ECON 899B: PS4

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Question 1 - Value Function Iteration

Construct the U_0 and U_1 as the current per-period payoff vectors (do not including the T1EV shocks):

$$U_0 = \alpha C \mathbb{1}(I > 1) + \lambda \mathbb{1}(I = 1)\mathbb{1}(C > 1)$$
$$U_1 = \alpha C - P$$

where C is the consumption shock vector, I is the inventory vector, and P is the price vector across states. Following the algorithm in slide 9 of the notes on dynamic discrete choice models we use value function iteration:

$$EV^0 = \log(\exp(U_0) + \exp(U_1)) + \gamma$$
$$EV^k = \log(\exp(U_0 + \beta F_0 EV^{k-1}) + \exp(U_1 + \beta F_1 EV^{k-1})) + \gamma$$

where F_0 is the transition matrix without investment and F_1 is the transition matrix with investment.

The resulting EV are reported in the following table:

I	C	P	U0	U1	EV
0	0	4	0.0	-4.0	61.128
1	0	4	0.0	-4.0	65.01
2	0	4	0.0	-4.0	68.482
3	0	4	0.0	-4.0	71.669
4	0	4	0.0	-4.0	74.63
5	0	4	0.0	-4.0	77.394
6	0	4	0.0	-4.0	79.959
7	0	4	0.0	-4.0	82.263
8	0	4	0.0	-4.0	84.073
0	1	4	-4.0	-2.0	58.491
1	1	4	2.0	-2.0	63.128
2	1	4	2.0	-2.0	67.01
3	1	4	2.0	-2.0	70.482
4	1	4	2.0	-2.0	73.669
5	1	4	2.0	-2.0	76.63
6	1	4	2.0	-2.0	79.394
7	1	4	2.0	-2.0	81.959
8	1	4	2.0	-2.0	84.263
0	0	1	0.0	-1.0	63.244
1	0	1	0.0	-1.0	66.895
2	0	1	0.0	-1.0	70.203
3	0	1	0.0	-1.0	73.26
4	0	1	0.0	-1.0	76.11
5	0	1	0.0	-1.0	78.766
6	0	1	0.0	-1.0	81.201
7	0	1	0.0	-1.0	83.282
8	0	1	0.0	-1.0	84.278
0	1	1	-4.0	1.0	61.025
1	1	1	2.0	1.0	65.244
2	1	1	2.0	1.0	68.895
3	1	1	2.0	1.0	72.203
4	1	1	2.0	1.0	75.26
5	1	1	2.0	1.0	78.11
6	1	1	2.0	1.0	80.766
7	1	1	2.0	1.0	83.201
8	1	1	2.0	1.0	85.282

Question 2 - Policy Function Iteration

In the table below, I compare the expected values from value function iteration "EV" and policy function iteration based on the estimated frequencies from the simulated data "EVhat". In addition, I include the estimated frequency based on the simulated data. I use the following algorithm from page 10 and 11 of the slides on dynamic discrete choice models:

$$
\begin{aligned}
e_0 &= \gamma - \ln \hat{P}_0 \\
e_1 &= \gamma - \ln \hat{P}_1 \\
P^0 &= (1 + \exp(-(\hat{P}_1 - \hat{P}_0)))^{-1} \\
F^k &= (1 - P^k) .* F_0 + R^k .* F_1 \\
EV^k &= (I - \beta F^k)^{-1}((1 - P^k) .* (U_0 + e_0) + P^k .* (U_1 + e_1)) \\
\tilde{V}^k &= (U_1 + \beta F_1 EV^k) - (U_0 + \beta F_0 EV^k) \\
P^{k+1} &= 1./(1 + \exp(-\tilde{V}^k))
\end{aligned}
$$

where $\hat{P}_0$ and $\hat{P}_1$ are the estimated state-level frequencies based on the simulated data and ".*" and "./" denotes element-by-element multiplication and division, respectively.

The estimates are very close. On average, the PFI estimates are about 8 tenths of a percent off from the VFI estimates. The largest difference was just less than three percent; this was one of the two states that were constrained by the probability bounds of (0.001, 0.999). The value function iteration took 3,248 iterations and the policy function iteration only took 15 iterations. Both were very fast, but scaling this problem up, the computational advantages of the policy function iteration could be substantial.

I	C	P	P0hat	P1hat	EV	EVhat
0	0	4	0.475	0.525	61.128	61.553
1	0	4	0.608	0.392	65.01	65.419
2	0	4	0.683	0.317	68.482	68.891
3	0	4	0.72	0.28	71.669	72.094
4	0	4	0.773	0.227	74.63	75.092
5	0	4	0.828	0.172	77.394	77.924
6	0	4	0.824	0.176	79.959	80.591
7	0	4	0.818	0.182	82.263	83.084
8	0	4	0.999	0.001	84.073	85.309
0	1	4	0.115	0.885	58.491	58.932
1	1	4	0.469	0.531	63.128	63.553
2	1	4	0.611	0.389	67.01	67.419
3	1	4	0.673	0.327	70.482	70.891
4	1	4	0.781	0.219	73.669	74.104
5	1	4	0.743	0.257	76.63	77.092
6	1	4	0.836	0.164	79.394	79.927
7	1	4	0.872	0.128	81.959	82.604
8	1	4	0.727	0.273	84.263	85.117
0	0	1	0.001	0.999	63.244	63.769
1	0	1	0.067	0.933	66.895	67.3
2	0	1	0.083	0.917	70.203	70.617
3	0	1	0.08	0.92	73.26	73.709
4	0	1	0.154	0.846	76.11	76.601
5	0	1	0.217	0.783	78.766	79.353
6	0	1	0.333	0.667	81.201	81.974
7	0	1	0.286	0.714	83.282	84.313
8	0	1	0.999	0.001	84.278	86.655
0	1	1	0.001	0.999	61.025	61.466
1	1	1	0.045	0.955	65.244	65.657
2	1	1	0.052	0.948	68.895	69.303
3	1	1	0.1	0.9	72.203	72.617
4	1	1	0.097	0.903	75.26	75.704
5	1	1	0.176	0.824	78.11	78.606
6	1	1	0.286	0.714	80.766	81.389
7	1	1	0.167	0.833	83.201	83.912
8	1	1	0.333	0.667	85.282	86.336

Question 3 and 4 - Maximum Likelihood Estimation

The log-likelihood function is:

$$\sum_i a_i \ln P(s_i) + (1 - a_i) \ln(1 - P(s_i))$$
$$\text{s.t. } P(x_i) = \Psi(x_i)$$

where $\Psi(\cdot)$ is the contraction mapping laid out in question 2.

I used Brent's method to find the MLE estimate of -4.020022 for λ. A figure of the log-likelihood over values for λ is below: