# ECON 899B - PS2

Alex von Hafften

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I verify the formula listed in the problem set. Notice that $\varepsilon_{i0}/\sigma_0 \sim N(0,1)$. The likelihood associated with each duration T_i:

$$
\begin{aligned}
P(T_i = 1 | X_i, Z_i, \theta) &= P(Y_{i0} = 1 | X_i, Z_i, \theta) \\
&= P(\alpha_0 + X_i\beta + Z_{i0}\gamma + \varepsilon_{i0} > 0 | X_i, Z_i, \theta) \\
&= P(\varepsilon_{i0} > -\alpha_0 - X_i\beta - Z_{i0}\gamma | X_i, Z_i, \theta) \\
&= P(\varepsilon_{i0} < \alpha_0 + X_i\beta + Z_{i0}\gamma | X_i, Z_i, \theta) \\
&= P(\varepsilon_{i0}/\sigma_0 < (\alpha_0 + X_i\beta + Z_{i0}\gamma)/\sigma_0 | X_i, Z_i, \theta) \\
&= \Phi((\alpha_0 + X_i\beta + Z_{i0}\gamma)/\sigma_0)
\end{aligned}
$$

$$
\begin{aligned}
P(T_i = 2 | X_i, Z_i, \theta) &= P(Y_{i0} = 0, Y_{i1} = 1 | X_i, Z_i, \theta) \\
&= P(Y_{i0} = 0 | X_i, Z_i, \theta) \times P(Y_{i1} = 1 | Y_{i0} = 0, X_i, Z_i, \theta) \\
&= P(\alpha_0 + X_i\beta + Z_{i0}\gamma + \varepsilon_{i0} < 0 | X_i, Z_i, \theta) \\
&\quad \times P(\alpha_0 + X_i\beta + Z_{i1}\gamma + \varepsilon_{i1} > 0 | \alpha_0 + X_i\beta + Z_{i0}\gamma + \varepsilon_{i0} < 0, X_i, Z_i, \theta) \\
&= P(\alpha_0 + X_i\beta + Z_{i0}\gamma + \varepsilon_{i0} < 0 | X_i, Z_i, \theta) \\
