Homework 3: Bewley Model with Labor Search

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In this homework you will solve a a Bewley style model of incomplete markets where agents engage in labor market search.

1 Data Assignment

- Using your PSID data, run a series of distributed lag regressions to examine how job loss impacts earnings.
  - There are other variables you can merge into your PSID data set to ID layoffs, but lets do something simple to get started. Define an individual to have been laid off if they worked full-time for at least three consecutive years and then had a decline in annual hours of at least 25 percent in the fourth year. To avoid other shocks, lets not include individuals who saw an increase in the number of household members (e.g., they had a child). Define the control group in an analogous way expect in the fourth year the individual did not have a 25 percent decline in hours. Using these definitions of a treatment and control group, estimate the impact of job loss on earnings. How do your estimates compare to the ones reported in Davis and von Wacther or Huckfeldt? Why do you think they may differ?

2 Model Overview

Time is discrete and runs forever. There is a unit measure of individuals, a continuum of potential risk-neutral lenders, and a continuum of potential entrant firms. There are $T \geq 2$ overlapping generations of risk averse individuals that face idiosyncratic risk, similar to Menzio, Telyukova, and Visschers [2012]. Each individual lives T periods. Let β be an individual's discount factor.

In each period, individuals direct their search for jobs (e.g. Moen [1997], Burdett et al. [2001], and Menzio and Shi [2011]). Individuals then participate in an asset market where they make asset accumulation, and borrowing/savings decisions. Let t denote age and t_0 denote birth cohort. The objective of an individual is to maximize the present discounted value of utility over non-durable consumption ($c_{i,t,t+t_0}$):

$$\mathbb{E}_{t_0}\left[\sum_{t=1}^{T} \beta_i^t u(c_{i,t,t+t_0})\right]$$

For the remainder of this note we focus on a recursive representation of the problem, dropping the time subscript $t + t_0$.

Individuals are heterogeneous along multiple dimensions. Individuals are either employed, or unemployed, with employed value functions denoted W, unemployed value functions denoted U. Let $e \in \{W, U\}$ denote employment status. Let $b \in \mathcal{B} \equiv [\underline{B}, \bar{B}] \subset \mathbb{R}$ denote the net asset position of the individual, where $b > 0$ indicates saving and $b < 0$ indicates borrowing. Let $\vec{h} \in \mathcal{H} \equiv [\underline{h}, \bar{h}]$ representing an individual's human capital. Human capital follows a Markov chain which depends on an individual's employment status, and it is calibrated to match earnings changes of the employed, as well as earnings losses following job loss. Workers differ with respect to their piece-rate $\omega \in [0, 1]$ which denotes the share of their per-period match output that they receive as a wage.

Labor Market Unemployed individuals direct their search for employment across vacancies which specify a fixed piece rate ω for the duration of the employment match. Let $M(u, v)$ denote the labor market matching function, and define labor market tightness to be the ratio of vacancies (v) to unemployed workers (u). Since search is directed, there is a separate labor market tightness for each submarket, defined by an agent's age (t), requested piece-rate (ω), and human capital ($\vec{h}$). In each submarket, the job finding rate for individuals, $p(\cdot)$, is a function of labor market tightness $\theta_t(\omega, \vec{h})$, such that $p(\theta_t(\omega, \vec{h})) = \frac{M(u_t(\omega,\vec{h}),v_t(\omega,\vec{h}))}{u_t(\omega,\vec{h})}$. On the other side of the market, the hiring rate for firms $p_f(\cdot)$ is also a function of labor market tightness and is given by $p_f(\theta_t(\omega, \vec{h})) = \frac{M(u_t(\omega,\vec{h}),v_t(\omega,\vec{h}))}{v_t(\omega,\vec{h})}$. Once matched with a firm, a worker produces $f(\vec{h}) : \mathcal{H} \to \mathbb{R}_+$ and keeps a share ω of this production as their wage. Matches end exogenously each period with probability δ. It is important to note that because we model piece-rate contracts, workers' wages grow over time with their human capital. This generates a motive for employed workers to borrow against future income.

Model Timing The timing is such that at the start of the period unemployment shocks are realized. Unemployed individuals then enter the labor market and apply for jobs. After the labor market closes individuals make borrowing, saving, and consumption decisions. Idiosyncratic human capital risk is then realized, and the next period begins.

