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# Econ 714A Exam

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1. ~~Both~~ ~~[illegible]~~
$x_t$, $\pi_t$, and $i_t$ are control variables in the planner's problem. $x_t$ and $\pi_t$ ~~are~~ control variable when the ZLB does not hold and $i_t$ is chosen by the monetary authority, so it is a control variable as well.

In period $t=0$, news arrives. ~~Thus, the Euler Equation~~ ~~[illegible]~~ As we discussed w/ the growth model, control variables can only jump when news arrives. Thus ~~they can~~ $x_t$ and $\pi_t$ can jump only in period $t=0$. And not future periods ~~[illegible]~~ They instead evolve continuously.

$$\pi_t = \kappa x_t + \beta \pi_{t+1}$$

$$\pi_{t+1} = \frac{\pi_t - \kappa x_t}{\beta}$$

(2)

We can't solve the primal problem because the ZLB holds. Thus, the Legrangian is

$$\mathcal{L} = \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left[ x_t^2 + \alpha \pi_t^2 \right]$$

$$+ \lambda_t \left[ i_t - \frac{\pi_t - \kappa x_t}{\beta} - r_t^n - \sigma \Delta x_{t+1} \right]$$

$$+ \mu_t i_t$$

$$\text{FOC}[x_t]: \beta^t x_t + \frac{\lambda_t \kappa}{\beta} + \sigma \lambda_t - \lambda_{t-1} \sigma = 0$$

$$\text{FOC}[\pi_t]: \beta^t \alpha \pi_t - \frac{\lambda_t}{\beta} = 0$$

$$\text{FOC}[i_t]: \lambda_t + \mu_t = 0$$

By complementary slackness, if $i_t > 0 \Rightarrow \mu_t = 0$
$\Rightarrow \lambda_t = 0$

$$\Rightarrow \begin{cases} \beta^t x_t = 0 \\ \beta^t \alpha \pi_t = 0 \end{cases} \Rightarrow \begin{cases} x_t = 0 \\ \pi_t = 0 \end{cases}$$

$$\text{NKIS} \Rightarrow \sigma(0) = i_t - (0) - r_t^n$$
$$\Rightarrow i_t = r_t^n$$

$\boxed{r_t^n = \bar{r} \text{ for } t \geq T}$

Similarly, by complementary slackness, if $i_t = 0$
$\Rightarrow \mu_t \neq 0 \Rightarrow \lambda_t \neq 0$.

$$\text{Thus, } i_t = \begin{cases} 0, & t < T \\ \bar{r}, & t \geq T. \end{cases}$$

(3) From (2), we found that if $i_t > 0$

~~[illegible]~~ $\Rightarrow \mu_t = 0$ by complementary slackness

By the FOC wrt to $i_t$ implied that

$\lambda_t = 0 \Rightarrow x_t = \pi_t = 0$.

4 Continuing from 2

We know, for $\lambda_t \neq 0$,

$$\beta^t x_t + \frac{\lambda_t \kappa}{\beta} + \sigma \lambda_t - \sigma \lambda_{t-1} = 0$$

$$\beta^t \alpha \pi_t - \frac{\lambda_t}{\beta} = 0 \Rightarrow \lambda_t = \beta^{t+1} \alpha \pi_t$$

$$0 = \beta^t x_t + \frac{\beta^{t+1} \alpha \pi_t \kappa}{\beta} + \sigma\left(\beta^{t+1} \alpha \pi_t\right) - \sigma\left(\beta^t \alpha \pi_{t-1}\right)$$

$$0 = x_t + \alpha \pi_t \kappa + \beta \alpha \sigma \pi_t - \sigma \alpha \pi_{t-1}$$

$$\Rightarrow 0 = x_t + \alpha(\kappa + \beta\sigma)\pi_t - \sigma \alpha \pi_{t-1}$$

~~In SS,~~ $\cancel{x_{t+1} = x_t = \bar{x}}$ ~~and~~ $\cancel{\pi_t = \pi_{t+1} = \bar{\pi}}$

$$\cancel{\Rightarrow 0 = \bar{x} + \alpha(\kappa + \beta\sigma - \sigma)\bar{\pi}}$$

