Lecture 31: Arrays- Introduction — chapter-04

23 March 2021 09:37

Antenna Arrays

→ chapter-03 → radⁿ charac. of single element antennas were discussed & analyzed.

→ wire antennas
- linear ant
  - dipole
    - infinite small dipole
    - small dip
    - half wave dipo
  - monopole
  - folded dipole
- loop
  - small
  - large
  - $\bar{E} = E_\phi$
  - $\bar{H} = H_r \& H_\theta$
- Helical

analysis of dipole:
1. c/n eqⁿ
2. $\bar{E}$ & $\bar{H}$
3. R_r
4. D

→ The radⁿ pattern of a single element is relatively wide & each element provides low value of directivity (gain)

→ Many appl → need to design antennas with very directive characteristics (very high gain)

⇓

so as to meet the requirement of long distance commⁿ

⇑

1. by increasing the electrical size of the antenna

Case (i)

→ Enlarging the dimensions of single elements → often lead to more directive charac.

(ii)

→ Another way to enlarge the dimensions of the antenna, w/o necessarily increasing the size of the individual element, is to form an assembly of the individual (radiating) elements in an electrical & geometrical configuration

⇓

This new antenna, formed by multielements is referred to as an ARRAY

Note:- ① In most cases, elements of an array are identical

② The individual elements of an array may of any form (wire, aperture, etc)

Lecture 32: Antenna Arrays

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Continuation

(i) The total field of the array is determined by the vector addition of the fields radiated by the individual elements.

⇓

This assumes that c/n in each element is the same as that of the isolated element.

(ii) To provide very directive patterns, it is necessary that the fields from the elements of the array interfere constructively (add) in the desired directions & interfere destructively (cancel each other) in the remaining space.

* The shape of overall pattern of the antenna (array) is influenced by

(i) The geometrical configuration of the overall array (linear, circular, rectangular, ...)

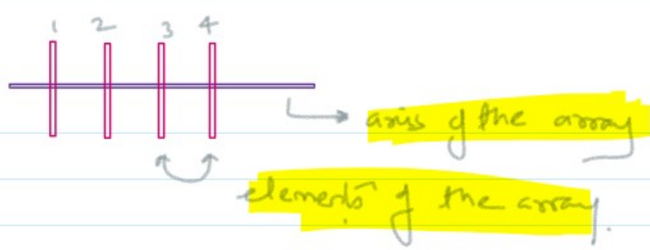
(ii) The relative displacement between the elements

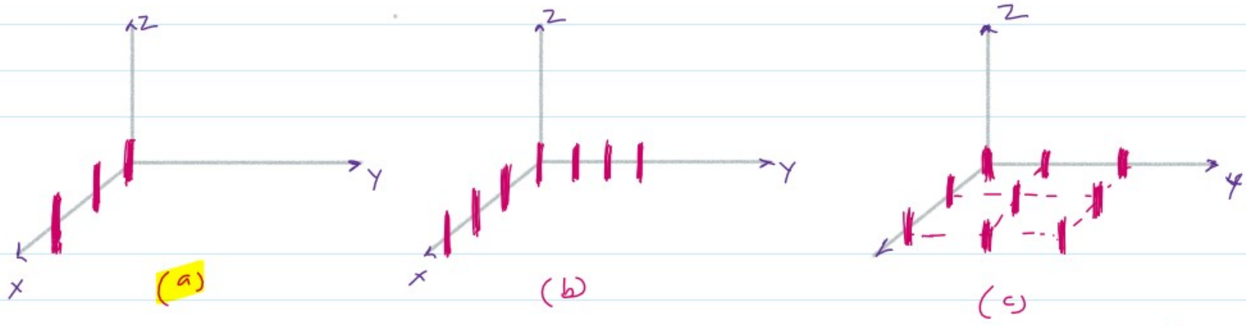
(iii) The excitation amplitude of the individual elements

(iv) The excitation phase of the individual elements

(v) The relative pattern of the individual elements.

* Note :- The antenna array is an arrangement of several individual antennas so placed & spaced that their individual electric field contributions combine in one preferred direction & get cancelled in all other directions.

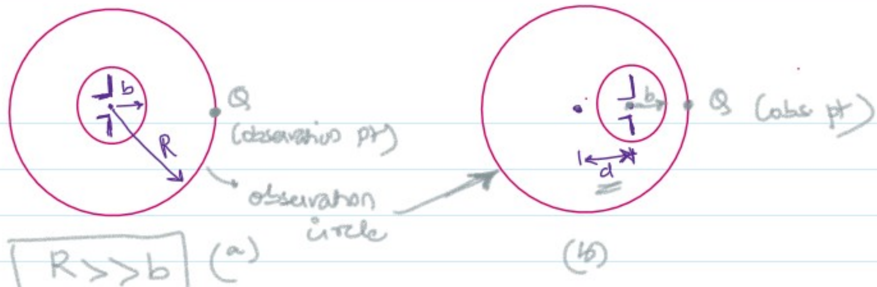




single dimensional array

two dimensional array

* Antenna as a point source.



$\boxed{R >> b}$

The distance 'd' has negligible effect on the field pattern provided

$$R >> b \quad ; \quad R >> d \; ; \quad R >> \lambda$$

However the phase pattern (i.e phase variation around the observation pt circle) will generally differ depending on the value

However the phase pattern (i.e phase variation around the observation pt circle) will generally differ depending on the value 'd'.

$\Downarrow$

It increases with $\uparrow$ in d

Lecture 33: Array of point sources

01 April 2021 10:40

→ The five cases involving two isotropic pt sources are as follows

(i) Two isotropic point sources of same amplitude & phase (BROADSIDE ARRAY)

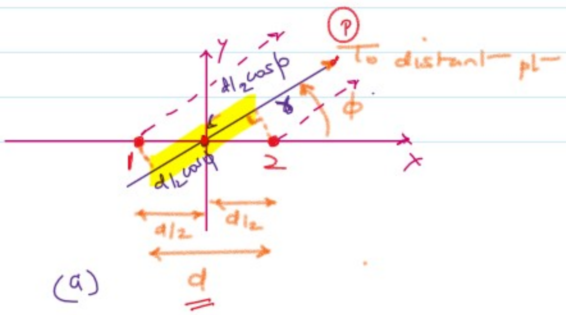
(ii) Two isotropic point sources of same amplitude but opposite phase (ENDFIRE ARRAY)

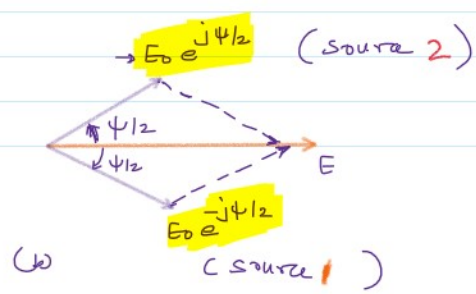
(iii) Two isotropic point sources of same amplitude & in phase quadrature

(iv) Two isotropic point sources of equal amplitude & any phase difference

(v) Two isotropic point sources of unequal amplitude & any phase difference

[I] Two isotropic point sources of same amplitude & phase.





