Computational Economics
Fall 2017, University of Wisconsin
Instructor: Dean Corbae

Problem Set #4b - Due 10/11/17

In this problem set you are to contrast pooling and "separating" equilibria with bankruptcy. For parameters, let: preference parameters be given by (coefficient of relative risk aversion $\alpha = 1.5$ and $\beta = 0.8$ - I know it's quite low for an annual model); earnings be given by a two-state Markov process ($y(e) = 1$, $y(u) = 0.05$, $\Pi(s^{'} = e|s = e) = \Pi(s^{'} = u|s = u) = 0.75$); open economy real interest rate ($r = 0.04$); pooling contract borrowing constraint ($\underline{a} = -0.525$); and legal record keeping technology parameter ($(1 - \rho) = 0.1$).

Compute steady state equilibria with pooling and separating contracts. Starting with pooling contracts, the algorithm is a lot like Huggett. In particular, starting with a guess for the pooling price of debt, say q^0, calculate default and asset decisions using the T operator, calculate the implied steady state cross-sectional disribution using the T^* operator, check if the intermediary is making zero profits on its loans (i.e. if $\left|q^0 - \frac{1-\Delta'(q^0)}{1+r}\right| <$ accepted tolerance) quit otherwise update to q^1 and start over. Since this difference tends to be very nonlinear, instead of bisection, start with a guess $q^0 = 1/(1+r)$ which will lead to a positive difference and set q^1 lower but move slowly down until you hit the accepted tolerance. For the separating equilibrium, start with a guess for the price of debt menu $q^0(a', s)$, calculate default and asset decisions using the T operator (to make the programs run faster, use the same lower bound as above $\underline{a}$ understanding this is not part of the separating contract), then check if the intermediary is making zero profits on each contract, if not update $q^1(a', s)$. This process is better behaved, so you can use bisection. Note that when you have finally found the equilibrium menu of debt prices, you can then calculate the cross-sectional distribution.

0. Plot value functions for each earnings level with pooling contracts. With separating contracts, are the value functions strictly concave?

1. Plot borrowing/savings functions for each earnings level with pooling contracts and then with separating contracts. What is the economwide debt to income level with pooling contracts? For separating contracts?

2. Plot default decision rules with pooling contracts. With separating contracts. What is the economywide default rate with pooling contracts? For separating contracts?

3. Plot bond prices in the two economies across debt holdings for each earnings level. What is the interest rate for pooling contracts? What is the equilibrium distribution of interest rates across borrowing levels for each earnings type? Note: while the menu is plotted in the first case, the second case should just graph the interest rates which are selected in equilibrium. What is the average interest rate in the separating equilibrium?

4. Plot the equilibrium cross-sectional distribution $\mu(s, a, h)$ for the pooling and separating contract economies.

5. Create a table which has pooling equilibrium and separating equilibrium columns with the following moments: average income, average savings, average loan, average default rate, average amount of default, and average bond price. These moments are an easy way to check your results for validity and then can later be used to compare to corresponding data moments. Eyeballing value and policy functions is just too imprecise as a check.

6. Starting in an economy with pooling contracts, compute consumption equivalents for the employed and unemployed for each asset position moving to an economy with separating contracts. Graph consumption equivalents for the 4 different types of people (s, h) across a. Who wins/loses from the change to a separating equilibrium? What is the welfare gain or loss? Note that while the supports of the cross-sectional distribution of assets with positive mass may differ between the 2 equilibria, the actual consumption equivalents depend on the support of the value functions, which need not have positive mass in equilibrium. That is, while people may choose to be at different asset positions in equilibrium, all the matters for the consumption equivalents is the value function calculations which can always be over the same support. When calculating the aggregate welfare gain, only the equilibrium support of the cross-sectional distribution matters, but that doesn't matter for the calculations of the consumption equivalents.