ECON 899A - Problem Set 1

Alex von Hafften*

9/15/2021

Assume that households have log preferences, the production technology satisfies $Y_t = Z_t K_t^\theta$ where $\theta = 0.36$; and capital depreciates at rate $\delta = 0.025$. We will assume technology shocks follow a 2 state Markov Process. The transition matrix is calibrated to NBER business cycle data where we take an expansion to be an instance of a positive technology shock and recession to be an instance of a negative technology shock. The transition matrix is given by

$$\Pi = \begin{bmatrix} 0.977 & 0.023 \\ 0.074 & 0.926 \end{bmatrix}$$

where, for instance, $Pr(Z_{t+1} = Z^g | Z_t = Z^g) = 0.977$.

You must expand the state space to add technology shocks from the set $\zeta = \{Z^g = 1.25; Z^b = 0.2\}$. Notice that these values satisfy that $\bar{Z} = 1$: To see this, note that Π implies an invariant distribution over the two states of $\bar{p}^g = 0.763$ and $\bar{p}^b = 0.237$. In that case, set $Z^g = 1.25$ and solved for Z^b in $\bar{Z} = \bar{p}^g Z^g + \bar{p}^b Z^b$.

1. State the dynamic programming problem.

The sequence formulation of the planners problem is:

$$\max_{(C_t, K_{t+1})_{t=1}^\infty} E_0 \sum_{t=0} \beta^t \log(C_t)$$

$$\text{s.t. } C_t + K_{t+1} = Z_t K_t^\theta + (1-\delta) K_t$$

The dynamic programming problem is:

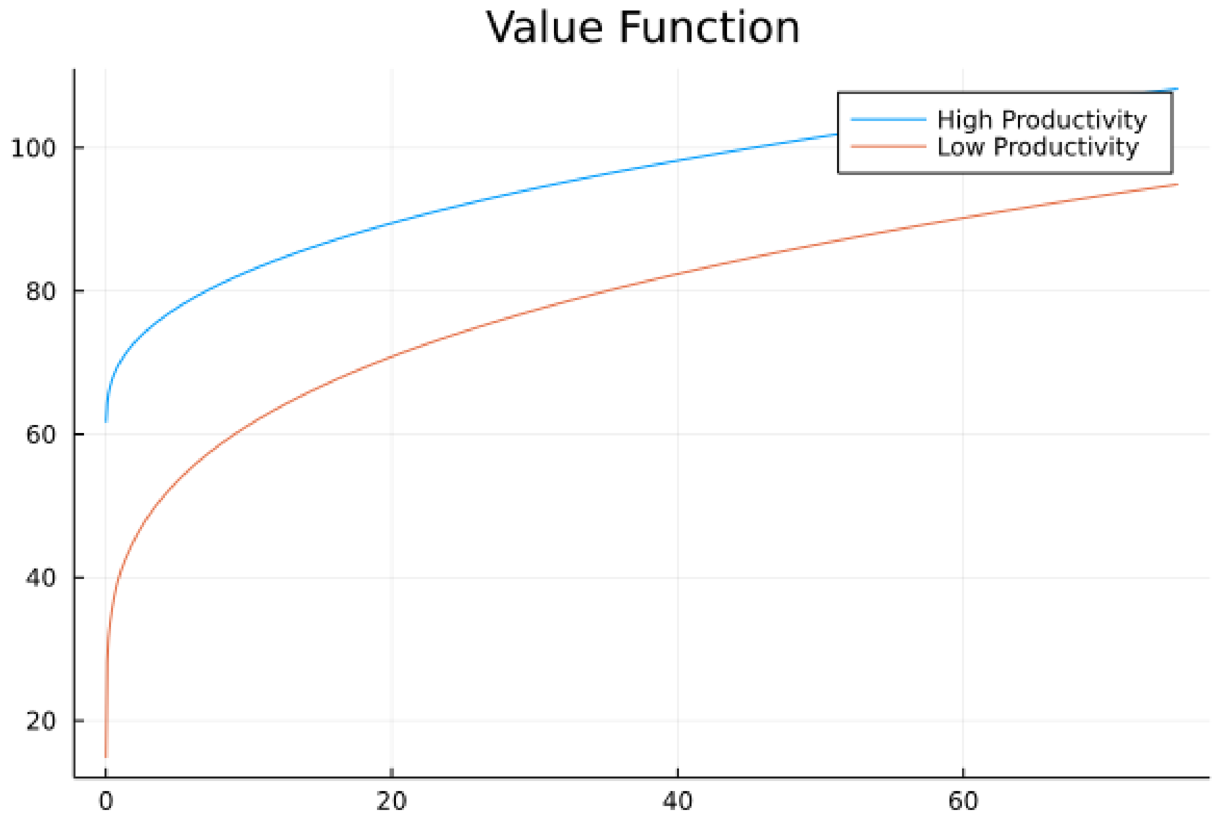
$$V(K, Z) = \max_{C, K'} \{\log(C) + \beta E[V(K', Z') | Z]\}$$

