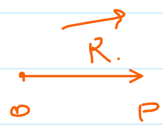
Lec 01: Prerequisite - Scalar and Vector

10 January 2022 09:05

Scalar → magnitude (eg – Temp, mass, ...)

Vector → magnitude & direction. (eg – force, velocity, acceleration)



$\vec{A}$ & $\vec{B}$

$\vec{A} = \vec{B}$

vector of unit mag → unit vector

vector $\vec{A}$ can be written as

$$\boxed{\vec{A} = A \cdot \vec{a}_A}$$

unit vector in the same direction

magnitude of $\vec{A}$

$$\vec{a}_A = \frac{\vec{A}}{A}$$

Textbooks:

1. Electromagnetic Waves and Radiating Systems- Jordan and Balmain, PHI, 2nd edition
2. Principles of Electromagnetics Engineering- Matthew N. O.Sadiku , S.V.Kulkarni, Oxford university press, 6th edition
3. Antenna Theory: Analysis and Design, Costantine A. Balanis, John Wiley Publication, 4th edition
4. Antenna and wave Propagation, John D Kraus, A S Khan, McGraw Hill, 4th edition
5. Antenna Theory and Design. Stutzman, Theile, John Wiley and Sons, 3rd edition

Reference Books:

1. Engineering Electromagnetics, William H Hayt and John A Buck, Tata McGraw-Hill Publishing Company Limited, 7th edition
2. Antennas and Radio Wave Propagation, R. E. Collin, McGraw Hill, International Student Edition

Introduction to Electrostatic Field

- Source:

Stationary Charge

- What is Charge?

Excess or deficiency of Electron in an atomic structure

- Example

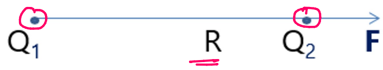
Rub the surface of the balloon with the cloth and hold the balloon a short distance above your head and watch your hair stick to it!

- The balloon gains electrons from the cloth and becomes negatively-charged, so it attracts your hair, which is positively-charged

- Coulomb's Law
- Electric Field Intensity,
- Electric Field due to
  - Point charge
  - Line charge and
  - Surface charge distributions

* Coulomb's Law.

- Experimental law deals with force between two point charges
- The law states that the force F between two point charges Q_1 and Q_2 is-
  - directly proportional to the products ($Q_1 \times Q_2$) of the charges
  - Inversely proportional to square of distance (R) between them
  - along the line joining them.
  - depends upon medium
  - depends upon the nature of charges (+ or –).



- The force is expressed as –
- $F = \frac{Q_1 Q_2}{4\pi\epsilon R^2}$
- where ε is permittivity of the medium. For free space $\varepsilon = \varepsilon_0\varepsilon_r = \varepsilon_0 = 8.854 \times 10^{-12}$ F/m i.e. $\varepsilon_r = 1$

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon R^2}$$

$\varepsilon = \varepsilon_0 \cdot \varepsilon_r$

$\varepsilon_r = 1$

$\varepsilon = \varepsilon_0$

Vector Form:

- The force $\mathbf{F_2}$ on Q_2 due to Q_1 is given by-

$$\therefore \bar{F}_2 = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{12}^2}\bar{a}_{12}$$

Where $\bar{a}_{12}$ is unit vector in the direction of $\bar{R}_{12}$

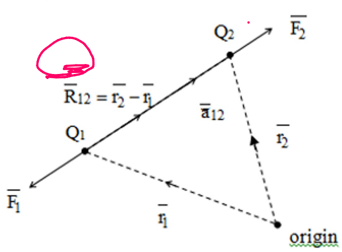
$$\bar{a}_{12} = \frac{\bar{R}_{12}}{|\bar{R}_{12}|} = \frac{\bar{r}_2 - \bar{r}_1}{|\bar{r}_2 - \bar{r}_1|}$$

Similarly, force $\bar{F}_1$ on Q_1 due Q_2 is given by

$$\bar{F}_1 = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{21}^2}\bar{a}_{21}$$

$$= \frac{-Q_1 Q_2}{4\pi\varepsilon_0 R_{12}^2}\bar{a}_{12}$$

$$\bar{F}_1 = -\bar{F}_2$$



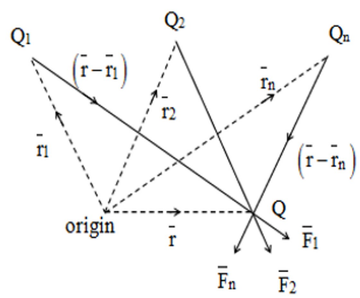
$\varepsilon = \varepsilon_0 \varepsilon_r$

$\varepsilon_r = 1$

$\varepsilon = \varepsilon_0$

Principle of Superposition:

- Consider system of n point charges. The total force is given as-



$$\bar{F} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 + \ldots + \bar{F}_n$$

$$= \frac{QQ_1(\bar{r}-\bar{r}_1)}{4\pi\varepsilon_0(|\bar{r}-\bar{r}_1|)^3} + \frac{QQ_2(\bar{r}-\bar{r}_2)}{4\pi\varepsilon_0(|\bar{r}-\bar{r}_2|)^3} + \ldots + \frac{QQ_n(\bar{r}-\bar{r}_n)}{4\pi\varepsilon_0(|\bar{r}-\bar{r}_n|)^3}$$

$$= \frac{Q}{4\pi\varepsilon_0}\sum_{k=1}^{n}\frac{Q_k(\bar{r}-\bar{r}_k)}{|\bar{r}-\bar{r}_k|^3}$$

Q Consider charge $Q_1 = 3\times10^{-4}$ C at $M(1,2,3)$ &
$Q_2 = -10^{-4}$ C at $N(2,0,5)$ → in a vacuum
we desire to find the force exerted on Q_2 by Q_1

⇒ The force exerted on Q_2 by Q_1 is given by

$$\vec{F}_2 = \frac{Q_1 Q_2}{4\pi\varepsilon_0 \cdot R_{12}^2} \cdot \bar{a}_{12} = F_{12}$$

$$R_{12} = r_2 - r_1 = (2-1)a_x + (0-2)a_y + (5-3)a_z$$

$$R_{12} = a_x - 2a_y + 2a_z$$

Find $|R_{12}|$ & the unit vector

$$|R_{12}| = 3$$

$$\vec{a}_{12} = \frac{\bar{r}_2 - \bar{r}_1}{|\bar{r}_2 - \bar{r}_1|} = \frac{1}{3}(a_x - 2a_y + 2a_z)$$

$$\therefore F_2 = \frac{3\times10^{-4} \times (-10^{-4})}{4\pi(1/36\pi)\times10^{-9} \times (3)^2} \times \frac{1}{3}(a_x - 2a_y + 2a_z)$$

$$\therefore \; F_2 = \frac{3\times10^{-4} \times (-10^{-4})}{4\pi\left(\frac{1}{36\pi}\right)\times10^{-9} \times (3)^2} \times \frac{1}{3}(a_x - 2a_y + 2a_z)$$

$$F_2 = -30\left(\frac{a_x - 2a_y + 2a_z}{3}\right) \; N$$

$$\boxed{F_2 = -10\,a_x + 20\,a_y - 20\,a_z \quad N}$$

P1. Three point charges Q_1=1 mC, Q_2=2 mC and Q_3=-3mC are respectively located at (0,0,4), (-2,6,1) and (3,-4,-8). Calculate force on Q_1.

