ECON 712 - PS 3

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1. Consider the following overlapping generations problem. In each period $t = 1, 2, 3, ...$ a new generation of 2 period lived households are born. Each generation has a unitary mass. There is a unit measure of initial old who are endowed with $\bar{M} > 0$ units of fiat money. Each generation is endowed with w_1 in youth and w_2 in old age of non-storable consumption goods where $w_1 > w_2$. There is no commitment technology to enforce trades. The utility function of a household of generation $t \geq 1$ is $U(c_t^t, c_{t+1}^t) = \ln(c_t^t) + \ln(c_{t+1}^t)$ where (c_t^t, c_{t+1}^t) is consumption of a household of generation t in youth (i.e., in period t) and old age (i.e., in period $t + 1$). The preference of the initial old are given by $U(c_1^0) = \ln(c_1^0)$ where c_1^0 is consumption by a household of the initial old.

(a) State and solve the planner problem.

(b) State the representative household's problem in period $t \geq 0$. Try to write the budget constraints in real terms.

(c) Define and solve for an autarkic equilibrium, assuming that it exists.

(d) Define and solve for a competitive equilibrium assuming valued money but with $w_2 = 0$.

(e) Compare the solutions to the plannners problem, the autarky equilibrium and the stationary monetary competitive equilibrium with valued money, all with $w_2 = 0$.

(f) What happens to consumption, money demand and prices in a competitive equilibrium with valued money if the initial money supply is halved, i.e. $\bar{M}' = \frac{\bar{M}}{2}$. Keep the assumption that $w_2 = 0$.

2. Plot the trade offer curves for the following utility functinos where the endowment is (w_1, w_2) for goods 1 and 2, respectively.

(a) $U = 10c_1 - 4c_1^2 + 4c_2 - c_2^2, (w1, w_2) = (0, 2)$

(b) $U = \min 2c_1 + c_2, c_1 + 2c_2, (w1, w_2) = (1, 0)$

(c) $U = \min 2c_1 + c_2, c_1 + 2c_2, (w1, w_2) = (1, 10)$

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