Intermediation Frictions in Incomplete Markets

ECON 810A - Project

Alex von Hafften

UW-Madison

March 9, 2022

Secondary Treasury Market in COVID-19 Crisis

- The U.S. Treasury market - widely considered the world's deepest and most liquid financial market - is intermediated by large U.S. banks acting as broker-dealers.

Secondary Treasury Market in COVID-19 Crisis

- The U.S. Treasury market - widely considered the world's deepest and most liquid financial market - is intermediated by large U.S. banks acting as broker-dealers.

- In March 2020, concerns about COVID-19 prompted many large investors (e.g., hedge funds and foreign governments) to liquidate their holdings of Treasuries.

Secondary Treasury Market in COVID-19 Crisis

- The U.S. Treasury market - widely considered the world's deepest and most liquid financial market - is intermediated by large U.S. banks acting as broker-dealers.
- In March 2020, concerns about COVID-19 prompted many large investors (e.g., hedge funds and foreign governments) to liquidate their holdings of Treasuries.
- Intermediaries in the secondary market for U.S. Treasuries were overwhelmed (Duffie 2020).

Secondary Treasury Market in COVID-19 Crisis

- The U.S. Treasury market - widely considered the world's deepest and most liquid financial market - is intermediated by large U.S. banks acting as broker-dealers.
- In March 2020, concerns about COVID-19 prompted many large investors (e.g., hedge funds and foreign governments) to liquidate their holdings of Treasuries.
- Intermediaries in the secondary market for U.S. Treasuries were overwhelmed (Duffie 2020).
  - Yield rose sharply.

Secondary Treasury Market in COVID-19 Crisis

- The U.S. Treasury market - widely considered the world's deepest and most liquid financial market - is intermediated by large U.S. banks acting as broker-dealers.
- In March 2020, concerns about COVID-19 prompted many large investors (e.g., hedge funds and foreign governments) to liquidate their holdings of Treasuries.
- Intermediaries in the secondary market for U.S. Treasuries were overwhelmed (Duffie 2020).
  - Yield rose sharply.
  - Space on the balance sheet of broker-dealers for warehousing additional trades diminished.

Secondary Treasury Market in COVID-19 Crisis

- The U.S. Treasury market - widely considered the world's deepest and most liquid financial market - is intermediated by large U.S. banks acting as broker-dealers.
- In March 2020, concerns about COVID-19 prompted many large investors (e.g., hedge funds and foreign governments) to liquidate their holdings of Treasuries.
- Intermediaries in the secondary market for U.S. Treasuries were overwhelmed (Duffie 2020).
  - Yield rose sharply.
  - Space on the balance sheet of broker-dealers for warehousing additional trades diminished.
  - Bid-offer spreads widened.

Secondary Treasury Market in COVID-19 Crisis

- The U.S. Treasury market - widely considered the world's deepest and most liquid financial market - is intermediated by large U.S. banks acting as broker-dealers.
- In March 2020, concerns about COVID-19 prompted many large investors (e.g., hedge funds and foreign governments) to liquidate their holdings of Treasuries.
- Intermediaries in the secondary market for U.S. Treasuries were overwhelmed (Duffie 2020).
  - Yield rose sharply.
  - Space on the balance sheet of broker-dealers for warehousing additional trades diminished.
  - Bid-offer spreads widened.
  - Settlement failures increased.

Secondary Treasury Market in COVID-19 Crisis

- The U.S. Treasury market - widely considered the world's deepest and most liquid financial market - is intermediated by large U.S. banks acting as broker-dealers.
- In March 2020, concerns about COVID-19 prompted many large investors (e.g., hedge funds and foreign governments) to liquidate their holdings of Treasuries.
- Intermediaries in the secondary market for U.S. Treasuries were overwhelmed (Duffie 2020).
  - Yield rose sharply.
  - Space on the balance sheet of broker-dealers for warehousing additional trades diminished.
  - Bid-offer spreads widened.
  - Settlement failures increased.
- Aggressive intervention by the Fed restored market liquidity.

Sources of Intermediation Frictions in Treasury Market

- This episode raised questions about the functioning of the secondary Treasury market, doubts about the safe-haven status of Treasuries, and calls for reform.

Sources of Intermediation Frictions in Treasury Market

- This episode raised questions about the functioning of the secondary Treasury market, doubts about the safe-haven status of Treasuries, and calls for reform.

- The growth in U.S. government debt outstanding may have outstripped the ability of broker-dealers to effective intermediate the secondary Treasury market.

Sources of Intermediation Frictions in Treasury Market

- This episode raised questions about the functioning of the secondary Treasury market, doubts about the safe-haven status of Treasuries, and calls for reform.
- The growth in U.S. government debt outstanding may have outstripped the ability of broker-dealers to effective intermediate the secondary Treasury market.
- Broker-dealers pointed to post-financial-crisis bank regulatory reform - in particular, the Supplementary Leverage Ratio requirement - as the source of the disruption.

Sources of Intermediation Frictions in Treasury Market

- This episode raised questions about the functioning of the secondary Treasury market, doubts about the safe-haven status of Treasuries, and calls for reform.
- The growth in U.S. government debt outstanding may have outstripped the ability of broker-dealers to effective intermediate the secondary Treasury market.
- Broker-dealers pointed to post-financial-crisis bank regulatory reform - in particular, the Supplementary Leverage Ratio requirement - as the source of the disruption.
- **Research Question:** How does the intermediation of illiquid assets affect portfolio choice?

