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Micro Exam

① (a)

Rounds	M1	M2	M3	M4
1	W2*	W2	W1*	W3*
2		W1		
3		W3		
4		W4*		

The rounds proceed per the above table. The astericks inticate an accepted proposals DAA takes 4 rounds

(b)

Rounds	W1	W2	W3	W4
1	M1	M1*	M1	M4*
2	M4		M4*	
3	M3*			M3
4				M1
5				M2*

The rounds proceed per the above table. The asterisks inticate accepted proposals. DAA takes 5 rounds

(1)(c) Nothing changes from part (a).

From (b).

Round	W1	W2	W3	W4
1	M1 → M2	M1 → M2*	M1 → M2	M4*
2	M4		M4	
3	M3*		M3	
4			M2 → M1*	

The table above summarizes the rounds of the DAA. It take 4 rounds to complete.

The match is
W1 ⟷ M3
W2 ⟷ M2
W3 ⟷ M1
W4 ⟷ M4

A match is unstable if there exists a blocking pair

- There is no blocking pair for (M3, W1) because M3 prefers W1 over W2, W3, W4.
- There is no blocking pair for (M2, W2) because M2 prefer W2 over W1, W3, W4.
- There is no blocking pair for (M1, W3) because W3 prefers M1 over M2, M3, M4.
- There is no blocking pair for (M4, W4) because W4 prefers M4 over M1, M2, M3.

② $U_m(x,y) = 2 + 2x + 2y - xy$
$U_w(x,y) = 2 + 2x + 2y - xy$

(a) $$\frac{\partial U_m(x,y)}{\partial x} = 2 - y > 0$$
$$\frac{\partial U_w(x,y)}{\partial y} = 2 - x > 0$$

$\Rightarrow$ PAM is stable.

(b) $$h(x,y) = U_m(x,y) + U_w(x,y)$$
$$= 4 + 4x + 4y - 2xy$$

$$\frac{\partial h(x,y)}{\partial x} = 4 - 2y$$
$$\frac{\partial^2 h(x,y)}{\partial x \, \partial y} = -2 < 0$$

$\Rightarrow$ NAM is efficient by Becker's Theorem

(c) Assuming that men & women's types are uniformly distributed on $[0,1]$, the CDFs are $F(x) = x$ and $G(y) = y$ on $[0,1]$. In the continuous case, ~~[illegible]~~ a mass of men match w/ the same sized mass of women.

$$\Rightarrow 1 - F(x) = 1-(1 - G(y))$$

$$\Rightarrow F(x) = 1 - G(y)$$

$$\Rightarrow x(y) = 1 - y$$
$$\text{or} \quad y(x) = 1 - x.$$

2 (d) Define $\pi(x,y) := h(x,y) - v(x) - w(y)$

$$= 4 + 4x + 4y - 2xy - v(x) - w(y)$$

At optimum, $\left(\frac{\partial \pi}{\partial x}\right)_{x(y)=1-y} = \left(\frac{\partial \pi}{\partial y}\right)_{y(x)=1-x} = 0$

$$\frac{\partial \pi}{\partial x} = 4 - 2y - v'(x) = 0$$

$\Rightarrow$ $v'(x) = 4 - 2y$

$\Rightarrow$ $= 4 - 2(1-x)$

$= 4 - 2 + x$

$= 2 + x$

$\Rightarrow$ $v(x) = 2x + \frac{x^2}{2} + c_x$

By symmetry, $w(y) = 2y + \frac{y^2}{2} + c_y$

By $x = y = 0 \Rightarrow h(0,0) = 4$

$$h(0,0) = v(0) + w(0)$$

$\Rightarrow$ $4 = 2(0) + \frac{(0)^2}{2} + c_x + 2(0) + \frac{0^2}{2} + c_y$

$\Rightarrow$ $4 = c_x + c_y$

From the zero outside option

$\Rightarrow c_x \geq 0$ and $c_y \geq 0$

Thus, wages are not uniquely determined.

$v(x) = 2x + \frac{x^2}{2} + c_x$ where $c_x, c_y \geq 0$ and $c_x + c_y = 4$.

$w(y) = 2y + \frac{y^2}{2} + c_y$

2 (d) [The second part d on 2.]

$$U_m(x,y) = U_w(x,y) = 2 + 2x + 2y - xy - t$$

If NTU: $\frac{\partial U_m}{\partial x} = 2 - y > 0$ $\Rightarrow$ PAM is still stable.

$\frac{\partial U_w}{\partial y} = 2 - x > 0$

$U_w(0,0) = U_m(0,0) = 2 - t > 0$, so matching is still better that not matching. Nothing changes except for a decrease in utility.

If TU,

$$h(x,y) = 4 + 4x + 4y - 2xy - 2t$$

$\frac{\partial h}{\partial x} = 4 - 2y$

$\frac{\partial^2 h}{\partial x \partial y} = -2 < 0$ $\Rightarrow$ NAM is still efficient.

For wages, the constants c_x and c_y change.

$\Rightarrow$ $v(x) = 2x + \frac{x^2}{2} + c_x$

$w(y) = 2y + \frac{y^2}{y} + c_y$

where $c_x + c_y = 4 - 2t$
and $c_x \geq 0$ & $c_y \geq 0$

Note that all men & women are still better off matching than ~~to~~ not matching.

③ (a) Demand

ID#	WTP	Q for student	Q demanded
30	60	4	4
28	56	4	8
26	52	4	12
24	48	4	16

From Demand

$\Rightarrow \quad 48 \leq \text{price} \leq 52 \qquad \text{At } Q^* = 12$

Supply

ID #	Utility	Q for Student	Q supplied
1	1	3	3
3	9	3	6
5	25	3	9
7	49	3	12
9	81	3	15

From Supply:

$\Rightarrow \quad 81 \geq \text{price} \geq 49 \qquad \text{At } Q^* = 12$

$\Rightarrow \quad Q^* = 12 \quad \text{and} \quad P^* \in [49, 52].$

(3) (b) Sellers collude.

At $p = 49 \Rightarrow Q = 12$

$TR = 49 \cdot 12 = 588$

$TC = 1 \cdot 3 + 9 \cdot 3 + 3 \cdot 25 + 3 \cdot 49$

$= 252$

$\Rightarrow \pi = 336$

At $p = 52 \Rightarrow Q = 12$

$TR = 52 \cdot 12 = 624$

$TC = 252$

$\Rightarrow \pi = 372$

At $p = 56 \Rightarrow Q = 8$

$TR = 56 \cdot 9 = 504$

$TC = 252 - 3 \cdot 49 - 25 = 80$

$\Rightarrow \pi = 424$

At $p = 60 \Rightarrow Q = 4$

$TR = 60 \cdot 4 = 240$

$TC = 3 \cdot 1 + 1 \cdot 9 = 12$

$\Rightarrow \pi = 238$

Thus $p = 56$ and $Q = 8$ maximizes the colluding sellers profit w/ a profit of $\pi = 424$.