ECON 711 - PS 4

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Question 1. Choice rules from preferences

Let X be a choice set and $\succsim$ a complete and transitive preference relation on X. Show that the choice rule induced by $\succsim$, $C(A, \succsim) = \{x \in A : x \succsim y \ \forall y \in A\}$, must satisfy the Weak Axiom of Revealed Preference (WARP).

Proof: $C(\cdot)$ satisfies WARP if for any sets $A, B \subset X$ and any $x, y \in A \cap B$, if $x \in C(A)$ and $y \in C(B)$, then $x \in C(B)$ and $y \in C(A)$. Since $x \in C(A)$ and $y \in C(B)$, $x \succsim y$ and $y \succsim x$. For any $w \in B$, $y \succsim w$ because $y \in C(B)$. By transitivity, $x \succsim w$, so $x \in C(B)$. For any $z \in A$, $x \succsim z$ because $x \in C(A)$. By transitivity, $y \succsim z$, so $y \in C(A)$. $\square$

Question 2. Preferences from choice rules

Let X be a choice set and $C : \mathcal{P}(X) \to \mathcal{P}(X)$ a nonempty choice rule. Show that if C satisfies WARP, then the preference relation $\succsim_C$ defined on X by "$x \succsim_C y$ iff there exists a set $A \subseteq X$ such that $x, y \in A$ and $x \in C(A)$" is complete and transitive, and that the choice rule it induces, $C(\cdot, \succsim_C)$, is equal to C.

Question 3. Choice over finite sets

Let X be a finite set, and $\succsim$ a complete and transitive preference relation on X. (Hint: for (a), fix X finite, and prove by induction. For (b) use induction on $|X|$ to prove the stronger result that when X is finite, a utility representation exists with range $\{1, 2, ..., |X|\}$)

(a) Show that the induced choice rule $C(\cdot, \succsim_C)$ is nonempty - that $C(A, \succsim_C) \neq \emptyset$ if $A = \emptyset$.

(b) Show that a utility representation exists.

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