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AN AGGREGATE ECONOMY WITH DIFFERENT SIZE HOUSES

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Abstract
We build an aggregate model with different size houses and liquid assets. Typical households are born, are subject to idiosyncratic earnings risk, and save for both life-cycle reasons and housing reasons. Typically, a subset of these households, after accumulating some assets, make a down payment and buy a small starter's house or flat. As time passes, some households upgrade to a larger and nicer house. Households with houses may also eventually downgrade to a flat or even to no house and flat owners may sell. Our specification attempts to replicate some important features of modern aggregate economies: The distribution of earnings and of housing and non-housing wealth as well as some macroeconomic aggregates, including features of the mortgage issuing sector. (JEL: E21)

1. Introduction

There is a literature in modern macroeconomics that uses models with a large number of households subject to idiosyncratic earnings shocks that use savings to smooth consumption across time (see, e.g., Aiyagari 1994; Castañeda, Díaz-Giménez and Ríos-Rull 2003; and Krusell and Smith 1997). In this article we extend these model economies to environments where there are multiple assets: financial assets that are perfectly divisible and are costless to buy or sell, and other assets that are bulky and indivisible and that can be traded only at a considerable cost. These assets can be partially purchased on credit, and they give their owners some additional advantages, perhaps because of the the tax system or for moral hazard reasons. In this article we call these assets houses.

The specific aim of this article is to put together the building blocks needed to study the variations in asset prices. An important distinguishing feature is that we do not consider the stock of these assets to be productive capital. Instead, we model these as Lucas trees, assume that their supply is fixed, and use our

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model economy to determine their prices and not their quantities. As we will see, the ability of our model economy to replicate the main macroeconomic facts is comparable to that of the standard growth model.

In the literature there are several papers related to ours. Gruber and Martin (2006) study an economy with consumption goods that are durable and illiquid. They show that the decision rules display areas of inaction, and that financial wealth displays more dispersion than wealth held in the form of consumer durables. They compare various model economies that differ only in their transaction costs and they find that higher transaction costs and higher relative down-payments result in higher savings.

Martin (2005) uses a representative household model to study the implications of changes in demographics and other characteristics for asset prices, in particular housing prices, and interest rates. Ortalo-Magne and Rady (2006) is another interesting paper that explores the chain effects that may arise from variations in the prices of houses of different sizes. These chain effects are due to the amplification effects of capital gains—small changes in the price of small houses can induce large increases in the equity of their highly leveraged owners, and this may lead them to switch to larger houses, and to start a new cycle of price increases.

One of the objectives of our research project is to study the quantitative relevance of this channel, if any. In this context, Nakajima (2005) studies the response of of housing prices to an increase in the volatility of individual earnings, and he finds that the level of housing prices is quite responsive to purely second moment variations that are completely unrelated to changes in the levels of economic activity. Davis and Heathcote (2005) are interested in the business cycle properties of housing construction, but they worry about housing quantities not prices. Díaz-Giménez and Puch (1998) document how the behavior of model economies relates to the down-payment requirements. Chambers, Garriga, and Schlagenhauf (2007) connect the increase in housing ownership to reductions in the down payment. Finally, Díaz and Luengo-Prado (2006) study the determinants of housing tenure choice. There are other papers that also work on housing prices, but their notion of houses lacks some of the features that we think are fundamental to capture their essence, namely, that their cost is large relative to their buyers' incomes, that they are costly to buy and sell, and that they provide advantages to their owners that make renting a poor substitute of ownership. Our model economy includes an additional feature, that we consider to be novel: because we have houses of different sizes and houses that are in fixed supply, perhaps our model can be interpreted to be more a model of housing lots rather than a model of housing structures.

Our work is most closely related to Gruber and Martin (2006) and Díaz and Luengo-Prado (2006), but our calibration results are different. For example in our model economy we have a sizable group of people who do not own any houses;

the Gini Index of the wealth distribution in our model economy is closer to the data than the one in Gruber and Martin's economy but is further form the data than the one in Díaz and Luengo-Prado's. Finally, whereas in our model economy there is too much indebtedness, in the model economies described in these two papers there is too little.

So what is our value added? First, we are laying the blocks of a structure that can be used to study the variations in housing prices assuming that the number of houses, or perhaps lots, is given and, what is more important, that the sizes of this houses are also limited and given. Second, our model economy can be used to study the dynamics of the purchases and upgrades of houses. In this respect we provide information of the effective down payments on houses for first-time and sequential home buyers. Third, our structure can be extended to include aggregate uncertainty and we provide some information how to do this. These features are novel in the literatures and they represent sizable theoretical and computational challenges.

