

Notes on Hennessy and Whited (2007)

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- Notation is a bit confusing in the original paper, so trying to clear up my confusion here.
- Notation: Current productivity is z , yesterday productivity is z^- , and tomorrow productivity is z'
- Cash flow from operations:

$$z\pi(k) \equiv zk^\alpha$$

- Corporate tax income bill:

$$T^C(k, b, z^-, z) \equiv [\tau_c^+ \chi + \tau_c^-(1 - \chi)] \cdot [z\pi(k) - \delta k - r(k, b, z^-)b]$$

where $\chi \equiv \mathbb{1}[z\pi(k) - \delta k - r(k, b, z^-)b > 0]$

- Individual tax rate τ_i
- Cash distribution taxes

$$T^d(X) \equiv \int_0^X \tau_d(x) dx$$

$$\text{where } \tau_d(x) \equiv \bar{\tau}_d \times [1 - e^{-\phi x}]$$

$$\Rightarrow T^d(X) \equiv \begin{cases} 0, & X \leq 0 \\ \frac{\bar{\tau}_d}{\phi} (\phi X + e^{-\phi X} - 1), & X > 0 \end{cases}$$

- Costly external equity financing

$$\Lambda(x) = \lambda_0 + \lambda_1 x + \lambda_2 x^2$$

$$\text{where } \lambda_0 \geq 0, \lambda_1 \geq 0, \lambda_2 \geq 0$$

- State variable is net worth w and choice variables are debt and capital for next period
- Equity value function

$$V(w, z) = \max_{k', b'} \left\{ \underbrace{\Phi[w + b' - k' - T^d(w + b' - k')]}_{\text{cash payment to equity holders}} - \underbrace{(1 - \Phi)[k' - w - b' + \Lambda(k' - w - b')]}_{\text{equity issuance}} + \left[\frac{1}{1 + r(1 - \tau_i)} \right] E \left[\left(V(w(k', b', z, z'), z') \right)^+ \middle| z \right] \right\}$$

where

$$\Phi \equiv \mathbb{1}(w + b' > k')$$

$$w(k', b', z, z') \equiv z' \pi(k') + (1 - \delta)k' - T^C(k', b', z, z') - (1 + r(k', b', z))b'$$

- Naive value function:

$$V(k, b, z, z^-) = \max_{(k', b')} \left\{ \underbrace{w + b' - k'}_{\text{cash dividend if (+) or equity issuance if (-)}} - \underbrace{T^d(w + b' - k')}_{\text{taxes on cash dividend}} - \underbrace{\Lambda(-(w + b' - k'))}_{\text{equity issuance cost}} + \frac{1}{1 + r(1 - \tau_i)} E \left[\underbrace{\max\{V(k', b', z', z), 0\}}_{\text{if } V \text{ is } (-) \text{ can default}} \right] \right\}$$

where

$$\underbrace{y}_{\text{taxable corporate income}} \equiv \underbrace{zk^\alpha}_{\text{operating profits}} - \underbrace{\delta k}_{\text{depreciation}} - \underbrace{\tilde{r}(k, b, z^-)b}_{\text{interest on debt}}$$

$$\underbrace{T^C(x)}_{\text{corporate income tax bill}} \equiv \begin{cases} \tau_c^+ x, & \text{if } x > 0 \\ \tau_c^- x, & \text{if } x \leq 0 \end{cases}$$

$$\underbrace{w}_{\text{realized net worth}} \equiv \underbrace{y - T^C(y)}_{\text{after-tax corporate income}} + \underbrace{k}_{\text{capital}} - \underbrace{b}_{\text{debt principal}}$$

$$\underbrace{T^d(x)}_{\text{taxes on cash dividend}} = \begin{cases} \frac{\bar{\tau}_d}{\phi} (\phi x + e^{-\phi x} - 1), & x > 0 \\ 0, & x \leq 0 \end{cases}$$

$$\underbrace{\Lambda(x)}_{\text{equity issuance cost}} = \begin{cases} \lambda_0 + \lambda_1 x + \lambda_2 x^2, & \text{if } x > 0 \\ 0, & \text{if } x \leq 0 \end{cases}$$