In the section below, we present the Bellman equations that govern the behavior of agents in the economy.

2.1 Bellman Equations

Unemployed workers

Let $U_t(b, \vec{h})$ denote the value of entering the consumption-saving stage for an unemployed, age t individual with net worth b, and human capital $\vec{h}$. The continuation value of an unemployed agent is,

$$U_t(b, \vec{h}) = \max_{b' \geq \underline{b}} u(c) + \beta_i \mathbb{E}\left[\max_{\tilde{\omega}} p(\theta_{t+1}(\tilde{\omega}, \vec{h}'))W_{t+1}(\tilde{\omega}, b', \vec{h}') + \left(1 - p(\theta_{t+1}(\tilde{\omega}, \vec{h}'))\right) U_{t+1}(b', \vec{h}')\right] \;\forall t \leq T$$

$$U_{T+1}(b, \vec{h}) = 0$$

subject to the budget constraint,

$$c + q(b')b' \leq z + b$$

and the law of motion for human capital, which is indexed by employment status U,

$$\vec{h}' = H(\vec{h}, U) \tag{1}$$

For ease of exposition, lets assume the bond price is such that agents can consume and borrow at the risk-free rate r_f,

$$q(b') = \frac{1}{1 + r_f}$$

Employed workers

Agents without credit. Let $W_t(\omega, b, \vec{h})$ denote the value of entering the consumption-savings stage for an employed worker without credit access. For an agent that did not not receive a credit contract, their consumption and savings problem is constrained in that the agent is not allowed to borrow. At the start of the next period with probability δ the agent looses their job, and is immediately able to search for a job.[1] The value function summarizing the payoffs of an employed agent without credit access is,

$$W_t(\omega, b, \vec{h}) = \max_{b' \geq 0} u(c) + \beta_i \mathbb{E}\left[(1 - \delta)W_{t+1}(\omega, b', \vec{h}') + \delta U_{t+1}(b', \vec{h}')\right] \quad \forall t \leq T$$

$$W_{i,T+1}(\omega, b, \vec{h}) = 0$$

subject to the budget constraint,

[1]Given the model period is 1 quarter we must allow individuals to search immediately in order for the model to match labor flows in the data.

$$c + q(b^{'})b^{'} \leq (1-\tau)\omega f(\vec{h}) + b$$

and law of motion for employed individuals' human capital,

$$\vec{h}^{'} = H(\vec{h}, W) \tag{2}$$

As before, lets assume the bond price is such that agents can consume and borrow at the risk-free rate r_f,

$$q(b^{'}) = \frac{1}{1+r_f}$$

2.2 Firms

Firms are assumed to have access to a linear production technology, and to have an exogenous job destruction rate δ. Firms use the discount factor β_f. The continuation value of a firm that has committed to pay piece rate ω to their age t employee with human capital $\vec{h}$ is

$$\begin{aligned} J_t(\omega, \vec{h}) &= (1-\omega)f(\vec{h}) + \beta_f \mathbb{E}\left[(1-\delta)J_{t+1}(\omega, \vec{h}^{'})\right] \quad \forall t \leq T \\ J_{T+1}(\omega, \vec{h}) &= 0 \end{aligned}$$

subject to the law of motion for human capital for employed individuals,

$$\vec{h}^{'} = H(\vec{h}, W)$$

Firms must pay cost κ to post a vacancy. A vacancy specifies a wage piece rate ω, as well as a human capital requirement $\vec{h}$, and age t. Free-entry requires that:

$$\kappa \geq p_f\left(\theta_t(\omega, \vec{h})\right) J_t(\omega, \vec{h}) \tag{3}$$

The free entry condition binds for all submarkets such that $\theta_t(\omega, \vec{h}) > 0$.

2.3 Government

- For now les not worry about closing the economy. Set the marginal tax rate to 20%.

