ECON 713A - Problem Set 2

Alex von Hafften*

3/6/2020

1. Assume that firm $k = 1, 2, ...$ in a competitive industry has cost $k^2 + q + q^2$ of output level q. (Here, k^2 is an escapable cost in the long run.) Then the long run industry saw tooth supply curve has positive output only for prices at least what level?

The average cost of firm k is

$$AC_k(q) = \frac{k^2 + q + q^2}{q} = \frac{k^2}{q} + 1 + q$$

Firm 1 has the lowest average cost:

$$AC_1(q) = \frac{1}{q} + 1 + q$$

Firm 1's average cost is minimized at $q = 1$. FOC:

$$0 = \frac{-1}{q^2} + 1 \implies q = 1$$

SOC:

$$\frac{\partial^2 AC_1}{\partial^2 q} = \frac{2}{q^3} > 0$$

For firm 1 to be in the market in the long run price needs to be at least average cost of producing $q = 1$:

$$P > \frac{1}{1} + 1 + 1 = 3$$

*I worked on this problem set with a study group of Michael Nattinger, Andrew Smith, and Ryan Mather. I also discussed problems with Sarah Bass, Emily Case, Danny Edgel, and Katherine Kwok.

2. Assume a continuum of potential iPhone developers indexed by the quality of their idea. Each developer has a fixed cost of 1 can produce software code according to the function $q = 2\theta x$, where x is quantity of variable input used by the developer. A unit of software code sells for a price of 1, the cost of using a quantity x of the input is x^2. The "number" (i.e. mass) of firms with ideas above $\theta > 0$ is given by $M(\theta) = \theta^{-\beta}$, where $\beta > 2$. Apple taxes developers' revenues at a percentage rate $0 < \tau < 1$.

(a) Derive the aggregate supply curve of developer code.

Developer profit is:

$$\pi(x) = (1-\tau)2\theta x - x^2 - 1$$

FOC $[x]$:

$$\frac{\partial \pi}{\partial x} = 0 \implies 2\theta(1-\tau) - 2x = 0 \implies x = \theta(1-\tau)$$

SOC:

$$\frac{\partial^2 \pi}{\partial^2 x} = -2 < 0$$

$$\pi(\theta) = 2\theta^2(1-\tau)^2 - \theta^2(1-\tau)^2 - 1 > 0 \implies \theta > \frac{1}{1-\tau}$$

The aggregate supply curve:

$$\int_{\frac{1}{1-\tau}}^{\infty} \theta\theta^{-\beta} d\theta = \left[\frac{\theta^{2-\beta}}{2-\beta}\right]_{\frac{1}{1-\tau}}^{\infty} = -\frac{(\frac{1}{1-\tau})^{2-\beta}}{2-\beta} = \frac{(1-\tau)^{\beta-2}}{\beta-2}$$

(b) When Apple raises its tax rate, what happens to the mass of developer firms, and the amount of code each produces?

When Apple raises its tax rate ($\uparrow \tau$), then the mass of developer firms decreases ($\uparrow \frac{1}{1-\tau}$) and the amount each produces decreases ($\downarrow \theta(1-\tau)$).

(c) What is Apple's revenue maximizing tax?

Apple's tax revenue is:

$$\frac{(1-\tau)^{\beta-2}}{\beta-2}\tau$$

FOC $[\tau]$:

$$-(1-\tau)^{\beta-3}\tau + \frac{(1-\tau)^{\beta-2}}{\beta-2} = 0 \implies \tau^* = \frac{1}{\beta-1}$$

(d) What happens if β rises (while τ is fixed)? Interpret this in terms of firm heterogeneity.

If β rises, the distribution of "idea" distribution shifts down. There is less heterogeneity in the quality of developer's ideas. If τ remains fixed, the number of developers decrease.

3. The only Ben and Jerry's in town faces different linear inverse demand curves $P = a - bQ$ for triple-chocolate-chunk ice cream and fruit-bowl-punch ice cream. It buys each at the same constant unit cost from its supplier. The inverse demand curves cross at an interior point, above marginal cost, with Fruit-bowl-punch having a higher price intercept than Triple-chunk. Does it follow that its price charged is higher on the Fruit-Bowl Punch?

Yes. The marginal revenue for each demand curve is:

$$PQ = aQ - bQ^2 \implies MR = a - 2bQ$$

Profit-maximization production is at $MR = MC$:

$$MC = a - 2bQ \implies Q^* = \frac{a - MC}{2b}$$

This quantity coorespondes to the following price:

$$P^* = a - b\frac{a - MC}{2b} = \frac{a - MC}{2}$$

If fruit-bowl-punch has a higher price intercept than Triple-chunk:

