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Macro Midterm (ECON 712)

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Question 1 Planner's Problem

① The planner's problem is

$$\max_{\{c_1, c_2, n, K\}} \ln(c_1 + c_2) + \ln(1-n)$$

s.t.

$c_1 + K = w$ Resource feasibility in $t=1$

$c_2 = n + RK - g$ Resource feasibility in $t=2$

$c_1 \geq 0$ Nonnegativity constraints

$c_2 \geq 0$

$n \geq 0$

$1-n \geq 0$

$K \geq 0$

② Notice that its better for the household to forgo consume in $t=1$ and consume at least Rw in $t=2$. $\boxed{c_1=0} \Rightarrow K=w \Rightarrow c_2 \geq Rw-g$. This is because consumption in $t=1$ & consumption in $t=2$ are perfect substitutes. So we can rewrite the planners problem as:

$$\max_{\{c_2, K, n\}} \ln(c_2) + \ln(1-n)$$

$$\text{s.t. } K = w$$
$$c_2 = n + RK - g \quad \Rightarrow \quad c_2 = n + Rw - g$$
$$c_2, n, 1-n, K \geq 0$$

$$\Rightarrow \max_{\{n\}} \ln(n + Rw - g) + \ln(1-n)$$
$$\text{s.t. } n, 1-n \geq 0$$

$$\text{FOC }[n]: \frac{1}{n+Rw-g} + \frac{-1}{1-n} = 0$$

$$\Rightarrow 1-n = n + Rw - g$$
$$\Rightarrow 2n = 1 - Rw + g$$
$$\Rightarrow \boxed{n = \frac{1-Rw+g}{2}}$$

By assumption, n is between 0 & 1.

$$c_2 = \left(\frac{1-Rw+g}{2}\right) + Rw - g$$

$$\Rightarrow \boxed{c_2 = \frac{1+Rw-g}{2}}$$

↳ For $c_2 \geq 0$, we need to assume $1+Rw \geq g$.

The planner's solution is $c_1=0$, $c_2 = \frac{1+Rw-g}{2}$, $K=w$, $n = \frac{1-Rw+g}{2}$.

Case 2

(3) Find GBC: Government revenue is $\tau n + \delta RK$. Government expenditure is g. So, the GBC is

$$\tau n + \delta RK = g.$$

(4) Given τ and δ, State the HH problem

$$\max_{\{c_1, c_2, n, k\}} \quad \ln(c_1 + c_2) + \ln(1-n)$$

s.t. $c_1 + k = w$ BC in $t=1$

$c_2 = (1-\tau)n + (1-\delta)Rk$ BC in $t=2$

$c_1 \geq 0$ Nonnegativity constraints

$c_2 \geq 0$

$n \geq 0$

$1-n \geq 0$

$k \geq 0$

⑤ Since c_1 & c_2 are perfect substitutes, the HH will consume all c_1 if the tax on storage returns is high enough and all c_2 if the tax is relatively low. More formally, if $(1-\delta)R \geq 1$, then $c_1 = 0$ and if $(1-\delta)R < 1$, then $K = 0$.

If $(1-\delta)R \geq 1$, $c_1 = 0$ and the HH problem becomes:

$$\max_{\{c_2, K, n\}} \{\ln(c_2) + \ln(1-n)\}$$

$$K = w$$

$$c_2 = (1-\tau)n + (1-\delta)RK \;\;\Rightarrow\;\; c_2 = (1-\tau)n + (1-\delta)Rw$$

$$c_2 \geq 0, \; K \geq 0, \; n \geq 0, \; 1-n \geq 0$$

$$\Rightarrow \max_{n} \{\ln((1-\tau)n + (1-\delta)Rw) + \ln(1-n)\}$$

$$\text{FOC}[n]: \quad \frac{(1-\tau)}{(1-\tau)n + (1-\delta)Rw} + \frac{-1}{1-n} = 0$$