$$\text{s.t. } C + K' = Z K^\theta + (1-\delta) K$$

$$\implies V(K, Z) = \max_{K'} \{\log(Z K^\theta + (1-\delta) K - K') + \beta E[V(K', Z') | Z]\}$$

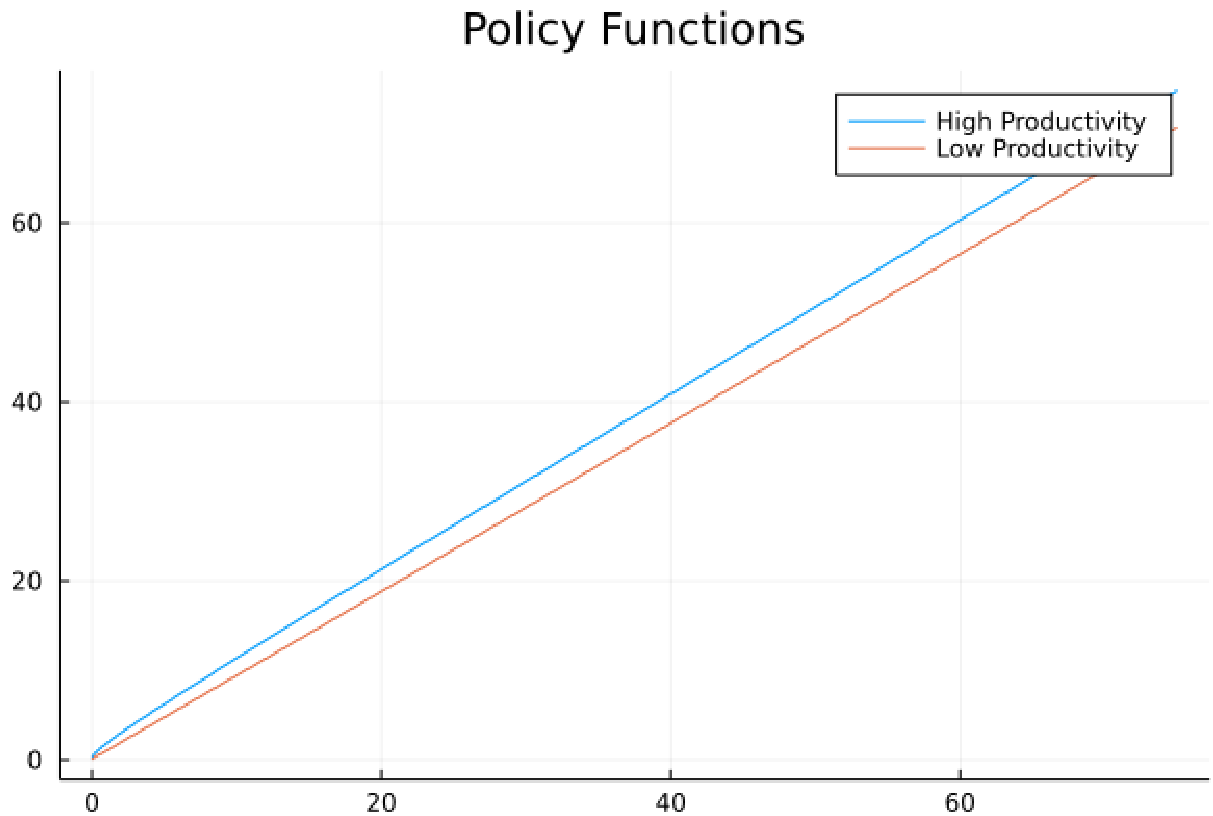
*This problem set is for ECON 899A Computational Economics taught by Dean Corbae with assistance from Phillip Coyle at UW-Madison. I worked on this problem set with a study group of Michael Nattinger and Xinxin Hu.

