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711 Final Exam

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① a $u(t) = -t$

Passing through A, with x people taking this route,

$$u(c(x)+1) = -x^6 - 1$$

If y fraction take route through B, the payoff is

$$u(1+c(y)) = -1-y^6$$

This game would be supermodular, if the more people took the route through A, the better it would be for a given player. I.e. Strategic complementaries. The game would be submodular, if the more people who took route A, the better it would be for a given player to take the route through B. Clearly, the game is submodular, because the more people take a given route, the longer it takes people to pass through the short narrow road segment.

Submodular

(b) Since the game is submodular, whichever route is faster will attract more drivers. This makes it slower. Thus, ~~the~~ in equilibrium, 1/2 the drivers take the route through A and 1/2 the drivers take the route through B.

Assume for sake of a contradiction that this is not the equilibrium. Then a driver on the more popular route could switch routes and get to T faster. Thus, this would be a profitable deviation. ~~Th~~ This allocation could not be an equilibrium $\Rightarrow \Leftarrow$.

Thus, the equilibrium commuting time is

$$1 + (1/2)^6 = \boxed{\frac{65}{64}}$$

(c) Based on similar logic to (b), ~~equi~~ for an allocation of drivers on each route to be an equilibrium, ~~all~~ drivers must be indifferent between each route. The payoffs for each route are below:

$$S \to A \to T : \quad x^6 + 1 \qquad (*)$$
$$S \to B \to T : \quad 1 + y^6 \qquad (**)$$
$$S \to A \to B \to T : \quad x^6 + y^6 \qquad (***)$$

$$(*) \& (**) \Rightarrow x^6 + 1 = 1 + y^6 \Rightarrow x = y \qquad (****)$$

$$(****) \& (*) \& (***) \Rightarrow 2x^6 = x^6 + 1$$
$$\Rightarrow x^6 = 1$$
$$\Rightarrow x = 1$$
$$\Rightarrow y = 1$$

Thus, all drivers take $S \to A \to B \to T$.

The new equilibrium commute time is

$$1^6 + 0 + 1^6 = \boxed{2.}$$

(d) No, the intuitively helpful action pushed the drivers all the long wide road to the short narrow roads. Thus, the avg commute time increased from $\frac{65}{64}$ to 2.

(2) (a) If both players are uninformed of the state, they will seek to maximize their expected payoff:

	L	M	R
T	1, 2/5	1, 3/10	1, 3/10
B	2, 2	0, 3/2	0, 3/2

L strictly dominates M & R.
By ISD, B strictly dominates T.

Thus, the Bayesian Nash Equilibrium is (B, L).

(2) (b) If the state is ω_1, ~~[illegible]~~

R strictly dominates L and M, By ISD, T strictly dominates B. So conditional on ω_1, the NE would be (T, R).

If the state is ω_2, M strictly dominates L and R. By ISD, T strictly dominates B. So conditional on ω_2, the NE would be (T, M)

Thus, the BNE is player 1 plays T and if the state is ω_1, player 2 plays R and if the state is ω_2, player 2 plays M.

(2) (c) If he is uninformed, player 2's expected payoff is 2. If he is informed his expected payoff is $\frac{1}{2}(3/5) + \frac{1}{2}(3/5) = 3/5$. No, player 2 does not gain from being informed because his expected payoff is lower.

(3) ~~[scribbled out]~~ The δ to support tit-for-tat.

Notice if both players play σ then the sequence of play will be
$(C,C), (C,C), (C,C), \ldots$

Thus let us consider a one-shot deviation from equilibrium play.

$$(1-\delta)\sum_{t=0}^{\infty}\delta^t(1) \geq (1-\delta)\left[5+\sum_{t=1}^{\infty}\delta^t(0)\right]$$

$$1 \geq 5-5\delta$$
$$\delta \geq 4/5$$

~~Thus~~ Let us consider a one-shot deviation from punishment play.

$$(1-\delta)\sum_{t=0}^{\infty}\delta^t(0) \geq (1-\delta)\left[(-4)+\sum_{t=1}^{\infty}\delta^t(1)\right]$$

