Quantifying Statistical Significance of Neural Network-based Image Segmentation by Selective Inference

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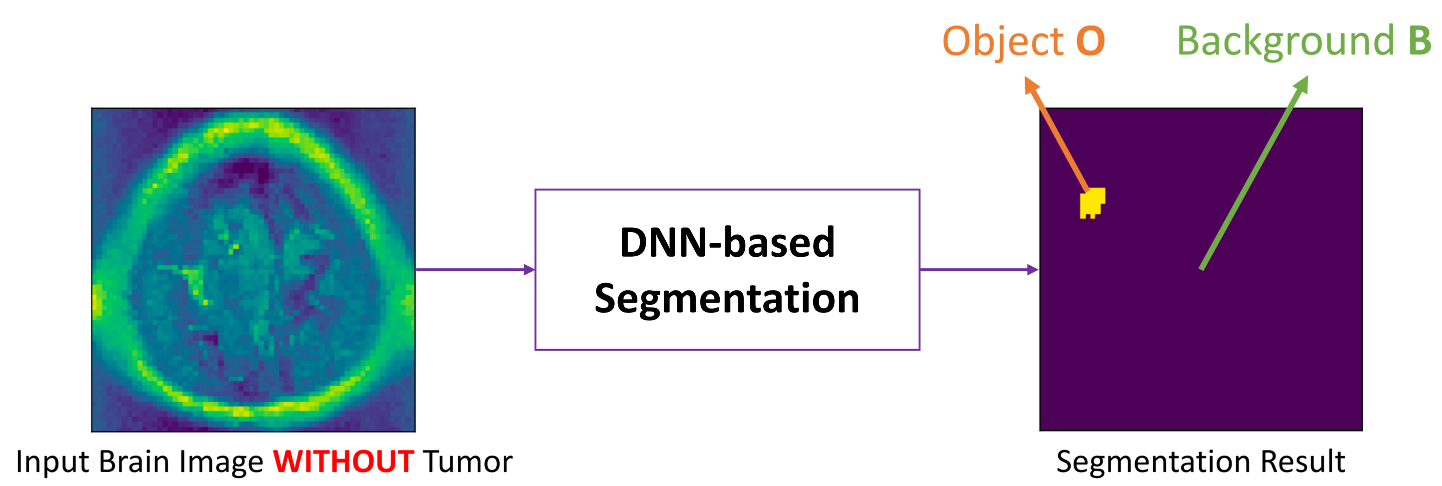
Brief Summary

- ✦ Deep neural network (DNN)-based image segmentation has become a popular model that achieves remarkable performance
- ✦ However, controlling the risk of obtaining incorrect segmentation results is a challenging task
- ✦ Therefore, we provide an exact inference method to quantify the statistical reliability of DNN-based image segmentation results

Motivating Example



Proposed p-value = 0.53

- ❖ Obviously, object **O** is falsely detected
  - ➡ Harmful for medical diagnosis task
- ❖ We propose a **statistical inference** method on the result obtained from a DNN. The p-value is used as a criterion for reliability evaluation
  - ➡ Large p-value indicates that NO object exists

Problem Setup - Two-sample Test

- ❖ An image is represented as a vector of pixel values $\boldsymbol{X} \in \mathbb{R}^n$

$$\underbrace{\boldsymbol{X}}_{\text{(random) vector}} = \underbrace{\boldsymbol{\mu}}_{\text{signal vector}} + \underbrace{\boldsymbol{\varepsilon}}_{\text{noise vector}} \qquad \underbrace{\boldsymbol{\varepsilon} \sim \mathbb{N}(\mathbf{0}, \Sigma)}_{\text{normally-distributed noise}}$$

