ECON 714B - Notes

Alex von Hafften

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Complete Markets

- Arrow-Debreu Market Structure and Sequential Markets are equivalent.

Limited Commitment

- Find participation constraint (PC) for planners' problem.
- Decentralize using borrowing constraint.

Ramsay Taxation

We use the primal approach:

1. Derive implementability constraint (IC) from HH problem.
2. Modify utility function using IC to get Ramsay Problem (RP).
3. Compare FOCs of RP to FOC of HH problems.

Setup:

- Infinite periods with consumptn/capital and labor.
- Feasibility (RC): $c(s^t) + g(s^t) + k(s^t) \leq F(k(s^{t-1}), \ell(s^t), s_t) + (1-\delta)k(s^{t-1})$.
- Preferences: $\sum_{t,s^t} \beta^t \mu(s^t) U(c(s^t), \ell(s^t))$.
- Exogenous gov't spending $g(s^t)$ funded by labor taxes τ, capital taxes θ, and gov't bonds $b(s^t)$ with return $R_b(s^t)$.
- HHBC: $c(s^t)+k(s^t)+b(s^t) \leq (1-\tau(s^t)w(s^t)\ell(s^t)+R_k(s^t)k(s^{t-1})$ where $R_k(s^t) = 1+(1-\theta(s^t))(r(s^t)-\delta)$.
- GBC: $b(s^t) = R_b(s^t)b(s^{t-1}) + g(s^t) - \tau(s^t)w(s^t)\ell(s^t) - \theta(s^t)[r(s^t) - \delta]k(s^{t-1})$
- Policy is $\{\tau(s^t), \theta(s^t)\}_{t=0}^{\infty}$.
- Allocation is $\{c(s^t), k(s^t), \ell(s^t), b(s^t)\}_{t=0}^{\infty}$.
- Prices are $\{w(s^t), r(s^t), R_b(s^t)\}\}_{t=0}^{\infty}$.

CE: A policy, allocation, and prices such that

1. Given policy and prices, the allocation solve HH problem.
2. Given policy and prices, the allocation solves firm problem.
3. GBC holds. 4. Markets clear.

Market clearing and firm problem $\implies r(s^t) = F_K(k(s^{t-1}), \ell(s^t))$ and $w(s^t) = F_L(k(s^{t-1}), \ell(s^t))$.

A Ramsay equilibrium is a policy, allocation, and prices that constitute a CE such that the policy maximizes HH utility.

CE $\iff$ RC and IC are satisfied: $U_C(s_0)[R_k(s_0)k_{-1} + R_b(s_0)b_{-1}] = \sum_{t,s^t} \beta^t \mu(s^t)[U_C(s^t)c(s^t) + U_L(s^t)\ell(s^t)]$.

$w(c(s^t), \ell(s^t), \lambda) := u(c(s^t), \ell(s^t)) + \lambda[U_c(s^t)c(s^t) + U_\ell(s^t)\ell(s^t)]$

RP: $\max \sum_{t,s^t} \beta^t \mu(s^t) w(c(s^t), \ell(s^t), \lambda)$ s.t. RC.

Inter. FOC: $\frac{w_\ell(s^t)}{w_c(s^t)} = F_\ell(s^t)$.

Intra. FOC: $w_c(s^t) = \sum_{s^{t+1}} \beta\mu(s^{t+1}|s^t) w_c(s^{t+1})[1 - \delta + F_K(s^{t+1})]$

SS capital taxes:

- RP $\implies 1 = \beta(1 - \delta + F_K)$.
- HHP $\implies 1 = \beta(1 + (1-\theta)(F_K - \delta))$.
- $\implies \theta = 0$.

Static Mirrleesian Problem

- Ramsay approach: Ruled out lump-sum taxes by assumptions.
- Proportional tax: τY.
- Lump-sum tax: $Y - \tau$.
- Mechanism design problem in which agents "abilities" are private information.
- Trade-off between efficiency and equity.

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- A two-type example with a continuum of HHs characterized by a productivity level θ:

$$\theta \in \Theta = \{\theta_H, \theta_L\} \text{ where } \theta_H > \theta_L$$

- $\pi(\theta)$ is the probability that a given HH is of type θ. Also the mass of such HH by LLN.
- Assume that a HH of type θ that works ℓ hours can produce: $y = \theta\ell$ units of output.
- HH prefs: $u(c, y, \theta) = u(c) - v(\ell) = u(c) - v(y/\theta)$ where u is increasing and concave and v is increasing and convex.
- Suppose first that θ is public information.
- An allocation is $(c(\theta), y(\theta))$.
- Let's consider a utilitarian planner's problem:

$$\max \pi(\theta_H)[u(c(\theta_H)) - v(y(\theta_H)/\theta_H)] + \pi(\theta_L)[u(c(\theta_L)) - v(y(\theta_L)/\theta_L)]$$

$$\text{s.t. } \pi(\theta_H)c(\theta_H) + \pi(\theta_L)c(\theta_L) \leq \pi(\theta_H)y(\theta_H) + \pi(\theta_L)y(\theta_L)$$