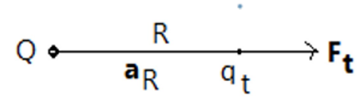
Electric Field Intensity(E):

- Consider source charge Q.
- Place test charge(q_t) in the region surrounding it
- The force experienced by test charge is –

$$\mathbf{F}_t = \frac{Qq_t\hat{\mathbf{a}}_R}{4\pi\epsilon R^2}$$

- Express it as force per unit charge
- $\frac{\mathbf{F}_t}{q_t}$ = Electric Field Intensity($\mathbf{E}$)

$$\boxed{\mathbf{E} = \frac{Q\,\hat{\mathbf{a}}_R}{4\pi\epsilon R^2}}$$



- The electric field intensity(E) is defined as the force per unit charge.
- Unit=Volts/meter

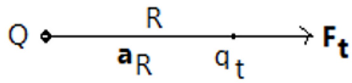
Electric Field Intensity(E):

- Consider source charge Q.
- Place test charge(q_t) in the region surrounding it
- The force experienced by test charge is –

$$\mathbf{F}_t = \frac{Qq_t\hat{\mathbf{a}}_R}{4\pi\epsilon R^2}$$

- Express it as force per unit charge
- $\frac{\mathbf{F}_t}{q_t}$ = Electric Field Intensity($\mathbf{E}$)

$$\mathbf{E} = \frac{Q\,\hat{\mathbf{a}}_R}{4\pi\epsilon R^2}$$



- The electric field intensity(E) is defined as the force per unit charge.
- Unit=Volts/meter

* Principle of superposition

The electric field intensity ($\bar{E}$) due to 'n' pt charges at a pt in free space = vector sum of the ($\bar{E}$) due to each charge acting alone.

The charges are located by $\bar{r}_1, \bar{r}_2, \ldots \bar{r}_n$ & the field pt is located by radius vector $\bar{r}$

$$\bar{E} = \frac{Q_1}{4\pi\varepsilon_0}\frac{(\bar{r}-\bar{r}_1)}{|r-r_1|^3} + \frac{Q_2}{4\pi\varepsilon_0}\frac{(\bar{r}-\bar{r}_2)}{|r-r_2|^3} + \cdots \frac{Q_n}{4\pi\varepsilon_0}\frac{(\bar{r}-\bar{r}_n)}{|r-r_n|^3}$$

$$\boxed{\bar{E} = \frac{1}{4\pi\varepsilon_0}\sum_{k=1}^{n} Q_k\frac{(r-r_k)}{|r-r_k|^3}} \longrightarrow (A)$$

Q) Two pt charges of 20nC & −20nC are situated at (1,0,0) & (0,1,0) in free space. Determine the $\bar{E}$ at (0,0,1)

$\rightarrow$ $Q_1 = 20\,nC$ at $(1,0,0) \rightarrow r_1 = a_x$
$Q_2 = -20\,nC$ at $(0,1,0) \rightarrow r_2 = a_y$

$\Rightarrow$ $r - r_1 = (a_z - a_x)$
$r - r_2 = (a_z - a_y)$

Cal E at $(0,0,1) \rightarrow r = a_z$

w.r.t.
$$E = \frac{1}{4\pi\varepsilon_0} \sum_{k=1}^{n} \frac{Q_k (r - r_k)}{|r - r_k|^3} = \frac{1}{4\pi\varepsilon_0} \sum_{k=1}^{2} \frac{Q_k (r - r_k)}{|r - r_k|^3}$$

$$= \frac{1}{4\pi\varepsilon_0} \left[\frac{20\times10^{-9}(a_z - a_x)}{(\sqrt{2})^3} + \frac{(-20\times10^{-9})(a_z - a_y)}{(\sqrt{2})^3} \right]$$

$$\boxed{E = -63.63\, a_x + 63.63\, a_y \quad V/m \quad (N/C)}$$

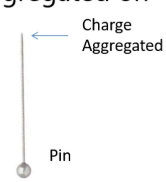
* Charge Configurations

1) Point charge.
2) line charge
3) Surface charge
4) Volume charge

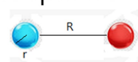
(1) Point Charge

- Point charge: A point charge mean a charge whose dimensions are much smaller than other relevant dimensions

- Examples
- Charge aggregated on pin head

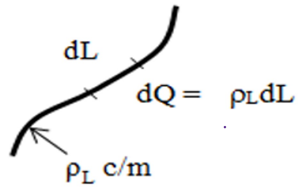


- Metallic spheres



2) Line charge

- It has filament like distribution of charge e.g. sharp beam in CRT, charged conductor of very small radius.

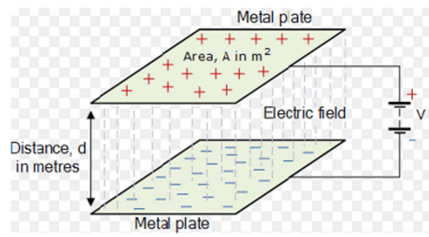


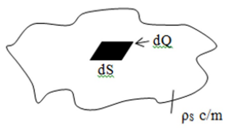
- Consider $dQ=\rho_L\, dL$
- We define the line charge density ρ_L C/m as charge per unit length.
- Hence

$$\therefore\ \rho_L = \lim_{dL \to 0} \frac{dQ}{dL}\ \text{C/m}$$

3) Surface charge

- An electric charge accumulated on a surface is called surface charge.
- Example: Parallel plate capacitor

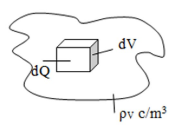




- Consider incremental charge $dQ=\rho_s\, ds$
- We define the surface charge density (ρ_S) as the charge per unit area.
- Hence $\therefore \rho_S = \lim_{dS \to 0} \frac{dQ}{dS}\ \text{C/ m}^2$

4) Volume charge

Here charge is confined within a volume.
It is volume like distribution of charge
e.g. Region surrounding cathode in CRT



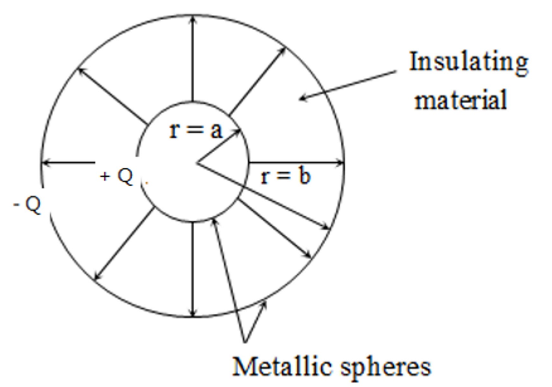
- Consider $dQ=\rho_v\, dV$
- We define the line charge density ρ_v C/m as charge per unit volume.
- Hence

$$\rho_v = \lim_{dv \to 0} \frac{dQ}{dv}\ \text{c/m}^3$$

Lec 04: Electric flux and Electric flux density

17 January 2022 08:59

Faraday's Experiment



- With the equipment dismantled, the inner sphere was given a known positive charge (+Q)
- The hemisphere were then clamped together around the charged sphere with dielectric material between them.
- The outer sphere was discharged.
- The charge on inner surface of outer sphere is measured.

