Vintage Human Capital, Growth, and the Diffusion of New Technology

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Vintage Human Capital, Growth, and the Diffusion of New Technology

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We develop a model of vintage human capital in which each technology requires vintage-specific skills. We examine the properties of a stationary equilibrium for our economy. The stationary equilibrium is characterized by an endogenous distribution of skilled workers across vintages. The distribution is shown to be single-peaked. Under general conditions, there is a lag between the appearance of a technology and its peak usage, a phenomenon known as diffusion. An increase in the rate of exogenous technological change shifts the distribution of human capital to more recent vintages, thereby increasing the diffusion rate.

I. Introduction

Why are new technologies often adopted slowly? Why do people often invest in old technologies even when apparently superior technologies are available? How are decisions to adopt new technologies affected by the prospect that even better technologies will arrive in the future? This paper attempts to answer these questions in a dynamic, general equilibrium model.

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Our model has three distinctive features. First, capital is specific to a particular technology. Second, the marginal product of investment in a technology increases with the existing capital specific to that technology so that new and old capital are complementary inputs in production. Third, new and superior technologies arrive continually. Because capital is technology specific, old technologies will continue to be used even though better technologies are available. This feature implies that there is a distribution of capital in use by vintages (see Solow [1960] for an early analysis of a vintage capital model and Jungenfelt [1986] for analyses of diffusion in vintage capital models). Because new and old capital are complementary inputs, people invest in old technologies even when newer ones are available. Therefore, the capital stock in a particular technology increases for some time after the technology is introduced. This phenomenon is known as diffusion. Because superior technologies arrive continually in the model, we are able to analyze the interplay between adoption rates and the rate of technological progress.

In our general equilibrium model, the diffusion rate depends on the current distribution of vintage capital, the relative superiority of the newest technology, and the quality of technologies expected to arrive in the future. The distribution of vintage capital in the future in turn depends on the rate of investment in current technologies. Rather than start at an arbitrary initial distribution, we examine the effect of changes in the quality of new technologies on the diffusion rate in a stationary equilibrium. We show that the diffusion rate in such an equilibrium increases with an increase in the relative superiority of new technologies over existing ones. We also show that the diffusion rate increases as the arrival rate of new technologies increases. In a classic paper, Rosenberg (1976) pointed to two opposing forces that determine how diffusion rates change with a change in the quality of new technologies. An increase in the relative superiority of currently available new technologies creates an incentive to switch to them. However, if substantially better technologies are expected to arrive in the future, firms have an incentive to wait for better technologies to arrive. In our model of ongoing technological change, the trade-off between these incentives is decisively resolved in the direction of quicker adoption and more rapid diffusion.

Our model is consistent with several empirical observations that have now attained the stature of stylized facts. First, while the adoption rates of new technologies vary widely across industries and over time, it is undeniable that new technologies are often adopted on a large scale only after a prolonged period of time. As Mansfield (1968, p. 136) points out, "it took 20 years or more for all of the major firms (in several industries) to install centralized traffic control, car retarders, by-product coke ovens and continuous annealing." None of these inventions was patentable, and so we must look to other sources to understand the apparently slow rate of adoption. Second, the empirical literature provides evidence that diffusion curves are generally S-shaped (see Jovanovic and Lach [1989] for a forceful expression of this view and also Griliches [1957], Davies [1979], Gort and Klepper [1982], and Jovanovic and MacDonald [1988]). Third, Davies (1979) finds evidence that diffusion rates increase with an increase in the growth rate of the adopting industry.

The existing theoretical literature on diffusion falls into three main categories. One avenue of research focuses on uncertainty about the quality of new technologies (Jensen 1983; Balcer and Lippman 1984). A second approach emphasizes strategic issues in technology adoption (Kamien and Schwartz 1972; Reinganum 1981; Spence 1984). A third approach stresses the role of spillover effects and learning by doing in generating slow diffusion (David 1969; David and Olson 1987; Jovanovic and MacDonald 1988; Jovanovic and Lach 1989). Uncertainty, strategic issues, and externalities are undoubtedly important; in this paper, however, we abstract away from these issues by considering a certainty model with competitive agents and without externalities.

Our model differs from those in the existing literature in several substantive ways. First, in our model not only do old technologies continue to be used when apparently superior technologies are available, but people also continue to invest in old technologies. The existing literature generally does not yield this implication. There is some evidence in Mansfield (1968, chap. 8) suggesting that firms using older techniques continued to grow, though obviously at a slower rate than firms using newer techniques. This evidence suggests that firms invested in old technologies. Second, in our general equilibrium model, both the cost of switching to a new technology and the cost of operating old technologies are endogenously determined rather than exogenously specified as in many existing models. In particular, this endogeneity allows us to analyze how the current distribution of technologies and the anticipated arrival rate of new technologies affect switching costs and operating costs. Third, much of the literature examines diffusion in environments in which technological change is a once-and-for-all event. For many industries (computers, for one), modeling technological change as a continual process seems more appropriate. Such a model allows us to analyze industries in which many different techniques coexist and gives insights into how the rates of decay of older technologies interact with the growth of newer ones.

In our model, all the capital is technology-specific human capital that is acquired by using a particular technology. Learning by doing is, in this sense, an important feature of the model. However, because there are no spillover effects across technologies, the competitive equilibrium of our model is also Pareto optimal.

In Section II we set up the model, define a competitive equilibrium, and prove existence and uniqueness of a stationary equilibrium. In Section III we characterize the stationary equilibrium, and in Section IV we show that competitive equilibria are Pareto optimal. Section V concludes the paper.

II. The Model

We consider an infinite-horizon, overlapping-generations model of agents who live for two periods. A new technology appears in every period. This technology is given by the production function $\gamma^t f(N, Z)$, where t denotes the period in which the technology appeared, N is the input of unskilled workers, Z is the input of experienced workers, and $\gamma > 1$. A given set of technologies with $t = \{0, -1, -2, \ldots\}$ is available in period 0. We assume that the production function f has constant returns to scale and that $f(\cdot, Z)$ is strictly increasing and strictly concave for each $Z > 0$. Of course, since f has constant returns to scale, $f(N, 0)$ is linear in N. Let $f(N, 0) = \omega_0 N$, where $\omega_0 \geq 0$. As will become apparent, the form of this production function plays an important role in determining the rate of diffusion. In particular, if the two types of labor are highly complementary, new technologies diffuse slowly. Note also that because $\gamma > 1$, in our economy output grows over time.

We shall adopt standard notation for vintage capital models. The letter t will index time and the letter τ will index the vintage of the technology, with the following interpretation: A technology of vintage τ in period t refers to the technology that appeared in period $t - \tau$. For example, $\tau = 2$ at date t denotes the technology that appeared in period $t - 2$. Notice also that the same technology in period $t + 1$ will have vintage $\tau = 3$.

A constant population of agents is born in every period. These agents live for two periods. We normalize the population size to be one. The preferences of these agents are given by the utility function

$$u(c_1, c_2) = c_1 + \beta c_2,$$

where c_1 denotes consumption when young, c_2 denotes consumption when old, and $0 < \beta < 1$. The old generation in period 0 cares only about current consumption. We allow individuals of a given generation to borrow and lend among themselves. Thus if the aggregate consumption of a generation is positive in both periods of life, then the market interest factor will be β.

Agents can work in only one vintage in each period of their lives. Experience is acquired by working in a firm using a particular technology as an unskilled worker for one period and is specific to that technology. In this sense, our model is one of learning by doing. The choices by young agents of which skills to learn induce a distribution of skilled workers across technologies in the following period. The distribution of old agents in period 0 is given.

Let μ_t be the distribution of experienced old agents at date t across vintages $\tau \in \{0, 1, 2, \ldots\}$. Thus $\mu_t(\tau)$ is the number (more precisely mass) of old agents with experience in vintage τ, that is, the old people who when young worked in the technology that appeared in period $t - \tau$. Since there are no experienced workers in the "just-born" vintage, $\mu_t(0) = 0$ for all t. We refer to μ_t as the state of the economy.

