : Using Equation (11.8.1) we relate P_{rad} and P_{in}, which is used to calculate P_{rad}.

$$K \text{ or } e_{cd} = \frac{P_{rad}}{P_{in}}$$

$$\therefore \quad P_{rad} = K \times P_{in} = 0.9 \times 10 = 9 \text{ (Watts)}$$

11.10 GAIN OF THE ANTENNA (G) (OR DIRECTIVE GAIN)

11.10.1 Definition

The gain(G) of an antenna is an actual or realized quantity which is less than the directivity (D) due to ohmic losses in the antenna.

11.10.2 Relation between G and D

To understand the difference in gain and directivity the expression for directivity is repeated here.

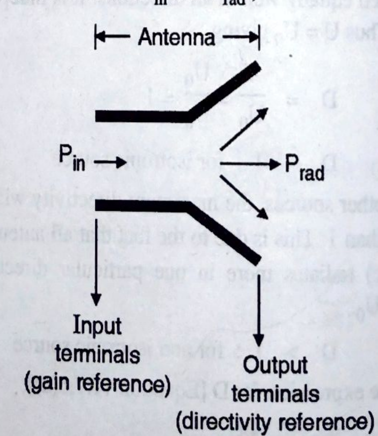
$$D = \frac{4\pi U}{P_{rad}} \quad ...(11.10.1)$$

Here we used output of the antenna i.e. P_{rad} as a reference. Instead if antenna input is used as reference, we get the expression for gain of the antenna. These two references with respect to antenna terminals is shown in Fig. 11.10.1.

The gain of the antenna is denoted by G, and using P_{in} instead of P_{rad} in the expression of D we get

$$G = \frac{4\pi U}{P_{in}} \quad ...(11.10.2(a))$$

Using the relation between P_{in} and P_{rad},



aw (4.24)Fig. 11.10.1 : Antenna reference terminals

$$P_{in} = \frac{P_{rad}}{e_{cd}\,(=K)} \quad \text{Thus,} \quad G = \frac{4\pi U}{P_{rad}/K} = K\frac{4\pi U}{P_{rad}}$$

or $\quad G = KD \quad$ ...(11.10.2(b))

If we consider maximum D i.e. D_0 then G is also maximum,

$$G_0 = K D_0 \quad \text{...(11.10.3)}$$

Here K or e_{cd} will take care of only conductor-dielectric loss.

When the reflection loss is considered K or e_{cd} should be replaced by e_r.

$$G_0 = e_t D_0 = e_r e_{cd} D_0 = e_{cd}(1 - |\Gamma|^2) D_0 \quad \text{...(11.10.3(a))}$$

Gain can be measured by comparing the radiation intensity of the antenna under test (AUT) with a reference antenna. The reference antenna is usually a dipole, horn or any other antenna whose gain can be calculated or is known.

$$G = \frac{U\,(AUT)}{U\,(\text{reference antenna})} \quad \text{...(11.10.4)}$$

In most of the cases, the reference antenna is a lossless isotropic antenna.

Thus $\quad G = \frac{U\,(AUT)}{U_0} = \frac{U\,(AUT)}{P_{in}/4\pi}$

or $\quad G = \frac{4\pi U\,(AUT)}{P_{in}\,(\text{lossless isotropic source})} \quad$ ...(11.10.5)

In practice, whenever the term 'gain' is used, it usually refers to the maximum gain as defined by Equation (11.10.3).

Usually the gain is expressed in terms of decibels instead of dimensionless quantity. The conversion formula is

$$G_0\,(dB) = 10 \log [\, G_0 \,(\text{dimensionless})\,] \quad \text{...(11.10.6)}$$

11.10.3 Comparison between G and D

Though the gain of the antenna is closely related to the directivity, the term gain takes into account the efficiency (K) of the antenna as well as directional capabilities (decided by P_{rad}).

But the directivity is described only in terms of directional properties, and it is therefore controlled only by the pattern.

Since the value of K lies between 0 and 1, we find the gain is less than or equal to D.

$$G \le D$$

Summary

For ideal antenna with no reflection loss,

$$G = D$$

For lossy antenna with no reflection loss,

$$G = KD = e_{cd} D$$

For lossy antenna with reflection loss,

$$G = e_t D = e_r e_{cd} D = e_{cd}(1 - |\Gamma|^2) D$$

UEx. 11.10.1 MU - May 10. 3 Marks

An antenna has a directivity of 20 and a radiation efficiency of 90%. Compute the gain in dBs.

☑ **Soln. :**

Given : $D = 20, K = 90\% = 0.9$

Using equation $\quad G = KD = 0.9 \times 20 = 18$

$$G(dB) = 10 \log G\,(\text{dimensionless}) = 10 \log (18)$$

Ex. 11.10.2 : The radiation resistance of an antenna is 72 Ω and the loss resistance is 8 Ω. What is the directivity in dB if the power gain is 16 ?

☑ **Soln. :**

Given : $R_r = 72\,\Omega, \quad R_L = 8\,\Omega, \quad G = 16$

The antenna radiation efficiency is given by

$$K = e_{cd} = \frac{R_r}{R_r + R_L} = \frac{72}{72 + 8} = 0.9 \rightarrow 90\%$$

The gain and directivity are related using

$$G = KD \quad \rightarrow \quad D = \frac{G}{K} = \frac{16}{0.9} = 17.77$$

In dBsD (dB) = 10 log (D) = 10 log (17.77) = 12.497 (dB)

Ex. 11.10.3 : An antenna has a loss resistance of 10 Ω, power gain of 20 and directivity 22, calculate the radiation resistance.

☑ **Soln. : Given :** $R_L = 10\,\Omega, \quad G = 20, \quad D = 22$

The gain and directivity are related using equation

$$G = KD \quad \rightarrow \quad K = \frac{G}{D} = \frac{20}{22} = 0.909$$

The radiation of efficiency is given by equation

$$K = \frac{R_r}{R_r + R_L} = \frac{1}{1 + (R_L / R_r)}$$

$$\therefore \quad 0.909 = \frac{1}{1 + (R_L / R_r)} \rightarrow \frac{R_L}{R_r} = \frac{1}{0.909} - 1 = 0.1$$

$$\therefore \quad R_r = \frac{R_L}{0.1} = \frac{10}{0.1} = 100\,\Omega$$

Ex. 11.10.4 (a) Calculate the directivity of an antenna with $\theta_{HP} = 2°, \phi_{HP} = 1°$, and

(b) Find the gain of this antenna if efficiency K = 0.5.

☑ **Soln. :**

(a) Using half power beamwidths we calculate directivity (approximate) by,

$$D_0 \simeq \frac{41253^{\square}}{\theta^{\circ}_{HP}\,\phi^{\circ}_{HP}} = \frac{41253}{(2)\,(1)} = 20.63 \times 10^3 \text{ (dimensionless)}$$

Module 3

In decibels directivity is

$$D_0\,(\text{dB}) = 10 \log D_0 = 10 \log (20.63 \times 10^3) = 43.14\ (\text{dB})$$

(b) The gain is related with D_0 as

$$G_0 = K D_0 = (0.5) \times 20.63 \times 10^3 = 10.31 \times 10^3$$

(dimensionless)

$$\text{In dBs, } G_0\,(\text{dB}) = 10 \log (G_0) = 10 \log (10.31 \times 10^3) = 40.13\ (\text{dB})$$

11.11 ANTENNA APERTURES

With each antenna, we can associate a number of equivalent areas (apertures). These are used to describe the power capturing characteristics of the antenna when a wave impinges on it. These areas or apertures are distinguished as :

(i) Physical aperture (ii) Effective aperture
(iii) Scattering aperture (iv) Loss aperture
(v) Collecting aperture

Let us first understand the concept of aperture with the help of physical aperture.

11.11.1 Concept of Aperture

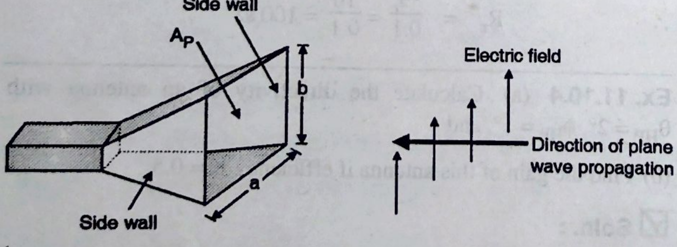
To understand the aperture concept consider a receiving antenna which is a horn antenna. Suppose this antenna is immersed in the field of a uniform plane wave as shown in Fig. 11.11.1.

As shown in Fig. 11.11.1 the horn is a rectangular with dimensions a and b. The area of opening called as **physical aperture (A_P)** given by

$$A_P = a \times b$$

Let the Poynting vector or power density of the plane incident wave be W_i (Watts / m^2). If the horn extracts all the power from the wave over its entire physical aperture, then the total power P absorbed from the wave is

$$P = W_i A_P \text{ (Watt)} \quad \text{...(11.11.1)}$$



aw (4.27)**Fig. 11.11.1 : To understand aperture concept**

But the problem is if we measure this power, it is less than the calculated one. The reason for this is the field response of the horn is not uniform over the opening. The side walls which are good conductor, the field E at the wall positions must be zero. Thus the opening of the horn is not having area A_P, it is slightly less than it. Why less because, in Equation (11.11.1) the power P is less when the area on the right side must be less than A_P. This area is called as **effective aperture** denoted by A_e. The ratio of A_e by A_P is called as **aperture efficiency (ϵ_{ap})**.

$$\epsilon_{ap} = \frac{A_e}{A_P} \text{ (dimensionless)} \quad \text{...(11.11.2)}$$

Ex. 11.11.1 : Calculate the radiation resistance of an antenna which is drawing 15 A current and radiating 5 kW.

Soln. :

Given : $I_{rms} = 15$ A, $P_{rad} = 5$ kW

In terms of rms current

$$P_{rad} = I_{rms}^2 \times R_r$$

$$\therefore R_r = \frac{P_{rad}}{I_{rms}^2} = \frac{5 \times 10^3}{15^2} = 22.22\ (\Omega)$$

Ex. 11.11.2 : How much current does an antenna draw when radiating 1000 Watts and is having a radiation resistance of 300 Ω.

Soln. :

Given : $P_{rad} = 100$ Watts, $R_r = 300\ \Omega$

$$P_{rad} = I_{rms}^2 \times R_r$$

$$\therefore I_{rms} = \sqrt{\frac{P_{rad}}{R_r}} = \sqrt{\frac{1000}{300}} = 1.8 \text{ Amp.}$$

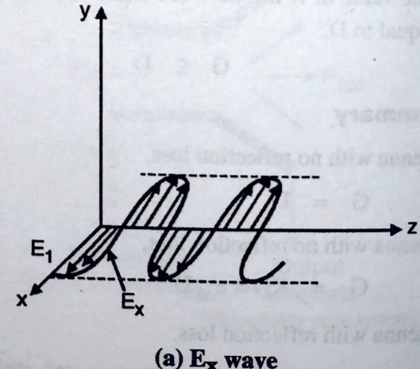
11.12 POLARIZATION

11.12.1 Polarization of Wave

UQ. Explain Polarization of antenna.
(MU - Q. 1(a), Dec. 19, 5 Marks)

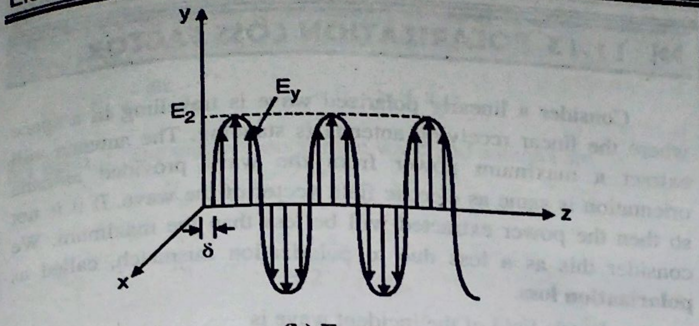
Definition

Polarization of a wave refers to the time varying behaviour of the electric field strength vector at some fixed point in space.



(a) E_x wave

aw (4.31)**Fig. 11.12.1**



(b) E_y wave

aw (4.31)Fig. 11.12.1

Let us consider the wave travelling in + z direction and hence it has E_x and E_y components as

$$E_x = E_1 \cos(\omega t - Kz)$$

$$E_y = E_2 \cos(\omega t - Kz - \delta)$$

Where, E_1 = amplitude of wave in x direction and

E_2 = amplitude of wave in y direction

δ = Phase angle between E_x and E_y

Now we can explain polarization. There are three types of polarization.

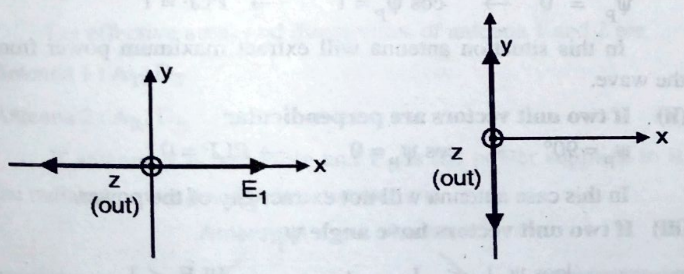
1) **Linear Polarization**

To explain this polarization we consider different cases as discussed below.

(i) Consider for a wave only E_x component is present and $E_y = 0$. Then the total electric field is consisting of only E_x component given by

i.e. $$\bar{E} = E_1 \cos(\omega t - Kz)\, a_x$$

The variation of wave is shown in Fig. 11.12.2(a), which is obtained by observing the Fig. 11.12.1(a) along the direction of propagation that is from left. This variation clearly shows that E is changing along a horizontal straight line. In this case the wave is called as linearly polarized in x direction.



(a) (b)

aw (4.32)Fig. 11.12.2

(ii) Let E has only E_y component and $E_x = 0$. Then the total electric field is

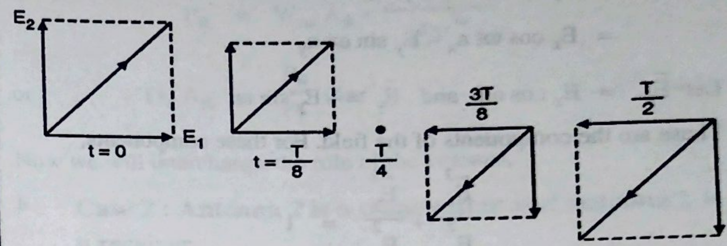
$$\bar{E} = E_2 \cos(\omega t - Kz - \delta)\, \bar{a}_y$$

The variation of electric field is shown in Fig. 11.12.2(b). Here again the variation of E is along a vertical straight line. The wave is called as linearly polarized in y direction.

(iii) Now consider both E_x and E_y are present and are in phase.

E_x and E_y are said to be in phase when they start at same time, they go to maximum at the same time and so on. The resultant electric field has a direction dependent on the relative magnitude of E_x and E_y.

The angle which this direction makes with the x axis is $\tan^{-1}(E_y / E_x)$ and this angle is always constant with time. This is shown in Fig. 11.12.3.



aw (4.33)Fig. 11.12.3 : Linear polarization

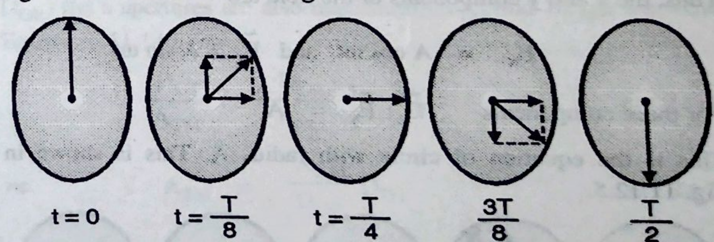
In all the above cases i.e. (i), (ii) and (iii), the direction of the resultant vector is constant with time and the wave is said to be linearly polarized.

2) **Elliptical Polarization**

In elliptical polarization E_x and E_y are not in phase i.e. they reach their maximum values at different instants of time. Because of this the direction of the resultant electric vector will vary with time. Consider

$$E_1 \neq E_2 \text{ and } \delta = 90^\circ$$

The phase difference of 90° indicates that when E_y is at its peak value E_2 then E_x is at its zero value. As E_y starts decreasing then E_x starts increasing. When E_y goes to zero then E_x reaches its maximum value E_1 and so on. This variation is shown in Fig. 11.12.4 at different timing.



aw (4.34)Fig. 11.12.4 : Elliptical polarization

In this case if you join tips of the phasors, the locus of the end point of the resultant E will be an ellipse and the wave is said to be **elliptical polarized**. This can be proved mathematically also as shown below.

Module 3

The electric field is a complex vector and is given by

$$E_0 = E_x \bar{a}_x + j E_y \bar{a}_y \; ...(\text{at } z = 0)$$

The j term indicates that y component leads the x component by 90° and E_x and E_y are positive real constants. The harmonic variation is obtained by multiplying the expression by $e^{j\omega t}$ and taking real or imaginary part of it.

$$\tilde{E}(o, t) = \text{Re}(E_x \bar{a}_x + j E_y \bar{a}_y) e^{j\omega t}$$

$$= \text{Re}\{(E_x \bar{a}_x + j E_y \bar{a}_y)(\cos \omega t + j \sin \omega t)\}$$

$$= E_x \cos \omega t \, \bar{a}_x - E_y \sin \omega t \, \bar{a}_y$$

Let $\tilde{E}_x = E_x \cos \omega t$ and $\tilde{E}_y = -E_y \sin \omega t$

These are the components of the field. For these components,

$$\frac{\tilde{E}_x^2}{E_x^2} + \frac{\tilde{E}_y^2}{E_y^2} = 1$$

This is the equation of ellipse. Thus really the locus of the end points of the resultant phasors is an ellipse.

3) Circular Polarization

The elliptical polarization can become circular polarization by applying the condition that

$$E_1 = E_2 \text{ and } \delta = 90°$$

If the y component leads the x component by 90° and if both components have amplitude A, then electric field is

$$E_0 = (\bar{a}_x + j \bar{a}_y) A \; ...(\text{at } z = 0)$$

Now the time varying field is

$$\tilde{E}(0,t) = \text{Re}\{(A \bar{a}_x + j A \bar{a}_y) e^{j\omega t}\}$$

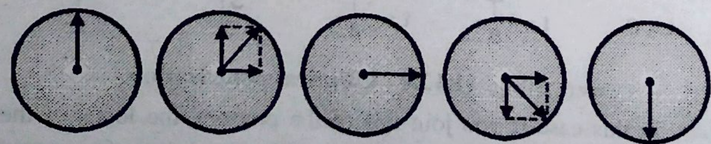
$$= (\bar{a}_x \cos \omega t - \bar{a}_y \sin \omega t) A$$

Thus, the x and y components of the field are

$$\tilde{E}_x = A \cos \omega t \text{ and } \tilde{E}_y = A \sin \omega t$$

For these components $\quad \tilde{E}_x^2 + \tilde{E}_y^2 = A^2$

This is the equation of circle with radius A. This is shown in Fig. 11.12.5.



aw (4.35) **Fig. 11.12.5 : Circular polarization**

11.13 POLARIZATION LOSS FACTOR

Consider a linearly polarized wave is travelling in a space where the linear receiving antenna is standing. The antenna will extract a maximum power from the wave provided antenna orientation is same as electric field vector of the wave. If it is not so then the power extracted will be less than the maximum. We consider this as a loss due to polarization mismatch, called as **polarization loss.**

Let the electric field of the incident wave is

$$\bar{E}_i = E_i \bar{a}_w \quad ...(11.13.1)$$

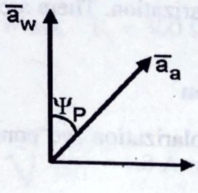
where $\bar{a}_w$ = unit vector of the electric field of the wave also called as unit vector of the wave.

Consider the polarization of the electric field of the receiving antenna is

$$E_a = E_a \bar{a}_a \quad ...(11.13.2)$$

where $\bar{a}_a$ = unit vector of the electric field of the antenna.

These two unit vectors are shown in Fig. 11.13.1.



aw (4.38) **Fig. 11.13.1 : Showing unit vectors $\bar{a}_w$ and $\bar{a}_a$**

The loss due to polarization mismatch is expressed in terms of polarization loss factor (PLF), given by

$$\text{PLF} = |\bar{a}_w \cdot \bar{a}_a|^2 = |\cos \psi_p|^2 \quad ...(11.13.3)$$

Where ψ_p = angle between two unit vectors.

From Equation (11.13.3) we note following points :

(i) If two unit vectors are perfectly aligned

$$\psi_P = 0 \rightarrow \cos \psi_P = 1 \rightarrow \text{PLF} = 1$$

In this situation antenna will extract maximum power from the wave.

(ii) If two unit vectors are perpendicular

$$\psi_P = 90° \rightarrow \cos \psi_P = 0 \rightarrow \text{PLF} = 0$$

In this case antenna will not extract any of the power.

(iii) If two unit vectors have angle ψ_P

$$|\cos \psi_P| < 1 \rightarrow \text{PLF} < 1$$

It gives power extracted less than maximum possible power. The PLF in Equation (11.13.3) is a dimensionless quantity.

