ECON 709B - Problem Set 4

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1. 7.28 Estimate the regression: $log(\hat{wage}) = \beta_1 education + \beta_2 experience + \beta_3 experience^2/100 + \beta_4$.[1]

```
library(tidyverse)
```

```
cps09mar <- read_delim("cps09mar.txt",
                       delim = "\t",
                       col_names = c("age", "female", "hisp", "education", "earnings",
                                     "hours", "week", "union", "uncov", "region", "race",
                                     "maritial"),
                       col_types = "dddddddddddd") %>%
  mutate(experience = age - education - 6,
         experience_2 = (experience^2)/100,
         wage = earnings / (hours*week),
         l_wage = log(wage),
         constant = 1) %>%
  filter(race == 4,
         maritial == 7,
         female == 0,
         experience < 45)

y <- cps09mar$l_wage

x <- cps09mar %>%
  select(education, experience, experience_2, constant) %>%
  as.matrix() %>%
  unname()

n <- dim(x)[1]
i <- diag(nrow = n, ncol = n)
```

(a) Report the coefficient estimates and robust standard errors.

...

(b) Let θ be the ratio of the return to one year of education to the return to one year of experience for $experience = 10$. Write θ as a function of the regression coefficients and variables. Compute $\hat{\theta}$ from the estimated model.

...

(c) Write out the formula for the asymptotic standard error for $\hat{\theta}$ as a function of the covariance matrix for $\hat{\beta}$. Compute $s(\hat{\beta})$ from the estimated model.

*I worked on this problem set with a study group of Michael Nattinger, Andrew Smith, and Ryan Mather. I also discussed problems with Emily Case, Sarah Bass, and Danny Edgel.

[1] Use the subsample of the CPS that you used for problems 3.24 and 3.25 (instead of the subsample requested in the problem)

...

(d) Construct a 90% asymptotic confidence interval for θ from the estimated model.

...

2. 8.1 In the model $y = X_1'\beta_1 + X_2'\beta_2 + e$, show directly from definition (8.3) that the CLS estimate of $\beta = (\beta_1, \beta_2)$ subject to the constraint that $\beta_2 = 0$ is the OLS regression of y on X_1.

...

3. 8.3 In the model $y = X_1'\beta_1 + X_2'\beta_2 + e$, with β_1 and β_2 each $k \times 1$, find the CLS estimate of $\beta = (\beta_1, \beta_2)$ subject to the constraint that $\beta_1 = \beta_2$.

...

4. 8.4(a) In the linear projection model $y = \alpha + X'\beta + e$ consider the restriction $\beta = 0$. Find the constrained least squares (CLS) estimator of α under the restriction $\beta = 0$.

...

5. 8.22 Take the linear model $y = X_1\beta_1 + X_2\beta_2 + e$ with $E[Xe] = 0$. Consider the resitriction $\beta_1/\beta_2 = 2$

(a) Find an explicit expression for the constrained least squares (CLS) estimator $\tilde{\beta} = (\tilde{\beta}_1, \tilde{\beta}_2)$ of $\beta = (\beta_1, \beta_2)$ under the restriction. Your answer should be specific to the restriction. It should not be a generic formula for an abstract general restriction.

...

(b) Derive the asymptotic distribution of $\tilde{\beta}_1$ under the assumption that the restriction is true.

...

6. 9.1 Prove that if an additional regressor X_{k+1} is added to X, Theil's adjusted $\bar{R}^2$ increases if and only if $|T_{k_1}| > 1$, where $T_{k-1} = \hat{\beta}_{k+1}/s(\hat{\beta}_{k+1}$ is the t-ratio for $\hat{\beta}_{k+1}$ and $s(\hat{\beta}_{k+1}) = (s^2[(X'X)^{-1}]_{k-1,k+1})^{1/2}$ is the homoskedasticity-formula standard error.

...

9.2 You have two independent samples (Y_{1i}, X_{1i}) and (Y_{2i}, X_{2i}) both with sample sizes n which satisfy $Y_1 = X_1\beta_1 + e_1$ and $Y_2 = X_2\beta_2 + e_2$, where $E[X_1e_1] = 0$ and $E[X_2e_2] = 0$. Let $\hat{\beta}_1$ and $\hat{\beta}_2$ be the OLS estimates of $\beta_1 \in R_k$ and $\beta_2 \in R_k$.

(a) Find the asymptotic distribution of $\sqrt{n}((\hat{\beta}_2 - \hat{\beta}_1) - (\beta_2 - \beta_1))$ as $n \to \infty$.

...

(b) Find an appropriate test statistic for $H0 : \beta_2 = \beta_1$.

...

(c) Find the asymptotic distribution of this statistic under H_0.

...

7. 9.4 Let W be a Wald statistic for $H_0 : \theta = 0$ versus $H_1 : \theta \neq 0$, where θ is $q \times 1$. Since $W \to_d \chi^2_q$ under H_0, someone suggests the test "Reject H_0 if $W < c_1$ or $W > c_2$ where c_1 is the $\alpha/2$ quantile of χ^2_q and c_2 is the $1 - \alpha/2$ quantile of χ^2_q."

(a) Show that the asymptotic size of the test is α.

...

(b) Is this a good test of H_0 versus H_1? Why or why not?

...

8. 9.7 Take the model $y = X\beta_1 + X^2\beta_2 + e$ with $E[e|X] = 0$ where y is wages (dollars per hour) and X is age. Describe how you would test the hypothesis that the expected wage for a 40-year-old worker is \$20 an hour.

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