Mitigating Disaster Risks in the Age of Climate Change*

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Abstract

Emissions control cannot address the consequences of global warming for weather disasters until decades later. We model regional-level mitigation, which reduces aggregate disaster risks to capital stock in the interim. Unexpected disaster arrivals increase belief regarding the adverse consequences of global warming and mitigation spending. Competitive markets underprovide such spending because of externalities. Capital taxes to fund mitigation restores first-best. We calibrate our model for seawalls that protect against Atlantic hurricanes. The optimal annual seawall tax is 1.3% of housing stock value. Welfare is 25% higher and coastal property prices are only 5% lower in the first-best compared to the competitive equilibrium because mitigation reduces aggregate risks.

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1 Introduction

Global costs of weather-related disasters have increased sharply in recent decades (see, e.g., Bouwer et al. (2007)). While this trend increase is partly due to economic growth and exposure of physical capital (Pielke et al. (2008), Bouwer (2011), Jongman et al. (2012)), recent climate research is increasingly confident in linking climate change to more frequent or severe natural disasters (National Academy of Sciences (2016)). For instance, climate models point to increased frequency and damage from hurricanes that make landfall (Grinsted, Ditlevsen, and Christensen (2019), Kossin et.al. (2020)). Similarly, the wildfires in the Western US states are also linked to climate change (Abatzoglou and Williams (2016)). Emissions control and carbon taxes, which have been the main focus of research using integrated assessment models (Nordhaus (2017)), will only impact such losses decades down the road to the extent they are even implemented globally.

At the same time, willingness to pay to avoid weather disasters are likely to be large given household risk preferences and permanence of such shocks (Pindyck and Wang (2013)). Hence, mitigation of natural disaster risks at the regional level, be it seawalls or land-use regulation, may need to play a major role going forward. But it has thus far been relatively under-emphasized both in climate change research and practice (Bouwer et al. (2007)). Among key questions are what determines mitigation, how valuable is it for social welfare, and what are the tax and asset pricing implications?

To answer these questions, we start by introducing costly mitigation into a continuous-time stochastic general-equilibrium model with disasters along the lines emphasized by Rietz (1988), Barro (2006), and Weitzman (2009). Disaster arrivals follow a Poisson process. Damages conditional on arrival are modeled as downward jumps in the capital stock.[1] The percentage losses of capital stock due to jump arrivals follow a Pareto distribution and are i.i.d. across arrivals (Gabaix, 2009). But spending today that comes at the cost of consumption and/or investment mitigates the fat-tailedness of damages in the sense of first-order

[1]For instance, the literature on weather disasters points to persistent declines in growth and productivity due to destruction of physical capital (Dell, Jones and Olken (2014)). Of course, weather disasters are related to extreme temperature and precipitation (Auffhammer, Hsiang, Schlenker, and Sobel (2013)).

stochastic dominance. Mitigation in our model is qualitatively in line with existing work on the value of protective investments like seawalls or locating assets away from hurricane or wildfire paths (Kousky et al. (2006), Schumacher and Strobl (2011), Hallegate (2017)). Our model of disasters and mitigation technology contribute to the literature in a number of dimensions as we detail below, particularly when it comes to quantitative calculations.

A defining aspect of costly mitigation in the age of climate change is that it depends on households learning about the consequences of global warming for disasters based on past arrivals. Each new disaster brings additional evidence that will result in belief updating regarding the consequences. This aspect is important for not only normative calculations since mitigation strategies as we will show crucially depend on perceived risks. For instance, scientific consensus on the impact of global warming on the frequency of hurricanes changed markedly in 2005, when a record number of hurricanes including Katrina made landfall (Emanuel (2005)). It also has positive predictions since recent weather disasters have moved public opinion on the consequences of climate change (see, e.g., Yale Climate Opinion Maps (2020)).

