ECON 712 - 2015 Midterm

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1. State and solve planner's problem:

The planner maximizes aggregate utility weighing each generation equally subject to resource feasibility and nonnegativity constraints.

$$\begin{aligned}
\max_{c_t^t, c_t^{t-1}, \ell_t^t \forall t \in \mathbb{N}} \quad & \ln(c_1^0) + \sum_{t=1}^{\infty} c_t^t - \frac{\gamma}{2}(\ell_t^t)^2 + \ln(c_t^{t-1}) \\
\text{s.t.} \quad & c_t^{t-1} + (1+n)c_t^t = A(1+n)\ell_t^t \\
& \ell_t^t \in [0,1] \\
& c_t^t \geq 0 \\
& c_t^{t-1} \geq 0
\end{aligned}$$

We can drop the nonnegativity constraint on c_t^{t-1} because it is logged and $\ln(0) = -\infty$. We can also drop the nonnegativity constraint on ℓ_t^t because that implies that output is zero, so c_t^{t-1} would be zero. Thus the legrangian is:

$$\mathcal{L} = \ln(c_1^0) + \sum_{t=1}^{\infty} c_t^t - \frac{\gamma}{2}(\ell_t^t)^2 + \ln(c_t^{t-1}) + \lambda_t^1(A(1+n)\ell_t^t - c_t^{t-1} - (1+n)c_t^t)$$

First order condition for c_t^t:

$$1 - \lambda_t^1(1+n) = 0 \implies \lambda_t^1 = \frac{1}{1+n}$$

First order condition for c_t^{t-1}:

$$\frac{1}{c_t^{t-1}} - \lambda_t^1 = 0 \implies c_t^{t-1} = 1+n$$

First order condition for ℓ_t^t:

$$-\gamma\ell_t^t + \lambda_t^1 A(1+n) = 0 \implies -\gamma\ell_t^t + \frac{1}{1+n}A(1+n) = 0 \implies \ell_t^t = \frac{A}{\gamma}$$

This implies

$$(1+n) + (1+n)c_t^t = A(1+n)\frac{A}{\gamma} \implies 1 + c_t^t = A\frac{A}{\gamma} \implies c_t^t = \frac{A^2}{\gamma} - 1$$