ECON 899A - Problem Set 3

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In this problem set we study the macroeconomic consequences of eliminating Social Security in the U.S. To do so, we set up and solve a simple general equilibrium overlapping generations model. This model is a simplified version of the model by Conesa and Krueger (1999), Social Security Reform with Heterogeneous Agents, *Review of Economic Dynamics*, 2, p.757-95.

Model Setup

Each period a continuum of agents is born and live for $N = 66$ periods. The population growth rate is $n = 0.011$ per year (which is the model period length). All agents die deterministically at age N; thus, there is no stochastic mortality. Newly born agents (age 1, which corresponds to real life age 20) hold no initial assets, $a_1 = 0$. Workers supply labor to a labor market. At age $J^R = 46$, workers retire and start receiving pension benefits, b. Pension benefits are financed by a proportional labor income tax $\theta = 0.11$.

The instantaneous utility function of a worker is given by:

$$u^W(c,l) = \frac{[c^\gamma(1-l)^{1-\gamma}]^{1-\sigma}}{1-\sigma}$$

with c - consumption and l - labor. The weight on consumption, γ, is 0.42, and the coefficient of relative risk aversion, σ, is 2. The instantaneous utility function of a retired agent reads:

$$u^R(c) = \frac{c^{(1-\sigma)\gamma}}{1-\sigma} \tag{1}$$

Worker's productivity at age j is given by $e(z,j) = z \times \eta_j$, where η_j is a deterministic age-efficiency profile and z - idiosyncratic productivity. Idiosyncratic productivity can be either High with $z_H = 3.0$ or Low with $z_L = 0.5$. At birth, each individual receives a realization of the random productivity z from its ergodic distribution $(0.2037, 0.7963)$. The persistence probabilities are $\pi_{HH} = 0.9261$ and $\pi_{LL} = 0.9811$ (this specification of the idiosyncratic risk is referred to as the "asymmetric case" in the paper).[1]

There is a constant returns to scale production technology $Y = K^\alpha L^{1-\alpha}$ with Y - aggregate output, K - aggregate capital stock and L - aggregate effective labor supply. The capital share, α, is 0.36 and depreciation rate, δ, is 0.06. The labor and capital markets are perfectly competitive.

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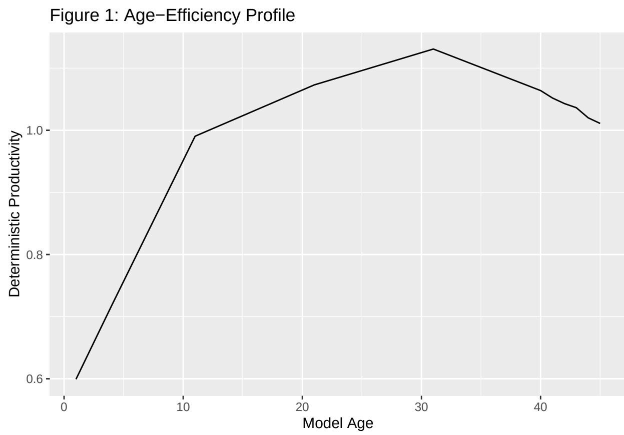
*This problem set is for ECON 899A Computational Economics taught by Dean Corbae with assistance from Philip Coyle at UW-Madison. I worked on this problem set with a study group of Michael Nattinger, Sarah Bass, and Xinxin Hu.

[1] The deterministic age-efficiency profile is plotted in figure 1; the corresponding data can be found in the file "ef.txt".



Dynamic Programming Problem

In the first part of the exercise you are asked to solve for the agents optimization problem. Agents face the interest rate, r, the wage, w, and the social security benefit, b, which they take as given. For now, just assume that $w = 1.05$, $r = 0.05$, and $b = 0.2$. We will see below, how these variables are determined in a general equilibrium.