2. Plot the value function over K for each state Z. Is it increasing (i.e. is $V(K_{i+1}, Z) \geq V(K_i, Z)$ for $K_{i+1} > K_i$)? Is it "concave" (in the sense that $V(K_{i+1}, Z) - V(K_i, Z)$ is decreasing)?

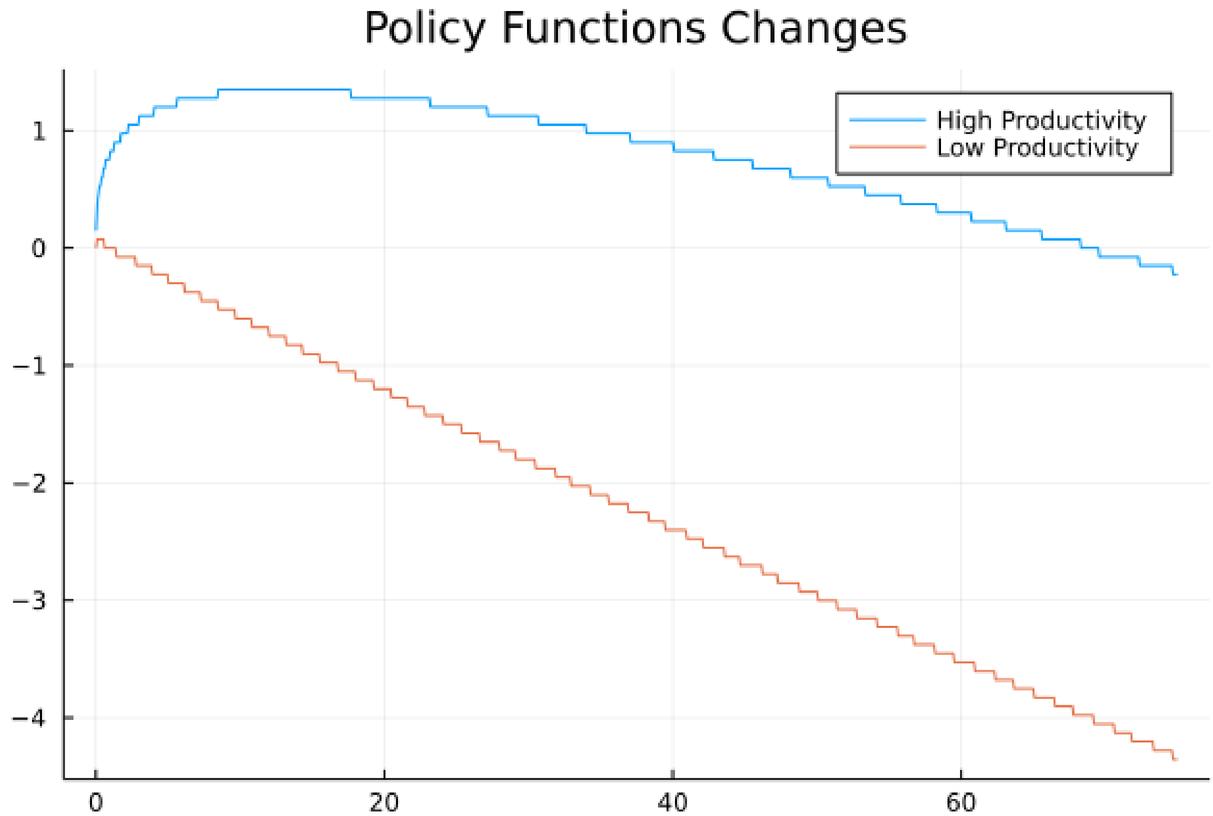


The value function is increasing and concave.

3. Is the decision rule increasing in K and Z (i.e. is $K'(K_{i+1}, Z) \geq K'(K_i, Z)$ for $K_{i+1} > K_i$ and is $K'(K, Z^g) \geq K'(K, Z^b)$)? Is saving increasing in K and Z (to see this, plot the change in the decision rule $K'(K, Z) - K$ across K for each possible exogenous state Z)?



The decision rule is increasing in K and Z.



Savings are increasing Z. For Z^g, savings are increasing in K for small K and then decreasing. And for Z^b, savings are decreasing for all levels of K.