ECON 717B: PS 1

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1 Part 1: Analytic Exercises

1. Returns to schoolings

   (a) ATE

   Marginal treatment effect is

   $$MTE(A) = Y_1(A) - Y_0(A) = 1 + 0.5A - A = 1 - 0.5A$$

   Average treatment effect is

   $$E[MTE(A)] = E[1 - 0.5A] = 1 - 0.5E[A] = 1 - 0.5 * 0.5 = 0.75$$

   (b) Fraction of treated population

   $$Pr\{D = 1\} = Pr\{-0.5 + A > 0\} = Pr\{A > 0.5\} = 0.5$$

   (c) Maximum and minimum treatment effect

   $$\max_{A \in [0,1]} MTE(A) = \max_{A \in [0,1]} [1 - 0.5A] = 1$$

   at $A = 0$.

   $$\min_{A \in [0,1]} MTE(A) = \min_{A \in [0,1]} [1 - 0.5A] = 0.5$$

   at $A = 1$.

   (d) $A \sim N(0, 1)$

   $$\sup_{A \in (-\infty,\infty)} MTE(A) = \sup_{A \in (-\infty,\infty)} [1 - 0.5A] = \infty$$

   as $A \to -\infty$.

   $$\inf_{A \in (-\infty,\infty)} MTE(A) = \inf_{A \in (-\infty,\infty)} [1 - 0.5A] = -\infty$$

   as $A \to \infty$.

   (e) ATET and ATEU

   $$ATET = E[MTE(A)|D = 1] = E[1-0.5A|A > 0.5] = 1-0.5E[A|A > 0.5] = 1-0.5*0.75 = 0.625$$

   $$ATEU = E[MTE(A)|D = 0] = E[1-0.5A|A < 0.5] = 1-0.5E[A|A < 0.5] = 1-0.5*0.25 = 0.875$$

(f) Why is ATEU > ATET?

ATEU > ATET because the marginal treatment effect is decreasing in A, but selection into treatment is increasing in A.

(g) OLS estimand

$$\beta(OLS) = E[Y|D=1] - E[Y|D=0] = E[1+0.5A|A>0.5] - E[A|A<0.5] = 1+0.5*0.75-0.25 = 1.125$$

(h) Why is OLS biased upward for ATE?

Because conditional independence fails due to selection effects. If treatment was random, then OLS would be unbiased.

2. Monotonicity

(a) Prove monotonicity holds.

For each observation i, define $V_{0,i} \equiv \delta_0 + U_{V,i}$ as the outcome without treatment and $V_{1,i} \equiv \delta_0 + \delta_1 + U_{V,i}$ as the outcome with treatment.

Case 1: $\delta_1 > 0 \implies \delta_0 + \delta_1 + U_{V,i} > \delta_0 + U_{V,i} \implies V_{1,i} > V_{0,i}$ for all i. Monotonicity holds.

Case 2: $\delta_1 < 0 \implies \delta_0 + \delta_1 + U_{V,i} < \delta_0 + U_{V,i} \implies V_{1,i} < V_{0,i}$ for all i. Monotonicity holds.

Case 3: $\delta_1 = 0 \implies \delta_0 + \delta_1 + U_{V,i} = \delta_0 + U_{V,i} \implies V_{1,i} = V_{0,i}$ for all i. Monotonicity holds.

(b) Define V such that monotonicity fails.

Consider heterogenous $\delta_{1,i} \in [-A, A]$:

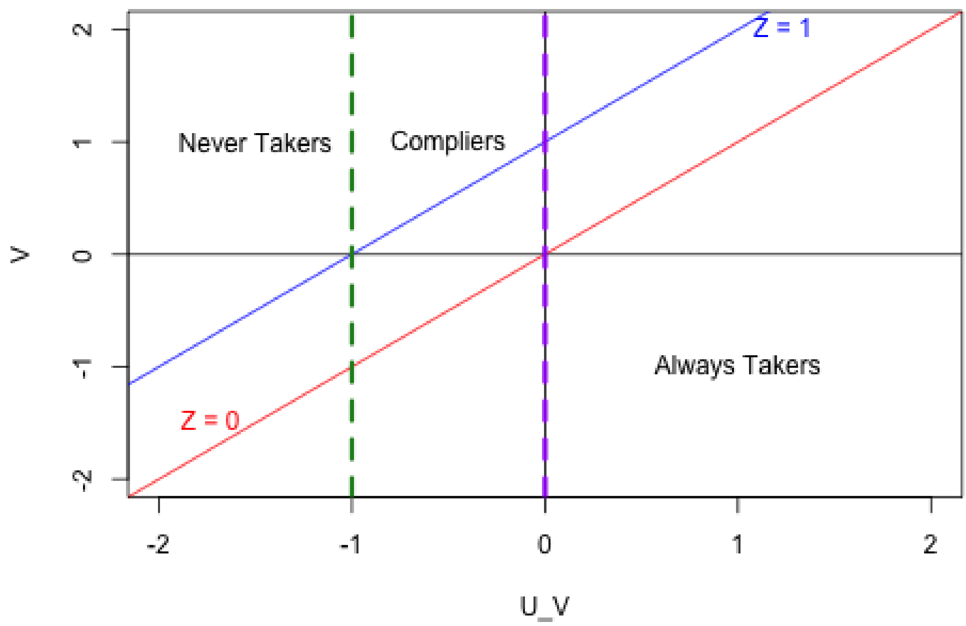
$$V_i = \delta_0 + \delta_{1,i} Z_i + U_{V,i}$$

Since $\delta_{1,i}$ can be positive or negative, defier will not choose the treatment even if they are exposed to the instrument.

3. Potential outcomes with uniform instrument

(a) Show range of always takers, compliers, defiers, and never takers.

Monotonicity holds, so there are no defiers. Always takers have $V > 0$ for both $Z = 0$ and $Z = 1$, so $U_V \in [0, 2]$. Compliers $V > 0$ for $Z = 1$, but $V < 0$ for $Z = 0$, so $U_V \in [-1, 0)$. Never takers have $V < 0$ for both $Z = 0$ and $Z = 1$, so $U_V \in [-2, -1)$. The figure below summarizes these ranges:



(b) Compute fraction of population in each group.
Using the uniform distribution, defiers are 0 percent, always takers are 50 percent, compliers are 25 percent, and never takers are 25 percent.

4. Two types
   (a) Compute ATE
   ...
   (b) Compute $Pr(D = 1|Z = 1)$ and $Pr(D = 1|Z = 0)$
   ...
   (c) Compute LATE
   ...

2 Monte Carlo Exercises

2.1 Question 1

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2.2 Question 2

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