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A SIMPLE GENERAL EQUILIBRIUM VERSION OF THE BAUMOL-TOBIN MODEL*

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This paper presents a simple general equilibrium model that includes optimizing choices of the frequency of trips to the bank. The model is used to analyze the effect of inflation on the capital stock, the interest elasticity of money demand, the optimum quantity of money, and the welfare costs of inflationary finance.

INTRODUCTION

How does inflation affect consumption, saving, and capital accumulation? The issues raised by this question have been important in monetary economics at least since the work of Tobin [1965]. Typically, the models used to address them introduce money either by making it an argument of the utility function or by assuming fixed cash-in-advance constraints.[1] But money demand and patterns of money holdings in fact arise, we suspect, not from a fixed Clower constraint or from the direct provision of utility by money, but rather from the kinds of considerations that govern the money holdings of the individuals of the classic Baumol-Tobin model: individuals make periodic conversions of a higher interest asset into money to make purchases until their next conversion; the frequency of conversions is determined by the tradeoff be-

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1. Models of inflation and capital accumulation in which money is an argument of the utility function or in which money demand functions are simply assumed include Tobin [1965], Sidrauski [1967], Brock [1974], Fischer [1979], Drazen [1981], Summers [1981], and Epstein and Hynes [1983]. For models employing cash-in-advance constraints see Stockman [1981] and Rotemberg [1984].

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tween the cost of conversions and the interest forgone by holding money [Baumol, 1952; Tobin, 1956]. It would therefore be extremely useful to develop a model that grounds money demand in considerations like those of the Baumol-Tobin model and that is rich enough in its modeling of other aspects of the economy to permit the analysis of the interactions between monetary and real phenomena. Such a model would permit us to investigate some of the central issues of monetary economics using a realistic model of money holdings.

The purpose of this paper is to develop and apply such a model. I present a simple general equilibrium model that includes optimizing calculations by agents concerning the frequency of trips to the bank. I use the model to investigate the effects of inflation on the capital stock and on the pattern of consumption, the interest elasticity of money demand, and the optimal rate of inflation in both first best and second best settings.

The paper naturally consists of two parts. Section I presents the model. The model is an overlapping generations model in which particular forms for the utility function and the transactions technology are assumed. The use of specific functional forms both makes the model tractable and makes it flexible along a variety of other dimensions. The model differs from traditional overlapping generations models in that it is set in continuous time. In other words, a new generation is born every instant, and at any time a continuum of generations are alive. The use of continuous time simplifies the analysis considerably. Furthermore, in the model, as in actual economies, money is both a store of value and medium of exchange. In particular, there is another asset that always provides a higher rate of return than does money, but money is held nonetheless because it provides a less costly way of making purchases.[2]

Section II applies the model to several of the classic questions of monetary economics. With regard to the Mundell-Tobin effect, I find, first, that in general the effect of inflation on the capital stock is ambiguous; second, that in some simple cases the conventional result that inflation increases the capital stock holds;

2. Cass and Yaari [1967] present a continuous time overlapping generations model. For overlapping generations models with the feature that money is rate-of-return dominated, see, for example, Bryant and Wallace [1979] and Woodford [1983].

and third, that in practice the magnitude of the effect of inflation on capital is likely to be small. For the interest elasticity of money demand, I find that the interest rate affects money holdings not only through the standard Baumol-Tobin channel of the frequency of withdrawals but also through the pattern of spending between withdrawals and through wealth effects; both of these additional channels have potentially important implications for the overall interest elasticity. Finally, I discuss the effect of inflation on the pattern of consumption and analyze the optimal rate of inflation. I find that conventional results concerning optimal money growth in a first-best setting hold without change but that the usual second-best finding that the existence of distortions in raising revenue implies that some money growth is desirable does not hold in this model.

Jovanovic [1982] also presents a general equilibrium model that includes endogenous frequency of the conversion of other assets into money. He uses the model to show that the usual "optimum quantity of money" result holds with this type of money demand. This paper both simplifies and extends Jovanovic's work. The key simplifying assumption is that the interest rate that an individual faces on the alternative asset is fixed. In Jovanovic's model each individual has access to a private storage technology for which the rate of return depends on the amount that the individual stores. In the model of this paper each individual faces the market interest rate, which he takes as given. In addition, the model extends Jovanovic's by making individuals' pattern of consumption endogenous: rather than assuming that individuals make real purchases at a constant rate, I assume a particular utility function and allow the path of consumption to depend on inflation and the real interest rate.[3]

3. Grossman [1982b] also develops a model similar to that of the present paper. Three other related types of models should be mentioned. First, Grossman and Weiss [1983] and Rotemberg [1984] develop models in which individuals make trips to the bank at regular intervals but in which the frequency of withdrawals is fixed. They use the models to analyze open-market operations. (See also Grossman [1982a].) Second, Bryant [1980] and Martins [1980] analyze overlapping generations models in which the distinguishing feature of money is that it can be held for a shorter time than other assets; again the holding periods are exogenous. Third, Gale and Hellwig [1984] establish the existence of equilibrium in a general equilibrium model that includes transactions costs like those of the Baumol-Tobin model.

I. THE MODEL

Overview

At each instant a new generation is born. Generations are of equal size, and individuals differ only in dates of birth. There is no uncertainty. Throughout, I consider only steady states. The focus on the steady state of an overlapping generations model simplifies the analysis greatly by providing a simple, if unrealistic, reason for having a constant number of individuals making withdrawals at any instant and by yielding a description of the general equilibrium of an economy that is tractable.