- Faraday found that the total charge on the outer sphere was equal in magnitude to the original charge placed on the inner sphere
- This was true regardless of the dielectric material separating the two spheres
- He concluded that, there was some sort of 'displacement' from the inner sphere to the

- From Faraday's experiment,
- Electric Flux (ψ) $\propto$ charge (Q)
- $\therefore\ \psi = KQ$
- In SI system, K = 1
- $\therefore \psi = Q$ coulomb

- Faraday found that the total charge on the outer sphere was equal in magnitude to the original charge placed on the inner sphere
- This was true regardless of the dielectric material separating the two spheres
- He concluded that, there was some sort of ‘displacement’ from the inner sphere to the outer which was independent of the medium
- We will refer this as displacement flux or simply electric flux(ψ)

- From Faraday's experiment,
- Electric Flux (ψ) $\propto$ charge (Q)
- $\therefore \psi = KQ$
- In SI system, $K = 1$
- $\therefore \psi = Q$ coulomb

Electric Flux Density(**D**)

- The electric flux density(**D**) at any point is defined as electric flux or electric displacement per unit area.
- It is a vector quantity.
- Its direction being taken as that of the normal to the surface element which makes the displacement through the element of area a maximum

- The flux density is-
- $\mathbf{D} = \dfrac{\text{Flux}}{\text{Area}} = \dfrac{Q}{4\pi R^2}\hat{\mathbf{a}}_{\mathbf{r}}$

Relation between **E** and **D**

E

The electric flux density ($\overline{D}$) at a point P which is at distance r meters from point charge Q is given as–

$$\overline{D} = \frac{Q}{4\pi r^2}\hat{a}_r \qquad (1)$$

Similarly $\overline{E}$ at a point P in free space is-

$$\overline{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{a}_r \qquad (2)$$

From above equations

$\therefore\ \overline{D} = \varepsilon_0\,\overline{E}$ (for free space)

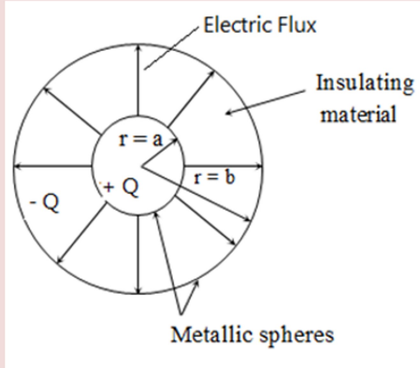
- The Electric field intensity(**E**) is dependent on the medium in which the charge is placed.
- Electric flux density(**D**) is independent of medium
- **D** provides the same information as **E** does

Maxwell's Equation for Static Field

Gauss's Law

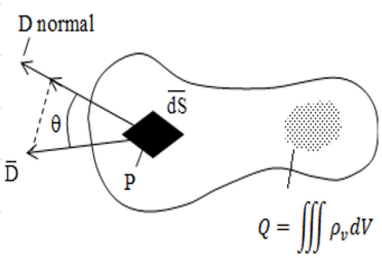
- From Faraday's experiment,
  - Electric Flux (ψ) $\propto$ charge (Q)
  - $\therefore \psi = KQ$
  - In SI system, $K = 1$
  - $\therefore \psi = Q$ coulomb
- Statement: The electric flux passing through any closed surface is equal to the total charge enclosed by that surface

- Faraday's Experiment



Mathematical Form

- Consider volume like distribution of charge, surrounded by a closed surface of any shape



- Let D be the flux density on surface which will vary in magnitude and direction from point to point on the surface.
- The incremental flux($d\Psi$) through incremental surface (**dS**) are given as-
- $d\Psi = D_{\text{normal}} dS = D \cos\theta dS = \mathbf{D} \cdot \mathbf{dS}$

$\therefore$ total flux through the closed surface

$$\psi = \int d\psi = \oint_{\text{closed surface}} \bar{D} \cdot \bar{dS} \quad \text{—— (3)}$$

$= Q =$ charge enclosed.

Lec 05: GAUSS'S LAW

18 January 2022 09:04

$$d\psi = D_{normal}\, ds$$

$$d\psi = D \cos\theta \cdot ds \quad \text{— } (1)$$

$$d\psi = \bar{D} \cdot \bar{ds} \quad \text{— } (2)$$

$$\Psi = \int d\psi = \oint \bar{D} \cdot \bar{ds} \quad \text{— } (3)$$

closed surface

$= \underline{\underline{Q}} =$ charge enclosed

For volume charge density (ρ_v), we can write

$$\boxed{\oint_S \bar{D} \cdot \bar{ds} = \int_{vol} \rho_v \cdot dv \quad \text{— } (4)}$$

↳ Eqn (4) represents

MAXWELL'S EQN → Integral form

* Differential or Point form of GAUSS's LAW

To obtain point form → use divergence theorem

⇓

relates surface integral with volume integral.

$$\oint_S \bar{A} \cdot \bar{ds} = \int_{vol} (\nabla \cdot \bar{A})\, dv$$

Applying Divergence theorem to eqn (4)

$$\therefore \oint_S \bar{D} \cdot \bar{ds} = \int_{vol} (\nabla \cdot \bar{D}) \cdot dv = \int_{vol} \rho_v dv \quad \text{— } (5)$$

Comparing two volume integrals

$$\boxed{\nabla \cdot \bar{D} = \rho_v \quad (6) \quad \text{— (Maxwell's eqn point-form)}}$$

Divergence defined in all coordinate system

$$\text{div } \mathbf{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} \qquad \text{(cartesian)}$$

$$\text{div } \mathbf{D} = \frac{1}{r}\frac{\partial}{\partial r}(r D_r) + \frac{1}{r}\frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z} \qquad \text{(cylindrical)}$$

$$\text{div } \mathbf{D} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r) + \frac{1}{r \sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, D_\theta) + \frac{1}{r \sin\theta}\frac{\partial D_\phi}{\partial \phi} \qquad \text{(spherical)}$$

* Applications of Gauss's Law

used to determine $\bar{D}$ & $\bar{E}$ → symmetric charge distributions

derivations $\bar{D}$ & $\bar{E}$
→ Point charge
→ Uniform line charge
→ Infinite surface charge

(i) Point charge

$$\bar{D} = \frac{Q}{4\pi r^2}\,\bar{a_r}$$

$$\bar{E} = \frac{Q}{4\pi \varepsilon_0 r^2}\,\bar{a_r} \qquad \{\vec{D} = \varepsilon_0 \vec{E}\}$$

(ii) Uniform line charge

$$\vec{D} = \frac{\rho_L}{2\pi\rho}\,\bar{a_\rho}$$

$$\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0\rho}\,\vec{a_\rho}$$

(iii) Infinite surface charge

$$\vec{D} = \frac{\rho_s}{2}\,\vec{a_z}$$

$$\bar{E} = \frac{\rho_s}{\varepsilon_0(2)}\,\bar{a_z}$$

Lec 06: Work done and electric potential

20 January 2022 13:29

$$\boxed{dW = -Q_t \bar{E} \cdot \bar{dL} \quad \text{Joules}} \quad \text{—— (6)}$$

The total work done → by integrating the diff work done. over the distance from initial position to the final position

∴ From eqn (6) we have

$$W = \int_{\text{Initial}}^{\text{final}} dW = \int_{\text{Initial}}^{\text{final}} -Q \bar{E} \cdot \bar{dL}$$

$$\boxed{\therefore W = -Q \int_{\text{initial}}^{\text{final}} \bar{E} \cdot \bar{dL}} \quad \text{—— (7)}$$

* Potential diff (V)

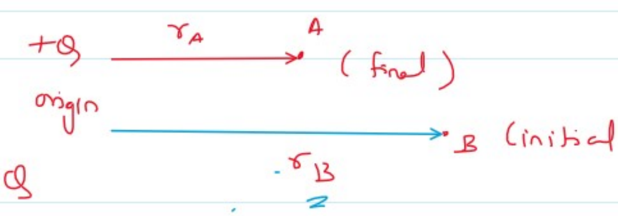
w.k.t. work done in moving the pt charge Q from pt B to pt A in the electric field $\bar{E}$ is given by

$$W = -Q \int_{B}^{A} \bar{E} \cdot \bar{dL} \quad \text{—— (1)}$$

Potential diff (V) is defn as the work done per unit charge in moving the unit charge from B to A in the field $\bar{E}$

$$\boxed{V_{AB} = V = \frac{W}{Q} = -\int_{B}^{A} \bar{E} \cdot \bar{dL}} \quad \text{—— (2)}$$

Eg:- Find the potential diff. betn pts A & B at radial distance r_A & r_B from the pt charge Q



spherical → (r, θ, ϕ)

$$\underbrace{dL = dr\,\hat{a}_r + r\,d\theta\,\hat{a}_\theta + r\sin\theta\,d\phi\,\hat{a}_\phi}$$