In decibels it is expressed as

$$\text{PLF(dB)} = 10 \log \text{PLF (dimensionless)}.$$

Ex. 11.13.1 : Find the polarization loss factor in dBs, when the electric field of the wave and antenna are expressed as

$$\bar{E}_i = E_0 e^{-j\beta Z} \bar{a}_x \qquad \bar{E}_a = E_1 (\bar{a}_x + \bar{a}_y)$$

☑ **Soln. :** The unit vectors for the wave and antenna are

$$\bar{a}_w = \bar{a}_x$$

$$\bar{a}_a = \frac{\bar{a}_x + \bar{a}_y}{\sqrt{2}}$$

$$\cos \psi_p = \bar{a}_w \cdot \bar{a}_a = \bar{a}_x \cdot \left(\frac{\bar{a}_x + \bar{a}_y}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}$$

$$\text{PLF} = |\bar{a}_w \cdot \bar{a}_a|^2 = |\cos \psi_p|^2 = \frac{1}{2}$$

(dimensionless)

$$\text{PLF (dB)} = 10 \log \text{PLF (dimensionless)}$$

$$= 10 \log \left(\frac{1}{2}\right) = -3$$

11.14 RELATION BETWEEN MAXIMUM DIRECTIVITY AND MAXIMUM EFFECTIVE AREA

Consider a radio communication link as shown in Fig. 11.14.1. The following derivation we will divide into two parts considering one antenna is a transmitter while other is receiver at a time.

While deriving the result we use following assumptions :

(i) Antennas are lossless ($e_{cd} = 1$)

(ii) Antennas are perfectly matched so that there is no reflection loss ($e_r = 1$)

(iii) Polarization loss factor is unity i.e. two antennas are aligned properly.

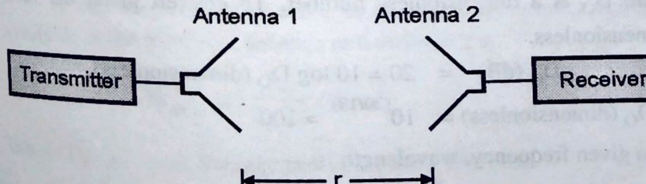
▶ **Case 1 : Antenna 1 is transmitter and antenna 2 is a receiver**

Let effective areas and directivities of antenna 1 and 2 are

Antenna 1 : A_T, D_T

Antenna 2 : A_R, D_R

If antenna 1 is **isotropic** and P_T is the power supplied to it, the radiated power density at a distance r is



aw (4.39)**Fig. 11.14.1 : Communication link**

$$W_{av} = \frac{P_T}{4\pi r^2} \text{ (W / m}^2\text{)} \qquad \text{...(11.14.1)}$$

The antenna 1 has a directivity of D_T, the actual power density is modified as

$$W_{av} = \frac{P_T D_T}{4\pi r^2} \text{ (W / m}^2\text{)} \qquad \text{...(11.14.2)}$$

This power density when multiplied by aperture of antenna 2. We get power collected and transferred to the receiver.

$$P_R = W_{av} A_R = \frac{P_T D_T A_R}{4\pi r^2}$$

or

$$D_T A_R = \frac{P_R}{P_T} (4\pi r^2) \qquad \text{...(11.14.3)}$$

Now we will interchange the role of the antenna.

▶ **Case 2 : Antenna 2 is a transmitter and antenna 1 is a receiver**

As in case 1 analysis we will arrive to the following expression

$$D_R A_T = \frac{P_R}{P_T} (4\pi r^2) \qquad \text{...(11.14.4)}$$

Comparing Equations (11.14.3) and (11.14.4), since right hand sides are equal then left hand sides must be equal, that is

$$D_T A_R = D_R A_T \quad \text{or} \quad \frac{D_T}{A_T} = \frac{D_R}{A_R} \qquad \text{...(11.14.5)}$$

Since the type of the antennas is mentioned while arriving to this expression it must be true for any type of antenna.

Or this ratio is a universal constant.

When the directivity of an antenna is increased its effective area will also increase in direct proportion, so that the ratio is a constant.

So when the directivities are maximum (denoted by D_{OT} and D_{OR}) the a apertures are also maximum (A_{TM} and A_{RM}), we write Equation (11.14.5) as

$$\frac{D_{OT}}{A_{TM}} = \frac{D_{OR}}{A_{RM}} \qquad \text{...(11.14.6)}$$

or

$$A_{TM} = \frac{A_{RM}}{D_{OR}} D_{OT} \qquad \text{...(11.14.7)}$$

To find the value consider any antenna for which aperture and directivity are known.

For a very short dipole acting as a receiver the effective aperture is

$$A_{RM} = \frac{3}{8\pi} \lambda^2 \qquad \text{...(11.14.8)}$$

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For the short dipole the directivity is also standard given by (derived in the next chapters).

$$D_{OR} = \frac{3}{2} \quad ...(11.14.9)$$

Using these values in Equation (11.14.7)

$$A_{TM} = \frac{A_{RM}}{D_{OR}} D_{OT} = \frac{(3/8\pi)\lambda^2}{(3/2)} D_{OT}$$

or $$A_{TM} = \frac{\lambda^2}{4\pi} D_{OT} \quad ...(11.14.10)$$

This is true for any antenna, so we write the general expression as

$$A_{em} = \frac{\lambda^2}{4\pi} D_O \quad ...(11.14.11)$$

The result in Equation (11.14.11) is valid only for ideal situation when there is no antenna loss, no reflection due to mismatch, no polarization loss. Now let us consider these losses to modify the expression for A_{em}.

For lossy antenna

The maximum aperture will get reduced by a antenna loss factor e_{cd} as

$$A_{em} = e_{cd}\left(\frac{\lambda^2}{4\pi}\right) D_O \quad ...(11.14.12)$$

For lossy antenna and impedance mismatch

For this case we replace e_{cd} by total antenna efficiency (e_t)

$$A_{em} = e_t\left(\frac{\lambda^2}{4\pi}\right) D_O \quad ...(11.14.13(a))$$

$$= e_{cd}(1-|\Gamma|^2) D_O \quad ...(11.14.13(b))$$

For lossy antenna with impedance mismatch and polarization loss

In addition to total efficiency, the loss factor is also introduced

$$A_{em} = e_{cd}(1-|\Gamma|^2)\left(\frac{\lambda^2}{4\pi}\right) D_O |\bar{a}_w \cdot \bar{a}_a|^2 \quad ...(11.14.14)$$

Equation (11.14.14) will take care of all losses present.

In these expressions the terms are having following meaning :

A_{em} = maximum effective aperture

e_{cd} = antenna radiation efficiency

e_t = total efficiency

Γ = voltage reflection coefficient

λ = wavelength of a wave incident

D_O = maximum directivity

$\bar{a}_w$ = unit vector in the direction of electric field of the wave

$\bar{a}_a$ = unit vector in the direction of antenna axis.

Summary

For antenna with no losses (ideal antenna)

$$A_{em} = \frac{\lambda^2}{4\pi} D_O \quad ...(11.14.15)$$

For antenna with all losses

$$A_{em} = e_{cd}(1-|\Gamma|^2)\left(\frac{\lambda^2}{4\pi}\right) D_O |\bar{a}_w \cdot \bar{a}_a|^2 \quad ...(11.14.16)$$

When no reflection

$$|\Gamma| = 0$$

When no polarization loss

$$\bar{a}_w \cdot \bar{a}_a = 1$$

When antenna is lossless

$$e_{cd} = 1$$

Putting these values in Equation (11.14.16) will reduce to Equation (11.14.15).

Obviously value of A_{em} in Equation (11.14.16) is less than in Equation (11.14.15).

If the power density of the incident wave (W_i) is know then power received is obtained by multiplying A_{em} with W_i. The power received is thus

$$P_r = A_{em} \times W_i \text{ (watts)} \quad ...(11.14.17)$$

This power is definitely maximum when there is no loss in the system.

Ex. 11.14.1 : Consider a lossless horn antenna with directivity of 20 dB. At a frequency of 10 GHz calculate

(i) the maximum effective aperture

(ii) the maximum power received when incident power density is 2×10^{-3} (W/m^2).

☑ **Soln. :**

(i) The antenna is lossless and no reflection is mentioned we write using Equation (11.14.11)

$$A_{em} = \frac{\lambda^2}{4\pi} D_O$$

Here D_O is a dimensionless number. To convert given dB into dimensionless.

$$D_O \text{ (dB)} = 20 = 10 \log D_O \text{ (dimensionless)}$$

$$\therefore D_O \text{ (dimensionless)} = 10^{(20/10)} = 100$$

At a given frequency, wavelength is

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{10 \times 10^9} = 3 \times 10^{-2} \text{ (m)}$$

Putting the values in the expression for A_{em}

$$A_{em} = \frac{(3 \times 10^{-2})^2}{4\pi} \times 100 = 7.162 \times 10^{-3} \ (m^2)$$

(ii) Using Equation (11.14.17), the maximum power received when there is no loss, given by

$$P_r = A_{em} \times W_i = (7.162 \times 10^{-3}) \times (2 \times 10^{-3})$$

$$P_r = 14.324 \ (\mu W)$$

UEx. 11.14.2 MU - Dec. 16, May 17, May 18, 5 Marks.

Find the gain of an Antenna when physical aperature is 5 m^2 at 2 GHz with efficiency 70%.

☑ Soln. :

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{2 \times 10^9} = 1.5 \times 10^{-1} = 0.15 \ (m)$$

$$A = \frac{\lambda^2}{4\pi} D$$

$$\therefore D = \frac{4\pi}{\lambda^2} A = \frac{4\pi}{(0.15)^2} \times 5 = 2792.53$$

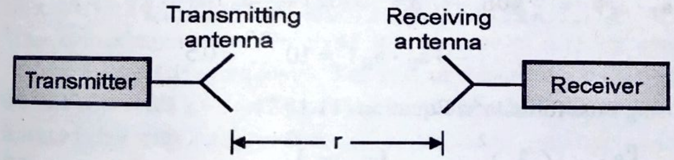
$$\therefore G = k D = 0.7 \times 2792.53 = 1954.8$$

11.15 FRISS TRANSMISSION FORMULA

UQ. Derive Friss transmission formula. State its significance in wireless communication. **(MU - May 15, Dec. 16, May 17, Dec. 17, May 18, 5 Marks)**

UQ. Derive the expression for FRIIS transmission equation. **(MU - Q. 1(e), Dec. 19, 5 Marks)**

Consider a simple transmit-receive communication link as shown in Fig. 11.15.1. The transmitting antenna transmits power P_T. In this section we are interested in calculating how much power of this receiving antenna can pick up.



aw (4.40)**Fig. 11.15.1 : A radio communication link**

If the transmitting antenna is isotropic, the power density available at the receiving antenna at a distance r is

$$W_{av} = \frac{P_T}{4\pi r^2} \ (W/m^2)$$

Where P_T = time average radiated power from the transmitting antenna. ...(11.15.1)

For a transmitting antenna that is not isotropic but has gain G_T, and is pointed for maximum power density in the receiver direction, the power per unit area available at the receiving antenna will be increased in proportion as,

$$W_{av} = \frac{P_T G_T}{4\pi r^2} \ (W/m^2) \quad ...(11.15.2)$$

Now the power collected by the lossless, matched receiving antenna of effective aperture A_{eR} is

$$P_R = W_{av} A_{eR} = \frac{P_T G_T A_{eR}}{4\pi r^2} \ (W) \quad ...(11.15.3)$$

The gain of the transmitting antenna can be expressed as

$$G_T = \frac{4\pi}{\lambda^2} A_{eT} \quad ...(11.15.4)$$

Putting this in Equation (11.15.3)

$$P_R = P_T \frac{A_{eT} A_{eR}}{r^2 \lambda^2} \quad \text{or} \quad \frac{P_R}{P_T} = \frac{A_{eT} A_{eR}}{r^2 \lambda^2} \quad ...(11.15.5)$$

The relation in Equation (11.15.5) is called as **Friss transmission formula.**

Here it is assumed that there is no loss present in the system. If all losses are present then each A_e should be replaced by using equation

$$A_e = e_{cd} (1 - |\Gamma|^2) \left(\frac{\lambda^2}{4\pi}\right) D_o |\bar{a}_w \cdot \bar{a}_a|^2$$

For all losses present

$$\frac{P_R}{P_T} = e_{cdT} e_{cdR} (1 - |\Gamma_T|^2)(1 - |\Gamma_R|^2) \left(\frac{\lambda}{4\pi r}\right)^2 D_T D_R |\bar{a}_T \cdot \bar{a}_R|^2 \quad ...(11.15.6)$$

Knowing the relation between gain and directivity

$$G = e_{cd} (1 - |\Gamma|^2) D$$

$$G_T = e_{cdT} (1 - |\Gamma_T|^2) D_T$$

$$G_R = e_{cdR} (1 - |\Gamma_R|^2) D_R$$

Thus Equation (11.15.6) can be written as

$$\frac{P_R}{P_T} = \left(\frac{\lambda}{4\pi r}\right)^2 G_T G_R |\bar{a}_T \cdot \bar{a}_R|^2 \quad ...(11.15.7)$$

Depending upon the situation the above Equation (11.15.6) will get modified, using

For no polarization loss

$$\bar{a}_T \cdot \bar{a}_R = 1$$

For no reflection loss

$$\Gamma = 0$$

For no loss in antenna

$$e_{cd} = 1$$

When no loss is present, Equation (11.15.6) will reduce to Equation (11.15.5).

Module 3

UEx. 11.15.1 MU - May 10, 5 Marks

A radio link has a 15 W transmitter connected to an antenna of 0.2 m^2 effective overture at 5 GHz. The receiving antenna has an effective aperture of 0.5 m^2 and is located at a 15 km line of sight distance the transmitting antenna. Assuming lossless matched antennas, find the power delivered to the receiver.

☑ **Soln. :**

Given : $P_T = 15\text{ W}, A_{eT} = 0.2\text{ m}^2, A_{eR} = 0.5\text{ m}^2$

$$f = 5\text{ GHz} \quad \rightarrow \quad \lambda = \frac{c}{f} = \frac{3\times10^8}{5\times10^9} = 0.6\text{ m}$$

$$r = 15\text{ km} = 15\times10^3\text{ m}$$

Using Equation (11.15.5)

$$P_R = P_T\frac{A_{eT}A_{eR}}{r^2\lambda^2} = 15\times\frac{0.2\times0.5}{(15\times10^3)^2\times0.06}$$

$$P_R = 1.66\times10^{-6}\text{ Watts}$$

UEx. 11.15.2 MU - May 13, 10 Marks.

The transmitting and receiving antennas are operating at 3 GHz with gains of 25 and 20 dB respectively, are separated by a distance of 500 m. Find the power delivered to the load when the input power is 100 W. Assume the PLF = 1.

☑ **Soln. :**

$$\text{Given :}f = 3\text{ GHz} \rightarrow \lambda = \frac{c}{f} = \frac{3\times10^8}{3\times10^9} = 0.1\text{ (m)}$$

$$r = 500\text{ m}; \quad P_T = 100\text{ W}$$

$$G_T = 25\text{ dB} \rightarrow 25 = 10\log G_T\text{ (dimensionless)}$$

$$\rightarrow G_T = 10^{2.5} = 316.26$$

$$G_R = 20\text{ dB} \rightarrow 20 = 10\log G_R\text{ (dimensionless)} \rightarrow G_R = 10^2 = 100$$

$$PLF = |\bar{a}_T\cdot\bar{a}_R| = 1$$

Using Equation (11.15.7)

$$P_R = P_T\left(\frac{\lambda}{4\pi r}\right)^2 G_T G_R |\bar{a}_T\cdot\bar{a}_R|^2$$

$$P_R = 100\left(\frac{0.1}{4\pi\times500}\right)^2 (316.26)(100)(1)$$

$$P_R = 8.01\times10^{-4}\text{ (Watts)}$$

UEx. 11.15.3 MU - May 15, 5 Marks.

What is maximum power received at a distance of 0.5 km over free space for 1 GHz frequency. The system consists of transmittin antenna with 2.5 dB gain and receiving antenna with 20 dB gain and antenna is fed with 150 W power.

☑ **Soln. :**

$$f = 1\text{ GHz} \rightarrow \lambda = \frac{c}{f} = \frac{3\times10^8}{1\times10^9} = 0.3\text{ (m)}$$

$$G_T = 2.5\text{ dB} \rightarrow 2.5 = 10\log G_T$$

$$\therefore \quad G_T = 10^{0.25} = 1.778$$

$$G_R = 20\text{ dB} \rightarrow 20 = 10\log G_R$$

$$\therefore \quad G_R = 10^2 = 100$$

$$P_T = 150\text{ W} \quad \text{and} \quad r = 0.5\text{ km} = 500\text{ (m)}$$

$$P_R = P_T\left(\frac{\lambda}{4\pi r}\right)^2 G_T G_R = 6.08\times10^{-5}\text{ (W)}$$

UEx. 11.15.4 MU - Dec. 12, 5 Marks.

A series of microwave repeater links operating are at 10 GHz are used to relay television signals into a valley that is surrounded by steep mountain ranges. Each repeater consists of a transmitter, receiver antennas and associated equipment. The transmitting and receiving antennas are identical horns with gains of 15 dB each. The repeaters are separated by 10 km. For acceptable signal to noise ratio, the power received at each repeater must be greater than 10 nW. Loss due to polarization mismatch should not exceed 3 dB. Assuming matched loads, calculated the minimum transmitter power that should be used.

☑ **Soln. :**

$$\text{Given : } f = 10\text{ GHz} \rightarrow \lambda = \frac{c}{f} = \frac{3\times10^8}{10\times10^9} = 0.03\text{ m.}$$

$$G_T = G_R = 15\text{ dB} \rightarrow 15 = 10\log G\text{ (dimensionless)}$$

$$\rightarrow G = 10^{1.5} = 31.62 = G_T = G_R$$

$$r = 10\text{ km} \qquad P_R \geq 10\text{ nW}$$

$$|\bar{a}_T\cdot\bar{a}_R|^2 = -3\text{ dB} \rightarrow -3 = 10\log|a_T\cdot a_R|^2$$

$$\rightarrow |a_R\cdot a_R|^2 = 10^{-0.3} = 0.5$$

Using Friss formula in Equation (11.15.7)

$$\frac{P_R}{P_T} = \left(\frac{\lambda}{4\pi r}\right)^2 G_T G_R |\bar{a}_T\cdot\bar{a}_R|^2$$

$$\frac{10\times10^{-9}}{P_T} = \left(\frac{0.03}{4\pi\times10\times10^3}\right)^2 \times 31.62\times31.62\times0.5$$

$$= 2.85\times10^{-11}$$

$$\text{or} \quad P_T = \frac{10\times10^{-9}}{2.85\times10^{-11}} = 350.98\text{ Watts}$$

$$\text{so for } P_R \geq 10\text{ nW} \quad \rightarrow (P_T)_{min} = 350.98\text{ (Watts)}$$

Ex. 11.15.5 : The identical transmitting and receiving antennas operating at 9 GHz are separated by 10 km. To meet the S / N ratio of the receiver, the received power must be at least 10 μW. Assuming that the antennas are polarization matched and aligned for maximum reception, find what should be gains (in dB) of the transmitting and receiving antennas be when the input power to the transmitting antenna is 10 W.

☑ **Soln. :**

Given : f = 9 GHz $\rightarrow \lambda = \frac{c}{f} = \frac{3\times 10^8}{9\times 10^9} = 0.033 \text{ (m)}$

$r = 10$ km, $P_R = 10\,\mu W$, $P_T = 10$ W,

$$|\bar{a}_T \cdot \bar{a}_R|^2 = 1$$

For identical antennas, $G_T = G_R = G$

Using Friss formula in Equation (11.15.7)

$$\frac{P_R}{P_T} = \left(\frac{\lambda}{4\pi r}\right)^2 G_T G_R |\bar{a}_T \cdot \bar{a}_R|^2$$

$$\frac{10\times 10^{-6}}{10} = \left(\frac{0.033}{4\pi\times 10\times 10^3}\right)^2 G^2 (1)$$

$\rightarrow G = 3769.91 \rightarrow$ G (dB) = 10 log (3769.91) = 35.76 (dB).

UEx. 11.15.6 MU - May 16, 10 Marks.

A radio link has 15 Watt transmitter connected to an antenna of 2.5 m^2 effective aperture at 5 GHz. The receiving antenna has an effective aperture of 0.5 m^2 and is located at a 15 km line of sight distance from transmitting antenna. Assume lossless antennas. Find power delivered to the receiver.

☑ **Soln. :**

$$\lambda = c/f = \frac{3\times 10^8}{5\times 10^9} = 0.06 \text{ m.}$$

$$P_R = P_T \frac{A_{eT} A_{eR}}{r^2 \lambda^2} = 15 \frac{2.5\times 0.5}{(15\times 10^3)^2\, 0.6^2} = 0.231\ (\mu W)$$

UEx. 11.15.7 MU - Dec. 16, May 17, May 18, 5 Marks.

What is maximum power received at a distance of 0.75 km over free space for 1 GHz frequency. The system consist of transmitting antenna with 3 dB gain and receiving antenna with 17 dB gain and antenna is fed with 200 W power.