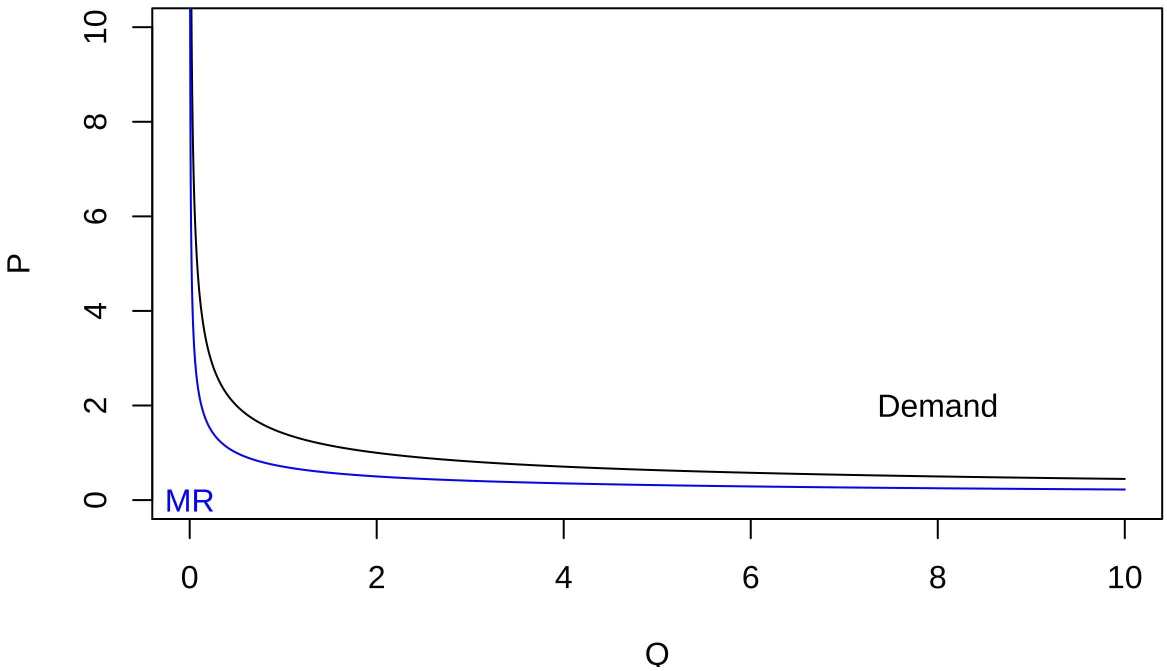
$$a_{FBP} > a_{TCC} \implies \frac{a_{FBP} - MC}{2} > \frac{a_{TCC} - MC}{2} \implies P^*_{FBP} > P^*_{TCC}$$

4. (a) Draw a negatively sloped demand curve with elasticity of magnitude greater than 1, and a corresponding marginal revenue curve.

From lecture, a demand (or supply) curve with constant elasticity takes the form:

$$
\begin{aligned}
Q &= kP^{\varepsilon} \\
\implies P &= Q^{1/\varepsilon}k^{-1/\varepsilon} \\
\implies TR = PQ &= Q^{1/\varepsilon+1}k^{-1/\varepsilon} \\
\implies MR &= (1/\varepsilon+1)Q^{1/\varepsilon}k^{-1/\varepsilon}
\end{aligned}
$$

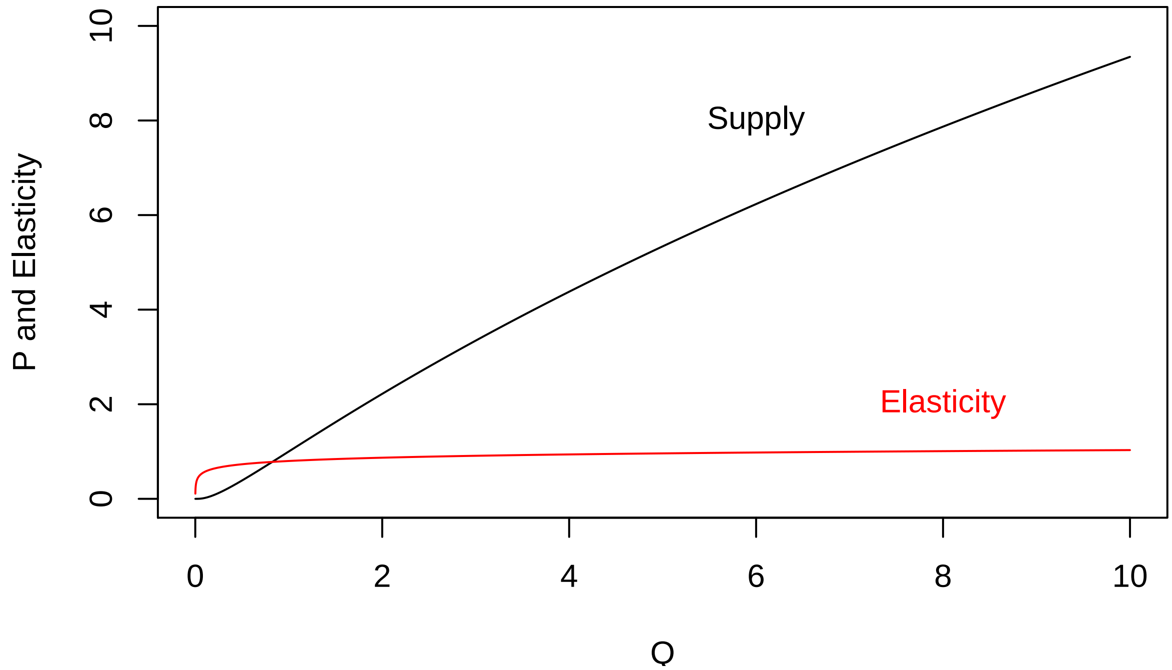
Below is plotted these demand and marginal revenue curves with $\varepsilon = -2$ and $k = 2$.



