Module 4

CHAPTER

13

Arrays

Module 4

13.1 INTRODUCTION (ARRAYS IN GENERAL)

In the previous chapters on linear wire antennas radiation characteristics of single element antennas. Usually the radiation pattern of a single element is relatively wide and provides low values of directivity (gain). For many applications like point-to-point communication at the higher frequencies the desired radiation pattern is a single narrow lobe or beam. This can be achieved by increasing the electrical size of the antenna.

Another way to enlarge the dimensions of the antenna, without increasing the size of the individual elements, is to form an **assembly** of radiating elements.

Thus high gain is achieved by using two ways :

1) increasing size of the element,

2) by using number of elements.

The second approach is studied in this chapter. This antenna is called as array.

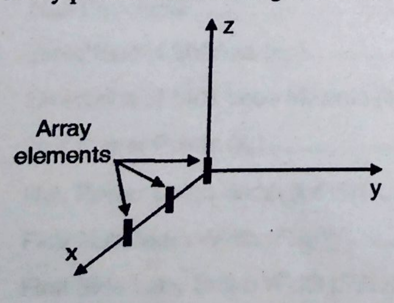
> **Antenna formed by multielements arranged to produce particular radiation pattern is called an array.**

In most cases elements of an array are identical. This is not necessary, but it is often convenient, simpler, and more practical.

To provide very directive patterns, it is often necessary that fields from the elements of the array interfere constructively (add) in the desired directions and interfere destructively (cancel each other) in the remaining directions. This is only ideal but can be achieved at least approximately.

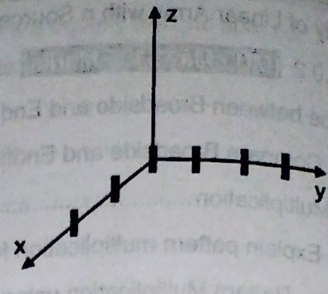
The array can be single dimension, two dimensional, or three directional. In Fig. 13.1.1(a), elements are arranged only along x-axis. They can also be arranged along y or z axis. This is single dimension array.

In two dimensional array elements can be arranged along x and y axis or in x-y plane as shown in Fig. 13.1.1(b) and (c).



(a)

Fig. 13.1.1 Contd...



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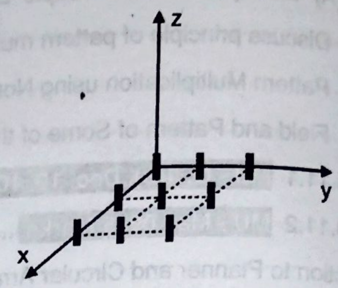
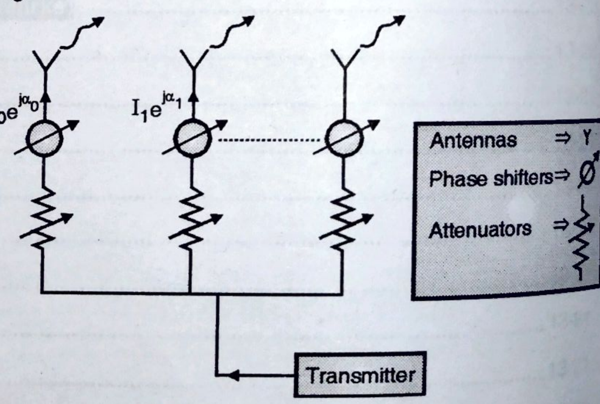


(c)

aw(8.1)**Fig. 13.1.1 : (a) single, (b) and (c) two dimensional array**

A typical linear array composed of similar element antennas is shown in Fig. 13.1.2. The output of each array element may be controlled in amplitude and phase by the use of attenuators and phase shifters as shown.

The amplitude of the current supplied to antenna determines the magnitude of the field radiated, while phase angle of the current will determine the phase of the field. The currents supplied to antennas are expressed as $I_0 e^{j\alpha_0}$, $I_1 e^{j\alpha_1}$, ... This is as shown in Fig. 13.1.2. Here I_0, I_1, ... are the amplitudes of the currents and α_0, α_1, ... are the phase angles. The field due to these antennas can be expressed as $E_0 e^{j\psi_0}$, $E_1 e^{j\psi_1}$ .... Here E_0, E_1, ... are the magnitudes of the field, while ψ_1, ψ_2, ..., are the phase angles.



aw(8.2)**Fig. 13.1.2 : A typical linear array**

The total field observed at some distant point P is obtained by superposition principle as

$$E_{total} = E_0 e^{j\psi_0} + E_1 e^{j\psi_1} + ... \qquad ...(13.1.1)$$

Note that if the currents supplied are equal that is,

$$I_0 e^{j\alpha_0} = I_1 e^{j\alpha_1} = \ldots \quad \ldots(13.1.2)$$

then $E_0 = E_1 = \ldots$, but $\psi_0, \psi_1, \ldots$, are not equal because ψ at observation point is not only decided by α but is also decided by distance of the antenna (d) from point P.

It is given by $$\psi = (\beta \times d) + \alpha \quad \ldots(13.1.3)$$

In general distance of each antenna from point P cannot be same as these antennas are physically separated from each other.

The array shown in Fig. 13.1.2 is a transmitting array; however, the transmitting pattern is the same as the receiving pattern by reciprocity.

13.1.1 Array Configurations

In general, arrays are found in many geometrical configurations. Some of these are :

(i) **Linear array :** In this configuration the array element centers lie along a straight line. The elements may be equally or unequally spaced. Refer Fig. 13.1.1(a) and (b).

(ii) **Planer array :** Here array element centers are located in a plane. Examples of planer arrays are circular and rectangular arrays in which the element centers are disposed on a circle, or contained within a rectangular area, respectively. Refer Fig. 13.1.1(c) for rectangular array.

(iii) **Conformal arrays :** In this array element locations must conform to some nonplanar surface such as found on an aircraft or missile.

13.1.2 Controlling Parameters of An Array

UQ. Describe five controls of array antenna. **(MU - Q. 1(d), Dec. 19, 5 Marks)**

The shape of the overall radiation pattern is decided by five controls. These are :

1) The geometrical configuration of the overall array that is whether elements are arranged linearly, circularly, rectangular, spherical, etc.
2) The relative displacement between the elements.
3) The excitation (current) amplitude of the individual elements.
4) The excitation phase of the individual elements.
5) The relative radiation pattern of the individual elements.

Understanding how these controls change the overall radiation pattern is really interesting. The simplest situation for study is to place these elements along a straight line (called as linear array) as shown in Fig. 13.1.1(a). In the beginning we will study two element array and then N number of element array.

In this chapter the antenna elements are taken as isotropic point sources which may represent different kinds of antennas.

13.2 ARRAY OF TWO ISOTROPIC POINT SOURCES

The simplest situation for study is to use two elements in the array. Each element is a point source. These two point sources 1 and 2 are placed along x-axis, separated by a distance d and located symmetrically with respect to origin of the coordinates as shown in Fig. 13.2.1(a). Here we are interested in finding field at a distance r along a line at an angle of ϕ with x axis. Note that in antenna theory we are not interested in radiation over a short distance, that is r is very large.

In this section we will study five cases involving two isotropic point sources as follows :

1) Two isotropic point sources of same amplitude and phase.
2) Two isotropic point sources of same amplitude but opposite phase.
3) Two isotropic point sources of same amplitude and in phase quadrature.
4) Two isotropic point sources of equal amplitude and any phase difference.
5) Most general case is two isotropic point sources of unequal amplitude and any phase difference.

These cases we shall study one by one to observe how five controls discussed in the previous section controls the radiation pattern of the array.

13.2.1 Case I : Two Isotropic Point Sources of Same Amplitude and Phase

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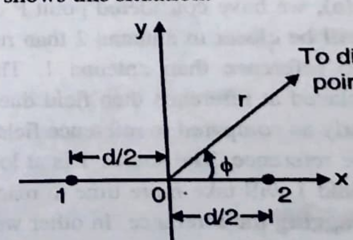
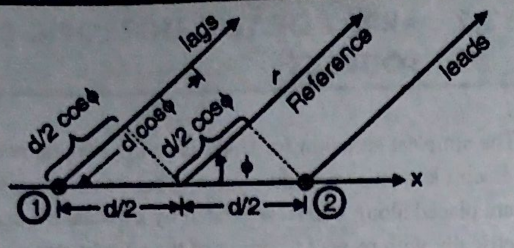
UQ. Derive an expression for array of two isotropic sources with same amplitude and in phase. **(MU - May 16, 5 Marks)**

Consider two isotropic point sources 1 and 2 having currents supplied with equal amplitudes and same phase. They are separated by a distance d and located symmetrically with respect to origin (O). We wish to find field at a distance r with angle ϕ from x-axis. For simplicity the observation point P is taken in x-y plane. Fig. 13.2.1(a) shows this situation.

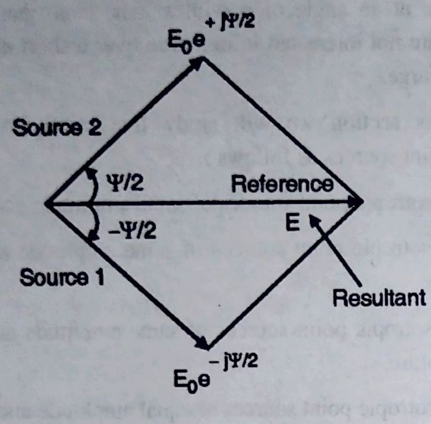


(a) Two element array

Fig. 13.2.1 Contd...



(b) To find path difference



(c) Phasor diagram

Fig. 13.2.1 : Study of two element array

To obtain the resultant field we should use some reference, there are two possibilities.

1) Origin of the coordinates as a reference

2) One of the source is a reference

Let us see now the effect of changing reference on the pattern.

13.2.1(A) Origin of The Coordinates is used as Reference

As the currents supplied to each antenna are equal in amplitude which results in same field strengths from the antenna, that is

$$E_1 = E_2 = E_0$$

To find phase of the fields w.r.t. reference, we join three points, antenna 1, reference, and antenna 2 with point P. If point P is far away from antenna system, all these three lines are almost parallel as shown in Fig. 13.2.1(b).

In Fig. 13.2.1(a), we have considered point P on the right of y-axis, so point P will be closer to antenna 2 than reference. Also point P is closer to reference than antenna 1. Thus if similar isotropic source is placed at reference then field due to antenna 2 will reach point P early as compared to reference field, so field 2 is said to be leading the reference. The source 1 is at longer distance than reference, the field 1 will take more time to reach point P, so field 1 is said to be lagging the reference. In other words source 2 results in leading field while source 1 gives lagging field.

The total path difference of two antennas from point P is

$$\text{Path difference} = d\cos\phi$$

So, Total phase difference $= \psi = \beta d \cos\phi$ ...(13.2.1)

As reference is exactly at the center of two antennas, field due to antenna 2 leads reference by $\psi/2$ and antenna 1 field lags reference by $\psi/2$.

Thus in terms of ψ, we write

$$\text{field due to source 1} = E_0 e^{-j\psi/2} \quad \text{(lagging)}$$

and

$$\text{field due to source 2} = E_0 e^{j\psi/2} \quad \text{(leading)}$$

The total field at point P is obtained using superposition principle. But these fields are expressed as phasor and the addition is a phasor addition.

We know that in phasor diagram, the angle measured anticlockwise from reference is considered to be positive (i.e. leading), while clockwise angles are considered negative (i.e. lagging). It gives rise to Fig. 13.2.1(c). In the Fig. 13.2.1(c), lengths of two phasors are shown equal, which is representing the magnitudes of the two fields. Also leading and lagging angles are same. So when we find resultant of two phasors, it is in the same direction as that of reference. The total electric field at point P is obtained using Superposition as

Total electric field,

$$E = E_0 e^{j\psi/2} + E_0 e^{-j\psi/2}$$

$$= E_0 (e^{j\psi/2} + e^{-j\psi/2})$$

i.e. $E = 2E_0 \cos(\psi/2)$ ...(13.2.2)

Let $2E_0 = 1$ or $E_0 = 1/2$, then the field is said to be normalized given by,

$$E_n = \cos(\psi/2) \qquad \text{...(13.2.2(a))}$$

Putting the value of ψ from Equation (13.2.1), that is,

$$\psi = \beta d \cos\phi$$

We get the normalized field,

$$E_n = \cos\left(\frac{\beta d \cos\phi}{2}\right) = \cos\left(\frac{2\pi}{\lambda}\cdot\frac{d\cos\phi}{2}\right)\left(\because \beta = \frac{2\pi}{\lambda}\right)$$

i.e. $E_n = \cos\left(\frac{\pi d}{\lambda}\cos\phi\right)$ ...(13.2.3)

The field pattern can be drawn by putting different values of d, and currents in phase ($\alpha = 0$).

i) CASE (I) : $d = \lambda/2$, $\alpha = 0$

Putting value of d in Equation (13.2.3),

$$E_n = \cos\left(\frac{\pi}{\lambda}\cdot\frac{\lambda}{2}\cos\phi\right) = \cos\left(\frac{\pi}{2}\cos\phi\right) \qquad \text{...(13.2.4)}$$

In the expression we find only one variable i.e. ϕ. By finding ϕ for field to maximum, minimum and half power we can draw the radiation pattern. These values of ϕ are obtained as follows :

Maxima direction (ϕ_{max})

E_n is maximum when

$$\cos\left(\frac{\pi}{2}\cos\phi\right) = \pm 1$$

or $$\frac{\pi}{2}\cos\phi_{max} = \pm n\pi \quad \text{where } n = 0, 1, 2, \ldots$$

or $$\frac{\pi}{2}\cos\phi_{max} = 0 \quad \text{if } n = 0$$

or $$\cos\phi_{max} = 0$$

or $$\phi_{max} = 90° \text{ and } 270°$$

Minima direction (ϕ_{min})

E_n is minimum when

$$\cos\left(\frac{\pi}{2}\cos\phi\right) = 0$$

or $$\frac{\pi}{2}\cos\phi = \pm(2n+1)\frac{\pi}{2} \;;\; n = 0, 1, 2, \ldots$$

or $$\frac{\pi}{2}\cos\phi_{min} = \pm\frac{\pi}{2}$$

or $$\cos\phi_{min} = \pm 1$$

or $$\phi_{min} = 0° \text{ and } 180°$$

Half power point direction (ϕ_{HP})

At half power point power is 1 / 2 but E_n or H_n is $1/\sqrt{2}$.

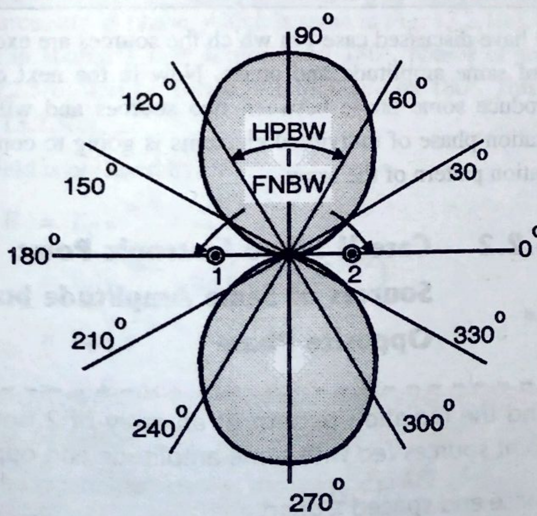
$$\cos\left(\frac{\pi}{2}\cos\phi\right) = \pm\frac{1}{\sqrt{2}}$$

or $$\frac{\pi}{2}\cos\phi_{HP} = \pm(2n+1)\frac{\pi}{4} \;;\; n = 0, 1, 2, \ldots$$

or $$\frac{\pi}{2}\cos\phi_{HP} = \pm\frac{\pi}{4}$$

or $$\cos\phi_{HP} = \pm 1/2$$

or $$\phi_{HP} = 60° \text{ and } 120°$$



aw(8.4)Fig. 13.2.2 : Radiation pattern with $d = \lambda/2$ and $\alpha = 0$

Remember that the radiation plot is nothing but magnitude plot, so though cosine term appear in Equation (13.2.4) (which takes positive as well negative values) we consider only positive values.

By plotting magnitude of E_n at different angles (ϕ), we get radiation pattern as below. This pattern is having two major lobes situated along $\phi = 90°$ and $-90°$ (i.e. 270°). Two nulls are present along $\phi = 0°$ and 180°.

Instead of going for the mathematical analysis given above, one can simply put different values of ϕ in Equation (13.2.4) and then the radiation pattern can be drawn. Table 13.2.1 gives the field values for different values of ϕ with increment of 30° right from 0° to 360°. If the increment is less, we get more values and the plot is more accurate but for this number of calculations are also increased.

Table 13.2.1 : Values of $|E_n|$ for different values of ϕ

ϕ	0	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°
$\lvert E_n \rvert$	0	0.2	0.707	1	0.707	0.2	0	0.2	0.707	1	0.707	0.2

The properties of this pattern are :

1) Figure eight (8) shape
2) A doughnut shape
3) Broadside couplet

Endfire and Broadside array

Two sources in the array are placed along x-axis, and at these sources the array ends. If the maxima of the radiation pattern is in the direction of ends that is, in + x or – x direction, then the array is said to be **endfire array**.

But if the maxima is perpendicular to the line of the array, then the array is said to be **broadside array**.