Multiple Assets

- To explore this question, I turned to Bewley-style models with multiple assets.

Multiple Assets

- To explore this question, I turned to Bewley-style models with multiple assets.
- In the baseline Bewley model (and models covered in ECON 810A), household can only invest in one-period risk-free bonds.

Multiple Assets

- To explore this question, I turned to Bewley-style models with multiple assets.
- In the baseline Bewley model (and models covered in ECON 810A), household can only invest in one-period risk-free bonds.
- In reality, households invest in many assets, including cash, real estate, stocks, bonds, etc.

Multiple Assets

- To explore this question, I turned to Bewley-style models with multiple assets.
- In the baseline Bewley model (and models covered in ECON 810A), household can only invest in one-period risk-free bonds.
- In reality, households invest in many assets, including cash, real estate, stocks, bonds, etc.
- Optimal portfolio choices change with wealth and age (Brandsas 2020).

Multiple Assets

- To explore this question, I turned to Bewley-style models with multiple assets.
- In the baseline Bewley model (and models covered in ECON 810A), household can only invest in one-period risk-free bonds.
- In reality, households invest in many assets, including cash, real estate, stocks, bonds, etc.
- Optimal portfolio choices change with wealth and age (Brandsas 2020).
- The curse of dimensionality quickly hampers rich portfolio choice problems in Bewley-style model.

Multiple Assets

- To explore this question, I turned to Bewley-style models with multiple assets.
- In the baseline Bewley model (and models covered in ECON 810A), household can only invest in one-period risk-free bonds.
- In reality, households invest in many assets, including cash, real estate, stocks, bonds, etc.
- Optimal portfolio choices change with wealth and age (Brandsas 2020).
- The curse of dimensionality quickly hampers rich portfolio choice problems in Bewley-style model.
- GE with aggregate uncertainty and heterogenous households is difficult (Bhandari 2022).

Literature Review - Kaplan and Violante (2014)

- Empirical literature finds that households spend about 25 percent of tax rebates on consumption immediately inconsistent with prediction from single-asset Bewley.

Literature Review - Kaplan and Violante (2014)

- Empirical literature finds that households spend about 25 percent of tax rebates on consumption immediately inconsistent with prediction from single-asset Bewley.
- Build a Bewley-style model with two assets: low return liquid asset and high return illiquid asset.

Literature Review - Kaplan and Violante (2014)

- Empirical literature finds that households spend about 25 percent of tax rebates on consumption immediately inconsistent with prediction from single-asset Bewley.
- Build a Bewley-style model with two assets: low return liquid asset and high return illiquid asset.
- HHs must pay fixed cost to adjust illiquid asset holding.

Literature Review - Kaplan and Violante (2014)

- Empirical literature finds that households spend about 25 percent of tax rebates on consumption immediately inconsistent with prediction from single-asset Bewley.
- Build a Bewley-style model with two assets: low return liquid asset and high return illiquid asset.
- HHs must pay fixed cost to adjust illiquid asset holding.
- Many HHs are optimally "wealthy hand-to-mouth" (i.e., hold very little liquid assets despite sizable amount of illiquid assets).

Literature Review - Kaplan and Violante (2014)

- Empirical literature finds that households spend about 25 percent of tax rebates on consumption immediately inconsistent with prediction from single-asset Bewley.
- Build a Bewley-style model with two assets: low return liquid asset and high return illiquid asset.
- HHs must pay fixed cost to adjust illiquid asset holding.
- Many HHs are optimally "wealthy hand-to-mouth" (i.e., hold very little liquid assets despite sizable amount of illiquid assets).
- Wealthy hand-to-mouth HHs have high MPC and rationalize empirical motivation.

Kaplan and Violante (2014) Household Value Function

- Household with age j. If $V_j^0(\mathbf{s}_j) \geq V_j^1(\mathbf{s}_j)$, the HH do not adjusted its illiquid assets:

$$V_j^0(\mathbf{s}_j) = \max_{c_j, h_j, m_{j+1}} [(1-\beta)(c_j^{\phi} s_j^{1-\phi})^{1-\sigma} + \beta\{C_j[V-j+1^{1-\gamma}]\}^{(1-\sigma)/(1-\gamma)}]^{1/(1-\sigma)}$$

$$\begin{aligned} \text{s.t. } (1+\tau^c)(c_j+h_j) + q^m(m_{j+1})m_{j+1} &= y_j + m_j - \tau(y_j, a_j, m_j) \\ s_j &= h_j + \zeta a_j \\ q^a a_{j+1} &= a_j \\ c_j &\geq 0, \\ h_j &\geq -\zeta a_j, \\ m_{j+1} &\geq -\underline{m}_{j+1}(y_j), \\ y_j &= \begin{cases} \exp(\xi_j + \alpha + z_j), & \text{if } j \geq J^w \\ p(\xi_{J^w}, \alpha, z_{J^w}), & \text{otherwise.} \end{cases} \end{aligned}$$

where $\mathbf{s}_j = (m_j, a_j, \alpha, z_j)$ and z_j evolves according to a conditional cdf Γ_j^z.

