

FIN 971: Problem Set 2¹

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Problem outlined in Dean's website. The profit function and the investment adjustment cost functional forms are:

$$\begin{aligned}\pi(k_t, z_t) &= z_t k_t^\theta \\ \psi(k_{t+1} - (1 - \delta)k_t, k_t) &= \frac{\psi_0(k_{t+1} - k_t)^2}{2k_t}\end{aligned}$$

Notice that:

$$\begin{aligned}\psi(k_{t+1} - (1 - \delta)k_t, k_t) &= \frac{\psi_0(k_{t+1} - (1 - \delta)k_t + (1 - \delta)k_t - k_t)^2}{2k_t} \\ \implies \psi(I_t, k_t) &= \frac{\psi_0(I_t + (1 - \delta)k_t - k_t)^2}{2k_t} \\ &= \frac{\psi_0(I_t - \delta k_t)^2}{2k_t} \\ \psi_I(I_t, k_t) &= \frac{\psi_0(I_t - \delta k_t)}{k_t} \\ \psi_k(I_t, k_t) &= \psi_0 \frac{(I_t - \delta k_t)^2(2) - 2k_t * 2(I_t - \delta k_t)}{(2k_t)^2} \\ &= \psi_0 \frac{I_t^2 - \delta^2 k_t^2}{2k_t^2}\end{aligned}$$

The model is characterized by the following equations:

$$\begin{aligned}q_t &= 1 + \frac{\psi_0(I_t - \delta k_t)}{k_t} \\ q_t &= E_t \left[M_{t+1} \left\{ z_{t+1} \theta k_{t+1}^{\theta-1} - \psi_0 \frac{I_{t+1}^2 - \delta^2 k_{t+1}^2}{2k_{t+1}^2} + q_{t+1}(1 - \delta) \right\} \right] \\ I_t &= k_{t+1} - k_t + \delta k_t \\ c_t &= z_t k_t^\theta - \frac{\psi_0(I_t - \delta k_t)^2}{2k_t} - I_t \\ z_t &= (1 - \rho_z) + \rho_z z_{t-1} + \varepsilon_t \\ M_t &= \frac{1}{1 + r} \left(\frac{c_t}{c_{t-1}} \right)^{-\gamma}\end{aligned}$$

¹Instructor: Dean Corbae

1. Steady State

In steady state, $x_t = x_{t+1} = \bar{x} \forall x$.

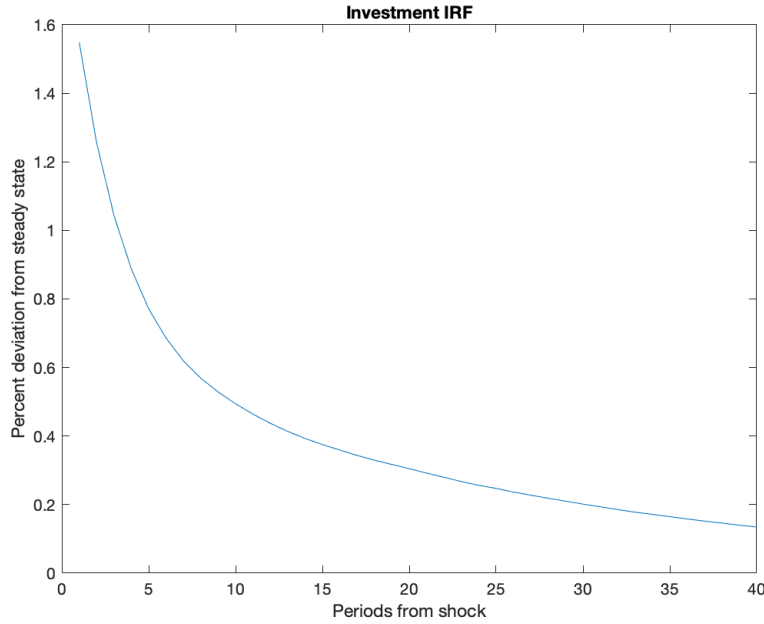
$$\begin{aligned}\bar{z} &= 1 \\ \bar{I} &= \delta \bar{k} \\ \bar{q} &= 1 \\ \bar{M} &= \frac{1}{1+r} \\ \bar{c} &= \bar{k}^\theta - \delta \bar{k} \\ \bar{k} &= \left(\frac{\theta}{r + \delta} \right)^{\frac{1}{1-\theta}}\end{aligned}$$