We now describe the evolution of the state of the economy. Let $N_1(t, \tau)$, $t = 0, 1, \ldots$, denote the number of young workers who enter vintage τ in period t. In period $t + 1$, these workers will be skilled in vintage $\tau + 1$. Therefore,

$$\mu_{t+1}(\tau + 1) = N_1(t, \tau) \quad \text{for } \tau = 0, 1, \ldots \text{ and for all } t. \tag{1}$$

Only experienced old workers in a particular vintage can supply the skilled labor input in that vintage. We allow experienced workers to move freely and to supply unskilled labor at any vintage. Let $N_2(t, \tau)$ denote the number of old workers who work as unskilled workers in vintage τ, and let $Z(t, \tau)$ denote the skilled labor input to vintage τ in period t. Then we have

$$0 \leq Z(t, \tau) \leq \mu_t(\tau) \quad \text{for } \tau = 0, 1, \ldots \text{ and for all } t. \tag{2}$$

We now turn to the decision problems of the workers. Let $w(t, \tau)$ and $v(t, \tau)$ denote the wage paid to unskilled and skilled workers, respectively, in vintage τ in period t. First consider the decision problem of old workers. Since old workers can work in any vintage as unskilled workers, they must be paid at least as much as unskilled workers in any vintage. That is, if $Z(t, \tau) > 0$, then $v(t, \tau) \geq w(t, s)$ for all vintages s. Furthermore, if the last inequality is strict, then workers skilled in vintage τ will supply only skilled labor, or $Z(t, \tau) = \mu_t(\tau)$. Clearly, old workers who work as unskilled workers will choose the vintage offering the highest wage rate; that is, if $N_2(t, \tau) > 0$, then $w(t, \tau) \geq w(t, s)$ for all s.

Now consider the decision problem of young workers. Young workers who enter vintage τ will earn at least $v(t + 1, \tau + 1)$ in the following period since they will then be skilled in vintage $\tau + 1$. Since young agents maximize discounted earnings, we have the following inequality:[1]

$$\begin{aligned} &w(t,\tau) + \beta \max\{v(t+1,\tau+1), w(t+1,0), w(t+1,1),\ldots\} \\ &\geq w(t,s) + \beta \max\{v(t+1,s+1), w(t+1,0), w(t+1,1),\ldots\} \end{aligned} \tag{3}$$

for all τ, s, and t such that $N_1(t, \tau) > 0$.

Profit-maximizing firms operate the production process in each vintage. Because we assume constant returns to scale, all firms make zero profits in equilibrium. In each vintage we have

$$\max_{N,Z} \{\gamma^{t-\tau} f(N,Z) - w(t,\tau)N - v(t,\tau)Z\} = 0 \quad \text{for all } \tau \geq 1, \text{for all } t. \tag{4}$$

In the just-born vintage, we have the profit maximization condition

$$\max_{N} \{\gamma^t f(N,0) - w(t,0)N\} = 0. \tag{5}$$

Recall that $f(N, 0) = \omega_0 N$. Thus (5) implies that if $N(t, 0) > 0$ for some t, then $w(t, 0) = \gamma^t \omega_0$.

A *competitive equilibrium* for this economy is a collection of wage functions $w(t, \tau)$ and $v(t, \tau)$; employment functions $N_1(t, \tau)$, $N_2(t, \tau)$, and $Z(t, \tau)$; and a sequence of distribution functions $\{\mu_t\}$ such that the following conditions hold:

i) Young workers are indifferent among vintages: The wage functions satisfy (3). Old workers maximize their income: $Z(t, \tau) > 0$ implies $v(t, \tau) \geq w(t, s)$ for all s, $v(t, \tau) > w(t, s)$ for all s implies $Z(t, \tau) = \mu_\tau$, and $N_2(t, \tau) > 0$ implies $w(t, \tau) \geq w(t, s)$ for all s.
ii) Profit maximization: The employment functions solve (4) and (5).
iii) Resource constraints: $\Sigma_{\tau=0}^{\infty} N_1(t, \tau) = 1$ and $\Sigma_{\tau=0}^{\infty} [N_2(t, \tau) + Z(t, \tau)] = 1$ for all t, and the employment and distribution functions satisfy (1) and (2).

Hereafter, we focus on the *stationary equilibrium,* which is a competitive equilibrium with the following additional conditions:

iv) $\mu_t = \mu$, $N_2(t, \tau) = N_{2\tau}$, $Z(t, \tau) = Z_\tau$, $w(t, \tau) = \gamma^t w_\tau$, and $v(t, \tau) = \gamma^t v_\tau$ for all t, where w_τ and v_τ denote the wage rates at period 0.

To interpret the additional conditions imposed by a stationary equilibrium, think of the technologies as lying on the real line. In a stationary equilibrium, the distribution of agents across technologies ad-

[1] We assume for now that the interest rate is $(1/\beta) - 1$ and show that this is true in equilibrium.

vances to the right at a constant rate; however, relative to the newest technology, the distribution is stationary. In each period, the economy is unchanged from the previous period, except that all the technologies are more productive by a factor of γ. We therefore require the labor allocations at each vintage to be the same over time in a stationary equilibrium and wages to rise at the rate γ.[2]

The key condition in an equilibrium is the present value condition in (3). This condition is somewhat unwieldy, but we can use our constant returns to scale assumption to simplify it considerably. To use this assumption, let us define the input of unskilled labor relative to skilled labor by n and the profit function $\pi_\tau(w)$ by

$$\pi_\tau(w) = \max_n \{\gamma^{-\tau} f(n, 1) - wn\}. \tag{6}$$

From the constant returns to scale assumption, it is clear that if $Z_\tau > 0$ for some τ, then $v_\tau = \pi_\tau(w_\tau)$. Of course, if $Z_\tau < \mu_\tau$, then v_τ equals the maximum wage. Therefore,

$$v_\tau = \max\{\pi_\tau(w_\tau), \max_\tau w_\tau\} \quad \text{for all } \tau \text{ such that } \mu_\tau > 0. \tag{7}$$

Since wages rise at rate γ in a stationary equilibrium, the present value conditions (3) can be simplified to read

$$w_{\tau-1} + \beta\gamma \max\{\pi_\tau(w_\tau), \max_\tau w_\tau\} \leq k \tag{8}$$

with equality for all τ such that $\mu_\tau > 0$, where k is the present value of income for a young worker in period 0.

In the Appendix we establish the following result.

PROPOSITION 1. *Unskilled workers' wages increase with vintage.*—In a stationary equilibrium, (i) the support of the distribution is finite; that is, there is some number T such that $\mu_\tau > 0$ for $\tau \leq T$ and $\mu_\tau = 0$ for $\tau > T$; (ii) $Z_\tau = \mu_\tau$ for $\tau \leq T - 2$ and $Z_{T-1} > 0$; and (iii) $w_\tau \geq w_{\tau-1}$ and $v_{\tau+1} \leq v_\tau$ for $\tau = 1, \ldots, T - 1$.

Proposition 1 says that, in a stationary equilibrium, only a finite number of vintages will be used. Thus all technologies are eventually discarded. It also says that young workers enter only vintages in which old workers supply skilled labor and that unskilled workers' wages increase with the age of a technology whereas skilled workers' wages decline with the age of a technology. Thus young workers who enter new technologies invest when young and reap the benefits of their human capital when they are old.

