ECON 712 - PS 2

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Problem 1: Two-dimensional non-linear system

Consider the Ramsey model of consumption c_t and capital k_t:

$$k_{t+1} = f(k_t) + (1-\delta)k_t - c_t \tag{1}$$

$$\beta u'(c_{t+1}) = \frac{u'(c_t)}{1-\delta+f'(k_{t+1})} \tag{2}$$

parametrized by: $f(k) = zk^{\alpha}, z = 1, \alpha = 0.3, \delta = 0.1, \beta = 0.97, u(c) = \log(c)$.

1. Solve for steady state $(\bar{k}, \bar{c})$.
2. Linearize the system around its steady state.

(a) Rewrite equations (1) and (2) as

$$k_{t+1} = g(k_t, c_t)$$
$$c_{t+1} = h(k_t, c_t)$$

(b) Analytically calculate Jacobian $J = \begin{pmatrix} dk_{t+1}/dk_t & dk_{t+1}/dc_t \\ dc_{t+1}/dk_t & dc_{t+1}/dc_t \end{pmatrix}$ (use provided functional forms, but don't plug in parameters yet).

(c) Using Taylor expansion (first-order approximatino here), systems can be written in terms of deviations from steady state $\bar{k}_t = k_t - \bar{k}$ and $\bar{c}_t = c_t - \bar{c}$:

$$\begin{pmatrix} \bar{k}_{t+1} \\ \bar{c}_{t+1} \end{pmatrix} = J \begin{pmatrix} \bar{k}_t \\ \bar{c}_t \end{pmatrix}$$

3. Compute numerically eigenvalues and eigenvectors of the Jacobian at the steay state. Verify that the system has a saddle path. What is the slope of the saddle path at the steady state?
4. On a phase diagram in (k_t, c_t) show how the system evolves after an unexpecetd permanant positive productivity shock at $t_0, z' > z$. (You don't need to plot lines precisely - do this by hand, but pay attention to vector field (arrows), relative position of old and new steady states, directions of saddle paths and system trajectory after the shock.)

*I worked on this problem set with a study group of Michael Nattinger, Andrew Smith, and Ryan Mather. I also discussed problems with Emily Case, Sarah Bass, and Danny Edgel.

5. (continuing from 4) Compute numerically and plot trajectories of k_t and c_t for $t = 1, 2, ..., 20$ if the productivity shock occurs at $t_0 = 5$ and $z = z + 0.1$. For this question, we will be looking at the linearized version of the nonlinear system around the new steady state.

(a) Compute the new steady state $(\bar{k}', \bar{c}')$ and Jacobian matrix at that point.

(b) Diagonalize the system using eigenvectors and rewrite it in terms of $\hat{k}_t$ and $\hat{c}_t$.

(c) Write down non-explosive solution for $(\hat{k}_t, \hat{c}_t)$, rewrite in terms of original variables (k_t, c_t).

(d) Pin down a particular saddle path trajectory using a boundary condition $k_{t_0} = \bar{k}$ (capital can't jump from the old steady state at the time of the stock, so pick suitable c_{t_0}).

(e) Use the particular solution to compute and graph k_t and c_t after the shock.

6. For this question, we explore the nonlinear nature of the system and numerically solve the actual transition path using the "shooting method".

(a) In the previous question, you solve c_{t_0} under the linear system. Put $(k_{t_0}, c_{t_0}$ into the nonlinear system (1) and (2). Compute and graph how the sysem evolves. Does it converge to a steady state?

(b) Use "shooting method" to find the actual c_{t_0} needed. The method is to try different values of c_{t_0} such that after long enough time, the system will converge to the new steady state.

Problem 2: Setting up a model

For the problems below, state the Social Planner Problem (SPP), the Consumer Problem (CP), and define the Competitive Equilibrium (CE). (Don't solve).

1. Consider an overlapping generations economy of 2-period-lived agents. There is a constant measure of N agents in each generation. New young agents enter the economy at each date $t \geq 1$. Half of the young agents are endowed with w_1 when young and 0 when old. The other half are endowed with 0 when young and w_2 when old. There is nosavings technology. Agents order their consumption stead by $U(c_t^t, c_{t+1}^t) = \ln c_t^t + \ln c_{t+1}^t$. There is a measure N of initial old agents. Half of them are endowed with w_2 and the other half endowed with 0. Each old agent order their consumption by c_1^0. Each old agent is endowed with M units of fiat currency. No other generation is endowed with fiat currency, and the stock of fiat currency is fixed over time.