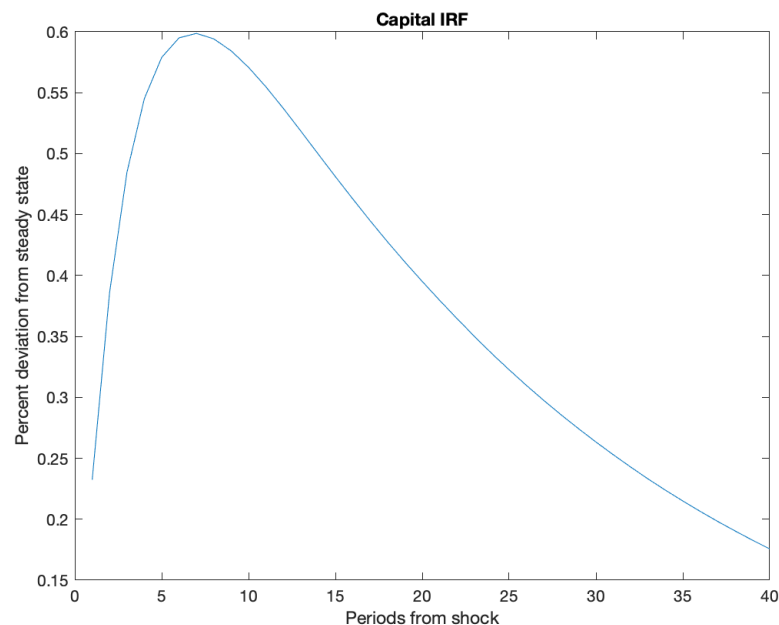
because $\psi(\bar{I}, \bar{k}) = \psi_I(\bar{I}, \bar{k}) = \psi_k(\bar{I}, \bar{k}) = 0$. For the parameter values from the problem set, the steady state values are:

\bar{z}	1.0
\bar{q}	1.0
\bar{M}	0.9615
\bar{k}	77.2351
\bar{I}	11.5853
\bar{c}	9.3785

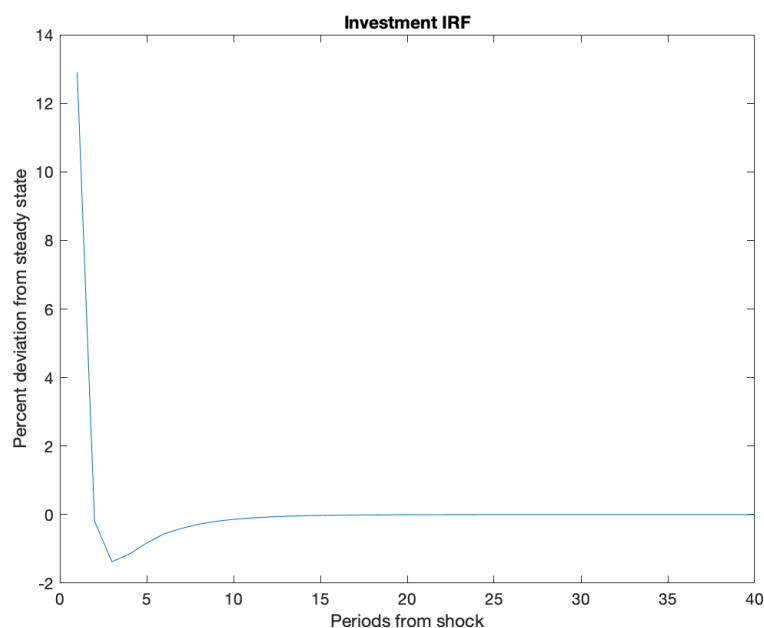
2. Impulse Response Functions

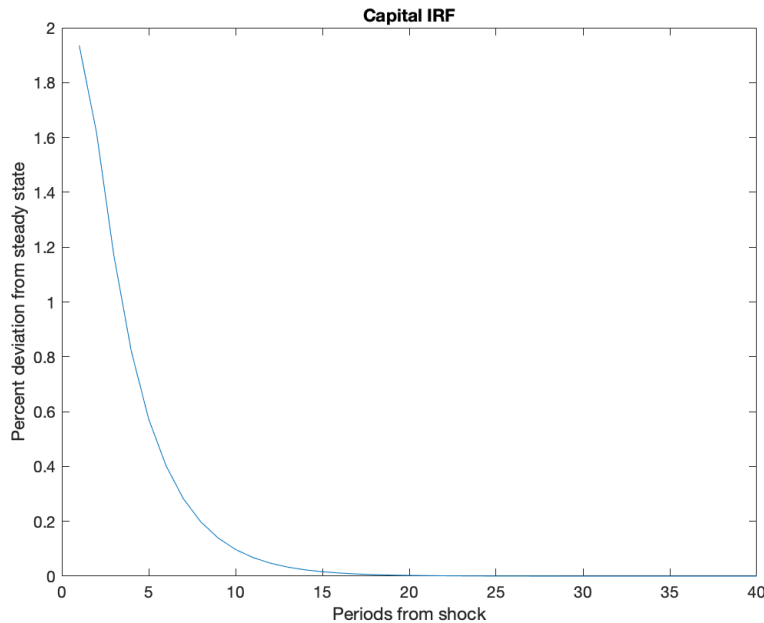
For $\gamma = 2.0$, the impulse response functions of investment and capital to a one standard deviation shock to productivity.





For $\gamma = 0.0$, the consumer is risk neutral and with an infinite intertemporal elasticity of substitution. The consumer does not care about consumption smoothing, so maximizes their total consumption. Thus, they invest everything now that production is more productivity. Thus, capital and investment spike and then return the steady state as productivity returns to the steady state level.





