# ECON 899A - Problem Set 6

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*This problem set is for ECON 899A Computational Economics taught by Dean Corbae with assistance from Philip Coyle at UW-Madison. I worked on this problem set with a study group of Michael Nattinger, Sarah Bass, and Xinxin Hu.

# Appendix - Static Labor Demand

$$\pi(s;p) = \max_{n\geq 0} psn^{\theta} - n - pc_f$$

FOC $[n]$:

$$\theta psn^{\theta-1} = 1 \implies n^* = (ps\theta)^{\frac{1}{1-\theta}}$$

# Appendix - Static Labor Supply

The HH problem:

$$\max_{C,N^s} \ln(C) - AN^s \text{ s.t. } pC \leq N^s + \Pi$$

$$\implies \max_{N^s} \ln\left(\frac{N^s + \Pi}{p}\right) - AN^s$$

FOC $[N^s]$:

$$\frac{p}{N^s + \Pi}\frac{1}{p} = A \implies N^s = \frac{1}{A} - \Pi$$

$$\implies C = \frac{(\frac{1}{A} - \Pi) + \Pi}{p} = \frac{1}{Ap}$$

# Appendix - Steady State Firm Distribution

In this appendix, I find $\boldsymbol{\mu^*} = \begin{pmatrix} \mu^*(s_1) \\ \vdots \\ \mu^*(s_5) \end{pmatrix}$ explicitly in term of exit decision rules $X$, transition function $F$, and stationary distribution $\nu$. From the problem set,

$$\mu^*(s') = \sum_s [1 - X(s)]F(s, s')\mu^*(s) + M \sum_s [1 - X(s)]F(s, s')\nu(s)$$

Stacking the five equations on top of each:

$$\begin{aligned}
\begin{pmatrix} \mu^*(s_1) \\ \vdots \\ \mu^*(s_5) \end{pmatrix} &= \begin{pmatrix} \sum_s [1 - X(s)]F(s, s_1)\mu^*(s) \\ \vdots \\ \sum_s [1 - X(s)]F(s, s_5)\mu^*(s) \end{pmatrix} + M \begin{pmatrix} \sum_s [1 - X(s)]F(s, s_1)\nu(s) \\ \vdots \\ \sum_s [1 - X(s)]F(s, s_5)\nu(s) \end{pmatrix} \\
\implies \begin{pmatrix} \mu^*(s_1) \\ \vdots \\ \mu^*(s_5) \end{pmatrix} &= \begin{pmatrix} [1 - X(s_1)]F(s_1, s_1) & \dots & [1 - X(s_5)]F(s_5, s_1) \\ \vdots & & \vdots \\ [1 - X(s_1)]F(s_1, s_5) & \dots & [1 - X(s_5)]F(s_5, s_5) \end{pmatrix} \begin{pmatrix} \mu^*(s_1) \\ \vdots \\ \mu^*(s_5) \end{pmatrix} \\
&\quad + M \begin{pmatrix} [1 - X(s_1)]F(s_1, s_1) & \dots & [1 - X(s_5)]F(s_5, s_1) \\ \vdots & & \vdots \\ [1 - X(s_1)]F(s_1, s_5) & \dots & [1 - X(s_5)]F(s_5, s_5) \end{pmatrix} \begin{pmatrix} \nu(s_1) \\ \vdots \\ \nu(s_5) \end{pmatrix} \\
&\implies \boldsymbol{\mu^*} = Z\boldsymbol{\mu^*} + MZ\boldsymbol{\nu} \\
&\implies \boldsymbol{\mu^*} = M(I - Z)^{-1}Z\boldsymbol{\nu}
\end{aligned}$$

where

$$Z = \begin{pmatrix} [1 - X(s_1)]F(s_1, s_1) & \dots & [1 - X(s_5)]F(s_5, s_1) \\ \vdots & & \vdots \\ [1 - X(s_1)]F(s_1, s_5) & \dots & [1 - X(s_5)]F(s_5, s_5) \end{pmatrix} = \begin{pmatrix} [1 - X(s_1)] & \dots & [1 - X(s_1)] \\ \vdots & & \vdots \\ [1 - X(s_5)] & \dots & [1 - X(s_5)] \end{pmatrix}' .\times F'$$