Proposition 1 together with the present value condition (8) suggests

[2] In an earlier version of the paper we showed that if the distribution is stationary, the equilibrium allocations are stationary and wages rise at rate γ. Proofs with this weaker condition are available on request.

a simple recursive procedure for computing the wage sequence in a stationary equilibrium. To develop this procedure, first note that using proposition 1 in (8) we have

$$w_{\tau-1} + \beta\gamma\pi_\tau(w_\tau) = k \quad \text{for } \tau = 1, \ldots, T-1. \tag{9}$$

Next, the result in proposition 1 that young workers enter the just-born vintage and (5) imply that

$$w_0 = \omega_0. \tag{10}$$

Thus if the present value of earnings, k, and the number of vintages, T, are known, we can use (9) and (10) to compute the wage sequence for unskilled workers. We turn now to determining k and T. Suppose first that $Z_T = 0$ so that all old workers skilled in vintage T work as unskilled workers. Any young workers who enter vintage $T - 1$ will then work as unskilled workers when old as well. Such workers will therefore choose the vintage paying the highest wages to unskilled workers in both periods of their life. Thus young workers entering vintage $T - 1$ receive a wage w_{T-1} when young and γw_{T-1} when old. The equation for determining k is then given by

$$w_{T-1} + \beta\gamma w_{T-1} = k. \tag{11}$$

To determine the number of vintages, T, note that if $Z_T = 0$, the wage of workers entering vintage $T - 1$, w_{T-1}, must be the highest wage paid to unskilled workers at any vintage. In particular, $w_{T-1} \geq w_T$. Furthermore, since workers skilled in vintage T also receive a wage of w_{T-1}, $\pi_T(w_T) \leq w_{T-1}$. Now $\pi_T(\cdot)$ is a decreasing function of the wage. Thus

$$\pi_T(w_{T-1}) \leq w_{T-1}. \tag{12}$$

Next, from proposition 1 we see that workers skilled in vintage $T - 1$ choose to work as skilled workers. Therefore,

$$\pi_{T-1}(w_{T-1}) \geq w_{T-1}. \tag{13}$$

Inequalities (12) and (13) can now be used together with equations (9)–(11) to construct the wage sequence. The algorithm is to guess at a value of T and then use (9)–(11) to construct a wage sequence. We then verify whether the constructed wage sequence satisfies (12). If it does not, we guess a larger value of T. If the wage sequence fails to satisfy (13), we guess a smaller value of T. Note that because $\pi_\tau(\cdot)$ is decreasing in τ, if the wage sequence fails to satisfy (12), it will satisfy (13).

We have derived this algorithm for the case in which $Z_T = 0$. A slight modification is required to accommodate the possibility that $Z_T > 0$. In this case, it could be that $w_{T-1}(1 + \beta\gamma) < k$, so that the algorithm described above apparently cannot be used. Suppose, provisionally, that the present value of earnings, k, is known. Let S denote the smallest vintage such that $\pi_s(k/(1+\beta\gamma)) \leq k/(1+\beta\gamma)$. For $\tau = T, T+1, \ldots, S-1$, we define w_τ so that $w_{\tau-1} + \beta\gamma\pi_\tau(w_\tau) = k$ and let $w_{S-1} = k/(1+\beta\gamma)$. Clearly, this constructed wage sequence satisfies all the conditions of a stationary equilibrium. We have established that, in a stationary equilibrium, there exists a vintage S and a wage sequence w_τ, $\tau = 0, \ldots, S-1$, satisfying

$$w_\tau + \beta\gamma\pi_{\tau+1}(w_{\tau+1}) = k \quad \text{for } \tau = 0, \ldots, S-2, \tag{14}$$

$$w_0 = \omega_0, \quad w_{S-1} = \frac{k}{1+\beta\gamma}, \tag{15}$$

$$\pi_{S-1}\left(\frac{k}{1+\beta\gamma}\right) \geq \frac{k}{1+\beta\gamma}, \tag{16}$$

and

$$\pi_S\left(\frac{k}{1+\beta\gamma}\right) \leq \frac{k}{1+\beta\gamma}. \tag{17}$$

Note that (16) together with (14) implies that $w_{\tau+1} \geq w_\tau$, $\tau = 0, \ldots, S-2$, and that $\pi_\tau(w_\tau) > k/(1+\beta\gamma)$ for $\tau = 1, \ldots, S-2$ since π is strictly decreasing in the vintage.

This construction shows that the algorithm is the same both when $Z_T = 0$ and when $Z_T > 0$. We turn now to the problem of uncovering the number of active vintages T from the wage sequence satisfying (14)–(17) and, more generally, constructing the employment allocations.

Suppose that we find some sequence of wages, $w_0, w_1, \ldots, w_{S-1}$, that satisfies (14)–(17). Let $n_\tau(w_\tau)$ denote the solution to the problem

$$\pi_\tau(w_\tau) = \max_n \gamma^{-\tau} f(n, 1) - w_\tau n.$$

Then, given the sequence of wages, the employment sequence is constructed as follows. Suppose first that $n_{S-1}(w_{S-1}) > 0$. Then let all young workers who enter vintage $S-1$ work as unskilled workers in the following period in vintage $S-1$. Therefore, $N_{1S-1} = N_{2S-1}$. In all preceding vintages, $\mu_{\tau+1} = \mu_1 \Sigma_{S=1}^{\tau} n_S$ follows from the fact that $\mu_{\tau+1} = N_{1\tau} = n_\tau\mu_\tau$.

We now show that μ_1 can be chosen to satisfy labor market clearing. The market-clearing condition can be written as

$$\sum_{\tau=1}^{S-2} \mu_\tau n_\tau + \mu_{S-1}\frac{n_{S-1}}{2} = 1 - \mu_1. \tag{18}$$

From the recursive construction of μ, the left side of (18) is a continuous, increasing function of μ_1 that equals zero if μ_1 equals zero, and the right side is a continuous, strictly decreasing function ranging from zero to one. Therefore, a unique value of μ_1 solves (18). From this value of μ_1, the remaining allocations can be constructed.

If $n_{S-1}(w_{S-1}) = 0$, let $T - 1$ denote the oldest vintage such that $n_{T-1}(w_{T-1}) > 0$. Replace the second term on the left side of (18) by $\mu_{T-1}n_{T-1}/2$. The same argument for the existence of a distribution then applies.

The problem of proving the existence of a stationary equilibrium then reduces to the problem of verifying that a wage sequence can be found that satisfies (14)–(17). The following proposition, which is proved in the Appendix, asserts the existence and uniqueness of such a wage sequence.

PROPOSITION 2. *Existence of a wage sequence.*—There exists a unique number S and a unique sequence of wages, $w_0, w_1, \ldots, w_{S-1}$, satisfying (14)–(17).

If $n_{S-1}(w_{S-1}) > 0$, then the number of active vintages T equals S. If $n_{S-1}(w_{S-1}) = 0$, then T is the largest vintage such that $n_{T-1}(w_{T-1}) > 0$. We have established the following theorem.[3]

THEOREM. *Equilibrium existence.*—A stationary equilibrium exists for the vintage capital model.

From proposition 2 the wage sequence is unique. The employment sequence is also unique except for two knife-edge cases. One case occurs when workers skilled in vintage T are indifferent between working in vintage T as skilled workers and working in vintage $T - 1$ as unskilled workers. Then there is an indeterminacy in allocating old workers skilled in vintage between vintage $T - 1$ and vintage T. The other case occurs when $w_{T-1} = v_{T-1}$, $N_{2T-1} > 0$, and $Z_{T-1} < \mu_{T-1}$. Then there is an indeterminacy in allocating old workers skilled in vintage $T - 1$ between skilled and unskilled tasks. Both cases are clearly knife-edge in nature. The equilibrium is therefore unique in general.[4]

[3] It is also clear from the proof of proposition 2 that the maximum wage rate is strictly positive. Therefore, no workers are at a corner in their consumption and the equilibrium interest rate is $(1/\beta) - 1$.

[4] An interesting question is whether the economy converges to a stationary equilibrium from an arbitrary initial distribution. This question is difficult to answer analytically for multiple capital goods models like this one. For the parametric examples described below, the dynamical system implied by the equilibrium conditions is locally stable. Indeed, we found that for $\beta\gamma$ sufficiently close to unity, all the examples we considered were locally stable.