Source 1 & 2 → Same amp. & phase

→ At a distant pt in the direction of ϕ the field from source 1 is retarded by $\frac{d_r}{2}\cos\phi$ while the field from source 2 is advanced by $\frac{d_r \cos\phi}{2}$

where d_r is the distance between the sources expressed in radians

$$\therefore \boxed{d_r = \beta d = \frac{2\pi}{\lambda} d}$$

The total field at a large distance r in the direction ϕ is given by

$$E = E_0 e^{j\psi/2} + E_0 e^{-j\psi/2} \quad \text{--- (1)}$$

where $\psi = d_r \cos\phi$

E_0 = amplitude of the field component

$$E = 2E_0 \left[e^{j\psi/2} + e^{-j\psi/2} \right]$$

$$E = 2E_0 \left[\frac{e^{j\psi/2} + e^{-j\psi/2}}{2} \right]$$

$$E = 2E_0 \cos(\psi/2) \quad \text{——} \quad (2)$$

$$E = 2E_0 \cos\left(\frac{d_r \cos\phi}{2} \right) \quad \text{——} \quad (3)$$

Normalizing eqn (3) $\rightarrow$ set $2E_0 = 1$

$$E = \cos\left(\frac{d_r \cos\phi}{2} \right)$$

$$d_r = \beta d = \frac{2\pi}{\lambda} \cdot d \quad \rightarrow \text{for } d = \lambda/2$$

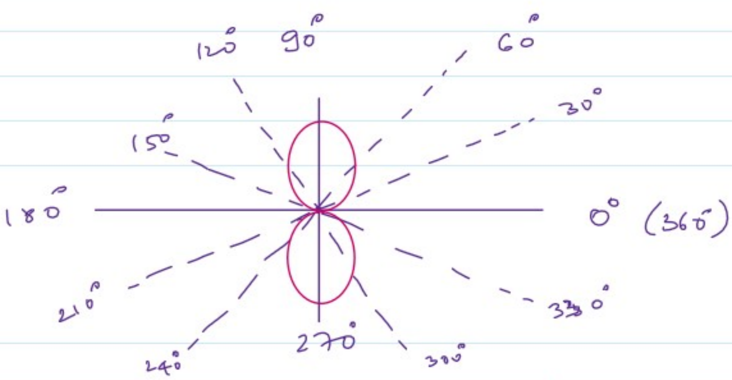
$$\therefore d_r = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} = \pi$$

$E = |E| e^{j\phi}$

phase factor

Array factor $\rightarrow$

$$\boxed{\underline{E} = \cos\left(\frac{\pi}{2} \cdot \cos\phi \right) \quad \text{——} \quad (4)}$$



for $\phi = 0°$

$$\bar{E} = \cos\left(\pi/2 \cdot \cos(0) \right)$$

$$\boxed{E = \cos\frac{\pi}{2} = 0}$$

for $\phi = 90°$

$$\boxed{E = 1}$$

$\rightarrow$ BROADSIDE ARRAY $\rightarrow$ 2 pt sources

Lecture 34: Array of point sources

05 April 2021 13:25

Continuation

Case (i) → Two isot. pt sources of same amp & phase

→ Two isotropic point sources of same amplitude & phase

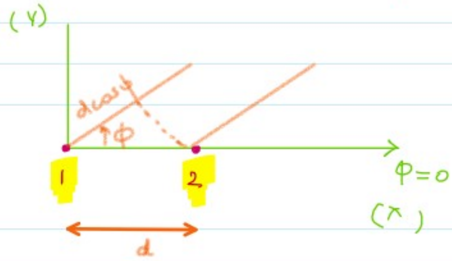


fig (a)

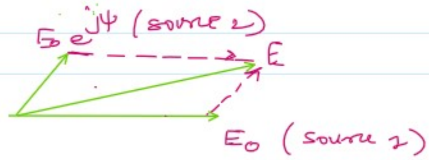


fig (b)

Source 1 → considered as reference

$$E = E_0 + E_0 e^{j\psi} \quad \text{--- (5)}$$

where $\psi = d_r \cos\phi$

$$E = E_0(1+e^{j\psi}) = \frac{2 \cdot E_0 e^{j\psi/2}\left(e^{-j\psi/2}+e^{j\psi/2}\right)}{2}$$

$$E = 2E_0 \cdot \cos(\psi/2)\, e^{j\psi/2}$$

$$\therefore E = 2E_0 \cdot e^{j\psi/2} \cos(\psi/2) \quad \text{--- (6)}$$

Normalizing eqn (6) ⇒ setting $2E_0 = 1$, in eqn (6) can be written as

$$E = e^{j\psi/2} \cdot \cos(\psi/2) \quad \text{--- (7)}$$

$$= \cos(\psi/2) \angle \psi/2$$

⇒ Thus, an observer at a fixed distance observes no phase change when the array is rotated (with respect to φ) around its midpoint, but a phase change is observed if the array is rotated with source 1 as the centre of rotation

(II) Two isotropic point sources of same amplitude & opposite phase.

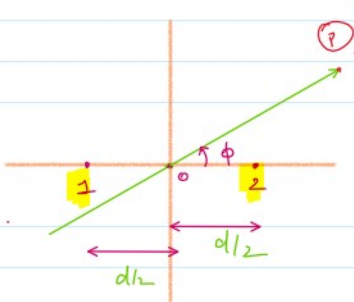


fig (c)

$$\rightarrow E = E_0 \cdot e^{j\psi/2} - E_0 e^{-j\psi/2} \quad \text{--- (1)}$$

$$E = 2E_0 \left(\frac{e^{j\psi/2} - e^{-j\psi/2}}{2}\right)$$

$$\therefore E = 2j\, E_0 \cdot \sin(\psi/2)$$

$$= 2j\, E_0 \cdot \sin\left(\frac{d_r \cos\phi}{2}\right) \quad \text{--- (2)}$$

where $\psi = d_r \cos\phi$

$= 2j\,E_0 \sin\left(\frac{\pi \cdot \cos\phi}{2}\right)$ —— (2)

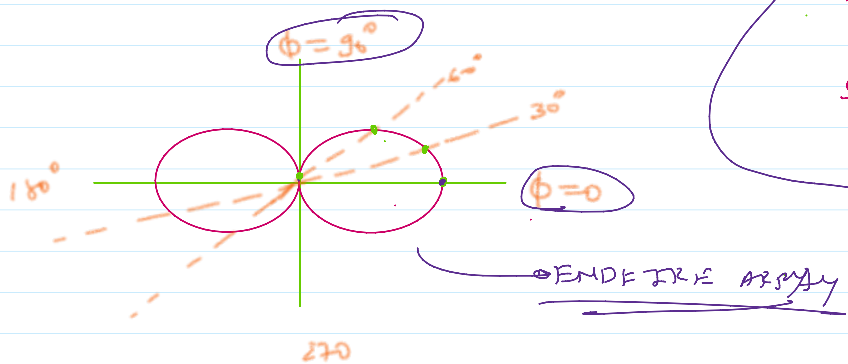
where $\psi = d_r \cos\phi$

$d_r = \beta d = \frac{2\pi}{\lambda} \cdot d.$

Setting $2E_0 j = 1$ (Normalizing)

$d_r = \beta d = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2}$

Array factor (AF) $\therefore$ $\boxed{E = \sin\left(\frac{d_r \cos\phi}{2}\right) = \sin\left(\frac{\pi}{2} \cdot \cos\phi\right)}$ —— (3)



$\phi = 30$

$\sin\left(\frac{\pi}{2} \cdot \cos 30^\circ\right)$

$\boxed{\phi = 0 \rightarrow E = 1}$

$\phi = 90 \rightarrow E = 0$

(III) Two isotropic point sources of same amplitude & in phase quadrature

(Two isotropic pt sources of the same amplitude & one is retarded by 45° & another source advanced by 45°.

