ECON 899A - Problem Set 5

Alex von Hafften*

10/13/2021

In this problem, we compute an approximate equilibrium of an Aiyagari (1994) paper with aggregate uncertainty using the techniques in Krusell and Smith (1998). There is a unit measure of agents, the time period is one quarter, preferences are given by

$$\sum_{t=0}^{\infty} \beta^t \ln(c_t)$$

where $\beta = 0.99$. The production technology is given by

$$y_t = z_t k_t^{\alpha} l_t^{1-\alpha}$$

where $\alpha = 0.36$, and aggregate technology shocks $z_t \in \{z_g = 1.01, z_b = 0.99\}$ are drawn from a Markov process to be described more fully below. Capital depreciates at rate $\delta = 0.025$. Agents have 1 unit of time and face idiosyncratic employment opportunities $\varepsilon_t \in \{0, 1\}$ where $\varepsilon_t = 1$ means the agent is employed an receives wage $w_t \bar{e}$ (where $\bar{e} = 0.3271$ denotes labor efficiency per unit of time worked) and $\varepsilon_t = 0$ means he is unemployed. The probability of transition from state (z, ε) to (z', ε'); denoted $\pi_{zz'\varepsilon\varepsilon'}$ must satisfy certain conditions:

$$\pi_{zz'00} + \pi_{zz'01} = \pi_{zz'10} + \pi_{zz'11} = \pi_{zz'}$$

and

$$u_z \frac{\pi_{zz'00}}{\pi_{zz'}} + (1 - u_z) \frac{\pi_{zz'10}}{\pi_{zz'}} = u_{z'}$$

where u_z denotes the fraction of those unemployed in state z with $u_g = 4\%$ and $u_b = 10\%$. The other restriction on $\pi_{zz'\varepsilon\varepsilon'}$ 0 necessary to pin down the transition matrix are that: the average duration of good and bad times is 8 quarters; the average duration of unemployment spells is 1.5 quarters in good times and 2.5 quarters in bad times; and

$$\frac{\pi_{gb00}}{\pi_{gb}} = 1.25 \cdot \frac{\pi_{bb00}}{\pi_{bb}}$$

and

$$\frac{\pi_{bg00}}{\pi_{bg}} = 0.75 \cdot \frac{\pi_{gg00}}{\pi_{gg}}$$

*This problem set is for ECON 899A Computational Economics taught by Dean Corbae with assistance from Philip Coyle at UW-Madison. I worked on this problem set with a study group of Michael Nattinger, Sarah Bass, and Xinxin Hu.

Capital is the only asset to self insure fluctuations; households rent their capital $k_t \in [0, 1)$ to firms and receive rate of return r_t. Without loss of generality, we can consider one firm which hires L_t units of labor efficiency units (so that $L_t = e(1u_t)$) and rents capital K so that wages and rental rates are given by their marginal products:

$$w_t \equiv w(K_t, L_t, z_t) = (1-\alpha)z_t\left(\frac{K_t}{L_t}\right)^{\alpha} \tag{1}$$

$$r_t \equiv r(K_t, L_t, z_t) = \alpha z_t\left(\frac{K_t}{L_t}\right)^{\alpha-1} \tag{2}$$

As in Krusell and Smith, approximate the true distribution Γ_t over (k_t, ε_t) in state z_t by I moments and let the law of motion for the moment be $m^t = h_I(m, z, z^t)$.

To start the Krusell-Smith algorithm, we need initial conditions. There's only 2 possibilities (z_t, ε_t) so choose the ones that are most likely (i.e. z_g and use $L_g = 1 - u_g = 0.96$ to generate $\varepsilon_{t=0}$). But to speed things along, we would like to have a good starting point for (k_t, K_t). To that end, we can solve for a steady state of the complete markets (representative agent) version of the model. Specifically we let $z = 1$, $L^{ss} = \pi L_g + (1-\pi)L_b$ where π is the long run probability of state g induced by $\pi_{zz'}$ and $L_g = 1 - u_g = 0.96$ and $L_b = 1 - u_b = 0.9$. The steady state solves the Euler equation

$$\begin{aligned}
u'(c) &= \beta u'(c)(r(K^{SS}, L^{SS}) + 1 - \delta) \\
\iff \frac{1}{\beta} &= \left(\alpha\left(\frac{K^{SS}}{K^{SS}}\right)^{\alpha-1} + 1 - \delta\right) \\
\iff K^{SS} &= \left(\frac{\alpha}{1/\beta + \delta - 1}\right)^{\frac{1}{1-\alpha}} L^{SS}
\end{aligned}$$