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ECON 711 Midterm

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Q1 $u(x) = \min\{x_1, x_2\}^\alpha (x_3 + x_4)^{1-\alpha}$

(a) Solve consumer problem, find $x(p,w)$ and $v(p,w)$.

Notice that the consumer will consume $x_1 = x_2$. Similar argument to Leontif utility (if $x_1 > x_2$, there's an incentive to reduce x_1 and increase x_2 to increase utility). So we can rewrite the utility function:

$$u(x) = x_1^\alpha (x_3 + x_4)^{1-\alpha}$$

~~The consumer problem is~~

$$\cancel{\max_{x_1, x_3, x_4 \geq 0} \{ x_1^\alpha (x_3 + x_4)^{1-\alpha} \}}$$

$$\cancel{\text{s.t. that } (p_1 + p_2) x_1 + p_3 x_3 + p_4 x_4 \leq w}$$

$$\cancel{\mathcal{L} = x_1^\alpha (x_3 + x_4)^{1-\alpha} + \lambda (w - (p_1 + p_2) x_1 - p_3 x_3 - p_4 x_4)}$$

$$\cancel{\text{FOC}[x_1]: \alpha x_1^{\alpha - 1} (x_3 + x_4)^{1-\alpha} - \lambda p_1 - \lambda p_2 = 0}$$

$$\cancel{\text{FOC}[x_3]: x_1^\alpha (1-\alpha)(x_3 + x_4)^{-\alpha} - \lambda p_3 = 0}$$

$$\cancel{\text{FOC}[x_4]: x_1^\alpha (1-\alpha)(x_3 + x_4)^{-\alpha} - \lambda p_4 = 0}$$

Furthermore notice that x_3 and x_4 are perfect substitutes. So, if $p_3 < p_4$, the consumer will not consume any x_4 and, if $p_4 < p_3$, the consumer will not consume any x_3.

$\boxed{Q1}$ (a) con't

If $p_3 < p_4$, the consumer problem becomes

$$\max_{x_1, x_3 \geq 0} x_1^{\alpha} x_3^{1-\alpha}$$

$$\text{st} \quad (p_1 + p_2) x_1 + p_3 x_3 \leq w$$

$$\mathcal{L} = x_1^{\alpha} x_3^{1-\alpha} + \lambda \left(w - (p_1 + p_2) x_1 - p_3 x_3 \right)$$

$$\text{FOC}[x_1]: \alpha x_1^{\alpha-1} x_3^{1-\alpha} = \lambda (p_1 + p_2)$$

$$\text{FOC}[x_3]: (1-\alpha) x_1^{\alpha} x_3^{-\alpha} = \lambda p_3$$

Utility function represents LNS preferences $\Rightarrow$ Walras' Law holds

$$\Rightarrow \frac{\alpha x_1^{\alpha-1} x_3^{1-\alpha}}{(p_1 + p_2)} = \frac{(1-\alpha) x_1^{\alpha} x_3^{-\alpha}}{p_3}$$

$$\Rightarrow \frac{\alpha x_3}{p_1 + p_2} = \frac{(1-\alpha) x_1}{p_3}$$

$$\Rightarrow x_3 = \frac{(1-\alpha)}{\alpha} \frac{(p_1 + p_2)}{p_3} x_1$$

Sub in BC $\Rightarrow$ $x_3 = \frac{(1-\alpha)}{\alpha} \frac{(p_1 + p_2)}{p_3} \left(\frac{w - p_3 x_3}{p_1 + p_2} \right)$

$$\Rightarrow x_3 = \frac{(1-\alpha)}{\alpha} \left[\frac{w}{p_3} - x_3 \right]$$

$$\Rightarrow x_3 = \frac{(1-\alpha)}{\alpha} \frac{w}{p_3} - \frac{(1-\alpha)}{\alpha} x_3$$

$\alpha x_3 + x_3 - \alpha x_3$
$= x_3$

$$\Rightarrow \frac{\alpha}{\alpha} x_3 + \frac{(1-\alpha)}{\alpha} x_3 = \frac{(1-\alpha)}{\alpha} \frac{w}{p_3}$$

$$\Rightarrow \quad \frac{x_3}{\alpha} = \frac{(1-\alpha)}{\alpha} \frac{w}{p_3}$$

$$\Rightarrow \quad x_3 = (1-\alpha) \frac{w}{p_3}$$

$$\Rightarrow (p_1 + p_2) x_1 + \cancel{p_3} (1-\alpha) \frac{w}{\cancel{p_3}} = w$$