- Smarter value function:

$$V(w, z) = \max_{(k', b')} \left\{ \underbrace{w + b' - k'}_{\text{cash dividend if (+) or equity issuance if (-)}} - \underbrace{T^d(w + b' - k')}_{\text{taxes on cash dividend}} - \underbrace{\Lambda(-(w + b' - k'))}_{\text{equity issuance cost}} + \frac{1}{1 + r(1 - \tau_i)} E \left[\underbrace{\max\{V(w', z'), 0\}}_{\text{if } V \text{ is } (-) \text{ can default}} \right] \right\}$$

where

$$\underbrace{y'}_{\text{taxable corporate income}} \equiv \underbrace{z'(k')^\alpha}_{\text{operating profits}} - \underbrace{\delta k'}_{\text{depreciation}} - \underbrace{\tilde{r}(k', b', z)b'}_{\text{interest on debt}}$$

$$\underbrace{T^C(x)}_{\text{corporate income tax bill}} \equiv \begin{cases} \tau_c^+ x, & \text{if } x > 0 \\ \tau_c^- x, & \text{if } x \leq 0 \end{cases}$$

$$\underbrace{w'}_{\text{realized net worth}} \equiv \underbrace{y' - T^C(y')}_{\text{after-tax corporate income}} + \underbrace{k'}_{\text{capital}} - \underbrace{b'}_{\text{debt principal}}$$

$$\underbrace{T^d(x)}_{\text{taxes on cash dividend}} = \begin{cases} \frac{\bar{\tau}_d}{\phi} (\phi x + e^{-\phi x} - 1), & x > 0 \\ 0, & x \leq 0 \end{cases}$$

$$\underbrace{\Lambda(x)}_{\text{equity issuance cost}} = \begin{cases} \lambda_0 + \lambda_1 x + \lambda_2 x^2, & \text{if } x > 0 \\ 0, & \text{if } x \leq 0 \end{cases}$$

- Smarter value function with bond prices instead of interest rates:

$$V(w, z) = \max_{(k', b')} \left\{ \underbrace{w + b'q(k', b', z) - k'}_{\text{cash dividend if (+) or equity issuance if (-)}} - \underbrace{T^d(w + b'q(k', b', z) - k')}_{\text{taxes on cash dividend}} - \underbrace{\Lambda(-(w + b'q(k', b', z) - k'))}_{\text{equity issuance cost}} + \frac{1}{1 + r(1 - \tau_i)} E \left[\underbrace{\max\{V(w', z'), 0\}}_{\text{if } V \text{ is } (-), \text{ firm can default}} \mid z \right] \right\}$$

where

$$\underbrace{y'}_{\text{taxable corporate income}} \equiv \underbrace{z'(k')^\alpha}_{\text{operating profits}} - \underbrace{\delta k'}_{\text{depreciation}} - \underbrace{(1 - q(k', b', z))b'}_{\text{interest on debt}}$$

$$\underbrace{T^C(x)}_{\text{corporate income tax bill}} \equiv \begin{cases} \tau_c^+ x, & \text{if } x > 0 \\ \tau_c^- x, & \text{if } x \leq 0 \end{cases}$$

$$\underbrace{w'}_{\text{realized net worth}} \equiv \underbrace{y' - T^C(y')}_{\text{after-tax corporate income}} + \underbrace{k'}_{\text{capital}} - \underbrace{q(k', b', z)b'}_{\text{debt principal}}$$

$$\underbrace{T^d(x)}_{\text{taxes on cash dividend}} = \begin{cases} \frac{\bar{\tau}_d}{\phi} (\phi x + e^{-\phi x} - 1), & x > 0 \\ 0, & x \leq 0 \end{cases}$$

$$\underbrace{\Lambda(x)}_{\text{equity issuance cost}} = \begin{cases} \lambda_0 + \lambda_1 x + \lambda_2 x^2, & \text{if } x > 0 \\ 0, & \text{if } x \leq 0 \end{cases}$$

- Zero profit condition:

$$b'(1 + r(1 - \tau_i)) = (1 + (1 - \tau_i)\tilde{r}(k', b', z))b' \int_{z_d(k', b', z)}^{\bar{z}} Q(z, dz') + \int_{\underline{z}}^{z_d(k', b', z)} R(k', z')Q(z, dz')$$