$$\Rightarrow \quad (1-\tau)n + (1-\delta)Rw = (1-\tau)(1-n)$$

$$\Rightarrow \quad (1-\delta)Rw = (1-\tau)(1-n) - (1-\tau)n$$

$$\Rightarrow \quad \frac{(1-\delta)Rw}{1-\tau} = 1 - 2n$$

$$\Rightarrow \quad 2n = 1 - \frac{(1-\delta)Rw}{1-\tau}$$

$$\Rightarrow \quad n = \frac{1-\tau-(1-\delta)Rw}{2(1-\tau)}$$

(5) con't

$$\Rightarrow c_2 = \cancel{(1-\tau)}\left(\frac{1-\tau - (1-\delta)Rw}{2\cancel{(1-\tau)}}\right) + (1-\delta)Rw$$

$$\Rightarrow c_2 = \frac{1-\tau}{2} - \frac{(1-\delta)Rw}{2} + (1-\delta)Rw$$

$$\Rightarrow c_2 = \frac{1-\tau + (1-\delta)Rw}{2}$$

If $(1-\delta)R < 1$, $K=0$ and the HH problem becomes

$$\max_{\{c_1, c_2, n\}} \{\ln(c_1 + c_2) + \ln(1-n)\}$$

$$c_1 = w$$
$$c_2 = (1-\tau)n$$
$$c_1, c_2, n, 1-n \geq 0$$

$$\Rightarrow \max_{n} \{\ln(w + (1-\tau)n) + \ln(1-n)\}$$

$$\text{FOC}[n]: \frac{(1-\tau)}{w + (1-\tau)n} + \frac{-1}{1-n} = 0$$

$$\Rightarrow (1-\tau)(1-n) = w + (1-\tau)n$$

$$\Rightarrow (1-\tau)(1-n) - (1-\tau)n = w$$

$$\Rightarrow (1-\tau)(1-2n) = w$$

$$\Rightarrow (1-2n) = \frac{w}{1-\tau}$$

⑤ Con't

$$\Rightarrow \quad 1 - \frac{w}{1-\tau} = 2n$$

$$\Rightarrow \quad \frac{1-\tau-w}{2(1-\tau)} = n$$ ← Note this is between 0 and 1 because $\tau \in [0, 1-w]$

$$\Rightarrow \quad c_2 = \frac{1-\tau-w}{2}$$ (for c_2 to be nonnegative assume $\tau + w < 1$.)

So, the solution to the HH problem is

$$c_1(\tau,\delta) = \begin{cases} 0 & \text{if } (1-\delta)R \geq 1 \\ w & \text{if } (1-\delta)R < 1 \end{cases}$$

$$c_2(\tau,\delta) = \begin{cases} \frac{1-\tau+(1-\delta)Rw}{2} & \text{if } (1-\delta)R \geq 1 \\ \frac{1-\tau-w}{2} & \text{if } (1-\delta)R < 1 \end{cases}$$

$$n(\tau,\delta) = \begin{cases} \frac{1-\tau-(1-\delta)Rw}{2(1-\tau)} & \text{if } (1-\delta)R \geq 1 \\ \frac{1-\tau-w}{2(1-\tau)} & \text{if } (1-\delta)R < 1 \end{cases}$$

$$K(\tau,\delta) = \begin{cases} w & \text{if } (1-\delta)R \geq 1 \\ 0 & \text{if } (1-\delta)R < 1 \end{cases}$$

(6) The government problem

Assume the government is benevolent and try to raise enough tax revenue to cover its expenditure g in a way that maximize HH utility:

$$\max_{\{\delta, \tau\}} \ln(c_1 + c_2) + \ln(1-n)$$

$$\text{s.t.} \quad \tau n + \delta R K = g$$

$$c_1 + K = w$$

$$c_2 = (1-\tau)n + (1-\delta)RK$$

$$K = \begin{cases} w & \text{if } (1-\delta)R \geq 1 \\ 0 & \text{if } (1-\delta)R < 1 \end{cases}$$

$$n = \begin{cases} \dfrac{1-\tau-(1-\delta)Rw}{2(1-\tau)} & \text{if } (1-\delta)R \geq 1 \\ \dfrac{1-\tau-w}{2(1-\tau)} & \text{if } (1-\delta)R < 1 \end{cases}$$

