Optimal Risk Weights

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Recap from March Presentation

- Moral hazard from deposit insurance creating limited liability
- Regulators address with risk-weighted capital requirements:

$$E \geq \mathbf{A} \cdot \mathbf{w}$$

where E is shareholder equity, $\mathbf{A}$ is assets, $\mathbf{w}$ is risk weights

- How $\mathbf{w}$ is determine has changed across time and Basel accords
- Key tradeoff:
  - Banks have better information about their riskiness than regulators
  - Banks have an incentive to underreport risk
- Question: How to design risk weights?

How are risk weights determined?

- *Standardized Approach* (SA)
  - Regulators stipulate buckets for assets and a risk weight for each bucket
- *Internal Ratings Based Approach* (IRB)
  - Bank develops credit risk model then approved by regulator
  - Estimates loan-level probability of default (PD) and loss given default

Problem with SA Risk Weights

- SA risk weights lack *risk sensitivity* (may not reflect economic risks)
- Result in asset substitution and capital misallocation:
  - *Across buckets* where banks make riskier loans across buckets
  - *Within bucket* where banks make riskier loans within a bucket
- Basel I risk weight on mortgages was 0.5 and corporate debt was 1.0
  - *Across:* If 0.5 is too low and 1 is too high $\implies$ hold more mortgages
  - *Within:* Risk weight not sensitive to LTV $\implies$ hold riskier mortgages

Problem with IRB Risk Weights

- Bank can manipulate IRB risk weights by underreporting risk
- Behn, Haselmann, and Vig (JF, 2022) find evidence of banks gaming
  - Delays in IRB model approval result in loans under both SA and IRB
  - In *absolute* terms, banks underreport PD when using IRB risk weights
  - And no downward bias in implied PD for SA loans
  - So, IRB loans have lower capital requirement *relative* to SA loans
  - Despite IRB loans having higher realized losses than SA loans
  - Higher interest rates on IRB loans $\implies$ bank aware IRB loans riskier
- BHV also find that lending by IRB banks grew relative to SA banks (consistent with effectively a lower capital requirement)

What do I want to do?

- Welfare analysis weighing costs and benefits of risk weight approaches
- An approach “closer” to IRB:
  - More underreporting
  - Lower capital requirements
- An approach “closer” to SA:
  - More capital misallocation

Outline

Related Literature

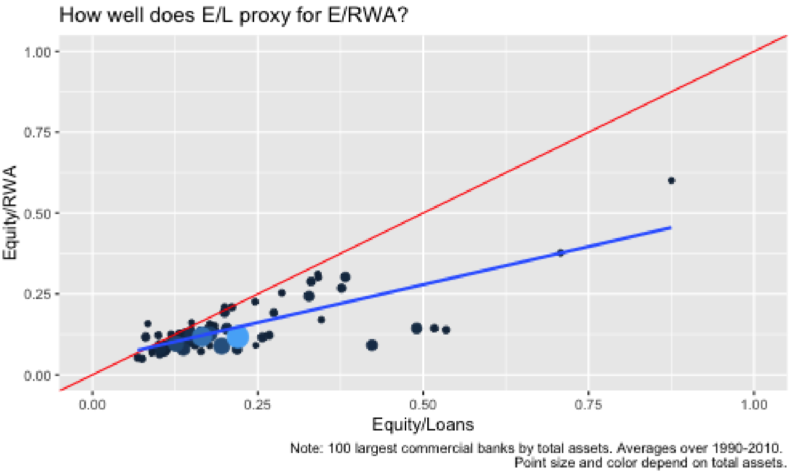
- Quantitative GE model of banking
  - Corbae and Levine (2022), Begenau and Landvoigt (2021), Bianchi and Bigio (2021), Corbae and D'Eramso (2021), Pandolfo (2021), Faira e Castro (2020), De Nicolo et al (2014), Van den Heuvel (2008)
- Risk weights
  - Begley, Purnanandam, Zheng (2017), Berg and Koziol (2017), Acharya, Engle, Pierret (2014), Gordy and Heifield (2012), Demirguc-Kunt et al (2010), Blum (2007), Gordy (2003)
- Bank opacity
  - Dang et al (2017)

How does the quantitative GE banking model risk weights?

