ECON 899B - PS2

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I verify the formula listed in the problem set. Notice that $\varepsilon_{i0}/\sigma_0 \sim N(0,1)$. The likelihood associated with each duration T_i:

$$\begin{aligned}
P(T_i = 1|X_i, Z_i, \theta) &= P(Y_{i0} = 1|X_i, Z_i, \theta) \\
&= P(\alpha_0 + X_i\beta + Z_{i0}\gamma + \varepsilon_{i0} > 0|X_i, Z_i, \theta) \\
&= P(\varepsilon_{i0} > -\alpha_0 - X_i\beta - Z_{i0}\gamma|X_i, Z_i, \theta) \\
&= P(\varepsilon_{i0} < \alpha_0 + X_i\beta + Z_{i0}\gamma|X_i, Z_i, \theta) \\
&= P(\varepsilon_{i0}/\sigma_0 < (\alpha_0 + X_i\beta + Z_{i0}\gamma)/\sigma_0|X_i, Z_i, \theta) \\
&= \Phi((\alpha_0 + X_i\beta + Z_{i0}\gamma)/\sigma_0)
\end{aligned}$$

$$\begin{aligned}
P(T_i = 2|X_i, Z_i, \theta) &= P(Y_{i0} = 0, Y_{i1} = 1|X_i, Z_i, \theta) \\
&= P(Y_{i0} = 0|X_i, Z_i, \theta) \times P(Y_{i1} = 1|Y_{i0} = 0, X_i, Z_i, \theta) \\
&= P(\alpha_0 + X_i\beta + Z_{i0}\gamma + \varepsilon_{i0} < 0|X_i, Z_i, \theta) \\
&\times P(\alpha_0 + X_i\beta + Z_{i1}\gamma + \varepsilon_{i1} > 0|\alpha_0 + X_i\beta + Z_{i0}\gamma + \varepsilon_{i0} < 0, X_i, Z_i, \theta) \\
&= P(\alpha_0 + X_i\beta + Z_{i0}\gamma + \varepsilon_{i0} < 0|X_i, Z_i, \theta) \\
&\times P(\alpha_0 + X_i\beta + Z_{i1}\gamma + \rho\varepsilon_{i0} + \eta_{i1} > 0|\alpha_0 + X_i\beta + Z_{i0}\gamma + \varepsilon_{i0} < 0, X_i, Z_i, \theta) \\
&= P(\varepsilon_{i0} < -\alpha_0 - X_i\beta - Z_{i0}\gamma|X_i, Z_i, \theta) \\
&\times P(\eta_{i1} > -\alpha_0 - X_i\beta - Z_{i1}\gamma - \rho\varepsilon_{i0}|\varepsilon_{i0} < -\alpha_0 - X_i\beta - Z_{i0}\gamma, X_i, Z_i, \theta) \\
&= P(\varepsilon_{i0} < -\alpha_0 - X_i\beta - Z_{i0}\gamma|X_i, Z_i, \theta) \\
&\times P(\eta_{i1} < \alpha_0 + X_i\beta + Z_{i1}\gamma + \rho\varepsilon_{i0}|\varepsilon_{i0} < -\alpha_0 - X_i\beta - Z_{i0}\gamma, X_i, Z_i, \theta) \\
&= .... \\
&= \Phi((-\alpha_0 - X_i\beta - Z_{i0}\gamma)/\sigma_0) \\
&\times \int_{-\infty}^{-\alpha_0 - X_i\beta - Z_{i0}\gamma} P(\eta_{i1} < \alpha_0 + X_i\beta + Z_{i1}\gamma + \rho w|X_i, Z_i, \theta)dw \\
&= \Phi((-\alpha_0 - X_i\beta - Z_{i0}\gamma)/\sigma_0) \\
&\times \int_{-\infty}^{-\alpha_0 - X_i\beta - Z_{i0}\gamma} \Phi(\alpha_0 + X_i\beta + Z_{i1}\gamma + \rho w)dw
\end{aligned}$$