III. Properties of the Equilibrium

The stationary distribution provides a picture of the rise and fall of a particular technology in our model. A new technology that arrives in period 0 is of vintage 1 in period 1 and therefore has exactly the same capital stock as the vintage 1 technology in period 0. In period 2, this technology has the same capital stock as the vintage 2 technology in period 0, and so on. Thus the rate of diffusion of new technology is closely related to the properties of the stationary distribution. We establish that the distribution is single-peaked and log concave in the following proposition.

PROPOSITION 3. *Single-peakedness and log concavity.*—There is a vintage R such that, for all $\tau \leq R$, $\mu_\tau \geq \mu_{\tau-1}$, and for $\tau > R$, $\mu_\tau \leq \mu_{\tau-1}$. Furthermore, log μ_τ is concave in τ.

Proof. Since w_τ is increasing in τ, it follows that n_τ is decreasing in τ. Recall that $\mu_\tau = \mu_{\tau-1}n_{\tau-1}$. Therefore, if $n_{\tau-1} > 1$, $\mu_\tau > \mu_{\tau-1}$; if $n_{\tau-1} < 1$, $\mu_\tau < \mu_{\tau-1}$, and all subsequent values of μ are also decreasing. Since the number of vintages in use is finite, the result follows.

Log concavity follows because the increment between log $\mu_{\tau+1}$ and log μ_τ is given by log n_τ, which is decreasing in τ. Q.E.D.

Proposition 3 illustrates the sense in which our model is one of diffusion. If the diffusion curve peaks at a vintage greater than 1, the distribution of skills (or capital) rises and then falls over time. This situation occurs if relative employment at vintage 1 is greater than unity. We examine this possibility below. Furthermore, the diffusion curve is log concave and the growth rate of capital decreases monotonically, as some of the empirical literature suggests.

We now discuss the factors determining the shape of the distribution function. In particular, we examine the conditions under which the distribution peaks at a vintage greater than 1. Whether relative employment in vintage 1 is greater than unity depends on the production function, $f(\cdot, \cdot)$, the rate of growth of the economy, γ, and the discount factor, β.

We first consider the role of the production function. A large number of young workers will enter vintage 1 and later vintages if the demand for their services is high, that is, if their marginal product is high in these vintages. These young workers will be willing to work as skilled workers when old, and their vintages will continue to attract many young workers if the marginal product of the young workers hired increases with the number of skilled workers. That is, if skilled and unskilled labor are complementary inputs in production, then adoption of new technologies is likely to be slow.

To illustrate this role of complementarities in generating diffusion, we conducted some simulations. We considered constant elasticity of

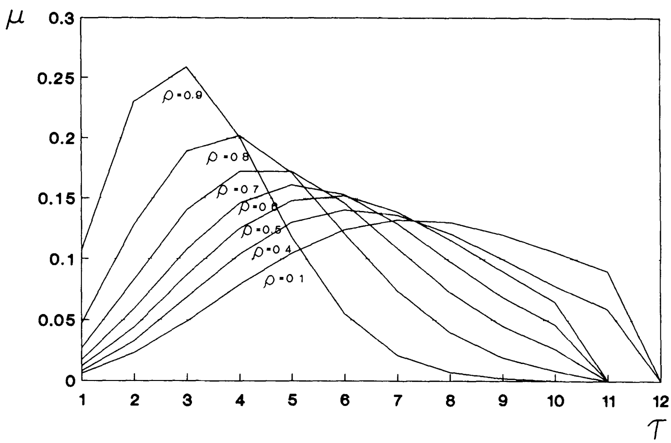

FIG. 1.—Diffusion curves. $\beta = 0.9$; $a_1 = 1.2$; $a_2 = 2.10$; $\gamma = 1.005$

substitution production functions of the form $(a_2 N^{\rho} + a_1 Z^{\rho})^{1/\rho}$, with the elasticity given by $1/(\rho - 1)$. As ρ decreases from one to zero, the inputs become more and more complementary. In figure 1 we plot the capital stock μ_{τ} at each vintage for a number of different values of ρ. Notice that the rate of diffusion slows as the inputs become more and more complementary. Again, the marginal gain to investing in an old technology is high when new investment is a complementary input to the existing capital stock. Few workers then join the newest technology, even though it is better. Notice also that the distribution is not symmetric about the peak. That is, the adoption rate of new technologies can be quite different from the decay rate of old technologies. Figure 2 reproduces figure 1 for $\rho = 0.1$ up to the peak of the diffusion curve. The adoption curve has a classic S-shape, as much of the empirical literature suggests it should.

We now examine the effect of a change in the rate of technological change on the stationary distribution. We consider two economies with growth rates γ' and γ ($\gamma' > \gamma$). Our main result is that the distribution associated with the higher growth rate, say μ', is dominated in the sense of stochastic dominance by the original distribution. In other words, when technological change accelerates, the distribution of skilled workers shifts to more recent vintages. This result also implies that the rate of diffusion of new technologies is higher if the economy grows more rapidly.

PROPOSITION 4. *Relative employment decreases with an increase in the growth rate.*—Consider two economies with growth rates γ' and γ ($\gamma' > \gamma$) and associated stationary distributions μ' and μ, respectively. Let $w(\tau, \mu')$ and $w(\tau, \mu)$ denote the wage rates in the two economies

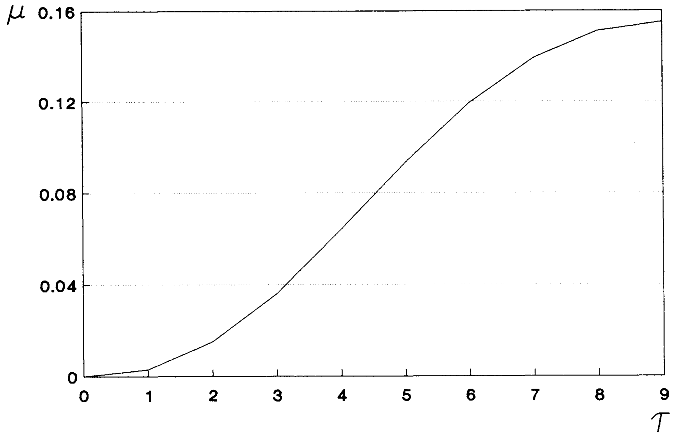

FIG. 2.—Diffusion curves up to adoption peak. $\beta = 0.84$; $a_1 = 1.05$; $a_2 = 1.0$; $\gamma = 1.005$; $\rho = 0.1$

and $n(\tau, \mu')$ and $n(\tau, \mu)$ denote the relative labor input decisions in the two economies. Then fewer vintages are used in the high-growth economy, the wages satisfy

$$w(\tau, \mu') \geq \left(\frac{\gamma}{\gamma'}\right)^{\tau} w(\tau, \mu) \quad \text{for all } \tau \text{ with } \mu'_\tau > 0,$$

and $n(\tau, \mu') \leq n(\tau, \mu)$ for all τ such that $n(\tau, \mu) > 0$.

We prove this proposition in the Appendix. We use proposition 4 to prove proposition 5.

PROPOSITION 5. *Stochastic dominance.*—Consider two economies with growth rates γ' and γ ($\gamma' > \gamma$). Let μ' and μ denote the respective stationary distributions. Then μ stochastically dominates μ'; that is,

$$\sum_{\tau=1}^{S} \mu_\tau \leq \sum_{\tau=1}^{S} \mu'_\tau \quad \text{for all } S.$$

Proof. We first establish that $\mu_1 \leq \mu'_1$. Suppose by way of contradiction that $\mu_1 > \mu'_1$. Then from proposition 4 it follows that $n(1, \mu_1)\mu_1 \geq n(1, \mu'_1)\mu'_1$ or, by definition of n, that $\mu_2 \geq \mu'_\tau$. From proposition 4 it follows that $\mu_\tau \geq \mu'_\tau$ for $\tau \geq 2$. Thus $\Sigma_{\tau=1}^{\infty} \mu_\tau > \Sigma_{\tau=1}^{\infty} \mu'_\tau$. Of course, both sides of this inequality sum to one, and we have a contradiction. Let T be the smallest number such that $\mu'_T < \mu_T$. The proposition follows immediately for $S < T$. Consider now the proposition for $S \geq T$. From proposition 4 it follows that $\mu'_\tau \leq \mu_\tau$ for all $\tau \geq T$. Thus, for any $S \geq T$, $\Sigma_{\tau=S}^{\infty} \mu'_\tau \leq \Sigma_{\tau=S}^{\infty} \mu_\tau$. But since $\Sigma_{\tau=1}^{\infty} \mu'_\tau = 1 - \Sigma_{\tau=1}^{\infty} \mu_\tau$, $1 - \Sigma_{\tau=1}^{S-1} \mu'_\tau \leq 1 - \Sigma_{\tau=1}^{S-1} \mu_\tau$, which establishes the result. Q.E.D.