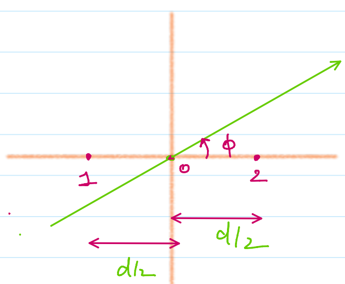


fig (a)

$$E = E_0 \cdot e^{j\left(\frac{\psi}{2} + \frac{\pi}{4}\right)} + E_0 \cdot e^{-j\left(\frac{\psi}{2} + \frac{\pi}{4}\right)} \quad \text{—— (1)}$$

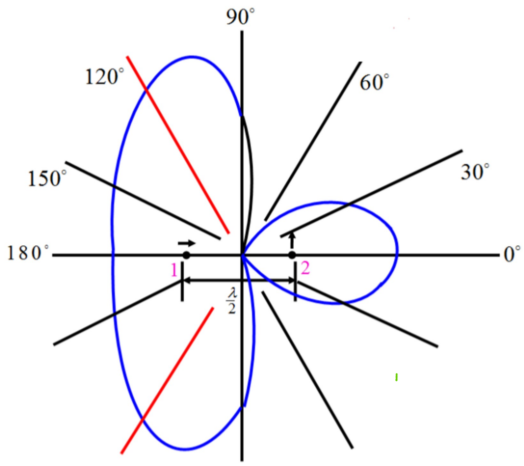
where $\psi = d_r \cos\phi$

$$E = 2E_0 \cos\left(\frac{\pi}{4} + \frac{d_r \cos\phi}{2}\right) \quad \text{—— (2)}$$

setting $2E_0 = 1$

$$E = \cos\Big(\underbrace{\frac{\pi}{4} + \frac{\pi}{2}\cos\phi}_{\text{argument}}\Big) \quad \{ d = \lambda/2 \} \quad \text{—— (3)}$$

$\therefore$ The field pattern of eqn (3) is as given below



$$E = \cos\left(\frac{\pi}{4} + \frac{\pi}{2}\cos\phi\right)$$

ϕ	0°	60°	90°	120°	180°
E	$1/\sqrt{2}$	0	$1/\sqrt{2}$	1	$1/\sqrt{2}$

The direction ϕ_M of max. field are obtained by setting argument of eq^n (3) equal to $K\pi$ where $K = 0, 1, 2, 3, \ldots\ldots$

$$\therefore \quad \frac{\pi}{4} + \frac{\pi}{2}\cos\phi_M = K\pi$$

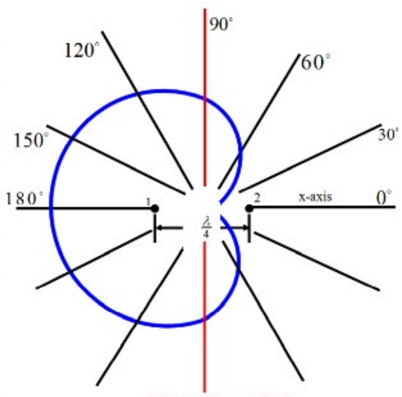
$$\therefore \quad \frac{\pi}{2}\cos\phi_M = -\frac{\pi}{4} \longrightarrow \{K=0\}$$

$$\boxed{\therefore \quad \phi_M = 120^\circ \ \& \ 240^\circ}$$

If $d = \lambda/4$, eq^n (3) becomes. $\left\{ E = \cos\left(\frac{\pi}{4} + \frac{d_r\cos\phi}{2}\right) \right\}$

for $d = \lambda/4 \Rightarrow d_r = \beta d = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi/2}{2}$

$$\boxed{E = \cos\left(\frac{\pi}{4} + \frac{\pi}{4}\cdot\cos\phi\right)}$$



HPBW = 180°

Spacing between the sources is reduced to λ/4

$$E = \cos\left(\frac{\pi}{4} + \frac{\pi}{4}\cos\phi\right)$$

ϕ	**0°**	**90°**	**120°**	**150°**	**180°**
E	**0**	$1/\sqrt{2}$	**0.924**	**0.994**	**1**

(IV) Two isotropic point sources of equal amplitude & any phase difference

The 2 pt sources are of equal amplitude & any phase difference (δ)

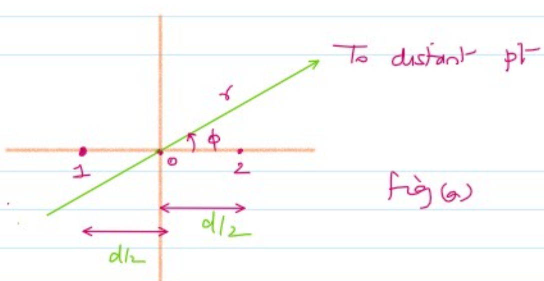


fig (a)

The total phase difference ψ is equal to

$$\psi = d_r\cos\phi + \delta \quad \text{--- (1)}$$

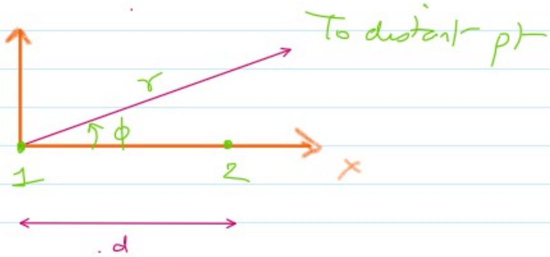
$$E = E_0\left[e^{j\psi/2} + e^{-j\psi/2}\right] = 2E_0\cos\psi/2 \quad \text{--- (2)}$$

Normalizing eq^n (2) $\{$setting $2E_0 = 1\}$

Normalizing eqn (2) {setting $2E_0 = 1$}

(AF) → $\boxed{\therefore\ E = \cos \psi/2}$ —— (3) {where value of ψ is given by eqn (1)}

V) Two isotropic pt sources of unequal amplitude & any phase difference



Assume source 1 has larger amplitude say E_0 at a larger distance 'r'. Field from source 2 has an amplitude aE_0 $(0 \le a \le 1)$

∴ The magnitude & phase of the total field (E) is given by

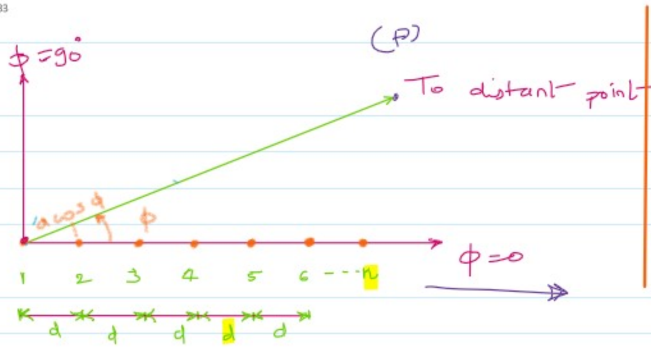
$$\boxed{\begin{aligned} E &= E_0\sqrt{(1+a\cos\psi)^2 + a^2\sin^2\psi} \\ &\angle \arctan\left[a\sin\psi\,(1+a\cos\phi)\right) \end{aligned}}$$ —— eqn (1)

where

$$\psi = dr\cos\phi + \delta$$

Lecture 36: Linear array of n isotropic point sources

08 April 2021 10:33



Consider n isotropic point source of equal amplitude & spacing arranged as linear array as shown in the fig.

Let E_0 be the amp. of source & ψ is the total phase diff of the fields from adjacent sources

The total field E at a large distance in the direction ϕ is given by (with source 1 as ref)

$$E = E_0 + E_0 e^{j\psi} + E_0 e^{j2\psi} + E_0 e^{j3\psi} + \cdots + E_0 e^{(n-1)j\psi} \quad \text{--- (1)}$$

where $\psi = \beta d \cos\phi + \delta$

$$\psi = d_r \cdot \cos\phi + \delta \quad \text{--- (2)} \qquad \left\{ \beta = \frac{2\pi}{\lambda} \right\}$$

Here $\delta \rightarrow$ phase diff of adjacent sources (i.e. source 2 w.r.t 1, source 3 w.r.t 2)

$$E e^{j\psi} = E_0 \left[e^{j\psi} + e^{2j\psi} + e^{3j\psi} + \cdots\cdots e^{jn\psi} \right] \quad \text{--- (3)}$$

Subtracting eq^n (3) from eq^n (1)

$$E(1 - e^{j\psi}) = E_0 \left[1 - e^{jn\psi}\right] \quad \text{--- (4)}$$

$$E = \frac{E_0 (1 - e^{jn\psi})}{(1 - e^{j\psi})}$$

Eq^n (4) can be written as

$$E = \frac{E_0 \cdot e^{jn\psi/2} \left(e^{jn\psi/2} - e^{-jn\psi/2}\right)}{e^{j\psi/2}\left(e^{j\psi/2} - e^{-j\psi/2}\right)} \quad \text{--- (5)}$$

$$E = E_0 \cdot \underbrace{e^{j(n-1)\psi/2}} \; \frac{\sin(n\psi/2)}{\sin(\psi/2)}$$

$$E = E_0 \cdot e^{j\xi} \frac{\sin(n\psi/2)}{\sin(\psi/2)} \quad \text{--- (6)}$$

where $\xi = \frac{(n-1)\psi}{2}$ --- (7) & ξ is referred as the field from source 1.