The pattern we obtained with $d = \lambda/2$ and $\alpha = 0$, has maximum in y-direction, which is perpendicular to x-axis. So this array is a **broadside array**. Remember this statement is valid only for $d = \lambda/2$ and $\alpha = 0$. If the separation (d) or α is changed, the array will no longer be broadside. This is proved in the next part of analysis. Now the question may arise, why it is broadside for $d = \lambda/2$ and $\alpha = 0$? Is there any physical reasoning for this behaviour. Try to understand it.

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ii) CASE (II) : $d = \lambda$, $\alpha = 0°$

Now we shall find radiation pattern by changing the separation between elements. Let the separation is $d = \lambda$. Putting this value in Equation (13.2.3),

$$E_n = \cos\left(\frac{\pi d}{\lambda}\cos\phi\right) = \cos\left(\frac{\pi\lambda}{\lambda}\cos\phi\right)$$

i.e. $$E_n = \cos(\pi\cos\phi) \qquad \ldots(13.2.5)$$

To obtain the radiation pattern obtain magnitude of E_n for different values of ϕ. It is given below in Table 13.2.2.

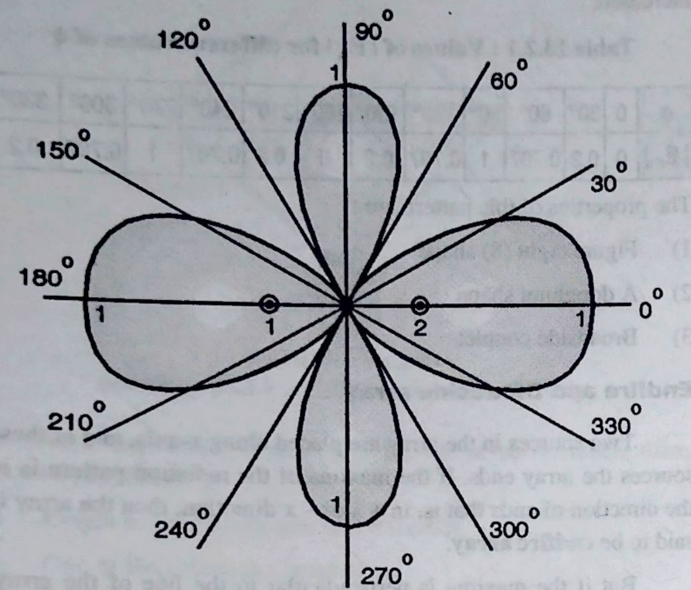
Table 13.2.2 : Values of $|E_n|$ for different values of ϕ

ϕ	0	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°
$\lvert E_n \rvert$	1	0.91	0	1	0	0.91	1	0.91	0	1	0	0.91

Plotting values of $|E_n|$ for different values of ϕ, the radiation pattern is obtained as shown in Fig. 13.2.4(A) :

This radiation pattern has 4 major lobes along $\phi = 0°, 90°, 180°$, and $270°$. This pattern is totally different from the pattern with $d = \lambda / 2$ and $\alpha = 0$ (Fig. 13.2.2). This proves that the separation between the array elements is one control which defines the radiation pattern, as mentioned in section 13.1.

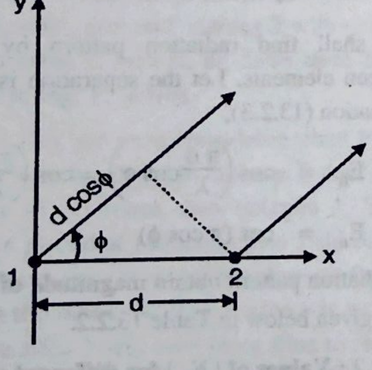
Now the second situation is to be studied where one of the source is considered as a reference. In the previous discussion we have taken center of the array as reference.



aw(8.7)Fig. 13.2.3(A) : Radiation pattern with $d = \lambda$ and $\alpha = 0$

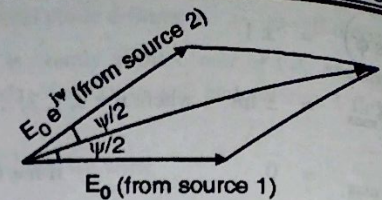
13.2.1(B) Source 1 is Used as A Reference

Two sources are again placed as shown in Fig. 13.2.4(a). As the reference is changed the phase diagram will change. We always show the reference in horizontal direction, so rotate Fig. 13.2.4(a) such that phasor of source 1 will become horizontal. The phasor diagram will look like as shown below, Fig. 13.2.4(b).



(a) Two element array

Fig. 13.2.4 Contd...



(b) Phasor diagram

aw(8.8)Fig. 13.2.4 : Study of two element array

As in the previous discussion, the total phase difference is given by Equation (13.2.1),

$$\psi = \beta d \cos \phi$$

The source 1 is used as reference, its field is expressed as

$$\text{Field of source 1} = E_0 e^{j0}$$

The field 2 is now leading the field 1 by an angle ψ, thus

$$\text{Field of source 2} = E_0 e^{j\psi}$$

The total electric field is

$$E = E_0 e^{j0} + E_0 e^{j\psi} = E_0 (1 + e^{j\psi})$$

$$= E_0 e^{j\psi/2} (e^{-j\psi/2} + e^{j\psi/2}) = 2 E_0 \cos(\psi/2) \angle \psi/2$$

Let $2 E_0 = 1$, then normalized field is

$$E_n = \cos(\psi/2) \angle \psi/2 \quad ...(13.2.6)$$

Comparing this equation with Equation (13.2.2(a)) we find that only difference is in phase angle. The magnitude plot is decided by the cosine term, which is same as in Equation (13.2.2(a)). Thus magnitude plot with Equation (13.2.6) is same as previous plot in Fig. 13.2.2. So,

Changing reference point will not have any effect on the radiation pattern (magnitude plot).

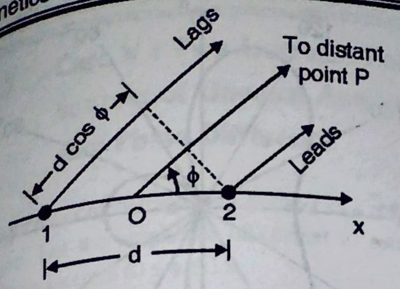
We have discussed case I in which the sources are excited by currents of same amplitude and phase. Now in the next case we shall introduce some phase between two sources and will prove that excitation phase of individual elements is going to control the total radiation pattern of the array.

13.2.2 Case II : Two Isotropic Point Sources of Same Amplitude but Opposite Phase

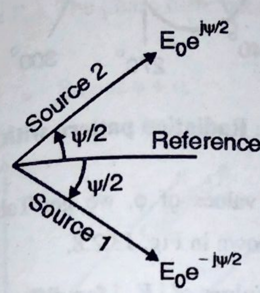
UQ. Find the radiation pattern of an array of 2 isotropic point sources fed with same amplitude and opposite phase and spaced $\frac{\lambda}{2}$ apart.

(MU - May 11, May 13, Dec. 17, 5 Marks)

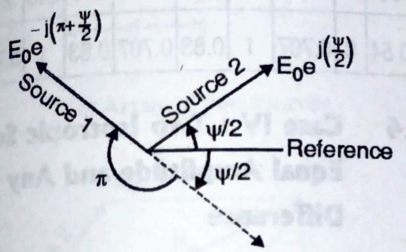
This case is similar to previous case except that the two sources in opposite phase instead of in the phase. Let the sources are placed as shown in Fig. 13.2.5(a).



(a) Two element array



(b) Currents in phase



(c) Currents out of phase

aw(8.10)**Fig. 13.2.5(a) : Two element array**

(b) Phase diagram when currents are in phase

(c) Phase diagram when current in source 1 lags by 180°

Fig. 13.2.5(b) shows the phasor diagram when currents in two sources are in phase, which is same in Fig. 13.2.1(c). When the current in source 1 lags source 2 by 180°, phasor of source 1 in Fig. 13.2.5(b) will get rotate anticlockwise by 180°. This is shown in Fig. 13.2.5(c).

Total field is obtained by,

$$E = E_0 e^{j\psi/2} + E_0 e^{-j(\pi+\psi/2)}$$

$$= E_0 e^{j\psi/2} + E_0 \left(e^{-j\pi} \times e^{-j\psi/2}\right)$$

$$= E_0 \left[e^{j\psi/2} - e^{-j\psi/2}\right] \qquad (\because e^{-j\pi} = -1)$$

$$= 2j\,E_0 \sin(\psi/2)$$

Where $\psi = \beta d \cos\phi$ ...(13.2.7)

When the separation between the sources is $\lambda/2$

$$\psi = \beta d \cos\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{2} \cos\phi = \pi\cos\phi$$

and the total field in Equation (13.2.7) becomes

$$E = 2j\,E_0 \sin\left(\frac{\pi}{2}\cos\phi\right)$$

Let $2j\,E_0 = 1$, then the normalized field is

$$E_n = \sin\left(\frac{\pi}{2}\cos\phi\right) \qquad ...(13.2.8)$$

Putting different values of φ we get total field at those angles. It is given in Table 13.2.3.

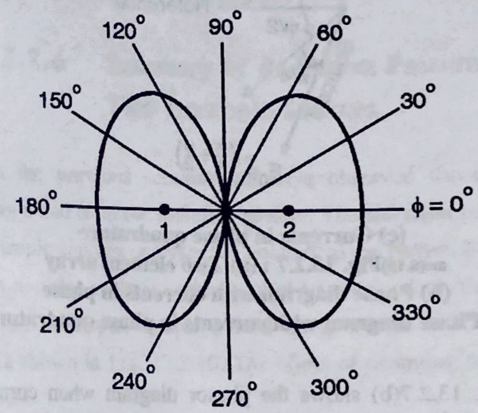
Table 13.2.3 : Values of $|E_n|$ for different values of φ

φ	0	30	60	90	120	150	180	210	240	270	300	330
$\lvert E_n \rvert$	1	0.97	0.707	0	0.707	0.97	1	0.97	0.707	0	0.707	0.97

The plot of it is as shown in Fig. 13.2.6.

The radiation pattern in Fig. 13.2.6 is a endfire pattern.

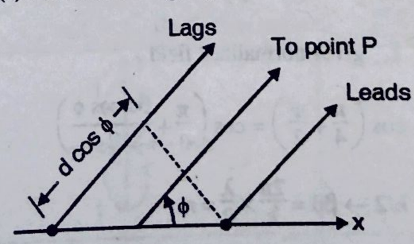
The effect of out of phase current in source 1 is the radiation pattern in case I (in phase currents, Fig. 13.2.2) gets rotated by 90°.



aw(8.11)**Fig. 13.2.6 : Radiation pattern for case II**

13.2.3 Case III : Two Isotropic Sources of Same Amplitude and in Phase Quadrature

Consider two isotropic sources placed as shown in Fig. 13.2.7 (a). Let source 1 lags by 45° and source 2 leads by 45°.



(a) Two element array

Fig. 13.2.7 Contd...

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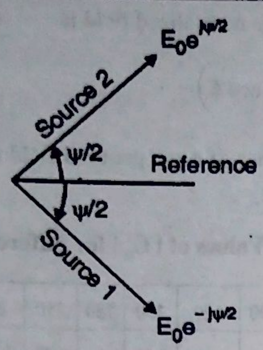
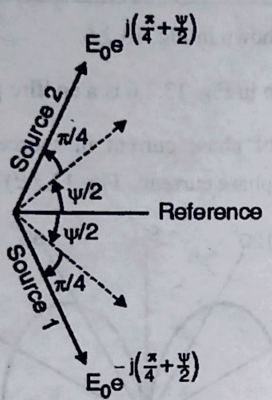


(b) Currents in phase



(c) Currents in phase quadrature

aw(8.12)Fig. 13.2.7 : (a) Two element array
(b) Phase diagram with currents in phase
(c) Phase diagram with currents in phase quadrature

Fig. 13.2.7(b) shows the phasor diagram when currents in two sources are in phase. But when current in source 2 leads while in source 1 lags by 45°, the phasor diagram changes as shown in Fig. 13.2.7(c).

The total field is

$$E = E_0 e^{j\left(\frac{\pi}{4}+\frac{\psi}{2}\right)} + E_0 e^{-j\left(\frac{\pi}{4}+\frac{\psi}{2}\right)}$$

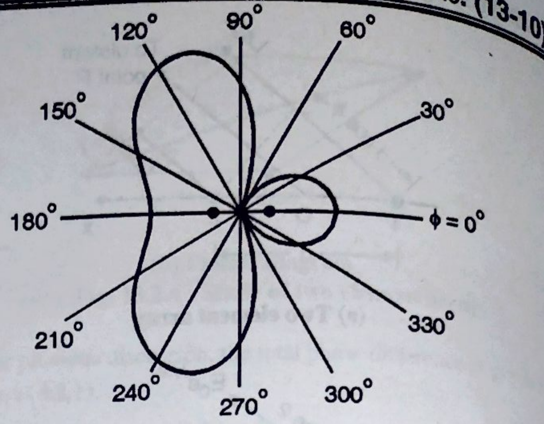
$$= 2E_0 \cos\left(\frac{\pi}{4}+\frac{\psi}{2}\right) \quad (\because e^{j\theta}+e^{-j\theta} = 2\cos\theta)$$

Let $2E_0 = 1$, gives normalized field

$$E_n = \cos\left(\frac{\pi}{4}+\frac{\psi}{2}\right) = \cos\left(\frac{\pi}{4}+\frac{\beta d\cos\phi}{2}\right)$$

Let $d = \lambda/2 \rightarrow \beta d = \frac{2\pi}{\lambda}\times\frac{\lambda}{2} = \pi$

$$\therefore \quad E_n = \cos\left(\frac{\pi}{4}+\frac{\pi}{2}\cos\phi\right) \qquad \text{...(13.2.9)}$$



aw(8.13)Fig. 13.2.8 : Radiation pattern with d = λ/2, α = 90°

By putting different values of φ, we get Table 13.2.4 and then radiation pattern as shown in Fig. 13.2.8.

Table 13.2.4 : Values of | E_n | for different values of φ

φ	0	30	60	90	120	150	180	210	240	270	300	330
$\lvert E_n \rvert$	0.707	0.54	0	0.707	1	0.83	0.707	0.83	1	0.707	1	0.83

13.2.4 Case IV : Two Isotropic Sources of Equal Amplitude and Any Phase Difference

Consider the current phase difference between the sources be α. Then the total phase difference is

$$\psi = \beta d\cos\phi + \alpha$$

With center of the array as reference, the total field is

$$E = E_0 e^{j\psi/2} + E_0 e^{-j\psi/2}$$

$$= 2E_0\cos(\psi/2) = 2E_0\cos\left(\frac{\beta d\cos\phi+\alpha}{2}\right)$$

The normalized field is obtained using $2E_0 = 1$

$$E_n = \cos\left(\frac{\beta d\cos\phi+\alpha}{2}\right) \qquad \text{...(13.2.10)}$$

Putting different values of α = 0°, 180°, and 90° we get results of case I, II and III respectively. So Equation (13.2.10) gives more general expression for two isotropic sources with any phase difference α.

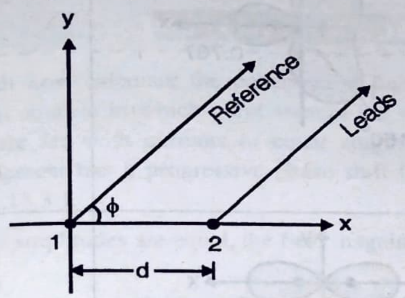
In all cases we studied, the same amplitudes of currents are used. Now let us change the amplitudes to have most general case.

13.2.5 Case V : Two Isotropic Point Sources of Unequal Amplitudes and Any Phase Difference

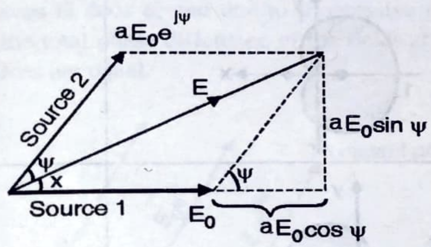
For simplicity consider source 1 is used as a reference and is placed at origin. Let the amplitude of current to source 2 is less than source 1 by a factor a $(0 \le a \le 1)$.