☑ **Soln. :**

3 dB = 10 log G_T $\therefore G_T = 10^{3/10} = 1.995$

17 dB = 10 log G_R $\therefore G_R = 10^{17/10} = 50.119$

Also $\lambda = \frac{c}{f} = \frac{3\times 10^8}{1\times 10^9} = 0.3$ (m)

Power received, $P_R = P_T \left(\frac{\lambda}{4\pi r}\right)^2 G_T G_R$

$$P_R = 200 \left(\frac{0.3}{4\pi\times 750}\right)^2 \times 1.995\times 50.119 = 20.3\ \mu W$$

11.16 INPUT IMPEDANCE (ANTENNA IMPEDANCE)

Every antenna is connected to a transmitter via a transmission line. The applied radio frequency voltage establishes a current distribution on the antenna structure. This causes power radiated into free space.

A small part of the input power is dissipated due ohmic/dielectric losses in the antenna. Also the applied voltage establishes a reactive field in the vicinity of the antenna.

One can think of the antenna as an equivalent complex impedance Z_A, which draws exactly the same amount of complex power from the transmission line as the antenna. This is known as the antenna input impedance.

The input impedance of an antenna is the impedance presented by the antenna at its terminals.

It is the complex quantity. The **real part** accounts for the radiated power and the power dissipated in the antenna. The **reactive part** accounts for the reactive-power stored in the near-field of the antenna. Thus

$$Z_A = R_A + j X_A \quad ...(11.16.1)$$

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Where, Z_A = Input impedance of an antenna = Z_{in}

R_A = Input resistance of an antenna = R_{in}

X_A = Input reactance of an antenna = X_{in}

First we shall discuss the input resistance. The average power dissipated in an antenna is

$$P_{in} = \frac{1}{2} R_{in} |I_{in}|^2 \quad ...(11.16.2)$$

where, I_{in} = Peak value of the current at the input terminals.

But this power dissipated consists of two parts.

(i) Power radiated (P_r)

(ii) Power dissipated representing losses (P_L)

$$P_{in} = P_r + P_L \quad ...(11.16.3)$$

Consider resistances R_r and R_L are responsible for power P_r and P_L respectively, then

$$P_r = \frac{1}{2} R_r |I_{in}|^2 \quad ...(11.16.4)$$

$$P_L = \frac{1}{2} R_L |I_{in}|^2 \quad ...(11.16.5)$$

So that $P_{in} = P_r + P_L$

$$= \frac{1}{2} R_r |I_{in}|^2 + \frac{1}{2} R_L |I_{in}|^2$$

$$= \frac{1}{2} (R_r + R_L) |I_{in}|^2 \quad ...(11.16.6)$$

Comparing Equation (11.16.6) with Equation (11.16.2)

$$R_{in} = R_r + R_L \quad ...(11.16.7)$$

Where R_r is called as radiation resistance of the antenna and R_L is the loss resistance. Using Equation (11.16.4) and Equation (11.16.5) we define radiation resistance as,

$$R_r = \frac{2P_r}{|I_{in}|^2} \quad ...(11.16.8)$$

And ohmic or loss resistance as

$$R_L = \frac{2P_L}{|I_{in}|^2} = \frac{2(P_{in}-P_r)}{|I_{in}|^2} \quad ...(11.16.9)$$

The radiation efficiency is defined as

$$K = \frac{P_r}{P_{in}} = \frac{P_r}{P_r+P_L} = \frac{R_r}{R_r+R_L} \quad ...(11.16.10)$$

The equivalent circuit of the antenna can be drawn as shown in Fig. 11.16.1.

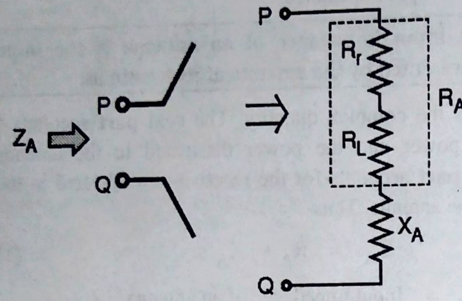


Fig. 11.16.1 : Antenna and its equivalent

> **Note :** Remember that there is no physical resistance or reactace at the antenna input terminals. The antenna impedance Z_A is the ratio of the input voltage to the input current at the antenna terminals.

11.16.1 Importance of Antenna Impedance

The antenna can be defined as a device that integrates the electric and magnetic fields to produce the voltages and currents required to actuate electrical devices.

In this transition from field to circuits it is necessary to know the impedance of antenna so that meaningful expressions involving both field and circuit quantities can be derived.

11.16.2 Radiation Resistance (R_r)

The antenna is a radiating device, which radiated the power into space in the form of electromagnetic waves. When current I is passed through antenna, the power dissipated by it is,

$$P = I^2 R$$

The energy when supplied to the antenna it is dissipated as,

(i) to radiate the electromagnetic waves (P_{rad}), and

(ii) as a ohmic loss. (P_{loss})

Thus P in the above expression consists of two parts

$$P = P_{rad} + P_{loss}$$

Though the radiation from an antenna is a desirable and useful phenomena but as far as antenna is concerned it is a loss. Let this loss is due to resistance R_r called as radiation resistance. Thus the power radiated is,

$$P_{rad} = I^2 R_r$$

The ohmic loss in the antenna is due to a resistance called as loss resistance (R_L). This loss is

$$P_{loss} = I^2 R_L$$

Putting the values the total power dissipated in antenna

$$P = P_{rad} + P_{loss}$$
$$= I^2 R_r + I^2 R_L = I^2 (R_r + R_L)$$

Note that the radiation resistance is not a real resistance inside the antenna. It simply represents a relation between the total energy radiated from the transmitting antenna and the current flowing in the antenna. It is defined as,

> **The radiation resistance (R_r) is the fictitious resistance which, when substituted in series with the antenna, will consume the same power as is actually radiated.**

The value of the radiation resistance depends on

(i) Configuration of an antenna.

(ii) Ratio of length to diameter of the conductor used.

(iii) The point where radiation resistance is considered.

(iv) Location of antenna w.r.t. ground and other objects.

(v) Corona discharge – a luminous discharge round the surface of an antenna due to ionization of air etc.

A half wave dipole has a radiation resistance of 73 Ω in free space. The free space means it is standing alone in the medium without any object close to it. Or it should be at least several wavelength away from the earth and other objects. This situation is not possible in practice and hence the radiation resistance vary from 73 Ω.

Unless otherwise mentioned the radiation resistance is at the point of current maximum in case of ungrounded antenna and to the current at the base of antenna in case of grounded antennas.

Importance of radiation resistance

The knowledge of R_r is important because it acts as a load to the transmitter or for the radio frequency transmission line connected between the transmitter and antenna.

For maximum power transfer the transmission line impedance should match with radiation resistance. For example, a half wave dipole with R_r equal to 73 Ω must be connected to the transmission line with Z_0 close to this value. Though exact match is not possible, a coaxial cable with 75 Ω can be used for this purpose.

Chapter Ends...

□□□

Module 3

CHAPTER 12

Wire Antennas

12.1 INTRODUCTION

Antennas which are in the form of straight wire are called as linear wire antennas.

Depending upon the length linear wire antennas are classified as,

i) Infinitesimal dipole $\left(l \leq \frac{\lambda}{50}\right)$

ii) Small dipole $\left(\frac{\lambda}{50} < l \leq \frac{\lambda}{10}\right)$

iii) $\frac{\lambda}{2}$ dipole $\left(l = \frac{\lambda}{2}\right)$

Here, l = the length of the antenna and

λ = the wavelength of the signal to be transmitted.

The radiation pattern of the dipole is decided by nature of current waveform present on the wire of the antenna. The current distribution along the length of the antenna depends upon the length of wire. In order to understand nature of current waveform following discussion is must.

12.1.1 Formation of Dipole

The simplest type of linear wire antenna is the dipole. It is formed using a two wire transmission line. The transmission line is not a radiating system. Some changes are required in it so that it starts radiating. It is explained below.

(i) Two wire transmission line

Consider a parallel two wire transmission line with far end open circuited as shown in Fig. 12.1.1(a). The transmission line has characteristic impedance Z_0 which mismatches with the open end impedance ($Z_L = \infty$). The result of mismatch is, current on each wire when arrives at the end of each of the wires, it undergoes a complete reflection (equal magnitude and 180° phase reversal).

Incident and reflected waves on the transmission line combine to give standing wave.

The current in each wire undergoes a 180° phase reversal between adjoining half cycles. Current in each wire radiates a field. If the spacing s << λ, the fields radiated by the current of each wire are essentially cancelled by those of the wire.

The net result is an almost non radiating transmission line. Thus the two wire parallel transmission line with small separation is not a radiating system.

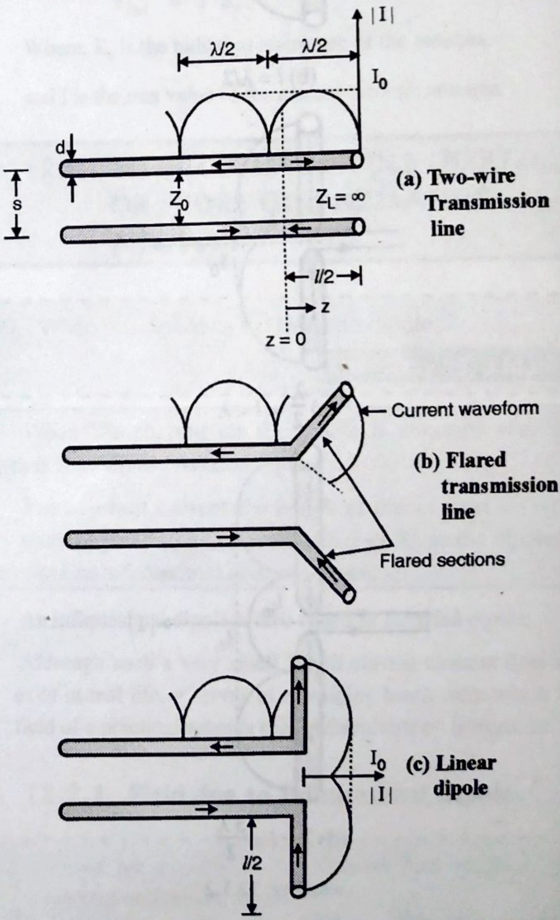
Flared transmission line : Fig. 12.1.1(b)

Consider the section of transmission line between $0 \leq z \leq \frac{l}{2}$ is flared as shown in Fig. 12.1.1(b). Then because the two wires of flared section are not close to each others, the field radiated by one is not cancelled by the other. Therefore it results in radiation. As the flaring is increased the radiation increases.

Dipole : Fig. 12.1.1(c)

In Fig. 12.1.1(c), the flaring is done to the maximum extent. The elements of the flared wires are aligned along a vertical line. This system is called as dipole. As the flaring is maximum, the resulting radiation is also maximum.

Note that the parallel horizontal wires on the left of the flared wires are not giving any radiation. Only the vertical flared sections are responsible for radiation. These two sections combinely is called as antenna. While the horizontal wires are simply feeding current to the antenna are called as feed lines.



aw(6.1)Fig. 12.1.1

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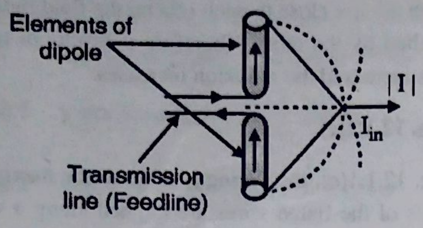
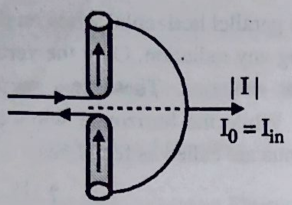
12.1.2 Current Distribution on the Dipole

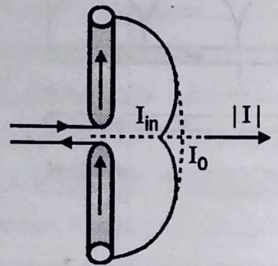
Current distributions for different lengths of a dipole is shown in Fig. 12.1.2.



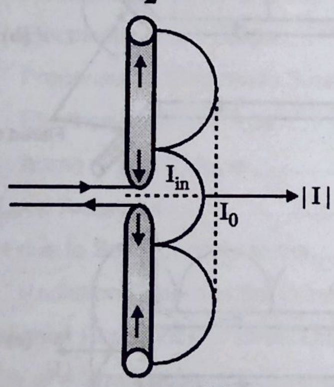
(a) $l << \lambda$



(b) $l = \lambda/2$



(c) $\frac{\lambda}{2} < l < \lambda$



(d) $\lambda < l < \frac{3\lambda}{2}$

aw(6.2)**Fig. 12.1.2**

Fig 12.1.2(a)

Here, dipole length is very small $\left(\frac{\lambda}{50} < l \leq \frac{\lambda}{10}\right)$, so current distribution is approximated by triangular distribution. Since

$$\sin\left(\frac{k\,l}{2}\right) \simeq \frac{k\,l}{2}; \text{ when } \frac{k\,l}{2} \text{ is very small.}$$

Fig. 12.1.2(d)

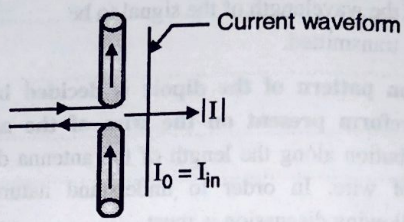
When $l < \lambda$, then currents in all parts of the dipole does not have the same phase along the length. Therefore the fields radiated by some parts of the dipole will not reinforce those of the others.

Fig. 12.1.2 (b) and (c)

Currents in upper and lower half are in phase so it radiates more.

When the length of the antenna is very small $\left(l \leq \frac{\lambda}{50}\right)$, the current along wire almost remain constant.

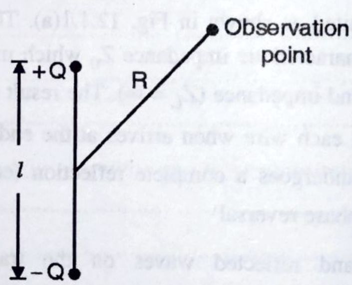
The field due to infinitesimal dipole is derived in the next section.



aw(6.3)**Fig. 12.1.3 : $l \leq \frac{\lambda}{50}$, infinitesimal dipole**

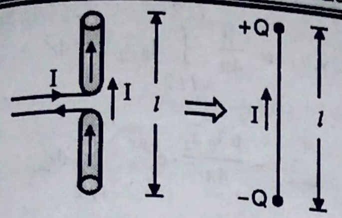
12.1.3 Why the term dipole ?

Basically a dipole is defined as a system of equal and opposite charges separated by a very small distance as compared to distance of observation point from the dipole i.e. R >> l. Refer Fig. 12.1.4.



aw(6.4)**Fig. 12.1.4 : Electric dipole**

A short dipole is initially in neutral condition and the moment a current starts to flow in one direction, one half of the dipole acquire an excess of charge and the other deficit. This is shown in Fig. 12.1.5.

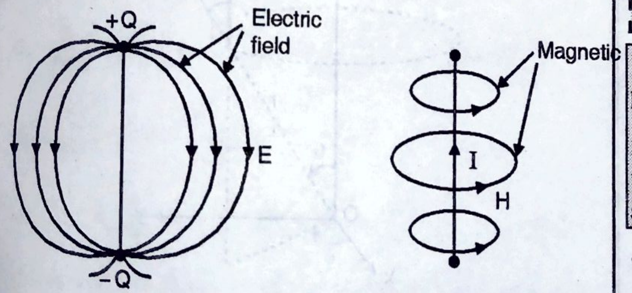


aw(6.5)**Fig. 12.1.5 : Short dipole and its equivalent**

Remember that the direction of current flow and direction of electrons are opposite. When current flows in upper half in upward direction, the electrons flow in downward direction which causes shortage of electrons at upper end making it behave like positive charge. Opposite is true for lower end which behaves like negative charge. When the current changes its direction (because of sinusoidal nature) this charge unbalance will first be neutralized and then changed. The upper end now behaves negatively charged while lower end is positively charged. The oscillating nature of current thus result in oscillating charge. Therefore the system is also called "oscillating electric dipole".

Due to charges, there will be voltage between the two halves of the dipole. The oscillating nature of current will thus result in oscillating voltage. If the current oscillation is sinusoidal, the voltage oscillation will also be sinusoidal and approximately 90° lagging the current in phase angle i.e. a short dipole is capacitive in nature from current voltage relation point of view.

The charges on the two halves of the dipole generates electric field, while current in the wire is responsible for magnetic field. So the dipole antenna is surrounded by an electric and magnetic field as shown in Fig. 12.1.6.



aw(6.6)**Fig. 12.1.6 : Field surrounding the dipole**

The phenomenon of radiation is rather complicated, so we have intentionally delayed its discussion until this chapter. In this chapter we will focus our attention only on linear wire antennas mentioned in the introduction. For each of these types, we will determine the radiation fields by taking the following steps :

12.1.4 Steps for Analysis of a Dipole

1) Select an appropriate coordinate system and determine magnetic vector potential $\bar{A}$.

2) Find $\bar{H}$ from $\bar{B} = \mu\bar{H} = \nabla \times \bar{A}$

3) Determine $\bar{E}$ from $\nabla \times \bar{H} = j\omega\epsilon\,\bar{E}$ or using intrinsic impedance given by,

$$\frac{E}{H} = \eta$$

Here, it is assumed that the medium is lossless ($\sigma = 0$)

4) To find the radiation resistance.

Initially we obtain the power radiated by using complex Poynting vector as

$$P_{rad} = \int \bar{W} \cdot d\bar{s} \text{ (Watts)}$$

Where, $$\bar{W} = \frac{1}{2}(\bar{E} \times \bar{H}^*) \text{ (Watts / m}^2\text{)}$$

Using P_{rad}, the radiation resistance of the antenna is determined.

$$P_{rad} = I^2 R_r$$

Where, R_r is the radiation resistance of the antenna.

and I is the rms value of the current through antenna.

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12.2 INFINITESIMAL DIPOLE (HERTZIAN OR SHORT OR OSCILLATING DIPOLE)

UQ. When is a dipole called Hertzian dipole. **(MU - May 15, 2 Marks)**

When the current on the dipole is constant over its length it is called as Hertzian dipole.

For constant current the length of dipole must be very very small as compared to wavelength ($l << \lambda$), so the dipole is also called as infinitesimal or short dipole.

- An infinitesimal dipole is also called as Hertzian dipole.
- Although such a very small length current element does not exist in real life, it serves as a building block from which the field of a practical antenna can be calculated by integration.

12.2.1 Field due to Infinitesimal Dipole

UQ. Derive an expression for E field and H field of infinitesimal dipole antenna. **(MU - May 16, 10 Marks)**

In this section we shall obtain field due to this dipole.

Assumptions

i) $l << \lambda$, i.e. length is very small $\left(l \le \frac{\lambda}{50} \right)$

ii) The radius of the wire, $a << \lambda$, i.e. it is very thin.

iii) The antenna is placed symmetrically at the origin of the coordinate system

and oriented along the z-axis as shown in Fig. 12.2.1.

iv) The current is assumed to be constant in z direction

i.e. $\bar{I}(z') = I_0 \bar{a}_z$

where I_0 = constant

v) The medium surrounding is air.

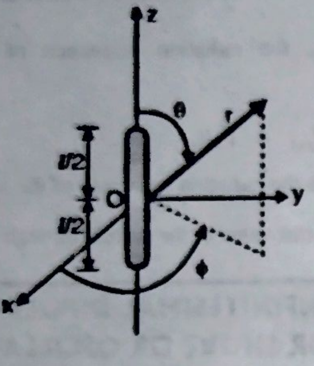

Fig. 12.2.1 : Infinitesimal dipole

Step 1 : To find magnetic vector potential $\bar{A}$

The general expression for the vector potential is

$$\bar{A}(x, y, z) = \frac{\mu}{4\pi} \int \bar{I}(x', y', z') \frac{e^{-jkR}}{R} dl' \quad ...(12.2.1)$$

where,

(x, y, z) represents the observation point,

(x′, y′, z′) represents the coordinates of the source.

R is the distance from source to the observation point.

When the current is flowing along wire along z-axis, then

$$\bar{I}(x', y', z') = \bar{I}(z') = I_0 \bar{a}_z \quad ...(12.2.2)$$

and, $$dl' = dz' \quad ...(12.2.3)$$

Also, the source is at origin and distance from source of the observation point means measuring distance from origin, so every R is replaced by r in spherical coordinates.

i.e. $$R = r \quad ...(12.2.4)$$

Now the expression for $\bar{A}$ (Equation (12.2.1)) is changed for our problem as

$$\bar{A}(x, y, z) = \frac{\mu}{4\pi} \int_{-l/2}^{l/2} I_0 \bar{a}_z \frac{e^{-jkr}}{r} dz'$$

$$= \frac{\mu I_0 \bar{a}_z}{4\pi r} \cdot e^{-jkr} \int_{-l/2}^{l/2} dz'$$

$$\therefore \quad \bar{A}(x, y, z) = \frac{\mu I_0 l}{4\pi r} e^{-jkr} \bar{a}_z = A_z \bar{a}_z \quad ...(12.2.5)$$

This vector magnetic potential is only in the direction of z as was expected since the current is in z-direction.