$$E = \underbrace{E_0 \frac{\sin(n\psi/2)}{\sin(\psi/2)}}_{\text{mag}} \; \underbrace{\angle \xi}_{\text{phase}} \quad \text{--- (8)}$$

In case the ref. pt is shifted to the centre of the array then phase (i.e. $\xi = (n-1)\psi$ is automatically eliminated & eq (8) reduces to

$\sin(\psi/2)$

In case the ref. pt is shifted to the centre of the array then phase $\angle$ ie $\xi = \frac{(n-1)\psi}{2}$ is automatically eliminated & eq^n (8) reduces to

$$\boxed{E = E_0 \frac{\sin(n\psi/2)}{\sin(\psi/2)}} \quad \text{——— (9)}$$

Now, when $\psi = 0$ eq^n (9) & eq^n (8) becomes indeterminant hence L Hospital rule is applied to evaluate the fn^c according to which the num. & denomi will be separately differentiated under $\psi \to 0$

$$\lim_{\psi \to 0} E = E_0 \lim_{\psi \to 0} \frac{d/d\psi \sin(n\psi/2)}{d/d\psi \sin(\psi/2)}$$

$$= E_0 \lim_{\psi \to 0} \frac{\cos(n\psi/2)\cdot(n/2)}{\cos(\psi/2)\cdot(1/2)}$$

'n' or 'N'

$$\therefore E_{max} = E_0 \cdot n = E_0 \cdot N \quad \text{——— (10)}$$

Thus max. value is N times from a single source. If E_0 is assumed to be unity

$$E_{max} = n \quad \text{——— (11)}$$

Hence normalized value is given by { eq^n (9) & (10) }

$$E_{nor.} = \frac{E}{E_{max}} = \frac{E_0 \sin(n\psi/2)}{E_0 \sin(\psi/2)\cdot n} \quad \text{——— (12)}$$

$$\boxed{\therefore E_{norm} = \frac{\sin(n\psi/2)}{n\cdot\sin(\psi/2)}} \quad \text{——— (13)}$$

The field given by eq^n (13) → referred to as ARRAY FACTOR

* Case (i) :- Broadside Array (sources in phase)

(i) elements are equally spaced
(ii) each element is fed with ~~un~~ of equal amp & same phase.

→ This arrangement gives max. field in a direction normal to the array (refer case (i) of 2 isotropic pt sources)

⇓ this cond^n results into

BROADSIDE type of array ($\delta = 0$)

For an array of n-isotropic pt sources, we have.

$$E = E_0 \cdot \frac{\sin(n\psi/2)}{\sin(\psi/2)} \quad \text{——— (1)}$$ { with centre pt of an array as ref }

from eq^n (9)

→ where $\psi = \beta d \cos\phi + \delta$

$\boxed{(\delta = 0) \text{ here}}$

$\psi = \beta d \cos\phi = \frac{2\pi}{\lambda} \cdot d \cos\phi$

Here → sources are in phase →

$\left.\begin{array}{l} \delta = 0 \\ \& \quad \psi = d_r \cos\phi \end{array}\right\}$ — (2)

For max field $\psi = 0$ — (3)

In order to satisfy (2) & (3)

$d_r \cos\phi = 0$

$\phi = 90^\circ$ or 270°

Thus the max. field is in a direction normal to the array.

Hence this condn, which is characterized by in phase source ($\delta = 0$)

⇓

results in a BROADSIDE TYPE OF ARRAY

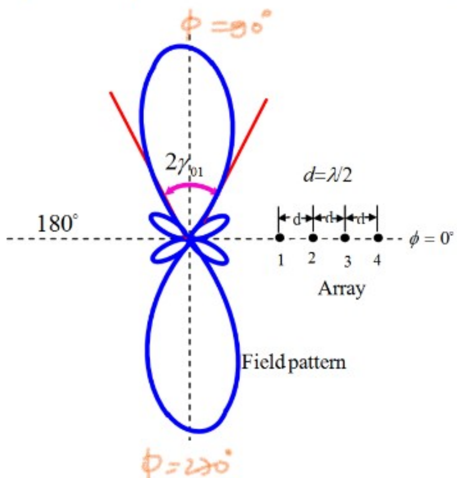
The pattern of 4 isotropic in phase pt sources with $d = \lambda/2$ & equal amplitude is shown below.

$$\psi = \frac{2\pi d}{\lambda}\cos\phi + \delta$$

$\delta = 0$, $d = \frac{\lambda}{2}$ and n = 4

$\psi = \pi \cos\phi \qquad E = \dfrac{\sin(n\psi/2)}{n \sin(\psi/2)}$

φ	Ψ	E
0°	π	0
60°	π/2	0
90°	0	1



n=4

Lecture 37: Broadside and Endfire arrays

08 April 2021 14:19

* Endfire Array

→ (i) Individual elements are equally spaced along a line
(ii) each element is fed with current of equal amplitude & opposite phase.

End fire may be defined as the arrangement in which the principal direction of radiation coincides with the direction of the array axis

w.k.t., for an array of n isotropic pt sources, total field is given by,

$$E = E_0 \frac{\sin(n\psi/2)}{\sin(\psi/2)} \quad \text{— (1)}$$

{from eqⁿ 9}
(with centre pt of an array as reference)

where

$$\psi = \beta d \cdot \cos\phi + \delta$$
$$= \frac{2\pi}{\lambda} d \cos\phi + \delta = d_r \cos\phi + \delta \quad \text{— (2)}$$

To obtain max field in the direction of array ($\phi = 0$)

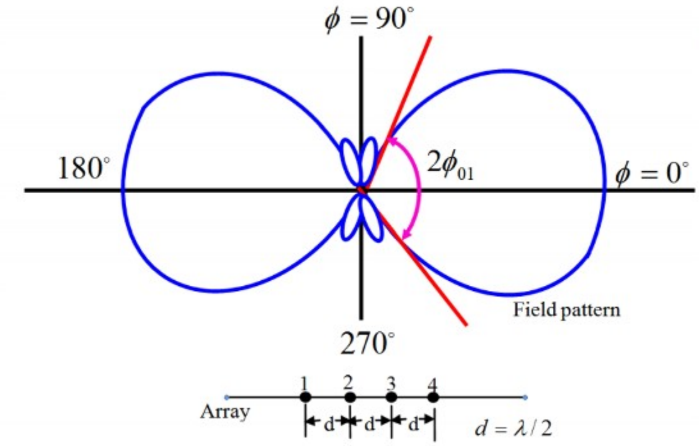
Note :- for max field $\psi = 0$

Substitute $\psi = 0$ & $\phi = 0$ in eqⁿ (2)

$$0 = d_r \cdot \cos 0 + \delta$$

$$\boxed{\delta = -d_r}$$

Hence, for an end-fire array the phase between the sources is retarded progressively by the same amount as the spacing between the sources in radians



[redacted]

$$\psi = \frac{2\pi d}{\lambda}\cos\phi + \delta$$

For $d = \lambda/2$, $\phi = 0^0$ and $\Psi = 0$

$$\delta = -\pi$$

$$\psi = \pi(\cos\phi - 1)$$

BWFN=120°

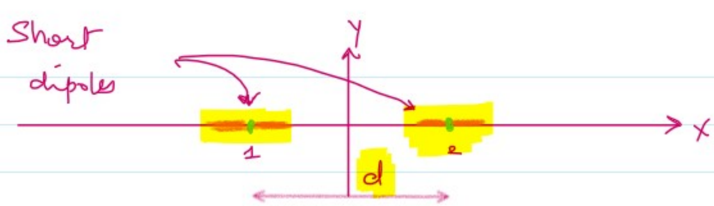
Field pattern of ordinary end-fire array of 4 isotropic point sources of same amplitude. Spacing is $\lambda/2$ and the phase angle $\delta = -\pi$.

