ECON 712 - PS 1

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Here we are interested in how the stock market may react to a future FOMC announcement of an increase in short term interest rates. To do so, consider the following simple perfect foresight model of stock price dynamics given by the following equation

$$p_t = \frac{d + p_{t+1}}{(1+r)} \tag{1}$$

where p_t is the price of a share at the beginning of period t before constant dividend d is paid out, and r is the short term risk free interest rate. The left hand side of (1) is the cost of buying a share while the right hand side is the benefit of buying the share (the owner receives a dividend and capital gain or loss from the sale of the share). Assume $r > 0$.

1. Solve for the steady state stock price $p^* = p_t = p_{t+1}$.

$$\begin{aligned} p^* &= \frac{d + p^*}{(1+r)} \\ p^* + rp^* &= d + p^* \\ p^* &= d/r \end{aligned}$$

2. Assume the initial price, p_0, is given. Solve the closed form solution to the first order linear difference equation in (1). Explain how price evolves over time (i.e. if $p_0 > p^*, p_0 < p^*, p_0 = p^*$) using both a phase diagram (i.e. p_{t+1} against p_t) as well as a graph of p_t against time t. If the initial stock price is away from the steady state, does it converge or diverge from the steady state. Explain why.

Rewriting (1), as $p_{t+1} = f(p_t)$:

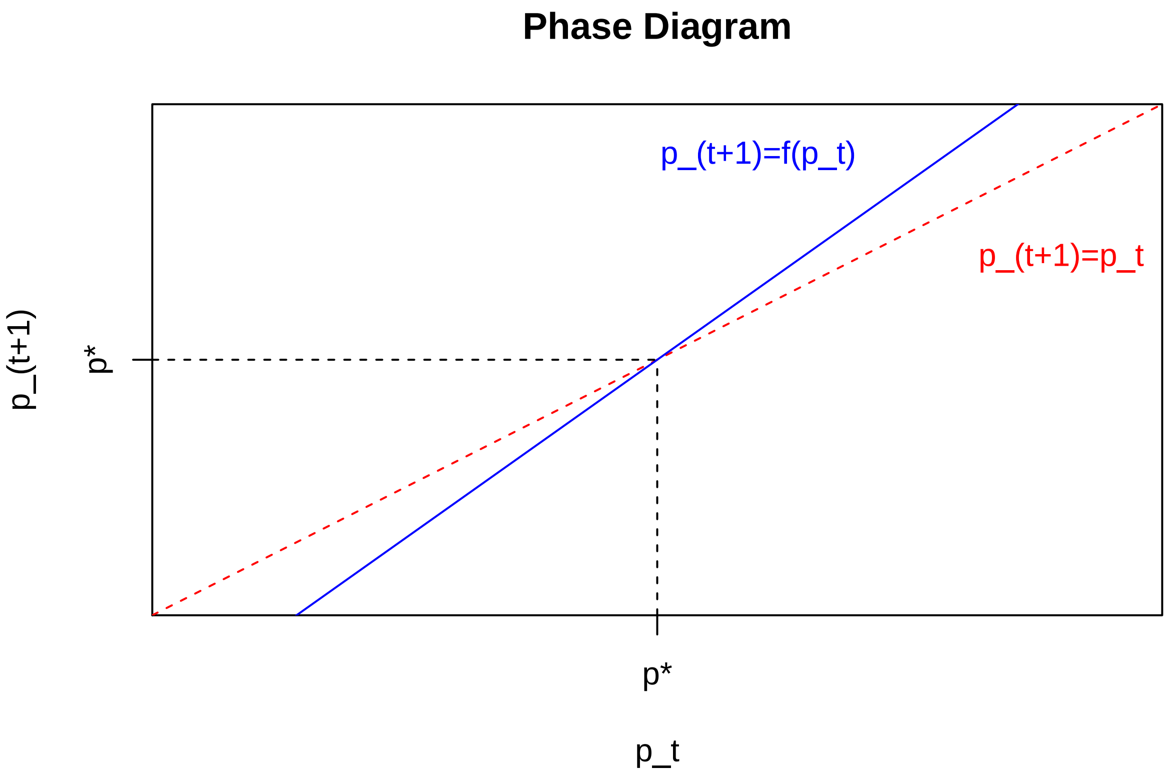
$$\begin{aligned} p_t &= \frac{d + p_{t+1}}{(1+r)} \\ \implies p_{t+1} &= (1+r)p_t - d \\ \implies p_{t+1} &= ap_t + b \text{ where } a = 1 + r \text{ and } b = -d. \end{aligned}$$