(b) Draw a supply curve with positive and rising elasticity.

A supply curve with constant elasticity takes the form: $Q = kP^{\varepsilon} \iff P = Q^{1/\varepsilon}k^{-1/\varepsilon}$. Let $\varepsilon(Q)$ be a positive increasing function, so $P = Q^{1/\varepsilon(Q)}k^{-1/\varepsilon(Q)}$ is a supply curve with positive and rising elasticity.

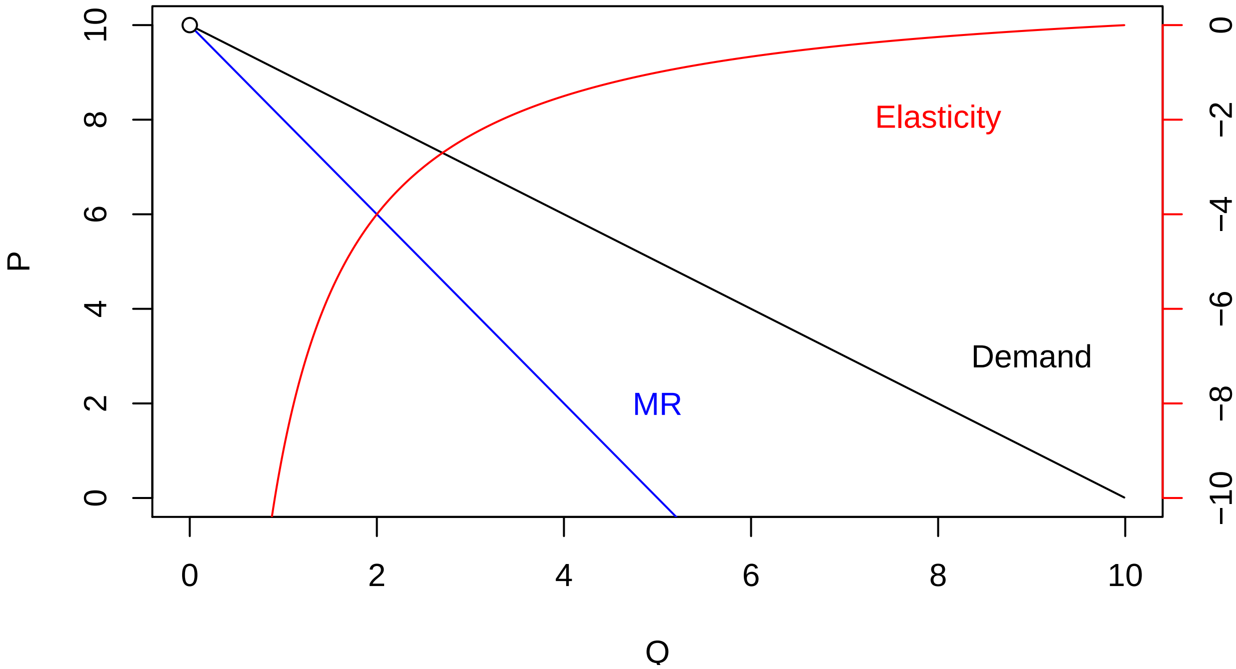
Below is plotted $\varepsilon(Q) = (\log(Q) + a)/b$, which is positive and increasing on $[.001, 10]$ with $a = 8$ and $b = 10$. In addition, the cooresponding supply curve is plotted with $k = 1$.



(c) Draw the marginal revenue curve for a monopolist facing a downward sloping demand curve that is continuous and linear everywhere except for an interval of quantities where it is perfectly elastic.

A linear demand function takes the form: $P = a - bQ \implies MR = a - 2bQ$. The elasticity of the demand curve is $\varepsilon = \frac{d\log(q)}{d\log(p)}$. Demand is perfectly elastic when $\varepsilon \to -\infty$.

Below is plotted a demand curve with $a = 10$ and $b = 1$, the associated elasticity, and the associated marginal revenue curve. Notice that as $Q \to 0$ $\varepsilon \to -\infty$, so demand and marginal revenue are not continuous at $Q = 0$.



5. There are two groups of Christmas shoppers. Group 2 is gun-shy and hates standing in lines. Group 1 has a lower linear demand, but are willing to take a bullet or stand in line for an hour shopping. Suppose the demands of the two groups are $P_1 = 3 - Q_1$ and $P_2 = 5 - Q_2$ respectively, and let $MC = 1$ be the firms marginal cost of production. What price should a monopolist charge each group, and how? Suppose, instead that the marginal cost was increasing: $MC = Q$ where $Q = Q_1 + Q_2$, is this problem still separable into two independent optimizations? What price should the monopolist charge to each group?

...