Each individual in this economy is faced with the problem that typically arises in an overlapping generations model: he is endowed only at birth but wishes to consume throughout his lifetime. Specifically, at birth he receives an endowment of amount E of the economy's single consumption good and a lump sum transfer of real amount S of fiat money. He wishes to consume throughout his life, which has length T. There are two means of storing wealth: bank deposits and money. Deposits pay interest i; money pays no interest. At the beginning of life the individual sells his endowment for money and then deposits some (perhaps none or all) of his money holdings in a bank. (This initial deposit is costless. Tobin discusses the case in which it is costly.) Purchases, however, can be made only with money. Each time an individual converts deposits into money, he must make a trip to the bank; in other words, he must sacrifice real resources or suffer some inconvenience. It is simplest to think of "money" as currency, but the model and its interpretation are essentially unchanged if money corresponds to demand deposits and i is the interest rate differential between demand deposits and the illiquid asset.

This way of modeling the coexistence of money and interest-bearing assets is closely related to several conventional approaches. First, the model can be thought of as embodying a "flexible Clower constraint"; goods can be purchased only with money, but there is no fixed length of time that the money must be held before the purchase. Second, the model is similar to the model of Bryant and Wallace [1979], in which the holding of bonds via intermediaries is costly. In their model the costs take the form of the use of some of the consumption good of the economy in "chopping up" bonds; here they take the form of trips to the bank. And third, the model follows the spirit of such writers as Tobin [1968] and Friedman [1969], for whom what distinguishes money and bonds is that the holding of bonds requires some inconvenience.

The precise reason that consumers do not purchase goods using claims on bank deposits is not important. One possibility is that this activity is simply illegal. Another is that for relevant levels of the nominal interest rate the costs of writing and verifying claims on bank accounts make money a superior means of exchange. In the latter case, we should keep in mind that the model would no longer apply with extreme interest rates.

Individual Behavior

Agents have the utility function,

$$U = \int_{t=0}^{T} \ln C(t)dt - c(n), \tag{1}$$

where $C(t)$ is consumption at age t (throughout, t indexes age rather than time) and n is the number of trips to the bank that the individual makes during his lifetime. $c(\cdot)$ is increasing and twice continuously differentiable. In much of what follows, I assume that it takes the particular form,

$$c(n) = an, \qquad a > 0. \tag{2}$$

The assumption of a linear effect of trips to the bank on utility can be justified, ignoring issues of timing and using the approximation, $e^{-an} \cong 1 - an$, by supposing that the endowment is produced using labor time and that each trip to the bank requires a fixed amount of that time. In this case, if L_0 is the endowment, q productivity, and a the fraction of L_0 required for a trip to the bank, net endowment of the consumption good is $qL_0\,(1 - an) \cong qL_0e^{-an}$ (see expression (7) below).[4]

I consider the utility-maximizing behavior of an individual in three steps. First, I consider his behavior between two trips to the bank. Second, I show that Tobin's result that trips to the bank are equally spaced holds in this model. Third, I consider the optimal size of withdrawals and the optimal number of trips to the bank.[5]

4. Karni [1973] generalizes the Baumol-Tobin model to the case in which trips to the bank involve both monetary costs and time. Below, in describing equilibrium in the goods market, I assume that the cost of trips to the bank is purely inconvenience; that is, I assume that no goods are lost as a result of trips to the bank.

5. Dixit, Mirrlees, and Stern [1975] and Hellwig [1973] analyze problems in which an agent faces this type of discrete transactions costs.

Let i be the interest rate on bank deposits and π the rate of inflation. Both are constant. If i is strictly positive, it follows immediately that the individual always withdraws from the bank exactly enough money to make purchases until his next withdrawal. Consider an individual who withdraws nominal quantity M_0 of money at age t_0 to make purchases until age t_1. Let $P(t)$ denote the nominal price of the consumption good when the individual is of age t. Utility maximization requires that at the margin the individual be indifferent between spending an additional unit of money at time t_0 and spending it at some later time t, $t_0 < t < t_1$. That is,

$$U'(C(t_0))/P(t_0) = U'(C(t))/P(t), \qquad t_0 < t < t_1,$$

or, since utility is logarithmic and the discount rate is zero,

$$P(t_0)C(t_0) = P(t)C(t), \qquad t_0 < t < t_1.$$

In words, the individual spends his money at a constant nominal rate until his next withdrawal. Since total spending between ages t_0 and t_1 is M_0, we have

$$C(t) = \frac{M_0}{t_1 - t_0}\frac{1}{P(t)} = \frac{M_0}{t_1 - t_0}\frac{1}{P(t_0)}\frac{1}{e^{\pi(t-t_0)}}, \quad \text{or} \quad t_0 < t < t_1.$$

It follows that utility from consumption over the interval (t_0,t_1) is

$$U_0 = \int_{t=t_0}^{t_1} \ln C(t)dt = (t_1 - t_0)\ln\left[\frac{M_0}{t_1 - t_0}\frac{1}{P(t_0)}\right] - \tfrac{1}{2}\pi(t_1 - t_0)^2. \tag{3}$$