Step 2 : To find the magnetic field intensity $\bar{H}$

We have,

$$\bar{B} = \mu \bar{H} = \nabla \times \bar{A}$$

In spherical coordinates

$$\nabla \times \bar{A} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \bar{a}_r & r\bar{a}_\theta & r\sin\theta\, \bar{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & r A_\theta & r\sin\theta\, A_\phi \end{vmatrix} \quad ...(12.2.6)$$

To solve $\nabla \times \bar{A}$ we require to find relation between A_z and A_r, A_θ, A_ϕ.

These components are shown in Fig. 12.2.2.

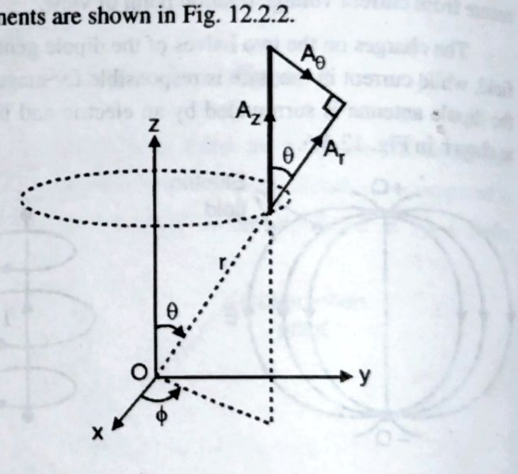

Fig. 12.2.2 : Conversion of A_z in spherical coordinates

From Fig. 12.2.2, we have

$$A_r = A_z \cos\theta \quad ...(12.2.7(a))$$

$$A_\theta = -A_z \sin\theta \quad ...(12.2.7(b))$$

(Negative sign in the above expression is due to θ which always increases away from z-axis. The conventional direction of A_θ is as shown in Fig. 12.2.2, but the projection of A_z in $\bar{a}_\theta$ direction is opposite.)

and, $$A_\phi = 0 \quad ...(12.2.7(c))$$

(Zero result is because A_z is vertical and its projection in horizontal direction (A_ϕ) must be zero.)

Also note that the value A_z at all points on the circle surrounding z-axis, (shown dotted) is same or in other words A_z does not vary with ϕ, giving all differentiations with respect to ϕ are zero, that is,

$$\text{all} \quad \frac{\partial}{\partial \phi} = 0 \qquad \text{...(12.2.8)}$$

Putting $A_\phi = 0$ and $\frac{\partial}{\partial \phi} = 0$ in the expression for $\nabla \times \bar{A}$ and expanding we have

$$\nabla \times \bar{A} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \bar{a}_r & r\bar{a}_\theta & r\sin\theta\, \bar{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & 0 \\ A_r & rA_\theta & 0 \end{vmatrix}$$

$$= \mu \left(H_r \bar{a}_r + H_\theta \bar{a}_\theta + H_\phi \bar{a}_\phi \right) \qquad \text{...(12.2.9)}$$

expanding and comparing two sides we get

$$\left.\begin{aligned} H_r &= 0 \\ H_\theta &= 0 \end{aligned}\right\}$$

$$\frac{1}{r^2 \sin\theta} \left(r\sin\theta\, \bar{a}_\phi \right) \left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial}{\partial \theta}(A_r) \right] = \mu H_\phi \bar{a}_\phi$$

$$\therefore \quad H_\phi = \frac{1}{\mu r} \left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial}{\partial \theta}(A_r) \right]$$

$$= \frac{1}{\mu r} \left[\frac{\partial}{\partial r}(-rA_z \sin\theta) - \frac{\partial}{\partial \theta}(A_z \cos\theta) \right]$$

Putting value of A_z from Equation (12.2.5), we get

$$H_\phi = \frac{1}{\mu r} \left\{ \frac{\mu I_0 l}{4\pi} \left[\frac{\partial}{\partial r}\left(-e^{-jkr} \cdot \sin\theta\right) - \frac{\partial}{\partial \theta}\left(\frac{e^{-jkr}}{r} \cdot \cos\theta \right) \right] \right\}$$

$$= \frac{I_0 l}{4\pi r} \left[(jk)\, e^{-jkr} \sin\theta + \frac{e^{-jkr}}{r} \sin\theta \right]$$

$$\text{i.e.} \quad H_\phi = \frac{I_0 l}{4\pi r} \sin\theta\,(jk) \left[1 + \frac{1}{jkr} \right] e^{-jkr} \qquad \text{...(12.2.10)}$$

Thus the magnetic field components due to infinitesimal dipole are

$$H_r = 0 \qquad \text{...(12.2.11(a))}$$

$$H_\theta = 0 \qquad \text{...(12.2.11(b))}$$

$$H_\phi = \frac{I_0 l}{4\pi r} \sin\theta\,(jk) \left[1 + \frac{1}{jkr} \right] e^{-jkr} \qquad \text{...(12.2.11(c))}$$

Step 3 : To find the electric field $\bar{E}$

The electric field can be obtained from the magnetic field using Maxwell's equation,

$$\nabla \times \bar{H} = j\omega\epsilon\, \bar{E}$$

The electric field components in spherical coordinates are E_r, E_θ and E_ϕ. Expanding $\nabla \times \bar{H}$ and comparing coefficients,

$$\nabla \times \bar{H} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \bar{a}_r & r\bar{a}_\theta & r\sin\theta\, \bar{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & 0 \\ 0 & 0 & r\sin\theta\, H_\phi \end{vmatrix}$$

$$= j\omega\epsilon \left(E_r \bar{a}_r + E_\theta \bar{a}_\theta + E_\phi \bar{a}_\phi \right) \qquad \text{...(12.2.12)}$$

expanding determinant

$$\frac{1}{r^2 \sin\theta} \left\{ \bar{a}_r \left[\frac{\partial}{\partial \theta}(r\sin\theta\, H_\phi) \right] - r\bar{a}_\theta \left[\frac{\partial}{\partial r}(r\sin\theta\, H_\phi) \right] \right\}$$

$$= j\omega\epsilon \left[E_r \bar{a}_r + E_\theta \bar{a}_\theta + E_\phi \bar{a}_\phi \right]$$

Comparing we get,

$$j\omega\epsilon\, E_\theta = \frac{1}{r^2 \sin\theta} \left\{ -r \cdot \frac{\partial}{\partial r}(r\sin\theta\, H_\phi) \right\} \qquad \text{...(12.2.12(a))}$$

$$j\omega\epsilon\, E_r = \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta}(r\sin\theta\, H_\phi) \qquad \text{...(12.2.12(b))}$$

$$= \frac{1}{r^2 \sin\theta} \frac{\partial}{\partial \theta} \left(r\sin\theta \frac{I_0 l}{4\pi r} \sin\theta\,(jk) \left(1 + \frac{1}{jkr} \right) e^{-jkr} \right)$$

$$= \frac{1}{r^2 \sin\theta} \frac{I_0 l}{4\pi} (jk) \left(1 + \frac{1}{jkr} \right) e^{-jkr} \frac{\partial}{\partial \theta}\left(\sin^2\theta\right)$$

$$= \frac{1}{r^2 \sin\theta} \frac{I_0 l}{4\pi} (jk) \left(1 + \frac{1}{jkr} \right) e^{-jkr} (2\sin\theta\cos\theta)$$

$$\therefore \quad E_r = \frac{I_0 l}{2\pi r^2} (\cos\theta) \left(\frac{k}{\omega\epsilon} \right) \left(1 + \frac{1}{jkr} \right) e^{-jkr} \qquad \text{...(12.2.13)}$$

We know, $k = \omega\sqrt{\mu\epsilon}$

$$\therefore \quad \frac{k}{\omega\epsilon} = \frac{\omega\sqrt{\mu\epsilon}}{\omega\epsilon} = \sqrt{\frac{\mu}{\epsilon}} = \eta \qquad \text{...(12.2.14)}$$

This is nothing but the intrinsic impedance, (η) of the surrounding media. Introducing η in the expression for E_r,

$$E_r = \frac{I_0 l}{2\pi r^2} (\cos\theta)\, \eta \left(1 + \frac{1}{jkr} \right) e^{-jkr} \qquad \text{...(12.2.15)}$$

Now, using Equation (12.2.12(a)) we get,

$$j\omega\epsilon\, E_\theta = \frac{1}{r^2 \sin\theta} \left\{ -r \cdot \frac{\partial}{\partial r}(r\sin\theta\, H_\phi) \right\}$$

$$= \frac{1}{r^2 \sin\theta} (-r) \frac{\partial}{\partial r} \left[r\sin\theta \frac{I_0 l}{4\pi r} \sin\theta\,(jk) \left(1 + \frac{1}{jkr} \right) e^{-jkr} \right]$$

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$$= \frac{I_0 l}{4\pi r}(\sin\theta)(jk)\frac{\partial}{\partial r}\left[\left(1+\frac{1}{jkr}\right)e^{-jkr}\right]$$

$$= -\frac{I_0 l}{4\pi r}(\sin\theta)(jk)\left[\left(1+\frac{1}{jkr}\right)(-jk)e^{-jkr} + e^{-jkr}\left(-\frac{1}{jkr^2}\right)\right]$$

Taking $(-jk)$ common out of square brackets,

$$j\omega\varepsilon E_\theta = \frac{+I_0 l}{4\pi r}(\sin\theta)(jk)^2\left[1+\frac{1}{jkr}+\frac{1}{(jkr)^2}\right]e^{-jkr}$$

$$\text{or } E_\theta = \frac{I_0 l}{4\pi r}(\sin\theta)\cdot(\eta)(jk)\left[1+\frac{1}{jkr}+\frac{1}{(jkr)^2}\right]e^{-jkr} \quad ...(12.2.16)$$

Thus the field components for infinitesimal dipole are :

$$E_r = \frac{I_0 l}{2\pi r^2}\cos\theta(\eta)\left[1+\frac{1}{jkr}\right]e^{-jkr}$$

$$E_\theta = \frac{I_0 l}{4\pi r}\sin\theta\cdot(\eta)(jk)\left[1+\frac{1}{jkr}+\frac{1}{(jkr)^2}\right]e^{-jkr} \quad ...(12.2.17)$$

$$E_\phi = 0$$

$$H_r = 0$$

$$H_\theta = 0$$

$$H_\phi = \frac{I_0 l}{4\pi r}\sin\theta(jk)\left[1+\frac{1}{jkr}\right]e^{-jkr}$$

These field components are presents in a region surrounding the dipole. A close observation of the field equations reveals that we have terms varying as $\frac{1}{r^3}$, $\frac{1}{r^2}$, and $\frac{1}{r}$.

- The $\frac{1}{r^3}$ term is called the **electrostatic field** since it corresponds to the field of an electric dipole. This term dominates over other terms in a region very close to infinitesimal dipole.
- The $\frac{1}{r^2}$ term is called the **inductive field**, and it is predictable from the Biot-Savart law. This term is important only at near field, that is at distances close to the current element.
- The $\frac{1}{r}$ term is called the **far field or radiation field** because it is the only term that remains at the far zone, that is at a point very far from the current element.

12.2.2 Field in the Far Field and Near Field Region

UQ. Explain near and far field radiation related to antenna. **(MU - Dec. 15, 5 Marks)**

In the previous section, we obtained general expressions. These can be modified in the region of interest.

Field expression for far field zone

If we restrict the observation point far away from the source (i.e. kr >> 1), we get far field expressions.

For the far field, kr >> 1 and $\frac{1}{jkr}$, $\frac{1}{(jkr)^2}$ terms can be neglected. Then set of equations in Equation (12.2.17) reduce to

$$\left.\begin{aligned} E_r &= 0, \\ E_\theta &= \frac{I_0 l}{4\pi r}(\sin\theta)\eta\cdot jk\cdot e^{-jkr} \\ E_\phi &= 0 \\ H_r &= 0 \\ H_\theta &= 0 \\ H_\phi &= \frac{I_0 l}{4\pi r}(\sin\theta)jk\cdot e^{-jkr} \end{aligned}\right\} kr > 1 \quad ...(12.2.18)$$

Field expressions for near field zone

If we restrict the observation point very close to the source, we get near field expressions.

For the near field, kr << 1 and $\frac{1}{(jkr)^2} >> \frac{1}{(jkr)} >> 1$.

Then the set of equations in Equations (12.2.17) reduce to

$$\left.\begin{aligned} E_r &\simeq -j\eta\frac{I_0 l}{2\pi kr^3}\cos\theta\cdot e^{-jkr} \\ E_\theta &\simeq -j\eta\frac{I_0 l}{4\pi kr^3}\sin\theta\cdot e^{-jkr} \\ E_\phi &= 0 \\ H_r &= 0 \\ H_\theta &= 0 \\ H_\phi &\simeq \frac{I_0 l}{4\pi r^2}\sin\theta\cdot e^{-jkr} \end{aligned}\right\} kr > 1 \quad ...(12.2.19)$$

At a certain distance r, near and far fields are equal. At this distance using Equations (12.2.18) and (12.2.19)

$|H_\phi|$(far field) = $|H_\phi|$(near field)

$$k = \frac{1}{r} \rightarrow r = \frac{1}{k} = \frac{v}{\omega} = \frac{1}{\beta} = \frac{\lambda}{2\pi} \simeq \frac{\lambda}{6} \quad ...(12.2.19(a))$$

12.2.3 Radiation Resistance of an Infinitesimal Dipole

UQ. Derive radiation resistance of infinitesimal dipole. Explain its significance. **(MU - May 15, Dec. 17, 5 Marks)**

▸ **Step 4 : To obtain the radiation resistance R_r**

Equation (12.2.17) says, the electric field E has components E_r and E_θ while the magnetic field has only one component H_ϕ. Then we write total $\bar{E}$ and $\bar{H}$ as,

$$\bar{E} = E_r \bar{a}_r + E_\theta \bar{a}_\theta$$

and

$$\bar{H} = H_\phi \bar{a}_\phi$$

The complex poynting vector is given by

$$\bar{W} = \frac{1}{2}\left(\bar{E} \times \bar{H}^*\right)$$

$$= \frac{1}{2}\left(E_r \bar{a}_r + E_\theta \bar{a}_\theta\right) \times \left(H_\phi^* \bar{a}_\phi\right)$$

$$= \frac{1}{2}\left(E_\theta H_\phi^* \bar{a}_r - E_r H_\phi^* \bar{a}_\theta\right) \quad \text{...(12.2.20)}$$

The complex poynting vector in general given by

$$\bar{W} = W_r \bar{a}_r + W_\theta \bar{a}_\theta + W_\phi \bar{a}_\phi \quad \text{...(12.2.21)}$$

Comparing Equations (12.2.20) and (12.2.21) we get

$$\left.\begin{aligned} W_r &= \frac{1}{2} E_\theta H_\phi^* \\ W_\theta &= -\frac{1}{2} E_r H_\phi^* \\ \text{and} \quad W_\phi &= 0 \end{aligned}\right\} \quad \text{...(12.2.22)}$$

Each component of $\bar{W}$ can now be obtained separately (Refer Equation (12.2.17))

$$W_r = \frac{1}{2} E_\theta H_\phi^* = \frac{1}{2}\frac{I_0 l}{4\pi r}\sin\theta \cdot (\eta)\, jk\left[1 + \frac{1}{jkr} + \frac{1}{(jkr)^2}\right] e^{-jkr} \times \frac{I_0 l}{4\pi r}\sin\theta\,(-jk)\left[1 - \frac{1}{jkr}\right] e^{jkr}$$

$$= -\frac{1}{2}\left(\frac{I_0 l}{4\pi r}\right)^2 \sin^2\theta\,(\eta)\,(jk)^2\left[1 + \frac{1}{jkr} + \frac{1}{(jkr)^2} - \frac{1}{jkr} - \frac{1}{(jkr)^2} - \frac{1}{(jkr)^3}\right]$$

$$= -\frac{1}{2}\left(\frac{I_0 l}{4\pi r}\right)^2 \sin^2\theta\,(\eta)\,(jk)^2\left[1 - \frac{1}{(jkr)^3}\right] = +\frac{I_0^2 l^2}{32\pi^2 r^2}\sin^2\theta\,(\eta)\left(\frac{2\pi}{\lambda}\right)^2\left[1 - \frac{j}{(kr)^3}\right] \quad \text{...}\left(k = \frac{2\pi}{\lambda}\right)$$

$$W_r = \frac{\eta}{8}\left|\frac{I_0 l}{\lambda}\right|^2 \frac{\sin^2\theta}{r^2}\left[1 - \frac{j}{(kr)^3}\right] \quad \text{...(12.2.23)}$$

The other component of W is obtained by

$$W_\theta = -\frac{1}{2} E_r H_\phi^* = -\frac{1}{2}\frac{I_0 l}{2\pi r^2}\cos\theta\,(\eta)\left[1 + \frac{1}{jkr}\right] e^{-jkr} \times \frac{I_0 l}{4\pi r}\sin\theta\,(-jk)\left[1 - \frac{1}{jkr}\right] e^{+jkr}$$

$$W_\theta = j\eta \frac{k\,|I_0 l|^2}{16\pi^2 r^3}\cos\theta \cdot \sin\theta\left[1 + \frac{1}{(kr)^2}\right] \quad \text{...(12.2.24)}$$

The complex power moving in radial direction is obtained by integrating $\bar{W}$ over a closed surface of radius r. The real part of it is the power transmitted by the antenna, which is to be used to find the radiation resistance. Integrating,

$$P = \oint_s \bar{W} \cdot \bar{ds} = \int_0^{2\pi}\int_0^{\pi}\left(W_r \bar{a}_r + W_\theta \bar{a}_\theta\right) \cdot \left(r^2 \sin\theta\, d\theta\, d\phi\, \bar{a}_r\right)$$

$$P = \int_0^{2\pi}\int_0^{\pi} W_r r^2 \sin\theta\, d\theta\, d\phi \qquad (\because \bar{a}_r \cdot \bar{a}_r = 1 \text{ and } \bar{a}_r \cdot \bar{a}_\theta = 0)$$

Using the value of W_r from Equation (12.2.23),

$$P = (\phi)_0^{2\pi}\int_0^{\pi}\frac{\eta}{8}\left|\frac{I_0 l}{\lambda}\right|^2 \frac{\sin^2\theta}{r^2}\left[1 - \frac{j}{(kr)^3}\right] r^2 \sin\theta\, d\theta.$$

$$= (2\pi)\frac{\eta}{8}\left|\frac{I_0 l}{\lambda}\right|^2\left[1-\frac{j}{(kr)^3}\right]\int_0^{\pi}\sin^3\theta\, d\theta$$

Note : Using Willi's formula we obtain the value of integration

$$\int_0^{\pi/2}\sin^n\theta\, d\theta = \frac{n-1}{n}\frac{n-3}{n-2}\cdots\frac{1}{2}\times\frac{\pi}{2} \quad \text{(For even n)}$$

$$= \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdots\frac{2}{3}\times 1 \quad \text{(For odd n)}$$

$$\therefore \int_0^{\pi}\sin^3\theta\, d\theta = 2\int_0^{\pi/2}\sin^3\theta\, d\theta = 2\cdot\frac{2}{3}\cdot 1 = \frac{4}{3} \quad ...(12.2.24(a))$$

$$\therefore \quad P = (2\pi)\cdot\frac{\eta}{8}\cdot\left|\frac{I_0 l}{\lambda}\right|^2\left[1-\frac{j}{(kr)^3}\right]\times\frac{4}{3}$$

$$= (\eta)\left(\frac{\pi}{3}\right)\left|\frac{I_0 l}{\lambda}\right|^2\left[1-\frac{j}{(kr)^3}\right] \quad ...(12.2.25)$$

This power consists of real part and imaginary part. When we talk about power radiated it is a real power. As antenna radiates this real power through the radiation resistance (R_r), the radiated power is given by real part of P.

$$\therefore \quad P_{rad} = \eta\cdot\left(\frac{\pi}{3}\right)\left|\frac{I_0 l}{\lambda}\right|^2 \quad ...(12.2.26)$$

In terms of radiation resistance (R_r) and current I_0, the power radiated is

$$P_{rad} = \frac{1}{2}|I_0|^2 R_r$$

Equating two expressions for P_{rad} we get

$$\frac{1}{2}|I_0|^2 R_r = \eta\cdot\left(\frac{\pi}{3}\right)\left|\frac{I_0 l}{\lambda}\right|^2$$

or
$$R_r = \eta\cdot\left(\frac{\pi}{3}\right)\left(\frac{l}{\lambda}\right)^2$$

Using $\eta = 120\pi$ for free space,

$$R_r = 80\,\pi^2\left(\frac{l}{\lambda}\right)^2 \quad ...(12.2.27)$$

12.2.4 Problems with Infinitesimal Dipole

UQ. Derive radiation resistance of infinitesimal dipole. Explain its significance. **(MU - May 15, 10 Marks)**

i) The infinitesimal dipole is assumed to have a very very small length ($l << \lambda$) particularly, $l \leq \frac{\lambda}{50}$. Let us calculate the radiation resistance R_r for the length equal to $\frac{\lambda}{50}$.

$$R_r = 80\,\pi^2\left(\frac{\lambda/50}{\lambda}\right)^2 = 0.316\,(\Omega) \quad ...(12.2.28)$$

The practical transmission lines with characteristic impedance of 50 Ω or 75 Ω when used as a feed line for the infinitesimal dipole, there is a large mismatch present at the feed point. This results in large reflection on the transmission line or power transmitted by antenna is very small. Thus efficiency of the antenna is decreased.