- Most recent paper proxy for risk-weighted capital ratio with equity-to-loans ratio
- Equivalent to unit risk weight on loans and zero on other assets
- E.g. the risk-weighted capital requirement from Pandolfo (2021):

$$\frac{\ell + s + c - [a + d]}{\ell} \geq \phi^{cr}$$

where ℓ is loans, s is securities, c is cash, a is wholesale funding, and d is deposits



- $\beta \approx 0.5$, $R^2 \approx 0.8$, size is significant and negative

Outline

What do I do?

- Build simple 2-period model in the spirit of Allen and Gale (2004)
- Bank is funded by insured deposits and invests in risky technology
- Optimal capital requirements with private info about loan riskiness
- How does private information change allocations and requirements?
- What do allocation look like if regulators ignore private information?

Outline

Environment

- Risky technology
- Insured deposits
- Unit mass of banks
- Regulator

Risky Technology

- Linear technology riskiness $S \in [0, 1]$
- In period 0, bank invests in X into the risky technology at S
- In period 1, the risky technology returns
  - $A \cdot S \cdot X$ with probability $p(S)$
  - Zero with probability $1 - p(S)$.

  with $A > 0$
- Assume $p'(S) < 0$ and $p''(s) \geq 0 \implies$ risk-return trade-off

Deposits

- Let $D \equiv \int_0^1 D_i di$ be aggregate deposits
- Inverse deposit supply curve is $r(D)$
- Assume $r'(D) > 0 \implies$ interior solution
- Deposits are insured $\implies$ limited liability

Bank i

- Bank i is born with equity $E_i > 0$ with $E \equiv \int_0^1 E_i di$
- Chooses its deposits quantity D_i and is a price-taker
- Chooses its loan riskiness S_i
- Invests $E_i + D_i$ into risky technology at S_i
- Maximizes expected equity holder return subject to limited liability

Regulator

- The regulator can subject banks to risk-weighted capital requirements:

$$\frac{E_i}{w(X)(D_i + E_i)} \geq \theta(X)$$

  where

  - $X \in \mathcal{X}$ is the vector of observables
  - $w : \mathcal{X} \to \mathbb{R}$ is the risk weight
  - $\theta : \mathcal{X} \to \mathbb{R}$ is the minimum ratio

- Equivalently,

$$\tilde{\theta}(X) E_i \geq D_i$$

  where $\tilde{\theta}(X) \equiv \frac{1 - w(X)\theta(X)}{w(X)\theta(X)}$

Information Structure

Full information:

- Regulator observes everything

$$X = \{(E_i, D_i, S_i)\}_{\forall i}$$

Partial information:

- Regulator observes E_i and D_i and does not observe S_i
- Bank i reports $\hat{S}_i$ to regulator

$$X = \{(E_i, D_i, \hat{S}_i)\}_{\forall i}$$

- Bank i can privately consume some output

Functional Forms and Parameters

- Risky technology: $p(S) = 1 - S^{\eta}$ and $\eta = A = 1$
- Inverse deposit supply curve: $r(D) = \gamma D^2 + \alpha$ with $\gamma = 1$ and $\alpha = 0$
- Representative bank: $E_i = E$

Outline

Full Information Problems

- Planners problem

$$\max_{S,D} p(S)AS(D+E) - r(D)D$$

- Bank i problem

$$\max_{S_i,D_i} p(S_i)[AS_i(D_i+E_i) - r(D)D_i]$$
$$\text{s.t. } \tilde{\theta}(X)E_i \geq D_i$$

Solutions[1]

