

ECON 711 - PS 2

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Question 1. Convex production sets, concave production functions, convex costs

Consider a production function $f : \mathbb{R}_+^m \rightarrow \mathbb{R}_+$ for a single-output firm.

- (a) Prove that if the production set $Y = \{(q, -z) : f(z) \geq q\} \subset \mathbb{R}^{m+1}$ is convex, the production function f is concave.

Proof:

- (b) Prove that if f concave, the cost function

$$c(q, w) = \min w \cdot z \text{ subject to } f(z) \geq q$$

is convex in q .

Proof:

Question 2. Solving for the profit function given technology...

Let $k = 2$, and let the production set be

$$Y = \{(y_1, y_2) : y_1 \leq 0 \text{ and } y_2 \leq B(-y_1)^{\frac{2}{3}}\}$$

where $B > 0$ is a known constant. Assume both prices are strictly positive.

- (a) Draw Y , or describe it clearly.
- (b) Solve the firm's profit maximization problem to find $\pi(p)$ and $Y^*(p)$. (It may help to set $z = -y_1$ as the amount of input used, explain why a profit-maximizing firm will set)

*I worked on this problem set with a study group of Michael Nattinger, Andrew Smith, Tyler Welch, and Ryan Mather. I also discussed problems with Emily Case, Sarah Bass, and Danny Edgel.