$$0 \geq (1-\delta)\left[(-4)+\frac{\delta}{1-\delta}\right]$$

$$0 \geq -4+4\delta+\delta$$

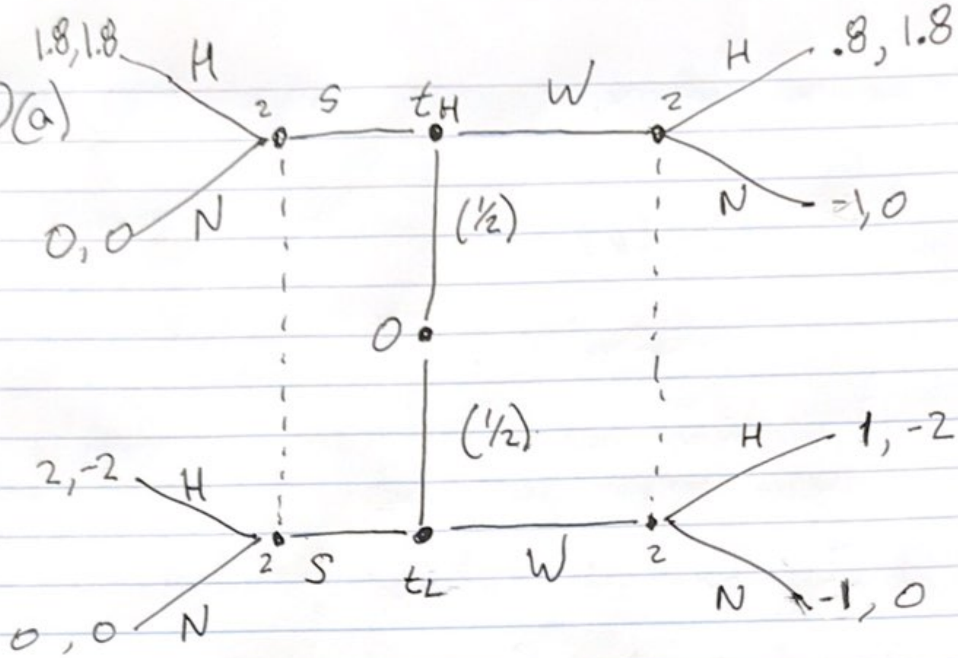
$$\frac{4}{5} \geq \delta$$

Thus, $\delta = \frac{4}{5}$ to support (σ,σ).

- If $\delta > \frac{4}{5}$, players are patient enough to support equilibrium play, but they are willing to play C to get back to $(C,C), \ldots$ ~~when~~ after D has been played.

- If $\delta < \frac{4}{5}$, the one-shot deviation from (C,C) is too tempting.

4 (a)



(9) (b) Notice that W is strictly dominated for t_L. Thus,

$$\sigma_{t_L}(W)=0 \qquad \sigma_{t_L}(S)=1$$

~~$\mu_2^W(t_H)=0$ $\mu_2^S(t_L)$~~

Now let us divide our analysis into the action of t_L & t_H ~~reprse~~ respectively.

$$(S, W) \to \mu_2^W(t_H)=1 \text{ and } \mu_2^S(t_L)=1$$

$$\Rightarrow \sigma_2^W(H)=1 \quad \text{and} \quad \sigma_2^S(N)=1.$$

t_H doesn't have incentive to deviate because that would lead to zero payoff.

Thus, this is a sequential equilibrium:

~~(W, S,~~

$$((\sigma_{t_L}(S)=1,\ \sigma_{t_H}(W)=1,\ \sigma_2^W(H)=1,$$
$$\sigma_2^S(N)=1),\ \mu_2^W(t_H)=\mu_2^S(t_L)=1).$$

④ (b) Con't

$-(S,S) \rightarrow \mu_2^S(t_H) = \mu_2^S(t_L) = \frac{1}{2}$

Thus expected payoff from H is $-.2$ for the firm.

So, $\sigma_2^S(N) = 1$.

Thus, $\sigma_{t_L}(S) = \sigma_{t_H}(S) = 1$.

The information set associated w/ W for the firm isn't hit.

Thus this is another sequential equilibrium:

$$\left(\left(\sigma_{t_L}(S) = 1,\ \sigma_{t_H}(S) = 1,\ \sigma_2^S(N) = 1 \right),\ \mu_2^S(t_H) = \mu_2^S(t_L) = \tfrac{1}{2} \right).$$

The second equilibrium is ruled out by the intuitive criterion because ~~[illegible]~~ if a candidate comes to the ~~[illegible]~~ interview well dress, it must be a t_H because it is known that W is dominated for t_L

(4)(C) ~~If~~ the option of hiring from within given the firm a payoff 3.6, it strictly dominates any of its payoff from choosing to interview candidate. the highest payoff associated with interviewing is 1.8. Thus, the firm would never interview and always hire from within.

~~Thus, the weak sequential equilibrium is any outcome~~

$$\{(P, \sigma_2^W(H), \sigma_2^S(H)), \sigma_{t_H}(W), \sigma_{t_H}(W)), \mu_2^S(t_H), \mu_2^W(t_H)\}$$

where $\sigma_2^W(H) \in \{1,0\}$
$\sigma_2^S(H) \in \{1,0\}$
$\sigma_{t_H}(W) \in \{1,0\}$
$\sigma_{t_H}(W) \in \{1,0\}$
$\mu_2^S(t_H) \in \{1,0\}$
$\mu_2^W(t$

~~The firm hiring~~

Thus, the weak sequential equilibria requires the firm from hiring within, but there can be anything in the subgame where they choose to interview because this subgame is never reached.