Now if amplitude of field due to source 1 is E_0 then due to source 2 will be $(a \times E_0)$. The phase difference between two source is

$$\psi = \beta d \cos \phi + \alpha$$



(a) Array of two sources



(b) Phasor diagram

aw(8.14)**Fig. 13.2.9 : (a) Array of two sources (b) Phasor diagram**

From Fig. 13.2.9(b), the amplitude of resultant is

$$|E|^2 = (E_0 + a E_0 \cos \psi)^2 + (a E_0 \sin \psi)^2$$

$$= E_0^2 [(1 + a \cos \psi)^2 + (a \sin \psi)^2]$$

or $$|E| = E_0 \sqrt{(1 + a \cos \psi)^2 + a^2 \sin^2 \psi} \quad ...(13.2.11)$$

The angle of resultant with respect to reference (source 1) is

$$\tan x = \frac{a E_0 \sin \psi}{E_0 + a E_0 \cos \psi} = \frac{a \sin \psi}{1 + a \cos \psi} \quad ...(13.2.12)$$

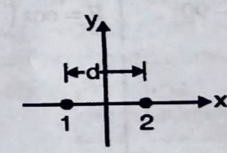
Thus the total field is expressed using Equations (13.2.11) and (13.2.12) as

$$E = |E| \angle x$$

$$= E_0 \sqrt{(1 + a \cos \psi)^2 + a^2 \sin^2 \psi} \angle \tan^{-1} [(a \sin \psi) / (1 + a \cos \psi)] \quad ...(13.2.13)$$

13.2.6 Summary of Radiation Patterns of Two Isotropic Sources

In the previous sections we have observed the effect of changing d and α on the radiation pattern. The radiation patterns of two isotropic sources analysed till now are given below. Some new patterns are also included which could be obtained on the similar lines. Two isotropic sources placed along x-axis with center at origin is shown in Fig. 13.2.10. The effect of changing d and α is as follows.



aw(8.15)**Fig. 13.2.10 : Array of two sources**

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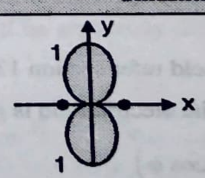
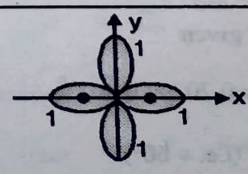
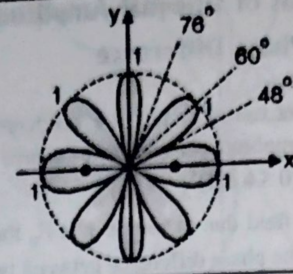
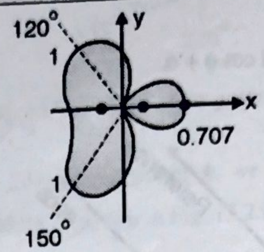
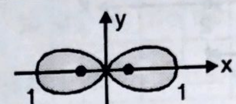
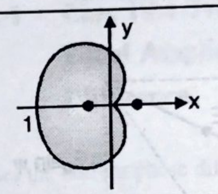
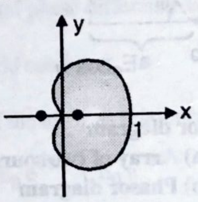
In the following discussion, positive α indicates that antenna 2 current leads antenna 1, while negative α indicates antenna 2 lags antenna 1.

Plot No.	d, α	Normalized field or Array Factor (AF)	Radiation pattern	
1	$d = \frac{\lambda}{2}, \alpha = 0$	$E = \cos\left(\frac{\pi}{2} \cos \phi\right)$		...(13.2.14)
2	$d = \lambda, \alpha = 0$	$E = \cos (\pi \cos \phi)$		...(13.2.15)

Plot No.	d, α	Normalized field or Array Factor (AF)	Radiation pattern
3	$d = 2\lambda, \alpha = 0$	$E = \cos(2\pi \cos\phi)$	 ...(13.2.16)
4	$d = \frac{\lambda}{2}, \alpha = 90°$	$E = \cos\left(\frac{\pi}{4} + \frac{\pi}{2}\cos\phi\right)$	 ...(13.2.17)
5	$d = \frac{\lambda}{2}$, $\alpha = 180°$	$E = \sin\left(\frac{\pi}{2}\cos\phi\right)$	 ...(13.2.18)
6	$d = \frac{\lambda}{4}$, $\alpha = +90°$ (ant. 2 leads ant. 1)	$E = \cos\left(\frac{\pi}{4} + \frac{\pi}{4}\cos\phi\right)$	 ...(13.2.19)
7	$d = \frac{\lambda}{4}, \alpha = -90°$ (ant. 2 legs ant. 1)	$E = \cos\left(\frac{\pi}{4} - \frac{\pi}{4}\cos\phi\right)$	 ...(13.2.20)

UEx. 13.2.1 (MU - May 11, May 13, Dec. 17, 5 Marks)
Explain and derive equations for total electric field, directivity, half power beam width in case of two isotropic point sources of same amplitude but opposite phase.

☑ **Soln. :**

For derivation of electric field refer section 13.2.2.

For separation of $d = \lambda/2$, the electric field is given by,

$$E_n = \sin\left(\frac{\pi}{2}\cos\phi\right)$$

The table values of E_n versus ϕ given

$$|E_n| = \frac{1}{\sqrt{2}} = 0.707 \text{ at } \phi = 60°$$

$$\text{and } \phi = 300° \text{ (i.e. } -60°)$$

$$\text{This gives HPBW} = 60 - (-60) = 120° = \phi_{HP}^{\circ}$$

The directivity is $D \cong \frac{41253^{\circ}}{\phi_{HP}^2} = \frac{41253}{(120)^2} = 2.86$

13.3 N-ELEMENT ARRAY OF EQUAL AMPLITUDE AND SPACING

Usually at high frequencies for point to point communication we require a single narrow beam of the radiation pattern. Such a radiation pattern cannot be obtained by using two element array studied in the previous section. To obtain such pattern we require to increase the number of elements (n) in the array. There are two terms related to n element array :

1) **Linear array :** An array is said to be linear, if the individual elements of the array are spaced equally along a line.

2) **Uniform array :** A linear array is said to be uniform if the elements are fed with currents of equal amplitude and having an uniform progressive phase shift along the line.

13.3.1 Expression for Field

UQ. Derive expression for array factor of array antenna.
MU - Q. 4(a), Dec. 19, 5 Marks

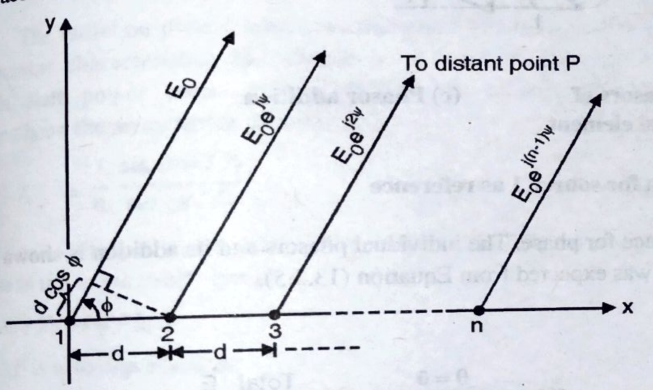
UQ. Derive array factor of N-element linear array, where all elements are equally fed and spaced. Also find the expression for the position of principle maxima, nulls and secondary maxima.
MU - May 15, May 16, 10 Marks

We shall now calculate the pattern of a linear array of n isotropic point sources in which point sources are spaced equally (say d) and are fed with currents of equal amplitudes but each succeeding element has a progressive phase shift (α). This is as shown in Fig. 13.3.1.

As the current amplitudes are equal, the field magnitudes are equal, that is,

$$E_1 = E_2 = \dots = E_n = E_0$$

As shown in Fig. 13.3.1, the path difference between the adjacent sources is d cos ϕ, and due to progressive phase shift in current (α), the total phase difference of the fields at point P from adjacent sources are equal.



aw(8.23)**Fig. 13.3.1 : Linear array with n isotropic point sources**

$$\psi_1 = \psi_2 = \dots = \psi$$

$$\text{where, } \psi = \beta d \cos \phi + \alpha \qquad \dots(13.3.1)$$

The total field E at a large distance in the direction ϕ (point P) is obtained by superposition as

$$E = E_0 e^{0j\psi} + E_0 e^{j\psi} + E_0 e^{j2\psi} + \dots + E_0 e^{j(n-1)\psi}$$

$$\text{i.e. } E = E_0 [1 + e^{j\psi} + e^{j2\psi} + \dots + e^{j(n-1)\psi}] \qquad \dots(13.3.2)$$

While writing this equation source 1 is assumed as a reference and fields from sources 2, 3, 4 etc. are leading in phase with respect to reference by angles ψ, 2ψ, 3ψ etc. respectively. Multiplying above equation by $e^{j\psi}$,

$$E e^{j\psi} = E_0 [e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \dots + e^{jn\psi}] \qquad \dots(13.3.3)$$

Subtracting Equation (13.3.3) from Equation (13.3.2),

$$E[1 - e^{j\psi}] = E_0 [1 - e^{jn\psi}]$$

$$\text{or } E = E_0 \left(\frac{1 - e^{jn\psi}}{1 - e^{j\psi}} \right) = E_0 \frac{e^{jn\psi/2} (e^{-jn\psi/2} - e^{jn\psi/2})}{e^{j\psi/2} (e^{-j\psi/2} - e^{j\psi/2})}$$

Knowing $e^{-j\theta} - e^{j\theta} = -2j \sin \theta$,

$$E = E_0 e^{j(n-1)\psi/2} \left[\frac{\sin (n\psi/2)}{\sin (\psi/2)} \right]$$

$$\text{or } E = E_0 \left[\frac{\sin (n\psi/2)}{\sin (\psi/2)} \right] \angle (n-1)(\psi/2) \qquad \dots(13.3.4)$$

This is the equation of total far field pattern of linear array of n-isotropic point sources with source 1 as a reference point for phase.

In case the reference point is shifted to the center of the array then the phase angle term is automatically eliminated and above equation reduces to

$$E = E_0 \left[\frac{\sin (n\psi/2)}{\sin (\psi/2)} \right] \angle 0 \qquad \dots(13.3.5)$$

13.3.2 Array Factor

UQ. Derive array factor of N-element linear array, where all elements are equally fed and spaced. Also find the expression for the position of principle maxima, nulls and secondary maxima.
MU - May 15, May 16, 10 Marks

UQ. Derive expression for array factor of array antenna.
MU - Q. 4(a), Dec. 19, 5 Marks

Now with $\psi = 0$, Equation (13.3.4) or Equation (13.3.5) becomes indeterminate and hence D' Hospital rule must be applied to evaluate the function. According to rule, the numerator and denominator will be separately differentiated under the limit when $\psi \to 0$.

$$\lim_{\psi \to 0} E = E_0 \lim_{\psi \to 0} \frac{d/d\psi (\sin n\psi/2)}{d/d\psi (\sin \psi/2)}$$

$$= E_0 \lim_{\psi \to 0} \frac{(n/2) \cos (n\psi/2)}{(1/2) \cos (\psi/2)} = n E_0$$

This is the maximum value E can attain. Thus

$$E_{max} = n E_0 \text{ when } \psi = 0 \qquad \dots(13.3.6)$$

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Thus the maximum of E is n times the field from a single source. The normalized field is then,

$$E_n = \frac{E}{E_{max}} = \frac{1}{n}\frac{\sin(n\psi/2)}{\sin(\psi/2)} \quad ...(13.3.7)$$

The field given by Equation (13.3.7) will be referred to as the "array factor".

$$\text{Array factor, AF} = \frac{1}{n}\frac{\sin(n\psi/2)}{\sin(\psi/2)} \quad ...(13.3.8)$$

It is also called as normalized array factor, $(AF)_n$.

The above expression for AF is obtained by considering the elements to be isotropic point sources. If the actual elements are not isotropic sources then the total field can be obtained by multiplying the array factor of the isotropic sources by the field of a single non-isotropic element. This is called as pattern multiplication rule.

Pattern multiplication rule

Total field of non instropic array =

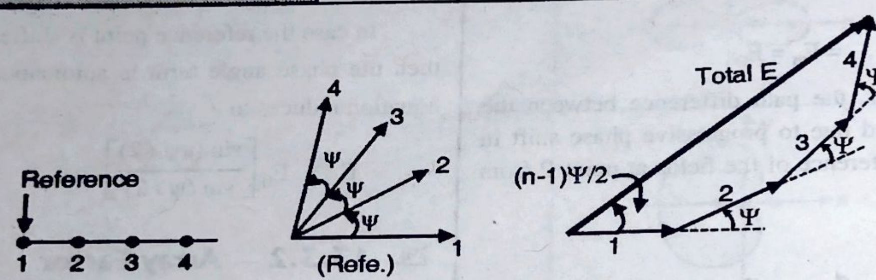
$(AF)_{isotropic}$ × Field of a single non isotropic element ...(13.3.9)

This rule we shall discuss more in detail in the next section. It is introduced here just to realize the importance of array factor (Equation (13.3.8)).

13.3.3 Phasor Diagrams

The total field at a distant point P can also be obtained by phasor addition of each term of Equation (13.3.1).

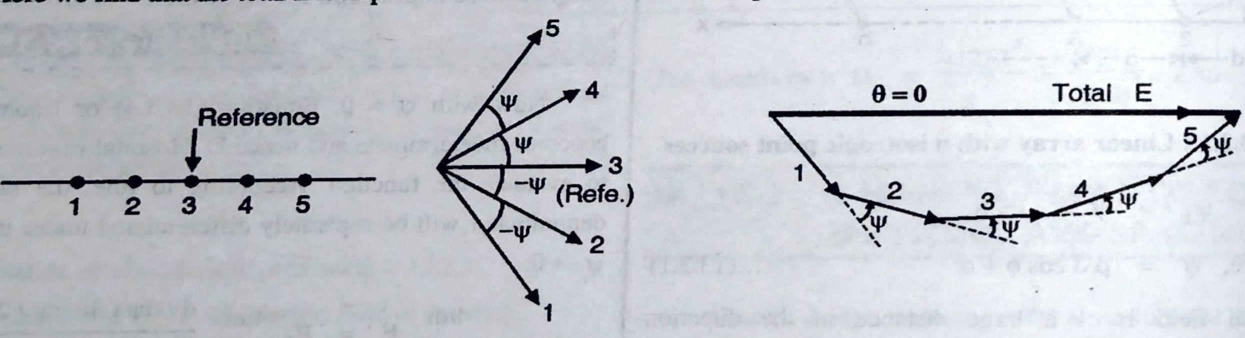
Consider a four element array with element 1 is used as a reference. The reference phasor is drawn horizontally in Fig. 13.3.2(b). The phasor addition of these phasors is shown in Fig. 13.3.2(c). The resultant field E is found to make an angle of $3\psi/2$ with the reference. This angle is nothing but $(n-1)\psi/2$ with n = 4 in Equation (13.3.4). This technique can be extended to any number of elements.



(a) Four element array with element 1 as a reference (b) Phasors of individual element (c) Phasor addition

aw(8.24)Fig. 13.3.2 : Phasor diagram for source 1 as reference

Consider now a five element array with center element 3 as a reference for phase. The individual phasors and its addition is shown in Fig. 13.3.3. Here we find that the total E is in phase with the reference, as was expected from Equation (13.3.5).



(a) Five element array with element 3 as a reference (b) Individual phasors (c) Phasor addition

aw(8.25)Fig. 13.3.3 : Phasor diagram for center of the array as reference

Ex. 13.3.1 : Prove that the n element array factor expression (Equation 13.3.8)

$$AF = \frac{1}{n}\frac{\sin\left(\frac{n\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)} \quad ...(i)$$

and normalized field expression for two element array (Equation 13.2.2(a)) given by

$$E_n = \cos\left(\frac{\psi}{2}\right) \quad ...(ii)$$

are identical.

☑ **Soln. :** Putting n = 2 in the expression (i)

$$AF = \frac{1}{2}\frac{\sin\left(\frac{2\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)} = \frac{1}{2}\frac{\sin(\psi)}{\sin\left(\frac{\psi}{2}\right)}$$

$$= \frac{1}{2}\frac{2\cdot\sin\left(\frac{\psi}{2}\right)\cdot\cos\left(\frac{\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)} = \cos\left(\frac{\psi}{2}\right)$$

This expression is same as that of expression (ii).

13.4 PROPERTIES OF RADIATION PATTERN

The radiation pattern which we discussed above have certain important characteristics like direction of maxima, direction of nulls, half power beam width (HPBW) etc., which certainly depends on the array factor expression.

$$AF = \frac{1}{n}\frac{\sin(n\psi/2)}{\sin(\psi/2)} \quad ...(13.4.1(a))$$

For small values of ψ, using the approximation that sine of a small angle is the angle itself, that is

$\sin(\psi/2) \rightarrow \psi/2$,

the AF is also expressed as

$$AF = \frac{\sin(n\psi/2)}{(n\psi/2)} \quad ...(13.4.1(b))$$

13.4.1 Null Directions (ϕ_0)

> **UQ.** Derive array factor of N-element linear array, where all elements are equally fed and spaced. Also find the expression for the position of principle maxima, nulls and secondary maxima.
>
> **MU - May 15, May 16, 10 Marks**

It is the direction in which there is zero field observed.

Thus AF in Equation (13.4.1(a)) or (13.4.1(b)) is zero when the numerator is zero but denominator should not be zero.

i.e.

$$\sin(n\psi/2) = 0, \quad \text{but } \sin(\psi/2) \neq 0 \quad ...(13.4.2)$$

or

$$\left.\frac{n}{2}\psi\right|_{\phi=\phi_0} = \pm N\pi \quad ...(13.4.2(a))$$

or

$$\frac{n}{2}(\beta d\cos\phi_0 + \alpha) = \pm N\pi$$

or

$$\cos\phi_0 = \frac{1}{\beta d}\left[\pm\frac{2N\pi}{n} - \alpha\right]$$

or

$$\phi_0 = \cos^{-1}\left[\frac{1}{\beta d}\left(-\alpha \pm \frac{2N\pi}{n}\right)\right] \quad ...(13.4.3)$$

where $N = 1, 2, 3, ...$ but $N \neq n, 2n, 3n, ...$

Why N = n, 2n, 3n, ... is not possible ? consider N = n. From Equation (13.4.2(a))

$$\frac{n}{2}\psi = \pm n\pi \quad \text{or} \quad \psi = \pm 2\pi$$

which gives $\sin(\psi/2) = \sin(\pm\pi) = 0$, resulting in denominator of Equation (13.4.1(a)) equal to zero, so that null condition of Equation (13.4.1) is not satisfied.

