ECON 713A - Problem Set 2

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3/6/2020

1. Assume that firm $k = 1, 2, ...$ in a competitive industry has cost $k^2 + q + q^2$ of output level q. (Here, k^2 is an escapable cost in the long run.) Then the long run industry saw tooth supply curve has positive output only for prices at least what level?

The average cost of firm k is

$$AC_k(q) = \frac{k^2 + q + q^2}{q} = \frac{k^2}{q} + 1 + q$$

Firm 1 has the lowest average cost:

$$AC_1(q) = \frac{1}{q} + 1 + q$$

Firm 1's average cost is minimized at $q = 1$. FOC:

$$0 = \frac{-1}{q^2} + 1 \implies q = 1$$

SOC:

$$\frac{\partial^2 AC_1}{\partial^2 q} = \frac{2}{q^3} > 0$$

For firm 1 to be in the market in the long run price needs to be at least average cost of producing $q = 1$:

$$P \geq \frac{1}{1} + 1 + 1 = 3$$

*I worked on this problem set with a study group of Michael Nattinger, Andrew Smith, and Ryan Mather. I also discussed problems with Sarah Bass, Emily Case, Danny Edgel, and Katherine Kwok.

2. Assume a continuum of potential iPhone developers indexed by the quality of their idea. Each developer has a fixed cost of 1 can produce software code according to the function $q = 2\theta x$, where x is quantity of variable input used by the developer. A unit of software code sells for a price of 1, the cost of using a quantity x of the input is x^2. The "number" (i.e. mass) of firms with ideas above $\theta > 0$ is given by $M(\theta) = \theta^{-\beta}$, where $\beta > 2$. Apple taxes developers' revenues at a percentage rate $0 < \tau < 1$.

(a) Derive the aggregate supply curve of developer code.

Developer profit is:

$$\pi(x) = (1-\tau)2\theta x - x^2 - 1$$

FOC $[x]$:

$$\frac{\partial \pi}{\partial x} = 0 \implies 2\theta(1-\tau) - 2x = 0$$

$$\implies x = \theta(1-\tau)$$

$$\implies q^*(\theta) = 2\theta^2(1-\tau)$$

SOC:

$$\frac{\partial^2 \pi}{\partial^2 x} = -2 < 0$$

$$\pi(\theta) = 2\theta^2(1-\tau)^2 - \theta^2(1-\tau)^2 - 1 > 0 \implies \theta > \frac{1}{1-\tau}$$

The aggregate supply curve:

$$\begin{aligned}
\int_{\frac{1}{1-\tau}}^{\infty} q^*(\theta)(-M'(\theta))d\theta &= \int_{\frac{1}{1-\tau}}^{\infty} 2\theta^2(1-\tau)(\beta\theta^{-\beta-1})d\theta \\
&= 2(1-\tau)\beta \int_{\frac{1}{1-\tau}}^{\infty} \theta^{1-\beta} d\theta \\
&= 2(1-\tau)\beta \left[\frac{1}{2-\beta}\theta^{2-\beta}\right]_{\theta=\frac{1}{1-\tau}}^{\infty} \\
&= 2(1-\tau)\beta\frac{1}{2-\beta}\left(\frac{1}{1-\tau}\right)^{2-\beta} \\
&= \frac{2\beta}{\beta-2}(1-\tau)^{\beta-1}
\end{aligned}$$

(b) When Apple raises its tax rate, what happens to the mass of developer firms, and the amount of code each produces?

When Apple raises its tax rate ($\uparrow \tau$), then the mass of developer firms decreases ($\uparrow \frac{1}{1-\tau}$) and the amount each produces decreases ($\downarrow \theta(1-\tau)$).

(c) What is Apple's revenue maximizing tax?

Apple's tax revenue is:

$$\frac{2\beta}{\beta-2}(1-\tau)^{\beta-1}\tau$$

Maximizing τ is equivalent to maximizing $(1-\tau)^{\beta-1}\tau$ because $\frac{2\beta}{\beta-2} > 0$.

FOC $[\tau]$:

$$(\beta-1)(1-\tau)^{\beta-2}(-1)\tau + (1-\tau)^{\beta-1} = 0 \implies \tau = 1/\beta$$

(d) What happens if β rises (while τ is fixed)? Interpret this in terms of firm heterogeneity.

If β rises, the distribution of "idea" distribution becomes more concentrated and shifts down. There is less heterogeneity in the quality of developer's ideas. If τ remains fixed, the number of developers decrease.