Consider charge Q at origin

$$\bar{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\,\hat{a}_r$$

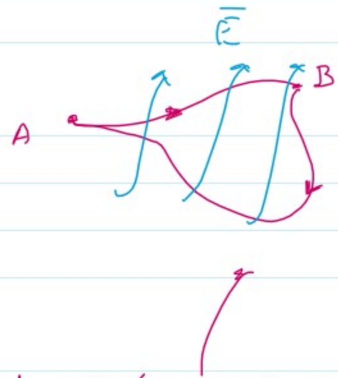
Lec 07: Relation between E and V, Current densities

24 January 2022 08:52

Relationship between $\bar{E}$ & $\bar{V}$ — Maxwells equation

The poten. diff between pt A & B is independent of the path. Hence,

$$V_{AB} = -V_{BA} \quad \text{—(1)}$$



$$\therefore \; V_{AB} + V_{BA} = \oint_L \bar{E} \cdot \bar{dL} \quad \text{—(2)}$$

OR

$$\boxed{\oint_L \bar{E} \cdot \bar{dL} = 0} \quad \text{—(3)}$$

This shows that the line integral of $\bar{E}$ along a closed path (ref. fig). must be zero.

⇒ Physically this implies that no net work is done in moving a charge along the closed path. in an electrostatic field

Apply Stoke's theorem to eqⁿ (3)

Stoke's theorem relates line integral with surface integral

$$\left[\; \oint_L \bar{A} \cdot dL = \int_S (\nabla \times \bar{A}) \cdot \bar{ds} \right.$$

$$\therefore \; \oint_L E \cdot \bar{dL} = \int_S (\nabla \times \bar{E}) \cdot \bar{d}s = 0$$

$$\boxed{\therefore \; \nabla \times \bar{E} = 0} \quad \text{——— (4)}$$

Any vector field that satisfy eqⁿ (3) & eqⁿ (4) is said to be conservative or irrotational field

* Potential gradient

w.k.t. $V = -\int \bar{E} \cdot \bar{dL}$

$$dV = -\bar{E} \cdot \bar{dL} = -E_x \, dx - E_y \, dy - E_z \, dz \quad \text{(a)}$$

Lec 08:Continuity equation

27 January 2022 10:29

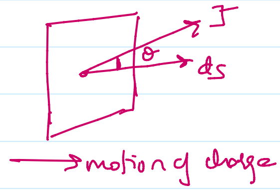
* Current, Current density and Continuity equation

(i) Current → Electric charge in motion constitutes c/n

$$I = \frac{dQ}{dt} \quad \text{—— (1)}$$

(ii) Current density ($\bar{J}$)

$\bar{J}$ at a given pt is defined as the c/n through a unit normal area at that pt. The incremental amount of c/n dI crossing an incremental surface ds is

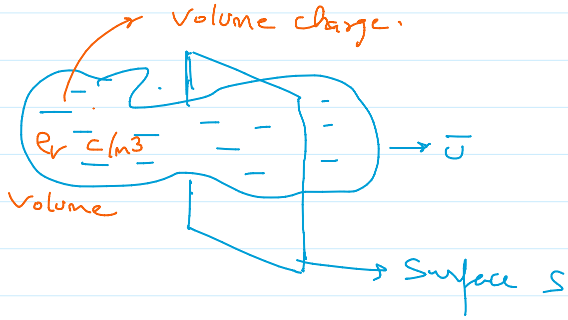


$$dI = J\,ds\cos\theta$$
$$= \bar{J}\cdot\bar{ds}$$

∴ The total c/n is given by

$$\boxed{I = \int_s \bar{J}\cdot\bar{ds}} \quad \text{—— (2)}$$

* Convection c/n density



when charge passes through surface 's' it constitutes a convection c/n with density ($\bar{J}$)

$$\bar{J} = \rho_v \bar{U} \quad \text{—— (3)}$$

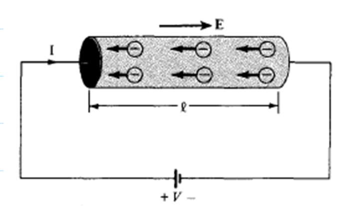
where ρ_v → vol. charge density

$\bar{U}$ → velocity to the right

* Do not satisfy Ohm's law

- It occurs when current flows through insulating medium such as liquid, vacuum etc. Example: Electron beam in CRT

* Conduction c/n density



POYNTING THEOREM

27 February 2023 20:03

Poynting Theorem :

When electromagnetic waves propagate through space from their source to distant receiving points, there is a transfer of energy from the source to the receivers. In order to find the power in an electromagnetic waves, it is necessary to develop a power theorem, which is known as Poynting theorem. It can be obtained from Maxwell's equation as follows

Consider Maxwell's equation from modifies Ampere's circuit law,

$$\nabla \times \overline{H} = \overline{J} + \frac{\partial \overline{D}}{\partial t} \qquad \ldots (1)$$

Dot each side of equation (1) with $\overline{E}$

$$\overline{E}.(\nabla \times \overline{H}) = \overline{E}.\overline{J} + E.\frac{\partial \overline{D}}{\partial t} \qquad \ldots (2)$$

Now make use of vector identity,

$$\nabla.(\overline{E} \times \overline{H}) = \overline{H}.\nabla \times \overline{E} - \overline{E}.\nabla \times \overline{H} \qquad \ldots (3)$$

from (2) and (3)

$$\therefore \quad \overline{H}.\nabla \times \overline{E} - \nabla.(\overline{E} \times \overline{H}) = \overline{E}.\overline{J} + \overline{E}.\frac{\partial \overline{D}}{\partial t} \qquad \ldots (4)$$

but $\quad \nabla \times \overline{E} = -\frac{\partial \overline{B}}{\partial t}$

and therefore,

$$-H.\frac{\partial \overline{B}}{\partial t} - \nabla.(\overline{E} \times \overline{H}) = \overline{E}.\overline{J} + \overline{E}.\frac{\partial \overline{D}}{\partial t} \qquad \ldots (5)$$

or

$$-\nabla.(\overline{E} \times \overline{H}) = \overline{E}.\overline{J} + \varepsilon \overline{E}.\frac{\partial \overline{E}}{\partial t} + \mu H.\frac{\partial \overline{H}}{\partial t} \qquad \ldots (6)$$

Lec 09: Biot Savart's Law

27 January 2022 10:29

Q) If a potential $V = x^2yz + Ay^3z$
(i) Find A so that Laplace's eqⁿ is satisfied.
(ii) With the value of 'A' determine elec. field at $(2,1,-1)$

$$\boxed{V = x^2yz + Ay^3z}$$

(i) Find A

Laplace's eqⁿ in CCS (RCS)

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0 \quad \text{--- (1)}$$

$$\frac{\partial V}{\partial x} = 2xyz \;; \quad \frac{\partial^2 V}{\partial x^2} = 2yz$$

$$\frac{\partial V}{\partial y} = x^2z + 3Ay^2z \;; \quad \frac{\partial^2 V}{\partial y^2} = 0 + 6Ayz$$

$$\frac{\partial V}{\partial z} = x^2y + Ay^3 \;; \quad \frac{\partial^2 V}{\partial z^2} = 0$$

Substituting above values in eqⁿ (1)

$$\nabla^2 V = 2yz + 6Ayz = 0$$

$$\boxed{\therefore A = -1/3}$$

(ii) $\bar{E}$ at $(2,1,-1)$

$$\bar{E} = -\nabla V = -\left[\frac{\partial V}{\partial x} \hat{a}_x + \frac{\partial V}{\partial y} \cdot \hat{a}_y + \frac{\partial V}{\partial z} \hat{a}_z\right]$$

w.r.t.