The values of N determines the order of the nulls (first, second, etc.). For zero to exist the argument of the $\cos^{-1}$ cannot exceed unity. Thus the number of nulls that can exist will be a function of the element separation d and the phase excitation α.

13.4.2 Directions of Maxima (ϕ_m)

> **UQ.** Derive array factor of N-element linear array, where all elements are equally fed and spaced. Also find the expression for the position of principle maxima, nulls and secondary maxima.
>
> **MU - May 15, May 16, 10 Marks**

These are the directions in which maximum of major lobe (ϕ_m) and minor or side lobes (ϕ_S) exists.

To find direction of major lobe maxima (ϕ_m)

The major lobe maxima occurs when denominator of Equation (13.4.1(a)) is zero. That is

$$\sin(\psi/2) = 0$$

or

$$\psi/2 = \pm m\pi$$

or

$$\left.\frac{1}{2}(\beta d\cos\phi + \alpha)\right|_{\phi=\phi_m} = \pm m\pi$$

or

$$\phi_m = \cos^{-1}\left[\frac{1}{\beta d}(-\alpha \pm 2m\pi)\right],$$

$$m = 0, 1, 2, ... \quad ...(13.4.4)$$

The array factor of Equation (13.4.1(b)) has only one maxima and occurs when m = 0. For other values of m denominator of AF is not equal to zero.

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For m = 0, Equation (13.4.4) reduces to

$$\phi_m = \cos^{-1}\left(\frac{\alpha}{\beta d}\right) \quad ...(13.4.5)$$

which is the observation angle that makes $\psi = 0$.

To find direction of side lobe maxima (ϕ_s)

The side lobe or minor lobe maxima occur approximately when the numerator of Equation (13.4.1(b)) attains its maximum value. ...(13.4.6)

$$\sin(n\psi/2) \simeq \pm 1$$

or $$\left.\frac{n}{2}\psi\right|_{\phi=\phi_S} \simeq \pm(2S+1)\frac{\pi}{2}$$

or $$\frac{n}{2}(\beta d\cos\phi_S + \alpha) \simeq \pm(2S+1)\frac{\pi}{2}$$

or $$\phi_S \simeq \cos^{-1}\left[\frac{1}{\beta d}\left(-\alpha \pm \left(\frac{2S+1}{n}\right)\pi\right)\right] \quad ...(13.4.7)$$

S = 1, 2, 3, ...

Here S = 0 is reserved for major lobe maxima. Above equation can also be written as

$$\phi_S \simeq \frac{\pi}{2} - \sin^{-1}\left\{\frac{1}{\beta d}\left[-\alpha \pm \left(\frac{2S+1}{n}\right)\pi\right]\right\}$$

For large values of d (d >> λ), above equation reduces to ...(13.4.8)

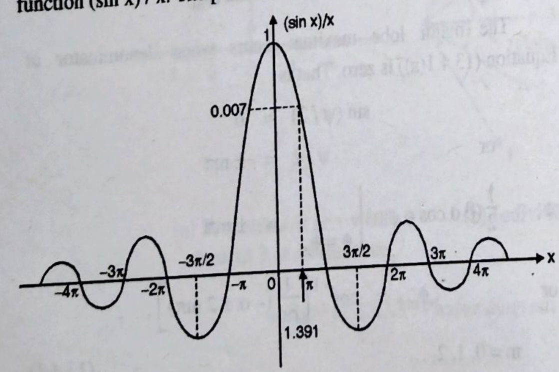
$$\phi_S \simeq \frac{\pi}{2} - \frac{1}{\beta d}\left[-\alpha \pm \left(\frac{2S+1}{n}\right)\pi\right]$$

13.4.3 To Find Side Lobe Level (SLL)

UQ. Derive array factor of N-element linear array, where all elements are equally fed and spaced. Also find the expression for the position of principle maxima, nulls and secondary maxima.

MU - May 15, May 16, 10 Marks

To obtain SLL we should have some knowledge of the function (sin x) / x. The plot of it is shown in Fig. 13.4.1.



aw(8.26)**Fig. 13.4.1 : Plot of (sin x) / x function**

From Fig. 13.4.1 we observe following properties :

(i) Plot is symmetric about vertical axis.

(ii) Maximum value is 1 at x = 0. This gives rise to major lobe maxima i.e. sin (0) / 0 is maximum equal to 1.

(iii) The amplitude decreases with increasing x.

(iv) The next highest value is observed when x = 3π / 2, this value results in maximum of the first minor lobe.

(v) The other peak amplitudes results in remaining minor lobes.

(vi) The function drops to a value of 0.707 (3 dB) at x = 1.391.

The Side Lobe Level (SLL) is defined as

$$\text{SLL} = \frac{|\text{Maximum value of largest side lobe}|}{|\text{Maximum value of major lobe}|}$$

The largest side lobe occurs approximately when

$$n\psi/2 \simeq \pm 3\pi/2 \quad ...(13.4.9)$$

i.e. $$\left.\frac{n}{2}(\beta d\cos\phi + \alpha)\right|_{\phi=\phi_S} = \pm 3\pi/2$$

or $$\phi_S \simeq \cos^{-1}\left\{\frac{1}{\beta d}\left[-\alpha \pm \frac{3\pi}{n}\right]\right\} \quad ...(13.4.10)$$

At this angle we can calculate AF using Equation (13.4.1(b)) by combining Equation (13.4.6) and (13.4.9) as,

$$\left.AF\right|_{\phi=\phi_S} = \frac{\sin(n\psi/2)}{(n\psi/2)} = \frac{1}{3\pi/2} = \frac{2}{3\pi} = 0.212$$

In decibels, $$AF = 20\log\left(\frac{2}{3\pi}\right) = -13.46 \text{ dB}$$

Thus maximum of the first minor lobe of the AF of Equation (13.4.1(b)) is 13.46 dB below the maximum of the major lobe.

Thus

$$\text{SLL} = -13.46 \text{ dB} \quad ...(13.4.11)$$

13.4.4 To Find Half Power Beam Width (HPBW)

From Fig. 13.4.1 we find that the function (sin x) x drops to 0.707 (3 dB) at x = 1.391. Using Equation (13.4.1(b)), the AF drops to 3 dB point at,

$$\left.\frac{n\psi}{2}\right|_{\phi=\phi_h} = \pm 1.391 \quad \text{i.e.} \left.\frac{n}{2}(\beta d\cos\phi + \alpha)\right|_{\phi=\phi_h} = \pm 1.391$$

or $$\phi_h = \cos^{-1}\left[\frac{1}{\beta d}\left(-\alpha \pm \frac{2.782}{n}\right)\right] \quad ...(13.4.12)$$

This can also be written as

$$\phi_h = \frac{\pi}{2} - \sin^{-1}\left[\frac{1}{\beta d}\left(-\alpha \pm \frac{2.782}{n}\right)\right] \quad ...(13.4.13)$$

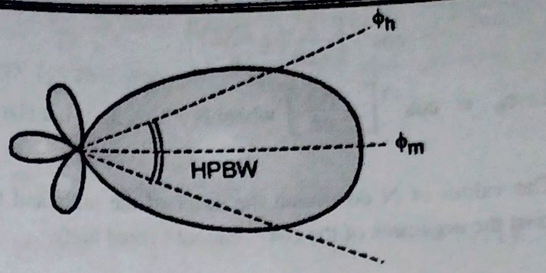
For large values of d (d >> λ), the term

$$\frac{1}{\beta d} = \frac{1}{(2\pi/\lambda)\,d} = \frac{\lambda}{2\pi d}$$

is very small, and ϕ_h reduces to

$$\phi_h \simeq \frac{\pi}{2} - \frac{1}{\beta d}\left(-\alpha \pm \frac{2.782}{n}\right) \quad ...(13.4.13(a))$$

For a symmetrical radiation pattern as shown in Fig. 13.4.2, we can obtain HPBW by knowing ϕ_m and ϕ_h.



aw(8.27)Fig. 13.4.2 : Relation between HPBW, ϕ_m and ϕ_h

For a symmetrical pattern

$$\text{HPBW} = 2\,|\phi_m - \phi_h| \quad ...(13.4.14)$$

13.4.5 Summary of Linear Array

	For n-element linear array along x-axis
1. Null directions	$\phi_0 = \cos^{-1}\left[\frac{1}{\beta d}\left(-\alpha \pm \frac{2N\pi}{n}\right)\right]$ N = 1, 2, 3, ... N ≠ n, 2n, 3n, ...
2. Maxima directions	$\phi_m = \cos^{-1}\left[\frac{1}{\beta d}(-\alpha \pm 2m\pi)\right]$ m = 0, 1, 2, ...
3. Side lobe maxima	$\phi_s = \cos^{-1}\left[\frac{1}{\beta d}\left(-\alpha \pm \left(\frac{2s+1}{n}\right)\pi\right)\right]$ s = 1, 2, 3, ...(13.4.15)
4. Half power points	$\phi_h = \cos^{-1}\left[\frac{1}{\beta d}\left(-\alpha \pm \frac{2.782}{n}\right)\right]$
5. Half Power Beam Width	HPBW = 2 | $\phi_m - \phi_h$ |
6. Side Lobe Level	SLL = – 13.46 dB

13.5 ARRAY OF N ISOTROPIC POINT SOURCES OF EQUAL AMPLITUDE AND SPACING (BROADSIDE CASE)

Definition

If the maximum radiation of an array is directed normal to the line of the array i.e. 90° and 270°, the array is called as broadside array.

13.5.1 Condition for Broadside Array

UQ. Write short note on Broad side array. **(MU - Dec. 15, 3 Marks)**

In the previous section we have studied condition for maximum radiation, which is

$$\psi = 0$$

Using the value of ψ from Equation (13.3.1) and for broadside case, φ = 90° or 270°,

$$\psi = \beta d \cos\phi + \alpha = 0$$

or

$$\beta d \cos(90° \text{ or } 270°) + \alpha = 0$$

i.e.

$$\alpha = 0 \quad ...(13.5.1)$$

Thus to have the maximum radiation of a uniform array directed normal to line of the array, the necessary condition is :

For broadside array, **all elements in the array should have the same phase excitation (α = 0) in addition to the same amplitude excitation.**

Module 4

13.6 IMPORTANT PROPERTIES OF BROADSIDE ARRAY

In the previous section we have seen different properties of linear array with phase excitation α, from which we obtain expressions for broadside array by putting α = 0. Refer section 13.4.5. These are as follows :

13.6.1 Null Directions

We have expression for null directions,

$$\phi_0 = \cos^{-1}\left[\frac{1}{\beta d}\left(-\alpha \pm \frac{2N\pi}{n}\right)\right] \quad \text{where N = 1, 2, 3,}$$

Putting α = 0 and the value of β (= 2π / λ), in the expression

$$\phi_0 = \cos^{-1}\left[\frac{1}{\beta d}\left(\pm \frac{2N\pi}{n}\right)\right]$$

$$= \cos^{-1}\left[\frac{\lambda}{2\pi d}\left(\pm\frac{2N\pi}{n}\right)\right]$$

i.e. $\phi_0 = \cos^{-1}\left[\pm\frac{N\lambda}{nd}\right]$ where N = 1, 2, 3, ... but $N \neq n, 2n, 3n,$ ...(13.6.1)

The values of N determine the order of the nulls and for null to exist the argument of the $\cos^{-1}$ cannot exceed unity.

13.6.2 Direction of Maxima

For linear array with phase excitation (α),

$$\phi_m = \cos^{-1}\left[\frac{1}{\beta d}(-\alpha \pm 2m\pi)\right]$$

with $\alpha = 0$,

$$\phi_m = \cos^{-1}\left[\pm\frac{2m\pi}{\beta d}\right]$$

i.e $\phi_m = \cos^{-1}\left(\pm\frac{m\lambda}{d}\right)$ where m = 0, 1, 2, ...(13.6.2)

The separation between the elements can be of any value.

Grating lobes

The interesting thing occurs when the separation is equal to multiples of a wavelength i.e. $d = N\lambda$, where N = 1, 2, 3, ... and $\alpha = 0$.

$$\psi = \beta d \cos\phi + \alpha = \frac{2\pi}{\lambda} N\lambda \times \cos\phi + 0$$

or $\psi = 2\pi N \cos\phi$

If we put $\phi = 0°$ or $180°$, $\psi = \pm 2\pi N$

This value of ψ when substituted in AF expression

$$AF = \frac{1}{n}\frac{\sin(n\psi/2)}{\sin(\psi/2)} = \frac{\sin(0)}{\sin(0)} \Rightarrow \text{maximum}$$

Thus in addition to having the maxima of the radiation pattern in broadside direction, there are additional maxima along the axis ($\phi = 0°$ and $180°$) of the array. These are the maxima in unwanted directions, called as grating lobes.

> **To avoid grating lobes the largest spacing between the elements should be less than one wavelength.**
>
> i.e. $d_{max} < \lambda$

13.6.3 Directions of Side Lobe Maxima (ϕ_S)

For linear array with phase excitation α,

$$\phi_S = \cos^{-1}\left[\frac{1}{\beta d}\left(-\alpha \pm \left(\frac{2S+1}{n}\right)\pi\right)\right]$$

For broadside case with $\alpha = 0$, the above equation reduces to

$$\phi_S = \cos^{-1}\left[\frac{1}{\beta d}\left(\pm\frac{2S+1}{n}\pi\right)\right]$$

$$\phi_S = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left(\pm\frac{2S+1}{n}\pi\right)\right]$$

or $\phi_S = \cos^{-1}\left[\pm\frac{(2S+1)\lambda}{2nd}\right]$ ...(13.6.3)

where S = 1, 2, 3, ...

13.6.4 Half Power Points (ϕ_h)

For linear array with phase excitation α,

$$\phi_h = \cos^{-1}\left[\frac{1}{\beta d}\left(-\alpha \pm \frac{2.782}{n}\right)\right]$$

For broadside array with $\alpha = 0$ the above equation reduces to

$$\phi_h = \cos^{-1}\left[\frac{\lambda}{2\pi d}\left(\pm\frac{2.782}{n}\right)\right]$$

i.e. $\phi_h = \cos^{-1}\left[\pm\frac{1.391\,\lambda}{\pi dn}\right]$ ...(13.6.4)

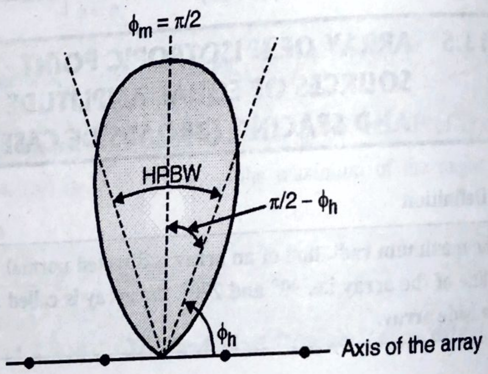
13.6.5 Half Power Beam Width (HPBW)

As shown in Fig. 13.6.1, the half power angle from axis of the array is ϕ_h, given in Equation (13.6.4). The angle of the major beam from axis is 90° (broadside case).

