ECON 714A - Problem Set 1

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Consider a neoclassical growth model with preferences $\sum_{t=0}^{\infty} \beta^t U(C_t)$, production technology $F(K_t)$, and the initial capital endowment K_0. Both $U(\cdot)$ and $F(\cdot)$ are strictly increasing, strictly concave and satisfy standard Inada conditions. The capital law of motion is $K_{t+1} = (1-\delta)K_t + I_t - D_t$ where D_t is a natural disaster shock that destroys a fixed amount of the accumulated capital.

1. Write down the social planner's problem and derive the inter-temporal optimality condition (the Euler equation).

The social planner's problem is to maximize the welfare of a representative agent subject to the resource constraint:

$$\max_{\{C_t, K_{t+1}\}} \sum_{t=0}^{\infty} \beta^t U(C_t)$$
$$\text{s.t. } C_t + D_t + K_{t+1} = F(K_t) + (1-\delta)K_t$$

Denote the lagrange multipler with $\beta^t \lambda_t$:

$$\sum_{t=0}^{\infty} \beta^t [U(C_t) + \lambda_t (F(K_t) + (1-\delta)K_t - C_t - D_t - K_{t+1})]$$

FOC $[C_t]$:

$$0 = \beta^t [U'(C_t) - \lambda_t] \implies U'(C_t) = \lambda_t$$

FOC $[K_{t+1}]$:

$$0 = -\beta^t \lambda_t + \beta^{t+1} \lambda_{t+1}(F'(K_{t+1}) + (1-\delta)) \implies \lambda_t = \beta \lambda_{t+1}(F'(K_{t+1}) + (1-\delta))$$

Thus, the Euler equation is:

$$U'(C_t) = \beta U'(C_{t+1})(F'(K_{t+1}) + (1-\delta))$$

*I worked on this problem set with a study group of Michael Nattinger, Andrew Smith, and Ryan Mather. I also discussed problems with Emily Case, Sarah Bass, and Danny Edgel.

2. Given the steady-state value of $D \geq 0$, write down the system of equations that determines the values of capital $\bar{K}(D)$ and consumption $\bar{C}(D)$ in the steady state. Draw a phase diagram with capital in the horizontal axis and consumption in the vertical axis, show the steady states, draw the arrows representing the direction of change, and the saddle path.

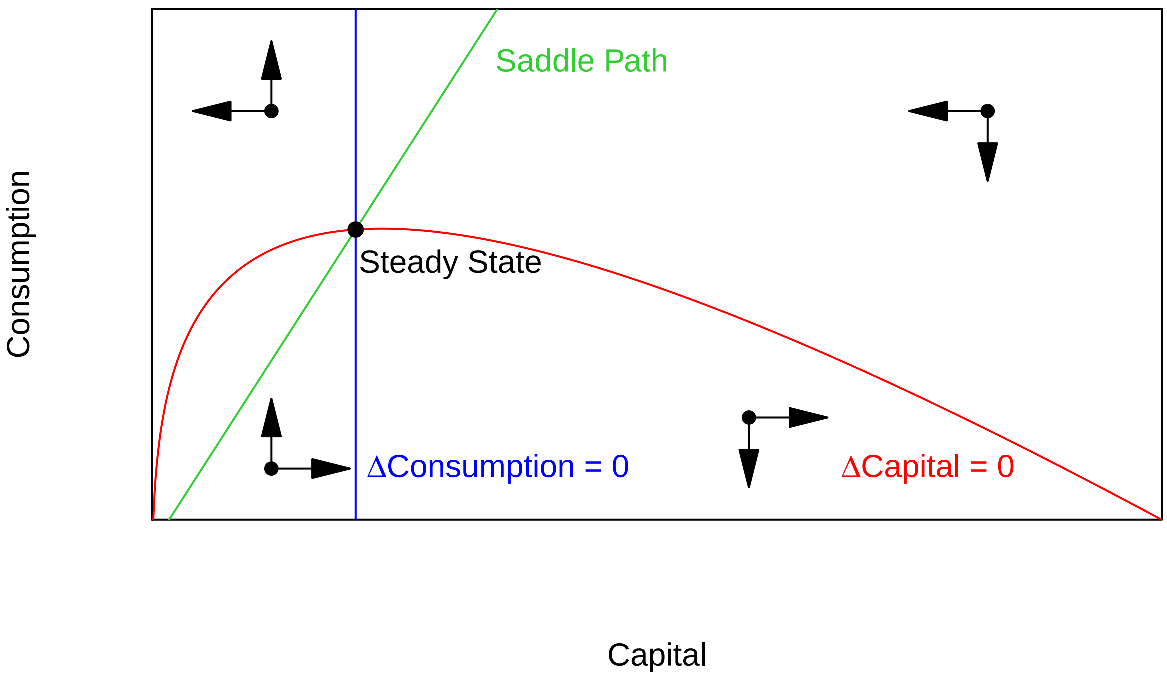
The system of equations can be derived from the Euler equation and the law of motion of capital. In a steady state, $C_t = C_{t+1} = \bar{C}$ and $K_t = K_{t+1} = \bar{K}$. In the steady state, the Euler equation implies