- ❖ Segmentation algorithm

$$\underbrace{\mathscr{A}}_{\text{algorithm}} : \underbrace{\boldsymbol{X}}_{\text{(random) image}} \mapsto \{ \underbrace{\mathscr{O}_{\boldsymbol{X}}}_{\text{set of object pixels}}, \underbrace{\mathscr{B}_{\boldsymbol{X}}}_{\text{set of background pixels}} \}$$

- ❖ Hypothesis testing

$$\mathrm{H}_0 : \underbrace{\frac{1}{|\mathscr{O}_{\boldsymbol{X}}|} \sum_{i \in \mathscr{O}_{\boldsymbol{X}}} \mu_i}_{\text{average signal value in object}} = \underbrace{\frac{1}{|\mathscr{B}_{\boldsymbol{X}}|} \sum_{i \in \mathscr{B}_{\boldsymbol{X}}} \mu_i}_{\text{average signal value in background}}$$

Problem Setup - Test Statistic

- ❖ Test statistic: difference between object and background regions

$$\boldsymbol{\eta}_{(\mathscr{O}_{\boldsymbol{X}}, \mathscr{B}_{\boldsymbol{X}})}^\top \boldsymbol{X} = \frac{1}{|\mathscr{O}_{\boldsymbol{X}}|} \sum_{i \in \mathscr{O}_{\boldsymbol{X}}} X_i - \frac{1}{|\mathscr{B}_{\boldsymbol{X}}|} \sum_{i \in \mathscr{B}_{\boldsymbol{X}}} X_i,$$

where

$$\boldsymbol{\eta}_{(\mathscr{O}_{\boldsymbol{X}}, \mathscr{B}_{\boldsymbol{X}})} = \frac{1}{|\mathscr{O}_{\boldsymbol{X}}|} \mathbf{1}^n_{\mathscr{O}_{\boldsymbol{X}}} - \frac{1}{|\mathscr{B}_{\boldsymbol{X}}|} \mathbf{1}^n_{\mathscr{B}_{\boldsymbol{X}}},$$

and $\mathbf{1}^n_C \in \mathbb{R}^n$ is a vector whose elements belonging to a set C are set to 1, and 0 otherwise

- ❖ Deriving the distribution of $\boldsymbol{\eta}_{(\mathscr{O}_{\boldsymbol{X}}, \mathscr{B}_{\boldsymbol{X}})}^\top \boldsymbol{X}$ is required to compute the p-value
- ❖ This task is difficult because $\boldsymbol{\eta}_{(\mathscr{O}_{\boldsymbol{X}}, \mathscr{B}_{\boldsymbol{X}})}$ is a random quantity

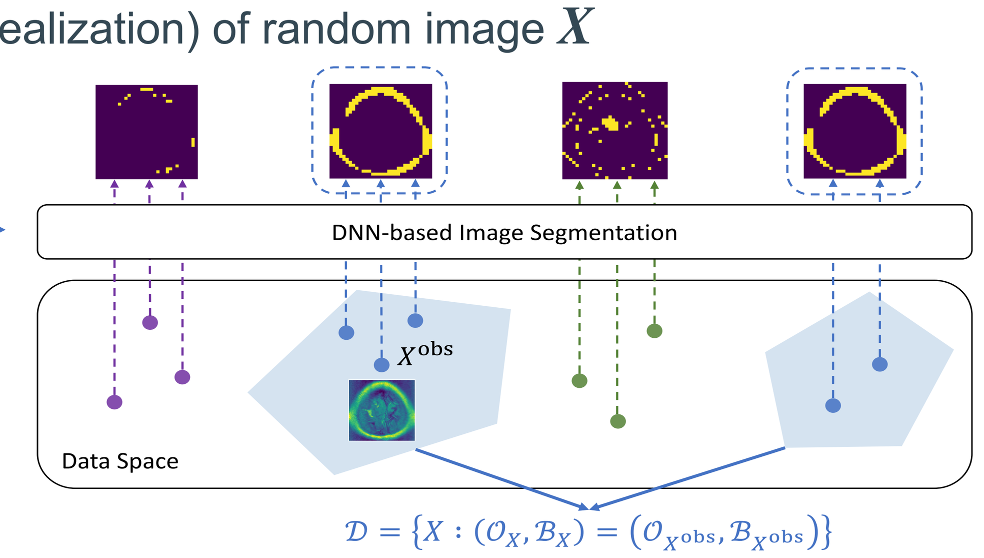
Proposed Method

- ❖ We introduce an approach to derive the distribution of the test statistic by **conditioning** on the **segmentation event**:

