

ECON 713B - Problem Set 3

Alex von Hafften*

4/25/2021

Problem 1

A risk-neutral principal hires an agent to work on a project, offering a wage of w . The agent exerts effort e . The agent's utility function is $v(w, e) = \sqrt{w} - g(e)$, where $g(e)$ is the disutility associated with effort e . The agent can choose one of two possible effort levels, e_1 or e_2 , with associated disutility levels $g(e_1) = 1$ and $g(e_2) = \frac{1}{2}$. If the agent chooses effort level e_1 , the project yields an output of 8 for the principal with probability $\frac{1}{2}$ and 0 with probability $\frac{1}{2}$. If he chooses e_2 , the project yields an output of 8 with probability $\frac{1}{4}$ and 0 with probability $\frac{3}{4}$. The agent's reservation utility is 0.

- (a) Suppose the effort level chosen by the agent is observable by the principal. A wage contract then specifies an effort level (e_1 or e_2) and an output-contingent wage schedule $\{w_H, w_L\}$. Here w_H is the wage paid if the project yields 8, and w_L is the wage paid if the project yields 0. If effort is observable, it is optimal for the principal to choose a fixed wage contract (that is, set $w_H = w_L$) for each effort level. Informally, explain the intuition for this result.

...

- (b) If effort is observable, which effort level should the principal implement? What is the principal's optimal wage contract?

...

- (c) If effort is not observable, which effort level should the principal implement? What is the principal's optimal wage contract?

...

*I worked on this problem set with a study group of Michael Nattinger, Andrew Smith, and Ryan Mather. I also discussed problems with Sarah Bass, Emily Case, Danny Edgel, and Katherine Kwok.

Problem 2

Consider a cashless entrepreneur who wants to borrow and carry out the following project. If he exerts an effort level of e_1 , he will produce an output of z with probability $P_1 > 0$ and 0 with probability $1 - P_1$. If he exerts an effort level of e_2 he will produce an output of z with probability P_2 ($P_2 < P_1$) and 0 with probability $1 - P_2$. Let $c_1 > 0$ be the cost of effort e_1 for the entrepreneur and $c_2 = 0$ be the cost of low effort e_2 . A monopolistic bank with a cost of fund of r offers a loan of 1 unit for a reimbursement of $z - x$ when the project is successful, where x is the share of output retained by the agent. Let the entrepreneur's utility with no project be 0. Assume $P_z < r$.

Determine the optimal loan contract of a bank which maximizes its expected profit subject to the incentive and participation constraints of the entrepreneur.

...

Problem 3

Consider a monopolist producing a good of quality q . The quality can be either high ($q = 1$; then, the marginal cost of production is $c_1 > 0$) or low ($q = 0$; then, the marginal cost is $c_0 = 0$). There is a mass one of identical consumers. Each consumer's payoff from purchasing one unit of a good with quality q at price p is $U = q - p$. Assume $c_1 < 1$ so that producing the high quality good is socially efficient.

- (a) Suppose that consumers do not observe the good's quality before purchasing it. The timing of the game is as follows:
- i. The monopolist chooses the quality of the good;
 - ii. The monopolist chooses the price;
 - iii. Consumers observe the price (but not the quality) and decide whether to buy (one unit of) the good.

Find a pure-strategy PBE of the game.

...

- (b) Suppose now that a proportion α of the consumers can observe the good's quality before purchasing it. The remaining $1 - \alpha$ consumers observe product quality only after the purchase. The timing of the game is modified in the third stage: the informed consumers observe q and the price and decide whether to buy the good, while the uninformed consumers only observe the price and decide whether to buy. Find a pure-strategy equilibrium of the game, in which all consumers buy a high-quality good.