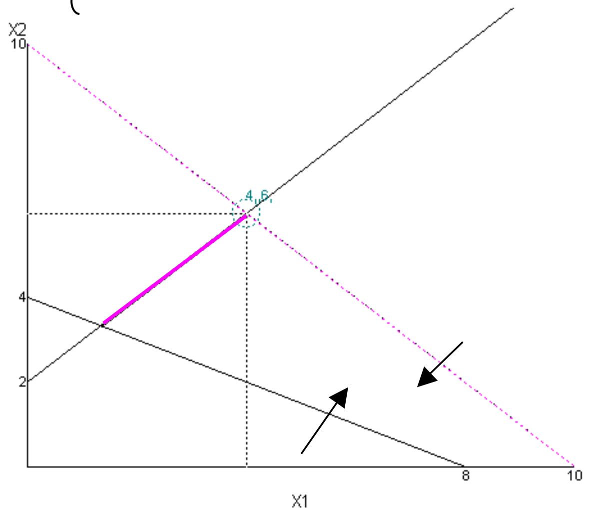
Aula 8 e 9 – SIMPLEX 2 fases

max $z = 2x_1 + 2x_2$

$$\text{S.A.}\quad \begin{cases} x_1 + x_2 \leq 10 \\ x_1 + 2x_2 \geq 8 \\ -x_1 + x_2 = 2 \\ x_1 ,\ x_2 \geq 0 \end{cases}$$



$x_1 = 4$
$x_2 = 6$
$z = 20$

x_1	x_2	x_3	x_4	y_1	y_2	b	
1	1	1	0	0	0	10	
1	2	0	-1	1	0	8	
-1	1	0	0	0	1	2	
-2	- 2	0	0	0	0	0	z
0	- 3	0	1	0	0	-10	w

2	0	1	0	0	-1	8	
3	0	0	-1	1	-2	4	
-1	1	0	0	0	1	2	
-4	0	0	0	0	2	4	z
-3	0	0		0	3	- 4	w

0	0	1	2/3	-2/3	1/3	16/3	
1	0	0	-1/3	1/3	-2/3	4/3	
0	1	0	-1/3	1/3	1/3	10/3	
0	0	0	-4/3	4/3	-2/3	28/3	z
0	0	0	0	1	1	0	w

Base inicial encontrada:
$x_1 = 4/3$
$x_2 = 10/3$

0	0	1	2/3	16/3	
1	0	0	-1/3	4/3	
0	1	0	-1/3	10/3	
0	0	0	-4/3	28/3	z

0	0	3/2	1	8	
1	0	1/2	0	4	
0	1	1/2	0	6	
0	0	2	0	20	z

Resposta:

$x_1 = 4$
$x_2 = 6$
$x_3 = 0$
$x_4 = 8$
$z = 20$

$\max z = 3x_1 + 2x_2$ S.A. $\begin{cases} 2x_1 + x_2 \geq 4 \\ x_1 - 2x_2 = 0 \\ x_1 + 2x_2 \leq 8 \\ x_1, x_2 \geq 0; \end{cases}$	Forma padrão $2x_1 + x_2 - x_3 + y_1 = 4$ $x_1 - 2x_2 + y_2 = 0$ $x_1 + 2x_2 + x_4 = 8$

x_1	x_2	x_3	x_4	y_1	y_2	b	
2	1	-1	0	1	0	4	
1	-2	0	0	0	1	0	
1	2	0	1	0	0	8	
-3	- 2	0	0	0	0	0	z
-3	1	1	0	0	0	-4	w

0	5	-1	0	1	-2	4	
1	-2	0	0	0	1	0	
0	4	0	1	0	-1	8	
0	- 8	0	0	0	3	0	z
0	-5	1	0	0	3	-4	w

0	1	-1/5	0	1/5	-2/5	4/5	
1	0	-2/5	0	2/5	1/5	8/5	
0	0	4/5	1	-4/5	3/5	24/5	
0	0	-8/5	0	8/5	-1/5	32/5	z
0	0	0	0	1	1	0	w

0	1	-1/5	0	4/5
1	0	-2/5	0	8/5
0	0	4/5	1	24/5
0	0	-8/5	0	32/5

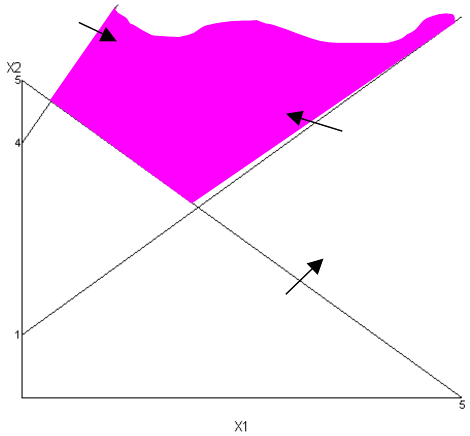
0	1	0	1/4	2
1	0	0	1/2	4
0	0	1	5/4	6
0	0	0	2	16

Resposta:

$x_1 = 4$
$x_2 = 2$
$x_3 = 6$
$x_4 = 0$
$z = 16$

$\max \ z = 2x_1 + 3x_2$

$$S.A. \begin{cases} -4x_1 + 2x_2 \leq 8 \\ -4x_1 + 4x_2 \geq 4 \\ x_1 + x_2 \geq 5 \\ x_1, \ x_2 \geq 0 \end{cases}$$



