## ECON 703 - PS 7

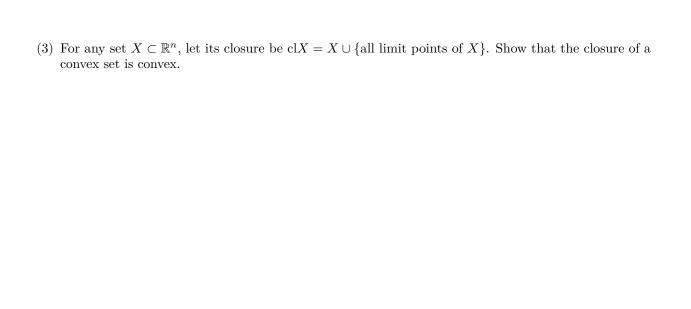
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(1) Let  $X \subset \mathbb{R}^n$  be a convex set, and  $\lambda_1, ..., \lambda_k \geq 0$  with  $\sum_{i=1}^k \lambda_i = 1$ . Prove that if  $x_1, ..., x_k \in X$ , then  $\sum_{i=1}^k \lambda_i x_i \in X$ .

<sup>\*</sup>I worked on this problem set with a study group of Michael Nattinger, Andrew Smith, and Ryan Mather. I also discussed problems with Emily Case, Sarah Bass, and Danny Edgel.

(2) The sum  $\sum_{i=1}^{k} \lambda_i x_i$  defined in Problem (1) is called a convex combination. The convex hull of a set S, denoted by co(S), is the intersection of all convex sets which contain S. Prove that the set of all convex combinations of the elements of S is exactly co(S).



(4) The function  $f: X \to \mathbb{R}$ , where X is a convex set in  $\mathbb{R}^n$ , is concave if  $\forall \lambda \in [0,1], x', x'' \in Xf((1-\lambda)x'+\lambda x'') \geq (1-\lambda)f(x')+\lambda f(x'')$ . Given a function  $f: X \to \mathbb{R}$ , its hypograph is the set of points (y,x) lying on or below the graph of the function: hyp  $f = \{(y,x)|x \in X, y \leq f(x)\}$ . Show that the function f is concave if and only if its hypograph is a convex set.

(5)	Let $X$ and $Y$ be of exists a hyperplane	disjoint, closed, an e $H(p,\alpha)$ that strice	d convex sets ctly separates	in $\mathbb{R}^n$ , one of $X$ and $Y$ .	f which is compa	ct. Show that there

(6) Call a vector  $\pi \in \mathbb{R}^n$  a probability vector if  $\sum_{i=1}^n \pi_i = 1$  and  $\pi_i \geq 0$  for all i=1,...,n. Interpretation is that there are n states of the world and  $\pi_i$  is the probability that state i occurs. Suppose that Alice and Bob each have a set of probability distributions ( $\Pi_A$  and  $\Pi_B$ ) which are nonempty, convex, and compact. They propose bids on each state of the world. A vector  $x = (x_1, ..., x_n) \in \mathbb{R}^n$ , where  $x_i$  denotes the net transfer Alice receives from Bob in state i, is called a trade (Thus, -x is the net transfer Bob receives in each state of the world.) A trade is agreeable if  $\inf_{\pi \in \Pi_A} \sum_{i=1}^n \pi_i x_i > 0$  and  $\inf_{\pi \in \Pi_B} \sum_{i=1}^n \pi_i (-x_i) > 0$ . The above means that both Alice and Bob expect to strictly gain from the trade. Prove that there exists an agreeable trade iff there is no common prior (i.e.,  $\Pi_A \cap \Pi_B = \emptyset$ ).