3. Q-Regression

Running this regression with the simulated data:

$$\frac{I_t}{k_t} = \alpha + \beta_1 q_{t-1} + \beta_2 \frac{\pi_{t-1}}{k_{t-1}}$$

I get the following OLS coefficients:

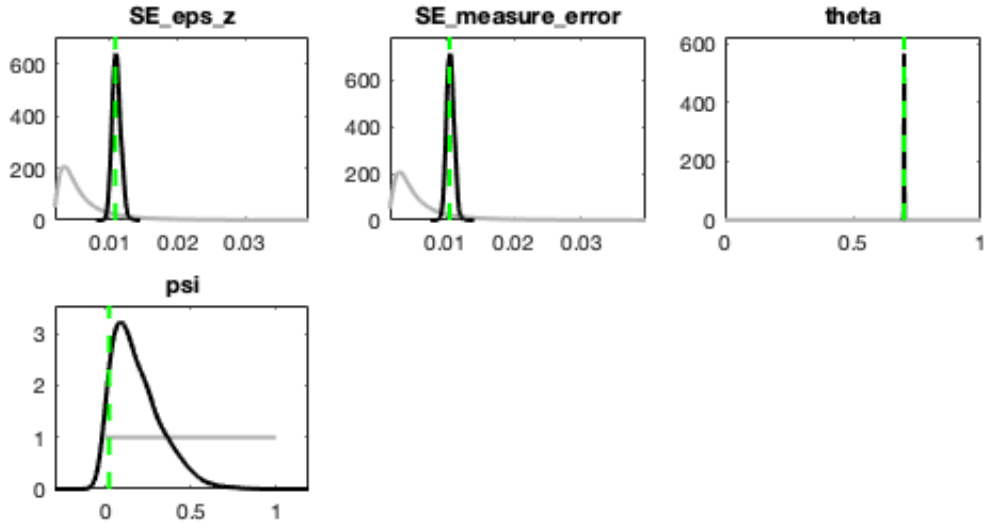
α	-27.9899
β_1	28.0687
β_2	0.2624

Since q_t is always right around 1, I don't think there's much to be gained looking at β_1 . But β_2 is positive. This means that if a firm relatively more profitable, they invest more.

3. Bayesian Estimation with Simulated Data

I changed "noadjustment_est.mod" to include capital adjustment costs. The following figure shows the priors and posteriors of $\theta, \psi_0, \sigma_\varepsilon, \sigma_q$. Similar to the no adjustment cost case, the posteriors are very different than the priors, indicating that this model is also well-identified.

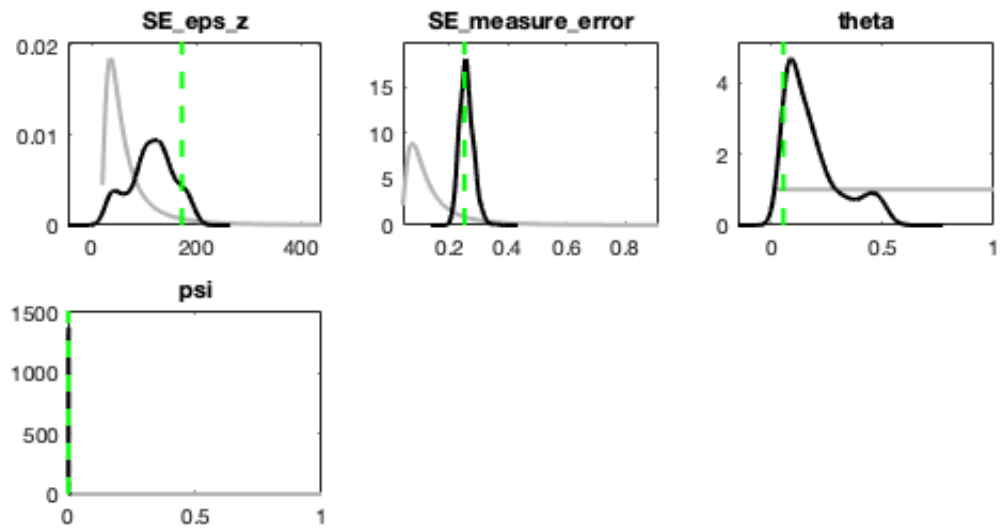
Parameter	Prior Mean	Posterior Mean	90 % HPD Interval
θ	0.5	0.7002	[0.6989, 0.7012]
ψ_0	0.5	0.1848	[0.0005, 0.3846]
σ_ε	0.007	0.0110	[0.0100, 0.0120]
σ_q	0.007	0.0107	[0.0099, 0.0116]



4. Bayesian Estimation with Real Data

I changed “noadjustment_est_D.mod” to include capital adjustment costs. The following figure shows the priors and posteriors of $\theta, \psi_0, \sigma_\varepsilon, \sigma_q$.

Parameter	Prior Mean	Posterior Mean	90 % HPD Interval
θ	0.5	0.1833	[0.0335, 0.4150]
ψ_0	0.5	0.0004	[0.0000, 0.0010]
σ_ε	78.133	114.3764	[41.4847, 185.1721]
σ_q	0.162	0.2595	[0.2202, 0.2943]



I think the estimated parameters are so different because this is a relatively simple model and does include a lot of important factors.