* Pattern Multiplication

→ Non isotropic but similar point sources ←

If the variation with absolute < le ϕ of both the amplitude & phase of the field is the same i.e. pattern have same shape & oriented in the same direction & the max amplitudes of the individual sources may be unequal are called as similar point sources

If however amplitudes are equal then the sources are not only similar but are identical



Consider 2 non-isotropic sources oriented parallel to the x-axis as shown in the fig above.

The individual field patterns are given by

$$E_0 = E_0' \sin\phi \quad \text{—— (1)}$$

The total field of 2 isotropic pt sources of equal amp & any phase diff is

The total field of 2 isotropic pt sources of equal amp & any phase diff is given by (refer case iv)

$$E = 2E_0 \cdot \cos\psi/2 \quad \text{--- (2)}$$

where $\psi = d_r\cos\phi + \delta$

Substitute eqn (1) in (2)

$$E = 2(\underbrace{E_0'\sin\phi}_{E_0}) \cdot \cos\psi/2$$

$$\boxed{E = 2E_0'\sin\phi\cos\psi/2} \quad \text{--- (3)}$$

setting $2E_0' = 1$

$$E = \underbrace{\sin\phi}_{\text{individual pt source}} \underbrace{\cos(\psi/2)}_{\text{rad pattern of case(4)}} \quad \text{--- (4)}$$

The same results can be obtained by multiplying the pattern of the individual source ($\sin\phi$) by the pattern of 2 isotropic pt sources ($\cos\psi/2$)

∴ The principle of PATTERN MULTIPLICATION

can be expressed as

The total field pattern of an array of non-isotropic similar sources is the product of the individual source pattern & the pattern of an array of isotropic pt sources each located at the phase centre of the individual source & having the same relative amp & phase, & the total phase pattern referred to the phase centre of the array is the sum of the phase patterns of the individual source & the array of isotropic pt source

The total field E is given by.

$$E = f(\theta,\phi) \cdot F(\theta,\phi) \; \Big/ \; f_p(\theta,\phi) + F_p(\theta,\phi) \quad \text{--- (5)}$$

where

$f(\theta,\phi) \rightarrow$ field pattern of individual source where

$f(\theta, \phi) \rightarrow$ field pattern of individual source

$f_p(\theta, \phi) \rightarrow$ phase — n. of individual source

$F(\theta, \phi) \rightarrow$ field pattern of array of isotropic source

$F_p(\theta, \phi) \rightarrow$ phase pattern of array of — n —

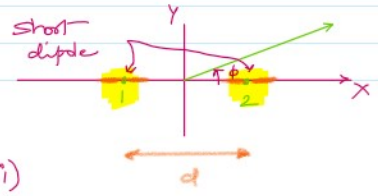
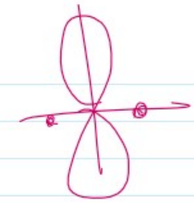


fig (i)



(i) Assume 2 identical pt sources seperated by a distance d,
(ii) Each source having a field pattern given by

$$E_0 = E_0' \sin\phi$$

Let $d = \lambda/2$ & $\delta = 0$

$d = \lambda/2$
δ

∴ The total field pattern is given by

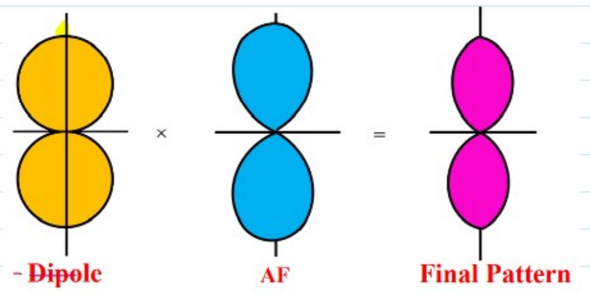
$$E = \sin\phi \cos\left(\frac{\pi}{2}\cos\phi\right) \longrightarrow \text{Normalized eq}^n$$

$\underbrace{\cos\left(\frac{\pi}{2}\cos\phi\right)}$ ⟶ Case(i)

2 $\overset{\text{iso}}{\text{pt}}$ src of same amp & same phase

$$E = \cos\left(\frac{\pi}{2}\cos\phi\right)$$

Note :-
The product of individual source pattern ($\sin\phi$) & the array pattern AF → ($\cos(\frac{\pi}{2}\cos\phi)$) ⟶ resultant field pattern



Horizontal Dipole: $E_0 = E'_0 \sin\phi$

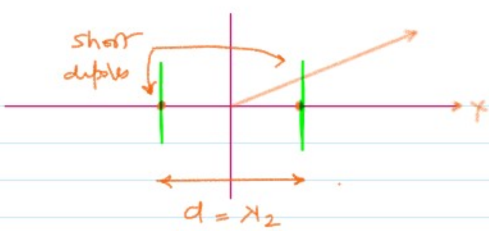
$$AF = \cos(\psi/2)$$

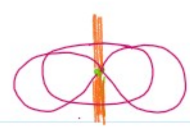
$$E = \sin\phi \cos\frac{\psi}{2}$$

where, $\psi = d_r\cos\phi + \delta$

For $\delta = 0$, Array Factor (AF) will give max. radiation in Broadside Direction

*

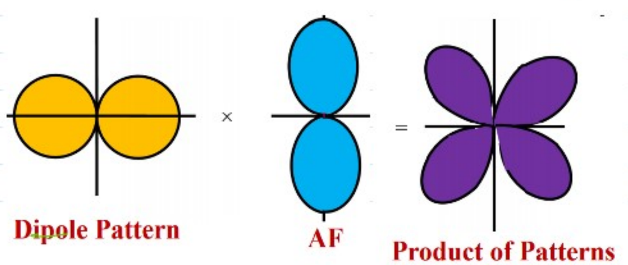




(i) Consider array of 2 non-isotropic sources seperated by a distance $d = \lambda/2$ & $\delta = 0$ (same as previous case)

(ii) The individual source pattern

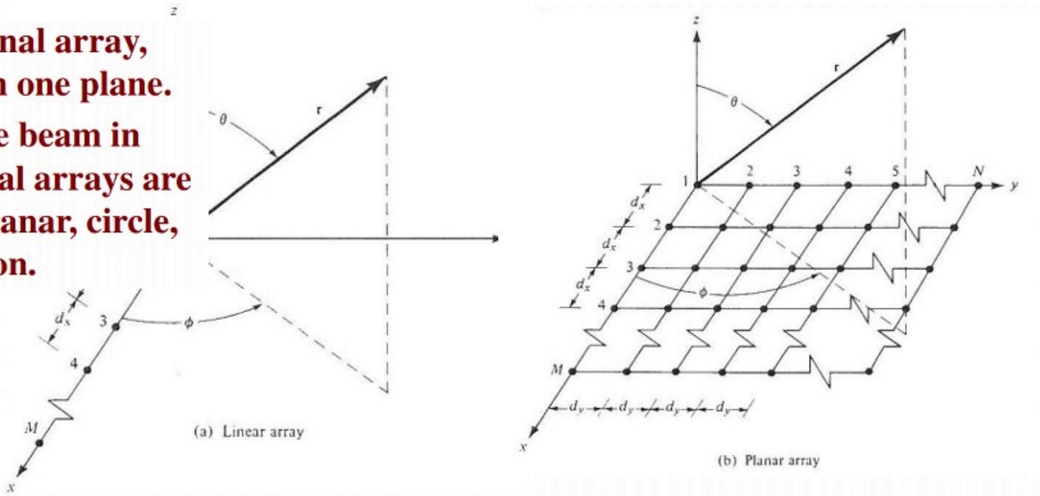
$$E_0 = E_0' \cdot \cos\phi$$



$\cos\phi$ $\quad\quad$ $\cos\left(\frac{\pi}{2}-\cos\phi\right)$

Planar Array

- **Linear Array = one-dimensional array, i.e., can scan the beam only in one plane.**
- **In order to be able to scan the beam in any direction, two-dimensional arrays are needed. Geometries can be planar, circle, cylindrical, spherical and so on.**



(a) Linear array

(b) Planar array

AF for each linear array along x-axis:

$$\mathrm{AF}=\sum_{m=1}^{M} I_{m1} e^{j(m-1)(kd_x\sin\theta\cos\phi+\beta_x)}$$

where $\sin\theta\cos\phi=\cos\gamma_x$ is the directional cosine with respect to the x-axis (γ_x is the angle between **r** and the x axis). It is assumed that all elements are equispaced with an interval of d_x and a progressive shift β_x. I_{m1} denotes the excitation amplitude of the element at the point with coordinates $x=(m-1)d_x,\ y=0$. In the figure above, this is the element of the m-th row and the 1st column of the array matrix. Note that the 1st row corresponds to $x = 0$.

