

# ECON 711 - PS 3

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## Question 1. Monotone Selection Theorems

Consider a single-output firm facing a tax  $\tau$  on revenue (not profit). The firm is not a price-taker in input markets, but its technology is still characterized by a weakly-increasing cost function  $c : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ , with  $c(q)$  the cost of producing  $q$  units of output.

- (a) Suppose the firm is a price taker in its output market. Show that its objective function  $(1 - \tau)pq - c(q)$  has strictly increasing differences in  $q$  and  $-\tau$ . Prove that this implies a monotone selection rule: an increase in  $\tau$  can never result in an increase in output. Explain why this is a stronger result than “baby Topkis”.

Look at lecture 5 page 3.

Now suppose the firm is not a price-taker in the output market, but faces an inverse demand function  $P(\cdot)$ , where  $P(q)$  is the price at which the firm can sell  $q$  units of output.

- (b) Show that the firm’s objective function  $(1 - \tau)P(q)q - c(q)$  does not necessarily have increasing differences in  $q$  and  $-\tau$ .

Take the derivative with respect to  $-\tau$ .

- (c) Show that if  $c(\cdot)$  is strictly increasing, the firm’s objective function still have strictly single-crossing differences; prove that an increase in  $\tau$  cannot result in an increase in output.

## Question 2. Robot Carwashes

A firm provides car washes using four inputs: unskilled labor ( $\ell$ ), managers ( $m$ ), robots ( $r$ ), and engineers ( $e$ ). Managers are required to supervise unskilled labor, and engineers are required to keep the robots running; the firm’s output is

$$q = f(\ell, m, r, e) = (\ell^{0.5}m^{0.3}) + r^{0.7}e^{0.1}z$$

with  $z = 1.1$ . Input costs are  $w_\ell, w_m, w_r$ , and  $w_e$ , so the firm’s problem is

$$\max_{\ell, m, r, e \geq 0} \{pf(\ell, m, r, e) - w_\ell \ell - w_m m - w_r r - w_e e\}$$

Suppose at each input price vector, the firm’s problem has a unique solution.

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\*I worked on this problem set with a study group of Michael Nattinger, Andrew Smith, Tyler Welch, and Ryan Mather. I also discussed problems with Emily Case, Sarah Bass, and Danny Edgel.

- (a) In an effort to encourage STEM education, a politician proposes subsidizing the wage of engineers. From the firm's point of view, this simply reduces the cost of engineers,  $w_e$ . What effect will this have on the firm's demand for each input?
- (b) Over time, the firm's technology shifts, with  $z$  changing from 1.1 to 0.9. With  $z = 0.9$ , what effect would the subsidy on engineers' wages have on the firm's demand for each input?
- (c) If the supply of managers is fixed in the short-run, would the subsidy's effect on unskilled labor be larger in the short-run or the long-run? Explain.