ECON 714A - Problem Set 3

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This problem asks you to update the CKM (2007) wedge accounting using more recent data. You are encouraged to use Matlab for the computations. Consider a standard RBC model with the CRRA preferences $E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t)$, $U(C, L) = \frac{C^{1-\sigma}-1}{1-\sigma} - \frac{L^{1+\phi}}{1+\phi}$, a Cobb-Douglas production function $Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$, a standard capital law of motion $K_{t+1} = (1-\delta)K_t + I_t$, and four wedges $\tau_t = \{a_t, g_t, \tau_{Lt}, \tau_{It}\}$. Each wedge τ_{it} follows an AR(1) process $\tau_{it} = \rho_i \tau_{it-1} + \varepsilon_{it}$ with innovations ε_{it} potentially correlated across i. One period corresponds to a quarter.

1. Download quarterly data for real seasonally adjusted consumption, employment, and output in the U.S. from 1980–2020 from FRED database. The series for capital are not readily available, but can be constructed using the "perpetual inventory method". To this end, download the series for (real seasonally adjusted) investment from 1950-2020.

Raw Data
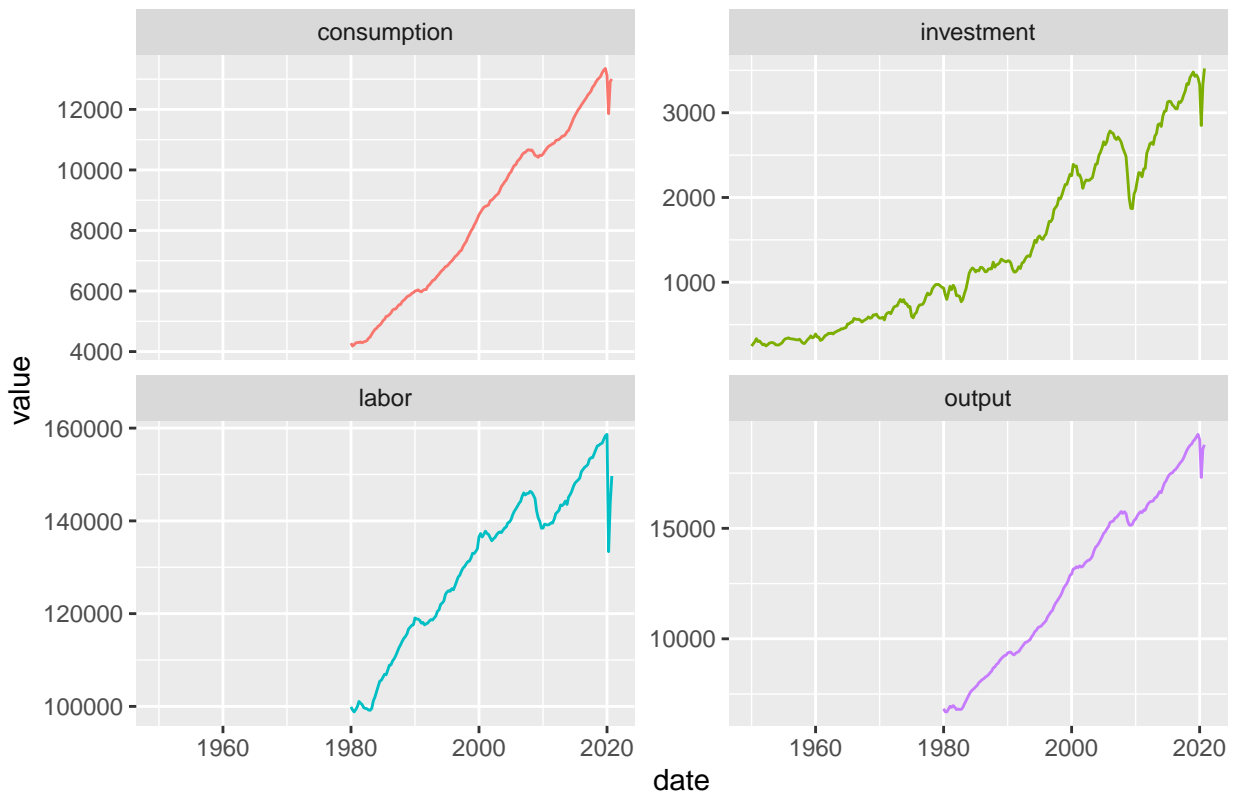
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2. Convert all variables into logs and de-trend using the Hodrick-Prescott filter.