When the length is further decreased then the efficiency is further decreased. Thus in practice **it is difficult to match infinitesimal dipole with a real transmission line.**

ii) We have also assumed current to be uniform, this requires current to be nonzero at the end points of the dipole. This is practically impossible since the surrounding medium is not conducting.

12.2.5 Use of Infinitesimal Dipole Analysis

UQ. Derive radiation resistance of infinitesimal dipole. Explain its significance. **(MU - May 15, 10 Marks)**

In spite of two problems discussed above we make analysis of infinitesimal dipole because we can use it as a building block to obtain field pattern due to any length antenna.

Also this dipole analysis teaches us that improvement in radiation resistance is possible by increasing the length of the dipole.

12.2.6 Radiation Pattern of the Infinitesimal (Hertzian) Dipole

The expression for the electric field in the far field is Equation (12.2.18).

$$E_\theta = \frac{I_0 l}{4\pi r}(\sin\theta)\,\eta\, jk\, e^{-jkr} \quad ...(12.2.29)$$

Using this expression for E, different plots can be made.

Field Pattern and Power Pattern

When the amplitude of the E is plotted it is called as **field pattern** or voltage pattern.

When the square of amplitude of E is plotted it is called as **power pattern.**

A three dimensional plot of an antenna pattern is avoided by plotting separately E plane and H plane patterns.

E-plane Pattern

In the spherical coordinate system a ϕ = constant plane is a vertical plane.

A plot of normalized |E| versus θ for a constant ϕ is called as E-plane pattern or vertical pattern. Normally this plot is made in ϕ = constant = 0 plane, which is nothing but xz plane.

H-plane Pattern

A plot of normalized |E| versus ϕ for $\theta = \frac{\pi}{2}$ is called as H plane pattern or horizontal pattern.

Note that, in the spherical coordinates angle θ is measured from z-axis and $\theta = \pi/2$ is a horizontal plane or xy plane.

Remember the normalized value is obtained by dividing the value by its maximum value.

Using Equation (12.2.29), the magnitude of E_θ is

$$|E_\theta| = \frac{I_0 l}{4\pi r} |\sin\theta| \, \eta \, k$$

Maximum value is obtained when $\theta = \frac{\pi}{2}$

$$|E_\theta|_{max} = \frac{I_0 l}{4\pi r} \eta \, k$$

Then the normalized value of $|E_\theta|$ is

$$|E_\theta|_n = \frac{|E_\theta|}{|E_\theta|_{max}} = |\sin\theta| \quad ...(12.2.30)$$

This is independent of ϕ and is a function of only θ,

$$|E_\theta|_n = f(\theta) = |\sin\theta| \quad ...(12.2.31)$$

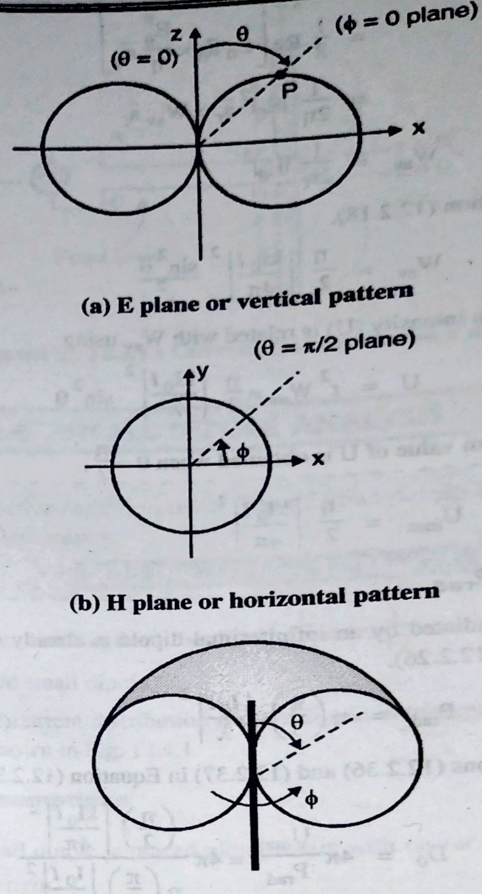
The plot of f (θ) for different values of θ in $\phi = 0$ plane is shown in Fig. 12.2.3(a), this is normalized E plane or vertical pattern for a Hertzian dipole. The plot is symmetric about z-axis.

For H plane pattern we set θ = constant = $\pi/2$, then f (θ) in Equation (12.2.31) becomes,

$$f(\theta) = 1 \quad ...(12.2.32)$$

Since the value of f (θ) is 1 for any value of ϕ, the locus will be a circle of radius equal to 1. It is shown in Fig. 12.2.3(b).

When the two plots are combined, we have a three dimensional plot which has a shape of doughnut (without hole at the centre). It is shown in Fig. 12.2.3(c).



(a) E plane or vertical pattern

(b) H plane or horizontal pattern

(c) Three dimensional pattern

aw(6.9)**Fig. 12.2.3 : Field patterns for a Hertzian dipole**

Module 3

12.2.7 Directivity of a Infinitesimal (Hertzian) Dipole

Directivity is obtained using

$$D_0 = 4\pi \frac{U_{max}}{P_{rad}} \quad ...(12.2.33)$$

where, U_{max} = Maximum radiation intensity (Watt / Sr) ;

P_{rad} = Power radiated (Watts)

To obtain U_{max}

The average Poynting vector is given by,

$$\bar{W}_{av} = \frac{1}{2} \operatorname{Re}\left[\bar{E} \times \bar{H}^*\right]$$

$$= \frac{1}{2} \operatorname{Re}\left[E_\theta \bar{a}_\theta \times H_\phi^* \bar{a}_\phi\right]$$

$$= \frac{1}{2} \text{Re}\left[E_\theta \bar{a}_\theta \times \frac{E_\theta^*}{\eta} \bar{a}_\phi \right]$$

$$= \frac{1}{2\eta} |E_\theta|^2 \bar{a}_r = W_{av} \bar{a}_r$$

where $$W_{av} = \frac{1}{2\eta} |E_\theta|^2 \quad ...(12.2.34)$$

Using Equation (12.2.18),

$$W_{av} = \frac{\eta}{2} \left| \frac{kI_0 l}{4\pi} \right|^2 \frac{\sin^2 \theta}{r^2} \quad ...(12.2.35)$$

The radiation intensity (U) is related with W_{av} using

$$U = r^2 W_{av} = \frac{\eta}{2} \left| \frac{kI_0 l}{\lambda} \right|^2 \sin^2 \theta$$

The maximum value of U is obtained when $\theta = \frac{\pi}{2}$,

$$U_{max} = \frac{\eta}{2} \left| \frac{kI_0 l}{4\pi} \right|^2 \quad ...(12.2.36)$$

To obtain P_{rad}

The power radiated by an infinitesimal dipole is already obtained in Equation (12.2.26).

$$P_{rad} = \eta \left(\frac{\pi}{3} \right) \left| \frac{I_0 l}{\lambda} \right|^2 \quad ...(12.2.37)$$

Using Equations (12.2.36) and (12.2.37) in Equation (12.2.33),

$$D_0 = 4\pi \frac{U_{max}}{P_{rad}} = 4\pi \frac{\left(\frac{\eta}{2}\right) \left| \frac{kI_0 l}{4\pi} \right|^2}{\eta \left(\frac{\pi}{3}\right) \left| \frac{I_0 l}{\lambda} \right|^2}$$

Using $K = \frac{2\pi}{\lambda}$, we get $D_0 = \frac{3}{2}$ ...(12.2.38)

The maximum effective aperture is,

$$A_{em} = \frac{\lambda^2}{4\pi} D_0 = \frac{3\lambda^2}{8\pi} \quad ...(12.2.39)$$

Ex. 12.2.1

(i) Find the power radiated by a 10 cm short dipole antenna operated at 50 MHz with an average current of 5 mA.

(ii) How much current is required to radiate a power of 1 W.?

☑ **Soln. :**

Given : l = 10 cm = 10×10^{-2} m, f = 50 MHz from which wavelength is

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{50 \times 10^6} = 6 \text{ (m)}$$

We can check whether the dipole is a Hertzian dipole, for which

$$l \leq \frac{\lambda}{50}$$

i.e. $$10 \times 10^{-2} \leq \frac{6}{50}$$

$$0.1 \leq 0.12$$

So the short dipole is a Hertzian dipole.

(i) For a short dipole the power radiated is obtained using

$$P_{rad} = \eta \left(\frac{\pi}{3} \right) \left| \frac{I_0 l}{\lambda} \right|^2$$

$$= (120\pi) \left(\frac{\pi}{3} \right) \left(\frac{5 \times 10^{-3} \times 10 \times 10^{-2}}{6} \right)^2$$

$$P_{rad} = 2.74 \text{ (}\mu\text{Watts)}$$

(ii) Now we use above relation in reverse way to find I_0.

$$P_{rad} = \eta \left(\frac{\pi}{3} \right) \left| \frac{I_0 l}{\lambda} \right|^2$$

$$1 = (120\pi) \left(\frac{\pi}{3} \right) \left(\frac{I_0 \times 10 \times 10^{-2}}{6} \right)^2$$

$$= 0.1096 I_0^2$$

or $$I_0 = 3.02 \text{ (A)}$$

UEx. 12.2.2 (MU - May 12, 4 Marks)

If operating frequency of a half wave is 400 MHz, find its effective area.

☑ **Soln. :**

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{400 \times 10^6} = 0.75 \text{ (m)}$$

$$A_{em} = \frac{\lambda^2}{4\pi} (D_0) = \frac{(0.75)^2}{4\pi} \times 1.5 = 0.067 \text{ (m}^2\text{)}$$

Ex. 12.2.3 : A 1 m long car radio antenna operates in the AM frequency of 1.5 MHz. How much current is required to transmit 4 W of power ?

☑ **Soln. :**

Given : $l = 1$ m, $P_{rad} = 4$ W

$$f = 1.5 \text{ MHz} \rightarrow \lambda = \frac{c}{f} = \frac{3 \times 10^8}{1.5 \times 10^6} = 200 \text{ (m)}$$

The car antenna is a monopole antenna. The length to wavelength ratio is

$$\frac{l}{\lambda} = \frac{1}{200} < \frac{1}{50}$$

Hence it is a Hertzian monopole. The radiation resistance of it is given by,

$$R_r = 40\pi^2\left(\frac{h}{\lambda}\right)^2 = 40\pi^2\left(\frac{1}{200}\right)^2 = 9.87\ \text{m}\Omega$$

...where h = 1(m)

The power radiated is expressed as

$$P_{rad} = \frac{1}{2} I_0^2 R_r$$

$$4 = \frac{1}{2} I_0^2 (9.87 \times 10^{-3})$$

$$\rightarrow I_0^2 = \frac{8}{9.87 \times 10^{-3}} = 810.56$$

$$\text{or} \quad I_0 = \sqrt{810.56} = 28.47 \text{ (Amp).}$$

UEx. 12.2.4 MU - May 15, 3 Marks.

At what distance from 50 cycle circuit is radiation field approximately equal to induction field.

Soln. :

Radiation and induction fields are approximately equal at a distance of

$$r \simeq \frac{\lambda}{6}$$

For a 50 cycle circuit,

$$r = \frac{\lambda}{6} = \frac{V}{f \times 6}$$

$$r = \frac{3 \times 10^8}{50 \times 6} = 10^6 \text{ (m)}$$

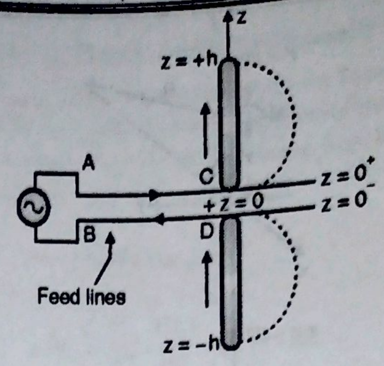
12.3 ASYMPTOTIC CURRENT DISTRIBUTION IN DIPOLE

Knowledge of current in a dipole is very much essential and it is simple and easy to determine. Let the dipole antenna be fed by a two wire transmission line. This transmission line feeds the current to the antenna at points C, D. These points are called feed points. The other end of the transmission lines is connected to generator at points AB.

Consider the antenna is placed along z-axis with center of the dipole at origin as shown Fig. 12.3.1.

Since current in each arm is sinusoidal, then these currents can be expressed as,

$$\left.\begin{aligned} I &= I(z) = I_0 \sin k(h-z) \quad ...z > 0^+ \\ I &= I(z) = I_0 \sin k(h+z) \quad ...z < 0^- \end{aligned}\right\} \quad ...(12.3.1)$$



aw(6.10)Fig. 12.3.1 : Current distribution on a dipole

12.4 SMALL DIPOLE ANALYSIS

UQ. Derive radiation resistance of small dipole. Explain its significance.

(MU - Dec. 16, May 17, May 18, 10 Marks)

Module 3

- When the length of the dipole, $\frac{\lambda}{50} < l \leq \frac{\lambda}{10}$ then the dipole is called small dipole.
- The current distribution on this dipole is triangular in nature as shown in Fig. 12.4.1.

Assumptions

(i) Small dipole is placed along z-axis with center of the dipole at origin.

(ii) Medium surrounding is air.

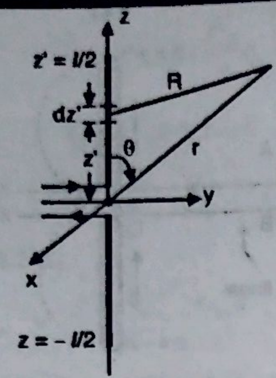
(iii) Diameter of the wire (d) is very less than λ.

(iv) The current is fed at the center of the dipole using transmission line.

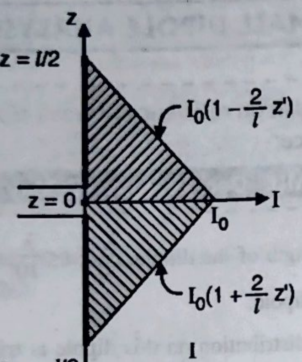
(v) The current distribution is assumed to be triangular with maximum value I_0 at the center and decreases linearly to a zero value at the ends of the dipole. The expression for these currents is expressed as,

$$\left.\begin{aligned} &\bar{I} = I_0\left(1 - \frac{2}{l} z'\right)\bar{a}_z \quad \text{for} \\ &\text{and } \bar{I} = I_0\left(1 + \frac{2}{l} z'\right)\bar{a}_z \\ &\text{for } -\frac{l}{2} \leq z' \leq 0 \end{aligned}\right\} \quad ...(12.4.1)$$

The small dipole and the current distribution on it is as shown in Fig. 12.4.1(a) and (b).



(a) Small dipole



(b) Current distribution

aw(6.13)**Fig. 12.4.1 : Small dipole**

▶ Step 1 : To find the vector magnetic potential ($\bar{A}$)

We have the expression for vector magnetic potential due to small current element dl is

$$d\bar{A} = \frac{\mu \bar{I}\, dl}{4\pi R} e^{-jkR}$$

For small section along z-axis, $dl = dz'$ and for current in z-direction the vector magnetic potential

$$\bar{A} = A_z \bar{a}_z$$

Then
$$dA_z \bar{a}_z = \frac{\mu I \bar{a}_z}{4\pi R} dz' e^{-jkR}$$

or
$$dA_z = \frac{\mu I}{4\pi R} e^{-jkR} dz'$$

Integrating dA_z within limits from $z = \frac{-l}{2}$ to $z = \frac{l}{2}$ we get

$$A_z = \int_{-l/2}^{l/2} dA_z$$

But the current is not same over the entire length of the antenna, for the lower half and upper half the currents are different. So we split the integral into two parts. Then

$$A_z = \frac{\mu}{4\pi}\left[\int_{-l/2}^{0} I_0\left(1 - \frac{2}{l} z'\right)\frac{e^{-jkR}}{R} \cdot dz' + \int_{0}^{l/2} I_0\left(1 + \frac{2}{l} z'\right)\frac{e^{-jkR}}{R} dz'\right]$$

Since the length of the dipole is very small $\left(\frac{\lambda}{50} < l \leq \frac{\lambda}{10}\right)$, then the difference between R and r is very small and can be neglected. This is valid for every R which is appearing alone as a distance or coming with phase constant k.

> **Note :** For small dipole the maximum length $l = \frac{\lambda}{10}$. Then the difference between R and r is
>
> $$\text{Path difference} = R - r = \frac{l}{2} \cos\theta = \frac{\lambda}{20} \cos\theta.$$
>
> $$\text{Then phase difference} = k(R - r) = k \frac{\lambda}{20} \cos\theta$$
>
> $$= \frac{2\pi}{\lambda} \cdot \frac{\lambda}{20} \cos\theta = \frac{\pi}{10} \cos\theta.$$
>
> The maximum phase difference is $\frac{\pi}{10} = 18°$ which is very small and can be neglected.

Then above expression changes to

$$A_z = \frac{\mu I_0}{4\pi r} e^{-jkr}\left[\int_{-l/2}^{0}\left(1 - \frac{2}{l} z'\right) dz' + \int_{0}^{l/2}\left(1 + \frac{2}{l} z'\right) dz'\right]$$

$$= \frac{\mu I_0}{4\pi r} e^{-jkr}\left[\left(z' - \frac{2}{l}\frac{z'^2}{2}\right)_{-l/2}^{0} + \left(z' + \frac{2}{l}\frac{z'^2}{2}\right)_{0}^{l/2}\right]$$

$$= \frac{\mu I_0}{4\pi r} e^{-jkr}\left[\left(\frac{l}{2} - \frac{l}{4}\right) + \left(\frac{l}{2} - \frac{l}{4}\right)\right]$$

$$\therefore \quad A_z = \frac{1}{2}\frac{\mu I_0 l}{4\pi r} e^{-jkr} \qquad ...(12.4.2)$$

The term $\left(\frac{1}{2}\right)$ is kept out in the expression purposely because the remaining expression is same as that of A_z for infinitesimal dipole, Equation (12.4.1). Then we can say A_z for small dipole is half that of A_z for infinitesimal dipole.

▶ Step 2 : To find the magnetic field intensity ($\bar{H}$)

We have,

$$\bar{B} = \mu \bar{H} = \nabla \times \bar{A} \quad \text{Or}$$

$$\mu \bar{H} = \frac{1}{r^2 \sin\theta}\begin{vmatrix} \bar{a}_r & r\,\bar{a}_\theta & r\sin\theta\,\bar{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & r A_\theta & r\sin\theta\, A_\phi \end{vmatrix} \qquad ...(12.4.3)$$

Using A_z component we can obtain A_r, A_θ and A_ϕ. These relations are similar to infinitesimal dipole given by,

$$\left.\begin{aligned} A_r &= A_z \cos\theta \\ A_\theta &= -A_z \sin\theta \\ A_\phi &= 0 \end{aligned}\right\} \quad ...(12.4.4)$$

Since the current filament is placed along z-axis, there is cylindrical symmetry, giving A_z independent of ϕ.

This results in all

$$\frac{\partial}{\partial\phi} = 0 \quad ...(12.4.5)$$

Using Equations (12.4.4) and (12.4.5) in Equation (12.4.3),

$$\mu\bar{H} = \frac{1}{r^2 \sin\theta}\begin{vmatrix} \bar{a}_r & r\,\bar{a}_\theta & r\sin\theta\,\bar{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & 0 \\ A_z\cos\theta & -r\,A_z\sin\theta & 0 \end{vmatrix}$$

$$= \frac{1}{r^2\sin\theta}\, r\sin\theta\,\bar{a}_\phi\left[\frac{\partial}{\partial r}(-r\,A_z\sin\theta) - \frac{\partial}{\partial\theta}(A_z\cos\theta)\right]$$

The right hand side have only $\bar{a}_\phi$ component, then left hand side will have only H_ϕ component of $\bar{H}$. Thus

$$\mu H_\phi = \frac{1}{r}\frac{\partial}{\partial r}(-r\,A_z\sin\theta) - \frac{1}{r}\frac{\partial}{\partial\theta}(A_z\cos\theta)$$

Note that the component A_z is inversely proportional to r refer Equation (12.4.2), then rA_z will not have r term in the denominator.

After differentiating w.r.t. r, when we multiply by $\left(\frac{1}{r}\right)$, then the first term on RHS will have r term in the denominator i.e. first term is inversely proportional to r.