$$V = x^2yz - \frac{1}{3}y^3z$$

$$\bar{E} = -(2xyz)\hat{a}_x + \left(x^2z - \frac{3y^2}{3}z\right)\hat{a}_y + \left(x^2y - \frac{y^3}{3}\right)\hat{a}_z$$

$$\boxed{\therefore \bar{E} \text{ at } (2,1,-1) = 4\hat{a}_x + 3\hat{a}_y - \frac{11}{3}\hat{a}_z}$$

* Steady Magnetic field

→ Biot Savart's Law (Coulombs Law)

Biot Savart's is also called as Ampere's law for the c/n element

Applications

(i) $\bar{H}$ due to infinitely long straight conductor

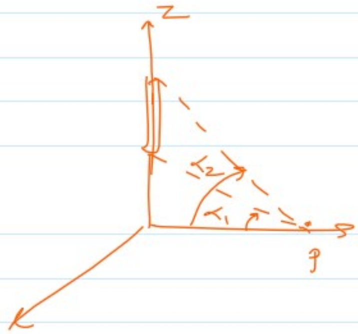
$$\rightarrow \bar{H} = \frac{I}{2\pi r}\hat{a}_\phi \quad A/m$$

$$\rightarrow \bar{B} = \mu\bar{H} = \frac{\mu I}{2\pi r}\hat{a}_\phi \quad wb/m^2$$

(ii) $\bar{H}$ due to straight conductor of finite length

$$\bar{H} = \frac{I}{4\pi r}\left[\sin\alpha_2 - \sin\alpha_1\right]\hat{a}_\phi \quad A/m$$

$$\bar{B} = \mu\bar{H} = \frac{\mu I}{4\pi r}\left[\sin\alpha_2 - \sin\alpha_1\right]\hat{a}_\phi \quad wb/m^2$$



(iii) $\bar{H}$ at the centre of a circular conductor

$$\bar{H} = \frac{I}{2R}\hat{a}_N \quad A/m$$

$$\bar{B} = \mu\bar{H} = \frac{\mu I}{2R}\hat{a}_N \quad wb/m^2$$

* Ampere's Circuital Law

Statement :-

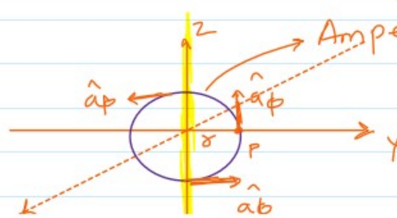
Ampere's circuital law states that the line integral of the tangential component of the magnetic field around a closed path is equal to the c/n enclosed by the path.

$$\oint \underbrace{\bar{H}\cdot\bar{dL}} = I_{enc} \quad \text{—— (A)}$$

concentric circle of radius 'r' as Amperian path

Proof :- Here $\bar{H} = H_\phi\hat{a}_\phi$

$$\therefore \bar{dL} = \underbrace{r\,d\phi\,\hat{a}_\phi} \quad \text{—— (1)}$$



while $\bar{H}$ obtained at pt P from Biot-Savart's

Lec 11: Chapter 02

01 February 2022 09:30

* Scalar Magnetic Potential (V_m), vector potential ($\bar{A}$)

Mag. potential — Scalar (V_m)
— vector ($\bar{A}$)

(1) Scalar Mag. Potential (V_M)

The mag. field intensity ($\bar{H}$)

$$\bar{H} = -\nabla V_M \quad \text{—— (1)}$$

Minus sign → analogy to the electric potential

This definition must satisfy results of magnetic fields.
i.e; example.

$$\nabla \times \bar{H} = \bar{J} \quad \text{—— (2)}$$

from (1) & (2)

$$\nabla \times (-\nabla V_M) = \bar{J} = 0$$

w.r.t. curl of gradient of any scalar = 0

Thus to define $\bar{H}$ as gradient of a scalar mag. potential, the c.n density must be zero through the region in which scalar mag. potential is defined.

$$\therefore \bar{H} = (-\nabla V_M) \text{ if } \bar{J} = 0 \quad \text{—— (3)}$$

* vector mag potential ($\bar{A}$)

From magnetostatic field w.r.t,

$$\nabla \cdot \bar{B} = 0 \quad \text{—— (1)}$$

Also from vector calculus, divergence of curl of any vector field is zero

i.e. $$\nabla \cdot (\nabla \times \bar{A}) = 0 \quad \text{—— (2)}$$

In order to satisfy eq^n (1) & (2) simultaneously

Lec 12: Maxwell's equations for time varying fields

04 February 2022 08:55

$$e = \oint \bar{E}\cdot d\bar{l} = -\frac{d}{dt}\int_s \bar{B}\cdot d\bar{s} \quad \text{—— (5)}$$

There are 2 conditions for the induced e.m.f as explained below

Case(i) :- A closed ckt stationary, while a mag. flux density varying with time. (Time changing flux linking a stationary closed path.

∴ from eqn (5) we have

$$\oint \bar{E}\cdot d\bar{l} = -\int \frac{\partial \bar{B}}{\partial t}\cdot d\bar{s} \quad \text{—— (6)}$$

This is similar to transformer action & emf ⇒ transformer e.m.f.

Using Stokes theorem, a line integral can be converted to surface integral

$$\oint_s (\nabla\times\bar{E})\cdot d\bar{s} = -\int \frac{\partial \bar{B}}{\partial t}\cdot d\bar{s} \quad \text{—— (7)}$$

Assuming that both the surface $\int^{s}$ taken over identical surfaces

$$(\nabla\times\bar{E})\cdot d\bar{s} = -\frac{\partial \bar{B}}{\partial t}\cdot d\bar{s}$$

Finally we have,

$$\boxed{\nabla\times\bar{E} = -\frac{\partial \bar{B}}{\partial t}} \quad \text{—— (8)}$$

Eqn (8) → represents one of the Maxwell's eqn.

If $\bar{B}$ is not varying with time, then eqn (6) & (8) → give the results obtained previously (electrostatic)

∴ $\oint \bar{E}\cdot d\bar{l} = 0$ &

$\nabla\times\bar{E} = 0$

Case (ii) :-

A magnetic flux density stationary, while a closed path moving.

Here the emf induced is called as generator emf (motional emf)

Consider that a charge Q is moved in a mag. field $\bar{B}$ with a velocity $\bar{v}$. Then the force on a charge is given by eg: $\bar{E} = |\bar{E}| \cos(\omega t + \phi)$ — mag; $2\pi f$; phase $\angle$ le

* Maxwell's eq^n — Phasor representation (Time Varying field)

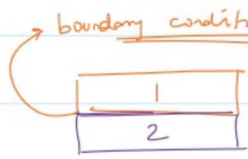
The partial derivative of a phasor w.r.t. time is equivalent to multiplying the corresponding phasor by $(j\omega)$

similarly → the second derivative is equivalent to $(j\omega)^2$

* Maxwell's eq^n in Phasor from (Time Varying field

Differential from	Integral from		PHASOR REPRESENTATION: Point form	PHASOR REPRESENTATION: Integral form
① $\nabla\cdot\bar{D} = \rho_v$	$\oint_S \bar{D}\cdot\bar{ds} = \int_V \rho_v dv$		$\nabla\cdot D = \rho_v$	$\oint_S \bar{D}\cdot\bar{ds} = \int_{vol} \rho_v dv$
② $\nabla\cdot\bar{B} = 0$	$\oint \bar{B}\cdot\bar{ds} = 0$	PHASOR REPRESENT. →	$\nabla\cdot\bar{B} = 0$	$\oint_S \bar{B}\cdot\bar{ds} = 0$
③ $\nabla\times\bar{E} = -\frac{\partial B}{\partial t}$	$\oint \bar{E}\cdot\bar{dl} = -\frac{\partial}{\partial t}\int \bar{B}\cdot\bar{ds}$		$\nabla\times\bar{E} = -j\omega\bar{B}$	$\oint_L \bar{E}\cdot\bar{dl} = -j\omega\int \bar{B}\cdot\bar{ds}$
④ $\nabla\times\bar{H} = \bar{J} + \frac{\partial D}{\partial t}$	$\oint \bar{H}\cdot\bar{dl} = \int_S \left(\bar{J} + \frac{\partial\bar{D}}{\partial t}\right)\cdot\bar{ds}$		$\nabla\times\bar{H} = \bar{J} + j\omega\bar{D}$	$\oint_L \bar{H}\cdot\bar{dl} = \int_S \bar{J}\cdot\bar{ds} + j\omega\int_S \bar{D}\cdot\bar{ds}$