The HPBW is now calculated as

$$HPBW = 2\left[\frac{\pi}{2} - \phi_h\right]$$

or $HPBW = 2\left[\frac{\pi}{2} - \cos^{-1}\left(\frac{1.391\,\lambda}{\pi dn}\right)\right]$ ...(13.6.5)



aw(8.28)**Fig. 13.6.1 : To find HPBW**

13.6.6 First Null Beam Width (FNBW)

From Equation (13.6.1), the null directions are given by

$$\phi_0 = \cos^{-1}\left(\pm\frac{N\lambda}{nd}\right)$$

For first null, substitute N = 1, which gives

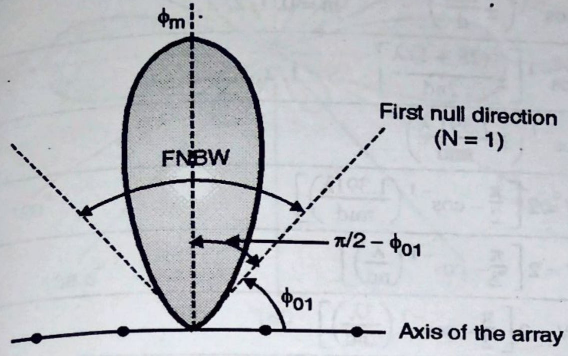
$$\phi_{01} = \cos^{-1}\left(\pm\frac{\lambda}{nd}\right)$$

So for broadside case, angle made by first null direction from major lobe direction is $\left(\frac{\pi}{2} - \phi_{01}\right)$.

$$\therefore \quad FNBW = 2\left[\frac{\pi}{2} - \phi_{01}\right]$$

or

$$FNBW = 2\left[\frac{\pi}{2} - \cos^{-1}\left(\frac{\lambda}{nd}\right)\right] \quad ...(13.6.6)$$



aw(8.29)**Fig. 13.6.2 : To find FNBW**

13.6.7 First Side Lobe Beam Width

Using Equation (13.6.3) we can obtain direction of first side lobe maxima (S = 1),

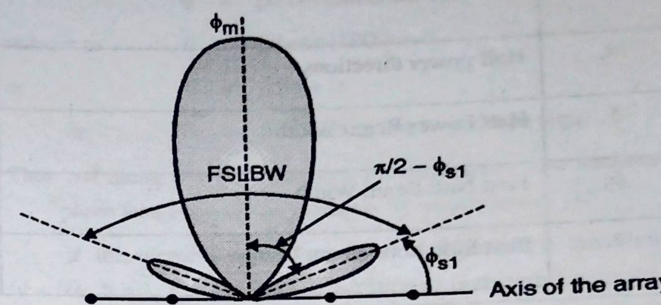
$$\phi_{S1} = \cos^{-1}\left[\pm\frac{(2S+1)\lambda}{2nd}\right] = \cos^{-1}\left[\pm\frac{3\lambda}{2nd}\right]$$

Angle made by this direction with major lobe direction is $\left(\frac{\pi}{2} - \phi_{S1}\right)$. So

$$FSLBW = 2\left[\frac{\pi}{2} - \phi_{S1}\right]$$

or

$$FSLBW = 2\left[\frac{\pi}{2} - \cos^{-1}\left(\frac{3\lambda}{2nd}\right)\right] \quad ...(13.6.7)$$

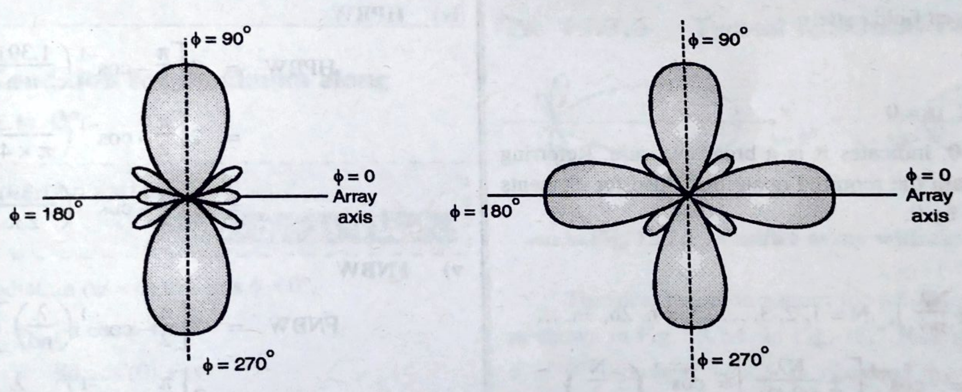


aw(8.30)**Fig. 13.6.3 : To find first side lobe beamwidth**

13.6.8 Typical Broadside Radiation Patterns

UQ. Write short note on Broad side array. **(MU - Dec. 15, 3 Marks)**

Module 4



(a) Only broadside case **(b) Broadside / endfire case**

aw(8.31)**Fig. 13.6.4 : Broadside radiation patterns**

As shown in Fig. 13.6.4(a), it is the desired broadside pattern with direction of maxima perpendicular to array axis. But if the separation $d = N\lambda$, N = 1, 2, 3, ..., then along with broadside maxima we get maxima along the axis, (called as endfire). Thus we get for

$d < \lambda$: Only broadside array

$d = N\lambda$: Broadside / endfire array

For spacing $d = \lambda$, the radiation pattern is as shown in Fig. 13.6.4(b).

If the spacing between elements is chosen between $\lambda < d < 2\lambda$, then the maximum of Fig. 13.6.4(b) along $\phi = 0°$ shifts towards the angular region $0° < \phi < 90°$ while the maximum along $\phi = 180°$ shifts towards $180° < \phi < 270°$.

13.6.9 Summary of Broadside Array

Sr. No.	Parameter	Expression
1.	Nulls	$\phi_0 = \cos^{-1}\left(\pm\frac{N\lambda}{nd}\right)$ N = 1, 2, 3, … N ≠ n, 2n, 3n, …
2.	Maxima	$\phi_m = \cos^{-1}\left(\pm\frac{m\lambda}{d}\right)$ m = 0, 1, 2, …
3.	Side lobe maxima	$\phi_S = \cos^{-1}\left[\pm\frac{(2S+1)\lambda}{2nd}\right]$ S = 1, 2, 3, …
4.	Half power directions	$\phi_h = \cos^{-1}\left(\pm\frac{1.391\lambda}{\pi nd}\right)$
5.	Half Power Beam Width	$HPBW = 2\left[\frac{\pi}{2} - \cos^{-1}\left(\frac{1.391\lambda}{\pi nd}\right)\right]$
6.	First Null Beam Width	$FNBW = 2\left[\frac{\pi}{2} - \cos^{-1}\left(\frac{\lambda}{nd}\right)\right]$
7.	First Side Lobe Beam Width	$FSLBW = 2\left[\frac{\pi}{2} - \cos^{-1}\left(\frac{3\lambda}{2nd}\right)\right]$

UEx. 13.6.1 MU - Dec. 12, 15 Marks, Dec. 15, Dec. 16, May 17, May 18, 10 Marks.

For an array of four isotropic sources along z-axis separated by a distance of λ/2 and a progressive phase shift α = 0, find

i) Null directions ii) Directions of maxima

iii) Directions of side lobe maxima

iv) Half power beam width (HPBW)

v) First null beam width (FNBW)

vi) Side lobe level (SLL)

vii) Rough normalized field pattern.

☑ **Soln. :**

Given : n = 4, d = λ/2, α = 0

The phase shift α = 0, indicates it is a broadside case. Referring section 13.6.9 we obtain the required quantities. Also for elements along z-axis replace φ by θ.

i) Null directions

$$\theta_0 = \cos^{-1}\left(\pm\frac{N\lambda}{nd}\right),\ N = 1, 2, 3, \ldots, N \neq n, 2n, 3n, \ldots$$

$$= \cos^{-1}\left[\pm\frac{N\lambda}{4(\lambda/2)}\right] = \cos^{-1}\left(\pm\frac{N}{2}\right)$$

For N = 1, $\theta_{01} = \cos^{-1}(\pm 0.5) = 60^\circ, 120^\circ$

For N = 2, $\theta_{02} = \cos^{-1}(\pm 1) = 0^\circ, 180^\circ$

N = 3 onwards is not possible since N/2 is greater than 1 whose $\cos^{-1}$ ( ) cannot be determined.

ii) Directions of maxima

$$\theta_m = \cos^{-1}\left(\pm\frac{m\lambda}{d}\right),\ m = 0, 1, 2, \ldots$$

$$= \cos^{-1}\left[\pm\frac{m\lambda}{(\lambda/2)}\right] = \cos^{-1}(\pm 2m)$$

For m = 0 ; $\theta_{m1} = \cos^{-1}(\pm 0) = 90^\circ$.

iii) Side lobe maxima

$$\theta_S = \cos^{-1}\left[\pm\frac{(2S+1)\lambda}{2nd}\right],\ S = 1, 2, 3, \ldots$$

For S = 1, $\theta_{S1} = \cos^{-1}\left[\pm\frac{3\lambda}{2(4)(\lambda/2)}\right]$

$$= \cos^{-1}(\pm 0.75) = 41.40^\circ,\ 138.6^\circ$$

Values for S = 2 onwards cannot be determined.

iv) HPBW

$$HPBW = 2\left[\frac{\pi}{2} - \cos^{-1}\left(\frac{1.391\lambda}{\pi n d}\right)\right]$$

$$= 2\left[\frac{\pi}{2} - \cos^{-1}\left(\frac{1.391\lambda}{\pi \times 4 \times (\lambda/2)}\right)\right]$$

$$= 2\left[\frac{\pi}{2} - \cos^{-1}\left(\frac{1.391}{2\pi}\right)\right] = 25.58^\circ$$

v) FNBW

$$FNBW = 2\left[\frac{\pi}{2} - \cos^{-1}\left(\frac{\lambda}{nd}\right)\right]$$

$$= 2\left[\frac{\pi}{2} - \cos^{-1}\left(\frac{\lambda}{4 \times \lambda/2}\right)\right] = 60^\circ$$

vi) SLL

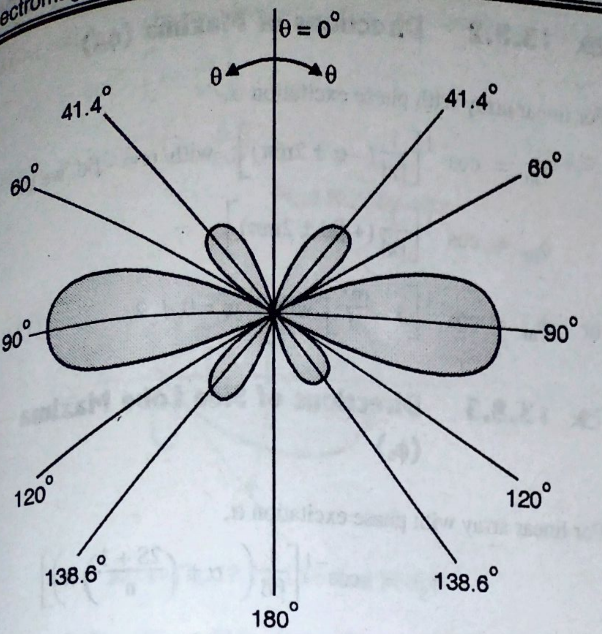
SLL = – 13.46 dB

vii) Radiation pattern

Null directions : 0°, 60°, 120°, 180°

Maxima directions : 90°

Side lobe maxima : 41.40°, 138.6°



aw(8.32)**Fig. Ex. 13.6.1**

13.7 ARRAY OF N ISOTROPIC POINT SOURCES OF EQUAL AMPLITUDE AND SPACING (ENDFIRE CASE)

Definition

An array is called as endfire if the maximum radiation is along one end of the axis of the array, i.e. $\phi = 0°$ or $180°$ with $\psi = 0$.

13.7.1 Condition for Maximum along $\phi = 0°$

UQ. Write short note on end fire array.
(MU - Dec. 15, 3 Marks)

To have maximum radiation ($\psi = 0$) towards $\phi = 0°$,

$$\psi = \beta d \cos \phi + \alpha$$

reduces to
$$0 = \beta d \cos (0) + \alpha$$

i.e.
$$\alpha = -\beta d \text{ (lagging due to negative sign)}$$

Thus end along $\phi = 0$ radiates provided there is a successive phase lag between the elements by an amount of βd.

For example if the spacing between the elements is $d = \lambda / 2$ then phase excitation

$$\alpha = -\beta d = -\frac{2\pi}{\lambda} \times \frac{\lambda}{2} = -\pi$$

It means for $\phi = 0°$ radiation, with separation between the elements as $\lambda / 2$, source 2 lags source 1 by 180°, source 3 lags source 2 by 180° and so on.

13.7.2 Condition for Maximum along $\phi = 180°$

UQ. Write short note on end fire array
(MU - Dec. 15, 3 Marks)

If the maximum is desired along $\phi = 180°$,

$$\psi = \beta d \cos \phi + \alpha$$

reduces to
$$0 = \beta d \cos (180°) + \alpha$$

or
$$0 = -\beta d + \alpha$$

or
$$\alpha = \beta d \text{ (leading due to positive sign)}$$

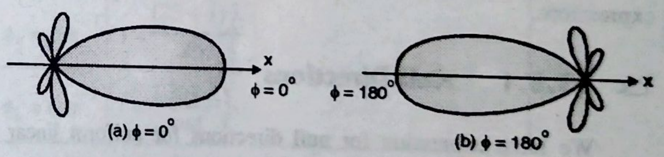
Thus end along $\phi = 180°$ radiates provided there is a successive phase lead between the elements by an amount of βd.

If the element separation is a multiple of a wavelength ($d = N\lambda$, $N = 1, 2, 3, \ldots$), then in addition to both ends firing we get radiation in broadside directions. Thus there exists four maxima, two in the broadside direction and two along the axis of the array.

Also as discussed in broadside case to avoid grating lobes the element separation should be less than one wavelength ($d_{max} < \lambda$).

End fire array			
For maximum along $\phi = 0°$	:	$\alpha = -\beta d$	...(13.7.1)
For maximum along $\phi = 180°$	:	$\alpha = +\beta d$	...(13.7.2)

13.7.3 Typical Radiation Pattern



aw(8.33)**Fig. 13.7.1 : Endfire array with (a) $\phi = 0°$, (b) $\phi = 180°$**

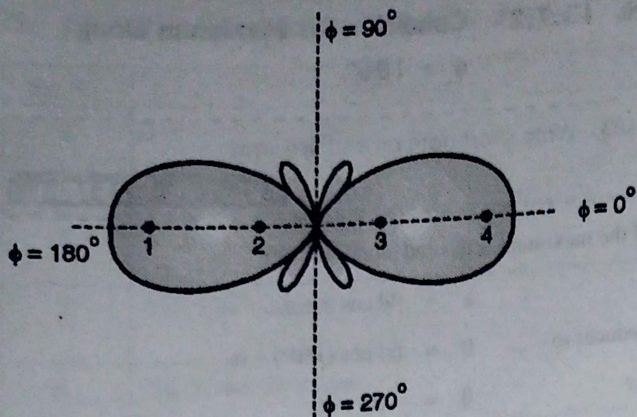
The ideal radiation pattern for which only one end is firing is as shown in Fig. 13.7.1. In Fig. 13.7.1(a), the major lobe is along $\phi = 0°$ direction, which is obtained by successive phase lag $\alpha = -\beta d$. In Fig. 13.7.1(b), the firing is along $\phi = 180°$ which is obtained with successive phase lead $\alpha = \beta d$.

As an example, if $d = \lambda / 2$, number of elements $n = 4$, and

$$\alpha = -\beta d = -\frac{2\pi}{\lambda} \times \frac{\lambda}{2} = -\pi.$$

When the radiation pattern is calculated it will be as shown in Fig. 13.7.2. Instead of $\alpha = -\pi$, if we use $\alpha = \pi$, the radiation pattern we obtain will be similar. It is a bi-directional pattern, not an ideal one.

Module 4



aw(8.34)**Fig. 13.7.2 : Radiation pattern of endfire with $n = 4$, $d = \lambda / 2$, $\alpha = -\pi$**

However, if the spacing is less than $\lambda / 2$, the maximum radiation is in the direction $\phi = 0$ when $\alpha = -\pi$ and in the direction $\phi = 180°$ when $\alpha = \pi$.

13.7.4 The Ordinary Endfire Array

In the previous discussion we have said that the endfire condition results when $\phi = 0°$ or $180°$, which corresponds to $\alpha = -\beta d$ or $+\beta d$. **Such arrays for which $\alpha = \pm\beta d$ are referred to as ordinary endfire arrays.** For this array we obtain expressions for different characteristics of the pattern in the next section.

13.8 PROPERTIES OF ENDFIRE ARRAYS

In section 13.4.5 we have listed some properties of linear array. In those by putting $\alpha = -\beta d$, we obtain the required expressions.

13.8.1 Null Directions

We have expression for null directions for uniform linear array with phase excitation α,

$$\phi_0 = \cos^{-1}\left[\frac{1}{\beta d}\left(-\alpha \pm \frac{2N\pi}{n}\right)\right]$$

Putting $\alpha = -\beta d$, we get

$$\phi_0 = \cos^{-1}\left[\frac{1}{\beta d}\left(+\beta d \pm \frac{2N\pi}{n}\right)\right] = \cos^{-1}\left[1 \pm \frac{1}{\beta d}\frac{2N\pi}{n}\right]$$

$$= \cos^{-1}\left[1 \pm \frac{\lambda}{2\pi d}\frac{2N\pi}{n}\right]$$

or $$\phi_0 = \cos^{-1}\left[1 - \frac{\lambda N}{nd}\right], \quad N = 1, 2, 3, \ldots \text{ but } N \neq n, 2n, 3n, \qquad \ldots(13.8.1)$$

We have selected negative sign in the bracket, otherwise with positive sign the bracket value will exceed unity and then $\cos^{-1}$ of it cannot be determined. Same argument is to be applied for rest of the analysis. But instead of $\alpha = -\beta d$ if we go for $\alpha = +\beta d$ then we will use positive sign in the bracket instead of negative sign.