2. The Stationary Model Economy

The model economy analyzed in this article is a modified version of a Lucas tree model economy with uninsured idiosyncratic risk. In the version that we describe here we abstract from aggregate uncertainty, although it lurks at the end of the tunnel. Unless we say otherwise, we are only considering its stationary equilibrium. The key features of our model economy are the following: (i) it includes a large number of households who die exponentially, who have identical preferences and who face an uninsured, household-specific shock to their endowments of efficiency labor units; and (ii) there are three type of assets in fixed supply in the economy, a standard divisible asset which provides a time-invariant dividend, that we call a bond, and two indivisible assets that provide their holders with a utility shift, and that we call flats or houses. We refer to flats or houses generically as dwellings. Whereas trades in bonds are costless, trades in dwellings are expensive, and each household is limited to owning at most one dwelling.

Preferences and shocks Households enter and exit the economy exponentially with probability π. Their preferences are given by $u_d(c)$ where d indicates the type of dwelling that the household owns. A household may own no dwellings, it may own a flat, or it may own a house, that is, $d \in \{0, f, h\}$. Houses are better than flats, and flats are better than owning nothing, that is $u_h(c) > u_f(c) > u_0(c)$.

Individuals are heterogeneous in their earnings ability, which is uncertain. We consider three different types of earnings groups, or social classes, e, that allow us to replicate the observed earnings inequality. Individuals may change their earnings group following to a Markov process. Moreover, each individual also faces another uninsurable shock to earnings, ε. These earnings shocks are drawn from a distribution $F(\varepsilon, e)$ with $e \sim \Gamma_{ee'}$ and

$$F(\varepsilon, e) = \left[\frac{\varepsilon - \underline{\varepsilon}(e)}{\bar{\varepsilon}(e) - \underline{\varepsilon}(e)}\right]^{\chi}.$$

Markets. There are various markets in this economy. There is a spot market for labor that is essentially irrelevant, and there are three spot markets for bonds, b, flats, and houses prices $\{p_b, p_f, p_h\}$, respectively. There is also an annuity market so that no assets disappear. This means that the assets owned by an individual who exits the economy are shared proportionally by all the owners of that particular asset.

Individuals are subject to borrowing constraints and they can use their dwellings as collateral. Specifically, they can borrow a fraction $1 - \alpha$ of the value of their dwelling. Dwellings are traded at a cost that is born by the buyer and that we write as $\phi(d, d') = p_{d'}(1+\delta)$ if $d = 0$ and $\phi(d, d') = p_{d'}(1+\delta) - p_d$ otherwise, where $\delta \geq 0$.

Household's problem. To write the problem of household in a convenient way given that it is non concave we use two different functions. Function $V_{e,d}(b)$ denotes the value function of an household that belongs to an earnings class e, owns a dwelling d and financial savings b, after the realization of the earning class shock, e, and before the realization of the earnings shock ε. Function $W_{e,d}(a)$ is the value function of a household who belongs to earnings class e, owns dwelling d and has cash-in-hand a. Consequently the relationship between functions V and W is

$$V_{e,d}(b) = \int_{\varepsilon} W_{e,d}[(p_b + r)b + \varepsilon] F(d\varepsilon, e).$$

Furthermore, let $W_{e,d}^{d'}(a)$ be the maximal utility of an household that belongs to earnings class e, starts the period owning dwelling d, ends the period owning dwelling d' and has cash-in-hand a. Then

$$W_{e,d}(a) = \max_{d'} \left\{W_{e,d}^{d}(a)\right\}.$$

We can now write the problem of an household conditional to not changing its dwelling as

$$\begin{aligned} W_{e,d}^{d}(a) = \max_{b} \quad & u_d(c) + \pi\beta E\left[V_{e',d}(b)/e\right], \\ \text{subj. to:} \quad & c + p_b b = a, \end{aligned}$$

whereas that of an household that changes its dwelling is

$$W^{d'}_{e,d}(a) = \max_b u_{d'}(c) + \pi\beta E\{V_{e',d'}(b)/e\},$$

$$\text{subject to:} \qquad c + p_b b - \phi(d, d') = a.$$

Stationary equilibrium. To describe this economy at a point in time we need a distribution of households, x, defined over dwellings, assets, and earnings shocks. A stationary equilibrium is a specific distribution of households, x^*, together with a set of asset prices $\{p_b^*, p_f^*, p_h^*\}$ such that, when the distribution of households is $x*$ and the individuals face a time invariant vector of prices, $\{p_b^*, p_f^*, p_h^*\}$, their choices (i) imply a distribution of households next period that is again x^*, and (ii) the vector of prices clears the assets markets, that is

$$\int_{e,d,b} b dx = 1, \quad \int_{e,f,b} dx = \mu_f, \quad \int_{e,h,b} dx = \mu_h.$$