Applying similar logic to second term we find the term is inversely proportional to r^2. In far field region when kr >> 1, the second term will be very small as compared to first term and is neglected. Then

$$\mu H_\phi = \frac{1}{r}\frac{\partial}{\partial r}(-r\,A_z\sin\theta)$$

Putting the value of A_z

$$\mu H_\phi = \frac{-1}{r}\cdot\frac{1}{2}\frac{\mu I_0 l}{4\pi}\sin\theta\cdot\frac{\partial}{\partial r}\left(e^{-jkr}\right)$$

$$\mu H_\phi = \frac{1}{2}(j)\frac{k\mu I_0 l}{4\pi r}\sin\theta\cdot e^{-jkr}$$

or $$H_\phi = \left(\frac{1}{2}\right)j\frac{k I_0 l}{4\pi r}\sin\theta\cdot e^{-jkr}\ (A/m) \quad ...(12.4.6)$$

This is the required expression for H_ϕ. This value is exactly half that of H_ϕ for infinitesimal dipole in the far field region.

Step 3 : To find the electric field intensity ($\bar{E}$)

From the infinitesimal dipole analysis we know that in far field region the component E_r is zero. The electric field E will have only E_θ component. Now E_θ can be obtained from H_ϕ using the concept of intrinsic impedance, given as

$$\frac{|E_\theta|}{|H_\phi|} = \eta = 120\pi \text{ (for free space)}$$

$$\therefore\ |E_\theta| = \eta\,|H_\phi| = \frac{1}{2}\eta\cdot\frac{k I_0 l}{4\pi r}\sin\theta\ (V/m) \quad ...(12.4.7)$$

Step 4 : To obtain R_r

We proceed similar to $\frac{\lambda}{2}$ dipole. The average power density W_{av} is obtained by using,

$$\bar{W}_{av} = \frac{1}{2}\mathrm{Re}\,[\bar{E}\times\bar{H}^*] = \frac{1}{2}\mathrm{Re}\,[E_\theta\,\bar{a}_\theta\times H_\phi^*\,\bar{a}_\phi]$$

$$= \frac{1}{2}\mathrm{Re}\left[E_\theta\,\bar{a}_\theta\times\frac{E_\theta^*}{\eta}\,\bar{a}_\phi\right]$$

$$\bar{W}_{av} = \frac{1}{2}\mathrm{Re}\left(\frac{|E_\theta|^2}{\eta}\,\bar{a}_r\right) \;...(\because\ \bar{a}_\theta\times\bar{a}_\phi = \bar{a}_r) \quad ...(12.4.8)$$

> **Note :** Let $A = x + jy$, then $A^* = x - jy$,
> and $AA^* = (x + jy)(x - jy) = x^2 + y^2$
> but $|A| = \sqrt{x^2 + y^2}$
> $\therefore\ AA^* = \left(\sqrt{x^2 + y^2}\right)^2$
> or $AA^* = |A|^2$

or $$W_{av} = \frac{1}{2\eta}|E_\theta|^2\,\bar{a}_r\ (W/m^2)$$

The power radiated is obtained by integrating W_{av} over a closed surface i.e.

$$P_{rad} = \oint \bar{W}_{av}\cdot\bar{ds}$$

$$= \int_0^{2\pi}\int_0^{\pi}\frac{1}{2\eta}|E_\theta|^2\,\bar{a}_r\cdot r^2\sin\theta\,d\theta\,d\phi\,\bar{a}_r$$

$$= \frac{1}{2\eta}\int_0^{2\pi}\int_0^{\pi}\left(\frac{\eta k I_0 l}{8\pi r}\sin\theta\right)^2 r^2\sin\theta\,d\theta\,d\phi$$

$$= \frac{15}{4}\pi\left(\frac{I_0 l}{\lambda}\right)^2(\phi)_0^{2\pi}\int_0^{\pi}\sin^3\theta\,d\theta$$

$$...\left(\because\ k = \frac{2\pi}{\lambda},\ \eta = 120\pi\right)$$

Module 3

The integral on LHS we have already solved, its value is $\frac{4}{3}$. Then

$$P_{rad} = \frac{15}{4}\pi I_0^2\left(\frac{l}{\lambda}\right)^2 \times 2\pi \times \frac{4}{3} = 10\pi^2 I_0^2\left(\frac{l}{\lambda}\right)^2 \quad ...(12.4.9)$$

The power radiated can also be written in terms of R_r and equating with above expression,

$$\frac{1}{2} I_0^2 R_r = 10\pi^2 I_0^2\left(\frac{l}{\lambda}\right)^2$$

or $$R_r = 20\pi^2\left(\frac{l}{\lambda}\right)^2 \ (\Omega) \quad ...(12.4.10)$$

This value is $\left(\frac{1}{4}\right)$ of that obtained for infinitesimal dipole.

UEx. 12.4.1 MU - May 15, 2 Marks.

Calculate radiation resistance of $\frac{\lambda}{10}$ dipole in free space.

Soln. :

$\lambda/10$ dipole is a small dipole. The radiation resistance of it is,

$$R_r = 20\pi^2\left(\frac{l}{\lambda}\right)^2$$

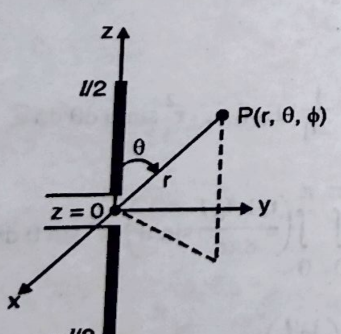
For $l = \lambda/10$

$$R_r = 20\pi^2\left(\frac{1}{10}\right)^2 = 1.97\ (\Omega)$$

12.5 FINITE LENGTH DIPOLE

In the previous sections, analysis of infinitesimal and small dipole is made. On the similar lines we can make analysis of finite length dipole.

Only results of the analysis are presented here for a dipole having some finite length l. This dipole is placed along z-axis from $-\frac{l}{2}$ to $\frac{l}{2}$, as shown in Fig. 12.5.1.



aw(6.14)**Fig. 12.5.1 : Finite length dipole of length l**

12.5.1 Field Expressions

The field radiated by the dipole at a distance r from it can be calculated on the similar lines as done in sections 12.2 and 12.4. The results of the far field analysis are

$$E_\theta = j\eta \frac{I_0 e^{-jKr}}{2\pi r}\left[\frac{\cos\left(\frac{Kl}{2}\cos\theta\right) - \cos\left(\frac{Kl}{2}\right)}{\sin\theta}\right] \quad ...(12.5.1)$$

and $$H_\phi = \frac{E_\theta}{\eta} = j\frac{I_0 e^{-jKr}}{2\pi r}\left[\frac{\cos\left(\frac{Kl}{2}\cos\theta\right) - \cos\left(\frac{Kl}{2}\right)}{\sin\theta}\right] \quad ...(12.5.2)$$

12.5.2 Field Pattern for Finite Length Dipole

The field expression for finite dipole is obtained in Equation (12.5.1) as

$$E_\theta = j\eta \frac{I_0 e^{-jKr}}{2\pi r}\left[\frac{\cos\left(\frac{Kl}{2}\cos\theta\right) - \cos\left(\frac{Kl}{2}\right)}{\sin\theta}\right]$$

$$|E_\theta| = \frac{\eta I_0}{2\pi r}\left|\frac{\cos\left(\frac{Kl}{2}\cos\theta\right) - \cos\left(\frac{Kl}{2}\right)}{\sin\theta}\right|$$

The maximum value of it is obtained using angle term equal to 1

$$|E_\theta|_{max} = \frac{\eta I_0}{2\pi r}$$

The normalized value of $|E_\theta|$ is

$$|E_\theta|_n = \frac{|E_\theta|}{|E_\theta|_{max}} = \left|\frac{\cos\left(\frac{Kl}{2}\cos\theta\right) - \cos\left(\frac{Kl}{2}\right)}{\sin\theta}\right| \quad ...(12.5.3)$$

This normalized value is not only depends upon θ but also on length (l). For different values of l, the plots can be made.

12.5.3(A) Radiation Pattern for λ/2 Dipole

With $l = \lambda/2$, and $K = 2\pi/\lambda$

$$\frac{Kl}{2} = \frac{1}{2}\left(\frac{2\pi}{\lambda}\right)\left(\frac{\lambda}{2}\right) = \frac{\pi}{2}$$

$$\therefore \quad \cos\left(\frac{Kl}{2}\right) = \cos\left(\frac{\pi}{2}\right) = 0$$

Equation (12.5.3) reduces to

$$|E_\theta|_n = \left| \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right| \quad ...(12.5.4)$$

Putting different values of θ, the E-plane pattern is obtained as shown in Fig. 12.5.2(a). The half power beamwidth of it is 78°.

$$\text{HPBW} = 78° \ \text{...for } \lambda/2 \text{ dipole} \quad ...(12.5.5)$$

The radiation pattern consist of two major lobes perpendicular to the dipole axis.

12.5.3(B) Radiation Pattern for λ Dipole

With $l = \lambda$

$$\frac{K\,l}{2} = \frac{1}{2}\left(\frac{2\pi}{\lambda}\right)(\lambda) = \pi$$

$$\cos\left(\frac{Kl}{2}\right) = \cos\pi = -1$$

Equation (12.5.3) reduces to

$$|E_\theta|_n = \left| \frac{\cos(\pi\cos\theta) - (-1)}{\sin\theta} \right|$$

$$= \left| \frac{\cos(\pi\cos\theta) + 1}{\sin\theta} \right| \quad ...(12.5.6)$$

The plot of it is shown in Fig. 12.5.2(b). The half power beam width of it is 47°.

$$\text{HPBW} = 47° \ \text{...for } \lambda \text{ dipole} \quad ...(12.5.7)$$

The radiation pattern has two major lobes perpendicular to the dipole axis.

12.5.3(C) Radiation Pattern for (3λ/2) Dipole

With $l = 3\lambda/2$

$$\frac{K\,l}{2} = \frac{1}{2}\left(\frac{2\pi}{\lambda}\right)\left(\frac{3\lambda}{2}\right)$$

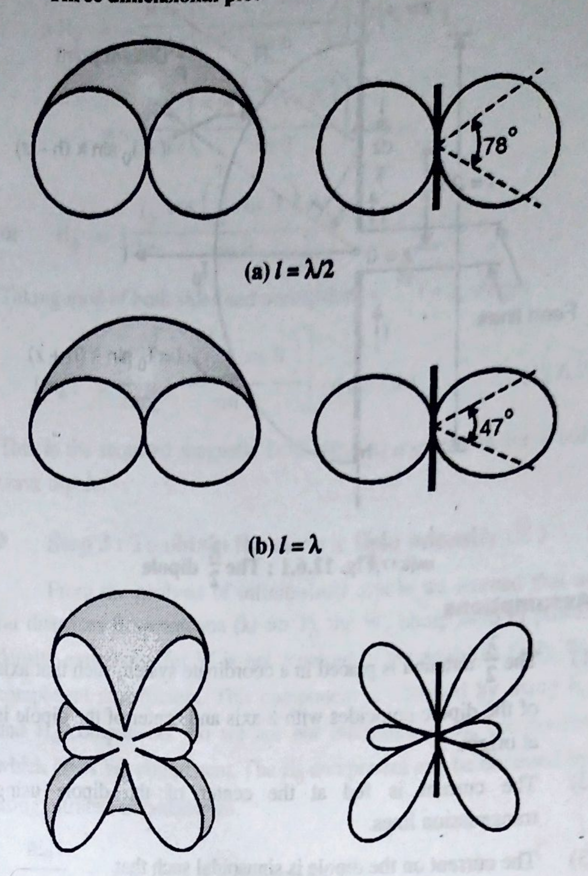
$$= \frac{3}{2}\pi$$

$$\cos\left(\frac{3}{2}\pi\right) = 0$$

Equation (12.5.3) reduces to

$$|E_\theta|_n = \left| \frac{\cos\left(\frac{3}{2}\pi\cos\theta\right)}{\sin\theta} \right| \quad ...(12.5.8)$$

The plot is shown in Fig. 12.5.2(c). The radiation pattern consists of four major lobes and two minor lobes.



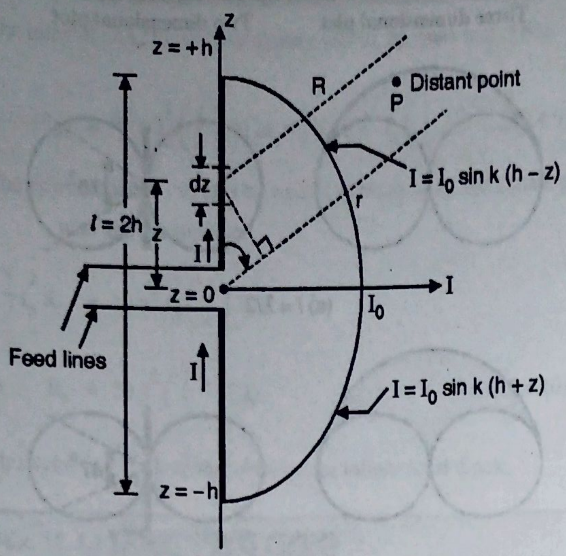
aw(6.16)**Fig. 12.5.2 : Radiation pattern for finite length dipoles**

12.6 HALF WAVE DIPOLE

UQ. Derive radiation resistance of half wave dipole antenna. **(MU - Dec. 15, 5 Marks)**

- Half wave length dipole or simply half wave dipole is one of the simplest antenna which has physical length of half wavelength.
- The $\frac{\lambda}{2}$ antenna is also known as Hertz antenna.
- A dipole antenna may be defined as symmetrical antenna in which two ends are at equal potentials relative to the midpoint.
- It is one of the most commonly used antenna. Because its radiation resistance is 73 Ω, it can be connected to 75 Ω transmission line more easily for matching purpose.
- Now we have to calculate the radiation field of half wave dipole and its radiation resistance.

Module 3



aw(6.17)**Fig. 12.6.1 : The $\frac{\lambda}{2}$ dipole**

Assumptions

1) The $\frac{\lambda}{2}$ antenna is placed in a coordinate system such that axis of the dipole coincides with z-axis and center of the dipole is at origin.
2) The current is fed at the center of the dipole using transmission lines.
3) The current on the dipole is sinusoidal such that

$$I = I_0 \sin k(h - z) \quad \text{...for } 0 \le z \le \frac{\lambda}{4}$$

$$I = I_0 \sin k(h + z) \quad \text{...for } -\frac{\lambda}{4} \le z \le 0$$

4) The medium surrounding the antenna is air.
5) The diameter of the wire is very small as compared to wavelength i.e. ($d \ll \lambda$)

▶ Step 1 : To find magnetic vector potential $\bar{A}$

The field due to dipole can be easily obtained if we consider it as consisting of a chain of Hertzian dipoles. The magnetic vector potential at P due to a differential length dl (= dz) of the dipole carrying current I is

$$dA_z = \frac{\mu I\, dz}{4\pi R} e^{-jkR}$$

where, R is the distance between Idz to distant point P.

When we add vector magnetic potential due to all differential elements present from z = − h to z = h, we get the total vector magnetic potential at point P. Summation is nothing but integration, thus

$$A_z = \int_{-h}^{h} dA_z = \int_{-h}^{h} \frac{\mu I\, dz}{4\pi R} e^{-jkR}$$

The problem in solving this integral is, the current is not same over the total length of the antenna. See current given in third assumption. The expression for current from z = − h to z = 0 is different than from z = 0 to z = h. Then the above integral we split into two parts as follows :

$$A_z = \int_{-h}^{0} \frac{\mu I_0 \sin k(h+z)}{4\pi R} e^{-jkR} dz + \int_{0}^{h} \frac{\mu I_0 \sin k(h-z)}{4\pi R} e^{-jkR} dz \quad \text{...(12.6.1)}$$

In the Fig. 12.6.1 the difference,

$$r - R = z\cos\theta \quad \text{or} \quad R = r - z\cos\theta$$

If r >> l (length of the antenna), we may substitute R ≃ r in the denominator of A_z expression, Equation (12.6.1) where the magnitude of the distance is needed. For the phase term in the numerator of Equation (12.6.1), the difference between kR and kr is significant (depends on the value of k), so we replace R by r − z cos θ and not r. Thus,

$$R = r - z\cos\theta \text{...in the numerator}$$

$$R = r \text{...in the denominator}$$

Now Equation (12.6.1) changes to

$$A_z = \frac{\mu I_0}{4\pi r}\left[\int_{-h}^{0} \sin k(h+z)\, e^{-jk(r - z\cos\theta)} dz + \int_{0}^{h} \sin k(h-z)\, e^{-jk(r - z\cos\theta)} dz\right]$$

$$A_z = \frac{\mu I_0 e^{-jkr}}{4\pi r}\left[\int_{-h}^{0} \sin k(h+z)\, e^{+jkz\cos\theta} dz + \int_{0}^{h} \sin k(h-z)\, e^{jkz\cos\theta} dz\right] \quad \text{...(12.6.2)}$$

For the $\frac{\lambda}{2}$ dipole, the length of the antenna

$$l = 2h = \frac{\lambda}{2} \quad \text{or} \quad h = \frac{\lambda}{4} \quad \text{...(12.6.3)}$$

then, using $k = \frac{2\pi}{\lambda}$, the sine terms in the above integrals reduces as

$$\sin k(h+z) = \sin(kh + kz) = \sin\left(\frac{2\pi}{\lambda}\cdot\frac{\lambda}{4} + kz\right)$$

$$= \sin\left(\frac{\pi}{2} + kz\right) = \cos kz$$

Similarly $\quad \sin k(h-z) = \cos kz$

Now Equation (12.6.2) reduces to,

$$A_z = \frac{\mu I_0 e^{-jkr}}{4\pi r}\left[\int_{-h}^{0} \cos(kz)\cdot e^{jkz\cos\theta} dz + \int_{0}^{h} \cos(kz)\cdot e^{jkz\cos\theta} dz\right] \quad \text{...(12.6.4)}$$

After making some mathematical analysis results in

i.e. $$A_z = \frac{\mu I_0 e^{-jkr}}{2\pi kr}\left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}\right] \quad \text{...(12.6.5)}$$

This is the required vector magnetic potential.

Step 2 : To obtain the magnetic field intensity ($\overline{H}$)

We have, $\nabla \times \bar{A} = \bar{B} = \mu \bar{H}$

In spherical coordinates,

$$\nabla \times \bar{A} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \bar{a}_r & r\,\bar{a}_\theta & r\sin\theta\,\bar{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & r\,A_\theta & r\sin\theta\,A_\phi \end{vmatrix}$$

$$= \mu \left[H_r \bar{a}_r + H_\theta \bar{a}_\theta + H_\phi \bar{a}_\phi \right] \quad ...(12.6.6)$$

To solve $\nabla \times \bar{A}$ we require A_r, A_θ, and A_ϕ components. From A_z we obtain these components using,

$$A_r = A_z \cos\theta$$

$$A_\theta = -A_z \sin\theta$$

$$A_\phi = 0$$

Also note that, the A_z in Equation (12.6.5) is a function of r and θ, that is, it is independent of ϕ. Then every differentiation

$$\frac{\partial}{\partial \phi} = 0$$

Putting these values in Equation (12.6.6),

$$\nabla \times \bar{A} = \frac{1}{r^2 \sin\theta} \begin{vmatrix} \bar{a}_r & r\,\bar{a}_\theta & r\sin\theta\,\bar{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & 0 \\ A_r & r\,A_\theta & 0 \end{vmatrix}$$

$$= \mu \left[H_r a_r + H_\theta a_\theta + H_\phi a_\phi \right]$$

expanding determinant and comparing coefficients we get,

$$\mu H_\phi = \frac{1}{r^2 \sin\theta} r\sin\theta \left[\frac{\partial}{\partial r}(r A_\theta) - \frac{\partial}{\partial \theta}(A_r) \right]$$

$$= \left[\frac{1}{r} \cdot \frac{\partial}{\partial r}(-r A_z \sin\theta) - \frac{1}{r}\frac{\partial}{\partial \theta}(A_z \cos\theta) \right]$$

After putting the value of A_z and differentiating, you can very easily imagine that the first term is inversely proportional to r while the second term is inversely proportional to r^2. At large distances the second term will be very small as compared to the first term and can be neglected. In practice we use the antenna to transmit the waves over large distances, so there is nothing wrong in making this approximation. Then

$$\mu H_\phi = \frac{1}{r}\frac{\partial}{\partial r}(-r A_z \sin\theta)$$

$$\mu H_\phi = \frac{1}{r}\frac{\partial}{\partial r}\left[-r \cdot \frac{\mu I_0 e^{-jkr}}{2\pi kr} \left(\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta} \right) \sin\theta \right]$$

$$\mu H_\phi = -\frac{\mu I_0}{2\pi kr} \left(\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right) \frac{\partial}{\partial r}(e^{-jkr})$$

$$= -\frac{\mu I_0}{2\pi kr} \left(\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right) (-jk)\, e^{-jkr}$$

or $$H_\phi = j\frac{I_0}{2\pi r} \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} e^{-jkr}$$

Taking mod of both sides and noting that $| e^{-jkr} | = 1$, we get

$$| H_\phi | = \frac{I_0}{2\pi r} \left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right] (\text{Amp / m}) \quad ...(12.6.7)$$

This is the required magnetic field intensity expression for a half wave dipole.