* Boundary conditions



→ Integral from of Maxwell's eq^n

(i) Electrostatic Boundary conditions

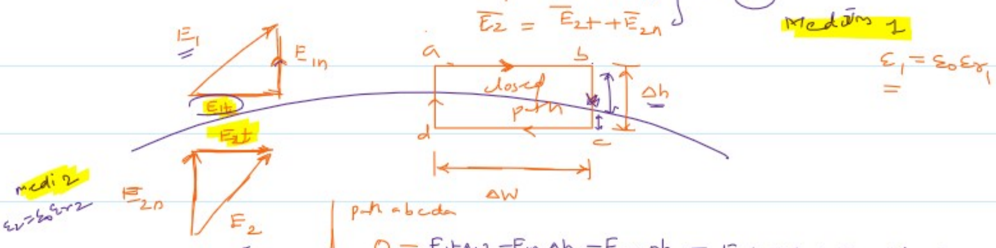
$$\oint_L \bar{E}\cdot\bar{dl} = 0 \quad \text{—①} \quad \&$$

$$\oint \bar{D}\cdot\bar{ds} = Q_{encl.} \quad \text{—②}$$

$$\bar{E} = \bar{E}_t + \bar{E}_n \quad \& \quad \bar{D} = \bar{D}_t + \bar{D}_n \quad \text{—③}$$

* Dielectric—Dielectric boundary cond^n

$$\left.\begin{aligned}\bar{E}_1 &= \bar{E}_{1t} + \bar{E}_{1n}\\ \bar{E}_2 &= \bar{E}_{2t} + \bar{E}_{2n}\end{aligned}\right\} \text{—④}$$



path abcda

$$0 = E_{1t}\Delta w - E_{1n}\Delta h - E_{2n}\frac{\Delta h}{2} - E_{2t}\Delta w + E_{2n}\frac{\Delta h}{2} + \frac{1}{2}E_{1n}\Delta h$$

As $\Delta h \to 0$

* Tangential component

⑤ — $\boxed{\bar{E}_{1t} = \bar{E}_{2t}}$ $\quad$ $\boxed{E_{1t} = E_{2t}}$

Tangential comp. of $\bar{E}$ are same on the 2 sides of boundary

bLec 14: Magnetic boundary conditions

10 February 2022 10:57

→ We make use of Gauss's Law for magnetic fields

$$\oint \bar{B} \cdot \bar{dS} = 0 \quad \text{— (1)}$$

& Ampere ckt law

$$\oint \bar{H} \cdot \bar{dL} = I \quad \text{— (2)}$$

$\bar{B} \to B_t + B_n$

$\bar{H} \to H_t + H_n$

$$\boxed{B_{1n} = B_{2n} \quad \text{or} \quad \mu_1 H_{1n} = \mu_2 H_{2n}}$$

normal component of $\bar{B}$ is continuous at the boundary whereas normal component of $\bar{H}$ is discontinuous at the boundary

$$\boxed{H_{1t} = H_{2t} \quad \text{or} \quad \frac{B_{1t}}{\mu_1} = \frac{B_{2t}}{\mu_2}}$$

The tangential component of $\bar{H}$ is continuous while that of $\bar{B}$ is discontinuous at the boundary.

(i) Derive wave equation (Helmholtz wave eqn)

(ii) Determine solution of vector wave equation.

* WAVE PROPAGATION

In the solution of any EM problem the fundamental relations that must be satisfied are the four propagation constant

$\gamma = \alpha + j\beta$

attenuation constant

phase constant (wave number)

Solving (13) and (16)

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}+1\right)} \qquad \dots (17)$$

and
$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}-1\right)} \qquad \dots (18)$$

* Solution of wave eqn

From eqn (9) we have.

$\nabla^2 E = \gamma^2 E$

(x, y, z)

$\nabla^2 E - \gamma^2 E = 0$ {from eqn (5)}

$\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$

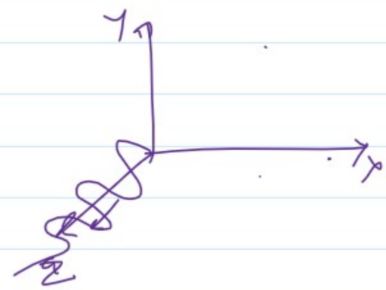
Expanding ∇^2 and $\overline{E}$ in Cartesian co–ordinate system.

$$\nabla^2 = \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}, \quad \text{and } \overline{E} = E_x\hat{a}_x + E_y\hat{a}_y + E_z\hat{a}_z$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\left(E_x\hat{a}_x + E_y\hat{a}_y + E_z\hat{a}_z\right) = \gamma^2\left(E_x\hat{a}_x + E_y\hat{a}_y + E_z\hat{a}_z\right)$$

Thus equation (9) is equivalent to three scalar equations

One for each component of $\overline{E}$ along $\hat{a}_x$, $\hat{a}_y$ and $\hat{a}_z$



The wave equation reduces to a very simple form if uniform plane wave is considered.

Consider uniform plane wave traveling in $+\hat{a}_z$ direction, $\overline{E}$ is independent of x and y and there is no component in the direction of propagation.

Therefore equation (9) reduces to,

$$\nabla^2\,\overline{E} = \frac{\partial^2 \overline{E}}{\partial z^2} = \gamma^2\overline{E} \qquad \dots (19)$$

which is equivalent to

$$\frac{\partial^2}{\partial z^2}E_x = \gamma^2 E_x \qquad \dots 20(a)$$

$$\frac{\partial^2}{\partial z^2}E_y = \gamma^2 E_y \qquad \dots 20(b)$$

$$\frac{\partial^2}{\partial z^2}E_z = \gamma^2 E_z = 0 \qquad \dots 20(c)$$

$\boxed{\nabla^2 E = \gamma^2 E}$

(E_x, E_y, E_z)

In general for uniform plane wave traveling in z direction, $\overline{E}$ may have x and y component but not E_z.

