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ECON 711B Midterm

(1)

$N = \{1, 2\}$
$S = S_1 \times S_2$
$S_1 = \{R, P, S\}$
$S_2 = \{r, p, s\}$

$u(s_1, s_2) =$

$s_1 = \backslash s_2 =$	r	p	s
R	(α, α)	$(-5+\alpha, 5)$	$(5+\alpha, -5)$
P	$(5, -5+\alpha)$	$(0, 0)$	$(-5, 5)$
S	$(-5, 5+\alpha)$	$(5, -5)$	$(0, 0)$

Case 1: $\alpha > 5$

Rock strictly dominates Paper. W/ paper dominated, Rock strictly dominates Scissors.

Ex: $\alpha = 7$

	r	p	s
R	7, 7	2, 5	12, -5
P	~~5, 2~~	~~0, 0~~	~~-5, 5~~
S	~~-5, 12~~	~~5, -5~~	~~0, 0~~

Thus, the only outcome not strictly dominated is (R, r).

(R, r) is a NE.

Case 2

$\alpha = 5$

No strategies are eliminated via ISD.
So all strategies are rationalizable.

	r	p	s
R	5,5	0,5	10,-5
P	5,0	0,0	-5,5
S	-5,10	5,-5	0,0

(R, r) is a NE.

Case 3 $\alpha < 5$

Ex: $\alpha = 1$

	r	p	s
R	1,1	-4,5	6,-5
P	5,-4	0,0	-5,5
S	-5,6	5,-5	0,0

No strategies are eliminated via ISD, so all strategies are rationalizable.

② a. $N = \{1, 2\}$

$S = S_1 \times S_2$

$S_1 = \{C, D\}$

$S_2 = \{c, d\}$

$u(s_1, s_2) =$

	c	d
C	$(.8, .8)$	$(-1, 1)$
D	$(1, -1)$	$(-v_1, -v_2)$

The first pure strategy NE is (D, c)
~~Both~~ Player 1 wouldn't Δ to C because $1 > .8$. Player 2 wouldn't Δ to d because $v_2 > 1$.

The second pure strategy NE is (C, d). Both player wouldn't Δ strategies based on same logic for other NE.

The mixed strategy NE is where player 1 plays C w/ prob that 2 is indifferent between c & d.

$$\sigma_1 (.8) + (1 - \sigma_1)(-1) = \sigma_1 (1) + (1 - \sigma_1)(-v_2)$$

$$.8\sigma_1 - 1 + \cancel{\sigma_1} = \cancel{\sigma_1} - v_2 + v_2 \sigma_1$$

$$(.8 - v_2)\sigma_1 = 1 - v_2$$

$$\sigma_1 = \frac{v_2 - 1}{v_2 - .8}$$

② a. con't

2 plays c w/ prob such that 1 is indifferent between C & D. By symmetry,

$$\sigma_2 = \frac{v_1 - 1}{v_1 - .8}$$

~~[illegible]~~

Thus $\left(\left(\frac{v_2 - 1}{v_2 - .8}\right)C + \left(1 - \frac{v_2 - 1}{v_2 - .8}\right)D, \left(\frac{v_1 - 1}{v_1 - .8}\right)c + \left(1 - \frac{v_1 - 1}{v_1 - .8}\right)d\right)$

is a mixed strategy NE.

If v_1 & v_2 increase, the probability that the other player chickens out increases.

$1 - \left(\frac{v_i - 1}{v_i - .8}\right)$ is convex on $(1, \infty)$, so the expected payoff is decreasing as v_i increases.

(3) a. $N = \{Alice, Bob\}$
$S = S_1 \times S_2$
$S_1 = \{R, S\}$
$S_2 = \{r, s\}$

$u(s_1, s_2) =$

	r	s
R	(10,10)	(0,6)
S	(6,0)	(6,6)

All strategies survive ISD, so they are all rationalizable.

There is two pure NE at (R,r) and (S,s).

There is one mixed strategy NE

$$10R + (1-R)(0) = 6R + (1-R)6$$

$$10R = \cancel{6R} + 6 - \cancel{6R}$$

$$R = 6/10$$

$\left(\frac{6}{10}R + \frac{4}{10}S, \frac{6}{10}r + \frac{4}{10}s\right)$ is a NE.

(b)

	r	s	t
R	10,10	0,6	10,8
S	6,0	6,6	6,4
T	8,10	4,6	8,8

~~Both~~ All strategies survive ISD, so all are rationalized.

Pure Strategy NE:
- (R, r),
- (S, s)

Same mixed strategy NE persists

$$\left(\tfrac{6}{10}R + \tfrac{4}{10}S,\ \tfrac{6}{10}r + \tfrac{4}{10}s\right)$$

No mix between just R and T

Mixing between S and T

$$6s + 6(1-s) = 4s + 8(1-s)$$
$$\cancel{6s} + 6 - \cancel{6s} = 4s + 8 - 8s$$
$$-2 = -4s$$
$$1/2 = s$$

$(1/2\,S + 1/2\,T,\ 1/2\,s + 1/2\,t)$ is NE

3(b) mix between R, S, & T

$$10R + 10T = 6R + 6S + 6T = 8R + 4S + 8T$$

$$\Rightarrow 10R + 10T = 6$$
$$\Rightarrow 8R + 4S + 8T = 6$$

$$R = T$$

$$R = T = \frac{6}{20} = \frac{3}{10}$$

$$\Rightarrow 2 \cdot 8 \left(\frac{6}{20}\right) + 4S = 6$$

$$\frac{24}{5} + 4S = 6$$

$$4S = \frac{6}{5}$$

$$S = \cancel{\frac{24}{5}}\ \frac{3}{10}$$

Thus $\frac{3}{10}$

(4)

$$u_i(b_i, b_j ; v_i) = \begin{cases} v_i - b_j, & \text{if } b_i > b_j \\ 0, & \text{otherwise} \end{cases}$$

Suppose player j bids by some function $b(v_j)$
Assume strictly increasing.

$$\Rightarrow U_i(b_i; v_i) = [v_i - b(v_j)] \Pr\{ b(v_j) < b_i \}$$

$$= [v_i - b(v_j)] \Pr\{ v_j < b^{-1}(b_i) \}$$

$$= [v_i - b(v_j)]\, b^{-1}(b_i)$$ By uniform distribution

$\boxed{\text{FOC}}$

$$0 = \frac{v_i - b(v_j)}{b'(b^{-1}(b_i))}$$

By linearity assumption we know

$$b(v) = \alpha v + \beta$$

$$\Rightarrow b'(v) = \alpha$$
$$\Rightarrow b^{-1}(b) = \frac{b - \beta}{\alpha}$$

$$\boxed{\text{FOC}} \Rightarrow 0 = \frac{v_i - [\alpha v_j + \beta]}{b'\left(\frac{b_i - \beta}{\alpha}\right)}$$

Thus, we get that $\alpha = 1$ and $\beta = 0$, so

$$\boxed{b(v_i) = v_i}$$