One can now show that withdrawals are evenly spaced. Consider some interval of the individual's life (t_0,t_2) that both begins and ends with a withdrawal. I claim that if exactly one withdrawal takes place between t_0 and t_2, utility is maximized by having the withdrawal occur at age $(t_0 + t_2)/2$. Let t_1 denote the age at which the intermediate withdrawal is made, and let M denote the nominal value, as of age t_0, of assets devoted to consumption between ages t_0 and t_2. In other words, the individual withdraws some M_0 at t_0 and $M_1 = (M - M_0)e^{i(t_1-t_0)}$ at t_1. Total utility from consumption over the interval (t_0,t_2) is

$$U_0 + U_1 = (t_1 - t_0)\ln\left[\frac{M_0/P(t_0)}{t_1 - t_0}\right] - \frac{1}{2}\pi(t_1 - t_0)^2$$

$$+ (t_2 - t_1) \ln \left[\frac{M_1/P(t_1)}{t_2 - t_1} \right] - \frac{1}{2} \pi (t_2 - t_1)^2$$

$$= (t_1 - t_0) [\ln M_0 - \ln(t_1 - t_0) - \ln P(t_0)]$$

$$+ (t_2 - t_1) [\ln(M - M_0) + i(t_1 - t_0) - \ln(t_2 - t_1)$$

$$- \ln P(t_0) - \pi(t_1 - t_0)] - \frac{1}{2} \pi (t_1 - t_0)^2 - \frac{1}{2} \pi (t_2 - t_1)^2,$$

where I have substituted for M_1 and $P(t_1)$. Maximizing this expression with respect to M_0 and t_1 yields the first-order conditions:

$$M_0/(t_1 - t_0) = (M - M_0)/(t_2 - t_1), \tag{4}$$

$$\ln[M_0/(t_1 - t_0)] - i(t_1 - t_0) = \ln [(M - M_0)/(t_2 - t_1)] - i(t_2 - t_1). \tag{5}$$

Substituting (4) into (5) implies that $t_1 - t_0 = t_2 - t_1$.[6]

Thus, any two consecutive intervals between trips to the bank must be of equal length. It follows immediately that all trips to the bank must be equally spaced. (The intervals between birth and the first trip to the bank and between the last trip and death, by the same reasoning, must be of the same length as the interval between trips.) τ, the interval between trips, is given by

$$\tau = T/(n + 1),$$

(where $n + 1$ instead of n appears in the denominator because of the assumption that no trip to the bank is needed at the beginning of life).

Let W denote the individual's initial real wealth of $E + S$, and define $w = W/T$. Suppose that the individual makes n withdrawals during his lifetime. Then (4) and the lifetime budget constraint imply that at time $j\tau$ (j a nonnegative integer less than or equal to n) the individual withdraws real amount $[W/(n + 1)]e^{rj\tau}$, where $r = i - \pi$ is the real interest rate. That is, the individual's real withdrawals grow at the real interest rate. Consumption at age t is thus given by

$$C(t) = we^{rj\tau}e^{-\pi(t - j\tau)}, \qquad j\tau < t < (j + 1)\tau. \tag{6}$$

If both π and r are positive, consumption jumps to successively higher levels at the time of each withdrawal and falls gradually between withdrawals.

6. If $i > 0$, second-order conditions are satisfied.

The contribution of consumption over the interval $(j\tau,(j+1)\tau)$ to lifetime utility is thus

$$U_j = \tau \ln w + rj\tau^2 - \tfrac{1}{2}\pi\tau^2.$$

Lifetime utility is therefore

$$V(\pi,i;n) = \sum_{j=0}^{n} [\tau \ln w + rj\tau^2 - \tfrac{1}{2}\pi\tau^2] - c(n)$$

$$(7) \qquad = (n+1)\cdot\tau \ln w + (n(n+1)/2)r\tau^2 - \tfrac{1}{2}(n+1)\pi\tau^2 - c(n) = T \ln w + (T/2)[rT - i\tau] - c(n).$$

If the cost function for trips to the bank is linear (in other words, if $c(n) = an$), the first-order condition for n is

$$(i/2)\,(T^2/(n+1)^2) = a.$$

Thus, if integer constraints are ignored, the optimal time between trips to the bank is

$$(8) \qquad \tau = T/(n+1) = \sqrt{2a/i}.$$

(Because $V_{nn} < 0$ when costs are linear, the optimal n when integer constraints are accounted for must be either the greatest integer less than or the least integer greater than the n that satisfies (8). In what follows, I treat n as a continuous variable.)

We thus have the well-known Baumol square root rule. Unlike Baumol, however, I find that the frequency of trips to the bank is independent of wealth. This arises because of the assumption that trips to the bank have a constant cost in utility units rather than in units of the consumption good.[7]

Substituting (2) and (8) into (7) yields the indirect utility function:

$$(9) \quad V^*(\pi,i) = T \ln w + (T/2)[rT - \sqrt{2ai}] - (T\sqrt{2ai}/2) + a.$$

In the case of the general cost function, it is helpful to write costs as a function of τ rather than n. Since $n = T/\tau - 1$, define $z(\tau) = c((T/\tau) - 1)$. I assume that $z'(\cdot) < 0$, $z''(\cdot) > 0$, $z'(0) = -\infty$, and $z'(\infty) = 0$. (If $c(n) = an$, for example, $z(\tau) = a[(T/\tau) - 1]$.) The first-order condition for τ is

7. Again, see Karni. If the cost of a trip to the bank took the form of a certain amount of the consumption good and if W were large relative to na, then the usual Baumol-Tobin formula (including wealth) would be approximately correct.

$$-z'(\tau) = iT/2. \tag{10}$$

Since $z''(\cdot) > 0$ by assumption, the second-order condition holds. A simple family of cost functions to consider is the constant elasticity class,

$$z(\tau) = -(T/2)\,[\alpha^{1/\sigma}]\,[\sigma/(\sigma - 1)]\,\tau^{(\sigma-1)/\sigma}, \quad \alpha > 0, \quad \sigma > 0. \tag{11}$$

Here

$$\tau = \alpha i^{-\sigma};$$

the interest elasticity of τ is thus $-\sigma$.