We can solve for the dynamic programming problem of the agents by starting in the last period of their life ($j = N$) and iterating backwards until $j = 1$. The dynamic programming problem of a retired agent reads:

$$V_j(a) = \max_{a' \geq 0} \{u^R((1 + r)a + b - a') + \beta V_{j+1}(a')\} \tag{2}$$

with $V_{N+1}(a) = 0$ for all a. In the expression above, V_j stands for the value function of agent at age j. A prime denotes tomorrow's variables. The discount factor $\beta = 0.97$. Observe that we have plugged in agent's budget constraint into the utility function. To solve this problem on the computer, we tabulate the value function, $V(a)$, in a finite number n_a of points. The maximization over a' then occurs over the values in the set $\{a^1, a^2, ..., a^{n_a}\}$ with $a_1 = 0$ (which means that we rule out borrowing). Note that in the very last period, the value function, $V_N(a)$, is given by (1) with $c = (1 + r)a + b$. For a given table of values for $V_{j+1}(a')$ on the capital grid, retired agent's saving at age j can be found by choosing a', which gives the largest value for the right-hand side of (2) given a, which we store as $a'_j(a)$.

The dynamic programming problem of a worker is given by:

$$V_j(a, z) = \max_{a' \geq 0, 0 \leq l \leq 1} \{u^W(w(1 - \theta)e(z, \eta_j)l + (1 + r)a - a', l) + \beta E[V_{j+1}(a', z')|(a, z, j)]\} \tag{3}$$

subject to $0 \geq l \geq 1$. The social security tax rate θ is 0.11. The dynamic programming problem of a worker involves maximization over an additional control, which is labor, l. We can, however, make use of the household's first-order condition with respect to l combined with the budget constraint to arrive at the following expression for l (derived in the appendix).

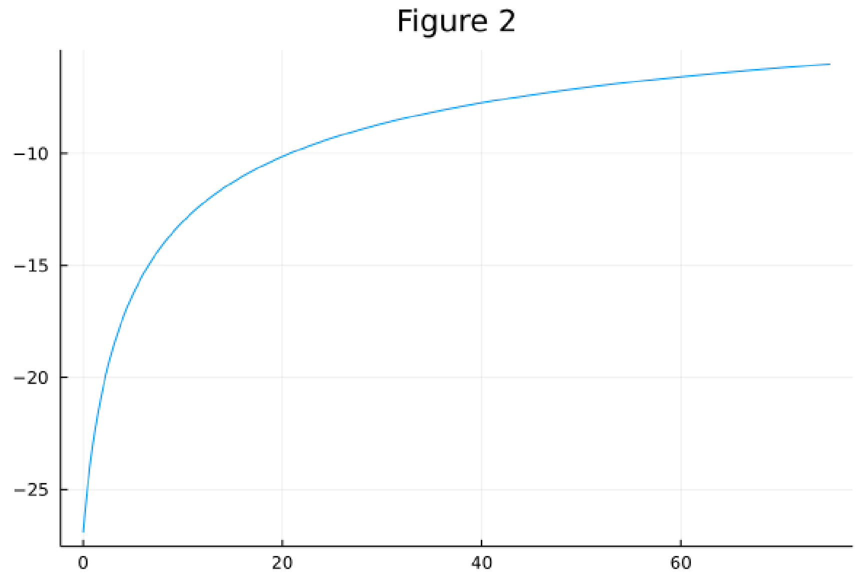
$$l = \frac{\gamma(1 - \theta)e(z, \eta_j)w - (1 - \gamma)[(1 + r)a - a']}{(1 - \theta)we(z, \eta_j)}$$

Thus, for a given combination of (z, a, a'), the expression above gives the optimal solution for labor. Then, for a given table of values for $V_{j+1}(a', z')$, worker's saving at age j can be found by choosing a', which gives the largest value for the right-hand side of (3) given a and z, which we store as $a'_j(a, z)$. We store the corresponding optimal labor supply as $l_j(a, z)$.