$$\boldsymbol{\eta}_{(\mathscr{O}_{\boldsymbol{X}}, \mathscr{B}_{\boldsymbol{X}})}^\top \boldsymbol{X} \;\Big|\; (\mathscr{O}_{\boldsymbol{X}}, \mathscr{B}_{\boldsymbol{X}}) = (\mathscr{O}_{\boldsymbol{X}^{\rm obs}}, \mathscr{B}_{\boldsymbol{X}^{\rm obs}}) \tag{1}$$

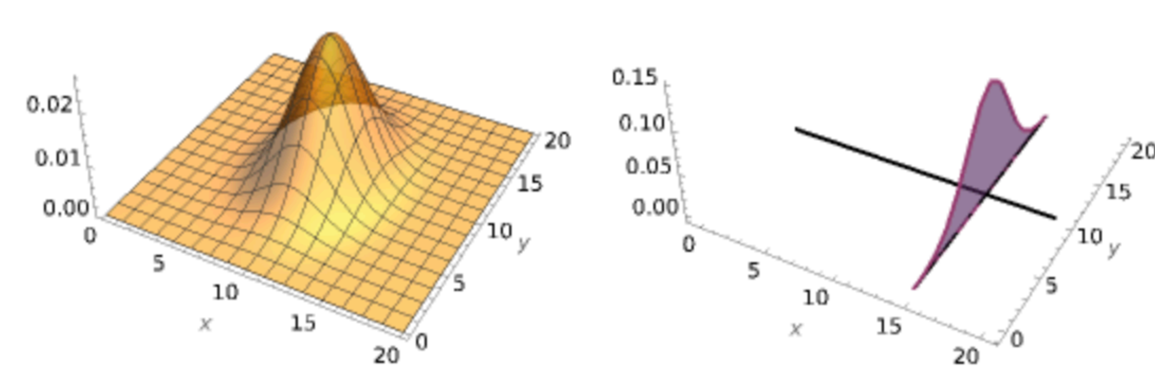
where $\boldsymbol{X}^{\rm obs}$ is an observation (realization) of random image $\boldsymbol{X}$

- ❖ Illustration of the conditional data space
- ❖ Characterization of the distribution of the test statistic become possible



Why Conditioning?

