ECON 712 - PS 1

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Here we are interested in how the stock market may react to a future FOMC announcement of an increase in short term interest rates. To do so, consider the following simple perfect foresight model of stock price dynamics given by the following equation

$$p_t = \frac{d + p_{t+1}}{(1+r)} \tag{1}$$

where p_t is the price of a share at the beginning of period t before constant dividend d is paid out, and r is the short term risk free interest rate. The left hand side of (1) is the cost of buying a share while the right hand side is the benefit of buying the share (the owner receives a dividend and capital gain or loss from the sale of the share). Assume $r > 0$.

1. Solve for the steady state stock price $p^* = p_t = p_{t+1}$.

$$\begin{aligned} p^* &= \frac{d + p^*}{(1+r)} \\ p^* + rp^* &= d + p^* \\ p^* &= d/r \end{aligned}$$

2. Assume the initial price, p_0, is given. Solve the closed form solution to the first order linear difference equation in (1). Explain how price evolves over time (i.e. if $p_0 > p^*, p_0 < p^*, p_0 = p^*$) using both a phase diagram (i.e. p_{t+1} against p_t) as well as a graph of p_t against time t. If the initial stock price is away from the steady state, does it converge or diverge from the steady state. Explain why.

Rewriting (1), as $p_{t+1} = f(p_t)$:

$$\begin{aligned} p_t &= \frac{d + p_{t+1}}{(1+r)} \\ \implies p_{t+1} &= (1+r)p_t - d \\ \implies p_{t+1} &= ap_t + b \text{ where } a = 1 + r \text{ and } b = -d. \end{aligned}$$

For complementary function, use $p_t^c = ca^t$ and use $p_t^p = p^*$ for the particular solution. Thus, the general solution is $p_t^g = p_t^c + p_t^p = ca^t + p^*$. Using $p_0^g = p_0$ as a boundary solution, $p_0 = ca^0 + p^* \implies c = p_0 - p^*$. Thus,

$$p_t = (p_0 - p^*)a^t + p^* \text{ where } a = 1 + r \text{ and } p^* = d/r.$$

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Because $|a| = |1 + r| > 1$, the price diverges unless $p_0 = p^*$. If $p_0 < p^*$, $p_t \to -\infty$ as $t \to \infty$ and, if $p_0 > p^*$, $p_t \to \infty$ as $t \to \infty$.