ECON 710A - Problem Set 1

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1. Suppose $(Y, X')'$ is a random vector with $Y = X'\beta_0 \cdot U$ where $E[U|X] = 1, E[XX']$ is invertible, and $E[Y^2 + ||X||^2] < \infty$. Furthermore, suppose that $\{(Y_i, X_i')'\}_{i=1}^n$ is a random sample from the distribution of $(Y, X')'$ where $\frac{1}{n}\sum_{i=1}^n X_i X_i'$ is invertible and let $\hat{\beta}$ be the OLS estimator, i.e., $\hat{\beta} = (\frac{1}{n}\sum_{i=1}^n X_i X_i')^{-1}\frac{1}{n}\sum_{i=1}^n X_i Y_i$.

(i) Interpret the entries of β_0 in this model?

...

(ii) Show that $Y = X'\beta_0 + \bar{U}$ where $E[\bar{U}|X] = 0$.

...

(iii) Show that $E[X(Y - X'\beta)] = 0$ iff $\beta = \beta_0$ and use this to derive OLS as a method of moments estimator.

...

(iv) Show that the OLS estimator is conditionally unbiased, i.e., that $E[\hat{\beta}|X_1, ..., X_n] = \beta_0$.

...

(v) Show that the OLS estimator is consistent, i.e., that $\hat{\beta} \to_p \beta_0$ as $n \to \infty$.

...

2. Let X be a random variable with $E[X^4] < \infty$ and $E[X^2] > 0$. Furthermore, let $\{X_i\}_{i=1}^n$ be a random cample from the distribution of X.

(i) For which of the following four statistics can you use the law of large numbers and continuous mapping theorem to show convergence in probability as $n \to \infty$,

$$\frac{1}{n}\sum_{i=1}^n X_i^3$$

...

$$\max_{1 \le i \le n} X_i$$

...

$$\frac{\sum_{i=1}^n X_i^3}{\sum_{i=1}^n X_i^2}$$

...

*I worked on this problem set with a study group of Michael Nattinger, Andrew Smith, and Ryan Mather. I also discussed problems with Emily Case, Sarah Bass, and Danny Edgel.

$$1\{\frac{1}{n}\sum_{i=1}^{n} X_i > 0\}$$

...

(ii) For which of the following three statistics can you use the central limit theorem and continuous mapping to show convergence in distribution as $n \to \infty$,

$$W_n := \frac{1}{\sqrt{n}}\sum_{i=1}^{n}(X_i^2 - E[X_1^2])$$

...

$$W_n^2$$

...

$$\frac{1}{\sqrt{n}}\sum_{i=1}^{n}(X_i^2 - \overline{X_n^2}) \text{ where } \overline{X_n^2} = \frac{1}{n}\sum_{i=1}^{n} X_i^2$$

...

(iii) Show that $\max_{1\le i\le n} X_i \to_p 1$ if $X \sim uniform(0,1)$.

...

(iv) Show that $\Pr(\max_{1\le i\le n} X_i > M) \to 1$ for any $M \ge 0$ if $X \sim exponential(1)$.

...

3. Suppose that $\{X_i\}_{i=1}^n$ is an iid sequence of $N(0,1)$ random variables. Let W be independent of $\{X_i\}_{i=1}^n$ with $\Pr(W=1) = \Pr(W=-1) = 1/2$. Let $Y_i = X_i W$.

(i) Show that $\frac{1}{\sqrt{n}}\sum_{i=1}^n X_i \to_d N(0,1)$ as $n \to \infty$.

...

(ii) Show that $\frac{1}{\sqrt{n}}\sum_{i=1}^n Y_i \to_d N(0,1)$ as $n \to \infty$.

...

(iii) Show that $Cov(X_i, Y_i) = 0$.

$$E[W] = (1)\Pr(W=1) + (-1)\Pr(W=-1) = (1)(1/2) + (-1)(1/2) = 0$$

$$E[Y_i] = E[X_i W] = E[X_i]E[W] = 0$$

$$Cov(X_i, Y_i) = E[(X_i - E[X_i])(Y_i - E[Y_i])] = E[X_i Y_i] = E[X_i^2 W] = E[X_i^2]E[W] = (1)(0) = 0$$

(iv) Does $V := \frac{1}{\sqrt{n}}\sum_{i=1}^n (X_i, Y_i)' \to_d N(0, I_2)$ as $n \to \infty$?

...

(v) How does this exercise related to the Cramer-Wold device introduced in lecture 2?

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