- Efficient allocation

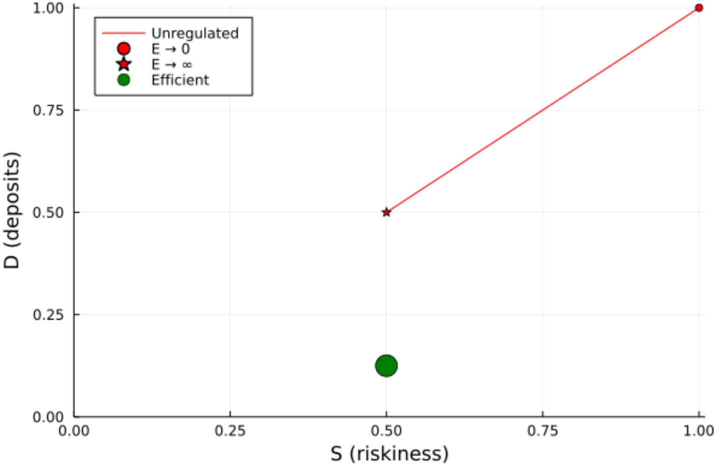
$$S^* = \frac{1}{2}$$
$$D^* = \frac{1}{8}$$

- Unregulated bank choice (i.e. $\tilde{\theta}(X) = \infty$)

$$S^U(E) = D^U(E) = \frac{1}{2}\left(\sqrt{4E^2+1} - 2E + 1\right)$$

[1] $p(S) = 1 - S$, $A = 1$, $r(D) = D^2$, and $E_i = E$.

Full Information Allocation ⏩ (E, S) ⏩ (E, D)



- Unregulated banks choose to be larger and riskier than is efficient.
- Banks take on less excessive risk with higher E.

Optimal Capital Requirements

- Capital requirements can implement the efficient allocation:

$$\tilde{\theta}(S_i, E) = \begin{cases} \frac{D^*}{E}, & \text{if } S_i = S^* \\ 0, & \text{otherwise.} \end{cases}$$

- One possible way to split up $\tilde{\theta}(S_i, E)$ is

$$w(S_i) = \begin{cases} 1, & \text{if } S_i = S^* \\ \infty, & \text{if } S_i \neq S^* \end{cases}$$

$$\theta(E) = \frac{E}{D^* + E}$$

Outline

Constrained Planner Problem

- Assume representative bank $E = E_i$
- Constrained planners problem

$$\max_{S,D} p(S)AS(D+E) - r(D)D$$
$$\text{s.t. } S = \max_{\tilde{S}}\{p(\tilde{S})[A\tilde{S}(D+E) - r(D)D]\}$$

- Isomorphic problem with limited commitment
  - I.e., bank cannot commit to S before planner give them D

Solution[2]

- Incentive compatibility constraint

$$S = \frac{1}{2}\left[\frac{D^2 + D + E}{D + E}\right]$$

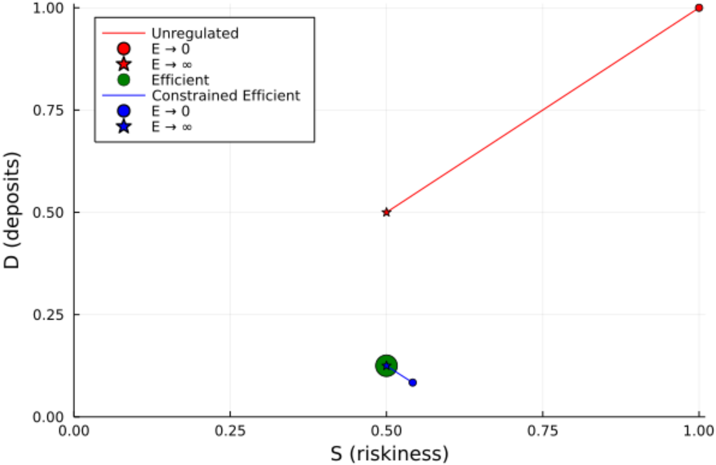
- Constrained efficient allocation

$$D^{**}(E) = \left\{ D \middle| \frac{D^4}{4(D+E)^2} - \frac{D^3}{D+E} - 2D + \frac{1}{4} = 0 \right\}$$

$$S^{**}(E) = \left\{ S \middle| S = \frac{1}{2}\left[\frac{D^2 + D + E}{D + E}\right], D = D^*(E) \right\}$$

[2] $p(S) = 1 - S$, $A = 1$, $r(D) = D^2$, and $E_i = E$.

