ECON 710A - Problem Set 1

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- 1. Suppose (Y,X')' is a random vector with $Y=X'\beta_0\cdot U$ where E[U|X]=1, E[XX'] is invertible, and $E[Y^2+||X||^2]<\infty$. Furthermore, suppose that $\{(Y_i,X_i')'\}_{i=1}^n$ is a random sample from the distribution of (Y,X')' where $\frac{1}{n}\sum_{i=1}^n X_iX_i'$ is invertible and let $\hat{\beta}$ be the OLS estimator, i.e., $\hat{\beta}=(\frac{1}{n}\sum_{i=1}^n X_iX_i')^{-1}\frac{1}{n}\sum_{i=1}^n X_iY_i$.
- (i) Interpret the entries of β_0 in this model?

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(ii) Show that $Y = X'\beta_0 + \bar{U}$ where $E[\bar{U}|X] = 0$.

. . .

(iii) Show that $E[X(Y - X'\beta)] = 0$ iff $\beta = \beta_0$ and use this to derive OLS as a method of moments estimator.

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(iv) Show that the OLS estimator is conditionally unbiased, i.e., that $E[\hat{\beta}|X_1,...,X_n]=\beta_0$.

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(v) Show that the OLS estimator is consistent, i.e., that $\hat{\beta} \to_p \beta_0$ as $n \to \infty$.

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- 2. Let X be a random variable with $E[X^4] < \infty$ and $E[X^2] > 0$. Furthermore, let $\{X_i\}_{i=1}^n$ be a random cample from the distribution of X.
- (i) For which of the following four statistics can you use the law of large numbers and continuous mapping theorem to show convergence in probability as $n \to \infty$,

$$\frac{1}{n} \sum_{i=1}^{n} X_i^3$$

. .

$$\max_{1 \le i \le n} X_i$$

. . .

$$\frac{\sum_{i=1}^{n} X_i^3}{\sum_{i=1}^{n} X_i^2}$$

. . .

^{*}I worked on this problem set with a study group of Michael Nattinger, Andrew Smith, and Ryan Mather. I also discussed problems with Emily Case, Sarah Bass, and Danny Edgel.

$$1\{\frac{1}{n}\sum_{i=1}^{n}X_{i}>0\}$$

. . .

(ii) For which of the following three statistics can you use the central limit theorem and continuous mapping to show convergence in distribution as $n \to \infty$,

$$W_n := \frac{1}{\sqrt{n}} \sum_{i=1}^n (X_i^2 - E[X_1^2])$$

. . .

$$W_n^2$$

. . .

$$\frac{1}{\sqrt{n}} \sum_{i=1}^{n} (X_i^2 - \overline{X_n^2})$$
 where $\overline{X_n^2} = \frac{1}{n} \sum_{i=1}^{n} X_i^2$

. . .

(iii) Show that $\max_{1 \le i \le n} X_i \to_p 1$ if $X \sim uniform(0,1)$.

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(iv) Show that $\Pr(\max_{1 \le i \le n} X_i > M) \to 1$ for any $M \ge 0$ if $X \sim exponential(1)$.

. . .

- 3. Suppose that $\{X_i\}_{i=1}^n$ is an iid sequence of N(0,1) random variables. Let W be independent of $\{X_i\}_{i=1}^n$ with $\Pr(W=1) = \Pr(W=-1) = 1/2$. Let $Y_i = X_i W$.
- (i) Show that $\frac{1}{\sqrt{n}} \sum_{i=1}^{n} X_i \to_d N(0,1)$ as $n \to \infty$.

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(ii) Show that $\frac{1}{\sqrt{n}} \sum_{i=1}^{n} Y_i \to_d N(0,1)$ as $n \to \infty$.

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(iii) Show that $Cov(X_i, Y_i) = 0$.

$$E[W] = (1) \Pr(W = 1) + (-1) \Pr(W = -1) = (1)(1/2) + (-1)(1/2) = 0$$

$$E[Y_i] = E[X_iW] = E[X_i]E[W] = 0$$

$$Cov(X_i, Y_i) = E[(X_i - E[X_i])(Y_i - E[Y_i])] = E[X_iY_i] = E[X_i^2W] = E[X_i^2]E[W] = (1)(0) = 0$$

(iv) Does $V := \frac{1}{\sqrt{n}} \sum_{i=1}^{n} (X_i, Y_i)' \to_d N(0, I_2)$ as $n \to \infty$?

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(v) How does this exercise related to the Cramer-Wold device introduced in lecture 2?

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