Kaplan and Violante (2014) Household Value Function (con't)

- If $V_j^1(\mathbf{s}_j) \geq V_j^0(\mathbf{s}_j)$, the HH adjusted its illiquid assets:

$$V_j^1(\mathbf{s}_j) = \max_{c_j, h_j, m_{j+1}, a_{j+1}} [(1-\beta)(c_j^{\phi} s_j^{1-\phi})^{1-\sigma} + \beta\{C_j[V - j + 1^{1-\gamma}]\}^{(1-\sigma)/(1-\gamma)}]^{1/(1-\sigma)}$$

$$\begin{aligned}
\text{s.t. } (1+\tau^c)(c_j + h_j) + q^m(m_{j+1})m_{j+1} + q^a a_{j+1} &= y_j + m_j + a_j - \kappa - \tau(y_j, a_j, m_j) \\
s_j &= h_j + \zeta a_j \\
c_j &\geq 0, \\
h_j &\geq -\zeta a_j, \\
m_{j+1} &\geq -\underline{m}_{j+1}(y_j), \\
y_j &= \begin{cases} \exp(\xi_j + \alpha + z_j), & \text{if } j \geq J^w \\ p(\xi_{J^w}, \alpha, z_{J^w}), & \text{otherwise.} \end{cases}
\end{aligned}$$

where κ is fixed cost of adjusting illiquid assets.

Literature Review - Rios-Rull and Sanchez-Marcos (2008)

- Build a Bewley-style model with financial assets and nonfinancial assets.

Literature Review - Rios-Rull and Sanchez-Marcos (2008)

- Build a Bewley-style model with financial assets and nonfinancial assets.
- Financial assets are perfectly divisible and costless to buy or sell.

Literature Review - Rios-Rull and Sanchez-Marcos (2008)

- Build a Bewley-style model with financial assets and nonfinancial assets.
- Financial assets are perfectly divisible and costless to buy or sell.
- Nonfinancial assets are bulky, indivisible, and have transaction costs.

Literature Review - Rios-Rull and Sanchez-Marcos (2008)

- Build a Bewley-style model with financial assets and nonfinancial assets.
- Financial assets are perfectly divisible and costless to buy or sell.
- Nonfinancial assets are bulky, indivisible, and have transaction costs.
- Label the nonfinancial assets as "houses".

Literature Review - Rios-Rull and Sanchez-Marcos (2008)

- Build a Bewley-style model with financial assets and nonfinancial assets.
- Financial assets are perfectly divisible and costless to buy or sell.
- Nonfinancial assets are bulky, indivisible, and have transaction costs.
- Label the nonfinancial assets as “houses”.
- Find reasonable lifecycle pattern: HHs accumulate some financial assets for downpayment, then buy a small house, then buy a large house.

Literature Review - Rios-Rull and Sanchez-Marcos (2008)

- Build a Bewley-style model with financial assets and nonfinancial assets.
- Financial assets are perfectly divisible and costless to buy or sell.
- Nonfinancial assets are bulky, indivisible, and have transaction costs.
- Label the nonfinancial assets as “houses”.
- Find reasonable lifecycle pattern: HHs accumulate some financial assets for downpayment, then buy a small house, then buy a large house.
- HHs pay a fixed cost to trade their house.

This Approach

- Similar to Kaplan and Violante (2014) and Rios-Rull and Sanchez-Marcos (2008), I use two assets.

This Approach

- Similar to Kaplan and Violante (2014) and Rios-Rull and Sanchez-Marcos (2008), I use two assets.
  - Here, cash and a long-term bond with stochastic maturity.

This Approach

- Similar to Kaplan and Violante (2014) and Rios-Rull and Sanchez-Marcos (2008), I use two assets.
  - Here, cash and a long-term bond with stochastic maturity.
- Instead of illiquidity of second asset coming from fixed transaction cost, I use random search for intermediary (i.e. broker-dealer).

Environment

- Two agents:

Environment

- Two agents:
  - Households.

Environment

- Two agents:
  - Households.
  - Intermediaries.

Environment

- Two agents:
  - Households.
  - Intermediaries.
- Two assets:

Environment

- Two agents:
  - Households.
  - Intermediaries.

- Two assets:
  - Cash/consumption good without no return.

Environment

- Two agents:
  - Households.
  - Intermediaries.

- Two assets:
  - Cash/consumption good without no return.
  - Long-term illiquid bonds with return r.

Environment

- Two agents:
  - Households.
  - Intermediaries.

- Two assets:
  - Cash/consumption good without no return.
  - Long-term illiquid bonds with return r.
  - Fraction $\delta \in (0, 1)$ of long-term bonds mature into cash each period.

Environment

- Two agents:
  - Households.
  - Intermediaries.

- Two assets:
  - Cash/consumption good without no return.
  - Long-term illiquid bonds with return r.
  - Fraction $\delta \in (0, 1)$ of long-term bonds mature into cash each period.

- Households randomly search for intermediaries:

Environment

- Two agents:
  - Households.
  - Intermediaries.