Step 3 : To obtain the electric field intensity ($\overline{E}$)

From the analysis of infinitesimal dipole we learned that at far distances from antenna (kr >> 1), the W_θ component in power density expression for W is not required in the analysis. Only W_r component is sufficient. This component is obtained by using E_θ and H_ϕ components. So we are not interested in E_r component which gives W_θ component. The E_θ component can be obtained by using intrinsic impedance as,

$$\frac{|E_\theta|}{|H_\phi|} = \eta = 120\pi \text{ (for free space)}$$

$$\therefore \quad |E_\theta| = \eta\, |H_\phi|$$

i.e. $$= (120\pi) \frac{I_0}{2\pi r} \left[\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right]$$

...Using Equation (12.6.7)

i.e. $$|E_\theta| = \frac{60 I_0}{r} \left| \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right| (\text{V / m}) \quad ...(12.6.8)$$

Notice that the radiation term of H_ϕ and E_θ are in time phase and orthogonal.

Step 4 : To obtain the radiation resistance (R_r)

Initially we will obtain the average power density by using the relation

$$\overline{W}_{av} = \frac{1}{2}\text{Re}\left[\bar{E} \times \bar{H}^* \right]$$

$$= \frac{1}{2}\text{Re}\left[E_\theta \bar{a}_\theta \times \frac{E_\theta^*}{\eta} \bar{a}_\phi \right] \quad ...(\because E_\theta = \eta H_\phi)$$

$$\overline{W}_{av} = \frac{1}{2\eta} |E_\theta|^2 \bar{a}_r \quad ...(\because \bar{a}_\theta \times \bar{a}_\phi = \bar{a}_r)$$

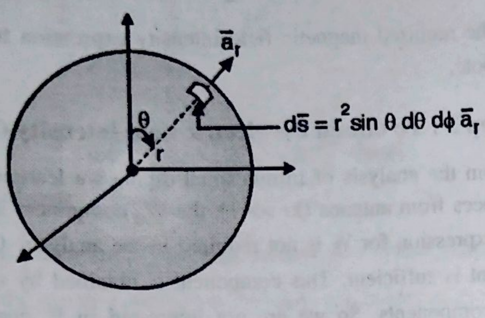
Module 3

> **Note :** The mathematical operation in the step above is,
> Let $A = x + jy$ then $A^* = x - jy$
> Now $A A^* = (x + jy)(x - jy) = x^2 + y^2 = |A|^2$

Putting the value of $|E_\theta|$ from Equation (12.6.8), and knowing that $\eta = 120\pi$,

$$\overline{W}_{av} = \frac{1}{2\eta} \frac{60^2 I_0^2}{r^2} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta} \bar{a}_r$$

$$= \frac{15 I_0^2}{\pi r^2} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta} \bar{a}_r \ (W/m^2) \qquad \text{...(12.6.9)}$$



aw(6.18)**Fig. 12.6.2**

Now we obtain power radiated in watts by using,

$$P_{rad} = \oint \overline{W}_{av} \cdot d\bar{s}$$

$$= \int_0^{2\pi}\int_0^{\pi} \left(\frac{15 I_0^2}{\pi r^2} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta} \bar{a}_r \right) \cdot (r^2 \sin\theta \, d\theta \, d\phi \, \bar{a}_r)$$

$$= \frac{15 I_0^2}{\pi} (\phi)_0^{2\pi} \int_0^{\pi} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{(\sin\theta)} d\theta \qquad \text{...(12.6.10)}$$

$$= 30 I_0^2 \int_0^{\pi} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} d\theta$$

$$= 60 I_0^2 \int_0^{\pi/2} \frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} d\theta$$

Solving this integral using numerical techniques

$$P_{rad} \simeq 36.56 I_0^2 \qquad \text{...(12.6.11)}$$

The power radiated can also be expressed in terms of radiation resistance (R_r) as,

$$P_{rad} = I_{rms}^2 R_r = \left(\frac{I_0}{\sqrt{2}}\right)^2 R_r = \frac{1}{2} I_0^2 R_r \qquad \text{...(12.6.12)}$$

Using Equation (12.6.11) and (12.6.12)

$$P_{rad} = \frac{1}{2} I_0^2 R_r = 36.56 I_0^2$$

$$\text{or } R_r = 73 \ (\Omega) \qquad \text{...(12.6.13)}$$

12.6.1 Input Impedance of $\frac{\lambda}{2}$ Dipole

When we observe the current distribution on this antenna we find, the current is maximum at the feed point. That is, input current and maximum current is same,

$$I_{in} = I_0$$

Then equating input power with radiated power

$$R_{in} = \left(\frac{I_0}{I_{in}}\right)^2 R_r = (1^2) R_r = R_r \qquad \text{...(12.6.14)}$$

Here it is also assumed that the antenna is lossless. The loss resistance, $R_L = 0$.

The input impedance of the antenna is,

$$Z_{in} = R_{in} + j X_{in}$$

The reactance at the input X_{in} is decided by the length of the antenna. For $l = \frac{\lambda}{2}$ the reactance is found to be 42.5. Thus for $\frac{\lambda}{2}$ dipole,

$$Z_{in} = R_{in} + j X_{in} = R_r + j X_{in} = 73 + j\,42.5 \qquad \text{...(12.6.15)}$$

If we decrease the length of the antenna, the reactive part also decreases. The radius of the wire also matters in this situation. Depending on the radius of the wire, the length of the dipole for first resonance (i.e. $Z_{in} = R_{in}$) is about $l = 0.47\,\lambda$ to $0.48\,\lambda$.

The thinner the wire, the closer the length is to $0.48\,\lambda$. For thicker wires, a larger segment of the wire has to be removed from $\frac{\lambda}{2}$ to achieve resonance.

In the analysis of $\frac{\lambda}{2}$ dipole, the diameter of the wire is assumed to be very small. In this case, instead of $l = 0.5\,\lambda$ if we use $l = 0.485\,\lambda$ then we achieve

$$Z_{in} = R_{in} = R_r$$

Once the reactive part is reduced to zero, the $\frac{\lambda}{2}$ dipole can be connected to 75 Ω coaxial cable (RG-59) at the input, which results in almost perfect match giving VSWR = S ≃ 1.

12.6.2 Radiation Pattern of a $\frac{\lambda}{2}$ Dipole

The electric field due to $\frac{\lambda}{2}$ dipole is Equation (12.6.8).

$$|E_\theta| = \frac{60\,I_0}{r}\left|\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right| \qquad ...(12.6.16)$$

The maximum value of it is obtained by setting angle term equal to 1.

$$|E_\theta|_{max} = \frac{60\,I_0}{r}$$

The normalized value is

$$|E_\theta|_n = \frac{|E_\theta|}{|E_\theta|_{max}} = \left|\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right| \qquad ...(12.6.17)$$

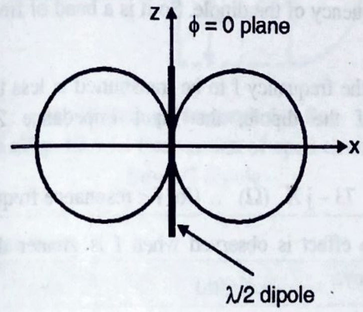
It is a function of only θ. For different values of θ the plot is obtained as shown in Fig. 12.6.3.

The E-plane or vertical pattern in $\phi = 0$ plane is shown in Fig. 12.6.3(a).

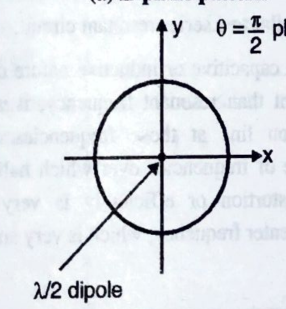
The H-plane or horizontal pattern is obtained using ($\theta = \pi/2$) in Equation (12.6.17).

$$|E_\theta|_n = 1$$

which is a circle of radius 1. The plot is shown in Fig. 12.6.3(b). Combining E plane and H plane plots we get three dimensional pattern as shown in Fig. 12.6.3(c). The 3-d plot is a doughnut shape (without hole at the center).

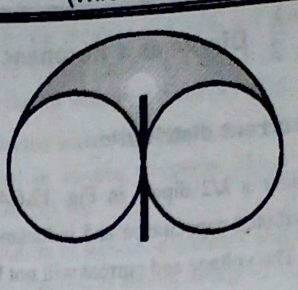


(a) E-plane pattern



(b) H-plane pattern

aw(6.19)**Fig. 12.6.3 Contd...**



(c) 3-d pattern

aw(6.19)**Fig. 12.6.3 : Radiation pattern of a λ/2 dipole**

12.6.3 Directivity of a $\frac{\lambda}{2}$ Dipole

Directivity of an antenna is given by,

$$D_0 = 4\pi\frac{U_{max}}{P_{rad}} \qquad ...(12.6.18)$$

Where U_{max} = maximum radiation intensity (Watt / Sr) ;

P_{rad} = power radiated (Watts)

To obtain U_{max}

From equation $W_{av} = \dfrac{15\,I_0^2}{\pi r^2}\dfrac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$

The radiation intensity (U) is related with W_{av} using

$$U = r^2 W_{av} = \frac{15\,I_0^2}{\pi}\frac{\cos^2\left(\frac{\pi}{2}\cos\theta\right)}{\sin^2\theta}$$

Maximum value of U is obtained when $\theta = \frac{\pi}{2}$,

$$U_{max} = \frac{15\,I_0^2}{\pi} \qquad ...(12.6.19)$$

To obtain P_{rad}

The power radiated by a λ/2 dipole is already obtained in Equation (12.6.11).

$$P_{rad} \simeq 36.56\,I_0^2 \qquad ...(12.6.20)$$

To obtained D_0

Using Equation (12.6.19) and Equation (12.6.20) in Equation (12.6.18).

$$D_0 = 4\pi\frac{\frac{15\,I_0^2}{\pi}}{36.56\,I_0^2} = 1.643$$

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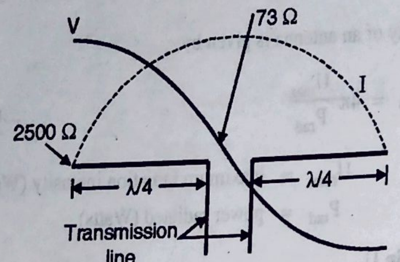
12.6.4 $\frac{\lambda}{2}$ Dipole as a Resonant Antenna

Voltage and current distribution

Now consider a λ/2 dipole in Fig. 12.6.4. This half wave antenna has distributed capacitance and inductance and acts like a resonant circuit. The voltage and current will not be in phase.

If an RF voltmeter is connected from the end of the antenna to ground, a large voltage will be measured. If the meter lead is moved towards the center, the voltage decreases. At the center it is minimum. The current at the center is maximum while at the ends is minimum as shown in Fig. 12.6.4.



aw(10.8)Fig. 12.6.4 : Voltage and current distribution on a half wave dipole

By referring this figure, it will become apparent that to connect a transmission line to this antenna configuration, we must observe the impedance at the connection points. The impedance varies along the length of the antenna, obtained from

$$Z = \frac{V}{I}$$

It is highest where the current is lowest (at the ends) and lowest where the current is highest (at the center).

At the center of this λ/2 dipole the impedance is approximately 73 Ω and increases to about 2500 Ω at either end.

In order to achieve maximum power transfer, this antenna must be connected to a 75 Ω transmission line at the center point.

For both transmitting and receiving, an antenna is often operated at its **resonant frequency.** Only at this frequency the antenna input impedance is purely resistive (73 Ω). Below this frequency the input impedance is capacitive while above this frequency it is inductive. This behavior is similar to series resonant circuit, hence this antenna is referred as resonant antenna. The resonant frequency is now defined as

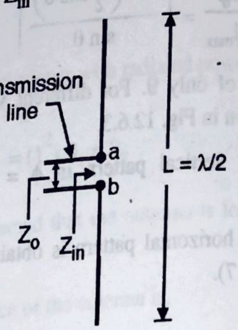
Resonant frequency of the antenna is the frequency at which the antenna input impedance is purely resistive.

12.6.5 Frequency Behaviour of Half Wave Dipole

The half wave dipole of L = λ/2 is shown in Fig. 12.6.5. The half wave dipole has input impedance (Z_{in}) purely resistive only at resonance frequency for which it is designed.

$$Z_{in} \simeq 73 + j\,0\ (\Omega) \qquad \text{... (at resonance)}$$

The connection of the transmission line of the characteristic impedance Z_o same as Z_{in} will result in maximum power radiated.



aw(11.5)Fig. 12.6.5 : Half wave dipole

The input signal applied to the dipole is usually a modulated signal, having carrier frequency (center frequency) equal to resonant frequency of the dipole. So it is a band of frequency to be transmitted.

When the frequency f to be transmitted is less than resonant frequency of the dipole, the input impedance Z_{in} becomes capacitive.

$$Z_{in} = 73 - j\,X\ (\Omega) \quad \text{...(for f < resonance frequency)}$$

The opposite effect is observed when f is greater than resonant frequency.

$$Z_{in} \simeq 73 + j\,X\ (\Omega) \quad \text{...(for f > resonant frequency)}$$

This behaviour is similar to a series resonant circuit.

Because of the capacitive or inductive nature of the antenna at frequency different than resonant frequency, it will mismatch with the transmission line at those frequencies. It results in distortion. The range of frequencies over which half wave dipole operates without distortion or efficiently is very small. It is typically 2% of the center frequency, which is very small.

Conclusion

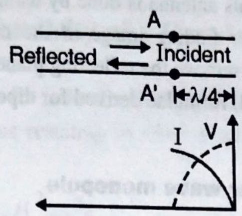
A half wave dipole :

- **Behaves like series resonant circuit.**
- **The bandwidth of operation is very small, typically 2 % of the center frequency.**

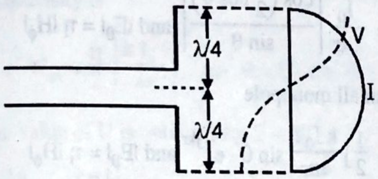
12.6.6 $\frac{\lambda}{2}$ Dipole as a Standing Wave Antenna

Consider a transmission line with characteristic impedance Z_o is open circuited at the far end ($Z_L = \infty$). This forms a mismatched transmission line having incident and reflected waves present on it. The resulting standing wave pattern is shown in Fig. 12.6.6(a).

This open circuit transmission line if flared out at points (A-A′) $\lambda/4$ from the end, the result is shown in Fig. 12.6.6(b). It is surprising to note that it is a $\lambda/2$ dipole.



(a) Open circuit transmission line



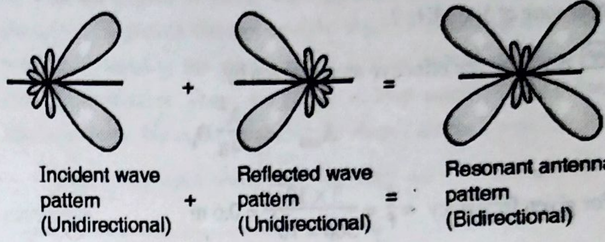
(b) Flared out transmission line

aw(10.6)Fig. 12.6.6 : Conversion of open circuit line into $\lambda/2$ dipole

Now consider the incident and reflected waves one at a time. The incident wave which travels towards right gives a radiation pattern similar to that of long wire antenna in Fig. 12.6.6.

For the reflected wave travelling towards left has a radiation pattern similar but in reverse direction. These two patterns are shown in Fig. 12.6.7(a), (b).

The combination of these two patterns will result in radiation pattern of resonant antenna as shown in Fig. 12.6.7(c). We find this pattern is bi-directional.



aw(10.7)Fig. 12.6.7 : (a) Radiation pattern of incident wave
(b) Radiation pattern of reflected wave
(c) Radiation pattern of $\lambda/2$ dipole

12.6.7 Comparison between Dipoles

A linear dipole is said to

Infinitesimal when $l \le \lambda / 50$

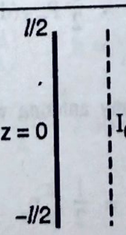
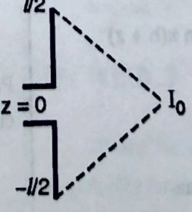
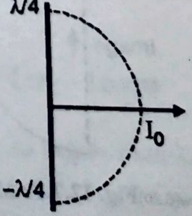
Small when $\lambda / 50 < l \le \lambda/10$

Finite when l is any length.

where l is the length of the dipole.

Comparison

Short Half wave

Criteria	Infinitesimal	Small	$\lambda/2$ dipole
Length	$l \le \lambda / 50$	$\lambda / 50 < l \le \lambda/10$	$l = \lambda/2$
Current over the length	Constant	Triangular	Sinusoidal
Expression for current	For $-l/2 \le z \le l/2$ $I = I_0$	For $0 \le z \le l/2$ $I = I_0\left(1 - \frac{2}{l}z\right)$ For $-l/2 \le z \le 0$ $I = I_0\left(1 + \frac{2}{l}z\right)$	For $0 \le z \le l/2$ $I = I_0 \sin k\left(\frac{\lambda}{4} - z\right)$ For $-l/2 \le z \le 0$ $I = I_0 \sin k\left(\frac{\lambda}{4} + z\right)$
Waveform			

Criteria	Infinitesimal	Small	λ/2 dipole
Radiation resistance R_r	$R_r = 80\pi^2 (l/\lambda)^2$	$R_r = 20\pi^2 (l/\lambda)^2$	$R_r = 73\ \Omega$
Effective length l_e	$l_e = l$	0.5 l	λ/π
Directivity D_0	$D_0 = 1.5$	1.5	1.643

Ex. 12.6.1 : What is the effective area of a half wave dipole operating at 500 MHz ?

Soln. : The effective area is given by

$$A_{em} = \frac{\lambda^2}{4\pi} D_0$$

For given frequency $= \frac{c}{f} = \frac{3 \times 10^8}{500 \times 10^6} = 0.6$ m

For half wave dipole

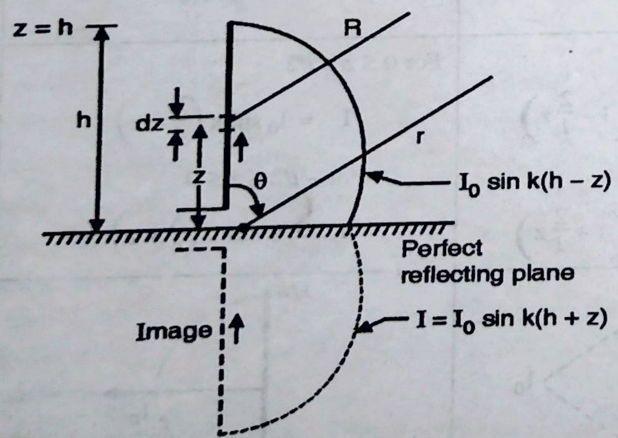
$$D_0 = 1.643$$

$$\therefore \quad A_{em} = \frac{(0.6)^2}{4\pi} \times 1.643 = 0.047\ (m^2)$$

12.7 MONOPOLE ANTENNA

In dipole antenna we have seen that it is center fed antenna. It has two sections of length $\frac{l}{2}$ each. One end of the dipole behaves like positive charge and the other like negative charge. The charge at the ends is continuously oscillating.

Monopole antenna consists of only one section of length, h. When the length $h = \frac{\lambda}{4}$, it is called monopole $\frac{\lambda}{4}$ antenna. The monopole antenna is placed vertically on a perfect conducting (reflecting) plane. It is fed against this plane as shown in Fig. 12.7.1.



aw(6.20)**Fig. 12.7.1 : Monopole antenna**

12.7.1 Field of a Monopole

Since monopole antenna is standing on a perfect reflecting plane, analysis of this antenna is done by using method of images. In this method after finding image of the monopole below the reflecting plane, we remove the reflecting plane. Now this structure looks like dipole. All formulae derived for dipole can be applied to monopole.

For a quarter wave monopole

Field is same as for λ/2 dipole given by

$$|H_\phi| = \frac{I_0}{2\pi r} \left| \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \right| \text{ and } |E_\theta| = \eta\ |H_\phi| \qquad ...(A)$$

For a small monopole

$$H_\phi = \frac{1}{2} j \frac{k I_0 l}{4\pi r} \sin\theta \cdot e^{-jkr} \text{ and } |E_\theta| = \eta\ |H_\phi| \qquad ...(B)$$

12.7.2 Radiation Resistance of a Monopole Antenna

UQ. Derive radiation resistance of a monopole antenna. **(MU - Dec. 15, 5 Marks)**

The monopole of height h produces the same field strengths above the plane as does the dipole of length $l = 2h$, when both are fed with the same current. However, the monopole radiates only through the hemispherical surface above the plane, so its radiated power is only one-half that of corresponding dipole.