Proposition 5 shows that at a higher growth rate, diffusion is more rapid. The distribution of capital shifts toward more recent vintages in the first-order stochastic dominance sense. This result is consistent with the evidence presented by Davies (1979). The intuition for the result is straightforward except for some complications induced by the general equilibrium structure of the model. At a higher growth rate, the relative inferiority of older technologies measured by $\gamma^{-\tau}$ is greater. Therefore, fewer young workers join older technologies and the distribution of capital shifts toward more recent vintages. However, one should not ignore general equilibrium effects. Young workers' decisions to join relatively new vintages offering lower wages drive down the wages of unskilled workers, thereby tending to make continued operation of older vintages profitable. Proposition 4 bounds the decline in wages of unskilled workers and limits this general equilibrium effect.

Our next result shows that the earnings profile becomes flatter as the growth rate of the economy increases. From proposition 4,

$$w(\tau, \mu') \geq \left(\frac{\gamma}{\gamma'}\right)^{\tau} w(\tau, \mu).$$

This result implies that

$$\begin{aligned} \pi'_{\tau}(w') &= \max_{n} \gamma'^{-\tau} f(n) - w'n \\ &\leq \left(\frac{\gamma}{\gamma'}\right)^{\tau} \max\left[\gamma^{\tau} f(n) - \left(\frac{\gamma'}{\gamma}\right)^{\tau} w'n\right] \leq \left(\frac{\gamma}{\gamma'}\right)^{\tau} w_{\tau}(w). \end{aligned} \tag{19}$$

Since $\gamma' > \gamma$, it follows that

$$\frac{\gamma' \pi'_{\tau+1}(w'_{\tau+1})}{w'_{\tau}} \leq \frac{\gamma \pi_{\tau+1}(w_{\tau+1})}{w_{\tau}}. \tag{20}$$

The denominator of (20) is the wage of a young unskilled worker. The numerator is the wage of that worker in the following period. Thus the wage profile is flatter at a higher growth rate. Because preferences are linear, the wage profile does not pin down the intertemporal allocation of workers' consumption. We examine how consumption profiles of a generation change with the growth rate of the economy. The aggregate (or per capita) consumption of young workers is defined by

$$C_y = \sum_{\tau=0}^{T-1} w_{\tau} \mu_{\tau+1}, \tag{21}$$

and the consumption of old workers is similarly defined. The consumption profile of a generation is given by

$$\frac{C_0}{C_y} = \frac{\sum_{\tau=1}^{T} \gamma v_\tau \mu_\tau}{\sum_{\tau=0}^{T-1} w_\tau \mu_{\tau+1}}. \tag{22}$$

In figure 3, we plot this consumption profile against the growth parameter γ for three values of the substitution parameter ρ. The plots are discontinuous when the number of vintages in use changes. As can be seen from the figure, the consumption profiles are not monotonic. There are two effects working in opposite directions. The wage profile at any given vintage becomes flatter when the growth rate increases, thus making the consumption profile flatter. However, the distribution of workers shifts toward more recent vintages in which wage profiles are steeper, thus making the consumption profile steeper. When an increase in the growth rate leaves the number of vintages unchanged, the consumption profile becomes flatter. The effect of a flattening wage profile can be clearly seen in such a case. The effect of a shift in the distribution can be seen most clearly when the number of vintages used changes. The consumption profile becomes steeper. It is interesting to note that the figure shows that consumption profiles are generally steep when complementarities are high even though more workers join older technologies with relatively flat wage profiles. The reason is that with higher complementarities the wage profiles at each vintage become steeper.

We can also use the model to study human capital accumulation. One measure of investment in human capital is forgone earnings. That is, the investment of a worker joining vintage τ is given by $w_{T-1} - w_\tau$. Total investment in the economy is then $\Sigma_{\tau=0}^{T-1} (w_{T-1} - w_\tau)\mu_{\tau+1}$.[5] Since the capital stock depreciates completely after one period of use, the investment is also the amount of capital in the economy. In figure 4, we plot the ratio of investment to output against the growth rate parameter for three values of the substitution parameter. The investment/output ratio and the consumption profile are obviously closely linked. Again, the discontinuities in figure 4 occur when the number of vintages changes. The flattening wage profile reduces measured

[5] Our measure of investment is the standard one in a multiple capital goods model and is formally identical to the way investment is measured in the National Income and Product Accounts. Aggregate investment is the sum of investment in all sectors at market prices and therefore measures the decrease in consumption at the margin due to the investment activity.

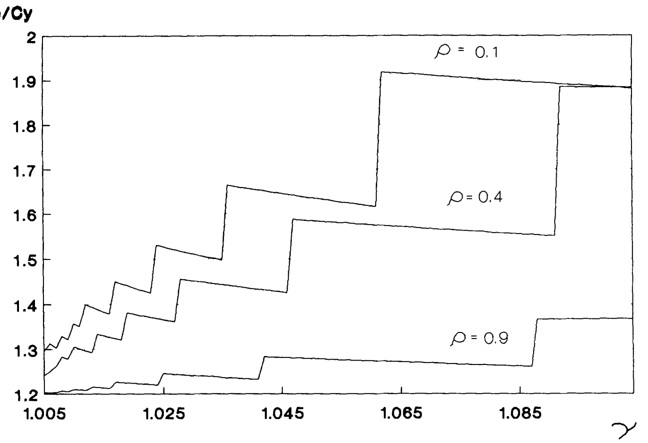

FIG. 3.—Consumption profiles and growth rates. $\beta = 0.9$; $a_1 = 1.2$; $a_2 = 1.0$

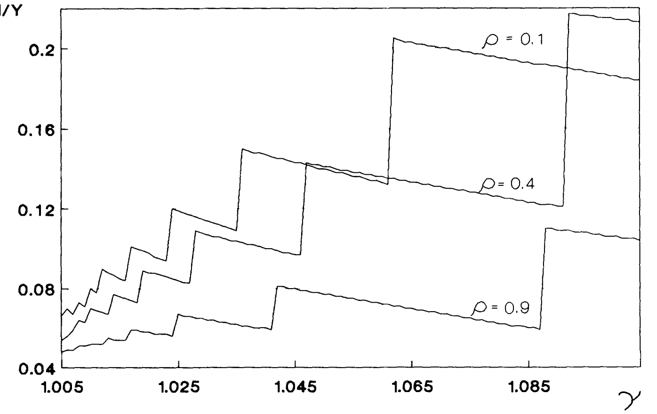

FIG. 4.—Investment/output ratios and growth rates. $\beta = 0.9$; $a_1 = 1.2$; $a_2 = 1.0$

investment at each vintage, whereas the shift of workers to more recent vintages with steeper wage profiles increases investment. As the figure shows, the latter effect eventually dominates, raising the investment/output or capital/output ratio. Stronger complementarities also tend to raise the amount of investment in the economy.

