ECON 712 - PS 3

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1. Consider the following overlapping generations problem. In each period $t = 1, 2, 3, ...$ a new generation of 2 period lived households are born. Each generation has a unitary mass. There is a unit measure of initial old who are endowed with $\bar{M} > 0$ units of fiat money. Each generation is endowed with w_1 in youth and w_2 in old age of non-storable consumption goods where $w_1 > w_2$. There is no commitment technology to enforce trades. The utility function of a household of generation $t \geq 1$ is $U(c_t^t, c_{t+1}^t) = \ln(c_t^t) + \ln(c_{t+1}^t)$ where (c_t^t, c_{t+1}^t) is consumption of a household of generation t in youth (i.e., in period t) and old age (i.e., in period $t+1$). The preference of the initial old are given by $U(c_1^0) = \ln(c_1^0)$ where c_1^0 is consumption by a household of the initial old.

(a) State and solve the planner problem.

In any period t, the social planner weights agents alive equally and optimally allocates resources between them given preferences and technologies:

$$\max_{(c_t^t, c_t^{t-1}) \in \mathbb{R}_+^2} \ln(c_t^t) + \ln(c_t^{t-1})$$
$$\text{s.t. } c_t^t + c_t^{t-1} \leq w_1 + w_2$$

Since utility is strictly increasing in consumption, we know that the maximum will occur at $c_t^t + c_t^{t-1} = w_1 + w_2 \implies c_t^{t-1} = w_1 + w_2 - c_t^t$. Thus, we can write the social planner's problem as an unconstrained maximization problem:

$$\max_{c_t^t \in \mathbb{R}_+} \ln(c_t^t) + \ln(w_1 + w_2 - c_t^t)$$

Setting the first order condition to zero:

$$\frac{1}{c_t^t} - \frac{1}{w_1 + w_2 - c_t^t} = 0$$
$$\implies c_t^t = \frac{w_1 + w_2}{2}$$

Plugging the solution into the equation for the consumption of old agents:

$$c_t^{t-1} = w_1 + w_2 - \frac{w_1 + w_2}{2}$$
$$= \frac{w_1 + w_2}{2}$$

*I worked on this problem set with a study group of Michael Nattinger, Andrew Smith, and Ryan Mather. I also discussed problems with Emily Case, Sarah Bass, and Danny Edgel.

(b) State the representative household's problem in period $t \geq 0$. Try to write the budget constraints in real terms.

I break the household problem into the problem facing an initial old agent and the problem facing agents born in period $t > 0$. Let p_t be the number of dollar per unit of consumption good in period t. Define $m_{t+1}^t \in \mathbb{R}_+$ as the monetary holdings of the generation born in t between periods t and $t+1$.

The problem facing the initial old agents is:

$$\begin{aligned}
&\max_{c_1^0 \in \mathbb{R}_+} \ln(c_1^0) \\
&\text{s.t. } c_1^0 \leq w_2 + \frac{\bar{M}}{p_1}
\end{aligned}$$

The problem facing agents born in period $t > 0$ is:

$$\begin{aligned}
&\max_{(c_t^t, c_{t+1}^t) \in \mathbb{R}_+^2} \ln(c_t^t) + \ln(c_{t+1}^t) \\
&\text{s.t. } c_t^t \leq w_1 - \frac{m_{t+1}^t}{p_t} \\
&\quad c_{t+1}^t \leq w_2 + \frac{m_{t+1}^t}{p_{t+1}}
\end{aligned}$$

(c) Define and solve for an autarkic equilibrium, assuming that it exists.

(d) Define and solve for a competitive equilibrium assuming valued money but with $w_2 = 0$.

(e) Compare the solutions to the plannners problem, the autarky equilibrium and the stationary monetary competitive equilibrium with valued money, all with $w_2 = 0$.

(f) What happens to consumption, money demand and prices in a competitive equilibrium with valued money if the initial money supply is halved, i.e. $\bar{M}' = \frac{\bar{M}}{2}$. Keep the assumption that $w_2 = 0$.

2. Plot the trade offer curves for the following utility functinos where the endowment is (w_1, w_2) for goods 1 and 2, respectively.

(a) $U = 10c_1 - 4c_1^2 + 4c_2 - c_2^2, (w1, w_2) = (0, 2)$

(b) $U = \min 2c_1 + c_2, c_1 + 2c_2, (w1, w_2) = (1, 0)$

(c) $U = \min 2c_1 + c_2, c_1 + 2c_2, (w1, w_2) = (1, 10)$