Exercise 1

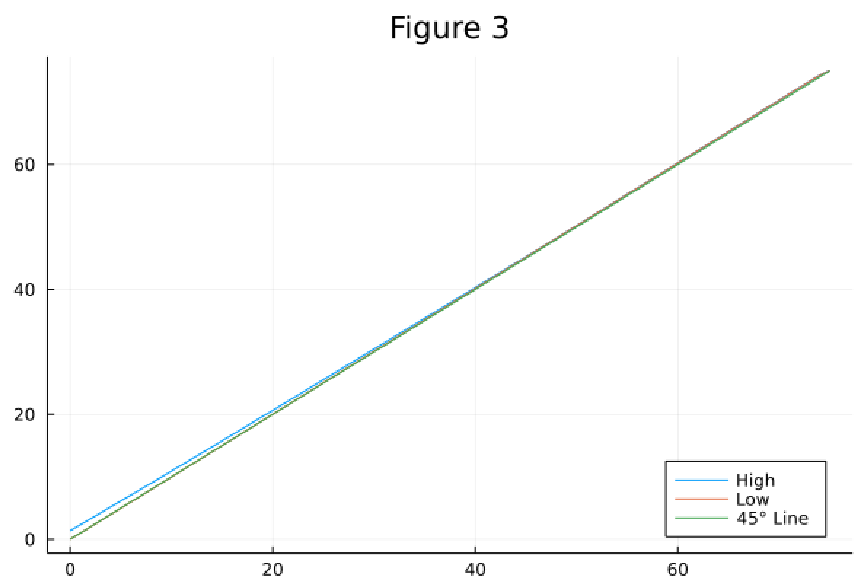
Solve the dynamic programming problem of retirees and workers. Plot the value function over a for a retired agent at the model-age 50. Is it increasing and concave?

Yes, the value function for a retired agent at the model-age 50 is increasing and concave (figure 2).



Plot the savings function for a worker at the model-age 20, $a'_{20}(z, a)$. Is saving increasing in a? Is it increasing in z?

The policy function for a worker at the model-age 20 $a'_{20}(z, a)$ is increasing in a and increasing in z (figure 3).



The savings function for a worker at the model-age 20 $a'_{20}(z, a) - a$ is weakly increasing in z and decreasing in a when $l > 0$ and increasing in a when $l = 0$ (figure 4). For both high and low productivity agents, at a certain asset level, they stop supplying labor. When they are still supplying labor, savings are increasing in z and decreasing in a. When they are not supplying labor, savings are constant in z and increasing in a. (Ignore the drop at the far rightside; that is an artifact of the asset grid.)

Figure 4

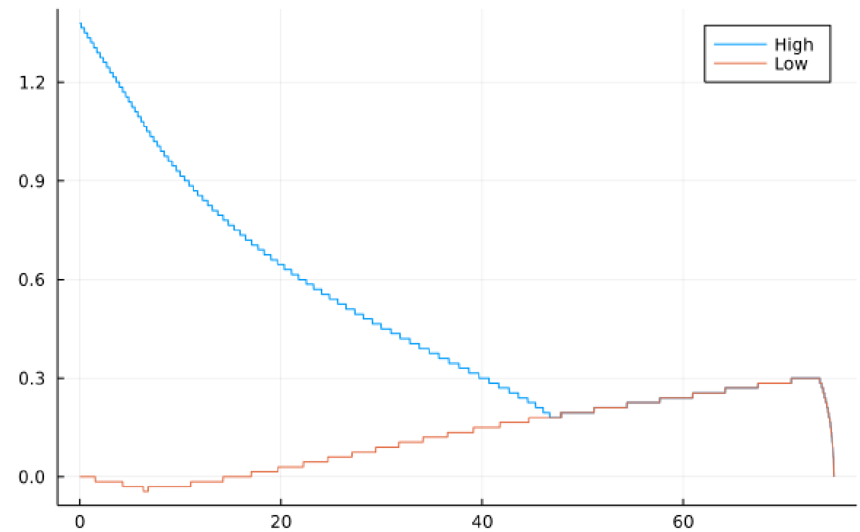
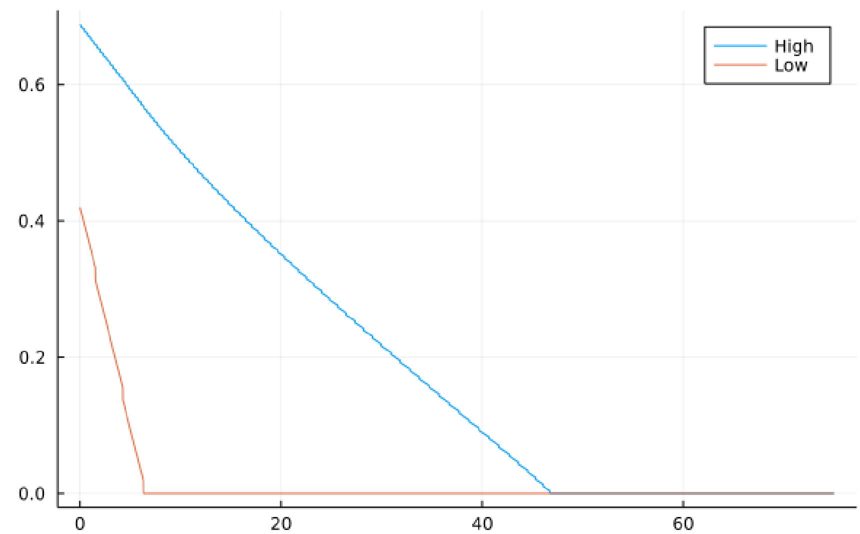