Data after HP Filter (in log deviations)
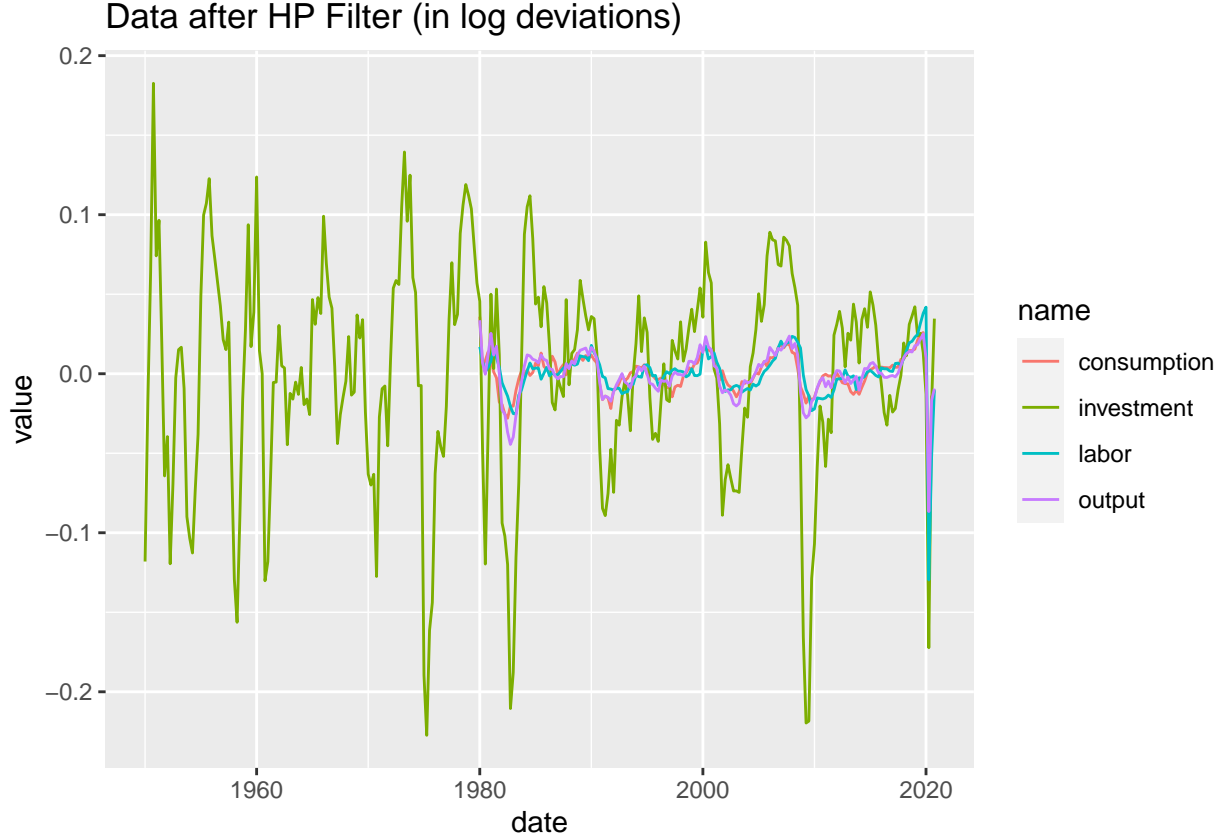
3. Assume that capital was at the steady-state level in 1950 and the rate of depreciation is $\delta = 0.025$ and use the linearized capital law of motion and the series for investment to estimate the capital stock (in log deviations) in 1980-2020. Justify this approach.