3. The only Ben and Jerry's in town faces different linear inverse demand curves $P = a - bQ$ for triple-chocolate-chunk ice cream and fruit-bowl-punch ice cream. It buys each at the same constant unit cost from its supplier. The inverse demand curves cross at an interior point, above marginal cost, with Fruit-bowl-punch having a higher price intercept than Triple-chunk. Does it follow that its price charged is higher on the Fruit-Bowl Punch?

Yes. The marginal revenue for each demand curve is:

$$PQ = aQ - bQ^2 \implies MR = a - 2bQ$$

Profit-maximization production is at $MR = MC$:

$$MC = a - 2bQ \implies Q^* = \frac{a - MC}{2b}$$

This quantity coorespondes to the following price:

$$P^* = a - b\frac{a - MC}{2b} = \frac{a + MC}{2}$$

If fruit-bowl-punch has a higher price intercept than Triple-chunk:

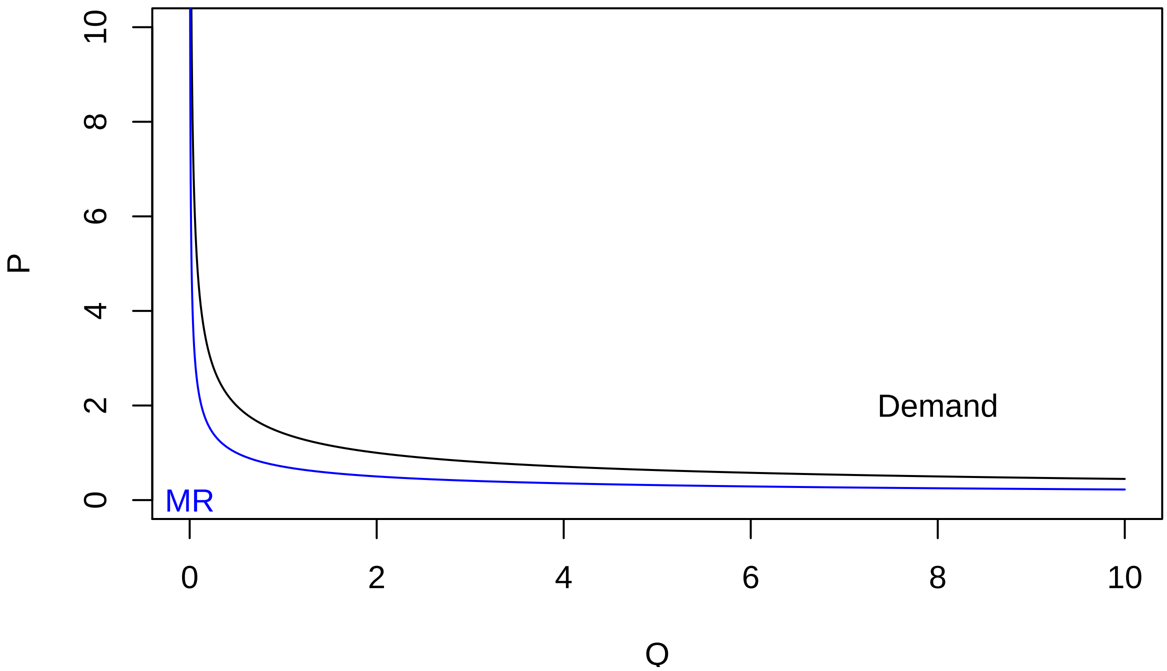
$$a_{FBP} > a_{TCC} \implies \frac{a_{FBP} + MC}{2} > \frac{a_{TCC} + MC}{2} \implies P^*_{FBP} > P^*_{TCC}$$

4. (a) Draw a negatively sloped demand curve with elasticity of magnitude greater than 1, and a corresponding marginal revenue curve.