Figure 5



Exercise 2

After solving for agent's dynamic programming problem, solve for the steady-state distribution of agents over age, productivity and asset holdings, $F_j(z, a)$. Find first the relative sizes of each cohort of age j (denoted by μ_j) using the expression below:

$$\mu_{i+1} = \frac{\mu_i}{1+n}$$

for $i = 1, ..., N-1$ with any $\mu_1 = \tilde{\mu}_1 > 0$. Then normalize μ, so that it sums up to 1 across all age groups. Finally, start with the newborn generation with zero wealth: given its distribution, $F_1(z^H, 0) = \mu_1 \times 0.2037$ and $F_1(z^L, 0) = \mu_1 \times 0.7963$, compute the distribution of agents over asset holdings at subsequent ages by applying the optimal decision rules.

Exercise 3

We have computed the decision rules and the stationary distribution for given prices. There is no guarantee that at these prices the supply of assets and labor by the agents equals the demand for capital and labor by the firms. In order to find the equilibrium prices, we use the "guess and verify" method. First, we make initial guesses on aggregate capital and aggregate labor, demanded by the firm, which we denote K^0 and L^0, respectively. They imply the interest rate r^0 and wage w^0, since markets are perfectly competitive. Observe that from the guess on K and L we can compute the pension benefit, b, using the government budget constraint:

$$b = \frac{\theta w^0 L^0}{\sum_{j=J^R}^{N} \mu_j}$$

Given r^0 and w^0, we compute the optimal decision rules and the stationary distribution. Finally, we verify, if our guess was correct by computing the aggregate assets and labor supplied by households:

$$K^{new} = \sum_{j=1}^{N} \sum_{m=1}^{n_a} \sum_{z \in \{z^L, z^H\}} F_j(z, a_m) a_m$$

$$L^{new} = \sum_{j=1}^{J^R-1} \sum_{m=1}^{n_a} \sum_{z \in \{z^L, z^H\}} F_j(z, a_m) e(z, \eta_j) l_j(z, a_m)$$

If the guess was "far off" the obtained values, we update our initial guess with $K^1 = 0.99K^0 + 0.01K^{new}$ and $L^1 = 0.99L^0 + 0.01L^{new}$ and repeat the procedure. We proceed so until the guess and the updated values for K and L are "sufficiently close".

You are now asked to evaluate the macroeconomic consequences of eliminating social security.