Log-linearizing the capital law of motion,

$$
K_{t+1} = (1 - \delta)K_t + I_t
$$
$$
\implies (1 + k_{t+1})\bar{K} = (1 - \delta)(1 + k_t)\bar{K} + \bar{I}(1 + i_t)
$$
$$
\implies \bar{K} + k_{t+1}\bar{K} = (1 - \delta)\bar{K} + (1 - \delta)\bar{K}k_t + \bar{I} + \bar{I}i_t
$$
$$
\implies k_{t+1} = (1 - \delta)k_t + \frac{\bar{I}}{\bar{K}}i_t
$$
$$
\implies k_{t+1} = (1 - \delta)k_t + \delta i_t
$$

If we assume that we're in the steady state in 1950, then $k_t = 0$. Thus, we can interate forward using i_t from the data over the next thirty years until 1980.

Estimated Capital Stock (in log deviations)
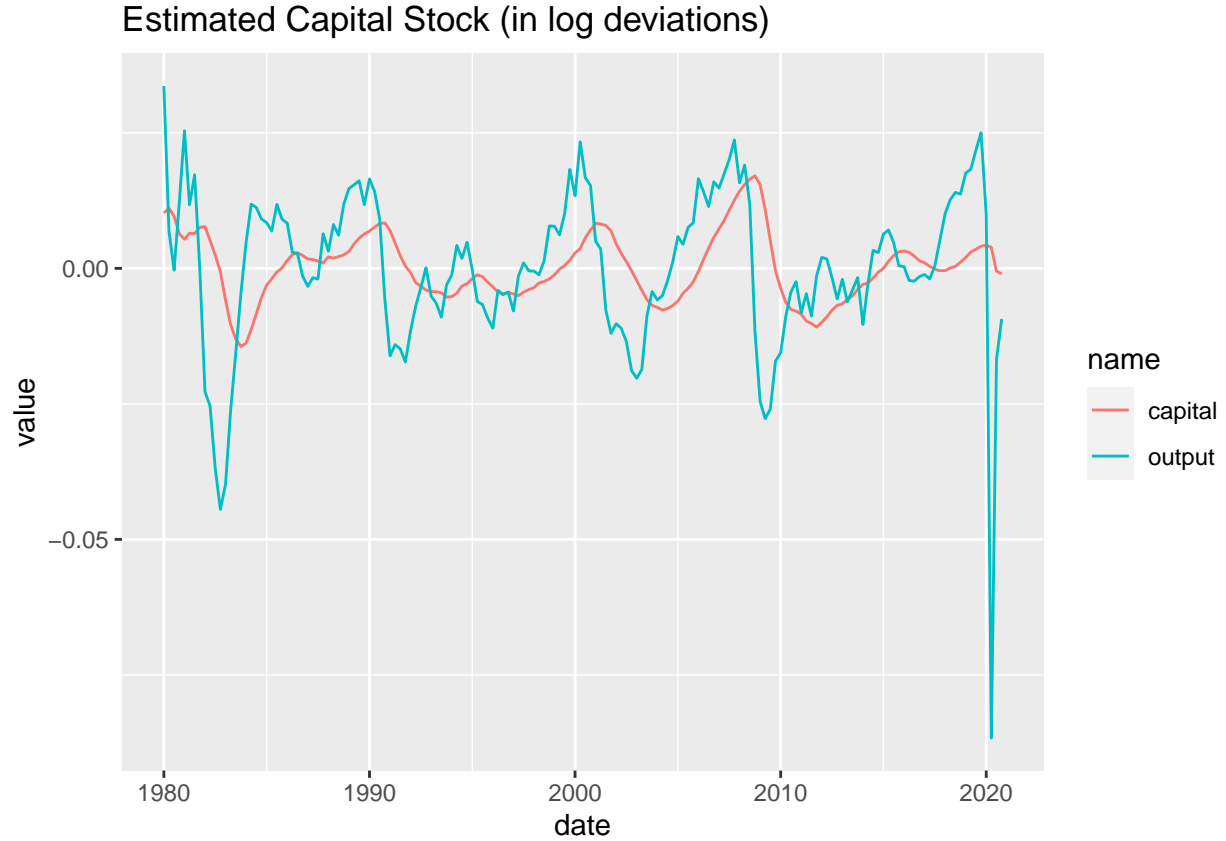
4. Linearize the equilibrium conditions. Assuming $\beta = 0.99$, $\alpha = 1/3$, $\sigma = 1$, $\phi = 1$ and the steady-state share of government spendings in GDP equal $1/3$, estimate a_t, g_t and τ_{Lt} for 1980-2020. Run the OLS regression for each of these wedges to compute their persistence parameters ρ_i.