- [Guess that $c(\theta_H) = c(\theta_L)$ and $y(\theta_H) > y(\theta_L)$.]
- FOC:

1. $c(\theta_H) = c(\theta_L)$
2. $u'(c(\theta)) = \frac{1}{\theta}v'(\ell(\theta))$
3. $\frac{v'(\ell(\theta_H))}{v'(\ell(\theta_L))} = \frac{\theta_H}{\theta_L} > 1 \implies v'(\ell(\theta_H)) > v'(\ell(\theta_L)) \implies \ell(\theta_H) > \ell(\theta_L)$

- All HH consume same amount, high type HH work more.
- Assume that θ is private information
- Incentive compatibility: High type will pretend to be low type in public information outcome.
- Defn: A direct revelation mechanism consists of actions/message set $A_i, i \in [0,1]$ where $\forall i, A_i = \Theta_i$ and outcome function (c, y): $c, y : \Theta \to \mathbb{R}_+$.
- Defn: A revelation mechanism is incentive compatible iff $u(c(\theta)) - v(y(\theta)/\theta) \geq u(c(\hat{\theta})) - v(y(\hat{\theta})/\theta)$ $\forall(\hat{\theta}, \theta)$.
- Defn: A revelation mechanism is resource feasible iff $\pi(\theta_H)c(\theta_H) + \pi(\theta_L)c(\theta_L) \leq \pi(\theta_H)y(\theta_H) + \pi(\theta_L)y(\theta_L)$.
- The planners' problem is:

$$\max \pi(\theta_H)[u(c(\theta_H)) - v(y(\theta_H)/\theta_H)] + \pi(\theta_L)[u(c(\theta_L)) - v(y(\theta_L)/\theta_L)]$$

$$\text{s.t. IC and RC}$$

- There are two IC constraints: H pretending to be L and L pretending to be H. We consider a "relaxed probelm in which we drop the IC constraint of the low type. We will check ex post if this constraint is satisfied.
- Relaxed problem:

$$\max \pi(\theta_H)[u(c(\theta_H)) - v(y(\theta_H)/\theta_H)] + \pi(\theta_L)[u(c(\theta_L)) - v(y(\theta_L)/\theta_L)]$$

$$\text{s.t. } u(c(\theta_H)) - v(y(\theta_H)/\theta_H) \geq u(c(\theta_L)) - v(y(\theta_L)/\theta_H)$$

$$\text{and } \pi(\theta_H)c(\theta_H) + \pi(\theta_L)c(\theta_L) \leq \pi(\theta_H)y(\theta_H) + \pi(\theta_L)y(\theta_L)$$

- Use λ as multipler on IC for H and use μ as multipler on RC.
- Then the FOCs are:

1. $\pi(\theta_H)u'(c(\theta_H)) + \lambda u'(c(\theta_H)) - \pi(\theta_H)\mu = 0$
2. $\pi(\theta_L)u'(c(\theta_L)) - \lambda u'(c(\theta_L)) - \pi(\theta_L)\mu = 0$
3. $-\frac{\pi(\theta_H)}{\theta_H}v'(y(\theta_H)/\theta_H) - \lambda\frac{1}{\theta_H}v'(y(\theta_H)/\theta_H) + \pi(\theta_H)\mu = 0$
4. $-\frac{\pi(\theta_L)}{\theta_L}v'(y(\theta_L)/\theta_L) + \lambda\frac{1}{\theta_H}v'(y(\theta_L)/\theta_H) + \pi(\theta_L)\mu = 0$

- Combine (1) and (3):

$$u'(c(\theta_H)) = \frac{1}{\theta_H}v'(y(\theta_H)/\theta_H)$$

- This is exactly what you would get if there is no private info.
- Allocation for the high type is "ex-post efficient".
- No distortion at the top.
- You don't want to distort the allocation for types that no other type wants to be. L doesn't want to be pretend to be H, so no need to distort H.
- Next, combine (1) and (2):

$$\frac{u'(c(\theta_H))}{u'(c(\theta_L))} = \frac{\pi(\theta_H)}{\pi(\theta_L)}\frac{(\pi(\theta_L) - \lambda)}{(\pi(\theta_H) + \lambda)} = \frac{\pi(\theta_H)\pi(\theta_L) - \pi(\theta_H)\lambda}{\pi(\theta_L)\pi(\theta_H) + \pi(\theta_L)\lambda} < 1$$

- $\implies c(\theta_H) > c(\theta_L)$ [public info this held with equality].
- But since the IC holds with equality:

$$u(c(\theta_H)) - v(y(\theta_H)/\theta_H) = u(c(\theta_H)) - v(y(\theta_L))$$