13.8.2 Directions of Maxima (ϕ_m)

For linear array with phase excitation α,

$$\phi_m = \cos^{-1}\left[\frac{1}{\beta d}(-\alpha \pm 2m\pi)\right] \quad \text{with } \alpha = -\beta d, \text{ we get}$$

$$\phi_m = \cos^{-1}\left[\frac{1}{\beta d}(+\beta d \pm 2m\pi)\right]$$

or $$\phi_m = \cos^{-1}\left[1 - \frac{m\lambda}{d}\right] \quad \text{where } m = 0, 1, 2, \ldots \qquad \ldots(13.8.2)$$

13.8.3 Directions of Side Lobe Maxima (ϕ_s)

For linear array with phase excitation α,

$$\phi_S = \cos^{-1}\left[\frac{1}{\beta d}\left(-\alpha \pm \left(\frac{2S+1}{n}\right)\pi\right)\right]$$

Now with $\alpha = -\beta d$, we get

$$\phi_S = \cos^{-1}\left[\frac{1}{\beta d}\left(+\beta d \pm \left(\frac{2S+1}{n}\right)\pi\right)\right]$$

or $$\phi_S = \cos^{-1}\left[1 - \frac{(2S+1)\lambda}{2nd}\right] \quad \text{where } S = 1, 2, 3, \ldots \qquad \ldots(13.8.3)$$

13.8.4 Half Power Points (ϕ_h)

For linear array with phase excitation α,

$$\phi_h = \cos^{-1}\left[\frac{1}{\beta d}\left(-\alpha \pm \frac{2.782}{n}\right)\right]$$

Now with $\alpha = -\beta d$, we get

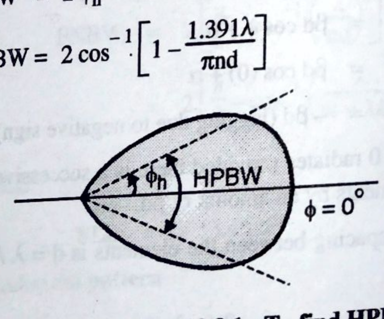
$$\phi_h = \cos^{-1}\left[1 - \frac{1.391\lambda}{\pi nd}\right] \qquad \ldots(13.8.4)$$

13.8.5 Half Power Beam Width (HPBW)

From Fig. 13.8.1 it is very clear that the half power beam width

$$\text{HPBW} = 2\,\phi_h$$

$$\text{i.e. HPBW} = 2\cos^{-1}\left[1 - \frac{1.391\lambda}{\pi nd}\right] \qquad \ldots(13.8.5)$$



aw(8.35)**Fig. 13.8.1 : To find HPBW**

13.8.6 First Null Beam Width (FNBW)

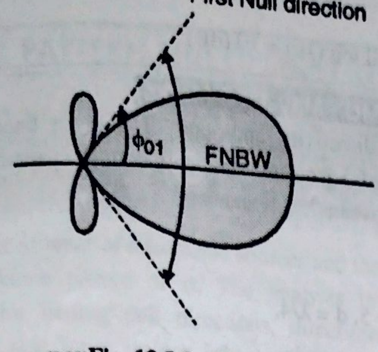
From Fig. 13.8.2 it is clear that the first null beam width

$$\text{FNBW} = 2\,\phi_{01}$$

The first null ϕ_{01} is obtained by putting N = 1 in Equation (13.8.1)

$$\phi_{01} = \cos^{-1}\left[1 - \frac{\lambda}{nd}\right]$$

$$\text{so FNBW} = 2\cos^{-1}\left(1 - \frac{\lambda}{nd}\right) \quad \text{...(13.8.6)}$$



aw(8.36)Fig. 13.8.2 : To find FNBW

13.8.7 First Side Lobe Beam Width (FSLBW)

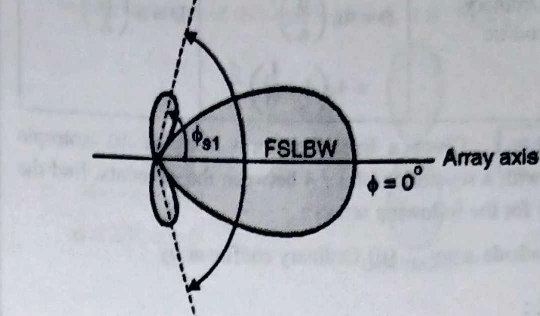
From Fig. 13.8.3 it is clear that the first lobe beam width

$$\text{FSLBW} = 2\phi_{S1}$$

The first side lobe maxima ϕ_{S1} is obtained by putting S = 1 in Equation (13.8.3),

$$\phi_{S1} = \cos^{-1}\left[1 - \frac{3\lambda}{2nd}\right]$$

$$\text{so, FSLBW} = 2\cos^{-1}\left[1 - \frac{3\lambda}{2nd}\right] \quad \text{...(13.8.7)}$$



aw(8.37)Fig. 13.8.3 : To find FSLBW

13.8.8 Summary of Ordinary Endfire Array

Sr. No.	Parameter	Expression
1.	Nulls	$\phi_0 = \cos^{-1}\left(1 - \frac{\lambda N}{nd}\right)$ N = 1, 2, 3, ... N ≠ n, 2n, 3n, ...
2.	Maxima	$\phi_m = \cos^{-1}\left(1 - \frac{m\lambda}{d}\right)$ m = 0, 1, 2, ...
3.	Side lobe maxima	$\phi_S = \cos^{-1}\left[1 - \frac{(2S+1)\lambda}{2nd}\right]$
4.	Half power directions	$\phi_h = \cos^{-1}\left(1 - \frac{1.391\lambda}{\pi nd}\right)$
5.	Half Power Beam Width	$\text{HPBW} = 2\cos^{-1}\left(1 - \frac{1.391\lambda}{\pi nd}\right)$
6.	First Null Beam Width	$\text{FNBW} = 2\cos^{-1}\left(1 - \frac{\lambda}{nd}\right)$
7.	First Side Lobe Beam Width	$\text{FSLBW} = 2\cos^{-1}\left(1 - \frac{3\lambda}{2nd}\right)$

UEx. 13.8.1 (MU - May 12, 10 Marks)

A uniform linear array is required to produce an endfire beam when it is operated at 12 GHz. It contains 50 radiations and are spaced 0.5 λ. Find the progressive phase shift to produce the endfire beam. Also find the array length.

☑ **Soln. :**

Given : n = 50, d = λ/2

$$\text{Phase shift, } \alpha = -\beta d = -\frac{2\pi}{\lambda} \times \frac{\lambda}{2} = -\pi$$

$$\text{Length, } L = (n-1)\,d = 49(\lambda/2) = 24.5\,\lambda \text{ (m)}$$

13.9 DIRECTIVITY OF LINEAR ARRAY WITH N SOURCES

In this section we shall see the expressions for directivity for different types of arrays studied in the previous sections. These are given by Table 13.9.1.

Module 4

Table 13.9.1 : Directivities

Sr. No.	Array	D for any length	D for large array (L >> d)
1.	Broadside	$D = 2n\left(\frac{d}{\lambda}\right)$ $= 2\left(1+\frac{L}{d}\right)\frac{d}{\lambda}$	$D = 2\left(\frac{L}{\lambda}\right)$
2.	Ordinary endfire	$D = 4n\left(\frac{d}{\lambda}\right)$ $= 4\left(1+\frac{L}{d}\right)\frac{d}{\lambda}$	$D = 4\left(\frac{L}{\lambda}\right)$

Ex. 13.9.1 : Given a linear, uniform array of 10 isotropic elements with a separation of $\lambda / 4$ between the elements, find the directivity for the following arrays :

(i) Broadside array (ii) Ordinary endfire array

☑ **Soln. :**

(i) For broadside array

$$D = 2n\left(\frac{d}{\lambda}\right) = 2\,(10)\,\frac{(\lambda/4)}{\lambda} = 5 \text{ (no unit)}$$

In decibels,

$$D = 10 \log (5) = 6.99 \text{ (dB)}$$

(ii) For ordinary endfire array

$$D = 4n\left(\frac{d}{\lambda}\right) = 4\,(10)\,\frac{(\lambda/4)}{\lambda} = 10 \text{ (no unit)}$$

In decibels,

$$D = 10 \log (10) = 10 \text{ (dB)}$$

UEx. 13.9.2 (MU - May 11, 5 Marks)

Calculate the directivity and gain of a given linear broadside uniform array of 5 isotropic elements with a separation of quarter wavelength between the elements.

☑ **Soln. :**

Given : $n = 5, d = \lambda/4$.

The length of the array is

$$L = (n-1)\,d = 4 \times (\lambda/4) = \lambda \text{ (m)}$$

The directivity of a broadside array is

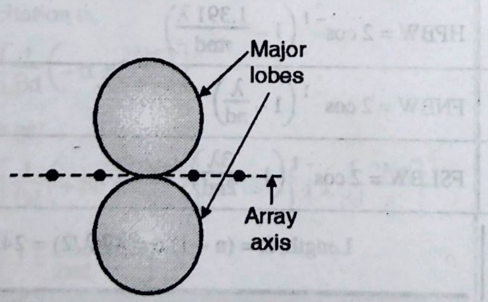
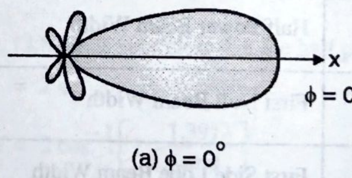
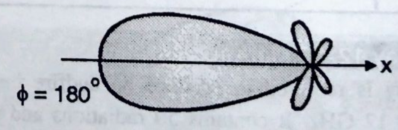
$$D = 2\left(\frac{L}{\lambda}\right) = 2.$$

For the lossless antenna

$$\text{Gain} = G = D = 2.$$

13.10 DIFFERENCE BETWEEN BROADSIDE AND END FIRE ARRAY

UQ. Compare Broadside and Endfire array. **(MU - Dec. 16, May 17, May 18, 5 Marks)**

Sr. No.	Broadside array	End fire array
i)	Here the maximum radiation is perpendicular to the axis of the array.	Here the maximum radiation is along one end of the axis of the array.
ii)	The major lobe is at ϕ equal to 90° or 270°, to the array axis.	The major lobe is along ϕ equal to 0° or 180°.
iii)	The typical radiation pattern is as shown 	The radiation pattern is as shown  (a) $\phi = 0°$  (b) $\phi = 180°$
iv)	To have broadside pattern all elements should have same phase excitation.	For maximum along $\phi = 0°$: $\alpha = -\beta d$ For maximum along $\phi = 180°$: $\alpha = \beta d$
v)	Direction of nulls : $\phi_0 = \cos^{-1}\left(\pm\frac{N\lambda}{nd}\right)$ N = 1, 2, 3, ... N ≠ n, 2n, 3n, ...	Direction of nulls : $\phi_0 = \cos^{-1}\left(1-\frac{N\lambda}{nd}\right)$ N = 1, 2, 3, ... N ≠ n, 2n, 3n, ...

Sr. No.	Broadside array	End fire array
vi)	Direction of maxima : $\phi_m = \cos^{-1}\left(\pm\frac{m\lambda}{d}\right)$ m = 0, 1, 2, ...	Direction of maxima : $\phi_m = \cos^{-1}\left(1 - \frac{m\lambda}{d}\right)$ m = 0, 1, 2, ...
vii)	Directivity is $D = 2n\left(\frac{d}{\lambda}\right) = 2\left(1 + \frac{L}{d}\right)\frac{d}{\lambda}$	Directivity is $D = 4n\left(\frac{d}{\lambda}\right) = 4\left(1 + \frac{L}{d}\right)\frac{d}{\lambda}$

13.11 PATTERN MULTIPLICATION

UQ. Explain pattern multiplication for antenna array. **(MU - Dec. 17, Q. 4(a), Dec. 19, 5 Marks)**

Consider an array of n isotropic sources and the problem is to find the radiation pattern of it. The analysis involves lot of calculations for finding null directions, directions of maxima, directions of side lobes (minor lobes) etc. After finding these directions one can plot the radiation pattern.

Sometimes instead of exact radiation pattern with accurate directions the rough patterns are also sufficient. This can be done very easily by a **pattern multiplication technique.**

With the help of this method it is possible to sketch rapidly, almost by inspection, the patterns of complicated arrays.

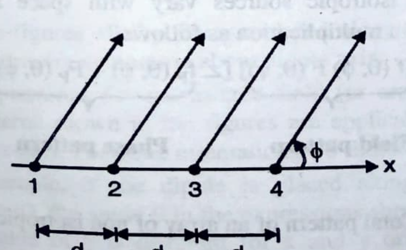
This method is not only applicable for isotropic sources but for non isotropic sources also. In the first part of the section we shall consider ~~only~~ isotropic sources and in the later part non isotropic sources.

13.11.1 Pattern Multiplication using Isotropic Sources

13.11.1(A) Example of Four Isotropic Sources

UQ. Discuss principle of pattern multiplication with example. **(MU - May 15, 5 Marks)**

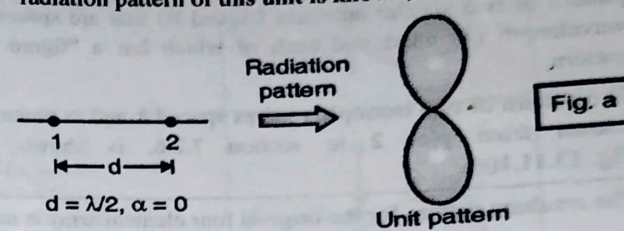
Consider an array of four isotropic sources with spacing (d) equal to λ/2 and phase (α) equal to zero. The resultant pattern can be obtained analytically by adding the four fields due to four elements.



aw(8.39)**Fig. 13.11.1 : Array of four isotropic sources**

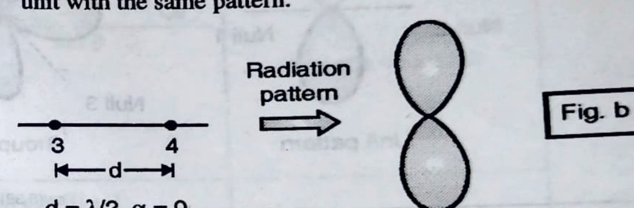
The same pattern can be obtained from the following considerations :

i) **Consider antenna 1 and 2 as one unit of two isotropic antennas separated by λ/2 and in phase (d = λ/2, α = 0). The radiation pattern of this unit is known, it is "figure 8" shape.**



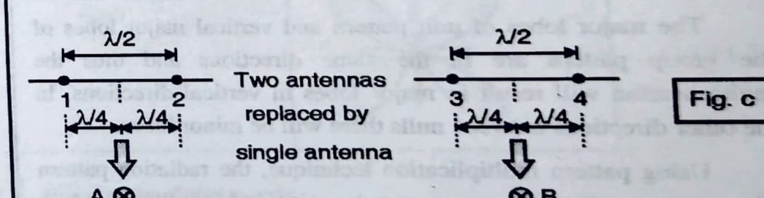
aw(8.39)**Fig. 13.11.1(a)**

ii) **Also antennas 3 and 4 may be considered as another similar unit with the same pattern.**



aw(8.39)**Fig. 13.11.1(b)**

iii) **Suppose we have one antenna whose radiation pattern is "figure 8" shaped. This single antenna (A) can be used to replace antennas 1 and 2 by locating it midway between them.**

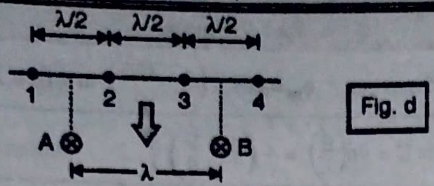


aw(8.39)**Fig. 13.11.1(c)**

iv) **Antennas 3 and 4 could similarly be replaced by a single antenna (B) having "figure 8" pattern, as shown in Fig. 13.11.1(c).**

v) **Now the system looks like as shown below, in which antenna A is representing antennas 1 and 2, while antenna B is representing antennas 3 and 4. As antennas A and B are placed midway of antennas 1-2 and 3-4 respectively, the separation between A and B is λ.**

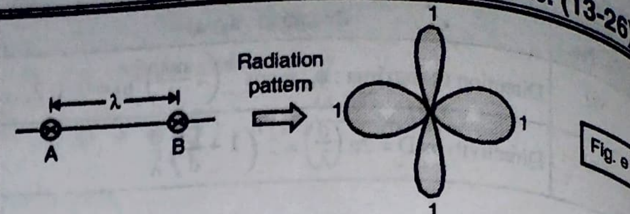
Module 4



aw(8.39)**Fig. 13.11.1(d)**

vi) The problem is now reduced to that of finding the radiation pattern of two similar antennas (A and B) that are spaced a wavelength (λ) apart and each of which has a "figure 8" pattern.

vii) The pattern of two isotropic sources spaced λ and in phase is known from plot 2 in section 7.2.6, is shown in Fig. 13.11.1(e).

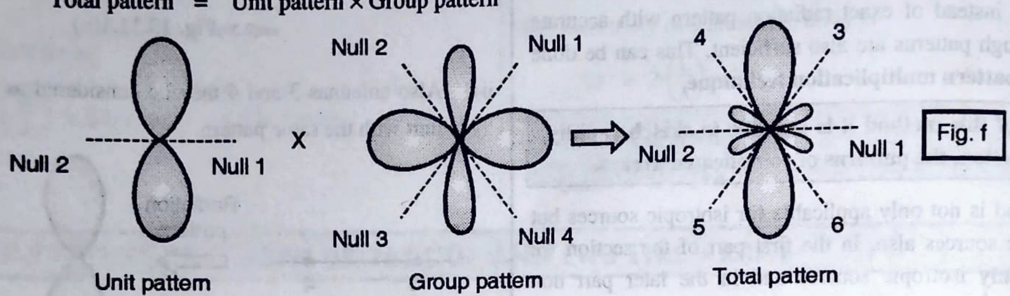


aw(8.39)**Fig. 13.11.1(e)**

viii) In the Fig. 13.11.1(e), group of two antennas is representing original four isotropic sources. The radiation pattern of the group (A and B) is called as group pattern (Fig. 13.11.1 (e)). And the radiation pattern of the unit of the group (A or B) is called as unit pattern (Fig. 13.11.1(a) or 13.11.1(b)).

ix) The resultant pattern for the original four element array is now obtained as the product of the unit pattern with group pattern.

$$\text{Total pattern} = \text{Unit pattern} \times \text{Group pattern} \quad ...(13.11.1)$$



aw(8.39)**Fig. 13.11.1(f)**

x) The property of the total pattern is that the number of nulls in the total pattern is the sum of the nulls in the unit and group patterns (assuming none of the nulls are coincident).

As shown in the diagram, the unit pattern consists of two null directions, the group pattern consists of four null directions. Thus the resultant is having six nulls. In between nulls, the lobes (major and minor) are present.

The major lobes of unit pattern and vertical major lobes of the group pattern are in the same directions and thus the multiplication will result in major lobes in vertical directions. In the other directions between nulls there will be minor lobes.