&\quad \times P(\alpha_0 + X_i\beta + Z_{i1}\gamma + \rho\varepsilon_{i0} + \eta_{i1} > 0 | \alpha_0 + X_i\beta + Z_{i0}\gamma + \varepsilon_{i0} < 0, X_i, Z_i, \theta) \\
&= P(\varepsilon_{i0} < -\alpha_0 - X_i\beta - Z_{i0}\gamma | X_i, Z_i, \theta) \\
&\quad \times P(\eta_{i1} > -\alpha_0 - X_i\beta - Z_{i1}\gamma - \rho\varepsilon_{i0} | \varepsilon_{i0} < -\alpha_0 - X_i\beta - Z_{i0}\gamma, X_i, Z_i, \theta) \\
&= P(\varepsilon_{i0} < -\alpha_0 - X_i\beta - Z_{i0}\gamma | X_i, Z_i, \theta) \\
&\quad \times P(\eta_{i1} < \alpha_0 + X_i\beta + Z_{i1}\gamma + \rho\varepsilon_{i0} | \varepsilon_{i0} < -\alpha_0 - X_i\beta - Z_{i0}\gamma, X_i, Z_i, \theta) \\
&= .... \\
&= \Phi((-\alpha_0 - X_i\beta - Z_{i0}\gamma)/\sigma_0) \\
&\quad \times \int_{-\infty}^{-\alpha_0 - X_i\beta - Z_{i0}\gamma} P(\eta_{i1} < \alpha_0 + X_i\beta + Z_{i1}\gamma + \rho w | X_i, Z_i, \theta) dw \\
&= \Phi((-\alpha_0 - X_i\beta - Z_{i0}\gamma)/\sigma_0) \\
&\quad \times \int_{-\infty}^{-\alpha_0 - X_i\beta - Z_{i0}\gamma} \Phi(\alpha_0 + X_i\beta + Z_{i1}\gamma + \rho w) dw
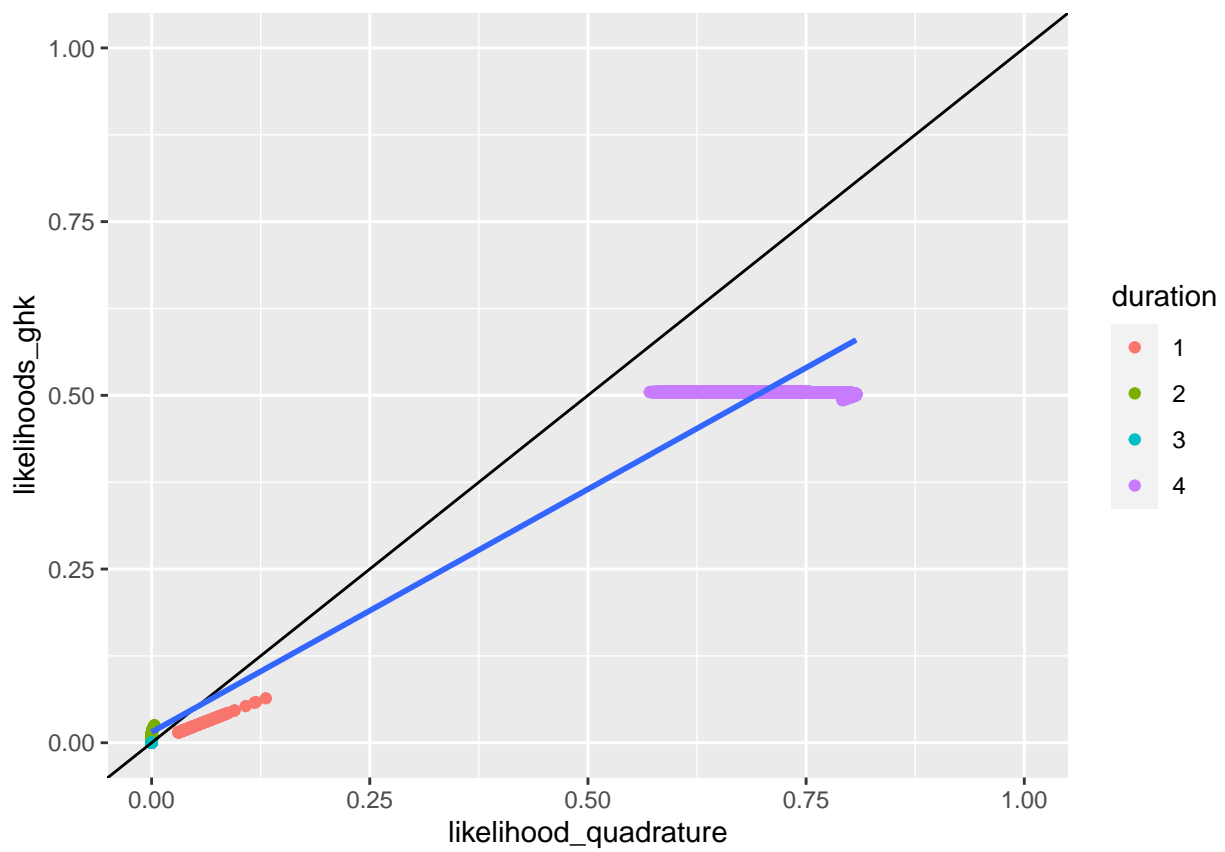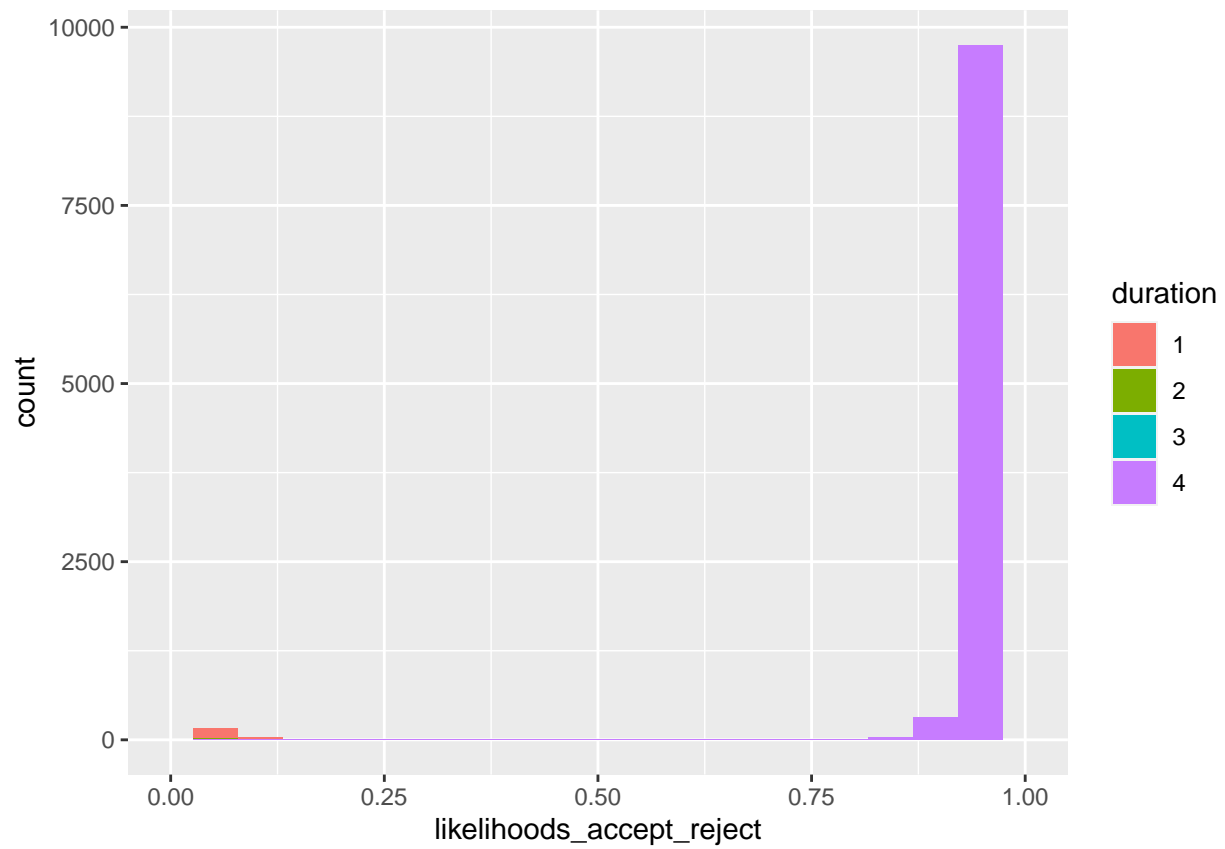\end{aligned}
$$

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## Warning: Removed 8 rows containing missing values (geom_bar).