- Two assets:
  - Cash/consumption good without no return.
  - Long-term illiquid bonds with return r.
  - Fraction $\delta \in (0, 1)$ of long-term bonds mature into cash each period.

- Households randomly search for intermediaries:
  - If an household and an intermediary meet, the household can sell long-term bonds.

Environment

- Two agents:
  - Households.
  - Intermediaries.
- Two assets:
  - Cash/consumption good without no return.
  - Long-term illiquid bonds with return r.
  - Fraction $\delta \in (0, 1)$ of long-term bonds mature into cash each period.
- Households randomly search for intermediaries:
  - If an household and an intermediary meet, the household can sell long-term bonds.
  - Nash bargain over price with $\theta \in (0, 1)$ being the bargaining power of the household.

Environment

- Two agents:
  - Households.
  - Intermediaries.

- Two assets:
  - Cash/consumption good without no return.
  - Long-term illiquid bonds with return r.
  - Fraction $\delta \in (0, 1)$ of long-term bonds mature into cash each period.

- Households randomly search for intermediaries:
  - If an household and an intermediary meet, the household can sell long-term bonds.
  - Nash bargain over price with $\theta \in (0, 1)$ being the bargaining power of the household.
  - Households can always buy long-term bonds (i.e. "on-the-run" Treasuries), but can only sell them through an intermediary (e.g. "off-the-run" Treasuries).

Model Timing

1. Exogenous labor income is drawn.

Model Timing

1. Exogenous labor income is drawn.
2. Long-term bonds return r.

Model Timing

1. Exogenous labor income is drawn.
2. Long-term bonds return r.
3. Households and intermediaries meet.

Model Timing

1. Exogenous labor income is drawn.
2. Long-term bonds return r.
3. Households and intermediaries meet.
4. Consumption good is eaten.

Model Timing

1. Exogenous labor income is drawn.
2. Long-term bonds return r.
3. Households and intermediaries meet.
4. Consumption good is eaten.
5. Households and intermediaries bargain and trade.

Model Timing

1. Exogenous labor income is drawn.
2. Long-term bonds return r.
3. Households and intermediaries meet.
4. Consumption good is eaten.
5. Households and intermediaries bargain and trade.
6. Value functions are evaluated.

Intermediaries

- Intermediaries are infinitely lived and risk-neutral.

Intermediaries

- Intermediaries are infinitely lived and risk-neutral.
- Discount factor β_I.

Intermediaries

- Intermediaries are infinitely lived and risk-neutral.
- Discount factor β_I.
- They have “deep pockets”.

Intermediaries

- Intermediaries are infinitely lived and risk-neutral.
- Discount factor β_I.
- They have "deep pockets".
- They consume long-term bonds as they mature.

Intermediaries

- Intermediaries are infinitely lived and risk-neutral.
- Discount factor β_I.
- They have "deep pockets".
- They consume long-term bonds as they mature.
- Their value for buying b long-term bonds from a HH:

$$W(b) = \underbrace{\beta_I \delta b}_{\text{consumption one period after trade}} + \underbrace{\beta_I^2 (1-\delta)\delta b}_{\text{consumption two period after trade}} + \underbrace{\beta_I^3 (1-\delta)^2 \delta b}_{\text{consumption three periods after trade}} + \ldots$$

$$= \frac{\beta_I \delta b}{1 - \beta_I (1-\delta)}$$

Households

- Live for T periods.

Households

- Live for T periods.
- Risk averse.

Households

- Live for T periods.
- Risk averse.
- Discount factor β.

Households

- Live for T periods.
- Risk averse.
- Discount factor β.
- Exogenous Markov process for labor earnings y.

Households

- Live for T periods.
- Risk averse.
- Discount factor β.
- Exogenous Markov process for labor earnings y.
- Make consumption-savings choice with zero borrowing limit.

Households

- Live for T periods.
- Risk averse.
- Discount factor β.
- Exogenous Markov process for labor earnings y.
- Make consumption-savings choice with zero borrowing limit.
- Hold cash a.

Households

- Live for T periods.
- Risk averse.
- Discount factor β.
- Exogenous Markov process for labor earnings y.
- Make consumption-savings choice with zero borrowing limit.
- Hold cash a.
- Hold long-term illiquid bonds b.

Households

- Live for T periods.
- Risk averse.
- Discount factor β.
- Exogenous Markov process for labor earnings y.
- Make consumption-savings choice with zero borrowing limit.
- Hold cash a.
- Hold long-term illiquid bonds b.
- Meet an intermediaries with probability γ and can liquidate fraction ℓ of their long-term illiquid bonds.