- ❖ Conditioning $\Longrightarrow$ Making random quantity to be FIXED

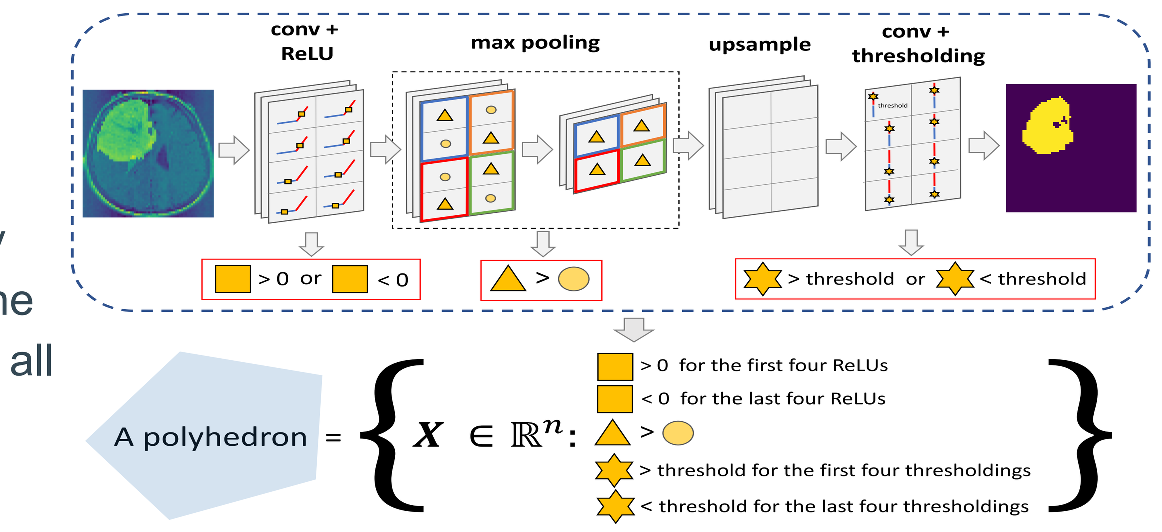


- ❖ By conditioning on the segmentation event
  - ➡ $\boldsymbol{\eta}_{(\mathscr{O}_{\boldsymbol{X}}, \mathscr{B}_{\boldsymbol{X}})}$ is **fixed**
  - ➡ Quantity in (1) follows **a truncated normal distribution**

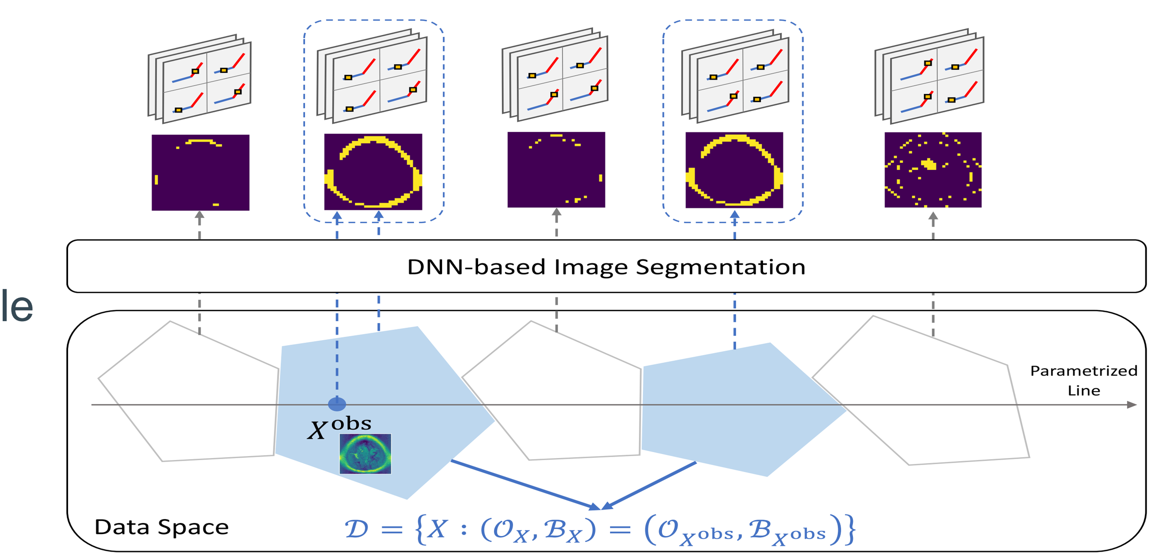
Conditional Data Space Identification

- ❖ Considering a class of **piecewise linear networks**, the conditional data space is characterized by **a union of polyhedrons**