```
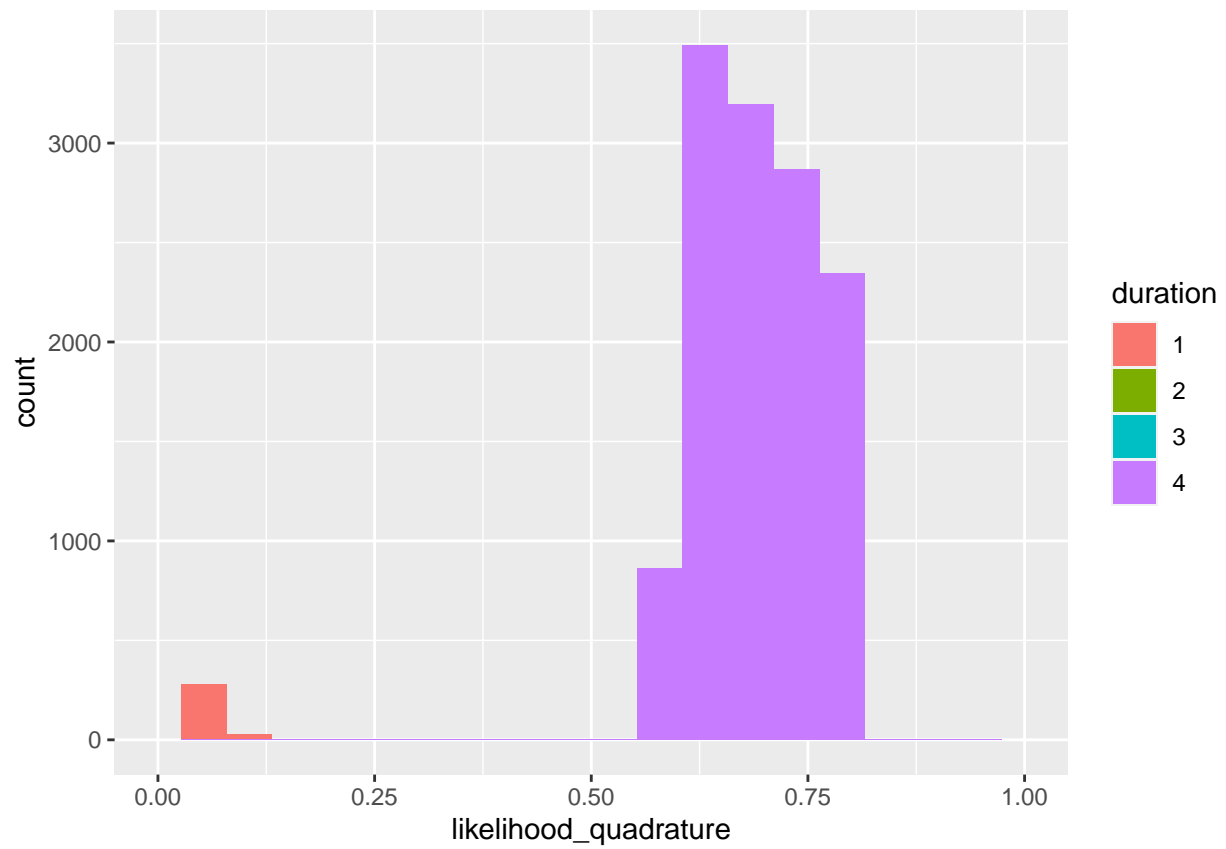
```
## Warning: Removed 8 rows containing missing values (geom_bar).
```
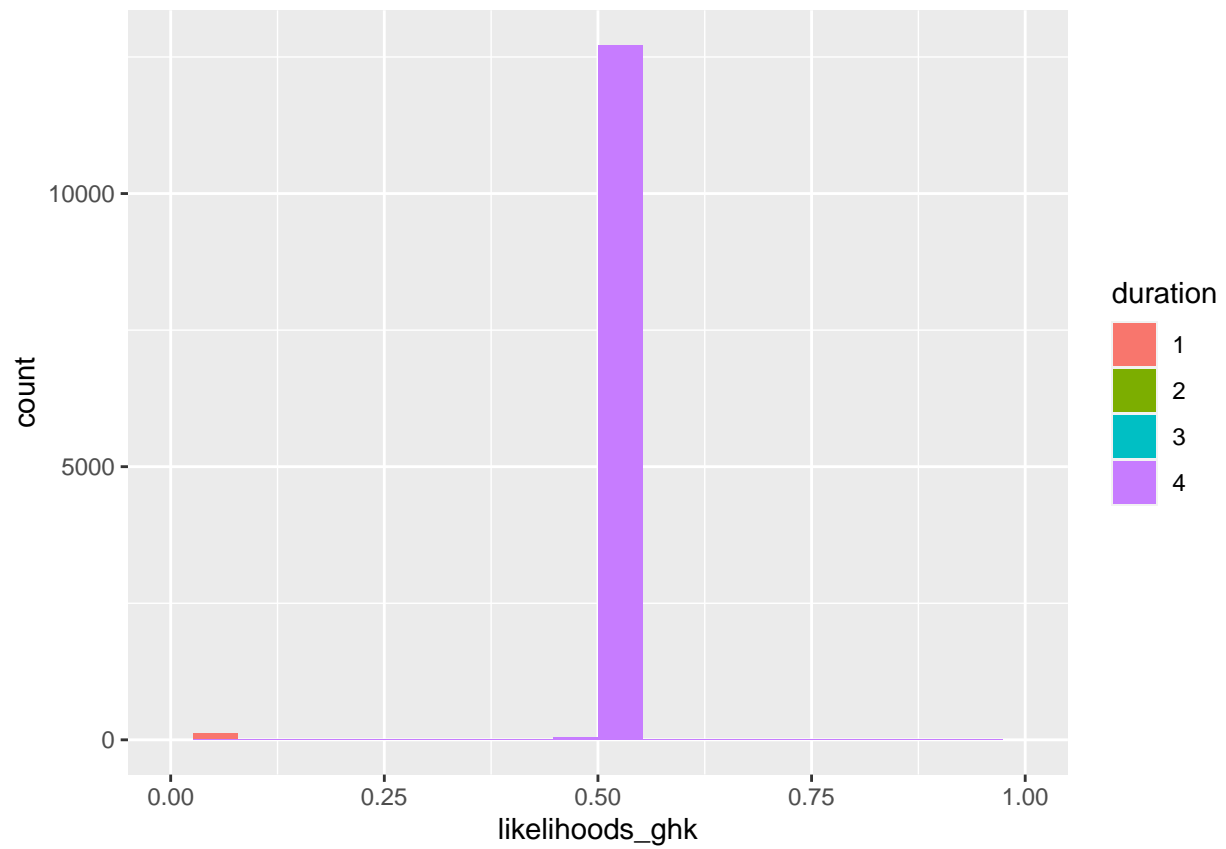
Warning: Removed 8 rows containing missing values (geom_bar).

Warning: Removed 8 rows containing missing values (geom_smooth).