Closing the Model

To close the model, we must specify how i and π are determined, characterize the behavior of banks, and describe equilibrium in the markets for money and goods. For simplicity, I assume that there is a constant returns to scale storage technology with rate of return r_0. Capital can only be held via banks. The competitive banking sector merely purchases and holds capital (on which it receives real return r_0) and issues deposits (on which it pays real return r_0). In what follows, for expositional convenience I often focus especially on the case of r_0 equal to zero, the growth rate of the economy. The model would be essentially unchanged if the alternative asset were bonds rather than capital; I discuss this case below.

The rate of inflation is determined by the rate of money growth. Specifically, since the growth rate of the economy is zero, π equals the rate of money growth. The nominal interest rate is determined by the requirement that the real interest rate equal r_0: $i = r_0 + \pi$.

If the rate of money growth is positive, the government earns seignorage. I assume that this seignorage is rebated to the generation just born; in other words, money is injected into the economy by transfers to newborn individuals. We can find the amount of these transfers, S, in either of two ways. First, S must equal the rate of money growth times real economy-wide money holdings per member of the new generation. Second, by Walras' Law, the money market clears if and only if the goods market does. That is, S must be such that the demand for goods (the consumption of all generations) equals the supply (the endowment of the young plus the return to storage).

I adopt the second method of deriving S. If we normalize the size of each generation to be $1/T$—so that the total population at any moment is one—equilibrium in the goods market requires that

$$\frac{1}{T}\int_{t=0}^{T} C(t)dt = \frac{E}{T} + r_0\frac{1}{T}\int_{t=0}^{T} K(t)dt, \tag{12}$$

where $K(t)$ is capital holdings at age t. Note that since the economy is in steady state and since the rate of population growth is zero, aggregate consumption equals the average consumption of an individual during his lifetime (and similarly for capital holdings).

Expression (6) implies that the contribution of generations in the ages $(j\tau,(j+1)\tau)$ to total consumption is[8]

$$\begin{aligned} C_j &= \frac{1}{T}\int_{t=j\tau}^{(j+1)\tau} we^{rj\tau}e^{-\pi(t-j\tau)}\,dt \\ &= \frac{1}{T}\frac{1-e^{-\pi\tau}}{\pi}e^{rj\tau}\,w. \end{aligned}$$

Thus, total consumption is

$$\begin{aligned} C &= \sum_{j=0}^{n} C_j \\ &= \left[\frac{1-e^{-\pi\tau}}{\pi\tau}\right]\left[\frac{e^{rT}-1}{e^{r\tau}-1}\frac{\tau}{T}\right]w. \end{aligned} \tag{13}$$

Similarly, one can show that aggregate capital holdings are

$$K = \frac{w}{r}\left[\frac{\tau}{T}\frac{e^{rT}-1}{e^{r\tau}-1} - 1\right]. \tag{14}$$

Substituting (13) and (14) into (12) implicitly defines w (and therefore S). If the real interest rate is zero, we obtain

$$w = \frac{E+S}{T} = \frac{\pi\tau}{1-e^{-\pi\tau}}\frac{E}{T}. \tag{15}$$

This completes the model.

8. I ignore the possibility of $\pi = 0$. In general, because of the relevant functions are appropriately continuous, where necessary their values for special cases can be found simply by using l'Hopital's rule.

Extensions and Variations

This basic model can be modified in a variety of ways. Here I consider several possibilities. First, one could assume that the government injects money into the economy not by transferring it to individuals but by using it to purchase goods. The goods could be thought of as being devoted to some public good. The condition for goods market equilibrium is then that the supply of goods equals the consumption of all generations plus the amount purchased by the government. That is, government purchases are determined by

$$G = \frac{E}{T} + r_0 \frac{1}{T} \int_{t=0}^{T} K(t)dt - \frac{1}{T} \int_{t=0}^{T} C(t)dt. \tag{12'}$$

Since there are no lump sum transfers in this version of the economy, W equals E. The remaining equations of the model are unchanged.

Second, government bonds can be introduced into the economy. If bonds and capital are perfect substitutes (that is, if both must be held via banks and involve the same costs of withdrawals) and if we assume that $r_0 = 0$, then the issue of bonds paying the market interest rate involves neither profits nor losses for the government; the description of equilibrium is therefore unaffected by the presence of bonds. Indeed, I could have assumed that storage and capital were absent from the model and that the only asset other than money was government bonds that yielded real return $r_0 = 0$. Of course, if storage is not possible, the government can choose both the rate of inflation and the nominal interest rate; it must then rebate any profits or losses from the issue of bonds. And if storage is possible, the government can offer real return on bonds greater than r_0 and drive capital out of the economy.

Third, and more significantly, the economy considered here is an endowment economy. One can add production by supposing that individuals supply labor that is combined with capital to produce output. The simplest assumption is that labor is supplied only at birth—an extreme version of the assumption that individuals work in the early part of life and are retired in the later part. Labor income then plays the role that the endowment plays in the basic version of the model. Both the real interest rate and the wage are thus endogenous.