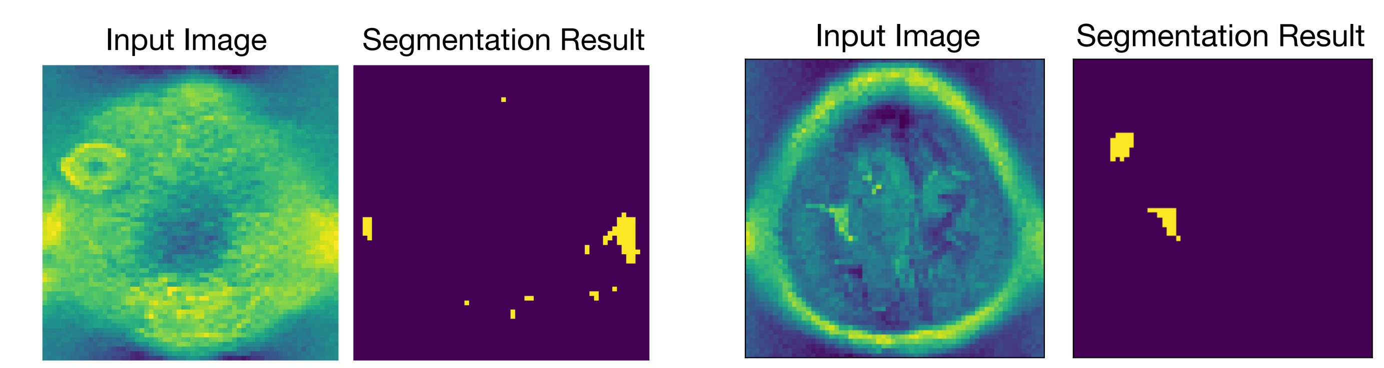
Step A: Considering a polyhedron by conditioning on the selected pieces of all hidden nodes



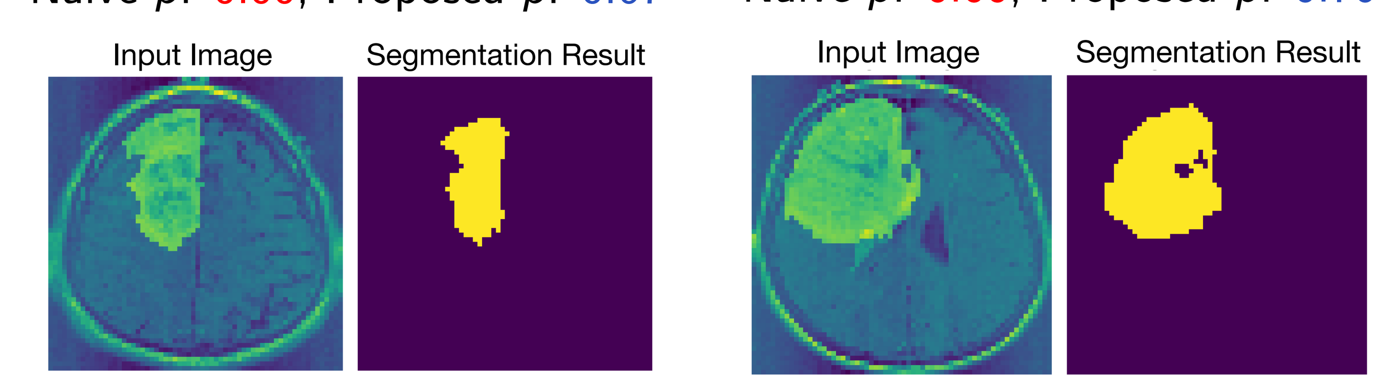
Step B: Combining multiple polyhedrons



Experimental Results



Naive-p: 0.00, Proposed-p: 0.67   Naive-p: 0.00, Proposed-p: 0.76



Naive-p: 0.00, Proposed-p: 0.00   Naive-p: 0.00, Proposed-p: 0.00

References

Lee et al. (2016). “Exact post-selection inference, with application to the lasso” In: The Annals of Statistics.

Duy et al. (2021). “Parametric programming approach for more powerful and general lasso selective inference”. In: International Conference on Artificial Intelligence and Statistics (AISTATS).