If N such arrays are placed at even intervals along the y direction, a rectangular array is formed. We assume again that they are equispaced at a distance d_y and there is a progressive phase shift β_y along each row. We also assume that the normalized current distribution along each of the x-directed arrays is the same but the absolute values correspond to a factor of I_{1n} $(n=1,...,N)$. Then, the AF of the entire $M\times N$ array is

$$AF = \sum_{m=1}^{M} I_{m1} e^{j(m-1)(kd_x \sin\theta\cos\phi + \beta_x)} \cdot \sum_{n=1}^{N} I_{1n} e^{j(n-1)(kd_y \sin\theta\sin\phi + \beta_y)},$$

or

$$AF = S_{x_M} \cdot S_{y_N},$$

where

$$S_{x_M} = AF_{x1} = \sum_{m=1}^{M} I_{m1} e^{j(m-1)(kd_x \sin\theta\cos\phi + \beta_x)}, \text{ and}$$

$$S_{y_N} = AF_{1y} = \sum_{n=1}^{N} I_{1n} e^{j(n-1)(kd_y \sin\theta\sin\phi + \beta_y)}.$$

In the array factors above,

$$\sin\theta\cos\phi = \hat{\mathbf{x}} \cdot \hat{\mathbf{r}} = \cos\gamma_x,$$

$$\sin\theta\sin\phi = \hat{\mathbf{y}} \cdot \hat{\mathbf{r}} = \cos\gamma_y.$$

The pattern of a rectangular array is the product of the array factors of the linear arrays in the x and y directions.

In the case of a uniform planar rectangular array, $I_{m1} = I_{1n} = I_0$ for all m and n, i.e., all elements have the same excitation amplitudes. Thus,

$$AF = I_0 \sum_{m=1}^{M} e^{j(m-1)(kd_x \sin\theta\cos\phi + \beta_x)} \times \sum_{n=1}^{N} e^{j(n-1)(kd_y \sin\theta\sin\phi + \beta_y)}.$$

The normalized array factor is obtained as

$$\boxed{AF_n(\theta,\phi) = \left[\frac{\sin\left(M\dfrac{\psi_x}{2}\right)}{M\sin\left(\dfrac{\psi_x}{2}\right)}\right] \cdot \left[\frac{\sin\left(N\dfrac{\psi_y}{2}\right)}{N\sin\left(\dfrac{\psi_y}{2}\right)}\right]},$$

where

$$\psi_x = kd_x \sin\theta\cos\phi + \beta_x,$$

$$\psi_y = kd_y \sin\theta\sin\phi + \beta_y.$$

Lecture 39: Endfire Array with Increased directivity and Beam scanning

12 April 2021 13:48

As discussed earlier, $\delta = -d\gamma$ ⇒ producing max field in the direct $\phi = 0°$

⇓

but directivity is not max

Hansen & Woodyard → a larger directivity is obtained by increasing the phase change between the sources.

$$\delta = -\left(d\gamma + \frac{\pi}{n}\right) \quad \text{— (1)}$$

→ cond^n for increased directivity

$$\psi = \underbrace{d\gamma(\cos\phi - 1)} - \frac{\pi}{n} \quad \text{— (2)}$$

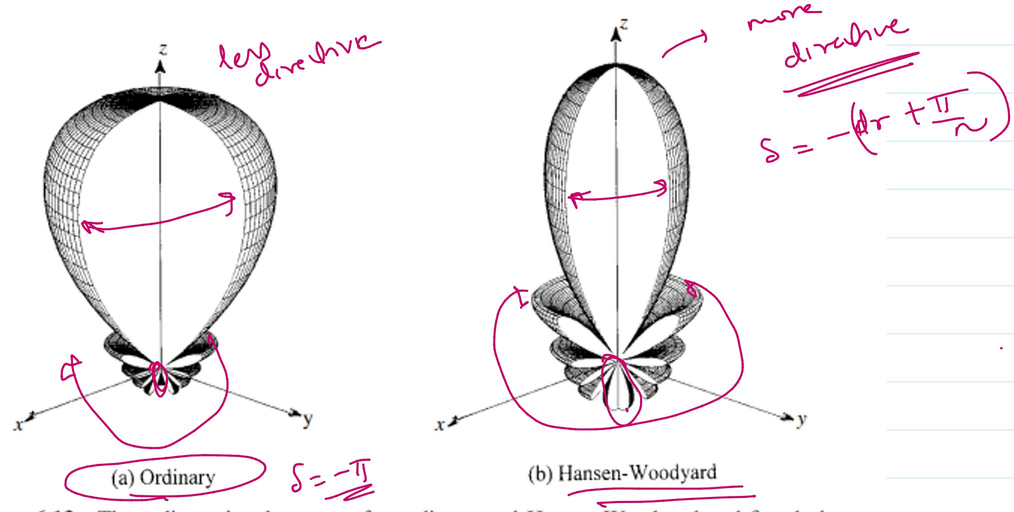


Figure 6.13 Three-dimensional patterns for ordinary and Hansen-Woodyard end-fire designs ($N = 10$, $d = \lambda/4$).

To realize the directivity increase afforded by additional phase difference requires that $|\psi|$ be restricted → value of $\frac{\pi}{n}$ at $\phi = 0$ & a value in the vicinity of π at $\phi = 180°$

⇓

This can be fulfilled

if the spacing is reduced

* Scanning Array (Array with max field in arbitrary direction)

Consider case of an array with a field pattern having a maximum value in some arbitrary direction ϕ_1 not equal to $k\frac{\pi}{2}$ where $k = 0, 1, 2$ or 3.
then,

$$\psi = d_r \cos\phi_1 + \delta \quad \text{—— (1)}$$

above eqn (1)

$$\psi = 0$$

$$0 = d_r \cos\phi_1 + \delta \quad \text{—— (2)}$$

$d_r = \beta d$
$= \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2}$
$= \pi$

For eg:- $n = 4$; $d = \lambda/2$, & that we wish to have a max field in the direction $\phi = 60°$

∴ using eqn (2)

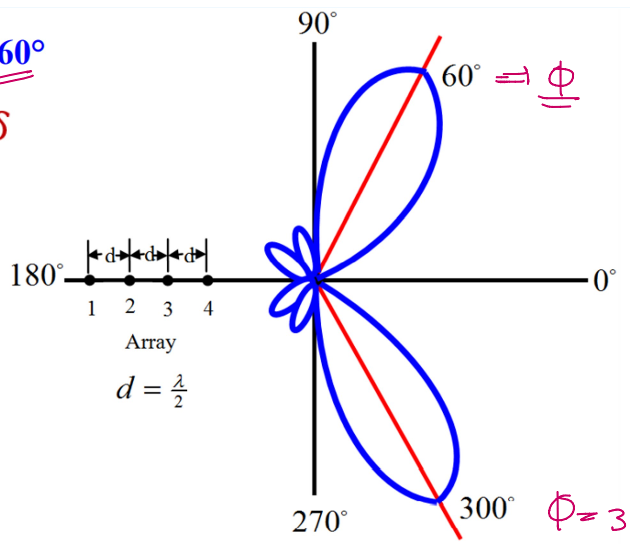
∴ $\delta = -\frac{\pi}{2}$ & the field pattern will be as shown below.

