Wave Propagation

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Maximum Usable Frequency Earlier, it was mentioned that the critical frequency f_c is the highest frequency that returns from an ionospheric layer at a vertical incidence. When the frequency exceeds f_c, the return will depend upon the angle of incidence at a particular ionospheric layer. Thus, for a specified angle of incidence, there will be a maximum frequency which will be reflected back. The maximum possible value of frequency for which reflection takes place for a given distance of propagation is termed as *maximum usable frequency (MUF)* for that distance and for the given ionospheric layer. Beyond MUF, the wave will not return. Figure 4–6 shows that at the point of reversal of path to return to the ground, the sky wave requires the angle of reflection to be 90°. Thus, if ϕ_0 is the incident angle and ϕ_r is the reflection angle, the refractive index n can be written as

$$n = \frac{\sin\phi_i}{\sin\phi_r} = \frac{\sin\phi_i}{\sin 90} = \sin\phi_i = \sqrt{1 - \frac{81N_{\max}}{f_{MUF}^2}} \tag{1}$$

$$\sin^2\phi_i = 1 - \frac{81N_{\max}}{f_{MUF}^2} \tag{2}$$

But as discussed earlier,

$$f_c^2 = 81N_{\max} \tag{3}$$

$$\text{Thus,} \quad \sin^2\phi_i = 1 - \frac{f_c^2}{f_{MUF}^2} \quad \text{or} \quad \frac{f_c^2}{f_{MUF}^2} = 1 - \sin^2\phi_i = \cos^2\phi_i \tag{4}$$

$$f_{MUF}^2 = \frac{f_c^2}{\cos^2\phi_i} = f_c^2\sec^2\phi_i \tag{5}$$

Finally, we get

$$f_{MUF} = f_c \sec\phi_i \tag{6}$$

Equation (6) is known as *secant law*. It indicates that f_{MUF} is greater than f_c by a factor $\sec\phi_i$. It gives the maximum frequency which can be used for sky wave communication for a given angle of incidence between two locations.

Figure 4–10 illustrates the maximum usable frequencies at different times destined for coverage of various distances.

Lowest Usable Frequency The frequency below which the entire power gets absorbed is referred to as *lowest usable frequency* (LUF).

Optimum Frequency The frequency at which there is optimum return of wave energy is called the *optimum frequency* (OF).

Figure 4–11 illustrates the LUF, OF, MUF and the critical frequency f_c on a frequency scale. Limits of all these frequencies are different for different layers.

Virtual Height It may be defined as 'the height to which a short pulse of energy sent vertically upward and traveling with the speed of light would reach taking the same two-way travel time as does the actual pulse reflected from the ionospheric layer.'

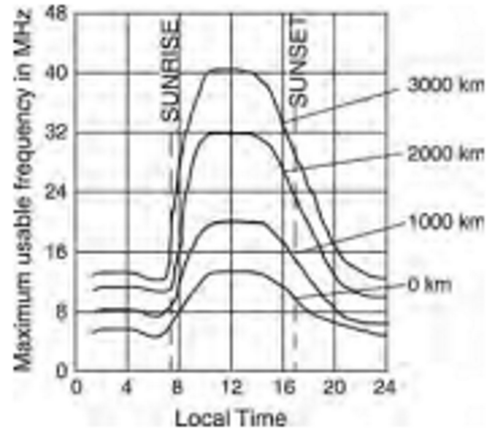

Figure 4–10 Maximum usable frequencies at different times of the day.

Figure 4–12 illustrates that there is no sharp change of the direction of wave and it starts bending down gradually (from the point E) through the process of refraction in the ionosphere. Just below the ionosphere (the point F), the incident and refracted rays follow exactly the same path as would have been followed by them if the reflection had taken place from a surface located at a greater height (the point B) which is often referred as the *virtual height*. If the virtual height of a layer is known, the angle of incidence required for the return of wave to the ground at a selected spot (the point C) can easily be calculated.

