ECON 709 - PS 4

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1. Suppose that another observation X_{n+1} becomes available. Show that

(a) $\bar{X}_{n+1} = (n\bar{X}_n + X_{n+1})/(n+1)$

(b) $s^2_{n+1} = ((n-1)s^2_n + (n/(n+1))(X_{n+1} - \bar{X}_n)^2)/n$

2. For some integer k, set $\mu_k = E(X^k)$. Construct an unbiased estiamtor $\hat{\mu}_k$ for μ_k, and show its unbiasedness.

3. Problem 3

Consider the central moment $m_k = E((X-\mu)^k)$. Construct an estimator $\hat{m}_k$ for m_k without assuming a known μ. In general, do you expect $\hat{m}_k$ to be biased or unbiased?

4. Problem 4

Calculate the variance of $\hat{\mu}_k$ that you proposed above, and call it $Var(\hat{\mu_k})$.

5. Problem 6

Show that $E(s_n) \leq \sigma$. (Hint: Use Jensen's inequality, CB Theorem 4.7.7).

6. Problem 8

Show algebraically that $\hat{\sigma}^2 = n^{-1}\sum_{i=1}^n (X_i - \mu)^2 - (\bar{X}_n - \mu)^2$.

7. Problem 9

Find the covariance of $\hat{sigma}^2$ and $\bar{X}_n$. Under what condition is this zero?

8. Problem 10

Suppose that X_i are i.n.i.d (independent but not necessarily identically distributed) with $E(X_i) = \mu_i$ and $Var(X_i) = \sigma_i^2$.

(a) Find $E(\bar{X}_n)$.

(b) Find $Var(\bar{X}_n)$.

9. Problem 12

Show that if $Q \sim \chi^2_r$, then $E(Q) = r$ and $Var(Q) = 2r$. (Hint: use the representation: $Q = \sum_{i=1}^n X_i^2$ with $X_i \sim N(0,1)$).

10. Problem 14

Suppose that $X_i \sim N(\mu_X, \sigma_X^2) : i = 1, ..., n_1$ and $Y_i \sim N(\mu_Y, \sigma_Y^2) : i = 1, ..., n_2$ are mutually independent. Set $\bar{X}_n = n_1^{-1}\sum_{i=1}^{n_1} X_i$ and $\bar{Y}_n = n_2^{-1}\sum_{i=1}^{n_2} Y_i$.

(a) Find $E(\bar{X}_n - \bar{Y}_n)$.

*I worked on this problem set with a study group of Michael Nattinger, Andrew Smith, and Ryan Mather. I also discussed problems with Emily Case, Sarah Bass, and Danny Edgel.

(b) Find $Var(\bar{X}_n - \bar{Y}_n)$.

(c) Find the distribution of $\bar{X}_n - \bar{Y}_n$.