The flattening of the wage profile is also associated with a change in the cost of switching from the oldest technology to the newest one. We measure these switching costs by $w_{T-1} - \omega_0$. The sense in which $w_{T-1} - \omega_0$ is a switching cost can be clarified if we assume that a single, competitive firm operates all the technologies. Then $w_{T-1} - \omega_0$ measures the cost to this firm of switching an unskilled worker

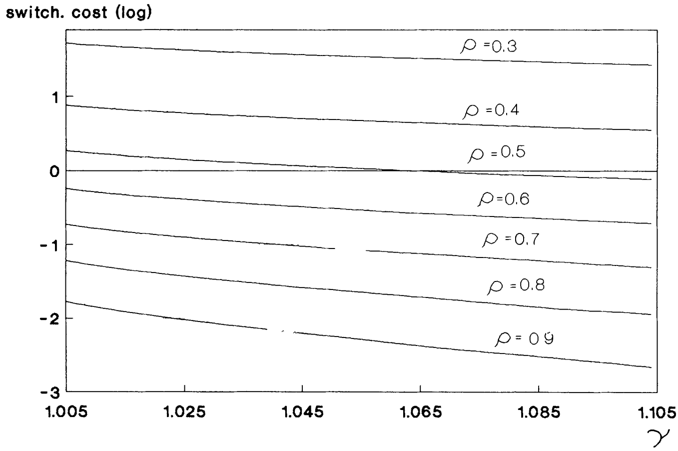

FIG. 5.—Switching costs and growth rates. $\beta = 0.9$; $a_1 = 1.2$; $a_2 = 1.0$

from the oldest to the newest technology. Note from (20) that $\gamma'\pi_1'(w_1') \leq \gamma\pi_1(w_1)$. Therefore, $\omega_0 + \beta\gamma'\pi_1'(w_1') \leq \omega_0 + \beta\gamma\pi_1(w_1)$. From (8), $w_{T-1}'(1 + \beta\gamma') \leq w_{T-1}(1 + \beta\gamma)$. Therefore, w_{T-1} falls with an increase in the growth rate γ. That is, switching costs decline with an increase in the growth rate. This decline in switching costs stimulates young workers to move toward more recent vintages. In figure 5, we plot the logarithm of switching costs against the growth rate and the substitution parameter. Note that switching costs fall with an increase in the growth rate and rise with an increase in the complementarity of the inputs.

We now examine the effect of a change in the discount factor β on the stationary distribution. We show in propositions 6 and 7 below that an increase in the discount factor results in a more rapid rate of diffusion. An increase in the discount factor can be interpreted as a decrease in the length of the time interval. An economy with a higher discount factor is, in this sense, one with more frequent technological change. The result in propositions 6 and 7 suggests that diffusion is more rapid in economies with more frequent technological change. But a note of caution is required. Shrinking the length of the time interval also shrinks the training time of young workers, so we are changing two variables at once.

Proposition 6 is proved in the Appendix. The proof of proposition 7 parallels that of proposition 5 and is omitted.

PROPOSITION 6. Consider two economies with discount factors $\beta' > \beta$ with associated stationary distributions μ' and μ, respectively. Let

$w(\tau, \mu')$ and $w(\tau, \mu)$ denote the wage rates in vintage τ and $n(\tau, \mu')$ and $n(\tau, \mu)$ denote the input decisions in vintage τ in the two economies. Then $w(\tau, \mu') \geq w(\tau, \mu)$ and $n(\tau, \mu') \leq n(\tau, \mu)$, all τ.

PROPOSITION 7. Let μ' and μ denote the steady-state distributions for two economies characterized by discount factors β' and β, respectively. Then μ stochastically dominates μ'.

IV. Optimality of the Competitive Equilibrium

In this section, we establish that if the economy does not grow too fast, the competitive equilibrium is Pareto optimal. In fact, the competitive equilibrium maximizes the discounted value of output. We shall need to assume that $\beta\gamma < 1$, a standard condition in models of economic growth. This condition ensures that the discounted consumption stream is bounded and that our social welfare function is well defined. Let $c_t = (c_{1t}, c_{2t})$ denote the consumption when young and old of a representative agent born at time t. We shall assume that a planner has preferences given by

$$W(c) = c_{2,-1} + \sum_{t=0}^{\infty} \beta^t U(c_{1t}, c_{2t}), \tag{23}$$

where $U(c_{1t}, c_{2t}) = c_{1t} + \beta c_{2t}$. Let $y_t = c_{2t-1} + c_{1t}$ denote total output at time t. The planner is assumed to have available a total labor endowment of two units in each period. Let $N_1(t, \tau)$ denote labor of young workers allocated to vintage τ at time t and $N_2(t, \tau)$ denote labor of old workers to unskilled tasks. Let $Z(t, \tau)$ denote skilled labor allocated to vintage τ at time t. In keeping with our assumptions, $Z(t, \tau) \leq N_1(t - 1, \tau - 1)$. The problem faced by the planner is then

$$\max \sum_{t=0}^{\infty} \beta^t y_t \tag{24}$$

subject to

$$0 \leq y_t \leq \sum_{\tau=0}^{\infty} \gamma^{t-\tau} f(N_1(t, \tau) + N_2(t, \tau), Z(t, \tau)), \tag{25}$$

$$\sum_{\tau=0}^{\infty} N_1(t, \tau) \leq 1 \quad \text{for all } t, \tag{26}$$

$$Z(t + 1, \tau + 1) \leq N_1(t, \tau) \quad \text{for all } t \text{ and for all } \tau \geq 0, \tag{27}$$

and

$$\sum_{\tau=0}^{\infty}[Z(t+1,\tau)+N_2(t+1,\tau)] \leq \sum_{\tau=0}^{\infty} N_1(t,\tau) \quad \text{for all } t, \tag{28}$$

given $N_1(-1, \tau)$.

Let λ_t denote the Lagrange multiplier on constraint (25). The necessary first-order conditions are then given after some simplification by

$$\begin{gathered}\beta^t\gamma^{t-\tau}f_1(N(t,\tau),Z(t,\tau)) \\ +\beta^{t+1}\gamma^{t+1-(\tau+1)}f_2(N(t+1,\tau+1),Z(t+1,\tau+1))=\lambda_t\end{gathered} \tag{29}$$

for all vintages with $N(t, \tau) > 0$ and $Z(t+1, \tau+1) > 0$. Let $\hat{\lambda}_t = (\beta\gamma)^{-\tau}\lambda_t$. Then equation (29) can be rewritten to read

$$\gamma^{-\tau}f_1(N(t,\tau),Z(t,\tau))+\beta\gamma^{-\tau}f_2(N(t+1,\tau+1),Z(t+1,\tau+1))=\hat{\lambda}_t. \tag{30}$$

Let $w(t, \tau) = \gamma^{t-\tau}f_1(N(t, \tau), Z(t, \tau))$ and $v(t, \tau) = \gamma^{t-\tau}f_2(N(t, \tau), Z(t, \tau))$. Equation (30) then implies that the present value of wages is equated across vintages in which young workers enter. For such vintages, (30) and (3) are equivalent. Similarly, for vintages in which Z or N equals zero, it can be shown that (3) holds.

The fact that $f(\cdot, \cdot)$ is homogeneous of degree one implies that $f_1(\cdot, \cdot)$ and $f_2(\cdot, \cdot)$ are homogeneous of degree zero. Let $n(t, \tau) = N(t, \tau)/Z(t-1, \tau-1)$. Euler's theorem implies that

$$f_2(n(t,\tau), 1) = f(n(t,\tau), 1) - n(t,\tau)f_1(n(t,\tau), 1).$$

Hence,

$$v(t,\tau) = f(n(t,\tau), 1) - w(t,\tau)n(t,\tau).$$

Therefore, profit maximization follows.

Finally, we verify that the competitive equilibrium satisfies the transversality condition that $(\beta\gamma)^t\gamma^{-\tau}f_2(N(t, \tau), Z(t, \tau))Z(t, \tau)$ goes to zero as t goes to infinity. But this obviously follows because $f_2(\cdot, \cdot)Z(\cdot, \cdot) \leq f(1, 1)$, which is bounded. Therefore, the competitive equilibrium yields the same labor allocations that the planner does. Consumers are indifferent in the competitive equilibrium about the timing of their consumptions. Hence, the competitive equilibrium is Pareto optimal. Clearly, any solution to the programming problem described above can also be supported as a competitive equilibrium. Note that these welfare theorems apply to a competitive equilibrium with an arbitrary initial distribution of skilled workers and not just to the stationary equilibrium.

