ECON 711 - PS 1

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Question 1: The Law of Supply

Suppose $k = 3$, and a firm uses goods one and two as inputs and produces good three as output. (Formally, $y \in Y$ requires $y_1, y_2 \leq 0$.) For each of the following, either give an example showing it's possible or prove that it's impossible. (Feel free to use examples where Y contains only a few points.)

(a) If p_3 falls and p_1 and p_2 stay the same, can the firm's output y_3 go up?

Proof: Let $p = (p_1, p_2, p_3)$ and $p' = (p_1, p_2, p_3 + \delta)$, so $\Delta p = (p_1 - p_1, p_2 - p_2, p_3 + \delta - p_3) = (0, 0, \delta)$ where $\delta > 0$. Furthermore, let $y = (y_1, y_2, y_3)$, $y' = (y_1', y_2', y_3')$ $\Delta y = (y_1' - y_1, y_2' - y_2, y_3' - y_3) = (\Delta y_1, \Delta y_2, \Delta y_3)$. By the law of supply,

$$\begin{aligned} \Delta p \cdot \Delta y &\geq 0 \\ \implies (0, 0, \delta) \cdot (\Delta y_1, \Delta y_2, \Delta y_3) &\geq 0 \\ \implies (0)\Delta y_1 + (0)\Delta y_2 + (\delta)\Delta y_3 &\geq 0 \\ \implies (\delta)\Delta y_3 &\geq 0 \\ \implies \Delta y_3 &\geq 0 \end{aligned}$$

So, yes, the firms output y_3 can increase or stay the same.

*I worked on this problem set with a study group of Michael Nattinger, Andrew Smith, Tyler Welch, and Ryan Mather. I also discussed problems with Emily Case, Sarah Bass, and Danny Edgel.