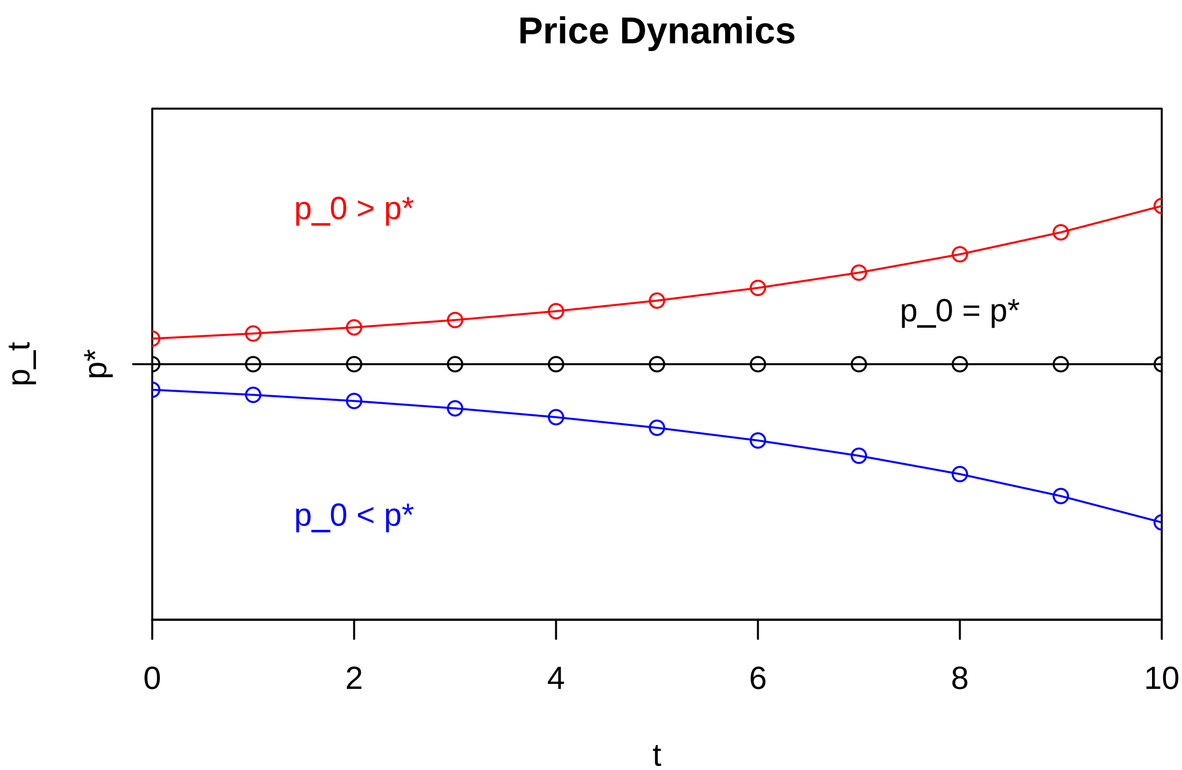
For complementary function, use $p_t^c = ca^t$ and use $p_t^p = p^*$ for the particular solution. Thus, the general solution is $p_t^g = p_t^c + p_t^p = ca^t + p^*$. Using $p_0^g = p_0$ as a boundary solution, $p_0 = ca^0 + p^* \implies c = p_0 - p^*$. Thus,

$$p_t = (p_0 - p^*)a^t + p^* \text{ where } a = 1 + r \text{ and } p^* = d/r.$$

*I worked on this problem set with a study group of Michael Nattinger, Andrew Smith, and Ryan Mather. I also discussed problems with Emily Case, Sarah Bass, and Danny Edgel.

Because $|a| = |1 + r| > 1$, the price diverges unless $p_0 = p^*$. If $p_0 < p^*$, $p_t \to -\infty$ as $t \to \infty$ and, if $p_0 > p^*$, $p_t \to \infty$ as $t \to \infty$.





3. [Matlab] Suppose the risk free rate is $r = 1\%$ and the stock pays constant dividend $d = 1$ per share per period. Open the Matlab code we provided. The code generates and plots the price dynamics given the first-order difference equation in part 2 given the initial share price $p_0 = 100$ at time $t = 0$. Modify the code (i.e. simply replace 100) with three different initial prices which respectively are below, at, and above the steady state price level implied by these parameters to plot the price dynamics over 100 periods.

See included the Matlab file, ps1_vonhafften.m.