Figure 4–12 illustrates two different cases containing (a) flat earth, and (b) curved earth. The above discussion is true for both cases. On comparison of Fig. 4–12 (*a*) and (*b*), it can easily be concluded that both the actual height and the virtual height in case of curved earth are less than that for flat earth. The virtual height, however, is always greater than the actual height of

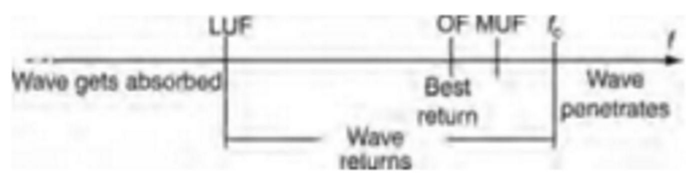

Figure 4–11 LUF, OF, MUF and f_c on a frequency scale.

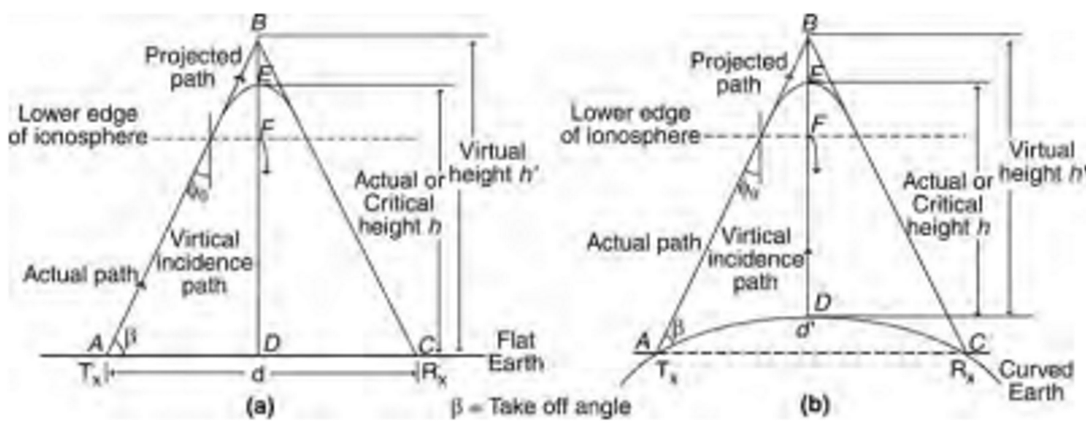

Figure 4–12 Actual and virtual height and the related parameters for (a) flat earth, and (b) spherical earth.

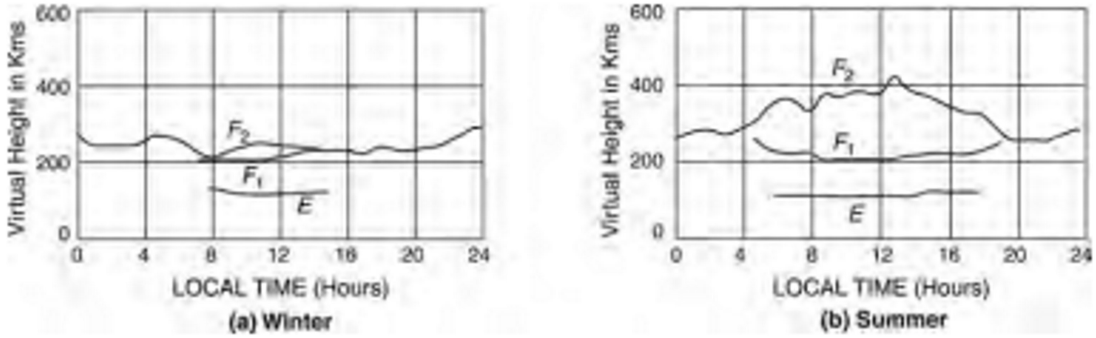

Figure 4–13 Virtual heights at different times of a day.

reflection because the exchange of energy that takes place between the wave and the electrons of the ionosphere causes the velocity of propagation to be reduced. The difference between virtual and true heights is influenced by the electron distribution in the regions below the level of reflection. It is usually quite small, but on occasions may be as large as 100 km or more. Figure 4–13 shows the virtual heights for different ionospheric layers at different instants of time in (a) winter, and (b) summer seasons.