From lecture, a demand (or supply) curve with constant elasticity takes the form:

$$\begin{aligned} Q &= kP^{\varepsilon} \\ \implies P &= Q^{1/\varepsilon}k^{-1/\varepsilon} \\ \implies TR = PQ &= Q^{1/\varepsilon+1}k^{-1/\varepsilon} \\ \implies MR &= (1/\varepsilon+1)Q^{1/\varepsilon}k^{-1/\varepsilon} \end{aligned}$$

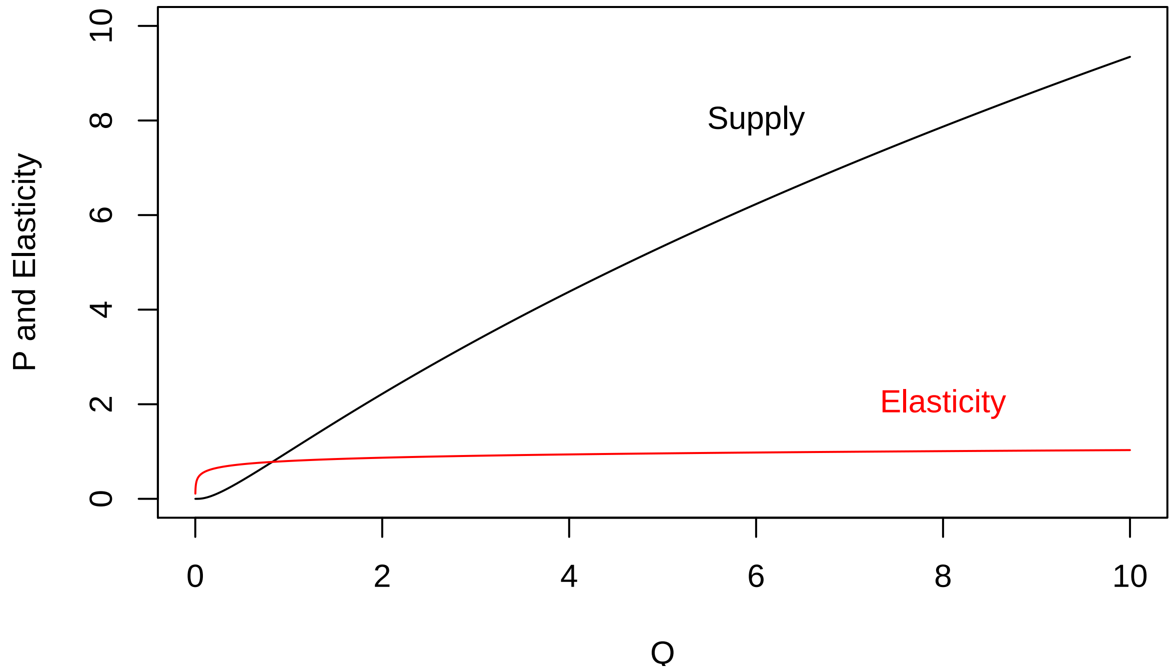
Below is plotted these demand and marginal revenue curves with $\varepsilon = -2$ and $k = 2$.



(b) Draw a supply curve with positive and rising elasticity.

A supply curve with constant elasticity takes the form: $Q = kP^{\varepsilon} \iff P = Q^{1/\varepsilon}k^{-1/\varepsilon}$. Let $\varepsilon(Q)$ be a positive increasing function, so $P = Q^{1/\varepsilon(Q)}k^{-1/\varepsilon(Q)}$ is a supply curve with positive and rising elasticity.