~~From NKPC~~ $\cancel{\Rightarrow \bar{\pi} = \kappa \bar{x} + \beta \bar{\pi}}$

$$\cancel{\Rightarrow (1-\beta)\bar{\pi} = \kappa \bar{x}}$$

$$\cancel{\bar{\pi} = \frac{\kappa}{1-\beta}\bar{x}}$$ ~~and~~ $$\cancel{\bar{x} = \frac{1-\beta}{\kappa}\bar{\pi}}$$

$$\cancel{0 = \frac{1-\beta}{\kappa}\bar{\pi} + \alpha(\kappa +}$$

④ $0 = x_t + \alpha(\kappa + \beta\sigma)\pi_t - \sigma\alpha\pi_{t-1}$

In SS, $\pi_t = \pi_{t+1} = \bar{\pi}$ and $x_t = x_{t+1} = \bar{x}$

$$0 = \bar{x} + \alpha(\kappa + \beta\sigma)\bar{\pi} - \sigma\alpha\bar{\pi}$$

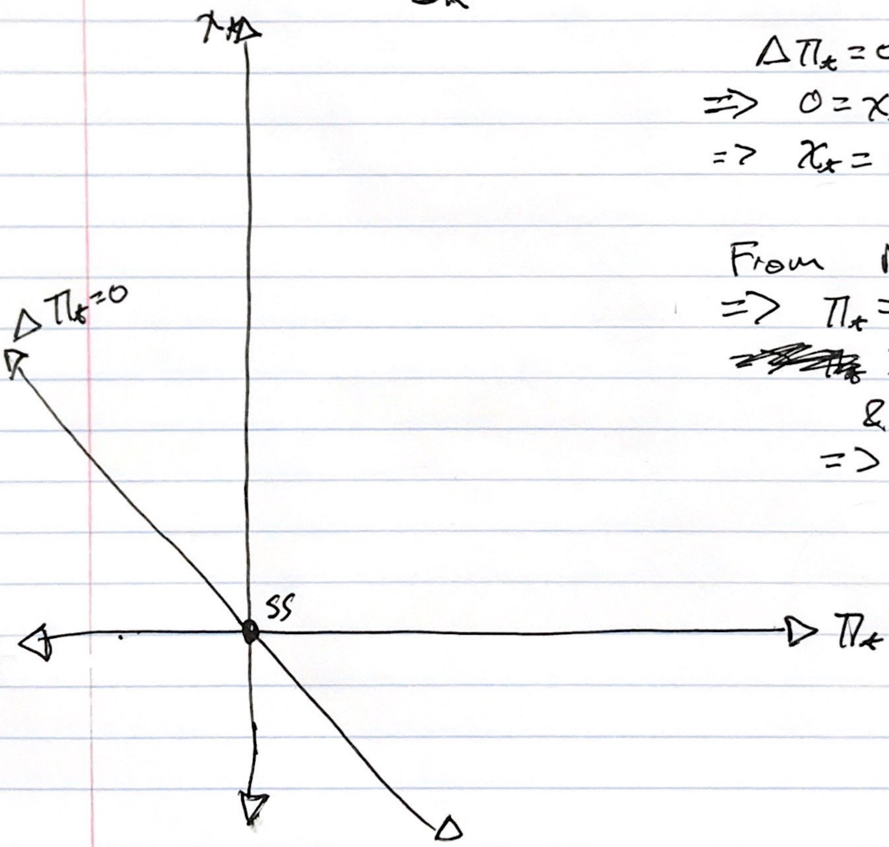
From NKPC, $\bar{x} = \frac{1-\beta}{\kappa}\bar{\pi}$

$$\Rightarrow 0 = \frac{1-\beta}{\kappa}\bar{\pi} + \alpha(\kappa + \beta\sigma)\bar{\pi} - \sigma\alpha\bar{\pi}$$

$$\Rightarrow 0 = \left(\frac{1-\beta}{\kappa} + \alpha(\kappa+\beta\sigma) - \sigma\alpha\right)\bar{\pi}$$

$$\Rightarrow \bar{\pi} = 0$$

$$\Rightarrow \bar{x} = \frac{1-\beta}{\kappa}(0) = 0$$




$\Delta\pi_t = 0$

$\Rightarrow 0 = x_t + \alpha(\kappa + \beta\sigma - \sigma)\pi_t$

$\Rightarrow x_t = -\alpha(\kappa + \beta\sigma - \sigma)\pi_t$

From NKIS and $\Delta x_t = 0$

$\Rightarrow \pi_t = i_{t-1} - r^n_{t-1}$

In $t < T$, $i_t = 0$

& $r^n_{t-1} = \underline{r} < 0$

$\Rightarrow \pi_t = -\underline{r} > 0$