Skip Distance The minimum distance at which the wave returns to the ground at a critical angle ϕ_c is termed the *skip distance*. Figure 4–8 illustrates two different skip distances which correspond to rays 2, 3 and 4. As mentioned earlier, the skip distance and the maximum usable frequency correspond to each other.

4–6 Relation Between MUF and the Skip Distance

Flat Earth Case Figure 4–14*a* illustrates the ionized layer which is assumed to be thin with sharp ionization density gradient so as to obtain mirrorlike reflections. For shorter distances, the earth can be assumed to be flat. In the figure, h is the height of the ionospheric layer, d is the skip distance, θ_i is the angle of incidence and θ_r is the angle of reflection. In view of the geometry of the configuration,

$$\cos\theta_i = \frac{OB}{AB} = \frac{h}{\sqrt{h^2 + d^2/4}} = \frac{2h}{\sqrt{4h^2 + d^2}} \tag{1}$$

In view of (4) of Sec. 4–5,

$$\frac{f_c^2}{f_{MUF}^2} = \cos^2\theta_i = \frac{4h^2}{4h^2 + d^2} \quad \text{or} \quad \frac{f_{MUF}^2}{f_c^2} = \frac{4h^2 + d^2}{4h^2} \tag{2}$$

$$\frac{f_{MUF}}{f_c} = \sqrt{\frac{4h^2 + d^2}{4h^2}} = \sqrt{1 + \frac{d^2}{4h^2}} \quad \text{or} \quad f_{MUF} = f_c\sqrt{1 + (\frac{d}{2h})^2} \tag{3}$$

Equation (3) gives MUF in terms of skip distance. Alternatively, from (1),

$$\frac{f_{MUF}^2}{f_c^2} = 1 + \frac{d^2}{4h^2} \quad \text{or} \quad \left(\frac{d}{2h}\right)^2 = \frac{f_{MUF}^2}{f_c^2} - 1 \quad \text{or} \quad d^2 = (2h)^2\left[\frac{f_{MUF}^2}{f_c^2} - 1\right]$$

$$d = (2h)\sqrt{[\frac{f_{MUF}^2}{f_c^2} - 1]} \tag{4}$$

Equation (4) gives skip distance with MUF

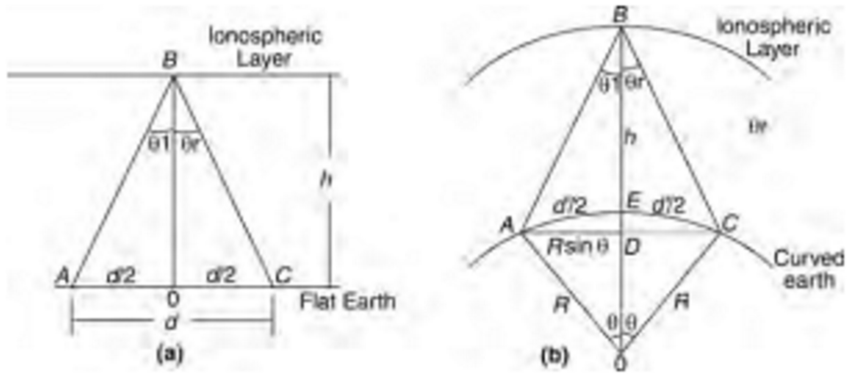

Figure 4–14 Skip distance and related geometrical parameters for (a) flat earth, and (b) spherical earth.

Curved earth case: Fig. 4–14*b* This shows the ionized layer and the curved earth. It is again assumed that the ionospheric layer is thin with sharp ionization density gradient so as to obtain mirror like reflections. In this figure, 2θ is the angle subtended by the skip distance d' at the center of the earth. From the geometry of Fig. 4–14*b*, the following relations are obtained:

$$\text{Arc}\, d' = 2R\theta \tag{5}$$

$$\text{Angle}\, 2\theta = d'/R \tag{6}$$

$$\begin{aligned} AD &= R\sin\theta, \quad OD = R\cos\theta, \quad BD = OE + EB - OD \\ &= R + h - R\cos\theta \end{aligned} \tag{7}$$

$$AB = \sqrt{(AD)^2 + (BD)^2} = \sqrt{(R\sin\theta)^2 + (R + h - R\cos\theta)^2} \tag{8}$$