Proof : In a region in which there is no charge density,

D & E

$\nabla \cdot E = 0$ $\quad D = \varepsilon E$

$$\nabla.\,\overline{E} = \frac{1}{\varepsilon}\,\nabla.\,\overline{D} = 0$$

i.e. $$\frac{\partial E_x}{\partial x}+\frac{\partial E_y}{\partial y}+\frac{\partial E_z}{\partial z} = 0$$

$\boxed{E = \frac{D}{\varepsilon}}$

For uniform plane in which $\overline{E}$ is independent of x and y, the first two terms of this relation are equal to zero so that it reduces to $\frac{\partial E_z}{\partial z} = 0$

There is no variation of E_z in z direction. So second derivative then must be zero. It requires that

Lec 16 & 17: Solution to wave equation

15 February 2022 09:04

Intrinsic Impedance (η)

η = $\frac{\text{Elec field}}{\text{Mag field}}$

Now taking ratio

$$\frac{E_x}{H_y} = \frac{E_0 e^{-\gamma z}}{\frac{\gamma E_0}{j\omega\mu} e^{-\gamma z}} = \frac{j\omega\mu}{\gamma}$$

$$= \frac{j\omega\mu}{\sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}} = \sqrt{\frac{j\omega\mu}{(\sigma + j\omega\varepsilon)}} = \text{complex quantity}$$

$$= \eta \text{ (intrinsic impedance)}$$

where η is complex quantity, which may be expressed as

$$\eta = \sqrt{\frac{j\omega\mu}{(\sigma + j\omega\varepsilon)}} = |\eta| < \theta_n = |\eta| e^{j\theta n}$$

$\therefore$ [redacted]

$$\therefore \quad \theta_n = \frac{1}{2}\tan^{-1}\frac{\sigma\omega\mu}{\omega^2\mu\varepsilon} = \frac{1}{2}\tan^{-1}\frac{\sigma}{\omega\varepsilon}$$

$$|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\left[1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}}$$

Hence the equation (27) may be written as –

$$H_y = \frac{E_0}{|\eta| < \theta_n} e^{\alpha z} \cos(\omega t - \beta z)$$

$$\overline{H}(z_n t) = \frac{E_0}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta n)\hat{a}_y \qquad \dots (28)$$

$$\therefore \overline{E}(z, t) = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\hat{a}_x \qquad \dots. (25)$$

From equation (25) and (28), as the wave propagates along $+\hat{a}_z$, it decreases or attenuates in amplitude by a factor $e^{-\alpha z}$, hence α is known as the attenuation constant or factor of the medium. It is a measure of the spatial rate of decay of the wave in the medium, measured in nepers per meter (Np/m) or in decibels per meter (dB/m). An attenuation of 1 neper denotes a reduction to e^{-1} of the original value whereas an increase of 1 neper indicates in increase by a factor of e.

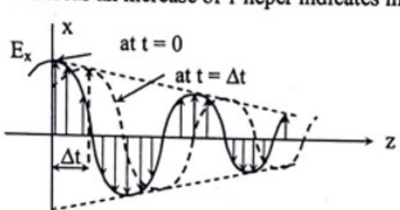


Fig : $\overline{E}$ field with y–comp. traveling in + $\hat{a}_x$ direction

The quantity β is measure of the phase shift per length and is called the phase constant or wave number.
In terms of β, the velocity of propagation and wavelength are given by,

$$v = \frac{\omega}{\beta}, \quad \lambda = \frac{2\pi}{\beta} \qquad \dots (29)$$

$$E_x = E_0 e^{-\alpha z} \cos(\omega t - \beta z) \qquad \dots (24)$$

$$\therefore \quad H_y = \frac{\gamma E_0}{j\omega\mu} e^{-\alpha z} \cos(\omega t - \beta z) \qquad \dots (27)$$

Lec 18: Wave motion in different mediums

17 February 2022 09:19

$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}+1\right)} \rightarrow 0 \quad — ① \quad \ldots (17)$$

$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left(\underbrace{\sqrt{1+\frac{\sigma^2}{\omega^2\varepsilon^2}}}_{\rightarrow 0}-1\right)} \quad — ② \quad \ldots (18)$$

$$\theta_n = \frac{1}{2}\tan^{-1}\frac{\sigma\omega\mu}{\omega^2\mu\varepsilon} = \frac{1}{2}\tan^{-1}\frac{\sigma}{\omega\varepsilon} \quad — ③$$

$\eta = |\eta|\angle\theta_n$

$$|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\left[1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}} \quad — ④$$

Wave Motion in Perfect (Lossless) Dielectrics :

It is a special case of lossy dielectrics.

Here $\sigma << \omega\varepsilon$,

$$\boxed{\therefore\ \sigma \cong 0,\ \ \varepsilon = \varepsilon_0\varepsilon_r,\ \ \mu = \mu_0\mu_r} \quad \ldots\ldots(31)$$

Substituting above relations into equation (17) , (18)

$$\alpha = 0 \quad \ldots\ldots.(32)$$

$$\beta = \omega\sqrt{\mu\varepsilon} \quad \ldots\ldots.(33)$$

$$v = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\varepsilon}} \quad \ldots\ldots(34)$$

$$\lambda = \frac{2\pi}{\beta} = \quad \ldots\ldots(35)$$

Also, $$\eta = \sqrt{\frac{\mu}{\varepsilon}}\angle 0° \quad \ldots\ldots(36)$$

Thus $\overline{E}$ and $\overline{H}$ are in time phase with each other.

16 February 2022 09:18

Intrinsic Impedance

Now taking ratio

$$\frac{E_x}{H_y} = \frac{E_0 e^{-\gamma z}}{\frac{\gamma E_0}{j\omega\mu} e^{-\gamma z}} = \frac{j\omega\mu}{\gamma}$$

where $\gamma^2 = j\omega\mu(\sigma + j\omega\varepsilon)$

$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$

$$= \frac{j\omega\mu}{\sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}} = \sqrt{\frac{j\omega\mu}{(\sigma + j\omega\varepsilon)}} = \text{complex quantity}$$

$$= \eta \text{ (intrinsic impedance)}$$

$\eta = |\eta| \angle \theta_n$

$e^{j\theta_n}$

where η is complex quantity, which may be expressed as

$$\eta = \sqrt{\frac{j\omega\mu}{(\sigma + j\omega\varepsilon)}} = |\eta| < \theta_n = |\eta| e^{j\theta n}$$

$\eta = \frac{E}{H}$

$\boxed{H = \frac{E}{\eta}}$

$|\eta| \angle \theta_n$

$$\therefore \quad \eta^2 = \frac{j\omega\mu(\sigma - j\omega\varepsilon)}{\sqrt{(\sigma + j\omega\varepsilon)(\sigma - j\omega\varepsilon)}} = \sqrt{\frac{\omega^2\mu\varepsilon + j\sigma\omega\mu}{\sigma^2 + \omega^2\varepsilon^2}}$$

$$\therefore \quad \theta_n = \frac{1}{2}\tan^{-1}\frac{\sigma\omega\mu}{\omega^2\mu\varepsilon} = \frac{1}{2}\tan^{-1}\frac{\sigma}{\omega\varepsilon}$$

$$|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\left[1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{1/4}}$$

$E =$

Hence the equation (27) may be written as –

$$H_y = \frac{E_0}{|\eta| < \theta_n} e^{\alpha z} \cos(\omega t - \beta z)$$

$$\overline{H}(z,t) = \frac{E_0}{|\eta|} e^{-\alpha z} \cos(\omega t - \beta z - \theta n)\hat{a}_y \quad \ldots (28)$$

$$\therefore \overline{E}(z,t) = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\hat{a}_x \quad \ldots (25)$$

Condition at a Boundary Surface

01 February 2023 12:08

Generalized Form of Maxwell's Equation:

Differential form	Integral form	Remarks
$\nabla.\bar{D} = \rho_v$	$\oint_S \bar{D}.\overline{dS} = \int_{vol} \rho_v dv$	Gauss's Law
$\nabla.\bar{B} = 0$	$\oint \bar{B}.\overline{dS} = 0$	Gauss's Law for magnetic field
$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$	$\oint \bar{E}.\overline{dL} = \frac{\partial}{\partial t} \int_S \bar{B}.\overline{dS}$	Faraday's law
$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$	$\oint \bar{H}.\overline{dL} = \int_S \left(\bar{J} + \frac{\partial \bar{D}}{\partial t} \right) . \overline{dS}$	Modified form of Ampere's circuital law

Maxwell's Equation in Phasor Form:

Point Form	Integral Form
$\nabla \times \bar{H} = \bar{J} + j\omega\bar{D}$	$\oint_L \bar{H}.\overline{dL} = \int_S \bar{J}.\overline{dS} + j\omega \int_S \bar{D}.\overline{dS}$
$\nabla \times \bar{E} = -j\omega\bar{B}$	$\oint_L \bar{E}.\overline{dL} = -j\omega \int \bar{B}.\overline{dS}$
$\nabla.\bar{D} = \rho_v$	$\oint_S \bar{D}.\overline{dS} = \int_{vol} \rho_v dV$
$\nabla.\bar{B} = 0$	$\oint_S \bar{B}.\overline{dS} = 0$
$\nabla.\bar{J} = -j\omega\rho_v$	$\oint_S \bar{J}.\overline{dS} = -j\omega \int_{vol} \rho_v dV$

Condition at a Boundary Surface:

- Dielectric (ε_{r1}) – Dielectric (ε_{r2})
- Conductor – Dielectric
- Conductor – free Space.