Formally, assume that labor supply is fixed as $1/T$ and that production involves constant returns to scale, and define k and y to be capital and output per laborer. We can then write output as

$$y = f(k).$$

I make the usual assumptions that $f'(\cdot) > 0$, $f''(\cdot) < 0$, $f'(0) = \infty$, and $f'(\infty) = 0$. The real interest rate is $r = f'(k)$, and the wage is $E = y - rk$. The capital stock is given by (14) as before, and the goods market equilibrium condition is

$$\frac{1}{T}\int_{t=0}^{T} C(t)dt = \frac{1}{T}y. \tag{16}$$

Combining (16) and (13) yields

$$\left[\frac{1 - e^{-\pi\tau}}{\pi\tau}\right]\left[\frac{e^{rT} - 1}{e^{r\tau} - 1}\frac{\tau}{T}\right][y - rk + S] = \frac{1}{T}y. \tag{17}$$

Equation (17) implicitly defines S. We thus have a version of the model in the spirit of Diamond's [1965] overlapping generations model (see also Cass and Yaari [1967]).

Fourth, the model can accommodate a general time pattern of endowments over individuals' lifetimes without difficulty, as long as the proceeds from selling the endowments are automatically deposited into individuals' bank accounts. Specifically, let $W(t)$ denote the endowment at age t (of both the consumption good and fiat money), and define

$$w^* = \frac{1}{T}\int_{t=0}^{T} W(t)e^{-rt}dt.$$

The description of individual behavior is the same as before, except that w^* replaces w. As long as the pattern of endowments is such that aggregate capital holdings are positive, it can be seen that goods market equilibrium is described by (12) as before. With obvious modifications this framework could be used to introduce a general time pattern of labor supply into the version of the model with production.

Finally, one dimension along which the model cannot easily be modified should be noted. The assumptions about preferences—specifically, the assumptions of logarithmic utility, no discounting, and separability between consumption and trips to the bank—are central to the tractability of the model. In general, with a different utility function, trips to the bank will not be evenly spaced. The equations describing equilibrium would therefore be extremely complicated, and the analysis of those equations essentially impossible. In other words, the result that trips to the bank are evenly spaced is important not in itself but as a tool for making the analysis tractable, and it is a result that is not robust.

II. Applications

Inflation and the Capital Stock

What is the long-run effect of inflation on the capital stock? The model of this paper provides a means of addressing this question for an economy in which money is held for transactions purposes. The usual baseline cases are that if agents have infinite horizons, inflation has no effect on the steady state capital stock [Sidrauski, 1967], and that if they have finite horizons, inflation increases the steady state capital stock [Drazen, 1981]. The natural presumption for this model would thus be that inflation has a positive effect on capital.[9] In what follows, for expositional simplicity I assume that the real interest rate is zero.

Substituting (15) into (14) from Section I shows that if $r_0 = 0$, aggregate capital holdings are

$$K = \left[\frac{1}{2}(T - \tau)\right]\left[\frac{\pi\tau}{1 - e^{-\pi\tau}}\frac{E}{T}\right]. \tag{18}$$

The first term is real capital holdings per unit of initial wealth; the second term is initial wealth. Expression (18) implies that the elasticity of the steady state capital stock with respect to the inflation rate ($= i$) is

$$\frac{\tau}{T - \tau}\sigma + \left[\frac{1 - e^{-\pi\tau} - \pi\tau e^{-\pi\tau}}{1 - e^{-\pi\tau}}\right](1 - \sigma), \tag{19}$$

where σ is (minus) the elasticity of τ with respect to i.

This model has three major implications for the effect of inflation on the steady state capital stock. First, in general the effect of inflation on capital is ambiguous; that is, the Mundell-Tobin effect may be reversed. Second, in some simple cases the Mundell-

9. Jovanovic finds that the rate of inflation affects the steady state capital stock ambiguously in his model, which is one of infinitely-lived agents. Essentially the complicated form of his assumptions about the return to capital makes it possible for the effect of inflation on the capital stock in his model to depart in either direction from the effect in the Sidrauski model.

Tobin effect is present. And third, in any event, plausibly the effect of inflation on the capital stock is very small. I discuss each of these implications in turn.

The first term of (19) is positive: increases in the rate of inflation cause individuals to make trips to the bank more frequently and thus to hold a greater proportion of their wealth in the form of capital. The second term of (19) captures effects operating through the rebating of seignorage. The expression in brackets is positive. It follows that if the interval between withdrawals is sufficiently responsive to the nominal interest rate—in particular, if σ is sufficiently greater than one—an increase in the rate of inflation can reduce the steady state capital stock. The mechanism that can cause this effect is the following. If money demand is sufficiently interest-elastic, increases in the rate of money growth reduce seignorage. If the resulting wealth effect on capital holdings outweighs the substitution effect, the steady state capital stock falls.