Using pattern multiplication technique, the radiation pattern of more complicated arrays can be obtained as illustrated in Example 13.11.1.

13.11.2 Pattern Multiplication using Non Isotropic but Similar Sources

The **similar** sources means their field amplitude and phase variation with angle ϕ is same. That is the radiation pattern must be of same shape and same orientation. The maximum amplitudes of the individual sources may be unequal.

When the radiation patterns, their orientation and amplitudes of individual sources is same, then they are said to be **identical**.

In the previous section the pattern multiplication is applied to isotropic sources. It can also be applied to non isotropic but similar sources. The rule here is,

The complete (normalized) pattern of an array with similar sources is

E (θ, φ) = (individual source pattern) × (pattern of an array of isotropic sources having the same locations, relative amplitudes, and phase as the non isotropic point sources)

In general, the amplitude as well as phase of the individual source and array of isotropic sources vary with space angle. So we consider both in multiplication as follows.

$$E(\theta, \phi) = \underbrace{[f(\theta, \phi)\, F(\theta, \phi)]}_{\text{Field pattern}}\ \underbrace{[\angle f_P(\theta, \phi) + F_P(\theta, \phi)]}_{\text{Phase pattern}} \quad ...(13.11.2)$$

Where,

$E(\theta, \phi)$ = Total pattern of an array of non isotropic sources

$f(\theta, \phi)$ = field pattern of the individual source

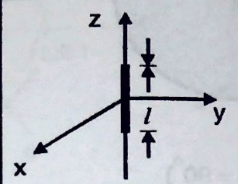
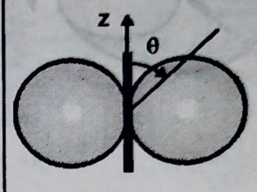
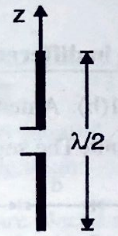
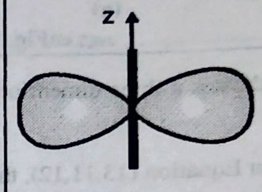
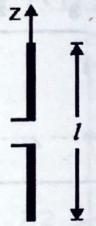
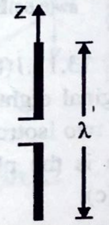
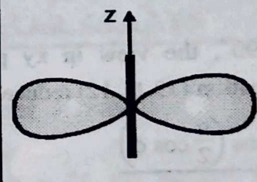
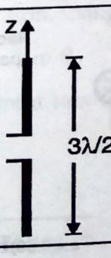
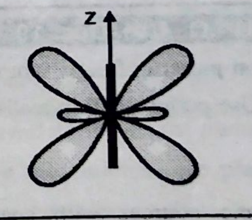
$f_P(\theta, \phi)$ = phase pattern of the individual source

$F(\theta, \phi)$ = field pattern of array of isotropic sources

$F_P(\theta, \phi)$ = phase pattern of array of isotropic sources

13.11.3 Field and Pattern of Some of the Non Isotropic Sources

The non isotropic sources in the previous discussion may be dipoles of any length. The field expressions and patterns of different dipoles are repeated here for fast reference.

Sr. No.	Type of dipole	Orientation	Normalized field expression	Radiation Pattern	
1.	Infinitesimal ($l \le \lambda/50$)		$E(\theta) = \sin\theta$		...(13.11.3)
2.	Half-wave ($l = \lambda/2$)		$E(\theta) = \dfrac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}$		...(13.11.4)
3.	Finite length (Any length)		$E(\theta) = \dfrac{\cos\left(\frac{\beta l}{2}\cos\theta\right) - \cos\left(\frac{\beta l}{2}\right)}{\sin\theta}$	Depends on length	...(13.11.5)
4.	$l = \lambda$		$E(\theta) = \dfrac{\cos(\pi\cos\theta) + 1}{\sin\theta}$		...(13.11.6)
5.	$l = 3\lambda/2$		$E(\theta) = \dfrac{\cos\left(\frac{3}{2}\pi\cos\theta\right)}{\sin\theta}$		...(13.11.7)

In these figures all dipoles are placed along z-axis and the radiation patterns are symmetric about dipole axis.

The expressions for normalized field (or array factor) and radiation patterns shown in the figures are applicable for dipoles placed along z-axis. For other orientations we should modify it.

For example, if the dipole is placed along x or y axis (or parallel to it), the angle θ in the expressions should be replaced by γ. The value of γ is different for x and y orientations. One should apply following changes.

For dipole along x-axis

$$\theta \rightarrow \gamma \quad \text{...(13.11.8)}$$

$$\cos\gamma = \sin\theta\cos\phi \quad \text{...(13.11.9)}$$

For dipole along y-axis

$$\theta \rightarrow \gamma \quad \text{...(13.11.10)}$$

$$\cos\gamma = \sin\theta\sin\phi \quad \text{...(13.11.11)}$$

For a λ/2 dipole along x-axis

$$E(\gamma) = E(\theta, \phi) = \frac{\cos\left(\frac{\pi}{2}\cos\gamma\right)}{\sin\gamma} = \frac{\cos\left(\frac{\pi}{2}\sin\theta\cos\phi\right)}{\sqrt{1-\cos^2\gamma}}$$

$$E(\theta, \phi) = \frac{\cos\left(\frac{\pi}{2}\sin\theta\cos\phi\right)}{\sqrt{1-\sin^2\theta\cos^2\phi}} \quad \text{...(13.11.12)}$$

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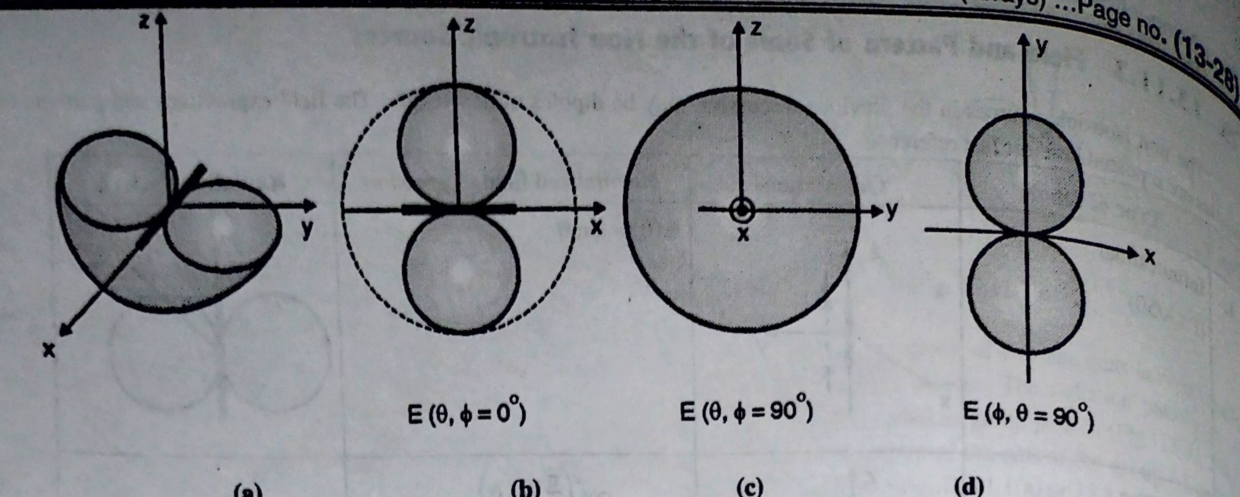


aw(8.49)**Fig. 13.11.2 : Radiation pattern of a λ/2 dipole in different planes**

Fig. 13.11.2(a) indicates a three dimensional view of the radiation pattern.

By putting $\phi = 0$ in Equation (13.11.12), the radiation pattern in xz plane is (Fig. 13.11.2(b)) obtained. Equation (13.11.12) now reduces to,

$$E(\theta, \phi = 0) = \frac{\cos\left(\frac{\pi}{2}\sin\theta\right)}{\cos\theta} \quad ...(13.11.13)$$

By putting $\phi = 90°$, the radiation pattern in yz plane is shown in Fig. 13.11.2(c). Equation (13.11.12) reduces to

$$E(\theta, \phi = 90°) = 1 \quad ...(13.11.14)$$

By putting $\theta = 90°$, the view in xy plane is Fig. 13.11.2(d) obtained. Also Equation (13.11.12) reduces to

$$E(\phi, \theta = 90°) = \frac{\cos\left(\frac{\pi}{2}\cos\phi\right)}{\sin\phi} \quad ...(13.11.15)$$

UEx. 13.11.1 MU - Q. 4(b). Dec. 19, 10 Marks

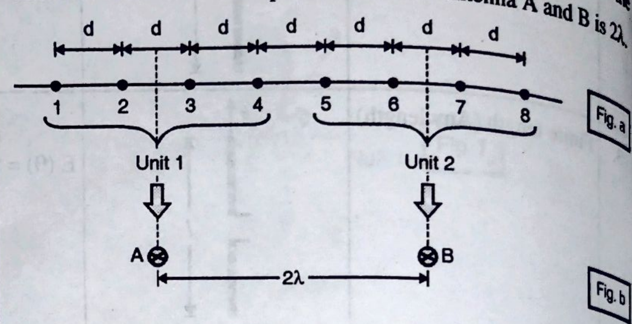
Obtain the radiation pattern of eight isotropic sources with spacing λ/2 and in phase. Use pattern multiplication technique.

☑ **Soln. :**

Eight isotropic sources with spacing λ/2 are shown in Fig. Ex. 13.11.1(a). Out of eight sources, we consider four as one unit and other four as another unit.

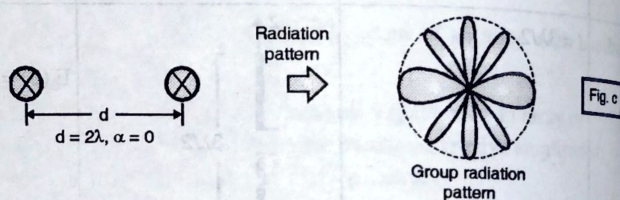
We can consider four antennas as one unit provided that radiation pattern of these antennas with $d = \lambda/2$ and $\alpha = 0$ is known to us. The radiation pattern is already known in section 13.11.1, otherwise we should calculate it.

Now four antennas of unit one will be replaced by antenna A having radiation pattern similar to four antennas. Similarly other unit of four antennas is replaced by antenna B. This is shown in Fig. Ex. 13.11.1(b). Antennas A and B are located exactly at the center of each unit. The separation between antenna A and B is 2λ.



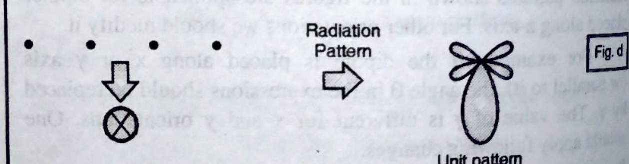
aw(8.50)**Fig. Ex. 13.11.1(a, b)**

In Fig. Ex. 13.11.1(b), group of two antennas (A and B) is representing original eight isotropic sources. The group radiation pattern is that of two isotropic sources with $d = 2\lambda$ and $\alpha = 0$. This radiation pattern is the plot 3 in section 13.2.6. It is shown in Fig. Ex. 13.11.1(c).



aw(8.50)**Fig. Ex. 13.11.1(c)**

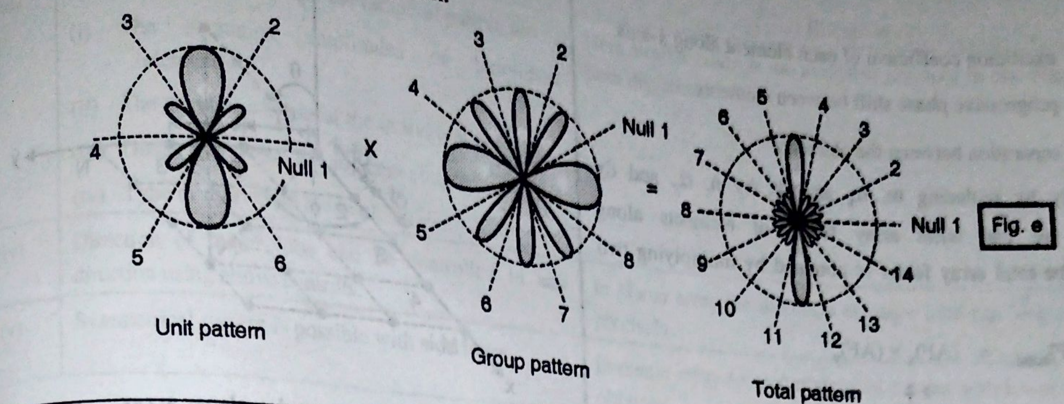
Radiation pattern of each unit in the group is known. It is the pattern of four isotropic source with $d = \lambda/2$, $\alpha = 0$ (Fig. f in section 13.11.1), repeated here in Fig. Ex. 13.11.1(d).



aw(8.50)**Fig. Ex. 13.11.1(d)**

The total radiation pattern is given by

Total pattern = Unit pattern × Group pattern

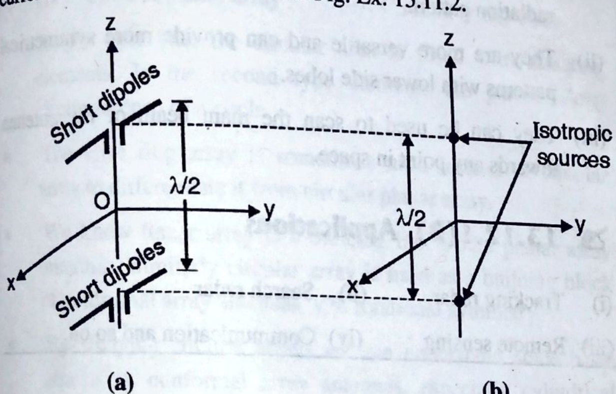


aw(8.50)**Fig. Ex. 13.11.1(e)**

UEx. 13.11.2 MU - May 09, 5 Marks.

Draw the radiation for an array of two parallel, half wavelength space short dipoles using pattern multiplication.

☑ Soln. : Consider the short dipoles are placed along z-axis with currents in x-direction as shown in Fig. Ex. 13.11.2.



aw(11.18)**Fig. Ex. 13.11.2**

For short dipole with current in z-direction, using

$$f(\theta, \phi) = \sin \theta$$

But the dipole in the problem have current in x-direction for which

$$\theta \rightarrow \gamma$$

$$\cos \gamma = \sin \theta \cos \phi$$

$$\sin \gamma = \sqrt{1-\cos^2 \gamma} = \sqrt{1-\sin^2 \theta \cos^2 \phi}$$

$$\therefore f(\theta, \phi) = \sqrt{1-\sin^2 \theta \cos^2 \phi} \quad \text{...(i)}$$

In Fig. Ex. 13.11.2(b), dipoles are replaced by isotropic sources having separation of λ/2 for which using equation

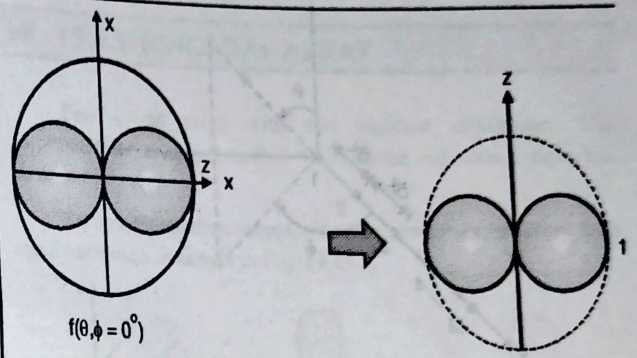
$$F(\theta, \phi) = \cos\left(\frac{\pi}{2} \cos \phi\right) \quad \text{...(ii)}$$

The radiation patterns for fields in Equation (i) and (ii) are

$$f(\theta, \phi) = \sqrt{1-\sin^2 \theta \cos^2 \phi}$$

$$F(\theta, \phi) = \cos\left(\frac{\pi}{2} \cos \phi\right)$$

Resultant pattern = $f(\theta, \phi) \times F(\theta, \phi)$



13.12 INTRODUCTION TO PLANNER AND CIRCULAR ARRAYS

13.12.1 Planer Array

In the previous sections we studied linear array where elements of the array are placed along a line. By changing excitation amplitude, phase or spacing between the elements the radiation pattern is controlled.

More controls are available when the elements are placed in a plane. For example when the elements are placed in the form of rectangular grid we get rectangular or planer array. It is shown in the Fig. 13.12.1(b). Fig. 13.12.1(a) shows a linear array with M elements placed along x axis with a separation of dx. In Fig. 13.12.1(b) rectangular grid is shown obtained by repeating M elements in Fig. 13.12.1(a), N times along y-direction with separation of dy.

For the elements placed along x axis in Fig. 13.12.1(a), the array factor is expressed as

$$AF' = \sum_{m=1}^{M} I_{m1} e^{j(m-1)(\beta dx \sin\theta \cos\phi + \alpha_x)} = (AF)_x$$

where,

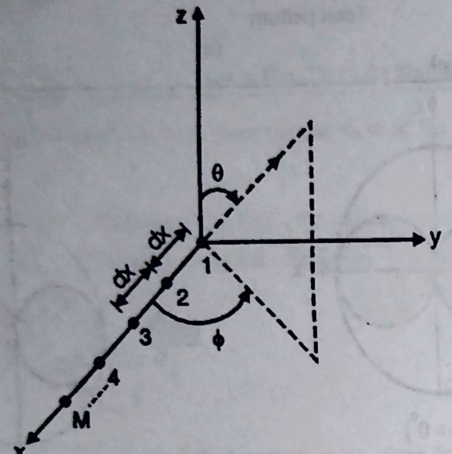
I_{m1} = excitation coefficient of each element along x-axis.