If the field exist in a region consisting of two different media, the conditions that must satisfy at the interface separating the media are called boundary conditions.

The integral form of Maxwell's equation can be used to determine boundary conditions.

Electric and Magnetic field boundary conditions

02 February 2023 08:48

Generalized Form of Maxwell's Equation:

Differential form	Integral form	Remarks
$\nabla.\bar{D} = \rho_v$	$\oint_S \bar{D}.\overline{dS} = \int_{vol} \rho_v dv$	Gauss's Law
$\nabla.\bar{B} = 0$	$\oint \bar{B}.\overline{dS} = 0$	Gauss's Law for magnetic field
$\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$	$\oint \bar{E}.\overline{dL} = \frac{\partial}{\partial t} \int_S \bar{B}.\overline{dS}$	Faraday's law
$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$	$\oint \bar{H}.\overline{dL} = \int_S \left(\bar{J} + \frac{\partial \bar{D}}{\partial t}\right).\overline{dS}$	Modified form of Ampere's circuital law

Maxwell's Equation in Phasor Form:

Point Form	Integral Form
$\nabla \times \bar{H} = \bar{J} + j\omega\bar{D}$	$\oint_L \bar{H}.\overline{dL} = \int_S \bar{J}.\overline{dS} + j\omega \int_S \bar{D}.\overline{dS}$
$\nabla \times \bar{E} = -j\omega\bar{B}$	$\oint_L \bar{E}.\overline{dL} = -j\omega \int \bar{B}.\overline{dS}$
$\nabla.\bar{D} = \rho_v$	$\oint_S \bar{D}.\overline{dS} = \int_{vol} \rho_v dV$
$\nabla.\bar{B} = 0$	$\oint_S \bar{B}.\overline{dS} = 0$
$\nabla.\bar{J} = -j\omega\rho_v$	$\oint_S \bar{J}.\overline{dS} = -j\omega \int_{vol} \rho_v dV$

Boundary conditions

If the field exist in a region consisting of two different media, the conditions that must satisfy at the interface separating the media are called boundary conditions.

Electrostatic Boundary conditions

Introduction to EM waves

11 February 2022 09:15

Waves are a means for transferring energy or information from one place to another

Electromagnetic waves as the name suggests, are a means for transferring electromagnetic energy

Plane wave :- If each field has same direction at every pt in any plane ⊥ar to the direction of wave travel, it is called as plane wave

Uniform Plane Wave :- If the field has the same direction and magnitude at every point in any plane ⊥ar to the direction of wave travel, it is called as Uniform plane wave

Electromagnetic (EM) waves.

All forms of EM energy share 3 fundamental characteristics

(i) They all travel at high velocity

(ii) In travelling, they assume the properties of waves.

(iii) They radiate outward from a source.

* EM wave propagation through various media

(i) Free Space ($\sigma = 0$; $\varepsilon = \varepsilon_0$, $\mu = \mu_0$)

(ii) Lossless dielectric ($\sigma << \omega\varepsilon$, $\varepsilon = \varepsilon_0\varepsilon_r$, $\mu = \mu_0\mu_r$)

Extra Numericals

27 January 2022 10:29

Q1) Consider 2 pt charges $Q_1 = 2\,mC$ at $(-3,7,4)$ & $Q_2 = 5\,mC$ at $(2,4,-1)$.
Find force F_2 on Q_2 (F_2 & F_1) ? Q

⇒

(i) Force F_2 on Q_2 is given by

$$F_2 = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{12}^2}\,\hat{a}_{12}$$

$$R_{12} = r_2 - r_1 = (2\hat{a}_x + 4\hat{a}_y - \hat{a}_z) - (-3\hat{a}_x + 7\hat{a}_y + 4\hat{a}_z)$$

$$= 5\hat{a}_x - 3\hat{a}_y - 5\hat{a}_z$$

$$\rightarrow \hat{a}_{12} = \frac{r_2 - r_1}{|R_{12}|} = \frac{5\hat{a}_x - 3\hat{a}_y - 5\hat{a}_z}{\sqrt{25+9+25}} = \frac{5\hat{a}_x - 3\hat{a}_y - 5\hat{a}_z}{7.6811}$$

$$\therefore \hat{a}_{12} = 0.651\,\hat{a}_x - 0.391\,\hat{a}_y - 0.651\,\hat{a}_z$$

$$\rightarrow \therefore \bar{F}_2 = \frac{2\times10^{-3}\times5\times10^{-3}}{4\pi\times8.85\times10^{-12}\times(7.6811)^2}\left(0.651\hat{a}_x - 0.391\hat{a}_y - 0.651\hat{a}_z\right)$$

$$\bar{F}_2 = \frac{10\times10^{-6}}{6561.45\times10^{-12}}\left(0.651\hat{a}_x - 0.391\hat{a}_y - 0.651\hat{a}_z\right)$$

$$= 1524.05\left(0.651\hat{a}_x - 0.391\hat{a}_y - 0.65\hat{a}_z\right)$$

$$\boxed{\bar{F}_2 = 992.158\,\hat{a}_x - 595.91\,\hat{a}_y - 990.63\,\hat{a}_z \quad N}$$

Note:- As $\hat{a}_{21} = -\hat{a}_{12}$

$\therefore \bar{F}_1 = -\bar{F}_2$

(ii) Force F_1 on Q_1

$$\boxed{\therefore \bar{F}_1 = -992.158\,\hat{a}_x + 595.91\,\hat{a}_y + 990.63\,\hat{a}_z \quad N}$$

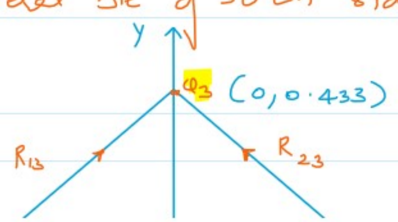
Q2) Three pt charges $Q_1 = 10^{-6}\,C$; $Q_2 = 10^{-6}\,C$ & $Q_3 = 0.5\times10^{-6}\,C$ are located at the corners of an equilateral Δle of 50 cm side. Determine force on charge Q_3

⇒

$Q_1 = 10^{-6}\,C$ at $(-0.25,\ )$

$Q_2 = 10^{-6}\,C$ at $(0.25, 0)$

$Q_3 = 0.5\times10^{-6}\,C$ at



- Experimental law deals with force between two point charges
- The law states that the force F between two point charges Q_1 and Q_2 is-
  - directly proportional to the products ($Q_1 \times Q_2$) of the charges
  - Inversely proportional to square of distance (R) between them
  - along the line joining them.
  - depends upon medium
  - depends upon the nature of charges (+ or –).

- The force is expressed as –
- $F = \frac{Q_1 Q_2}{4\pi\epsilon R^2}$
- where ε is permittivity of the medium. For free space $\varepsilon = \varepsilon_0\varepsilon_r = \varepsilon_0 = 8.854 \times 10^{-12}$ F/m i.e. $\varepsilon_r = 1$