⑦ Provide one equation in one unknown that characterizes the government problem

The government taxes storage at $(1-\delta)R=1$

$\Rightarrow (1-\delta) = \frac{1}{R} \Rightarrow \delta = 1 - 1/R$. Thus to cover g, the government need the GBC to hold:

$$\tau n + (1 - 1/R)Rk = g$$

$$\tau = \frac{g - (1-1/R)Rk}{n}$$

Inputting the HH decision rules for k and n:

$$\tau = \frac{g - (1 - 1/R)Rw}{\left(\frac{1 - \tau - w}{2(1-\tau)}\right)}.$$

$\boxed{\text{Case 3}}$

⑧ Given k and (τ, δ) what is the HH decision rule for labor supply at $t=2$?

$$\max_{\{c_2, n\}} \{\ln(c_1 + c_2) + \ln(1-n)\}$$

s.t. $c_1 + k = w$

$$c_2 = (1-\tau)n + (1-\delta)Rk$$

$$\Rightarrow \max_{n} \{\ln(w - k + (1-\tau)n + (1-\delta)Rk) + \ln(1-n)\}$$

FOC $[n]$: $$\frac{(1-\tau)}{w-k+(1-\tau)n+(1-\delta)Rk} + \frac{-1}{1-n} = 0$$

$$(1-\tau)(1-n) = w - k + (1-\tau)n + (1-\delta)Rk$$

$$(1-\tau)(1-2n) = w - k + (1-\delta)Rk$$

$$1 - 2n = \frac{w - k + (1-\delta)Rk}{1-\tau}$$

$$n^*(\tau, \delta, k) = \frac{1 - \tau - w + k - (1-\delta)Rk}{2(1-\tau)}$$

(9) What characterizes the optimal choice for government?

Given K and decision rule from 8, the government wants to maximize government revenue:

$$\max_{\{\tau,\delta\}} \left\{ \delta RK + \tau \left(\frac{1-\tau-2w+k-(1-\delta)RK}{2(1-\tau)} \right) \right\}$$

$$\text{FOC } [\tau]: \; 0 = \left(\frac{1-\tau-w+k-(1-\delta)RK}{2(1-\tau)} \right)$$

$$+ \tau \left(\frac{2(1-\tau)(-1) + (1-\tau-2w+k-(1-\delta)RK)(-2)}{4(1-\tau)^2} \right)$$

$$\text{FOC } [\delta]: \; 0 = RK + \frac{-RW}{2(1-\tau)} \implies 0 = 1 + \frac{-1}{2(1-\tau)}$$

$$\implies 1 = \frac{1}{2(1-\tau)}$$

$$\implies 1 = 2(1-\tau)$$

$$\implies \frac{1}{2} = 1-\tau$$

$$\implies \tau = 1/2$$

$$\implies 0 = (\cancel{1/2} - \cancel{2w} + \cancel{k} - (1-\delta)RK + (\cancel{k} + (\cancel{1/2} - \cancel{w} + \cancel{k} - (1-\delta)RW)$$

$$\implies 0 = (1-\delta)R(w-k)$$

(10) Given Government decision rules what is the households decision rule for k?

Since the government decision rule imply a high enough δ, it is better for the household to consume their endowment ($c_1 = 25$) in $t=1$ and not save ($k=0$).

(11) Suppose finite number of repetition. Can the Ramsey equilibrium.

No, the last repitition of the problem collapses the one-shot version of the problem. Since both the government and HH know that the last repitition will collapse to the no commitment equilibrium, Theres no reason for the Ramsey equilibrium to be supported in earlier repititions.