The equilibrium conditions from the lecture notes are:

$$Y_t = A_t K_t^\alpha L_t^{1-\alpha}$$
$$A_t F(K_t, L_t) = C_t + I_t + G_t$$
$$-\frac{U_{L_t}}{U_{C_t}} = (1 - \tau_{L_t}) A_t F_{L_t}$$
$$U_{C_t}(1 + \tau_{I_t}) = \beta E_t U_{C_{t+1}}(A_{t+1} F_{K_{t+1}} + (1 - \delta)(1 + \tau_{I_{t+1}}))$$

The functional forms from the prompt:

$$U(C_t, L_t) = \ln(C_t) - \frac{L_t^2}{2}$$
$$\implies U_{C_t} = \frac{1}{C_t}$$
$$\implies U_{L_t} = -L_t$$
$$F(K_t, L_t) = K_t^\alpha L_t^{1-\alpha}$$
$$\implies F_{L_t} = (1 - \alpha) K_t^\alpha L_t^{-\alpha}$$
$$\implies F_{K_t} = \alpha K_t^{\alpha-1} L_t^{1-\alpha}$$

3

Thus, the equilibrium conditions become:

$$A_t K_t^\alpha L_t^{1-\alpha} = C_t + I_t + G_t$$
$$C_t L_t = (1 - \tau_{L_t}) A_t (1 - \alpha) K_t^\alpha L_t^{-\alpha}$$
$$\frac{1}{C_t}(1 + \tau_{I_t}) = \beta E_t \frac{1}{C_{t+1}}(A_{t+1}\alpha K_t^{\alpha-1} L_t^{1-\alpha} + (1 - \delta)(1 + \tau_{I_{t+1}}))$$

In the steady state, $K_t = K_{t+1} = \bar{K}$, $C_t = C_{t+1} = \bar{C}$, $L_t = L_{t+1} = \bar{L}$, $I_t = I_{t+1} = \delta\bar{K}$, $G_t = G_{t+1} = \bar{G} = \bar{Y}/3$, and $A_t = A_{t+1} = \bar{A} = 1$.

$$(2/3)\bar{K}^\alpha \bar{L}^{1-\alpha} = \bar{C} + \delta\bar{K}$$
$$\bar{C} = (1 - \bar{\tau}_L)(1 - \alpha)\bar{K}^\alpha \bar{L}^{-1-\alpha}$$
$$1 + \bar{\tau}_I = \beta(\alpha\bar{K}^{\alpha-1}\bar{L}^{1-\alpha} + (1 - \delta)(1 + \bar{\tau}_I))$$