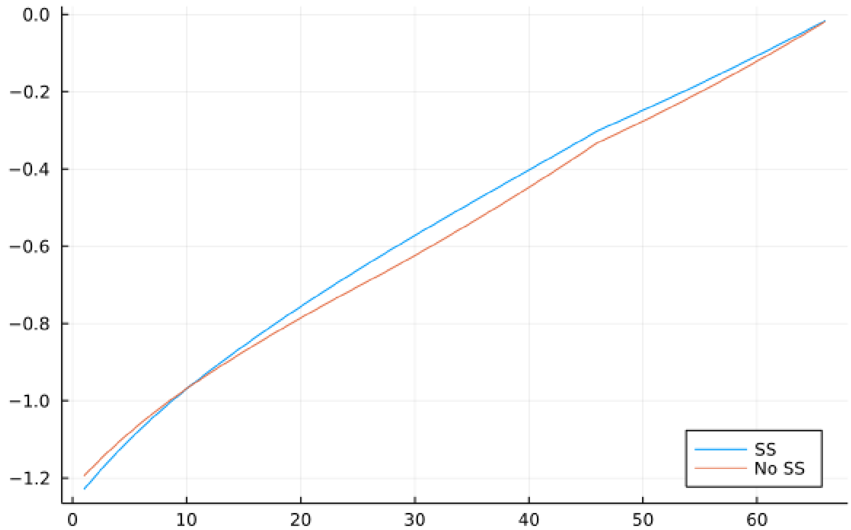
Table 1: Results Comparison

	(1)	(2)	(3)	(4)	(5)	(6)
theta	0.110	0.000	0.110	0.000	0.110	0.000
gamma	0.420	0.420	0.420	0.420	1.000	1.000
z_H	3.000	3.000	0.500	0.500	3.000	3.000
k	3.390	4.596	1.060	1.303	7.446	10.455
l	0.344	0.365	0.160	0.169	0.754	0.754
w	1.459	1.593	1.263	1.335	1.459	1.649
r	0.023	0.011	0.048	0.037	0.023	0.007
b	0.226	0.000	0.091	0.000	0.496	0.000
welfare	-35.896	-37.419	-45.105	-45.114	-23.160	-25.753
cv	0.603	0.669	1.263	1.107	0.611	0.709

1. First, solve for the benchmark model with social security. Is this economy dynamically efficient (compare the interest rate with the implicit return from social security, which is equal to the population growth rate)? Now eliminate social security by setting $\theta = 0$. Observe how aggregate capital accumulation and labor supply change as a result of the tax reform. Provide intuition in terms of insurance and output efficiency. How does aggregate welfare change? Who benefits and who loses due to this reform? How does the reform affect cross-sectional wealth inequality? You can use table 1 to support your answers.

- Column (1) of Table 1 shows the benchmark model with social security.
- Yes, the economy is dynamically efficient. The interest rate is $r \approx 0.023$, which exceeds the population growth rate of $n = 0.011$.
- Column (2) of Table 1 shows the benchmark model without social security.
- Aggregate capital increases dramatically from 3.390 to 4.596 when social security is removed. Without social security benefits during retirement, households save more when they are working.
- Aggregate labor increases slightly from 0.344 to 0.365 when social security is removed. In removing social security, we have also removed the distortionary labor tax that pays for it, so labor supply increases.
- Aggregate welfare drops from -35.896 to -37.419 when social security is removed.
- Figure 5 shows the average value function by age. The young (model-age of 10 or less) prefer the world without social security; everyone else preferred the world with social security.
- The coefficient of wealth variation is increases from 0.603 to 0.669, so the lack of social security increases cross-sectional wealth inequality.

Figure 6



2. In the second experiment, there is no idiosyncratic risk. Assume that at each age $j, z^L = z^H = 0.5$. First, compute the aggregate variables for the case with social security. How does the aggregate capital stock change relative to the benchmark model? Provide intuition in terms of capital as a buffer stock. Then, eliminate social security. How does the aggregate welfare change? What can you conclude about social security as an insurance device against idiosyncratic risk? Comment on the extent, to which these welfare comparisons across steady states are meaningful or misleading.