α_x = progressive phase shift between elements

dx = separation between the elements

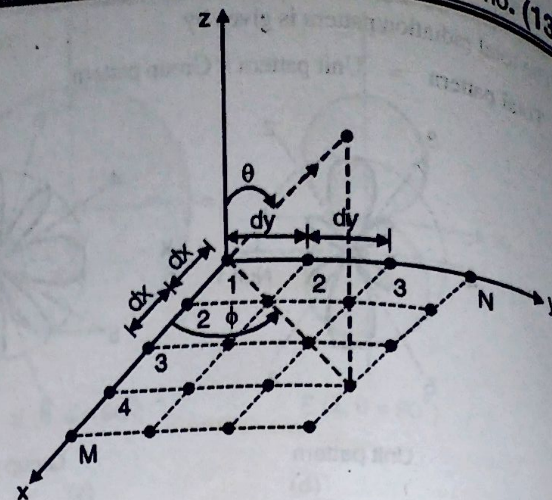
Similarly by replacing m, α_x and dx by n, α_y and dy respectively, we can write array factor of elements along y-direction. The total array factor is obtained by multiplying two array factors.

$$(AF)_{total} = (AF)_x \times (AF)_y$$



(a) Linear array

Fig. 13.12.1 Contd..



(b) Rectangular planer array

Fig. 13.12.1

13.12.1(A) Advantages of Planer Array

(i) Planer array provides more controls for controlling the radiation pattern.

(ii) They are more versatile and can provide more symmetrical patterns with lower side lobes.

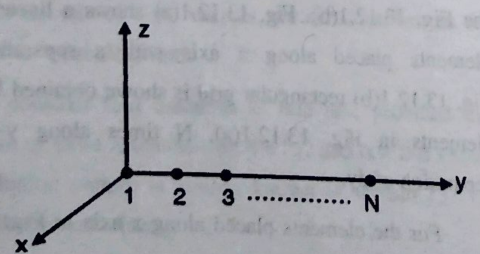
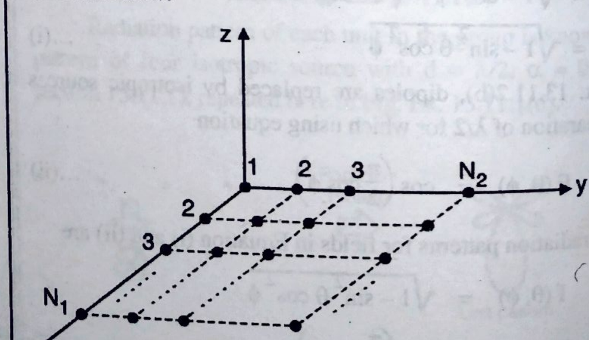
(iii) They can be used to scan the main beam of the antenna towards any point in space.

13.12.1(B) Applications

(i) Tracking radar (ii) Search radar

(iii) Remote sensing (iv) Communication and so on.

13.12.1(C) Linear Array versus Planer Array

Sr. No.	Linear array	Planar array
(i)	An array is said to be linear if elements of the array are spaced equally along a straight line.	Here elements of the array are placed in a plane.
(ii)	Linear array with N-number of elements along y-axis is as shown below. 	Planar array with $N_1 \times N_2$ number of elements placed in xy plane is shown below. 

Sr. No.	Linear array	Planar array
(iii)	Controls available to shape the radiation pattern are (i) The excitation amplitudes of individual elements. (ii) The excitation phase of the individual elements. (iii) Distance between the adjacent elements. (iv) Type of individual element.	Here similar controls are available not only in one direction but in both directions.
(iv)	Direction of major lobe can be controlled in any direction using above controls.	In planar array the direction of major lobe can be controlled more precisely.
(v)	Symmetrical pattern is possible with side lobes.	In planar array more symmetrical pattern with lower side lobes are obtained.

13.12.2 Circular Array

- Two types of circular array are shown in Fig. 13.12.2.
- Two types are :

  (i) Circular planar array (ii) Circular (ring) array
- In the first one, the circular area is filled with radiating elements. In the second type elements are placed along circumference of a circle.
- The term ring array is sometimes used instead of circular array to differentiate it from circular planar array.
- We know linear array is a building block of a planar array antennas. Similarly circular array is used as a building block of conformal array antennas with rotational symmetry.
- By studying circular arrays we can understand some basic aspects of conformal array antennas, especially cylindrical and conical arrays and other shapes with rotational symmetry.

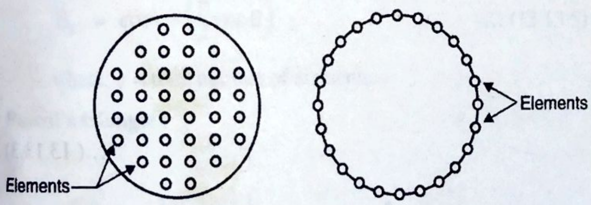


Fig. 13.12.2 : (a) Planar circular array ; (b) Circular array

13.12.2(A) Applications

(i) Radio direction finding

(ii) Sonar

(iii) To obtain omnidirectional pattern in the array plane.

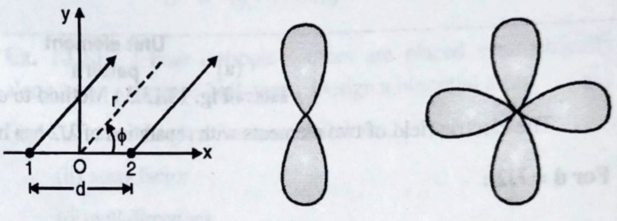
(iv) To obtain phase steered direction beams.

(v) Can produce several simultaneous beams.

13.13 BINOMIAL ARRAY

This is an array with non uniform amplitudes. The amplitudes are arranged so that the resultant radiation pattern has no minor lobes.

To understand the concept, consider a two element array with equal excitation as shown in Fig. 13.13.1.



aw(8.72)**Fig. 13.13.1 : (a) Two element array (b) radiation pattern with d = λ/2, λ**

This array has normalized electric field as,

$$E_n = \cos\left(\frac{\pi d}{\lambda}\cos\theta\right) \quad \text{...(13.13.1)}$$

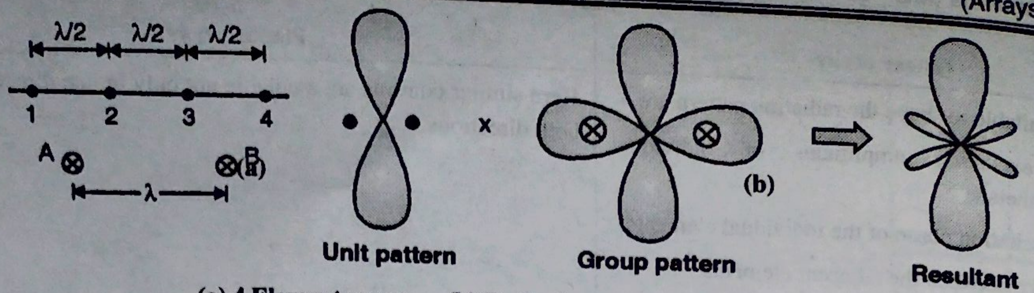
For different spacing (d), the radiation pattern is shown in Fig. 13.13.1(b) and (c). The pattern with d = λ/2, has figure eight shape pattern with no minor lobes.

Separation of λ/2 between the elements results in eight shape pattern with no minor lobes.

But with a uniform linear array, to increase the directivity when the length of array is increased, the resultant pattern will have minor lobes in it. For some applications a single narrow lobe without minor lobe is desired.

Consider now a four element array with a separation of (λ/2). Using the pattern multiplication technique the resultant of pattern is obtained as shown in Fig. 13.13.2.

In the figure, since group elements (A and B) are separated by a distance of λ, it has radiation pattern as given in Fig. 13.13.1(c). This creates a problem. The resultant pattern will have major lobes along with minor lobes.

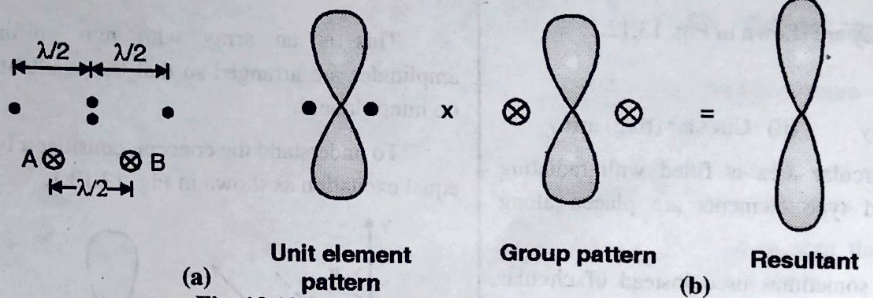


(a) 4 Element array (b) Radiation pattern using pattern multiplication

aw(8.73)**Fig. 13.13.2**

If by any means the group pattern is made figure 8 shape, then the resultant will be "figure eight multiplied by figure eight, that is" figure 8 squared pattern". But this requires separation between group elements to be less than or equal to λ/2.

For this purpose, start moving group element B towards A. The group element B is representing unit elements 3 and 4. When B is moved, elements 3 and 4 also moves with it. Now stop when B is at a distance of λ/2 from A. The array now looks like as shown in Fig. 13.13.3, having elements 2 and 3 overlapping.



(a) (b)

aw(8.74)**Fig. 13.13.3 : Method to obtain radiation pattern without minor lobes**

The electric field of two elements with separation of λ/2 can be obtained using Equation (13.13.1).

For d = λ/2 :

$$E_n = \cos\left(\frac{\pi(\lambda/2)}{\lambda}\cos\phi\right) = \cos\left(\frac{\pi}{2}\cos\phi\right) \qquad \text{...(13.13.2)}$$

Thus the resultant electric field for four element array in Fig. 13.13.3 is

$$(E_R)_n = \text{Field of unit pattern} \times \text{Field of group pattern}$$

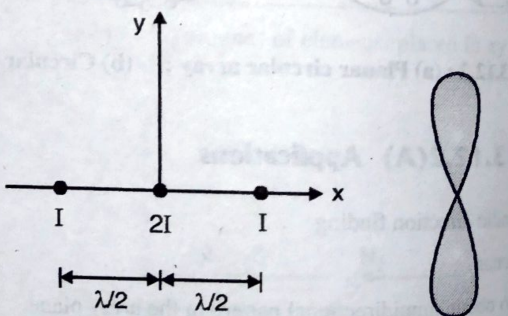
$$= \cos\left(\frac{\pi}{2}\cos\phi\right) \times \cos\left(\frac{\pi}{2}\cos\phi\right)$$

i.e.

$$(E_R)_n = \cos^2\left(\frac{\pi}{2}\cos\phi\right) \qquad \text{...(13.13.3)}$$

13.13.1 Three Element Non Uniform Array

The antennas 2 and 3 which are overlapping can be replaced by a single antenna carrying double the current in the outer elements. In other words, the result is a three element array that has a currents I, 2I and I, or current ratios 1 : 2 : 1.



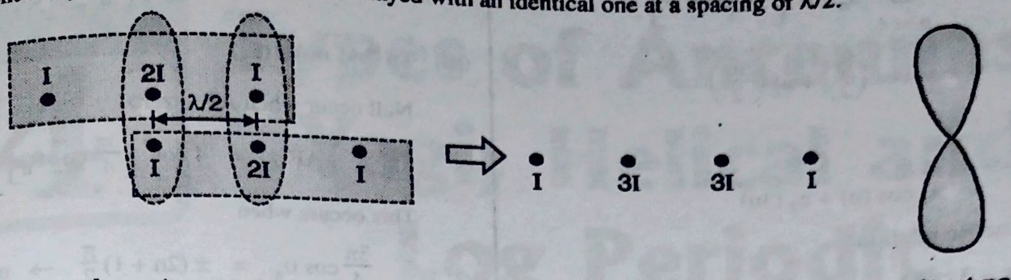
(a) Three element array (b) Resultant pattern

aw(8.75)**Fig. 13.13.4**

Note that currents in the elements of the array are not equal, so this array is a **non uniform array.**

13.13.2 Four Element Non Uniform Array

The three element array in Fig. 13.13.4(a) can be arrayed with an identical one at a spacing of λ/2.



(a) Two-three element arrays grouped togehter (b) Resultant four element array (c) The resultant pattern

aw(8.76)Fig. 13.13.5

The elements overlapping can be replaced by a single antenna, carrying current of 3I as shown in Fig. 13.13.5 (b) so the current ratios are

1 : 3 : 3 : 1

The resultant electric field will be,

$$(E_R)_n = \cos^3\left(\frac{\pi}{2}\cos\phi\right) \quad ...(13.13.4)$$

13.13.3 n-Element Non Uniform Array

This process may be continued to obtain a pattern having any desired degree of directivity and no secondary lobes if the amplitudes of the sources in the array correspond to the coefficients of a binomial series. These coefficients are conveniently displaced by **Pascal's triangle.** Each internal integer in the triangle is the sum of the adjacent ones above. The pattern of the array is then,

$$E_n = \cos^{n-1}\left(\frac{\pi}{2}\cos\theta\right) \quad ...(13.13.5)$$

where, n = total number of elements

Pascal's triangle :

```
            1
          1   1
        1   2   1
      1   3   3   1
    1   4   6   4   1     ... (13.13.6)
```

Although the above array has no minor lobes, its directivity is less than that of an array of the same size with equal amplitude sources. In practice most arrays are designed as a compromise between binomial and uniform arrays.

13.13.4 Design of Binomial Array

To design a binomial array with a desired half-power beamwidth or directivity, the required expressions are,

$$\text{HPBW } (d = \lambda/2) = \frac{1.06}{\sqrt{n-1}} = \frac{1.06}{\sqrt{2L/\lambda}} = \frac{0.75}{\sqrt{L/\lambda}}$$

$$D_o \simeq 1.77\sqrt{n} = 1.77\sqrt{1+2(L/\lambda)}\,.$$

Where,

n = no. of elements in the array

L = length of the array in terms of λ.

L = (n – 1) (λ/2)

Ex. 13.13.1 : Four isotropic sources are placed symmetrically along z-axis a distance 3λ/4 apart. Design a binomial array.

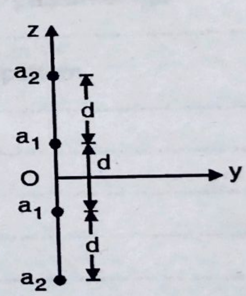
Find (i) normalized excitation coefficients,

(ii) array factor

(iii) null directions.

☑ **Soln. :**

(i) Normalized excitation coefficients



aw(8.77)**Fig. : For example 13.13.1**

For four element binomial array the excitation coefficients are obtained using Pascal's triangle Equation (13.13.6).

1 : 3 : 3 : 1

Thus elements of the array have excitations

$a_1 = 3, a_2 = 1$

(ii) Array factor

For four element array (even number)

2M = 4 or M = 2.

Module 4

Using equation

$$(AF)_4 = \sum_{n=1}^{2} a_n \cos\left[\frac{(2n-1)}{2}\psi\right]$$

$$= a_1 \cos\left(\frac{\psi}{2}\right) + a_2\left(\frac{3\psi}{2}\right)$$

Let $\frac{\psi}{2} = u$

$$(AF)_4 = a_1 \cos(u) + a_2 (3u)$$

Using trigonometric relation

$$\cos(3x) = 4\cos^3 x - 3\cos x$$

$$(AF)_4 = a_1 \cos(u) + a_2 [4\cos^3(u) - 3\cos(u)]$$

With $a_1 = 3$ and $a_2 = 1$

$$(AF)_4 = 3\cos(u) + 4\cos^3(u) - 3\cos(u)$$

$$= 4\cos^3(u) = 4\cos^3(\Psi/2)$$

Using $\Psi = \beta d \cos\theta,\ \beta = 2\pi/\lambda,\ d = 3\lambda/4$

$$(AF)_4 = 4\cos^3\left[\frac{\beta d \cos\theta}{2}\right] = 4\cos^3\left[\frac{(2\pi/\lambda)(3\lambda/4)\cos\theta}{2}\right]$$

$$= 4\cos^3\left[\frac{3\pi}{4}\cos\theta\right]$$

(iii) To find nulls (θ_n)

Null occur when AF is zero

$$(AF)_4 = 4\cos^3\left[\frac{3\pi}{4}\cos\theta_n\right] = 0$$

This occurs when

$$\frac{3\pi}{4}\cos\theta_n = \pm(2n+1)\frac{\pi}{2} \quad \rightarrow \text{ odd multiples of } \pi/2.$$

$$\cos\theta_n = \pm(2n+1)\frac{2}{3}, \quad n = 0, 1, 2, \ldots$$

$$\theta_n = \cos^{-1}\left[\pm(2n+1)\frac{2}{3}\right]$$

For n = 0 : $\theta_0 = \cos^{-1}[\pm(2/3)] = 48.19^\circ, 131.81^\circ$

For higher values of n, no null is obtained.

Chapter Ends...

□□□