For small dipole with length l, the power radiated is given

$$P_{rad}\text{ (short dipole)} = 10\pi^2 I_0^2 \left(\frac{l}{\lambda}\right)^2 \qquad ...(12.7.1)$$

$$P_{rad}\text{ (monopole)} = \frac{1}{2} P_{rad}\text{ (dipole)} = \left(\frac{1}{2}\right) 10\pi^2 I_0^2 \left(\frac{l}{\lambda}\right)^2 \qquad ...(12.7.2)$$

Power radiated by any antenna with radiation resistance R_r and current I_0 is

$$P_{rad} = \frac{1}{2} I_0^2 R_r \qquad ...(12.7.3)$$

Equating Equation (12.7.2) with Equation (12.7.3)

$$\frac{1}{2} I_0^2 R_r = \left(\frac{1}{2}\right) 10 \pi^2 I_0^2 \left(\frac{l}{\lambda}\right)^2 ; R_r = 10 \pi^2 \left(\frac{l}{\lambda}\right)^2$$

For monopole the length is only half that of dipole,

i.e. $l = 2h$

$$\therefore \quad R_r = 10 \pi^2 \left(\frac{2h}{\lambda}\right)^2 = 40 \pi^2 \left(\frac{h}{\lambda}\right)^2 \quad ...(12.7.4)$$

This formula is applicable for very short antennas only, but they are good approximations for dipoles of length up to quarter wavelength, and monopoles of height upto one-eighth wavelength.

12.7.3 Directivity of a Hertzian Monopole

The expression for field of a Hertzian monopole and Hertzian dipole are same, thus resulting in same average poynting vector given by equation

$$W_{av} = \frac{\eta}{2} \left|\frac{k I_0 l}{4\pi}\right|^2 \frac{\sin^2 \theta}{r^2}$$

The radiation intensity is

$$U = r^2 W_{av} = \frac{\eta}{2} \left|\frac{k I_0 l}{4\pi}\right|^2 \sin^2 \theta = U_0 \sin^2 \theta$$

The maximum value of U is obtained for $\theta = 90°$

$$U_{max} = U|_{\theta = 90°} = U_0$$

Knowing U, we obtain power radiated by,

$$P_{rad} = \int U \, d\Omega = \int_0^{2\pi} \int_0^{\pi/2} U_0 \sin^2 \theta \sin \theta \, d\theta \, d\phi$$

$$P_{rad} = U_0 \times 2\pi \times \frac{2}{3} \times 1 = \frac{4\pi}{3} U_0$$

The limits 0 to $\pi/2$ over θ is due to monopole radiates only in upper hemispheres. Using the definition of directivity

$$D_0 = \frac{4\pi U_{max}}{P_{rad}} = \frac{4\pi U_0}{(4\pi / 3) U_0} = 3$$

UEx. 12.7.1 MU - May 10, 3 Marks.

The height of a monopole antenna is $\lambda/100$. What is the radiation resistance ?

Soln. :

$$R_r = 40 \pi^2 \left(\frac{h}{\lambda}\right)^2 ; \quad h = \frac{\lambda}{100}$$

$$R_r = 40 \pi^2 \left(\frac{\lambda/100}{\lambda}\right)^2 = 40 \pi^2 \left(\frac{1}{100}\right)^2$$

$$R_r = 0.04 \, \Omega$$

12.8 LOOP ANTENNAS, IN GENERAL

UQ. Explain important features of loop antenna. **(MU - May 15, Dec. 16, May 17, May 18, 5 Marks)**

UQ. Write short note on loop antenna. **(MU - May 16, 5 Marks)**

In the chapter on linear wire antennas we showed that when the current is passed through straight wires, it results in radiation. Instead of keeping the wire straight if it is turned into a loop, results in radiation. These are called as loop antennas. Loop can take any shape, but square and circular shapes are very popular.

Depending upon the dimension they are divided into two categories.

1. Electrically small loops
2. Electrically large loops

12.8.1 Electrically Small Loops

When the overall length of the loop is less than one-tenth of a wavelength, it is called as electrically small loops.

For a circular loop with N turns, it is electrically small if $N \times \text{circumference} < (\lambda/10)$.

12.8.2 Properties of Electrically Small Loops

(a) The radiation pattern of these loops will have null direction perpendicular to the plane of the loop. This will be proved in the articles to be followed.

(b) Electrically small loop antennas have small radiation resistance, thus they are poor radiators. Hence very rarely used in transmission.

(c) But they can be used as receiving antennas in radios and pagers.

(d) The radiation resistance can be increased by increasing its size or number of turns.

(e) Another way to increase radiation resistance is to insert within its circumference or perimeter, a ferrite core of very high permeability (μ). This is called as ferrite loop. It increases the magnetic field intensity and hence radiation resistance.

12.8.3 Electrically Large Loops

When the overall length of the loop is about λ, it is called as electrically large loop.

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These loops will have maximum radiation along perpendicular to the loop.

12.8.4 Areas of Applications

Loop antennas are used in applications involving HF (3-30 MHz), VHF (30-300 MHz) and UHF (300-3000 MHz).

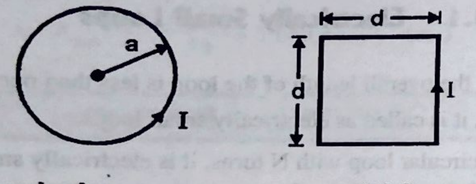
They can also be used in microwave range (1 GHz-100 GHz) as field probes.

12.9 SMALL LOOP ANTENNAS

A closed loop of current whose maximum dimension is less than about a tenth of a wavelength is called a small loop antenna.

It is found that the field pattern of a small circular loop of radius 'a' may be determined from the square loop of same area.

The circular loop of radius 'a' and square loop with side d are shown in Fig. 12.9.1.



(a) Circular loop Area, $A = \pi a^2$ **(b) Square loop** $A = d^2$

aw(7.1)**Fig. 12.9.1 : Circular and square loop**

The radiation pattern of square and circular small loops is same when

$$d^2 = \pi a^2 \qquad \text{...(12.9.1)}$$

12.10 E AND H DUE TO SMALL LOOP ANTENNA

The radiation fields of small loops are independent of the shape of the loop and depend only on the area of the loop. Therefore we will select a square loop as shown in Fig. 12.10.1 to simplify the analysis.

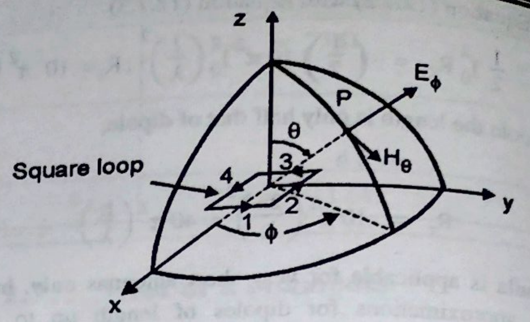
Assumptions

(i) The square loop in the figure is placed in the coordinate system such that center of the loop is at origin and sides parallel to x and y axis.

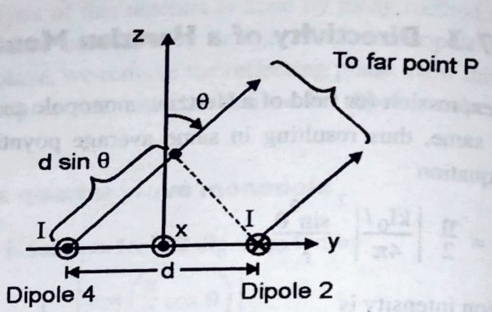
(ii) The current through the loop has constant amplitude I_0 and zero phase around the loop.

(iii) The analysis is restricted within far field region.

(iv) Each side of the square loop is a short uniform electric current segment which is modeled as an ideal dipole.



(a) Square loop at the origin



(b) To find field due to loop

aw(7.2)**Fig. 12.10.1**

At point P in the far field region the field due to dipole 1 and 3 will get cancel. The total field at P is due to dipole 2 and 4 only.

In the dipole analysis, when it is placed along z axis (vertical) it given E_θ and H_ϕ components. Dipoles 2 and 4 are parallel to x axis (horizontal) so the fields due to these will have E_ϕ and H_θ components as shown in Fig. 12.10.1(a).

For simplicity consider point P in yz plane. The radiation due to dipoles 2 and 4 is non-directional in yz plane and can be considered as isotropic sources.

The path difference and phase difference between the radiation due to 2 and 4 is

$$\text{Path difference} = d \sin\theta \qquad \text{...(12.10.1)}$$

$$\text{Phase difference} = K\, d \sin\theta = \psi \qquad \text{...(12.10.2)}$$

Note that K is nothing but β

$$K = \beta = \frac{2\pi}{\lambda} \qquad \text{...(12.10.3)}$$

In the analysis consider origin is the reference for phase. The field due to element 2 is leading since it is closer to P than origin. But due to element 4 it is lagging. Now the fields can be expressed as

$$\text{The field due to dipole 2} = -E_{\phi 0}\, e^{j\psi/2} \qquad \text{....(12.10.4)}$$

The field due to dipole 4 = $E_{\phi 0}\, e^{-j\psi/2}$ ...(12.10.5)

where $E_{\phi 0}$ = magnitude of field due to current I in the dipoles.

The current in element 4 is in + x direction, if it results in positive field then due to 2 is negative, as is done in above two expressions.

Total field is obtained using Superposition

$$E_\phi = -E_{\phi 0}\, e^{j\psi/2} + E_{\phi 0}\, e^{-j\psi/2}$$

$$= -E_{\phi 0}\,[e^{j\psi/2} - e^{-j\psi/2}]$$

$$E_\phi = -E_{\phi 0}\,[2\,j \sin(\psi/2)]$$

$$= -2\,j\,E_{\phi 0} \sin(\psi/2) = -2\,j\,E_{\phi 0} \sin\left[\frac{\beta\, d \sin\theta}{2}\right] \quad ...(12.10.6)$$

In the above result use is made of Equations (12.10.2) and (12.10.3).

The j term in E_ϕ indicates it is in phase quadrature with the individual fields due to dipoles.

$$\text{As } \sin_{x \to 0} x \simeq x$$

When d << λ (due to small loop) then

$$\sin\left(\frac{\beta d \sin\theta}{2}\right) \simeq \frac{\beta\, d \sin\theta}{2}$$

Thus the total field in Equation (12.10.6) reduces to

$$E_\phi = -2\,j\,E_{\phi 0}\left(\frac{\beta\, d \sin\theta}{2}\right)$$

$$= -j\,E_{\phi 0}\,\beta\, d \sin\theta \quad ...(12.10.7)$$

To obtain the value of $E_{\phi 0}$

For the small dipole of length L along z axis as shown in Fig. 12.10.2(a), the electric field is

$$E_\theta = \frac{j\,60\,\pi\, I_0 \sin\theta}{r}\,\frac{L}{\lambda}$$

Here angle θ is measured from axis of the dipole which is along z-axis. But in the present case the dipole axis is parallel to x-axis, resulting in

$$\theta = 90^\circ$$

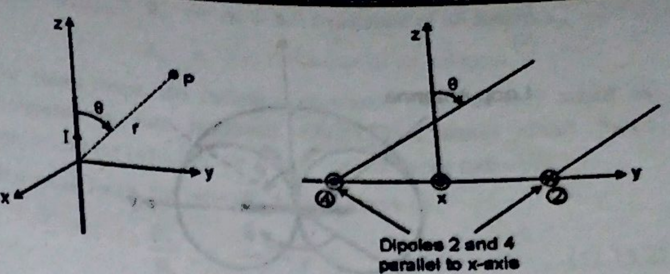
and $\sin 90^\circ = 1$

Thus

$$E_{\phi 0} = \frac{j\,60\,\pi\, I_0\, L}{r\,\lambda}$$

Also the length of dipoles for square loop is d thus replacing L by d

$$E_{\phi 0} = \frac{j\,60\,\pi\, I_0\, d}{r\,\lambda} \quad ...(12.10.8)$$



(a) Short dipole along z-axis (b) Dipoles parallel to x-axis

aw(7.3)**Fig. 12.10.2**

Using Equation (12.10.8) in Equation (12.10.7) and knowing area of the square loop $A = d^2$,

$$E_\phi = \frac{120\,\pi^2\, I_0 \sin\theta}{r}\left(\frac{A}{\lambda^2}\right) \quad ...(12.10.9)$$

From electric field, the magnetic field in free space (η = 120 π) can be obtained as

$$H_\theta = \frac{E_\phi}{\eta} = \frac{1}{(120\pi)} \times \frac{120\,\pi^2\, I_0 \sin\theta}{r}\left(\frac{A}{\lambda^2}\right)$$

or

$$H_\theta = \frac{\pi\, I_0 \sin\theta}{r}\left(\frac{A}{\lambda^2}\right) \quad ...(12.10.10)$$

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So the small loop placed in xy plane results in

For small square or circular loop :

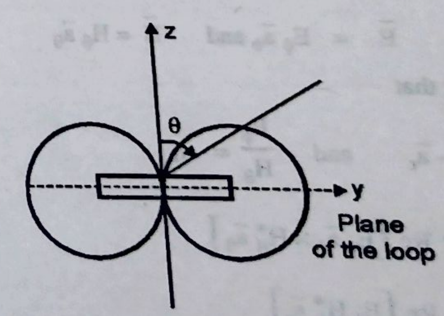
$$E_\phi = \frac{120\,\pi^2\, I_0 \sin\theta}{r}\left(\frac{A}{\lambda^2}\right) \quad ...(12.10.11)$$

$$H_\theta = \frac{\pi\, I_0 \sin\theta}{r}\left(\frac{A}{\lambda^2}\right) \quad ...(12.10.12)$$

Square loop ($A = d^2$); Circular loop ($A = \pi a^2$)

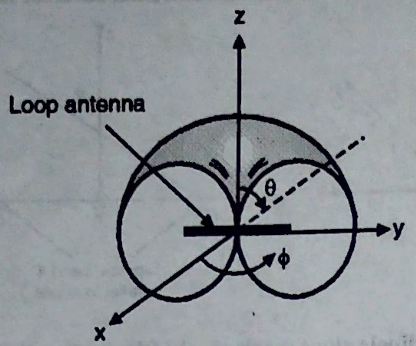
Note that these are the peak values of field components since I_0 is the peak value of current.

12.10.1 Radiation Pattern of the Small Loop



(a) Pattern in yz plane

aw(7.4)**Fig. 12.10.3 Contd...**



(b) Three dimensional view

Fig. 12.10.3

As the field component E_ϕ in Equation (12.10.11) is proportional to $\sin\theta$:

(i) The field should have zero value along $\theta = 0$ (i.e. z-axis)

(ii) The field is maximum when $\theta = 90°$ i.e. along the plane of the loop.

The radiation pattern is shown in Fig. 12.10.3, it has null in the direction of normal to the plane of the loop and maximum in the plane of the loop. It is a typical doughnut shape as shown in Fig. 12.10.3(b).

12.11 THE RADIATION RESISTANCE OF SMALL LOOP

The radiation resistance of any antenna is important from point of view of connection with the transmission line. This section is used to calculate it. We will follow the same steps used to calculate R_r for linear antennas.

Step 1 : To obtain power density (W_{rad})

The average complex Poynting vector

$$\overline{W}_{rad} = \frac{1}{2}\,\mathrm{Re}\left(\overline{E}\times\overline{H}^*\right)$$

From Equations (12.10.11) and (12.10.12) the field components are

$$\overline{E} = E_\phi\,\overline{a}_\phi \text{ and } \quad \overline{H} = H_\theta\,\overline{a}_\theta$$

Also knowing that

$$\overline{a}_\phi \times \overline{a}_\theta = -\overline{a}_r \quad \text{and} \quad \frac{E_\phi}{H_\theta} = -\eta$$

$$\overline{W}_{rad} = \frac{1}{2}\,\mathrm{Re}\left[E_\phi\,\overline{a}_\phi \times H_\theta^*\,\overline{a}_\theta\right]$$

$$= \frac{1}{2}\,\mathrm{Re}\left[E_\phi H_\theta^*\,\overline{a}_r\right]$$

$$= \frac{1}{2}\,\mathrm{Re}\left[\eta\, H_\theta H_\theta^*\,\overline{a}_r\right]$$

$$= \frac{1}{2}|H_\theta|^2\,\mathrm{Re}(\eta)\,\overline{a}_r = \frac{1}{2}\,\eta\,|H_\theta|^2\,\overline{a}_r$$

or
$$W_{rad} = \frac{1}{2}\,\eta\,|H_\theta|^2$$

$$= \frac{1}{2}(120\pi)\left(\frac{\pi I_0 \sin\theta}{r}\right)^2\left(\frac{A}{\lambda^2}\right)^2 \quad \text{...using Equation (12.10.12)}$$

$$W_{rad} = \frac{60\pi^3 I_0^2 \sin^2\theta}{r^2}\left(\frac{A}{\lambda^2}\right)^2 \quad \text{...(12.11.1)}$$

Step 2 : To obtain power radiated (P_{rad})

$$P_{rad} = \oint_{4\pi} W_{rad}\, ds = \int_0^{2\pi}\int_0^{\pi} W_{rad}\,(r^2 \sin\theta\, d\theta\, d\phi)$$

$$= \int_0^{2\pi}\int_0^{\pi} \frac{60\pi^3 I_0^2 \sin^2\theta}{r^2}\left(\frac{A}{\lambda^2}\right)^2 r^2 \sin\theta\, d\theta\, d\phi$$

$$= 60\pi^3 I_0^2\left(\frac{A}{\lambda^2}\right)^2 \int_0^{2\pi}\int_0^{\pi} \sin^3\theta\, d\theta\, d\phi$$

$$P_{rad} = 60\pi^3 I_0^2\left(\frac{A}{\lambda^2}\right)^2 \times 2\pi \times \int_0^{\pi} \sin^3\theta$$

Solving the integral

$$P_{rad} = 60\pi^3 I_0^2\left(\frac{A}{\lambda^2}\right)^2 \times 2\pi \times \left(\frac{4}{3}\right)$$

$$P_{rad} = 160\pi^4 I_0^2\left(\frac{A}{\lambda^2}\right)^2 \quad \text{...(12.11.2)}$$

Step 3 : To obtain R_r

Equating Equation (12.11.2) with $P_{rad} = \frac{1}{2} I_0^2 R_r$

$$\frac{1}{2} I_0^2 R_r = 160\pi^4 I_0^2\left(\frac{A}{\lambda^2}\right)^2 \quad \text{or} \quad R_r = 320\pi^4\left(\frac{A}{\lambda^2}\right)^2$$

or
$$R_r \simeq 31{,}200\left(\frac{A}{\lambda^2}\right)^2 \;(\Omega) \quad \text{...(12.11.3)}$$

For N number of turns in the loop, the total resistance is

$$R_r = 31200\left(\frac{NA}{\lambda^2}\right)^2 \;(\Omega) \quad \text{...(12.11.4)}$$

12.12 DIRECTIVITY OF A SMALL LOOP

The directivity is given by

$$D_0 = \frac{4\pi U_{max}}{P_{rad}} \quad \text{...(12.12.1)}$$

where U_{max} = Maximum radiation intensity (W/Sr) ;

P_{rad} = Power radiated (W)

The radiation intensity is obtained using Equation (12.11.1) and

$$U = r^2 W_{rad} = 60\pi^3 I_0^2 \sin^2\theta \left(\frac{A}{\lambda^2}\right)^2 = U_0 \sin^2\theta$$

The maximum value is achieved when $\theta = 90°$. Thus

$$U_{max} = U_0 \quad ...(12.12.2)$$

The power radiated is given by,

$$P_{rad} = \oint U d\Omega = \int_0^{2\pi}\int_0^{\pi} (U_0 \sin^2\theta) \sin\theta \, d\theta \, d\phi$$

$$= U_0 (\phi)_0^{2\pi} \int_0^{\pi} \sin^3\theta \, d\theta$$

$$P_{rad} = 2\pi U_0 \times 2 \int_0^{\pi/2} \sin^3\theta \, d\theta$$

$$= 4\pi U_0 \times \frac{2}{3} \times 1 = \frac{8\pi U_0}{3} \quad ...(12.12.3)$$

Putting values of U_{max} and P_{rad} in Equation (12.12.1)

$$D_0 = \frac{4\pi \times 60\pi^3 I_0^2 (A/\lambda^2)^2}{160\pi^4 I_0^2 (A/\lambda^2)^2} = \frac{3}{2} \quad ...(12.12.4)$$

12.13 RADIATION EFFICIENCY

The radiation efficiency of an antenna is given by,

$$K = \frac{R_r}{R_r + R_L} = \frac{1}{1 + (R_L/R_r)} \quad ...(12.13.1)$$

where R_r = radiation resistance of an antenna ;

R_L = loss resistance of an antenna

For small loops, the radiation resistance (R_r) is very small as compared to loss resistance (R_L). This makes small loops inefficient for radiation. Or small loops are poor radiators.

For small loops, the ratio (R_L / R_r) is given by,

$$\frac{R_L}{R_r} = \frac{3430}{C^3 f_{MHz}^{3.5} d} \quad ...(12.13.2)$$

where C = perimeter or circumference of a loop (m) ;

d = wire diameter (not loop diameter) (m)

f_{MHz} = frequency in MHz

In spite of low efficiency, there are many applications where such loops are useful in receiving applications provided the received S/N ratio is acceptable.

The radiation resistance can be increased by using number of turns. It is clear that by using N turns, the radiation resistance (R_r) increases in proportion to n^2. But the loss resistance increases in proportion to N. For this loop,

$$\frac{R_L}{R_r} = \frac{3430}{C^3 f_{MHz}^{3.5} N d}$$

Thus as n increases, the ratio R_L / R_r decreases and the efficiency in Equation (12.13.1) increases.

$$K = \frac{1}{1 + (R_L / R_r)}$$

Module 3

Chapter Ends...

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