- Column (3) of Table 1 shows the results without idiosyncratic productivity risk.
- Aggregate capital drops from 3.390 to 1.060. Since there is no idiosyncratic productivity risk, agents do not want to a capital buffer to insure against a low shock.
- Column (4) of Table 1 shows the results without both idiosyncratic productivity risk or social security.
- Aggregate welfare stays approximately the same at around -45.105 to -45.114.
- This suggests that the major benefit of social security is to insure against idiosyncratic productivity risk. Without idiosyncratic productivity risk, there is not any gain from social security.
- Comparisons between steady states are a good starting point, but a more complete analysis would include the transition between steady states. In this context, adding the transition will likely make things worse off because there are retired agents that have to deal with suddenly losing their expected social security benefit, so comparing steady states can provide a likely upper bound on the gains from welfare from the reform.

3. Consider the case, when labor supply is exogenous ($\gamma = 1$). Compare the distortionary effect of social security on the aggregate labor supply. How does the support for social security change with exogenous labor supply?

- Columns (5) and (6) of Table 1 shows the results when labor is exogenous with and without social security.
- All working agents provide 1 unit of labor, so the aggregate is 0.754 (i.e., the fraction of working agents).
- In addition, labor is exogenous so social security has no distortionary effect on labor supply.
- Aggregate welfare is drops from -23.160 to -25.753, so this does not support changing social security even with exogenous labor supply.

Appendix - Optimal Labor Supply

The dynamic programming problem of a worker is given by:

$$V_j(a,z) = \max_{a' \geq 0, 0 \leq l \leq 1} \{u^W(w(1-\theta)e(z,\eta_j)l + (1+r)a - a', l) + \beta E[V_{j+1}(a',z')|(a,z,j)]\}$$

$$\implies V_j(a,z) = \max_{a' \geq 0, 0 \leq l \leq 1} \left\{ \frac{[(w(1-\theta)e(z,\eta_j)l + (1+r)a - a')^{\gamma}(1-l)^{1-\gamma}]^{1-\sigma}}{1-\sigma} + \beta E[V_{j+1}(a',z')|(a,z,j)] \right\}$$

FOC [l]:

$$\begin{aligned}
0 &= [(w(1-\theta)e(z,\eta_j)l + (1+r)a - a')^{\gamma}(1-l)^{1-\gamma}]^{-\sigma} \\
&\quad \times [\gamma(w(1-\theta)e(z,\eta_j)l + (1+r)a - a')^{\gamma-1}(1-l)^{1-\gamma}w(1-\theta)e(z,\eta_j) \\
&\quad - (1-\gamma)(w(1-\theta)e(z,\eta_j)l + (1+r)a - a')^{\gamma}(1-l)^{-\gamma}] \\
\implies 0 &= [c^{\gamma}(1-l)^{1-\gamma}]^{-\sigma}[\gamma c^{\gamma-1}(1-l)^{1-\gamma}w(1-\theta)e(z,\eta_j) - (1-\gamma)c^{\gamma}(1-l)^{-\gamma}] \\
\implies c &= \frac{\gamma(1-l)w(1-\theta)e(z,\eta_j)}{1-\gamma}
\end{aligned}$$

Substitute into budget constraint:

$$\begin{aligned}
w(1-\theta)e(z,\eta_j)l + (1+r)a &= c + a' \\
\implies w(1-\theta)e(z,\eta_j)l + (1+r)a &= \frac{\gamma(1-l)w(1-\theta)e(z,\eta_j)}{1-\gamma} + a' \\
\implies l\left[w(1-\theta)e(z,\eta_j) + \frac{\gamma w(1-\theta)e(z,\eta_j)}{1-\gamma}\right] &= \frac{\gamma w(1-\theta)e(z,\eta_j)}{1-\gamma} + a' - (1+r)a
\end{aligned}$$

$$\begin{aligned}
\implies l &= \left[\frac{\gamma w(1-\theta)e(z,\eta_j)}{1-\gamma} + a' - (1+r)a\right]\left[w(1-\theta)e(z,\eta_j) + \frac{\gamma w(1-\theta)e(z,\eta_j)}{1-\gamma}\right]^{-1} \\
&= \frac{\gamma(1-\theta)e(z,\eta_j)w - (1-\gamma)[(1+r)a - a']}{(1-\theta)we(z,\eta_j)}
\end{aligned}$$