Households

- Live for T periods.
- Risk averse.
- Discount factor β.
- Exogenous Markov process for labor earnings y.
- Make consumption-savings choice with zero borrowing limit.
- Hold cash a.
- Hold long-term illiquid bonds b.
- Meet an intermediaries with probability γ and can liquidate fraction ℓ of their long-term illiquid bonds.
- The HHs value function is:

$$V_t(y,a,b) \equiv \gamma \underbrace{V_t^M(y,a,b)}_{\text{value if matched}} + (1-\gamma) \underbrace{V_t^U(y,a,b)}_{\text{value if matched}}$$

Unmatched Households Value Function

- A HH that is not matched with an intermediary choose consumption c, cash tomorrow a', and purchase new long-term bonds $\tilde{b}'$ to maximize utility:

$$V_t^U(y, a, b) = \max_{c, a', \tilde{b}'} \left\{ \underbrace{u(c)}_{\text{instantaneous value}} + \underbrace{\beta E[V_{t+1}(y', a', b')]}_{\text{continuation value}} \right\}$$

subject to

$$c + a' + \underbrace{\tilde{b}'}_{\text{new LT bonds}} = y + a + \underbrace{\delta b(1+r)}_{\text{matured LT bonds}}$$

$$b' = \underbrace{\tilde{b}'}_{\text{new LT bonds}} + \underbrace{(1-\delta) b(1+r)}_{\text{unmatured LT bonds}}$$

$$a', \tilde{b}' \geq 0$$

Matched Households Value Function

- A HH that is matched with an intermediary can either buy LT bonds (same problem as unmatched) or sell LT bonds:

$$V_t^M(y,a,b) = \max\left\{ \underbrace{V_t^U(y,a,b)}_{\text{buying LT bonds}}, \underbrace{\max_{c,a',\ell}\left\{u(c) + \beta E[V_{t+1}(y', a' + \overbrace{P(\ell(1-\delta)b(1+r))}^{\text{proceeds from selling LT bonds}}, b')]\right\}}_{\text{selling LT bonds}} \right\}$$

subject to

$$c + a' = y + a + \delta b(1+r)$$
$$b' = \underbrace{(1-\ell)(1-\delta)b(1+r)}_{\text{unsold, unmatured LT bonds}}, \quad a' \geq 0, \ell \in [0,1]$$

Nash Bargaining

- The matched household has $\hat{b}' \equiv \ell(1-\delta)b(1+r)$ LT bonds to sell.

Nash Bargaining

- The matched household has $\hat{b}' \equiv \ell(1-\delta)b(1+r)$ LT bonds to sell.
- The value to the intermediary is $W(\hat{b}') - P$.

Nash Bargaining

- The matched household has $\hat{b}' \equiv \ell(1-\delta)b(1+r)$ LT bonds to sell.
- The value to the intermediary is $W(\hat{b}') - P$.
- The outside option for the intermediary is zero.

Nash Bargaining

- The matched household has $\hat{b}' \equiv \ell(1-\delta)b(1+r)$ LT bonds to sell.
- The value to the intermediary is $W(\hat{b}') - P$.
- The outside option for the intermediary is zero.
- The value to the household is $\beta E[V_{t+1}(y', a' + P, b')]$.

Nash Bargaining

- The matched household has $\hat{b}' \equiv \ell(1-\delta)b(1+r)$ LT bonds to sell.
- The value to the intermediary is $W(\hat{b}') - P$.
- The outside option for the intermediary is zero.
- The value to the household is $\beta E[V_{t+1}(y', a' + P, b')]$.
- The outside option for the household is the unmatched value function: $\beta E[V_{t+1}(y', a', b' + \hat{b}')]$.

Nash Bargaining (con't)

- Nash bargaining solves:

$$\max_{P} \left[\underbrace{\beta E[V_{t+1}(y', a' + P, b') - \beta E[V_{t+1}(y', a', b' + \hat{b}')]}_{\text{surplus of HH}} \right]^{\theta}$$

$$\times \left[\underbrace{W(\hat{b}') - P}_{\text{surplus of intermediary}} \right]^{1-\theta}$$

FOC

Nash Bargaining (con't)

$$\underbrace{\begin{aligned}&\theta E[V_{a,t+1}(y', a' + P, b')] \left[W(\hat{b}') - P \right] \\ &-(1-\theta) \left[E[V_{t+1}(y', a' + P, b') - E[V_{t+1}(y', a', b' + \hat{b}')] \right]\end{aligned}}_{\equiv g(P)} = 0$$

We can solve this pricing condition numerically by:

- Numerically calculating $E[V_{a,t+1}(y', a' + P, b')]$.

Nash Bargaining (con't)

$$\underbrace{\theta E[V_{a,t+1}(y', a' + P, b')] \left[W(\hat{b}') - P \right] - (1-\theta) \left[E[V_{t+1}(y', a' + P, b') - E[V_{t+1}(y', a', b' + \hat{b}')] \right]}_{\equiv g(P)} = 0$$

We can solve this pricing condition numerically by:

- Numerically calculating $E[V_{a,t+1}(y', a' + P, b')]$.
- Using root solver.

Nash Bargaining (con't)

$$\underbrace{\theta E[V_{a,t+1}(y', a' + P, b')]\left[W(\hat{b}') - P\right] -(1-\theta)\left[E[V_{t+1}(y', a' + P, b') - E[V_{t+1}(y', a', b' + \hat{b}')]\right]}_{\equiv g(P)} = 0$$

We can solve this pricing condition numerically by:

- Numerically calculating $E[V_{a,t+1}(y', a' + P, b')]$.
- Using root solver.
- This step really slows down solving the model.