3 Assignment

To complete the assignment, please do the following:

1. Write down the definition of equilibrium for this economy.

2. Prove that the equilibrium is *Block Recusrive.*
   (a) For this proof assume you have a given tax rate τ.
3. Solve the model above using the suggested parameters below and reprot the following:
   (a) Make a histogram of the distribution of assets from your simulated data.
   (b) Make a histogram of the distribution of wages from your simulated data.
   (c) What is the unemployment rate in your economy?
   (d) Plot average earnings and assets over the life-cycle from your model. How do the earnings data compare to your age earnings profile from the PSID?
   (e) In your simulated data, what is the average gain in earnings when employed? How does this compare to your data estimate from last week's assignment?
   (f) In your simulated data, make a graph of earnings around job loss (following an individual for 4 quarters before job loss to 8 quarters after job loss). How does the graph compare to the estimates from Davis and von Wacther and your estimates from part (1) of the assignment?
   (g) In your simulated data, make a graph of consumption around job loss.
   (h) Increase the transfer to the unemployed by 10%. How do the above graphs of earnings and consumption around job loss change? What happens to the unemployment rate in your economy?

4 Calibration

- The period is one quarter. We set the annualized risk free rate to 4%, and the corresponding quarterly discount factor for firms and lenders is $\beta_{lf} = 0.99$. Set $\beta = .99$
- Set the job destruction rate to a constant 10% per quarter, $\delta = 0.1$ (Shimer [2005]). For the labor market matching function, use a constant returns to scale matching function that yields well-defined job finding probabilities:

$$M(u, v) = \frac{u \cdot v}{(u^\zeta + v^\zeta)^{1/\zeta}} \in [0, 1)$$

  The matching elasticity parameter is chosen to be $\zeta = 1.6$ as measured in Schaal [2012].
- Set the the vacancy posting cost $\kappa = .995$.
- Set $z = 0.4$

- Human capital evolves following a Markov chain . Assume the production function is linear in the human capital of the worker, $f(\vec{h}) = \tilde{h}$. The process for human capital is governed by two parameters $p_{\tilde{h},L}$ and $p_{\tilde{h},H}$.

$$H(\vec{h}, U) = \tilde{h}' = \begin{cases} \tilde{h} - \Delta & \text{w/ pr. } p_{\tilde{h},L} \text{ if unemployed} \\ \tilde{h} & \text{w/ pr. } 1 - p_{\tilde{h},L} \text{ if unemployed} \end{cases}$$

$$H(\vec{h}, W) = \tilde{h}' = \begin{cases} \tilde{h} + \Delta & \text{w/ pr. } p_{\tilde{h},H} \text{ if employed} \\ \tilde{h} & \text{w/ pr. } 1 - p_{\tilde{h},H} \text{ if employed} \end{cases}$$

Set $p_{\tilde{h},L} = 0.5$ and $p_{\tilde{h},H} = 0.05$.

- A worker's life span is set to $T = 120$ quarters (30 years). Newly born individuals enter as unemployed workers, with zero assets. Set their initial human capital to the lowest point on the human capital grid.

- Individual preferences over non-durable consumption are given by:

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}$$

Set the risk aversion parameter to a standard value, $\sigma = 2$.

5 Suggested solution algorithm

Solving the model proceeds in the following steps:

1. **Taxes:** Guess τ.

2. **Firms Bellman:** Compute the value to a firm of being in a match in the terminal period $J_T(\omega, h)$. Using the value of a firm in the terminal period, invert the free entry condition to obtain labor market tightness $\theta_T(\omega, h)$.

3. **Individual Problem:** Solve the individual problem in the terminal period.

   (a) Compute the value to the individual of being employed and unemployed, $W_T(\omega, b, \vec{h})$, $U_T(b, \vec{h})$ respectively.

4. **Individual's Job Search:** Use the estimate of $\theta_T(\omega, h)$ and to $W_T(\omega, b, \vec{h})$, $U_T(b, \vec{h})$ solve the individual's job search problem.

5. **Repeat for ages** $T - 1$, $T - 2, ..., 1$.

6. **Budget Balance:** Simulate a mass of individuals and check that the government's budget constraint is satisfied. Update guess of τ until the government budget is balanced.

Some additional suggestions:

- Solve the model on grids, like we discussed for the first homework. Choice variables go down the rows $(\omega_1 b_1, \omega_1 b_2 ..., \omega_1 b_N, \omega_2 b_1 ..., \omega_M b_N)'$ and variables subject to a stochastic process are along the columsn $(h_1, h_2, ..., h_P)$.
- I would make a grid of human capitals between 0.5 and 1.5 with even spacing.
- Set a grid of wage piece rates between 0 and 1 with even spacing (typically agents search in the top part of the grid, so you could make the minimum grid point higher).

References

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