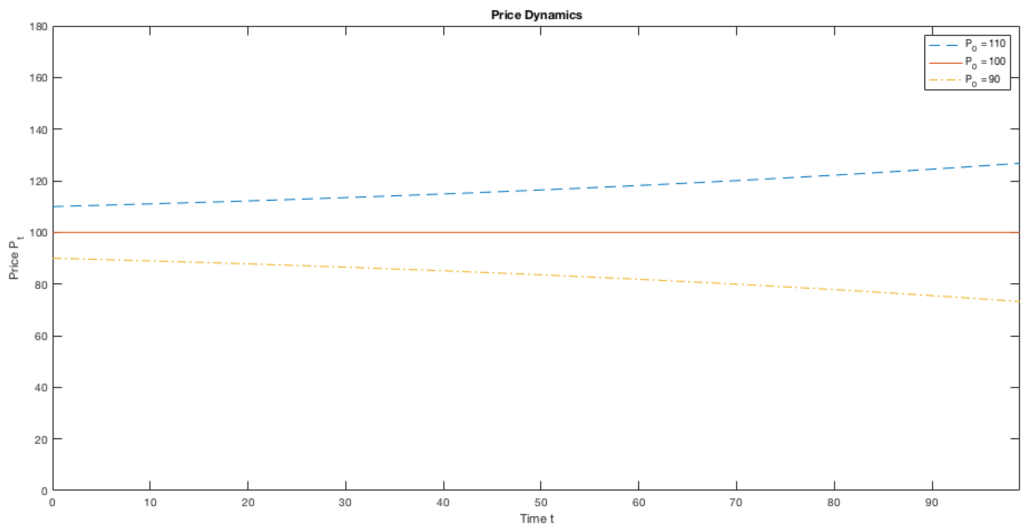


Figure 1: Matlab output for problem 3.

4. [Analytical and Matlab] Suppose the Federal Reserve announces at $t = 20$ to raise the federal funds rate from 1% to 2% at $t = 50$ and remain at the new level forever. Using (1) with $d = 1$, how does the price respond to the policy announcement and the interest rate change over time? Plot the price dynamics from $t = 0$ to $t = 99$. Please assume that we rule out rational bubbles (i.e., agents know and believe that prices will be at its fundamental value, the steady state, at $t = 50$).

First, modify (1), so that r can be different in different periods:

$$p_t = \frac{d + p_{t+1}}{(1 + r_t)} \implies p_{t+1} = (1 + r_t)p_t - d \implies p_t^* = d/r_t$$

Thus, the steady state price, p_t^*, also depends on the period.

For $t \in \{1, ..., 49\}$, the risk-free rate is 1% and $p_t^* = 1/.01 = 100$. For $t \in \{50, ..., 99\}$, the risk-free rate is 2% and $p_t^* = 1/.02 = 50$. Similar to problem (3), prices above the steady state price diverge upward and prices below the steady state price diverge downward.

For example, in the Matlab code, I plotted two lines illustrating the steady states in each half of the periods. The first line starts at $p_0 = 100$. For the first 49 periods, this line stays at the steady state price. Then, in period 50, this line starts to move upward because 100 is above the steady state price based on the higher risk-free rate. The second line starts at $p_0 \approx 70$ so that $p_50 = 50$. In periods 0 through 49, since this line is below the steady state price of 100 and it steadily falls. At period $t = 50$, this line equals the new steady state price of 50, so the line stays at 50 for the next 50 periods.

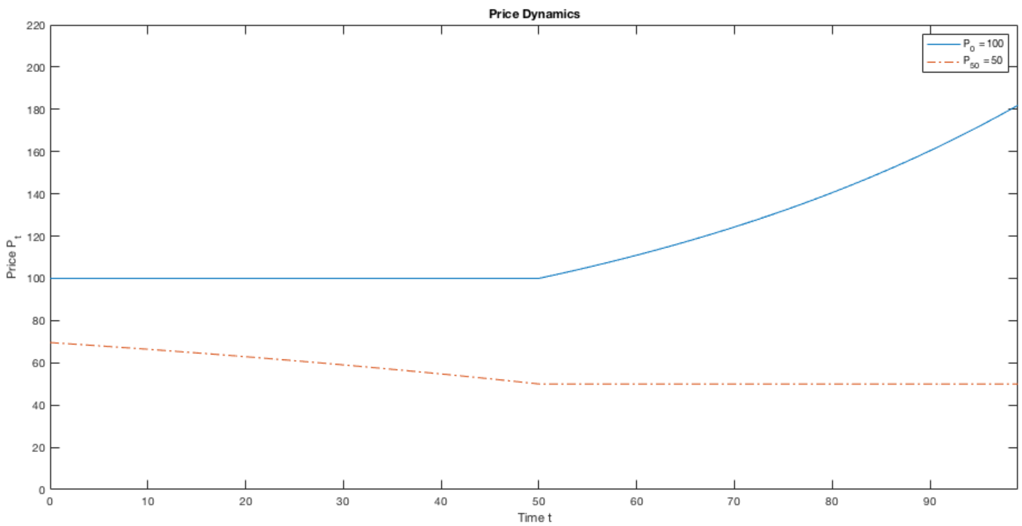


Figure 2: Matlab output for problem 4.