Private Information Allocation ⏩ (E, S) ⏩ (E, D)



- Constrained efficient has higher S and lower D than efficient
- Constrained efficient converge to the efficient as $E \to \infty$

Optimal Capital Requirements

- Capital requirements can implement the efficient allocation:

$$\tilde{\theta}(D_i, E) = \begin{cases} \frac{D^{**}(E)}{E}, & \text{if } D_i = D^{**}(E) \\ 0, & \text{otherwise.} \end{cases}$$

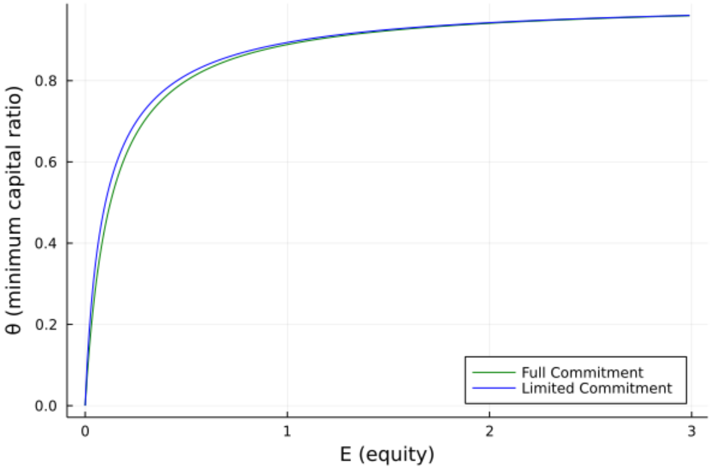
- One possible way to split up $\tilde{\theta}(D_i, E)$ is

$$w(D_i, E) = \begin{cases} 1, & \text{if } D_i = D^{**}(E) \\ \infty, & \text{if } D_i \neq D^{**}(E) \end{cases}$$

$$\theta(E) = \frac{E}{D^{**}(E) + E}$$

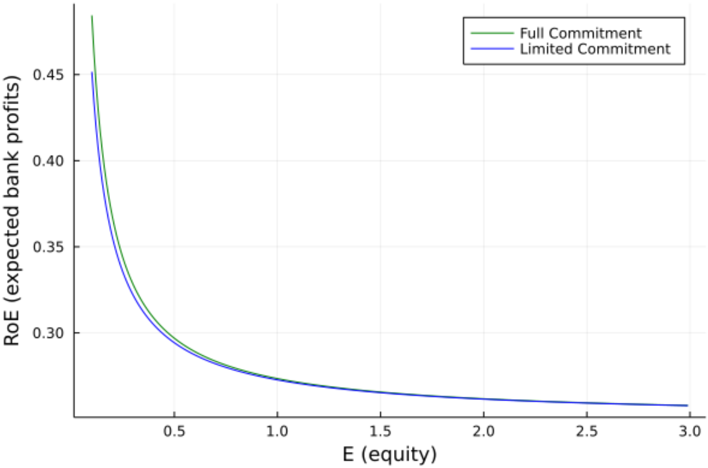
Outline

Optimal Capital Requirements



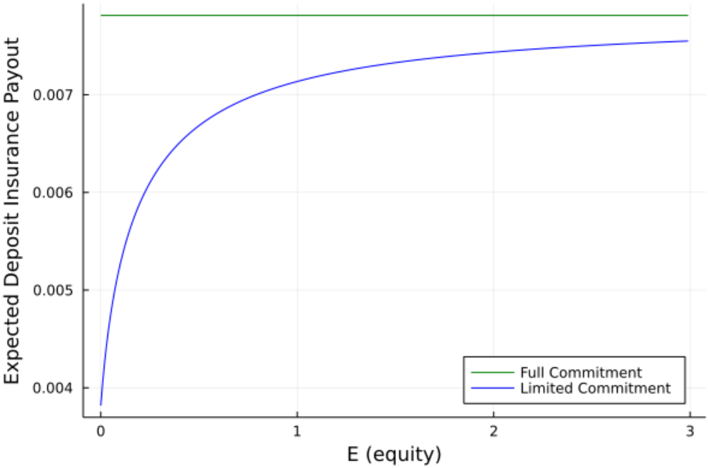
- Optimal capital requirements are higher with partial information