We can solve for the labor steady state:

$$(2/3)\bar{K}^\alpha \bar{L}^{1-\alpha} - \delta\bar{K} = (1 - \bar{\tau}_L)(1 - \alpha)\bar{K}^\alpha \bar{L}^{-1-\alpha}$$
$$\implies \delta = (2/3)\bar{K}^{\alpha-1}\bar{L}^{1-\alpha} - (1 - \bar{\tau}_L)(1 - \alpha)\bar{K}^{\alpha-1}\bar{L}^{-1-\alpha}$$
$$\implies \bar{K} = (\frac{\delta}{(2/3)\bar{L}^{1-\alpha} - (1 - \bar{\tau}_L)(1 - \alpha)\bar{L}^{-1-\alpha}})^{1/(\alpha-1)}$$
$$\implies 1 + \bar{\tau}_I = \beta(\alpha\frac{\delta}{(2/3)L^{1-\alpha} - (1 - \bar{\tau}_L)(1 - \alpha)\bar{L}^{-1-\alpha}}\bar{L}^{1-\alpha} + (1 - \delta)(1 + \bar{\tau}_I))$$
$$\implies (1/\beta - 1 + \delta)(1 + \bar{\tau}_I) = \frac{\alpha\delta}{(2/3) - (1 - \bar{\tau}_L)(1 - \alpha)\bar{L}^{-2}}$$
$$\implies \bar{L} = \sqrt{\frac{(1 - \bar{\tau}_L)(1 - \alpha)}{(2/3) - \frac{\alpha\delta}{(1/\beta-1+\delta)(1+\bar{\tau}_I)}}}$$

The steady state values of $\bar{Y}, \bar{K}, \bar{C}$, and $\bar{G}$ are implied by the equations above. For $\bar{\tau}_L = \bar{\tau}_I = 0.5$, the steady state values are:

variable	value
L_bar	0.810
K_bar	12.899
I_bar	0.322
Y_bar	2.037
C_bar	1.036
G_bar	0.679

4

Now we can turn to the wedges. The efficiency wedge is:

$$Y_t = A_t K_t^\alpha L_t^{1-\alpha}$$
$$\implies y_t = a_t + \alpha k_t + (1-\alpha)l_t$$
$$\implies a_t = y_t - \alpha k_t - (1-\alpha)l_t$$

The government consumption wedge is:

$$Y_t = C_t + I_t + G_t$$
$$\implies \bar{Y}(1+y_t) = \bar{C}(1+c_t) + \bar{I}(1+i_t) + \bar{G}(1+g_t)$$
$$\implies \bar{Y}y_t = \bar{C}c_t + \bar{I}i_t + \bar{G}g_t$$
$$\implies g_t = \frac{\bar{Y}y_t - \bar{C}c_t - \bar{I}i_t}{\bar{G}}$$
$$= \frac{\bar{Y}}{\bar{G}}y_t - \frac{\bar{C}}{\bar{G}}c_t - \frac{\bar{I}}{\bar{G}}i_t$$

The labor wedge is:

$$C_t L_t^{1+\alpha} = (1 - \tau_{L_t})A_t(1-\alpha)K_t^\alpha$$
$$\implies c_t + (1+\alpha)l_t = \hat{\tau_{L_t}}\frac{-\bar{\tau}_L}{1 - \bar{\tau}_L} + a_t + \alpha k_t$$
$$\implies \hat{\tau_{L_t}} = \frac{1 - \bar{\tau}_L}{\bar{\tau}_L}a_t + \alpha\frac{1 - \bar{\tau}_L}{\bar{\tau}_L}k_t - \frac{1 - \bar{\tau}_L}{\bar{\tau}_L}c_t - (1+\alpha)\frac{1 - \bar{\tau}_L}{\bar{\tau}_L}l_t$$