Nash Bargaining (con't)

$$\underbrace{\begin{aligned}&\theta E[V_{a,t+1}(y', a' + P, b')]\left[W(\hat{b}') - P\right] \\ &-(1-\theta)\left[E[V_{t+1}(y', a' + P, b') - E[V_{t+1}(y', a', b' + \hat{b}')]\right]\end{aligned}}_{\equiv g(P)} = 0$$

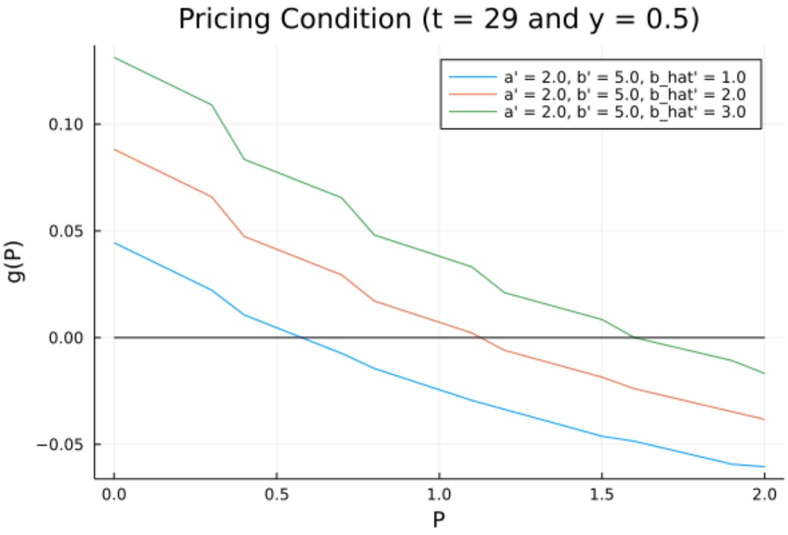
We can solve this pricing condition numerically by:

- Numerically calculating $E[V_{a,t+1}(y', a' + P, b')]$.
- Using root solver.
- This step really slows down solving the model.
- Use first-order Newton approximation around zero?

$$P \approx \frac{-g(0)}{g'(0)}$$

(needs 2d cubic interpolation instead of bilinear)

Pricing Condition



Computational Strategy

- Start at period T, where households eat all labor earnings, cash, and matured long-term bonds. There's no $T+1$ period, so matched HHs do not liquidate any LT bonds.

Computational Strategy

- Start at period T, where households eat all labor earnings, cash, and matured long-term bonds. There's no $T+1$ period, so matched HHs do not liquidate any LT bonds.
- For any period t,

Computational Strategy

- Start at period T, where households eat all labor earnings, cash, and matured long-term bonds. There's no $T+1$ period, so matched HHs do not liquidate any LT bonds.
- For any period t,
  - Solve unmatched HH problem.

Computational Strategy

- Start at period T, where households eat all labor earnings, cash, and matured long-term bonds. There's no $T+1$ period, so matched HHs do not liquidate any LT bonds.

- For any period t,
  - Solve unmatched HH problem.
  - Solve matched HH problem conditional on selling LT bonds. Guess ℓ and a', solve price condition (with root solver) for P, evaluate value function, and choose optimum.

Computational Strategy

- Start at period T, where households eat all labor earnings, cash, and matured long-term bonds. There's no $T+1$ period, so matched HHs do not liquidate any LT bonds.
- For any period t,
  - Solve unmatched HH problem.
  - Solve matched HH problem conditional on selling LT bonds. Guess ℓ and a', solve price condition (with root solver) for P, evaluate value function, and choose optimum.
  - Matched HH problem solution is max of unmatched solution and matched solution conditional on selling.

Computational Strategy

- Start at period T, where households eat all labor earnings, cash, and matured long-term bonds. There's no $T + 1$ period, so matched HHs do not liquidate any LT bonds.
- For any period t,
    - Solve unmatched HH problem.
    - Solve matched HH problem conditional on selling LT bonds. Guess ℓ and a', solve price condition (with root solver) for P, evaluate value function, and choose optimum.
    - Matched HH problem solution is max of unmatched solution and matched solution conditional on selling.
- Use linear interpolations due to two continuous state variables.

Computational Strategy

- Start at period T, where households eat all labor earnings, cash, and matured long-term bonds. There's no $T+1$ period, so matched HHs do not liquidate any LT bonds.
- For any period t,
  - Solve unmatched HH problem.
  - Solve matched HH problem conditional on selling LT bonds. Guess ℓ and a', solve price condition (with root solver) for P, evaluate value function, and choose optimum.
  - Matched HH problem solution is max of unmatched solution and matched solution conditional on selling.
- Use linear interpolations due to two continuous state variables.
- Coarse grid for solving the model; finer grid for simulating the model.

Calibration

- Average maturity of outstanding U.S. Treasury is 70 months, so $\delta = 0.17143$. more

Calibration

- Average maturity of outstanding U.S. Treasury is 70 months, so $\delta = 0.17143$. more
- CRRA utility with coefficient of relative risk aversion equal to 2.

Calibration

- Average maturity of outstanding U.S. Treasury is 70 months, so $\delta = 0.17143$. more
- CRRA utility with coefficient of relative risk aversion equal to 2.
- Annual frequency.