$$(p_1 + p_2) x_1 = w - (1-\alpha) w$$

$$x_1 = \frac{\alpha w}{p_1 + p_2}$$

Check BC:

$$\cancel{(p_1+p_2)} \left(\frac{\alpha w}{\cancel{p_1+p_2}} \right) + \cancel{p_3} \left((1-\alpha) \frac{w}{\cancel{p_3}} \right) = w$$

$$\alpha w + w - \alpha w = w$$

Similar logic if $p_4 < p_3$. So Marshallian demand is

$$x(p, w) = \begin{cases} \left(\frac{\alpha w}{p_1 + p_2}, \frac{\alpha w}{p_1 + p_2}, (1-\alpha) \frac{w}{p_3}, 0 \right) & \text{if } p_3 < p_4 \\ \left(\frac{\alpha w}{p_1 + p_2}, \frac{\alpha w}{p_1 + p_2}, 0, (1-\alpha) \frac{w}{p_4} \right) & \text{if } p_4 < p_3 \end{cases}$$

$\boxed{Q1}$ (a)

$$v(p, w) = \begin{cases} \left(\frac{\alpha w}{p_1+p_2}\right)^{\alpha} \left(\frac{(1-\alpha)w}{p_3}\right)^{1-\alpha} & \text{if } p_3 < p_4 \\ \left(\frac{\alpha w}{p_1+p_2}\right)^{\alpha} \left(\frac{(1-\alpha)w}{p_4}\right)^{1-\alpha} & \text{if } p_4 < p_3 \end{cases}$$

~~x_1 is a normal good~~

$$\frac{\partial x_i}{\partial w} = \frac{\alpha}{p_1+p_2} > 0$$, so x_1 is a normal good.

(b) Find $e(p,u)$

If $p_3 < p_4$,

$u = v(p, e(p,u))$

$$u = \left(\frac{\alpha}{p_1+p_2}\right)^{\alpha} \left(\frac{1-\alpha}{p_3}\right)^{1-\alpha} e(p,u)$$

$$e(p,u) = \begin{cases} u\left(\frac{p_1+p_2}{\alpha}\right)^{\alpha}\left(\frac{p_3}{1-\alpha}\right)^{1-\alpha}, & \text{if } p_3 < p_4 \\ u\left(\frac{p_1+p_2}{\alpha}\right)^{\alpha}\left(\frac{p_4}{1-\alpha}\right)^{1-\alpha} & \text{if } p_4 < p_3 \end{cases}$$

$$h_1(p,u) = \frac{\partial e(p,u)}{\partial p_1}$$

$$\Rightarrow h_1(p,u) = \begin{cases} u\alpha\left(\frac{p_1+p_2}{\alpha}\right)^{\alpha-1}\left(\frac{p_3}{1-\alpha}\right)^{1-\alpha} & \text{if } p_3 < p_4 \\ u\alpha\left(\frac{p_1+p_2}{\alpha}\right)^{\alpha-1}\left(\frac{p_4}{1-\alpha}\right)^{1-\alpha} & \text{if } p_4 < p_3 \end{cases}$$

$$\frac{\partial h_1(p,u)}{p_2} = u\alpha(\alpha-1)\left(\frac{p_1+p_2}{\alpha}\right)^{\alpha-2}\left(\frac{p_3}{1-\alpha}\right)^{1-\alpha} < 0$$

$\Rightarrow$ Good 2 is a complement for good 1

(b) con't

$$\frac{\partial h_1(p,u)}{\partial p_3} = u\,\alpha\left(\frac{p_1+p_2}{\alpha}\right)^{\alpha-1}(1-\alpha)\left(\frac{p_3}{1-\alpha}\right)^{-\alpha} > 0$$

$\Rightarrow$ Good 3 is a substitute for good 1

$\Rightarrow$ Swap p_4 for p_3 in the equation above

$\hookrightarrow$ Good 4 is also a substitute for good 1

$\boxed{Q1}$

(c) As we found in (a), if $P_3 > P_4$ the consumer will not consume any of good 3. So, the consumer would be a net seller of good 3.

