Approximating Probability Distributions Done Right

Disclaimer: This post is written based on the syllabus for Cambridge International AS and A Levels, and may not be applicable for higher level studies.

Approximation

1. Binomial to Normal:
   - Conditions*: $np > 5$ and $nq > 5$ (n is sufficiently large).
   - Apply continuity correction.
2. Binomial to Poisson:
   - A Poisson distribution can be used to model a discrete probability distribution in which the events
   - occur singly,
   - at random and independently,
   - in a given interval of space or time.
   - The mean and variance of a Poisson distribution are equal.
   - Conditions*: $n > 50$ and $np < 5$ (n is large, p is small/rare event).
3. Poisson to Normal:
   - Conditions*: $\lambda > 15$
   - Apply continuity correction.

Central Limit Theorem

> The central limit theorem (CLT) states that, provided n is large, the distribution of **sample means** of size n is:
>
> $$\overline{X}(n) \sim N\left(\mu, \frac{\sigma^2}{n}\right),$$
>
> where the original population has mean μ and variance σ^2.

- CLT can be used for sample size $n > 50$* (or $n > 30$**).

*Rule of thumb

**For Further Statistics

References:

1. Chalmers, Dean. Cambridge International AS & A Level Mathematics: Probability & Statistics 1 - Coursebook. Cambridge University Press, 2018.

2. Kranat, Jayne. Cambridge International AS & A Level Mathematics: Probability & Statistics 2 - Coursebook. Cambridge University Press, 2018.
3. McKevley, Lee, and Crozier, Martin. Cambridge International AS & A Level Further Mathematics - Coursebook. Cambridge University Press, 2018.