For Beam Maxima at φ = 60°

$$\psi = 0 = d_r \cos 60° + \delta$$

For d = λ / 2, $d_r = \pi$

$$\delta = -\frac{\pi}{2}$$



→ **Field pattern of array of 4 isotropic point sources of equal amplitude with phase adjusted to give the maximum at φ = 60° for spacing d = λ/2**

* Yagi-Uda antenna.

dipole antenna

parasitic elements

Directors — 5% shorter

Reflectors — 5% longer

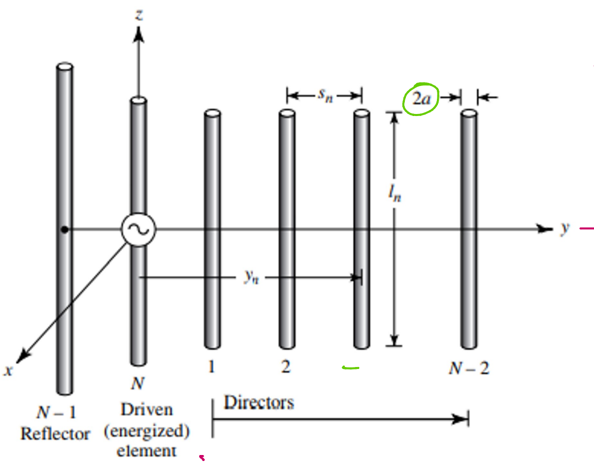
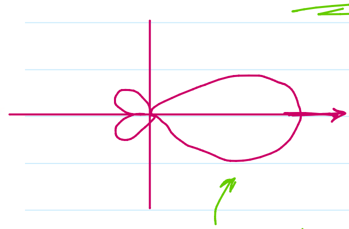


Figure 10.19 Yagi-Uda antenna configuration.



Used in the HF, VHF & UHF applications

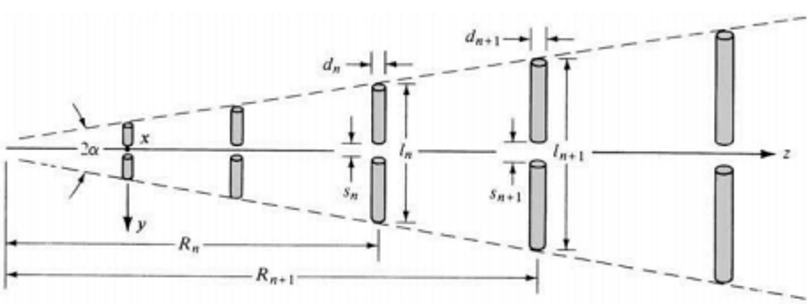
Endfire radiation pattern.

* Log Periodic Antenna.

→ freq. independent antennas

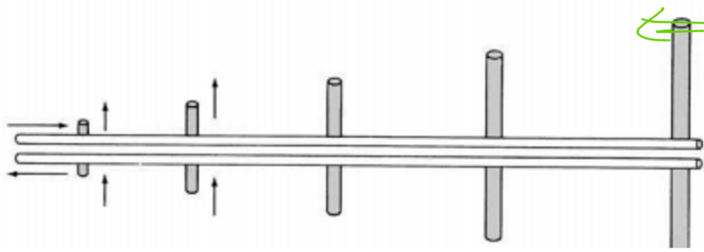
→ broadband system → antenna design → simple, small, light-weight & economical

General log periodic ant structure (LPDA)



(a) Dipole array

linear dipoles



Basic structure of a LPDA

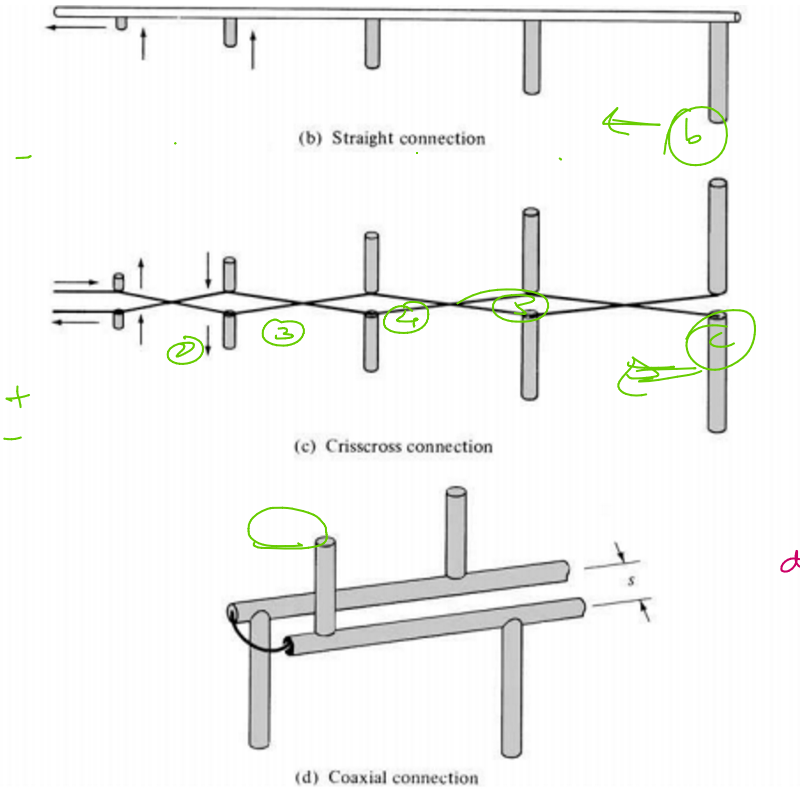


Figure 11.9 Log-periodic dipole array and associated connections.

Freq. independent antenna

↓

If the impedance & the pattern of an antenna do not change significantly over a $\boxed{\text{BW of } 10:1}$ or more the antenna is called as freq. independent antenna

MB - antennas

WB - antenna

① MB antenna $\longrightarrow BW = \frac{f_U - f_L}{f_0} \times 100$

% BW

eg → BW of 5%

↓

the freq. difference of acceptable operation is 5% of the f_0 of BW

② WB antennas

$BW = \frac{f_U}{f_L}$ → for wide band

eg: → BW = 10:1

it indicates that f_U is 10 times greater than the lower cut of freq.

The lengths & spacing of adjacent elements (LPDA) must be related by a constant scale factor (τ) so that

The lengths & spacing of adjacent elements (LPDA) must be related by a constant scale factor (τ) so that

