ECON 713B - Problem Set 1

Alex von Hafften*

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1 All Pay Auction

Consider a symmetric IPV (independent private values) setting with N bidders. Find an equilibrium of the all-pay auction when each bidder's valuation is an iid draw from $F(x) = x^a$ for $a \in (0; \infty)$ and $x \in [0; 1]$.

(a) Define this auction as a Bayesian game.

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(b) Find equilibrium strategies of all players.

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(c) Verify that the strategies that you have found do constitute an equilibrium.

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(d) Does the bidding become more competitive when a increases? Explain.

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(e) Compute the expected payment from each bidder before and after she learns her value.

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*I worked on this problem set with a study group of Michael Nattinger, Andrew Smith, and Ryan Mather. I also discussed problems with Sarah Bass, Emily Case, Danny Edgel, and Katherine Kwok.

2 Tricky Seller

Two people are interested in one object. Their valuations are drawn independently from $F(x) = x$ and $F(x) = x^2$, respectively, with $x \in [0; 1]$. The seller's value (a cost, perhaps) for the object is known, $c \in [0; 1]$.

(a) Describe outcome of the First-Price Auction with a reserve price r.

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(b) Describe outcome of the Second-Price Auction with a reserve price r.

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(c) What auction and what r will the seller choose? Which player wins more often?

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(d) Suppose now that $c = 0$ and there is no reserve price. Suppose that a seller can offer discount of α to one of the bidders in the second-price auction. If a bidder is offered a discount $\alpha \in [0; 1]$, then, if she wins, she pays only a fraction α of what she had to pay otherwise. Who should be offered a discount? Compute the optimal discount and expected revenues.

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3 Third Price Auction

Consider a third-price auction with three players: an auction in which bidder with the highest value wins, but pays only the third highest bid. Assume that valuation of players are iid from the uniform distribution on [0, 1].

(a) Define the auction as a Bayesian game.

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(b) Prove that a bid of $b_i(v_i) = \frac{n-1}{n-2} v_i$ is a symmetric Bayes-Nash equilibrium of the third-price auction.

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(c) Show that the expected revenue of a seller in the third-price auction is $R_3 = \frac{n-1}{n+1}$.

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(d) What is the symmetric Bayes-Nash equilibrium strategy in a kth price auction? (You need only state how each bidder bids; you need not provide a detailed analysis.)

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