The possibility that the Mundell-Tobin effect may be reversed does *not* depend on the assumption that transfers are received at the beginning of life. Consider the opposite extreme of an economy in which individuals receive amount S at the end of life. In other words, individuals deplete their bank balances to $-S$ at age T and then use their transfers to settle their debts to the bank. Thus, for a given τ, the greater S is, the lower capital holdings are at any point in life. It follows that if inflation *increases* seignorage, an increase in inflation can reduce steady state capital holdings (see Drazen). Specifically, if the real interest rate is zero, S is again defined by (15), and $K(t)$ is simply S less than what it is when transfers are received at birth. Expressions (15) and (18) imply that in this case aggregate capital holdings are

$$(18') \qquad K = \left[\frac{\frac{1}{2}(T - \tau)\pi\tau + T(1 - \pi\tau) - Te^{-\pi\tau}}{1 - e^{-\pi\tau}}\right]\frac{E}{T}.$$

I assume that the numerator is positive, so $K > 0$. For a fixed value of τ, K is decreasing in π. Thus, if τ is sufficiently inelastic with respect to i $(= \pi)$, increases in inflation reduce the capital stock. Indeed, if $\sigma < 1$, the Mundell-Tobin effect is reversed if T is sufficiently large relative to τ.

In short, in this extremely simple transactions-based model of money with finitely-lived agents, it is far from clear that the Mundell-Tobin effect will be present. Specifically, the interaction of the effect of inflation on seignorage and the timing of monetary injections is crucial: the Mundell-Tobin effect may be reversed either if transfers take place early in life and seignorage is decreasing in inflation or if transfers take place late in life and seignorage is increasing in inflation.[10]

Despite this ambiguity, there are important cases in which an increase in inflation increases the steady state capital stock. Specifically, there is always a substitution effect at work; for example, if seignorage is not rebated or if the economy is at the seignorage-maximizing rate of inflation, the Mundell-Tobin effect is present. In addition, the wealth effect may reinforce rather than counteract the substitution effect; this occurs if τ is inelastic and seignorage is rebated at the beginning of life or if τ is elastic and seignorage is rebated at the end of life.

Finally, observe that regardless of its sign the effect of inflation on the capital stock is likely to be small. If T is large relative to τ, the first term of (19) is approximately $(\tau/T)\,\sigma$; for $\pi\tau$ small, the second term is approximately $\frac{1}{2}\,\pi\tau(1 - \sigma)$. Realistically, both expressions are likely to be very small.

Money Demand

How does the demand for money depend on the interest rate? Begin by considering money demand between two trips to the bank. Suppose that at age $j\tau$ an individual makes a real withdrawal of amount W_j to make purchases until his next withdrawal at age $(j + 1)\tau$. As shown above, he spends his money at a constant rate in nominal terms. Thus, real money holdings as a function of age are

$$(20)\qquad m(t) = W_j \frac{(j + 1)\tau - t}{\tau} e^{-\pi(t-j\tau)}, \qquad j\tau < t < (j + 1)\tau.$$

Aggregate real money holdings between ages $j\tau$ and $(j + 1)\tau$ are thus

$$m_j = \int_{t=j\tau}^{(j+1)\tau} W_j \frac{(j + 1)\tau - t}{\tau} e^{-\pi(t-j\tau)} dt$$
$$= W_j \frac{e^{-\pi\tau} + \pi\tau - 1}{\pi^2\tau}.$$

10. For other models in which the Mundell-Tobin effect may be reversed, see Stockman [1981] and Gale [1983, pp. 109–118].

Using the fact that $W_j = w\tau e^{rj\tau}$, we may give total real money holdings in the economy by

$$(21) \qquad \begin{aligned} m &= \frac{1}{T}\sum_{j=0}^{n} m_j \\ &= \frac{e^{-\pi\tau} + \pi\tau - 1}{\pi^2\tau^2}\,\frac{e^{rT} - 1}{e^{r\tau} - 1}\,\frac{\tau}{T}\,w\tau. \end{aligned}$$

Finally, if we consider the case $r_0 = 0$, using (15) to substitute for w yields

$$(22) \qquad m = \left[\frac{e^{-\pi\tau} + \pi\tau - 1}{\pi^2\tau^2}\right]\left[\frac{\pi\tau}{1 - e^{-\pi\tau}}\,\frac{E}{T}\right]\tau.$$

For general r_0, E/T is replaced by $C = E/T + r_0K$. Note that, since C equals output, aggregate money demand in this economy can be written as a function of the nominal interest rate (since τ is determined by i), the real interest rate (since $\pi = i - r$), and aggregate output. (Output is in turn an endogenous variable, depending on r, and, if $r \neq 0$, on i.)

More important, the model provides insight into the nature of the money demand function. Expression (22) is the product of three terms. The first term is the ratio of average money holdings during an interval between two trips to the bank to the amount withdrawn at the beginning of the interval. In the Baumol-Tobin case this term is simply one half. The second term is the average amount withdrawn per unit of time during an individual's lifetime; the analogous Baumol-Tobin expression would be Y, the flow rate of expenditures. Here the term reflects the rebating of government seignorage. The final term in (22) is simply the interval between trips to the bank. The corresponding Baumol-Tobin term is $\sqrt{2a'/iY}$, where a' is the monetary cost of a trip to the bank.