Expected Bank Profit



- Expected bank profit is lower with partial information

Expected Deposit Insurance Payout



- Expected deposit insurance payout is lower with partial information

Outline

Naive regulator

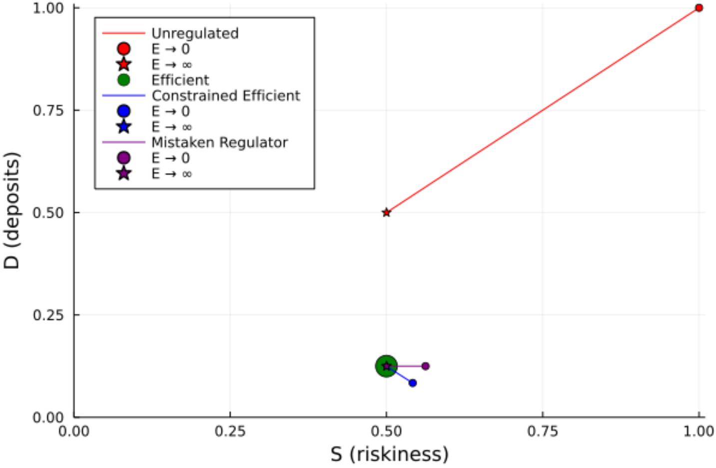
- What is the allocation if the regulator neglects private information?
- Regulator imposes full info requirement taking report of $\hat{S}_i$ as true
- Deposits are pinned to full information allocation

$$D^{MR} = 1/8$$

- Incentive compatibility pins down loan riskiness

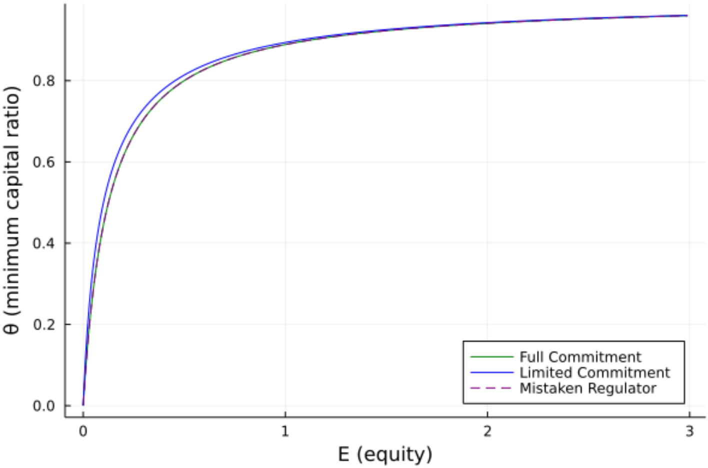
$$S^{MR} = \frac{1}{2}\left[\frac{(1/2)^2 + (1/2) + E}{(1/2) + E}\right]$$

Allocation with Naive Regulator (E, S) (E, D)



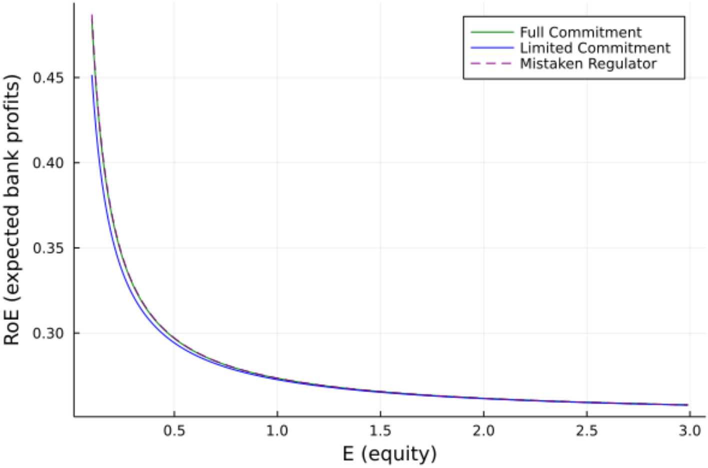
- Allocation is efficient quantity of deposit but excessively risky