Solução ótima ilimitada

x_1	x_2	x_3	x_4	x_5	y_1	y_2	b
-4	2	1	0	0	0	0	8
-4	4	0	-1	0	1	0	4
1	1	0	0	-1	0	1	5
-2	-3	0	0	0	0	0	0
3	-5	0	1	1	0	0	-9

-2	0	1	½	0	-½	0	6
-1	1	0	-¼	0	¼	0	1
2	0	0	¼	-1	-¼	1	4
-5	0	0	-¾	0	¾	0	3
-2	0	0	-¼	1	5/4	0	-4

0	0	1	¾	-1	-¾	1	10
0	1	0	-1/8	-½	1/8	½	3
1	0	0	1/8	-½	-1/8	½	2
0	0	0	-1/8	-5/2	1/8	5/2	13
0	0	0	0	0	1	1	0

Final Fase I, base inicial :

$X_1 = 2$, $x_2 = 3$

0	0	1	¾	-1	10
0	1	0	-1/8	-½	3
1	0	0	1/8	-½	2
0	0	0	-1/8	-5/2	13

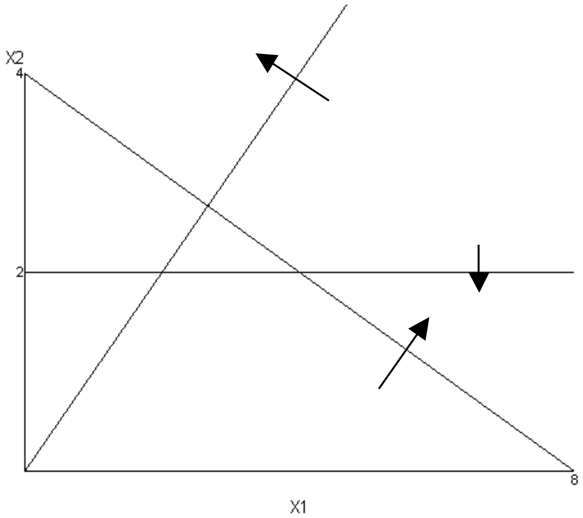
Solução ótima Ilimitada

Fazendo mais uma iteração :

0	0	4/3	1	-4/3	40/3
0	1	1/6	0	-2/3	14/3
1	0	-1/6	0	-1/3	1/3
0	0	1/6	0	-8/3	44/3

$\max \; z = 2x_1 + x_2$

$$
\text{S. A.} \begin{cases} x_1 - x_2 \leq 0 \\ x_2 \leq 2 \\ x_1 + 2x_2 \geq 8 \\ x_1, \; x_2 \geq 0 \end{cases}
$$



Não há solução viável (não existe intersecção possível entre as restrições)

Forma padrão:

$$
\begin{aligned}
x_1 - x_2 + x_3 \qquad\qquad &= 0 \\
x_2 \quad + x_4 \qquad &= 2 \\
x_1 + 2x_2 \qquad - x_5 + y_1 &= 8
\end{aligned}
$$

x_1	x_2	x_3	x_4	x_5	y_1	b
1	-1	1	0	0	0	0
0	1	0	1	0	0	2
1	2	0	0	-1	1	8
-2	-1	0	0	0	0	0
-1	-2	0	0	1	0	-8

1	0	1	1	0	0	2
0	1	0	1	0	0	2
1	0	0	-2	-1	1	4
-2	0	0	1	0	0	2
-1	0	0	2	1	0	-4

1	0	1	1	0	0	2
0	1	0	1	0	0	2
0	0	-1	-3	-1	1	2
0	0	2	3	0	0	6
0	0	1	3	1	0	-2

Final Fase I e $w \neq 0$, $\therefore$ não há solução viável

$\min z = 5x_1 + 8x_2 + 4x_3$

$$\text{S.A.} \begin{cases} x_1 + x_2 + x_3 \geq 2 \\ x_1 + x_2 \geq 1 \\ x_1, x_2, , x_3 \geq 0 \end{cases}$$

Forma padrão:

$x_1 + x_2 + x_3 - x_4 + y_1 = 2$
$x_1 + x_2 + x_3 - x_5 + y_2 = 1$

$\min z \equiv \max -z$

➔ $\max -z = -5x_1 - 8x_2 - 4x_3$

x_1	x_2	x_3	x_4	x_5	y_1	y_2	b
1	1	1	-1	0	1	0	2
1	1	0	0	-1	0	1	1
5	8	4	0	0	0	0	0
-2	-2	-1	1	1	0	0	-3

0	0	1	-1	1	1	-1	1
1	1	0	0	-1	0	1	1
0	3	4	0	5	0	- 5	- 5
0	0	-1	1	-1	0	2	- 1

0	0	1	-1	1	1	-1	1
1	1	0	0	-1	0	1	1
0	3	0	4	1	- 4	- 1	- 9
0	0	0	0	0	1	1	0

Final fase I, base inicial:
$x_1 = 1, x_2 = 0, x_3 = 1$

0	0	1	-1	1	1
1	1	0	0	-1	1
0	3	0	4	1	- 9

A solução inicial encontrada já é a solução ótima

$-z = -9$ ➔ $z = 9$