$$\cos\theta_i = \frac{BD}{AB} = \frac{R + h - R\cos\theta}{\sqrt{(R\sin\theta)^2 + (R + h - R\cos\theta)^2}} \text{ or}$$

$$(\cos\theta_i)^2 = \frac{(R + h - R\cos\theta)^2}{(R\sin\theta)^2 + (R + h - R\cos\theta)^2} \tag{9}$$

$$\text{Since } \frac{f_c^2}{f_{MUF}^2} = (\cos\theta_i)^2$$

the skip distance d' is maximum when θ is maximum. The curvature of the earth limits both the MUF and the skip distance. This limit is obtained when a wave leaves the transmitter at a grazing angle $OAB = 90°$. Under this condition,

$$\cos\theta = \frac{OA}{OB} = \frac{R}{R + h} \tag{10}$$

Since the actual value of θ is very small, this relation can be expanded as

$$\cos\theta = \frac{R}{R + h} = \frac{R}{R(1 + h/R)} = (1 + h/R)^{-1} \approx (1 - h/R), \text{ since } h/R << 1 \tag{11}$$

$$\cos\theta = \sqrt{1 - \sin^2\theta} \approx (1 - \theta^2)^{1/2} = 1 - \theta^2/2 \text{ for small } \theta \tag{12}$$

From (11) and (12),

$$1-\frac{\theta^2}{2}=1-\frac{h}{R} \quad \text{or} \quad \theta^2=\frac{2h}{R} \tag{13}$$

In view of (5) and (13),

$$d'^2=4R^2\theta^2=4R^2\frac{2h}{R}=8hR \text{ or } d'=\sqrt{8hR} \tag{14}$$

$$\text{From (14), } h=\frac{d'^2}{8R} \tag{15}$$

In (14) and (15), d' is the maximum skip distance.
Equation (11) can now be rewritten in view of (15) as

$$\cos\theta=1-\frac{h}{R}=1-\frac{d'^2}{8R^2} \tag{16}$$

Also in view of (14) and (16)

$$\sin\theta\approx\theta=\sqrt{\frac{2h}{R}}=\sqrt{\frac{2d'^2/8R}{R}}=\sqrt{\frac{d'^2}{4R^2}}=\frac{d'}{2R} \tag{17}$$

Again in view of (4), of Sec. 4–5, and (9), (16), and (17)

$$\frac{f_c^2}{f_{MUF}^2}=\cos^2\theta_i=\frac{[R+h-R(1-d'^2/8R^2)]^2}{(R^2\frac{d'^2}{4R^2})+\{R+h-R(1-\frac{d'^2}{8R^2})\}^2}$$

$$=\frac{(h+d'^2/8R)^2}{(d'^2/4)+(h+d'^2/8R)^2} \tag{18}$$

$$\frac{f_{MUF}^2}{f_c^2}=\frac{(d'^2/4)+(h+d'^2/8R)^2}{(h+d'^2/8R)^2}=1+\frac{d'^2/4}{(h+d'^2/8R)^2} \tag{19}$$

$$f_{MUF}=f_c[1+\{\frac{d'^2/4}{h+(d'^2/8R)^2}\}]^{1/2} \tag{20}$$

Equation (20) gives maximum usable frequency in terms of skip distance. To get the expression of skip distance in terms of maximum usable frequency, (19) can be rewritten as

$$\frac{d'^2}{4}=(h+\frac{d'^2}{8R})^2[(\frac{f_{MUF}}{f_c})^2-1] \tag{21}$$

$$d'=2(h+\frac{d'^2}{8R})\left[\left(\frac{f_{MUF}}{f_c}\right)^2-1\right]^{1/2} \tag{22}$$

Similarly, (22) is of quadratic form which will yield the value of the skip distance in terms of maximum usable frequency as given below.

$$d'=\frac{2R}{X}\pm 2\sqrt{(R/X)^2-2hR} \tag{23}$$

$$\text{where} \quad X=[(f_{MUF}/f_c)^2-1]^{1/2} \tag{24}$$

EXAMPLE 4–6.1 Calculate the maximum single hop distance for D, E, F_1 and F_2 layers if their heights are assumed to be 70, 130, 230 and 350 km respectively above the earth and the angle of incidence is 10° in all cases.