Calibration

- Average maturity of outstanding U.S. Treasury is 70 months, so $\delta = 0.17143$. more
- CRRA utility with coefficient of relative risk aversion equal to 2.
- Annual frequency.
- Two state Markov earnings process with $y_e = 1.0$ and $y_u = 0.5$ with $\pi_{ee} = 0.9$ and $\pi_{uu} = 0.5$ to match employment rate and employment duration.

Calibration

- Average maturity of outstanding U.S. Treasury is 70 months, so $\delta = 0.17143$. more
- CRRA utility with coefficient of relative risk aversion equal to 2.
- Annual frequency.
- Two state Markov earnings process with $y_e = 1.0$ and $y_u = 0.5$ with $\pi_{ee} = 0.9$ and $\pi_{uu} = 0.5$ to match employment rate and employment duration.
- $T = 30$.

Calibration

- Average maturity of outstanding U.S. Treasury is 70 months, so $\delta = 0.17143$. more
- CRRA utility with coefficient of relative risk aversion equal to 2.
- Annual frequency.
- Two state Markov earnings process with $y_e = 1.0$ and $y_u = 0.5$ with $\pi_{ee} = 0.9$ and $\pi_{uu} = 0.5$ to match employment rate and employment duration.
- $T = 30$.
- $\beta = \beta_I = 0.99$

Calibration

- Average maturity of outstanding U.S. Treasury is 70 months, so $\delta = 0.17143$. more
- CRRA utility with coefficient of relative risk aversion equal to 2.
- Annual frequency.
- Two state Markov earnings process with $y_e = 1.0$ and $y_u = 0.5$ with $\pi_{ee} = 0.9$ and $\pi_{uu} = 0.5$ to match employment rate and employment duration.
- $T = 30$.
- $\beta = \beta_I = 0.99$
- $\theta = 0.5$

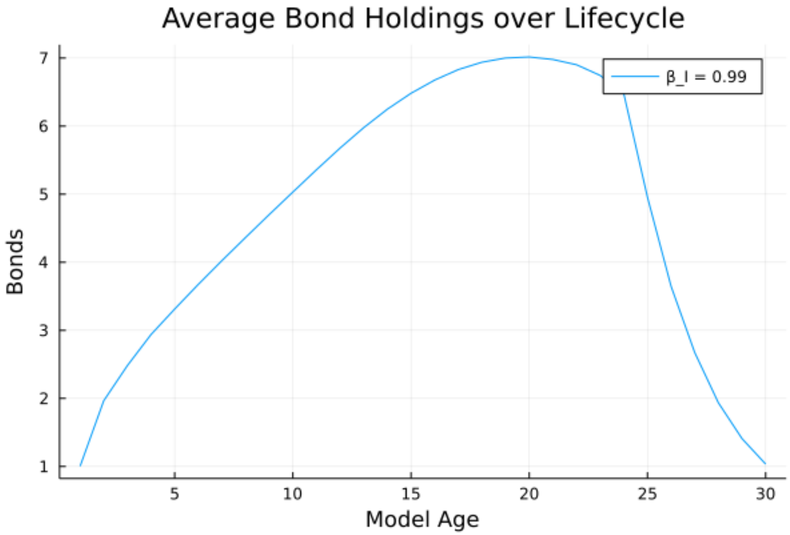
Calibration

- Average maturity of outstanding U.S. Treasury is 70 months, so $\delta = 0.17143$. more
- CRRA utility with coefficient of relative risk aversion equal to 2.
- Annual frequency.
- Two state Markov earnings process with $y_e = 1.0$ and $y_u = 0.5$ with $\pi_{ee} = 0.9$ and $\pi_{uu} = 0.5$ to match employment rate and employment duration.
- $T = 30$.
- $\beta = \beta_I = 0.99$
- $\theta = 0.5$
- $r = 0.1$

Calibration

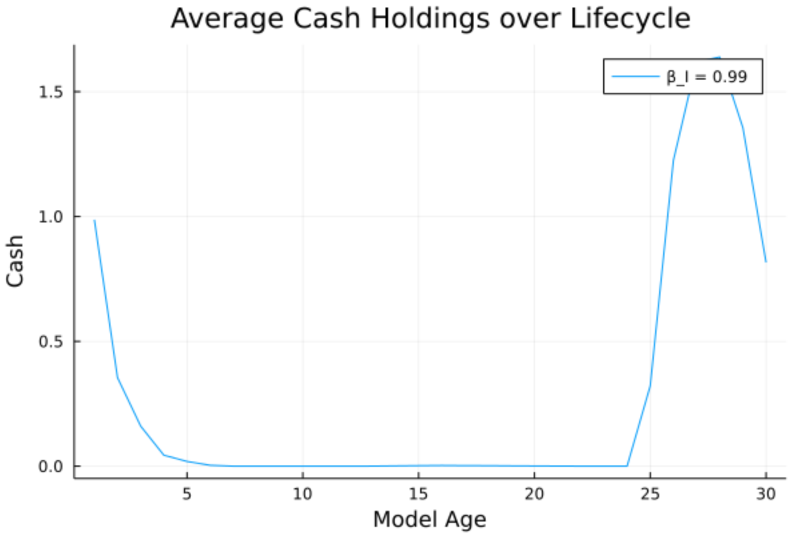
- Average maturity of outstanding U.S. Treasury is 70 months, so $\delta = 0.17143$. more
- CRRA utility with coefficient of relative risk aversion equal to 2.
- Annual frequency.
- Two state Markov earnings process with $y_e = 1.0$ and $y_u = 0.5$ with $\pi_{ee} = 0.9$ and $\pi_{uu} = 0.5$ to match employment rate and employment duration.
- $T = 30$.
- $\beta = \beta_I = 0.99$
- $\theta = 0.5$
- $r = 0.1$
- $\gamma = 0.2$

