“How Costly is External Financing? Evidence from a Structural Estimation”
Christopher Hennessy and Toni Whited (2007)
Journal of Finance

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Motivation

- Modigliani-Miller Irrelevance Theorem (1958, 1963)

  In frictionless world, financing decisions like capital structure (debt vs. equity), payout policy, cash holding, etc. do not matter.

- Why? No arbitrage

- MM assumes there are no financial friction:
  - Perfect and complete capital markets
  - No taxes
  - Bankruptcy is not costly
  - Capital structure does not affect investment policy or cash flows
  - Symmetric information

Hennessy and Whited (2007)

- HW (2007) formulate a dynamic structural model of optimal financial and investment policy for a firm facing
  - Corporate and personal taxes
  - Bankruptcy costs
  - Costs to issue external equity
- HW (2007) estimate parameters describing production technology and financial frictions using simulated method of moments (SMM)

Environment - Production and Debt

- Firm produces with k capital
- Productivity follows discretized AR(1) process in logs:

$$\ln z' = \rho \ln z + \sigma_\varepsilon \varepsilon$$

  where $\varepsilon \sim N(0,1)$. Tauchen discretization $\implies Q(z,z')$ transition probability and finite min/max
- Operating profits are zk^α where $\alpha \in (0,1)$
- Firm also has b net debt
  - $b > 0$ is one-period defaultable debt with interest rate $\tilde{r}$ that depends on k, b, and z (not contingent on z')
  - $b \leq 0$ is cash that returns risk-free rate r
- Firm defaults on debt if continuation value is negative

Environment - Taxes and Equity Issuance

- Personal tax rate $\tau_i \implies$ firms discounts using $\frac{1}{1+r(1-\tau_i)}$
- Corporate taxable income is operating profits net of depreciation and interest:

$$y \equiv zk^{\alpha} - \delta k - \tilde{r}(k, b, z^{-})b$$

- Corporate tax schedule has "kink" around zero

$$T^{C}(x) \equiv \begin{cases} \tau_c^{+} x, & \text{if } x > 0 \\ \tau_c^{-} x, & \text{if } x \leq 0 \end{cases}$$

- Shareholder tax liability on dividend:

$$T^{d}(X) = \int_0^X \tau_d(x)dx \text{ where } \tau_d(x) \equiv \bar{\tau}_d * [1 - e^{-\phi x}]$$

- Firm bears cost for external equity issuance:

$$\Lambda(x) \equiv \begin{cases} \lambda_0 + \lambda_1 x + \lambda_2 x^2, & \text{if } x > 0 \\ 0, & \text{if } x \leq 0 \end{cases}$$

"Naive" Way to Write Firm Value Function

$$V(k,b,z,z^-) = \max_{(k',b')} \Bigg\{ \underbrace{w+b'-k'}_{\text{cash dividend } (+) \text{ or equity issuance } (-)} - \underbrace{T^d(w+b'-k')}_{\text{taxes on cash dividend}} - \underbrace{\Lambda(-(w+b'-k'))}_{\text{equity issuance cost}}$$

$$+ \frac{1}{1+r(1-\tau_i)} E\Big[\underbrace{\max\{V(k',b',z',z),0\}}_{\text{if } V(\cdot) \text{ is } (-) \implies \text{ default}} \Big] \Bigg\}$$

where

$$\underbrace{y}_{\text{taxable corporate income}} \equiv \underbrace{zk^\alpha}_{\text{operating profits}} - \underbrace{\delta k}_{\text{depreciation}} - \underbrace{\tilde{r}(k,b,z^-)b}_{\text{interest on debt}}$$

$$\underbrace{T^C(x)}_{\text{corporate income tax bill}} \equiv \begin{cases} \tau_c^+ x, & \text{if } x > 0 \\ \tau_c^- x, & \text{if } x \le 0 \end{cases}$$

$$\underbrace{w}_{\text{realized net worth}} \equiv \underbrace{y - T^C(y)}_{\text{after-tax corporate income}} + \underbrace{k}_{\text{capital}} - \underbrace{b}_{\text{debt principal}}$$

$$\underbrace{T^d(x)}_{\text{taxes on cash dividend}} \equiv \begin{cases} \frac{\bar{\tau}_d}{\phi}(\phi x + e^{-\phi x} - 1), & \text{if } x > 0 \\ 0, & \text{if } x \le 0 \end{cases}$$

$$\underbrace{\Lambda(x)}_{\text{equity issuance cost}} \equiv \begin{cases} \lambda_0 + \lambda_1 x + \lambda_2 x^2, & \text{if } x > 0 \\ 0, & \text{if } x \le 0 \end{cases}$$