$$\frac{L_1}{L_2} = \frac{L_2}{L_3} = \frac{L_n}{L_{n+1}} = \tau$$

$$\tau = \frac{D_1}{D_2} = \frac{D_2}{D_3} = \frac{D_n}{D_{n+1}}$$

apex angle

$$\tan \alpha/2 = \frac{L_1}{2D_1}$$

Note:- $20^\circ \leq \alpha \leq 90^\circ$

$0.7 \leq \tau \leq 0.95$

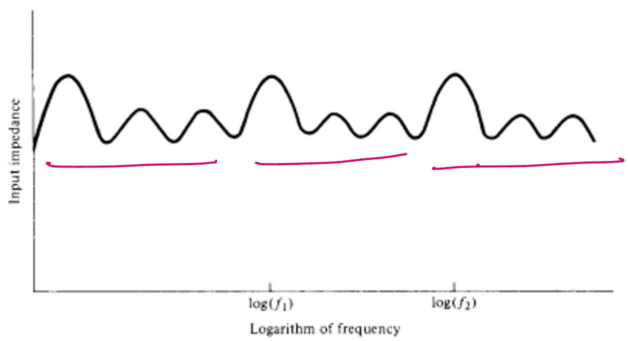
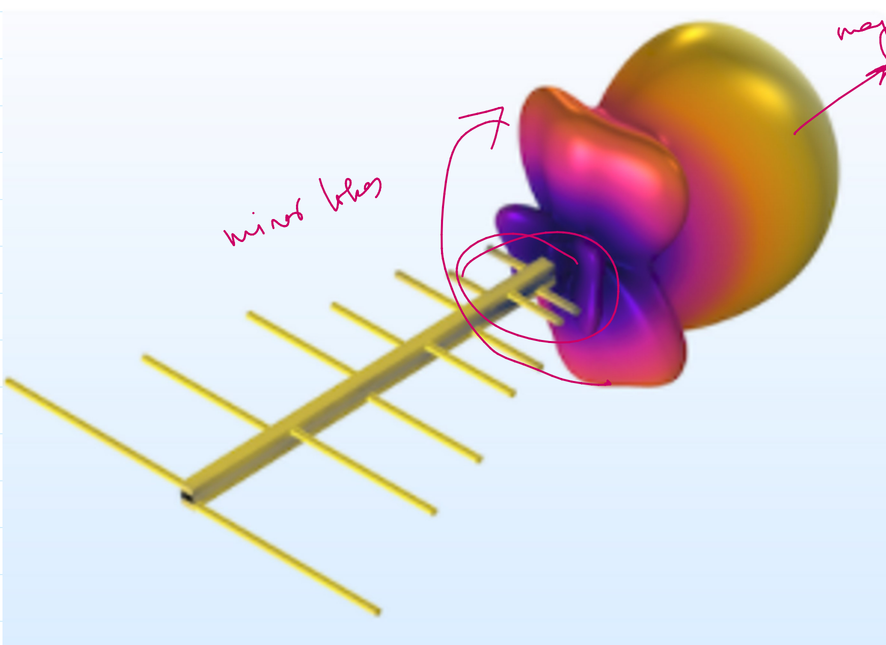


Figure 11.11 Typical input impedance variation of a log-periodic antenna as a function of the logarithm of the frequency.



minor lobes

major lobe

max rad

active region of LPDA is towards the apex. (shorter element) for high freq. &

near longest element for low freq. &

middle for intermediate freq

Lecture 41: Microstrip Antenna (MSA) design

16 April 2021 09:36

Balanis

TABLE 6.1 Nulls, Maxima, Half-Power Points, and Minor Lobe Maxima for Uniform Amplitude Broadside Arrays

NULLS	$\theta_n = \cos^{-1}\left(\pm \frac{n}{N}\frac{\lambda}{d}\right)$ $n = 1, 2, 3, \dots$ $n \neq N, 2N, 3N, \dots$
MAXIMA	$\theta_m = \cos^{-1}\left(\pm \frac{m\lambda}{d}\right)$ $m = 0, 1, 2, \dots$
HALF-POWER POINTS	$\theta_h \simeq \cos^{-1}\left(\pm \frac{1.391\lambda}{\pi N d}\right)$ $\pi d/\lambda \ll 1$
MINOR LOBE MAXIMA	$\theta_s \simeq \cos^{-1}\left[\pm \frac{\lambda}{2d}\left(\frac{2s+1}{N}\right)\right]$ $s = 1, 2, 3, \dots$ $\pi d/\lambda \ll 1$

TABLE 6.2 Beamwidths for Uniform Amplitude Broadside Arrays

FIRST-NULL BEAMWIDTH (FNBW)	$\Theta_n = 2\left[\frac{\pi}{2} - \cos^{-1}\left(\frac{\lambda}{Nd}\right)\right]$
HALF-POWER BEAMWIDTH (HPBW)	$\Theta_h \simeq 2\left[\frac{\pi}{2} - \cos^{-1}\left(\frac{1.391\lambda}{\pi N d}\right)\right]$ $\pi d/\lambda \ll 1$
FIRST SIDE LOBE BEAMWIDTH (FSLBW)	$\Theta_s \simeq 2\left[\frac{\pi}{2} - \cos^{-1}\left(\frac{3\lambda}{2dN}\right)\right]$ $\pi d/\lambda \ll 1$

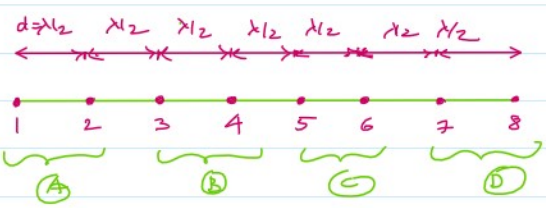
TABLE 6.3 Nulls, Maxima, Half-Power Points, and Minor Lobe Maxima for Uniform Amplitude Ordinary End-Fire Arrays

NULLS	$\theta_n = \cos^{-1}\left(1 - \frac{n\lambda}{Nd}\right)$ $n = 1, 2, 3, \dots$ $n \neq N, 2N, 3N, \dots$
MAXIMA	$\theta_m = \cos^{-1}\left(1 - \frac{m\lambda}{d}\right)$ $m = 0, 1, 2, \dots$
HALF-POWER POINTS	$\theta_h \simeq \cos^{-1}\left(1 - \frac{1.391\lambda}{\pi d N}\right)$ $\pi d/\lambda \ll 1$
MINOR LOBE MAXIMA	$\theta_s \simeq \cos^{-1}\left[1 - \frac{(2s+1)\lambda}{2Nd}\right]$ $s = 1, 2, 3, \dots$ $\pi d/\lambda \ll 1$

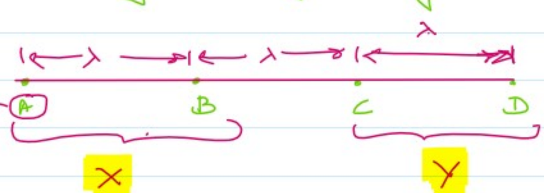
TABLE 6.4 Beamwidths for Uniform Amplitude Ordinary End-Fire Arrays

FIRST-NULL BEAMWIDTH (FNBW)	$\Theta_n = 2\cos^{-1}\left(1 - \frac{\lambda}{Nd}\right)$
HALF-POWER BEAMWIDTH (HPBW)	$\Theta_h \simeq 2\cos^{-1}\left(1 - \frac{1.391\lambda}{\pi d N}\right)$ $\pi d/\lambda \ll 1$
FIRST SIDE LOBE BEAMWIDTH (FSLBW)	$\Theta_s \simeq 2\cos^{-1}\left(1 - \frac{3\lambda}{2Nd}\right)$ $\pi d/\lambda \ll 1$

* Pattern multiplication of 8 isotropic point-sources → Rad. pattern.



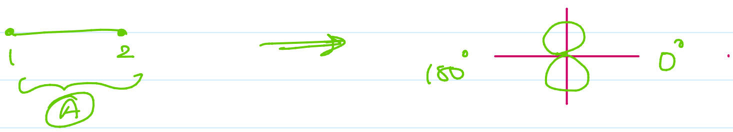
(i) Divide the array into a group of 2 elements each

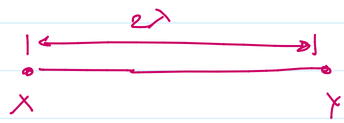


Here, now again further divide the array into group of 2 elements each

Here, now again further divide the array into group of 2 elements each

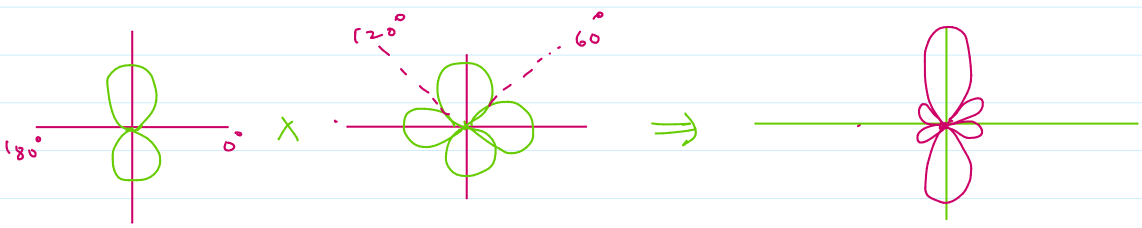
(ii) Now determine pattern (group pattern) for A (ie 1 & 2)





(iii). Now determine the pattern of X

∴ pattern of X = Unit pattern of (A or B) * group pattern of (A & B)



(iv) Determine the radiation pattern for an array of 8 isotropic pt sources

Resultant radiation pattern = unit pattern of (X or Y) * group pattern of (X & Y)