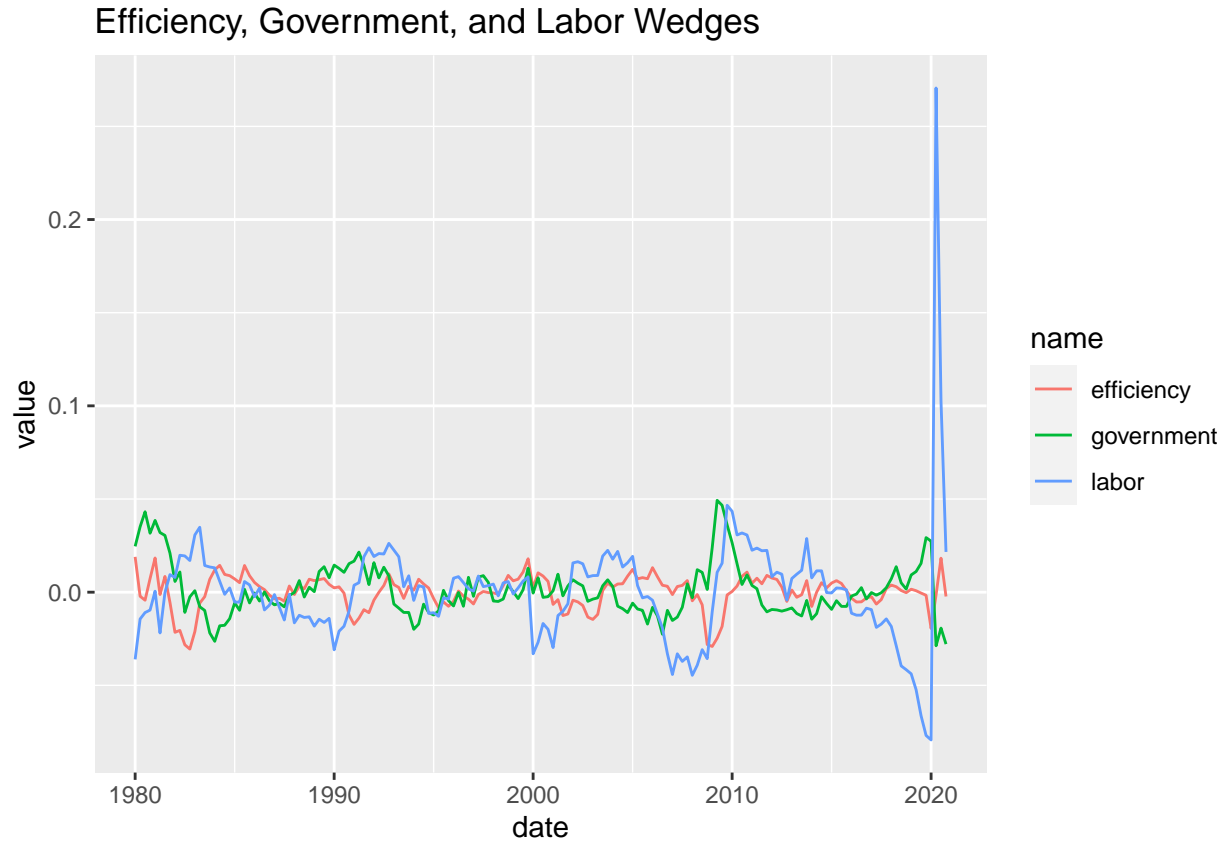## Efficiency, Government, and Labor Wedges

Table 2: Efficiency, Government, and Labor Wedge Persistance (OLS)

	Dependent variable:		
	efficiency	government	labor
	(1)	(2)	(3)
lag(efficiency)	0.711***		
	(0.054)		
lag(government)		0.808***	
		(0.047)	
lag(labor)			0.451***
			(0.070)
Observations	163	163	163
Adjusted R^2	0.517	0.647	0.200
Note:			*p<0.1; **p<0.05; ***p<0.01

5. Write down a code that implements the Blanchard-Kahn method to solve the model. Use the values of parameters, including ρ_a, ρ_g, and ρ_{τ_L}, obtained above, and assume $\rho_{\tau_I} = 0$ for now.

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Appendix

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# PCECC96 Real Personal Consumption Expenditures   Billions of Chained 2012 Dollars,
#                                                   Seasonally Adjusted Annual Rate
# GDPC1    Real Gross Domestic Product              Billions of Chained 2012 Dollars
#                                                   Seasonally Adjusted Annual Rate
# CE16OV   Employment Level                         Thousands of Persons
#                                                   Seasonally Adjusted
# GPDIC1   Real Gross Private Domestic Investment   Billions of Chained 2012 Dollars
#                                                   Seasonally Adjusted Annual Rate
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