Smarter Way to Write Firm Value Function

$$V(w,z) = \max_{(k',b')} \Bigg\{ \underbrace{w + b' - k'}_{\text{cash dividend } (+) \text{ or equity issuance } (-)} - \underbrace{T^d(w + b' - k')}_{\text{taxes on cash dividend}} - \underbrace{\Lambda(-(w + b' - k'))}_{\text{equity issuance cost}}$$

$$+ \frac{1}{1 + r(1-\tau_i)} E\Big[\underbrace{\max\{V(w',z'),0\}}_{\text{if } V \text{ is } (-) \text{ can default}} \Big] \Bigg\}$$

where

$$\underbrace{y'}_{\text{taxable corporate income}} \equiv \underbrace{z'(k')^{\alpha}}_{\text{operating profits}} - \underbrace{\delta k'}_{\text{depreciation}} - \underbrace{\tilde{r}(k',b',z)b'}_{\text{interest on debt}}$$

$$\underbrace{T^C(x)}_{\text{corporate income tax bill}} \equiv \begin{cases} \tau_c^+ x, & \text{if } x > 0 \\ \tau_c^- x, & \text{if } x \le 0 \end{cases}$$

$$\underbrace{w'}_{\text{realized net worth}} \equiv \underbrace{y' - T^C(y')}_{\text{after-tax corporate income}} + \underbrace{k'}_{\text{capital}} - \underbrace{b'}_{\text{debt principal}}$$

$$\underbrace{T^d(x)}_{\text{taxes on cash dividend}} \equiv \begin{cases} \frac{\bar{\tau}_d}{\phi}(\phi x + e^{-\phi x} - 1), & \text{if } x > 0 \\ 0, & \text{if } x \le 0 \end{cases}$$

$$\underbrace{\Lambda(x)}_{\text{equity issuance cost}} \equiv \begin{cases} \lambda_0 + \lambda_1 x + \lambda_2 x^2, & \text{if } x > 0 \\ 0, & \text{if } x \le 0 \end{cases}$$

Default and Interest Rates

- Firm defaults on debt if w below z'-specific threshold:

$$\underline{w}(z') = V^{-1}(0, z') < 0 \implies z_d(k', b', z) \text{ threshold}$$

- If firm defaults, outside investor gets recovery value

$$R(k', z') = \underbrace{(1-\xi)(1-\delta)k'}_{\text{depreciated capital}} + \underbrace{z'(k')^{\alpha}}_{\text{operating profit}} - \underbrace{T_c(z'(k')^{\alpha} - \delta k')}_{\text{corporate tax bill}} - \underbrace{\underline{w}(z')}_{\text{going-concern}}$$

- Interest rates on debt determined by zero-profit condition for outside investor

$$\underbrace{(1 + r(1-\tau_i))b'}_{\text{risk-free investment}} = \underbrace{(1 + (1-\tau_i)\tilde{r}(k', b', z))b' \int_{z_d(k',b',z)}^{\bar{z}} Q(z, dz')}_{\text{return on non-defaulted debt}}$$

$$+ \underbrace{\int_{\underline{z}}^{z_d(k',b',z)} R(k', z') Q(z, dz')}_{\text{return on defaulted debt}}$$

Computation

- No closed form solution $\implies$ solve numerically
- Computational strategy:
  - Guess $\tilde{r}(k', b', z) = r$
  - Solve V with value function iteration
  - Compute $z_d(k', b', z)$
  - Update $\tilde{r}(k', b', z)$ using zero-profit condition
  - Repeat until convergence

Some things to keep in mind

1. Discount bond prices are bounded whereas interest rates are not:

$$V(w,z) = \max_{(k',b')} \Bigg\{ \underbrace{w + b'q(k',b',z) - k'}_{\text{dividend if } (+) \text{ or equity issuance if } (-)} - \underbrace{T^d(w + b'q(k',b',z) - k')}_{\text{taxes on dividend}}$$

$$- \underbrace{\Lambda(-(w + b'q(k',b',z) - k'))}_{\text{equity issuance cost}} + \frac{1}{1 + r(1-\tau_i)} E\Big[\underbrace{\max\{V(w',z'),0\}}_{\text{if } V \text{ is } (-)\text{, default}} |z \Big] \Bigg\}$$

$$\text{where} \quad \underbrace{y'}_{\text{taxable income}} \equiv \underbrace{z'(k')^\alpha}_{\text{operating profits}} - \underbrace{\delta k'}_{\text{depreciation}} - \underbrace{(1 - q(k',b',z))b'}_{\text{interest on debt}}$$

$$\underbrace{w'}_{\text{realized net worth}} \equiv \underbrace{y' - T^C(y')}_{\text{after-tax corporate income}} + \underbrace{k'}_{\text{capital}} - \underbrace{q(k',b',z)b'}_{\text{debt principal}}$$

2. What is the w grid?
   - HW (2007) are specific about z, b, k grids, but quiet about the w grid
   - My solution: Loop over z, b, k grids and solve w for $q = 0$ and $q = 1/(1+r)$, then linear interpolate between min and max

3. No contraction mapping for bond prices $\implies$ update q slowly

Parameters

- External Parameters
- Estimated Parameters