3. Mapping the Model to the Data

To specify the model economy we must choose some functional forms and parameters. Some details of the specification of the model are independent of the equilibrium and can be set beforehand. The equilibrium allocation depends on the parameter choices and our aim is to choose the values of the parameters so that some statistics of the model economy have certain desired values, typically the same values that their counterpart variables in the data. The process of parameter selection can be referred either as calibration or as a method of moments estimator.

3.1. Description of Parameters

Parameters that can be set independently. The parameters that can be determined independently include the population turnover features, 1.5% per year; the average duration of adult life, which we choose to be 67 years (this choice implies that $\pi = 0.985$); some features of the financial system such as a 1.0% mortgage premium (this is the difference between the borrowing rate and the lending rate), a 20.0% down payment and a 10.0% cost when buying a dwelling (this cost stands in for real estate commissions, taxes, and the time and hassle required to purchase a dwelling).

Preferences (3). Preferences are time-separable with a discount rate β and a standard coefficient of relative risk aversion $\sigma = 2$ (one of those parameters that is hard to pin down) and two utility shifters $\gamma^d = \gamma^f, \gamma^h$.

$$u_d(c) = \frac{c^{1-\sigma}}{1-\sigma}\gamma^d.$$

Earnings Shocks (11). We choose an earnings process with three earnings classes and within each earnings class there is an interval of earnings with the continuous density $F(\varepsilon, e)$. This gives us five parameters for the intervals of earnings (one of them is a normalization condition), four possible parameters of the transition matrix $\Gamma_{e,e'}$ (because we assume zero the probabilities of going from the top group to the bottom group and vice-versa), and the additional parameter χ that allows us to vary the mean to median ratio within each earnings class. To achieve a life cycle earnings profile where individuals increase their earnings on average over time, we assume that every individual enters the economy being poor. In model economies without housing estimating these parameters is complicated (see, e.g., Castañeda, Díaz-Giménez, and Ríos-Rull 2003; and Díaz, Pijoan-Mas, and Ríos-Rull 2003). In this article we use a process that is similar to the one used in Díaz, Pijoan-Mas, and Ríos-Rull (2003).

Asset parameters (3). Although we normalize the size of the bond to be one, the size of its dividend d has to be specified. We must also determine the numbers of flats and houses relative to the population, μ^f and μ^h.

3.2. Description of Targets/Moments to Match

1. We target a labor share out of income of 0.84. Note that the absence of depreciation in our model makes the labor share larger than usual.
2. Average earnings of individuals aged 60–31 to those aged 30–20: 1.82.
3. Financial asset wealth to income: 2.18.
4. Owner occupied housing wealth times relative to income: 2.3.
5. Fraction of households that own a house: 0.35.
6. Fraction of people who own a flat: 0.30
7. Relative prices of houses and flats p_h/p_f: 2.0.
8. Risk-free interest rate: 5%.
9. Down payment for first-time dwelling buyers: 16.3%.
10. Down-payment for repeated dwelling buyers: 26.5%.
11. Ratio of mortgage debt to income: 34%.
12. Fraction of people with debt in the model economy (that is those who have negative financial assets is 40%.
13. Average ratio of financial debt to housing value: 50%. This number has been increasing in the last few years and our target is appropriate for the early 1990s.

14. General properties of the Lorenz curve of earnings.
15. General properties of the Lorenz curve of assets.

3.3. Performance of the Model Economy

Our model economy does quite well in matching our targets despite having 17 parameters: 12 used to pin down specific targets, and 5 used to determine the general properties of the Lorenz curves. Moreover, of those 17 parameters, four play small roles (the three parameters that that define the bounds of the earning classes and the parameter that governs the curvature of the density of earnings within each earning class). The actual values of the model economy statistics relative to our targets are 18.0% for the down payment required from first-time buyers of dwellings and 27.9% for repeated buyers. The ratio of mortgages to income is 33.2%. Also the actual fraction of people with debt is 41.6% and the the average ratio of debt to housing value is 56.5%. The interest rate in the economy is a little bit too high, 7%.

