

FIN 971: PS6

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1. Prove that in this particular problem, all the constraints should be binding

If (3) holds for both H and L , then

$$\begin{aligned} & \begin{cases} d_H(a) + (1 - p_H(a)) \cdot \delta \cdot a'_H(a) + p_H(a) \cdot R \geq \lambda \cdot (Y^H - Y^L) + d_L(a) + (1 - p_L(a)) \cdot \delta \cdot a'_L(a) + p_L(a) \cdot R \\ d_L(a) + (1 - p_L(a)) \cdot \delta \cdot a'_L(a) + p_L(a) \cdot R \geq \lambda \cdot (Y^L - Y^H) + d_H(a) + (1 - p_H(a)) \cdot \delta \cdot a'_H(a) + p_H(a) \cdot R \end{cases} \\ \implies & \begin{cases} d_H(a) + (1 - p_H(a)) \cdot \delta \cdot a'_H(a) + p_H(a) \cdot R \geq \lambda \cdot (Y^H - Y^L) + d_L(a) + (1 - p_L(a)) \cdot \delta \cdot a'_L(a) + p_L(a) \cdot R \\ d_L(a) + (1 - p_L(a)) \cdot \delta \cdot a'_L(a) + p_L(a) \cdot R + \lambda \cdot (Y^H - Y^L) \geq d_H(a) + (1 - p_H(a)) \cdot \delta \cdot a'_H(a) + p_H(a) \cdot R \end{cases} \\ \implies & d_H(a) + (1 - p_H(a)) \cdot \delta \cdot a'_H(a) + p_H(a) \cdot R = \lambda \cdot (Y^H - Y^L) + d_L(a) + (1 - p_L(a)) \cdot \delta \cdot a'_L(a) + p_L(a) \cdot R \end{aligned}$$

Thus, (3) holds with equality.

Clearly, (4) does not hold with equality for H (see policy functions).

2. After solving the problem, graph the value function $b(a)$ and decision rules $d_i(a), p_i(a), a'_i(a)$.

I ended up coding this problem set in Julia instead of Matlab. I used the JuMP package for forming the optimization problem and the GLPK package for optimization. Unfortunately, due to my understanding of JuMP, I was unable to include an interpolated object in the objective function. Thus, I resorted to using grid search over $a'_H(a)$ and $a'_L(a)$ and then using the constrained optimization methods to get $d_L(a), d_H(a), p_L(a), p_H(a)$. It wasn't ideal, but I used a number of tricks to speed up the grid search (starting at the previous maximizer and stopping when the value function starts to decrease). As you can see I find very similar results to slides in class that presumably used `fmincon` in Matlab. The waviness of the policy functions come from the grid search.

If the agent reports high cash flow, the principal promises higher future utility (i.e. the a'_H policy function is above the 45 degree line) until a point when the principal starts to statically reward the agent (i.e. d_H policy function starts to increase from zero at high enough a). The principal never terminates after a high report.

If the agent reports low cash flow, the principal decreases the promised future utility (i.e. the a'_L policy function is below the 45 degree line). At a certain point, the principal terminates the project after enough low reports (i.e. p_L is positive for low enough a). The principal never rewards the agent after a bad report.

Value Function



