ECON 711 - PS 3

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Question 1. Monotone Selection Theorems

Consider a single-output firm facing a tax τ on revenue (not profit). The firm is not a price-taker in input markets, but its technology is still characterized by a weakly-increasing cost function $c : \mathbb{R}_+ \to \mathbb{R}_+$, with $c(q)$ the cost of producing q units of output.

(a) Suppose the firm is a price taker in its output market. Show that its objective function $(1-\tau)pq - c(q)$ has strictly increasing differences in q and $-\tau$. Prove that this implies a monotone selection rule: an increase in τ can never result in an increase in output. Explain why this is a stronger result than "baby Topkis".

Now suppose the firm is not a price-taker in the output market, but faces an inverse demand function $P(\cdot)$, where $P(q)$ is the price at which the firm can sell q units of output.

(b) Show that the firm's objective function $(1-\tau)P(q)q - c(q)$ does not necessarily have increasing differences in q and $-\tau$.

(c) Show that if $c(\cdot)$ is strictly increasing, the firm's objective function still have strictly single-crossing differences; prove that an increase in τ cannot result in an increase in output.

Question 2. Robot Carwashes

A firm provides car washes using four inputs: unskilled labor (ℓ), managers (m), robots (r), and engineers (e). Managers are required to supervise unskilled labor, and engineers are required to keep the robots running; the firm's output is

$$q = f(\ell, m, r, e) = (\ell^{0.5} m^{0.3}) + r^{0.7} e^{0.1})^z$$

with $z = 1.1$. Input costs are w_ℓ, w_m, w_r, and w_e, so the firm's problem is

$$\max_{\ell,m,r,e \geq 0} \{p f(\ell, m, r, e) - w_\ell \ell - w_m m - w_r r - w_e e\}$$

Suppose at each input price vector, the firm's problem has a unique solution.

(a) In an effort to encourage STEM education, a politician proposes subsidizing the wage of engineers. From the firm's point of view, this simply reduces the cost of engineers, w_e. What effect will this have on the firm's demand for each input?

*I worked on this problem set with a study group of Michael Nattinger, Andrew Smith, Tyler Welch, and Ryan Mather. I also discussed problems with Emily Case, Sarah Bass, and Danny Edgel.

(b) Over time, the firm's technology shifts, with z changing from 1.1 to 0.9. With $z = 0.9$, what effect would the subsidy on engineers' wages have on the firm's demand for each input?

(c) If the supply of managers is fixed in the short-run, would the subsidy's effect on unskilled labor be larger in the short-run or the long-run? Explain.