Solution

In view of (1),

$$\cos\theta_i = \frac{OB}{AB} = \frac{h}{\sqrt{h^2 + d^2/4}} = \frac{2h}{\sqrt{4h^2 + d^2}}$$

$d = 2h\sqrt{(\sec\theta_i)^2 - 1} = 2h\sqrt{(\sec 10)^2 - 1} = 2h \times 0.176 = 0.352\text{h}$

For D layer, $d = 0.352h = 0.352 \times 70 = 24.5$ km

For E layer, $d = 0.352h = 0.352 \times 130 = 45.76$ km

For F_1 layer, $d = 0.352h = 0.352 \times 230 = 80.96$ km

For F_2 layer, $d = 0.352h = 0.352 \times 350 = 123.2$ km

EXAMPLE 4–5.1 Calculate the skip distance for flat earth with MUF of 10 MHz if the wave is reflected from a height of 300 km where the maximum value of n is 0.9.

Solution

In view of (1),

$$
\begin{aligned}
n^2 &= 0.81 = (1 - 81N/f^2) \\
N_{\max} &= (1 - n^2)f^2/81 = [(1 - 0.81)\times 10^{14}]/81 = (19/81)\times 10^{12} \\
&= 23.45\times 10^{10} \\
f_c &= 9\sqrt{N_{\max}} = 9\sqrt{(23.45\times 10^{10})} = 9\times 4.8425\times 10^5 = 4.36\,\text{MHz} \\
d_{\text{skip}} &= 2h\sqrt{[(f_{\text{MUF}}/f_c)^2 - 1]} = 2\times 300\sqrt{[(10/4.36)^2 - 1]} = 600\times 6.527 \\
&= 3916.2\,\text{km}
\end{aligned}
$$

EXAMPLE 4–5.2 The critical frequencies at an instant observed for E, F_1 and F_2 layers were found to be 3, 5 and 9 MHz. Find the corresponding concentration of electrons in these layers.

Solution

$$
\begin{array}{ll}
f_c = 9\sqrt{N_{\max}} \quad \text{or} & N_{\max} = f_c^2/81 \\
\text{For E layer, } f_c = 3\text{MHz} & N_{\max} = f_c^2/81 = 9\times 10^{12}/81 = 0.111\times 10^{12} \\
\text{For } F_1 \text{ layer, } f_c = 5\text{MHz} & N_{\max} = f_c^2/81 = 25\times 10^{12}/81 = 0.3086\times 10^{12} \\
\text{For } F_2 \text{layer, } f_c = 9\text{MHz} & N_{\max} = f_c^2/81 = 81\times 10^{12}/81 = 10^{12}
\end{array}
$$

EXAMPLE 4–5.3 Calculate the critical frequencies for E, F_1 and F_2 layers if N_{max} for each corresponding layer reduces to 80% of the values obtained in Problem 4–5.1.

Solution

$$
\begin{array}{ll}
\text{For E layer, } N_{\max} = 0.8\times 0.111\times 10^{12} & f_c = 9\sqrt{N_{\max}} = 9\sqrt{0.0888\times 10^{12}} = 2.68\,\text{MHz} \\
\text{For } F_1 \text{layer, } N_{\max} = 0.8\times 0.3086\times 10^{12} & f_c = 9\sqrt{N_{\max}} = 9\sqrt{0.24688\times 10^{12}} = 4.47\,\text{MHz} \\
\text{For } F_2 \text{layer, } N_{\max} = 0.8\times 10^{12} & f_c = 9\sqrt{N_{\max}} = 9\sqrt{0.8\times 10^{12}} = 8.05\,\text{MHz}
\end{array}
$$

Critical Frequency The highest frequency that returns from an ionospheric layer at a vertical incidence is called the *critical frequency* for that particular layer. For a regular layer, it is proportional to the square root of maximum electron density in the layer. Figure 4–9 shows the critical frequencies for different ionospheric layers at different instants of time in (a) winter, and (b) summer seasons.