V. Concluding Remarks

We have presented a model of investment in technology-specific human capital. Such specificities lead to a lag between the time at which a new technology becomes available and its peak usage. In other words, technologies diffuse slowly. We showed that technologies diffuse more rapidly in high-growth economies, that an increase in the rate of technology arrival implies an increase in the rate of diffusion, and that the wage profiles over time are flatter in older technologies than in newer ones. In that sense, people who learn newer technologies invest in technology-specific skills, and our model is one of human capital accumulation. The equilibrium we describe is Pareto optimal.

An obvious extension of our model would be to allow for uncertainty in the rate of technological innovation. We conjecture that in such situations, a technological innovation that is above average will attract a large number of young workers and lead to above-average investment in the newest technology. Since this capital is specific to the technology, in subsequent periods relatively few young workers will be attracted to even newer technologies. These technologies will then be adopted and diffused at a slower rate than average. Thus this extension of the model can account for bursts in technological advance followed by a slowdown.

Our assumption of exogenous technical change obviously does not reflect the reality of the process of innovation, which requires the use of resources. It would be interesting to examine a model in which technological innovation and adoption are jointly and endogenously determined. One possible modification of our model would be to let the productivity of the newest vintage relative to that of the previous one be determined by the number of workers who enter the newest industry. In such a case, workers in the newest vintages can be thought of as engaging in innovative activity.

Another possibility is to think of spillover effects as implying that if workers are skilled in relatively new technologies, then the productivity of the newest technology is higher. In such a model, the rate of growth of the economy and the rate of adoption of new technologies are both determined endogenously. The resulting equilibrium is not necessarily Pareto optimal, and the model can be used to study the effect of policy interventions on the growth rate of the economy.

We conjecture that an exogenous improvement in the technology of innovation will lead, as in the model in this paper, to an increase in the rate of diffusion. The earnings profiles will also likely become flatter as a result of such an improvement.

Appendix

Proof of Proposition 1

We prove this proposition by proving a series of claims.

CLAIM 1. *No holes in the stationary distribution.*—If $Z_T = 0$ for some T, then $\mu_{T+1} = 0$.

Proof. Suppose $Z_T = 0$ and $\mu_{T+1} > 0$. Then $w_T = \gamma^{-T}\omega_0$ for all t from (5). Furthermore, $w_{T-1} \geq \gamma^{-T+1}\omega_0$ and $w_{T+1} \geq \gamma^{-T-1}\omega_0$ since firms could make infinite profits otherwise. Now if $\omega_0 = 0$, $\pi_T(w_T)$ is infinite, implying that the present value of income is infinite, which is inconsistent with the fact that output is bounded. If $\omega_0 > 0$, $w_T < w_{T-1}$. From (8) this implies that $\pi_{T+1}(w_{T+1}) > \pi_T(w_T)$. But $w_{T+1} > \gamma w_T$, which implies that $\pi_{T+1}(w_{T+1}) \leq \pi_T(w_T)$. We have a contradiction.

CLAIM 2. *The support of μ is finite.*—There exists a T such that $\mu_\tau = 0$ for all $\tau > T$.

Proof. The argument is proved by contradiction. Using claim 1 repeatedly and noting that $0 \leq Z_\tau \leq \mu_\tau$ imply that $\mu_\tau > 0$ for all τ and that $Z_\tau > 0$ for all τ. Therefore, from (8),

$$w_\tau + \beta\gamma\pi_{\tau+1}(w_{\tau+1}) = k \quad \text{for all } \tau. \tag{A1}$$

Suppose first that, for some τ, $w_\tau > w_{\tau+1}$. Then, clearly, from (A1), $\pi_S(w_S) < \pi_{S+1}(w_{S+1})$ for all $S \geq \tau$ and $w_S > w_{S+1}$. We next show that there is some T such that $n_\tau(w_\tau) \geq 1$ for all $\tau \geq T$. Define T as the smallest number that satisfies $\gamma^{-T}f(1, 1) \leq w_\tau$. Since $w_\tau > 0$, such a number exists. Since $\pi_T(w_T) \geq w_\tau$ and $\pi_T(w_T) \leq \gamma^{-T}f(n_T(w_T), 1)$, clearly $n_T(w_T) \geq 1$. It also follows that $n_\tau(w_\tau) \geq 1$ for all $\tau \geq T$. Since wages are strictly decreasing, $N_{2\tau} = 0$ and $Z_\tau = \mu_\tau$. Therefore, $\mu_{\tau+1} = n_\tau(w_\tau)\mu_\tau$ and μ_τ is an increasing sequence. But $\Sigma_\tau\, \mu_\tau = 1$. We have a contradiction. Consider next the case in which, for all τ, $w_\tau \leq w_{\tau+1}$. But π_τ is a decreasing function of the vintage, and so for τ sufficiently large, $\pi_\tau(w_\tau) < w_1$, implying that $Z_\tau = 0$. This contradiction establishes the desired result. Q.E.D.

CLAIM 3. *Unskilled workers' wages are nondecreasing.*—Let T denote the last vintage. In a stationary equilibrium, $w_\tau \geq w_{\tau-1}$ and $v_{\tau+1} \leq v_\tau$ for $\tau = 1, \ldots, T-1$. Furthermore, $Z_\tau = \mu_\tau$, $\tau \leq T-2$, and $Z_{T-1} > 0$.

Proof. Let T be the last vintage. We show that the wage sequence is nondecreasing up to vintage $T-1$. Suppose, by way of contradiction, that $w_{T-1} < w_{T-2}$. Then from equation (8),

$$\pi_T(w_T) > \pi_{T-1}(w_T). \tag{A2}$$

Therefore, $w_T < w_{T-1}$, which implies that $N_{2T} = 0$. Since $N_{1T} = 0$, inequality (A2) cannot hold and we have a contradiction. Therefore, $w_{T-1} \geq w_{T-2}$. This implies that $Z_{T-1} > 0$ and $\pi_{T-1}(w_{T-1}) < \pi_{T-2}(w_{T-2})$. From the present value conditions, $w_{T-2} > w_{T-3}$. By induction, the wages of unskilled workers are strictly increasing for $\tau = 0, \ldots, T-2$. Furthermore, since the wages of skilled workers are strictly decreasing for $\tau = 1, \ldots, T-1$, $Z_\tau = \mu_\tau$. Q.E.D.

Proof of Proposition 2

The proposition is proved by construction. To construct the wage sequence, fix a number T. Define a sequence of wages recursively as functions of w_{T-1} as follows. Let the wage at vintage $T-2$ be given by

$$w_{T-2}(w_{T-1}) = w_{T-1}(1 + \beta\gamma) - \beta\gamma\pi_{T-1}(w_{T-1}). \tag{A3}$$

Clearly, $w_{T-2}(\cdot)$ is a strictly increasing function, and for w_{T-1} sufficiently large, $w_{T-2} \geq 0$. Define the rest of the wage sequence recursively as

$$w_\tau(w_{T-1}) = w_{T-1}(1 + \beta\gamma) - \beta\gamma\pi_{\tau+1}(w_{\tau+1}(w_{T-1})). \tag{A4}$$