In Tables 1 and 2 we report selected statistics from the Lorenz curves for earnings and for wealth in both our model economy and in the data (as reported by Budría et al. 2002). We think that our model economy is close enough to the data given the small number of degrees of freedom at our disposal.

4. Conclusion and a Preview of Aggregate Uncertainty

In this paper we have described the main building blocks needed to study housing prices in model economies where houses are actively traded, and which models

TABLE 1. The Distribution of earnings: model and data (1998 Survey of Consumer Finance).

	Quintiles					
	1st	2nd	3rd	4th	5th	Gini
Model earnings share	3.5	5.0	7.7	11.1	72.3	.65
Data earnings share	−0.2	4.0	13.0	22.9	60.2	.61

TABLE 2. Total, financial, and housing wealth distribution.

		Quintiles					
		1st	2nd	3rd	4th	5th	Gini
Total	Model	0.24	1.30	2.27	9.92	86.27	.82
	US Eco	−0.29	1.35	5.14	12.37	81.43	.80
Finan	Model	−22.34	−17.43	−1.39	2.33	138.83	1.568
	US Eco	−9.53	−1.07	0.36	5.84	104.39	0.953
Housing	Model	0.00	5.69	20.92	31.56	41.84	0.457
	US Eco	0.22	2.92	12.78	22.45	61.63	0.626

explicitly the main features of the housing market in modern economies. The next step in this research project is to study versions of this model economy that include aggregate uncertainty on earnings, interest rates, mortgage premia, demographics and so on. The problem is that the extended state space includes the distribution of households across financial wealth, dwellings and earnings classes. To get around this problem, we use the approach pioneered by Krusell and Smith (1998). This allows us to use the aggregate shocks and the current market clearing prices as the only state variables. This approach seems to be working and we expect to be able to report the stochastic properties of housing and equity prices sometime in the near future.

Appendix A: Parameter Values

Preferences. The preference parameter is $\beta = 0.892127$, and the utility modifiers are $\gamma^h = 0.19215$ and $\gamma^f = 0.0637$.

Earnings process. We chose three earnings classes with earnings in the intervals $\{[0.5, 2.0], [1.5, 12, 5], [41.9, 51.2]\}$ respectively, and with transition matrix

$$\Gamma_{ee'} = \begin{bmatrix} 0.992 & 0.008 & 0.000 \\ 0.009 & 0.980 & 0.011 \\ 0.000 & 0.083 & 0.917 \end{bmatrix}$$

The value of the parameter χ is 0.5.

Appendix B: Computational Procedures

The problem that the households solve is difficult because it is not concave and first order conditions cannot be used to solve the model. Moreover, not only do we have to solve the problem of the household and compute the stationary distribution for given parameters and prices, but we also have to do it many times to ensure that both the economy is in equilibrium (markets clear) and that the value of the model statistics are close to our targets.

There are three states in the household's problem: (i) financial assets, (ii) dwelling position, and (iii) earnings class. There are two decisions: savings (a continuous choice) and dwelling (that can take three values house, flat, or nothing). This combination implies that in general the value function W is not concave which means that the first-order conditions are not sufficient to find the optimum. We tried to get around this problem by using continuous earnings realizations and taking expectations over the following period earnings (and thus defining function V) hoping that the kinks would be smoothed out. Although this approach works very often, it does not work always (the earnings cannot be ensure to be spread enough so that the integration enforces concavity) which causes serious problems when the value function has to be solved many times as is the case when we are estimating parameters and even more in stochastic environments. Consequently we opted for the discretization of the state space, that is computationally very costly but also very robust. Still, we maintained the continuous support of earnings because the distribution of households and the decision rules oh households become smoother which makes for an easier estimation process. We use 700 points in the grid for the financial asset for function V so there are 6,300 possible individual states. The points in the grid are not equally spaced (they are closer for low values of wealth) and they are located in different locations for each dwelling. We then use successive approximations to find the solution to the household problem. Not that within each iteration the function W has to be constructed, and the decision rules found. The distribution of households over assets and earnings class is a now a vector and can be easily stored. We have done our work in a powerful Beowulf cluster with 26 processors.

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