Baseline

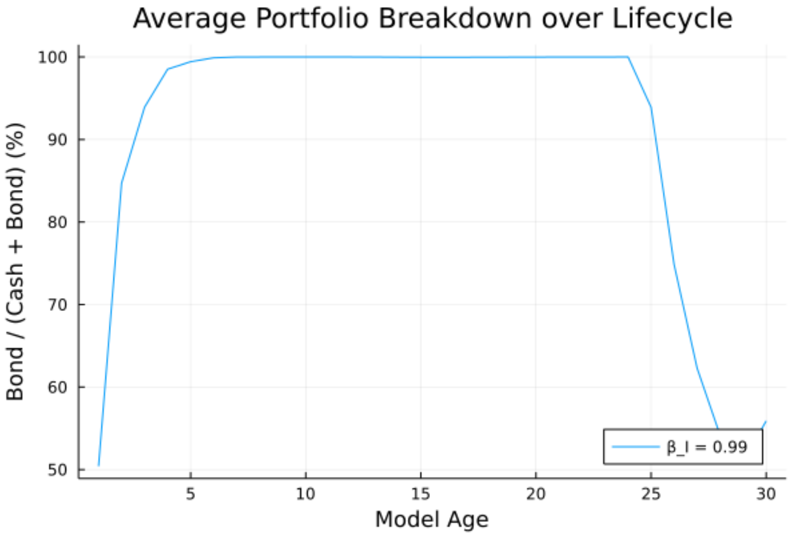


Average Bond Holdings over Lifecycle
β_l = 0.99
Bonds
Model Age
1
2
3
4
5
6
7
5
10
15
20
25
30

Baseline


Average Cash Holdings over Lifecycle
β_l = 0.99
Cash
1.5
1.0
0.5
0.0
5
10
15
20
25
30
Model Age

Baseline



Average Portfolio Breakdown over Lifecycle
Bond / (Cash + Bond) (%)
100
90
80
70
60
50
5
10
15
20
25
30
Model Age
β_l = 0.99

Policy Experiments changing Intermediation Frictions

	Baseline	Policy Experiment #1	Policy Experiment #2
β_I	0.99	0.9	0.99
γ	0.2	0.2	0.1

- Policy Experiment #1: Intermediaries are less patient $\implies$ consuming b over time less $\implies$ price drops

$$\frac{\partial W(b)}{\partial \beta_I} = \frac{\delta b}{[1 - \beta_I(1-\delta)]^2} > 0$$

Policy Experiments changing Intermediation Frictions

	Baseline	Policy Experiment #1	Policy Experiment #2
β_I	0.99	0.9	0.99
γ	0.2	0.2	0.1

- Policy Experiment #1: Intermediaries are less patient $\implies$ consuming b over time less $\implies$ price drops

$$\frac{\partial W(b)}{\partial \beta_I} = \frac{\delta b}{[1 - \beta_I(1-\delta)]^2} > 0$$

- Policy Experiment #2: HH is less likely to meet intermediate.

References

Brandsas, Eirik (2020) “Stock Market Participation and Exit: The Role of Homeownership,” Working Paper.

Duffie, Darrell (2020) “Still the World’s Safe Haven? Redesigning the U.S. Treasury Market After the COVID-19 Crisis,” Hutchins Center Working Paper #62, June 2020.

Kaplan, Greg and Giovanni L. Violante (2014). “A Model of the Consumption Response to Fiscal Stimulus Payments,” Econometrica, Vol. 82, No. 4, July 2014, 1199-1239.

Rios-Rull, Jose-Victor and Virginia Sanchez-Marcos (2008). “An Aggregate Economy with different Size Houses,” Journal of European Economic Association, Vol. 6, No. 2/3, Proceedings of the Twenty-Second Annual Congress of the European Economic Association (Apr. - May, 2008), pp. 705-714

Nash Bargaining (con't)

- First order condition:

$$\theta \left[\beta E[V_{t+1}(y', a' + P, b') - \beta E[V_{t+1}(y', a', b' + \hat{b}')] \right]^{\theta - 1}$$

$$\times \beta E[V_{a,t+1}(y', a' + P, b')] \left[W(\hat{b}') - P \right]^{1-\theta}$$

$$= (1 - \theta) \left[W(\hat{b}') - P \right]^{-\theta}$$

$$\times \left[\beta E[V_{t+1}(y', a' + P, b') - \beta E[V_{t+1}(y', a', b' + \hat{b}')] \right]^{\theta}$$

Calibration δ

Average maturity of LT bond is:

$$\delta + 2(1-\delta)\delta + 3(1-\delta)^2\delta + ... = \delta \sum_{t=1}^{\infty} t(1-\delta)^{t-1}$$

For average maturity of $70/12 \approx 5.833 \implies \delta \approx 0.17143$