Clearly, $w_\tau(w_{T-1})$ is a continuous, strictly increasing function, which is positive and therefore well defined for sufficiently large w_{T-1}. In particular, $w_0(w_{T-1})$ is a continuous, strictly increasing function. Therefore, there is a unique value of w_{T-1} for each T such that $w_0(w_{T-1}) = \omega_0$. Denote the wage sequence at this value of w_{T-1} by $(w_0(T), w_1(T), \ldots, w_{T-1}(T))$ and the associated present value by $k(T)$. If $\pi_1(\omega_0) \leq \omega_0$, the number of vintages is one. So suppose $\pi_1(\omega_0) > \omega_0$ and let $T = 2$. Then (A3) reads $\omega_0 + \beta\gamma\pi_1(w_1(2)) = w_1(2)(1 + \beta\gamma)$. If $\pi_1(w_1(2)) \leq w_1(2)$, then $\omega_0 \geq w_1(2) \geq \pi_1(w_1(2))$, which contradicts the assumption that $\pi_1(\omega_0) > \omega_0$. So $\pi_1(w_1(2)) > w_1(2)$, and the constructed sequence at $T = 2$ satisfies (14)–(16). We now use induction to argue that if a sequence satisfies (14)–(16) but does not satisfy (17) at T, the sequence at $T + 1$ also satisfies (14)–(16). We have constructed wage sequences satisfying (14) and (15) for each T. Suppose $\pi_T(w_T(T + 1)) < w_T(T + 1)$. Since $\pi_T(w_{T-1}(T)) > w_{T-1}(T)$, it follows that $w_{T-1}(T) < w_T(T + 1)$. Therefore, $k(T) < k(T + 1)$. Furthermore, because $\pi_T(w_T(T + 1)) < w_T(T + 1)$, from (A3), $w_{T-1}(T) < w_{T-1}(T + 1)$. Using the result that $k(T) < k(T + 1)$, we can use (A4) recursively to show that $w_0(T) < w_0(T + 1)$, which clearly contradicts the fact that $w_0(T) = \omega_0 = w_0(T + 1)$. We have established that $\pi_T(w_T(T + 1)) \geq w_T(T + 1)$.

We now show that $w_T(T + 1) \geq w_{T-1}(T)$ if the sequence at T satisfies (14)–(16). Suppose that $w_T(T + 1) < w_{T-1}(T)$. Then $k(T + 1) < k(T)$ and $w_{T-1}(T + 1) < w_{T-1}(T)$. Again, with (A4) it is easy to show that $w_{T-2}(T + 1) < w_{T-2}(T)$ and recursively that $w_0(T + 1) < w_0(T)$, establishing a contradiction. Since $w_T(T + 1) \geq w_{T-1}(T)$ and $\pi_{T+1}(\cdot) < \pi_T(\cdot)$, it is clear that, for T sufficiently large, (17) is satisfied.

We now establish the uniqueness of the wage sequence. Let $w_\tau(T)$ satisfy (9)–(12) and let $w_\tau(T^*)$ satisfy the same conditions for a larger number, T^*. Since $\pi_{T^*-1}(w_{T^*-1}(T^*)) \geq w_{T^*-1}(T^*)$ and $\pi_T(w_{T-1}(T)) \leq w_{T-1}(T)$, it follows that $w_{T^*-1}(T^*) < w_{T-1}(T)$. Therefore, $k(T^*) < k(T)$. It is also straightforward to show that $w_{T-1}(T^*) < w_{T-1}(T)$. With (A4) used recursively, it is easy to establish a contradiction. Q.E.D.

Proof of Proposition 4

We shall show that

$$w(\tau, \mu') \geq \left(\frac{\gamma}{\gamma'}\right)^\tau w(\tau, \mu) \quad \text{for } \tau \leq T'.$$

Since $n'_\tau[(\gamma/\gamma')^\tau w] = n_\tau(w)$, this suffices to prove the result. We shall assume throughout that $Z_{T'} = Z_T = 0$. The proof goes through with straightforward modifications if $Z_{T'}$ or Z_T is positive.

We start by assuming that $T' \leq T$. Suppose by way of contradiction that $w'_1 < (\gamma/\gamma')w_1$. Then, by definition of π, $\pi'_1(w'_1) > (\gamma/\gamma')\pi_1(w_1)$, which in turn implies, if $T' \geq 2$, that

$$k' = \omega_0 + \beta\gamma'\pi'_1(w'_1) > \omega_0 + \beta\gamma\pi_1(w_1) = k.$$

But then also,

$$k' = w'_1 + \beta\gamma' v'_2 > w_1 + \beta\gamma v_2 = k.$$

Since $w_1' < w_1$, $v_2' > v_2$, which in turn implies, if $T' \geq 3$, that $\gamma' v_2' = \gamma' \pi_2'(w_2') > \gamma \pi_2(w_2) = \gamma v_2$. From the definition of π, $w_2' < (\gamma/\gamma')^2 w_2$. When this argument is repeated,

$$w'_{T'-1} < \left(\frac{\gamma}{\gamma'}\right)^{T'-1} w_{T-1}. \tag{A5}$$

But also,

$$k' = w'_{T'-1} + \beta\gamma' w'_{T'-1} > k \geq w_{T'-1} + \beta\gamma w_{T'-1}. \tag{A6}$$

We have a contradiction, and $w_1' \geq (\gamma/\gamma')w_1$.

We now complete the argument with an inductive step. Suppose that there is some τ such that $w'_{\tau-1} \geq (\gamma/\gamma')^{\tau-1} w_{\tau-1}$ but $w'_\tau < (\gamma/\gamma')^\tau w_\tau$. Then $\pi'_\tau(w'_\tau) > (\gamma/\gamma')^\tau w_\tau$ and

$$k' = w'_{\tau-1} + \beta\gamma v_\tau > \left(\frac{\gamma}{\gamma'}\right)^{\tau-1} (w_{\tau-1} + \beta\gamma v_\tau) = \left(\frac{\gamma}{\gamma'}\right)^{\tau-1} k. \tag{A7}$$

Also note that $v'_{\tau+1} = (k' - w'_\tau)/\beta\gamma' > (\gamma/\gamma')^\tau[(k - w_\tau)/\beta\gamma] = (\gamma/\gamma')^\tau v_{\tau+1}$. Therefore, $w'_{\tau+1} < (\gamma/\gamma')^\tau w_{\tau+1}$. Repeating this argument, we get $w'_{T'-1} < (\gamma/\gamma')^{T'-1} w_{T-1}$, and using (A6) and (A7), we get a contradiction. Therefore, if $T' \leq T$, then $w'_\tau \geq (\gamma/\gamma')^\tau w_\tau$ for all $\tau \leq T'$.

We now show that $T' \leq T$. Suppose $T' > T$. With the same argument as above, it can be shown that $w'_T \geq (\gamma/\gamma')^T w_T$. But then $\pi'_T(w'_T) \leq (\gamma/\gamma')^T \pi_T(w_T)$ so that $\pi_T(w_T) > w_T$. This obviously contradicts a necessary condition of a stationary equilibrium. Q.E.D.

Proof of Proposition 6

Suppose $T' \leq T$ and $w'_\tau \leq w_1$. We shall show that this leads to a contradiction. From (8),

$$\omega_0 + \beta\gamma\pi_1(w_1) = w_1 + \beta\gamma\pi_2(w_2) = k \tag{A8}$$

and

$$\omega_0 + \beta'\gamma\pi_1(w_1') = w_1' + \beta'\gamma\pi_2(w_2') = k'. \tag{A9}$$

Subtracting the first equation from the second and noting that $w_1' \leq \omega_1$, after some rearranging we get

$$\beta'[\pi_1(w_1') - \pi_2(w_2')] \leq \beta[\pi_1(w_1) - \pi_2(w_2)]. \tag{A10}$$

Recall that $\beta' > \beta$ and $\pi(\cdot)$ is a decreasing function. Hence $w_2' < w_2$. By induction it is easy to show that $w'_\tau < w_\tau$, all τ. In particular, $w'_{T'-1} < w_{T'-1}$. It follows that $k' < k$. But

$$w'_{T'-1}(1 + \beta'\gamma) = k' < k = w_{T'-1}(1 + \beta\gamma). \tag{A11}$$

Subtracting the left side of (A11) from (A8) and the right side of (A11) from (A9), we can easily establish a contradiction. Therefore, $w_1' \geq w_1$.

When we repeat the same argument, it follows that $w'_\tau \geq w_\tau$ for all τ. The argument that $T' \leq T$ follows exactly the same lines as in the proof of proposition 4. Q.E.D.

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