$$U'(\bar{C}) = \beta U'(\bar{C})(F'(\bar{K}) + (1-\delta)) \implies \bar{K}(D) = \bar{K} = (F')^{-1}\left(\frac{1}{\beta} - (1-\delta)\right)$$

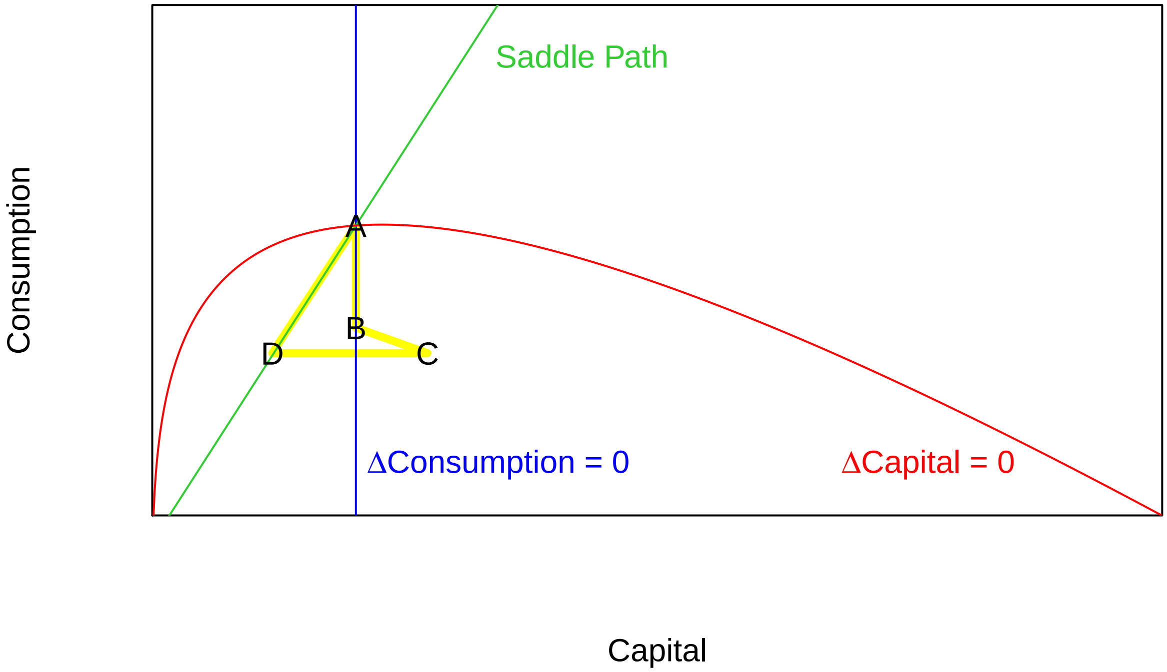
In the steady state, the law of motion of capital implies:

$$\bar{K} = (1-\delta)\bar{K} + F(\bar{K}) - \bar{C} - D \implies \bar{C}(D) = F(\bar{K}(D)) - D - \delta\bar{K}(D)$$

The phase diagram below shows the line where consumption does not change (blue), the line where capital does not change (red), the steady state (green), draw the arrows representing the direction of change, and the saddle path (black).



3. The scientists forecast an earthquake T periods from now that will destroy $D > 0$ units of capital. Assuming that economy starts from a steady state with $D = 0$, draw a phase diagram that shows the optimal transition path. Make two separate graphs showing the evolution of capital and consumption in time.



4. Assume that $U(C) = \frac{C^{1-\sigma}-1}{1-\sigma}$ and $F(K) = K^{\alpha}$ and the values of parameters are $\sigma = 1$, $\alpha = 1/3$, $\beta = 0.99^{1/12}$ (monthly model), $\delta = 0.01$, $T = 12$, $D = 1$. Using a shooting algorithm, solve numerically for the optimal transition path and plot dynamics of consumption and capital.