Capital Requirements



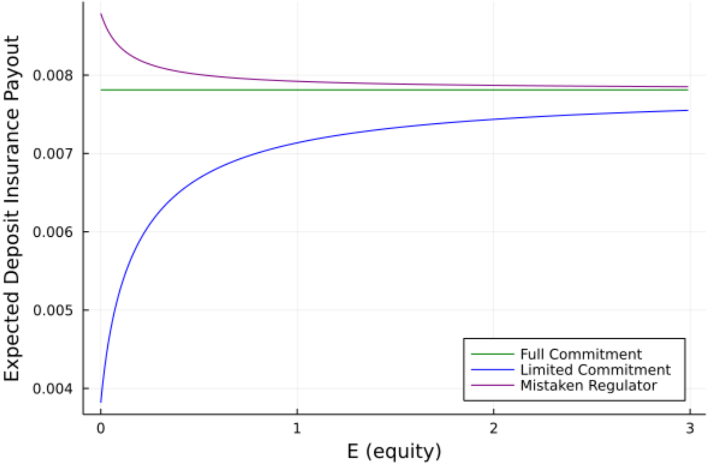
- By construction, capital requirements are the same as full info

Expected Bank Profit



- Expected bank profits are (slightly) higher than under full info

Expected Deposit Insurance Payout



- Expected deposit insurance payout are higher than with full info

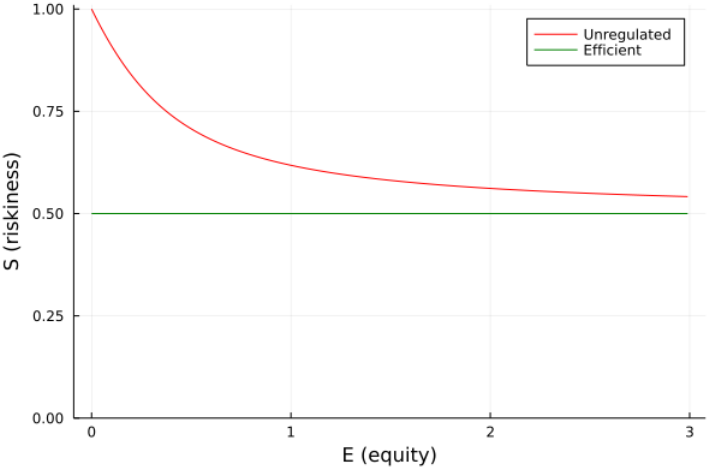
Outline

Next Steps

- Current approach does not really speak to risk weights
- Options for moving forward:
  - Take current framework and introduce second risky technology
  - Banks observe private noisy signal of return before making loan

Outline

Full Information Allocation in (E, S)



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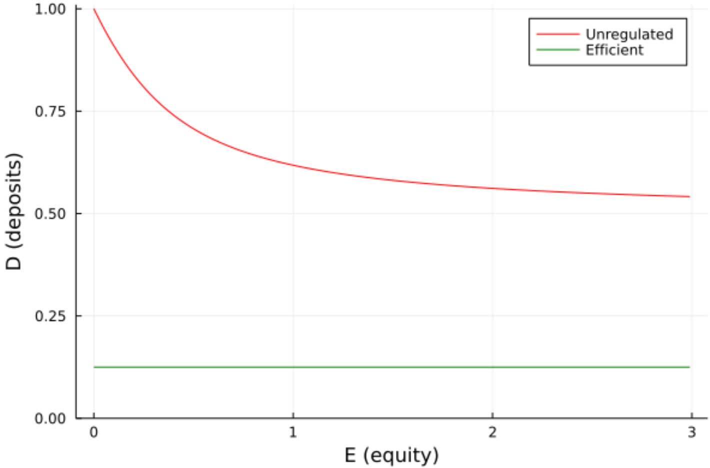
Full Information Allocation in (E, D)