In the Baumol-Tobin model the first and second terms are independent of the interest rate, and the third has interest elasticity minus one half. The interest elasticity of money demand is thus always minus one half. Here all three terms, in general, depend on i, and thus the situation is more complicated. Indeed, it is possible for the effects of the first two terms to be such that steady state money demand is *increasing* in the nominal interest rate. One can show that the product of the first two terms is everywhere increasing in $\pi\tau$. It follows that if τ is sufficiently unresponsive to i, then money demand is increasing in i (recall that since we are considering the case $r_0 = 0, i = \pi$). For example, if $\pi\tau = 1$, money demand is increasing in the interest rate if the elasticity of τ with respect to i is less (in absolute value) than approximately 0.12.[11]

In several cases of interest, however, the interest elasticity of money demand converges to the interest elasticity of the interval between trips. As $\pi\tau$ becomes large, the elasticity of the first term with respect to $\pi\tau$ approaches minus one and that of the second term approaches plus one; the overall interest elasticity of money demand therefore converges to the interest elasticity of τ. As $\pi\tau$ approaches zero, the elasticities of each of the first two terms with respect to $\pi\tau$ converge to zero, and the overall interest elasticity again converges to the elasticity of τ. Thus, if costs are linear, in both cases the interest elasticity of money demand converges to its Baumol-Tobin value of minus one half. Perhaps more important, in one realistic case the interest elasticity of money demand is again governed by the behavior of τ. Consider the behavior of money demand as a, the cost of the trips to the bank, approaches zero, for a fixed value of i. τ approaches zero, and so the first term converges to one half, and the second to E/T; money demand is thus simply $\frac{1}{2}(E/T)\,\tau$.

In general, the money demand function in equation (22) shows that changes in the nominal interest rate affect money demand through three channels—the frequency of trips to the bank, the pattern of spending between trips, and wealth effects. The Baumol-Tobin model focuses on the first channel. But in general, this channel may not dominate. The examples above show that the other channels may have important effects on the interest elasticity.[12]

Inflation, the Pattern of Consumption, and the Optimal Rate of Inflation

The model identifies clearly the nature of the distortion caused by inflation: inflation distorts the time pattern of consumption by making the real interest rate between withdrawals differ from the real interest rate across withdrawals. In a nonmonetary version of the model—that is, if money were not needed to purchase goods and all wealth were therefore held as bank deposits—an

11. One can show that if the interest elasticity of τ is minus one half—that is, if costs are linear—m is necessarily decreasing in i.

12. Johnson [1970] shows that wealth effects may influence the interest elasticity of money demand in the Baumol-Tobin model.

individual's consumption would grow smoothly at the real interest rate. In the model with money, assuming a positive nominal interest rate, consumption instead follows a sawtooth pattern: the real interest rate relevant for the pattern of consumption across withdrawals is that on bank deposits, while the real interest rate relevant for the pattern of consumption between withdrawals is that on money. In a partial equilibrium setting—that is, if we neglect the rebating of seignorage—consumption at times of withdrawals is equal to what it would be in the nonmonetary version of the model. Once we account for the rebating of seignorage, we find that consumption follows a sawtooth pattern around the path it would have in the nonmonetary model. Of course, if the time between trips to the bank is small—specifically, if $i\tau$ is small—the path of consumption is close to what it would be in the nonmonetary economy.

The well-known "Optimum Quantity of Money" rule states that the monetary authority should cause deflation at the real interest rate. In the model of this paper this rule would eliminate the sawtooth pattern of consumption. Here I show that if $r_0 = 0$ the welfare-maximizing rate of inflation is zero and thus that the usual result holds in this case.[13]

Substituting the expression for lump sum transfers (15) into the indirect utility function (7) yields the "reduced-form" indirect utility function,

$$(23)\qquad V^0\left(\pi; \frac{E}{T}\right) = T \ln\left[\frac{\pi\tau}{1 - e^{-\pi\tau}}\frac{E}{T}\right] - \frac{T}{2}\pi\tau - z(\tau).$$

Let $V^c(\pi\tau)$ be utility from consumption (that is, utility neglecting the cost of trips to the bank, $z(\tau)$). Differentiating yields

$$(24)\qquad \frac{\partial V^c(\pi\tau)}{\partial \pi\tau} = \frac{2(1 - e^{-\pi\tau}) - \pi\tau(1 + e^{-\pi\tau})}{2\pi\tau(1 - e^{-\pi\tau})}T.$$

One can show that this expression equals zero when $\pi\tau = 0$ and that the second derivative is everywhere negative. Since $\pi\tau = 0$ if and only if $\pi = 0$, utility from consumption attains a global maximum when inflation is zero. In addition, the cost of trips to the bank is minimized at $\pi = 0$. Thus, zero inflation maximizes

13. The issue of optimal monetary policy in an overlapping generations model is considerably more complex when the real return on capital may exceed the growth rate of the economy. See Weiss [1980] and Abel [1984]. Jovanovic discusses optimal monetary policy for an economy similar to that of this paper with infinitely lived agents.

welfare: the government's best policy is to set inflation to zero and to cause all wealth to be held in money. Note that this analysis requires neither the neglect of integer constraints nor an assumption of a particular functional form for the cost function $z(\cdot)$; it is thus quite general.

The Welfare Cost of Inflationary Finance

It is well-known that the "optimum quantity of money" rule may no longer be optimal if the government is attempting to finance a budget without the possibility of lump sum taxes. Here I analyze optimal inflation in a second best setting. As before, I consider only the case of a zero real interest rate; in addition, for simplicity I assume that τ obeys the square root rule. If the government uses its revenue from money creation to purchase goods (in other words, if it injects money by purchasing goods), then W equals E, and (13) implies that government purchases are

$$\begin{aligned} G &= E/T - C \\ &= \frac{E}{T} - \frac{1 - e^{-\pi\tau}}{\pi\tau}\frac{E}{T}. \qquad (25) \\ &= \frac{e^{-\pi\tau} + \pi\tau - 1}{\pi\tau}\frac{E}{T}. \end{aligned}$$

Our interest is in the size of the distortions associated with money finance. Define

$$q = \frac{-V_{\pi}/V_{E/T}}{G_{\pi}},$$

where subscripts denote partial derivatives. q measures the welfare cost of inflationary finance: raising one unit of revenue using money finance reduces welfare, at the margin, by the same amount as reducing w by q. If money finance were perfectly nondistortionary, q would equal one.