~~The demand for good 1 is increasing in P_3. This is a substitution effect~~

~~The Marshallian demand for good 1 is constant wrt to P_3, but the Hicksian demand is increasing. Thus, the Hicksian demand represents the substitution effect and there must be an offsetting wealth effect.~~

~~(d)~~ Since the consumer will not consume any good 3. The increase in P_3 will lead to the consumer having more wealth and thus can increasing their demand for good 1. This is a wealth effect

(d) If $P_3 < P_4$, ~~she is always a net seller of good 3.~~

She is a net seller if ~~$e_3 > (1-\alpha)\frac{\ldots}{P_3}$~~

$$e_3 > (1-\alpha)\,\frac{P_3 e_3 + P_4 e_4}{P_3}$$

She is a net buyer if $e_3 < (1-\alpha)\,\dfrac{P_3 e_3 + P_4 e_4}{P_3}$

Q2

(a) The data is consistent w/ profit-maximizing firm iff it satisfies WAPM:

p = \ y =	$(10,-3,-4)$	$(15,-6,-8)$	$(8,-5,-1)$
$(1,1,1)$	3	1	2
$(2,1,1)$	13	16	10
$(1,1,2)$	-1	-7	1

Yes, the data satisfy WAPM because for any $y \in y(p)$ and $y' \in y(p')$, $p \cdot y \geq p \cdot y'$.

A production set that rationalizes the data is the inner bound:

$$Y^I = \{(10,-3,-4),(15,-6,-8),(8,-5,-1)\}$$

$\boxed{Q2}$

(b) Consistent w/ convex production set?

~~No, notice that y^1 is between y^2 and y^3 for good 1 and good 3, but not good 2. This implies the production set would not be convex.~~

~~Yes, since in (a) we the data is rationalizable,~~

Yes, in (a), we rationalized the data by checking whether any ~~points~~ production sets were more profitable at the observed price vector. Effectively, we drew a hyperplane at each point perpendicular to the price vector. Since the data was rationalizable, it is consistent with a convex production set the outer bound:

$\cancel{Y^O = \{\mathbb{R}^3 \geq p}$

$$Y^O = \{ y \in \mathbb{R}^3 : p \cdot y \leq \pi(p),\ p \in \{(1,1,1), (2,1,1), (1,1,2)\}\}$$

$$\text{where } \pi(p) = \begin{cases} 3 & \text{if } p = (1,1,1) \\ 16 & \text{if } p = (2,1,1) \\ 1 & \text{if } p = (1,1,2). \end{cases}$$

Q2

(c) No, going from y^1 to y^2, output increases, z_1 increases, and z_2 increases.

Going from y^1 to y^3, output decreases, z_1 increases, and z_2 decreases

Going from y^2 to y^3, output decreases, z_1 decreases, and z_2 decreases.

For f to be supermodular, all choice variable need to have increasing differences

If $y^3 = (8, -2, -3)$, then f would be supermodular.

Everything needs to move in same direction. Or opposite direction, but consistently.

Q3

(a) Is $\succsim_{min}$ complete?

Yes, because every lottery has a worst-case outcome. ~~When comparing two lotteries the worst-case outcome can either be ..~~
L_* can either equal L'_* or $L_* > L'_*$ or $L_* < L'_*$. Thus $\succsim_{min}$ is complete.

Transitive? Yes, consider three lotteries L, L', L''. Let L_*, L'_*, L''_* be the worst-case outcomes. If $L_* \geq L'_*$ and $L'_* \geq L''_*$, then $L \succsim_{min} L'$ and $L' \succsim_{min} L''$. Since $L_*, L'_*, L''_* \in \mathbb{R}_+$, $L_* \geq L''_*$, so $L \succsim_{min} L''$.

Continuity? Yes, L, L', L'' w/ worst-case outcomes $L_* \geq L'_* \geq L''_*$ so $L \succsim_{min} L' \succsim_{min} L''$.

$\boxed{Q3}$ (a) continued.

Independence? Yes, let $L_* \geq L_*'$ so $L \succsim_{max} L'$.
Consider L'' w/ worst-case outcome L_*''. For $\alpha \in (0,$

$$L_* \geq L_*'$$

$$\alpha L_* \geq \alpha L_*'$$

$$\alpha L_* + (1-\alpha) L_*'' \geq \alpha L_*' + (1-\alpha) L_*''$$

$\Rightarrow \alpha L + (1-\alpha) L'' \succsim_{max} \alpha L' + (1-\alpha) L''$.

Yes, by the von Neumann and Morgenstern Theorem, $\succsim_{max}$ can be represented by an expected utility function $U(L) = \Sigma_{i: x_i \in X} p_i u(x_i)$.

$\boxed{Q3}$ (b) $u(x) = 1 - e^{-cx}$

$\cancel{\frac{\partial u}{\partial x} = ce^{-cx}}$

$\cancel{\frac{\partial^2 u}{\partial^2 x} = -c^2 e^{-cx}}$

If $L \succsim_{max} L'$, then $L_* \geq L'_*$.

Under CARA utility,

$$U(L) = \sum_{i : x_i \in X} p_i \left(1 - e^{-cx_i}\right)$$