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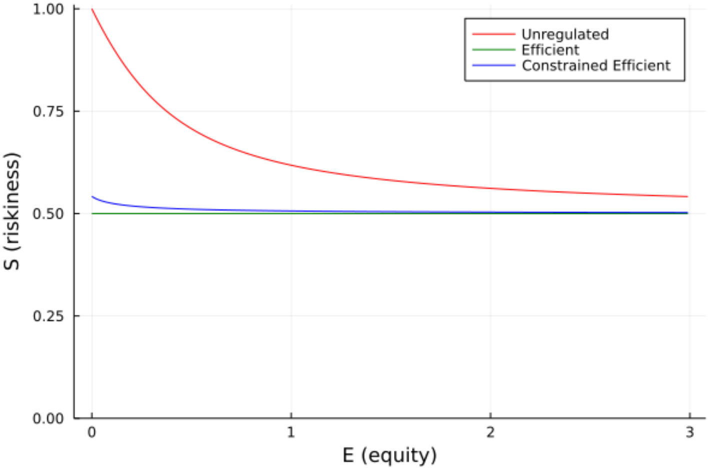
Private Information Allocation in (E, S)



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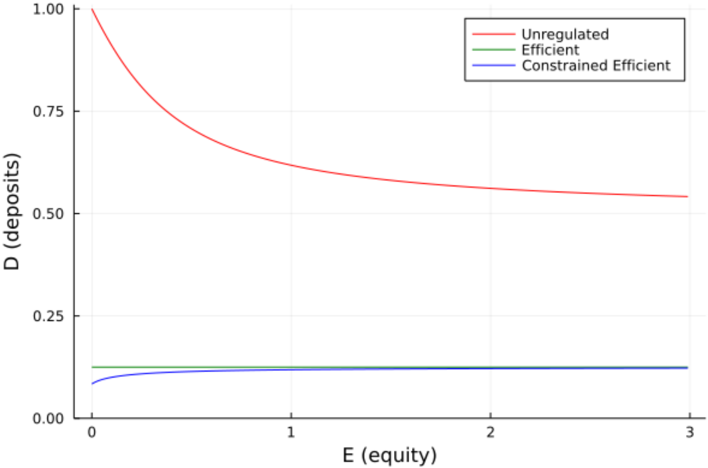
Private Information Allocation in (E, D)



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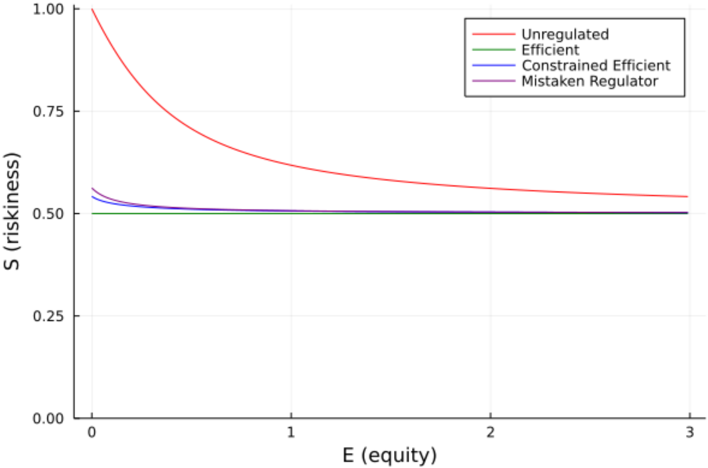
Private Information Allocation in (E, S)



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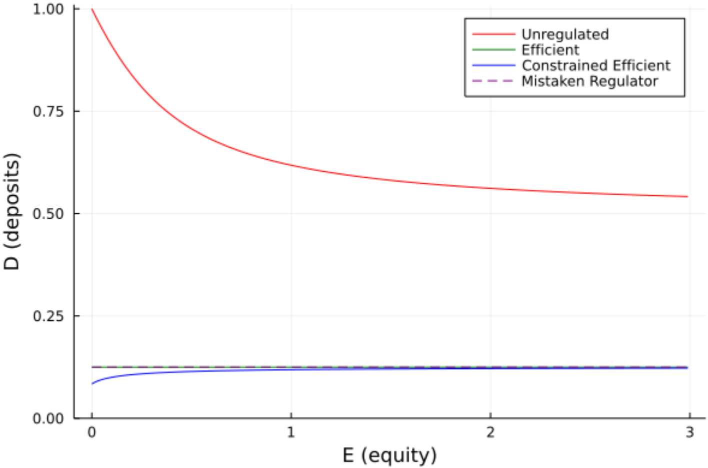
Private Information Allocation in (E, D)



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