Suppose that there is some exogenous shadow value of government revenue (normalized by $V_{E/T}$, the marginal utility of income), denoted λ. In a more general setting in which the government had a variety of distortionary means of raising revenue and in which it was attempting to maximize welfare subject to financing a target level of spending, λ would be the Lagrange multiplier associated with the government purchase constraint. In this case, under fairly general assumptions, λ would be greater than one if government spending were positive and would equal one if spending were zero: a small (infinitesimal) amount of revenue can be raised without distortions.

If government revenue has shadow value λ, the social planner's optimal policy is clearly to pursue money finance to the point where $q = \lambda$. Differentiation of (9) and (25) yields

$$q = \frac{\pi^2\tau^2}{1 - e^{-\pi\tau} - \pi\tau e^{-\pi\tau}}. \tag{26}$$

Using l'Hopital's rule twice establishes that the limit of q as $\pi\tau$ approaches zero (and thus, since $\pi\tau = \sqrt{2a\pi}$, as π approaches zero) is two. Differentiation of (26) shows that q is increasing in $\pi\tau$, and thus increasing in π.

The traditional result is that as long as λ is greater than one—in the more general model, as long as any government revenue is being raised in distortionary ways—then at least some money growth is desirable (see Phelps [1973] and Helpman and Sadka [1979]). This model has a very different implication: as long as the shadow value of government revenue is less than two, a small positive nominal interest rate is strictly undesirable. Inspection of (9), the expression for indirect utility, reveals the source of this finding. Increases in i reduce utility both by making consumption more costly in the intervals between trips to the bank and by increasing the number of trips to the bank. The first effect increases government revenue; the second is deadweight loss. The two are equal in size. Thus, q equals two.[14]

In other words, the mere existence of distortions in raising revenue is not enough to justify a positive rate of inflation (and thus a positive nominal interest rate) in this economy. Instead, a positive interest rate is desirable only if the shadow value of government revenue exceeds a critical value of two. Stuart [1984] and Ballard, Shoven, and Whalley [1985] conclude that for the United States economy λ is in the vicinity of 1.2 to 1.5; a critical value of $\lambda = 2$ is thus quite high.

This analysis treats the number of trips to the bank as a continuous variable. Accounting for integer constraints requires a very slight modification of our result. Individuals must make

14. With the more general cost function for trips to the bank (11), if $\sigma < 1$, then $q = 1/(1 - \sigma) > 1$ at $\pi = 0$. In contrast to both the implication of this model and the traditional result, Drazen [1979] presents a model in which a positive implicit tax on money holdings is undesirable for any level of government spending.

an integral number of trips to the bank; as a result, for sufficiently low but still positive values of i, they make no trips at all. It is therefore possible to use inflationary finance to a *very* small extent without imposing any social costs in the form of trips to the bank. Specifically, one can show that for $1 < \lambda < 2$, the optimal rate of inflation is positive but sufficiently low that consumers do not make even a single trip to the bank during their lives.

Conclusion

The model presented in this paper is a simple general equilibrium model in which money enters in the way that it seems to enter actual economies: money is both a store of value and a medium of exchange, and individuals' money holdings are determined by the tradeoff between the inconvenience of making trips to the bank and the interest forgone by holding money instead of some higher interest asset. By assuming that individuals have finite lifespans and a particular type of preferences, I obtain a general equilibrium model that incorporates endogenous frequency of trips to the bank and that is simple and flexible. The model can easily accommodate, for example, a general time pattern of endowments, bonds and capital, government purchases, and production.

The model is used to analyze the Mundell-Tobin effect, money demand, the effect of inflation on consumption, the optimum quantity of money, and efficient deficit finance. The applications of the model suggest two broad conclusions. The first is that careful consideration of general equilibrium effects and of the implications of endogenous timing of conversions of higher interest assets into money in many cases requires significant modifications to standard conclusions or suggests new possibilities. For example, I find that in this simple economy with a transactions-based model of money and finitely lived individuals, effects operating through the rebating of seignorage and the adjustment of frequency of conversions can cause increases in the rate of inflation to reduce the steady state capital stock. In the case of money demand, the model identifies three channels through which changes in the interest rate affect money holdings: the frequency of conversions, the pattern of spending between conversions, and wealth effects. Conventional partial equilibrium analyses emphasize only the first channel. With regard to efficient government finance, the model implies that the government's desire to finance positive spending without lump sum taxes is not enough to justify a positive rate of money growth for optimal tax reasons; instead there is a threshold level of the shadow value of government revenue at which some money growth becomes desirable.

The second broad conclusion is that as long as the rate of inflation is relatively low and individuals go to the bank relatively frequently, the real long-run effects of inflation are likely to be slight, and conventional shortcuts are likely to yield good approximations. In particular, the capital stock is little different from what it would be in the absence of inflation; the path of consumption departs only slightly from the path it would follow if money were not needed to make purchases; welfare differs little from its non-inflationary level; and money demand is well approximated by conventional partial equilibrium formulas. Thus, although the effects of permanent inflation on the steady state of the economy are genuine, in practice they are small.

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