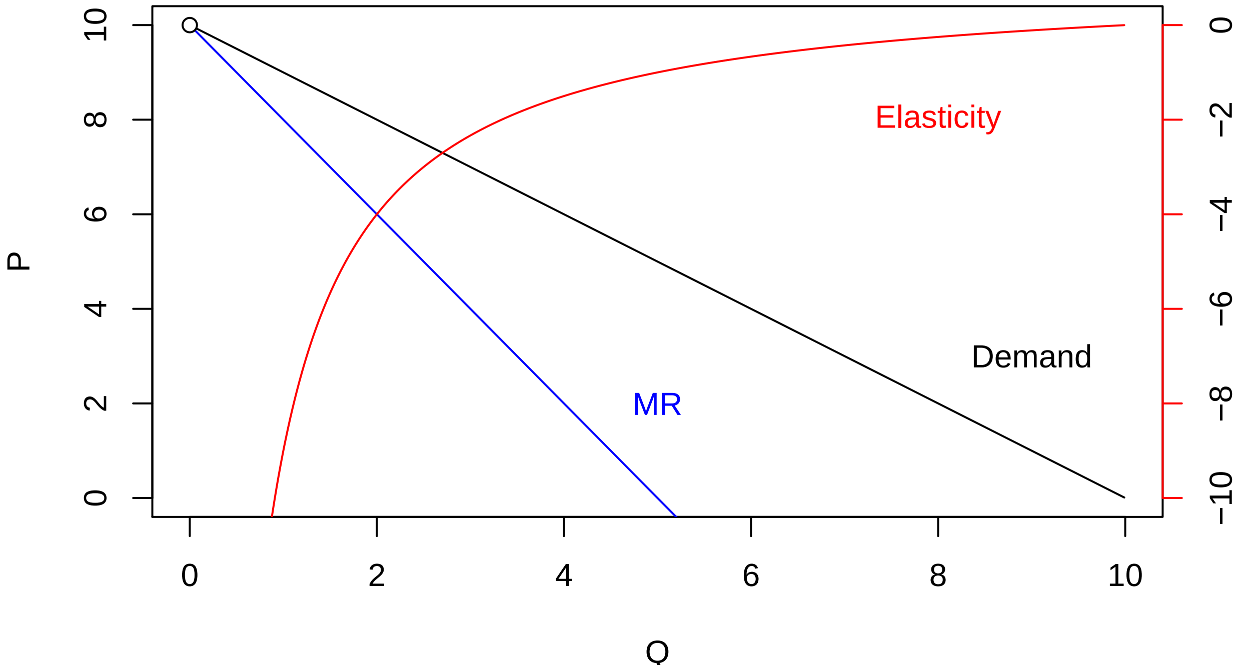
Below is plotted $\varepsilon(Q) = (\log(Q) + a)/b$, which is positive and increasing on $[.001, 10]$ with $a = 8$ and $b = 10$. In addition, the cooresponding supply curve is plotted with $k = 1$.



(c) Draw the marginal revenue curve for a monopolist facing a downward sloping demand curve that is continuous and linear everywhere except for an interval of quantities where it is perfectly elastic.

A linear demand function takes the form: $P = a - bQ \implies MR = a - 2bQ$. The elasticity of the demand curve is $\varepsilon = \frac{d\log(q)}{d\log(p)}$. Demand is perfectly elastic when $\varepsilon \to -\infty$.

Below is plotted a demand curve with $a = 10$ and $b = 1$, the associated elasticity, and the associated marginal revenue curve. Notice that as $Q \to 0$ $\varepsilon \to -\infty$, so demand and marginal revenue are not continuous at $Q = 0$.



5. There are two groups of Christmas shoppers. Group 2 is gun-shy and hates standing in lines. Group 1 has a lower linear demand, but are willing to take a bullet or stand in line for an hour shopping. Suppose the demands of the two groups are $P_1 = 3 - Q_1$ and $P_2 = 5 - Q_2$ respectively, and let $MC = 1$ be the firms marginal cost of production. What price should a monopolist charge each group, and how? Suppose, instead that the marginal cost was increasing: $MC = Q$ where $Q = Q_1 + Q_2$, is this problem still separable into two independent optimizations? What price should the monopolist charge to each group?

The monopolist should charge the price where $MC = MR_i = 1$. For group 1,

$$\begin{aligned} P_1 &= 3 - Q_1 \\ \implies MR_1 &= 3 - 2Q_1 \\ \implies 1 &= 3 - 2Q_1 \\ \implies Q_1 &= 1 \\ \implies P_1 &= 2 \end{aligned}$$

For group 2,

$$\begin{aligned} P_2 &= 5 - Q_2 \\ \implies MR_2 &= 5 - 2Q_2 \\ \implies 1 &= 5 - 2Q_2 \\ \implies Q_2 &= 2 \\ \implies P_2 &= 3 \end{aligned}$$

The store can charge two prices for the good: 2 dollars and 3 dollars. To get the price of 2 dollars, customers have to stand in a line for an hour. Customers that are charged 3 dollars do not have to stand in line. Thus, group 1 will be willing to stand in line and group 2 will forgo the line and pay the higher price. Alternately, the store could charge two dollars and shoot each customer, but give customers an opportunity to pay an extra dollar to avoid being shot.

Setting $MR_1 = Q$ and $MR_2 = Q$ gives us two equations in two unknowns, so the problem is not separable:

$$Q_1 + Q_2 = 3 - 2Q_1 \implies Q_2 = 3 - 3Q_1$$

$$Q_1 + Q_2 = 5 - 2Q_2 \implies Q_1 = 5 - 3Q_2$$

The solution is $Q_1 = 1/2$ and $Q_2 = 3/2 \implies P_1 = 5/2$ and $P_2 = 7/2$.