Problem 1: GMM Estimation of a Linear Regression Model

1. Write a code to implement a GMM estimation of a linear regression model, $Y_t = \beta' X_t + u_t$. The code should produce the point estimates and the Newey-West standard errors of β and the regression R^2. We will use this code in later assignments to evaluate statistical significance of predictability evidence.

Problem 2: Bayesian Estimation of an Autoregressive Model

Consider an AR(1) model for $y^T = \{y_t\}_{t=1}^T$:

$$y_{t+1} = \mu + \rho y_t + \sigma \epsilon_{t+1},$$

where $\epsilon \sim$ i.i.d. $\mathcal{N}(0,1)$.

1. Consider independent conjugate priors for the model parameters,

$$\mu \sim \mathcal{N}(m, s^2), \quad \rho \sim \mathcal{N}(\tilde{\rho}, \omega^2), \quad \sigma^2 \sim \mathcal{IG}(\alpha/2, \beta/2).$$

Show that the conditional posteriors are given by,

$$\mu|y^T, \rho, \sigma \sim \mathcal{N}, \quad \rho|y^T, \mu, \sigma \sim \mathcal{N}, \quad \sigma^2|y^T, \mu, \rho \sim \mathcal{IG}.$$

Find the parameters of the posterior distributions in terms of the parameters of the prior and the data.

2. The Excel file Longyielddata.xls contains the annual data for long-term 10-year US government bond yields from Global Financial Data (GFD) database.

Chose the priors for the model parameters. For example, we can use fairly uninformative priors:

	Prior Mean	Prior Std. Dev.
μ	0.3	0.5
ρ	0.95	0.2
σ^2	1	1

Design and implement an MCMC algorithm to draw a long sample from the joint posterior distribution of the three parameters, where the sampling of each parameter is implemented via Gibbs sampler. (I don't think Matlab provides a direct way to draw from an Inverse Gamma distribution. However, *gamrnd* allows you to generate a random variable from a Gamma distribution, and there is a direct relation between Gamma and Inverse Gamma random variables.)

3. Run a long MCMC chain and discard appropriate number of initial draws due to burn-in. Plot the three chains. Do the chains mix well? Does it look like they come from a stationary distribution? How large is the persistence in the chains? Plot the autocorrelation plots for the chains, and the scatter plots of each parameter against the others. You might find that the most problematic is the persistence and cross-correlation between μ and ρ, and there's a good reason for it. Notice that μ is an intercept, and not the unconditional mean of y_t : $E(y) = \mu/(1 - \rho)$. So every time we change ρ, intuitively, the draw for μ has to adjust to target the unconditional mean of the process. How would you change the problem to break up this almost mechanical correlation between the parameters?

4. Report the posterior means and standard deviations of the parameters — how different the posterior means are from the prior, and from the standard OLS estimates in the data? How do the prior and posterior standard deviations compare? Overall, do you think your results look reasonable?

5. If you carefully examine your chain for ρ, you might find that some draws are above 1. This does not look reasonable, as we have good reasons to believe that interest rates are stationary. Let's correct that. First, let's use a truncated Normal prior for ρ to ensure that this parameter is always in $(-1, 1)$:

$$f(\rho) \propto \mathcal{N}(\tilde{\rho}, \omega^2) \text{ if } \rho \in (-1, 1); \quad f(\rho) = 0 \text{ otherwise.}$$

Further, instead of using Gibbs to draw ρ, now let's use Independence Metropolis-Hastings where the proposal density is equal to the Normal conditional posterior of ρ you found in part 1). Design and implement the new MCMC algorithm. Notice that a new algorithm (known as a Rejection Sampling) should be a very simple and intuitive modification of the previous one.

6. Examine the chain, report the posterior means and standard deviations. Overall, do you think your results look reasonable?

7. Having simulation output from the MCMC chains makes it easy to compute, and assess significance of, any complicated non-linear functions of the parameters and the data. Suppose we are interested in $N = 5$ year forecast of yields from the model. Show that the forecast is given by,

$$E_t y_{t+N} = \mu \frac{1 - \rho^N}{1 - \rho} + \rho^N y_t.$$

Fix t. You can compute the implied forecast $(E_t y_{t+N})^i$ for each parameter draw from the chain θ^i, for i from 1 to M. Having the distribution of time-t forecasts $\left\{ (E_t y_{t+N})^i \right\}_{i=1}^{M}$, you can numerically compute their mean, and $2.5\% - 97.5\%$ confidence band. Now do it for all t, and plot the posterior mean and the confidence band for the yield forecasts from the model.

Problem 3: Latent Drift Model

Consider the following specification for consumption dynamics:

$$\Delta c_{t+1} = \mu + x_t + \sigma_c \eta_{t+1},$$
$$x_{t+1} = \rho x_t + \sigma_x e_{t+1},$$

where η and e are independent (over time and from each other) shocks with mean zero and variance one.

1. We are interested in estimating the parameters of the model: $\mu, \sigma_c, \rho, \sigma_x$. Consider the following four moments: $E(\Delta c_t), Var(\Delta c_t), Cov(\Delta c_t, \Delta c_{t-1}), Cov(\Delta c_t, \Delta c_{t-2})$. Show that these four moments exactly identify the four unknown parameters. Describe how you would design and implement a GMM estimation of the model parameters based on these four moments. How would you infer the unobserved x_t based on these estimates?

2. The shocks η and e are assumed to be independent from each other. Can we estimate the correlation between the shocks in the data? If so, show what data moments would identify it.