(4)(d) The equilibrium of the firm ~~[illegible]~~ hiring from within ~~[illegible]~~ and the subgames outlined in (b) are sequential equilibria.

(5) (a)

	X	Y	~~Z~~
A	12, −12	−2, 2	~~−1, −4~~
B	−2, 2	1, −1	~~−1, −2~~
~~C~~	~~−3, −1~~	~~−3, −1~~	~~$-\frac{1}{2}$, −2~~

Y strictly dominates Z.
B strictly dominates C.

There are no pure-strategy NE.

$$12X + (-2)Y = (-2)X + (1)Y$$
$$14X = 3Y$$
$$X = \frac{3}{14}Y$$
$$X = \frac{3}{14}(1-X)$$
$$X = \frac{3}{14} - \frac{3}{14}X$$
$$\frac{17}{14}X = \frac{3}{14}$$
$$X = \frac{3}{17} \quad Y = \frac{14}{17}$$

$$(-12)A + 2B = 2A + (-1)B$$
$$3B = 14A$$
$$A = \frac{3}{14}B$$
$$\Rightarrow A = \frac{3}{17} \text{ and } B = \frac{14}{17}$$

the NE is $\left(\frac{3}{17}A + \frac{14}{17}B, \frac{3}{17}X + \frac{14}{17}Y\right)$.

(5) (b) If 2 plays X, max payoff of 1 is 12
If 2 plays Y, max payoff of 1 is 1
If 2 plays Z, max payoff of 1 is $-1/2$.

$\Rightarrow \underline{v}_1^P = -1/2$

If 1 play A, max payoff of 2 is 2
If 1 play B, max payoff of 2 is 2
If 1 play C, max payoff of 2 is -1

$\Rightarrow \underline{v}_2^P = -1$

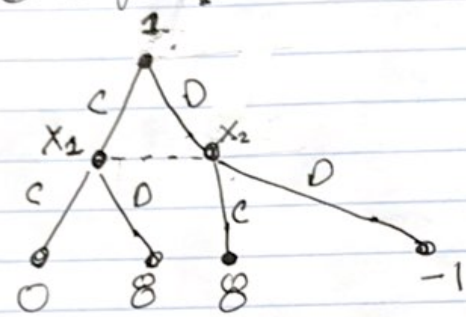
By the folk theorem, $v \in \{v \in F ; v_1 \geq \underline{v}_1, v_2 \geq \underline{v}_2 \ \forall i \in P\}$
is supportable with a high enough δ.

The expected payoff from the stage game is 8/17 for player 1 and $-8/17$ for player 2.

Notice that payoffs are negative if player 1 ~~plays~~ plays C or player 2 plays Z. Furthermore, the 2×2 game that survives ISD ~~is~~ is zero sum. Thus, it is impossible to ~~...~~ get payoffs that are strictly better for both players in a repeated game. If there are outcome w/ ~~1~~ 1 plays C and/or ~~2~~ plays Z both players are worse off. ~~...~~ Otherwise, one player is better off and the other is worse off.

(6) (a) $N = \{1\}$

$S = \{CC, CD, DC, DD\}$



6 (b) Call w/ probability α

$$\max_{\alpha} \ (0)\alpha^2 + \alpha(1-\alpha)8 + \alpha(1-\alpha)8 + (1-\alpha)^2(-1)$$

$$= \max_{\alpha} \ 16\alpha(1-\alpha) - (1-\alpha)^2$$

FOC: $0 = 18 - 34\alpha^*$

$$\alpha^* = \frac{18}{34} = \boxed{\frac{9}{17}}$$

6 (c) Picks call probability α and reoptimizes in the information set:

~~The probability you hit the first node.~~

α probability you hit the left node

$1-\alpha$ probability you hit the right node

$$\mu(x_1|\alpha) = \frac{\alpha}{\alpha + 1 - \alpha} = \alpha$$

$$\mu(x_2|\alpha) = \frac{1-\alpha}{\alpha + 1 - \alpha} = 1-\alpha$$

$$\max_{\beta} \ \alpha\left[(0)\beta + 8(1-\beta)\right] + (1-\alpha)\left[8\beta + (-1)(1-\beta)\right]$$

$$= \max_{\beta} \ \alpha 8(1-\beta) + (1-\alpha)\left[9\beta - 1\right]$$

$$= \max_{\beta} \ 8\alpha - 8\alpha\beta + 9\beta - 1 - 9\alpha\beta + \alpha$$

FOC $[\beta]$ $\quad -8\alpha + 9 - 9\alpha = 0$

$$9 = 17\alpha$$

$$\boxed{\frac{9}{17} = \alpha}$$

This probability is the same as in 6(b) because the Bayesian probability you're at the nodes in the same info set equals α.