Hence, our model features households learning from natural disaster arrivals about whether Poisson arrival rates are high or low (i.e., what we refer to as a bad versus good state). The bad state corresponds to more frequent arrival rates due to global warming, while the good state corresponds to no or mild effects of climate change. Unexpected arrival of a disaster leads to a jump in belief in the bad state (i.e. perceived risk). Absent any arrivals, this belief drifts down toward the good state (i.e., no news is good news when it comes to no arrival of disasters). Such a model is consistent with uncertainty regarding hurricane arrival rates (Nordhaus (2010)) that will be resolved over time and the importance of modeling uncertainty of climate models more generally (Barnett, Brock and Hansen (2020)). An important feature of our learning model is that "bad" news leads to abrupt and discontinuous change of belief, as a disaster arrival is a discrete event also serving as a discrete signal.[2]

[2]Our model generates time-varying disaster arrival rates via learning. Learning (Colin-Dufresne, Johannes, and Lochstoer (2016)) and disasters with time-varying arrival rates (as in Gabaix (2012), Gourio (2012), and Wachter (2013)) have been shown to be quantitively important to simultaneously explain business cycles and asset price fluctuations.

Output is determined by an AK growth function augmented with capital adjustment costs.[3] Output can include housing services and capital stock is composed of both physical and housing capital. Households are endowed with the widely-used non-expected utility proposed by Epstein and Zin (1989) and Weil (1990), which separates risk aversion from the elasticity of intertemporal substitution. Recent work in the context of valuing emissions curtailment points to the importance of using such risk preferences in generating a high social cost of carbon.[4] It will similarly play an important role in generating a high willingness-to-pay for mitigation that depends on perceived risks. There are convex adjustment costs to capital that make capital stock illiquid and hence give rise to rents for installed capital and the value of capital (Tobin's average q) fluctuates as households' beliefs about the disaster likelihood change over time.

Despite the novelty of introducing both belief updating and mitigation technology, our model is tractable. The planner's solution is characterized by an endogenously derived non-linear ordinary differential equation for the value function (the certainty equivalent wealth) together with first-order conditions for investment and mitigation spending that depend on household belief regarding disaster arrivals. The boundary conditions are given by solutions when the household belief is permanently in the low or high arrival state.[5]

Our model emphasizes mitigation externalities. Because mitigation changes the distribution of damages conditional on arrival, which benefits all firms and households, aggregate risk mitigation cannot be decentralized due to the positive externalities of mitigation. We show households and firms optimally choose no mitigation in a competitive equilibrium. Even though there are complete markets, the competitive economy has an extreme form of underspending on mitigation and over-investment in capital from the societal perspective

[3] There are pros and cons of using an AK model for our climate-change analysis. For analyzing weather disasters such as hurricanes which have been shown to have permanent effects on capital and output (Hsiang and Jina (2014)), an AK model setup is natural. But an AK setup might miss important features of growth rate dynamics in other settings (Jones (1995)).

[4] See, e.g., Jensen and Traeger (2014), Hambel, Kraft and Schwartz (2018), Cai and Lontzek (2019), Daniel, Litterman and Wagner (2019), and Barnett, Brock and Hansen (2020) for recent contributions.

[5] The solutions for the two special cases (at the boundaries) generalize the model in Pindyck and Wang (2013), which originally examined the general-equilibrium effects of disasters in a continuous-time production model with Poisson arrivals of disasters, by allowing for mitigation.

since firms do not internalize the benefits of aggregate risk mitigation.

Taxing capital effectively lowers the firm's marginal product of capital thereby addressing its over-investment motive, which in turn lowers the firm's average q in equilibrium. By using the tax proceeds and fully reimbursing the firm for its mitigation spending, the first-best solution can be achieved while still maintaining a balanced budget. This is similar to optimal Pigouvian taxes to address negative externalities of carbon emissions for climate change (Golosov, Hassler, Krusell and Tsyvinski (2014)).

Our model can be applied to different weather disasters. We use it here to value mitigation to reduce the risks to housing capital stock of more frequent hurricanes in the US Atlantic region due to global warming. Mitigation proposals typically include erecting seawalls (and improving drainage) to guard against storm surges and excess precipitation that come with hurricanes that hit landfall. The dangers of rising-sea levels such as flooding or beach erosion are also likely to happen during a storm (Kirezci et al. (2020)). While our planner's solution can be applied to even localities, such as New York City that can afford to fund its seawall, the high fixed costs of these measures likely require coordination of local and federal authorities over a larger area — which we take to be the Atlantic region.

Historically, Atlantic states are exposed to between one to two major landfall hurricanes per year, which we take as the good state. We calibrate the power law governing losses to target a historical conditional loss of around 0.40% of Atlantic coastal property (Nordhaus (2010)). The bad state would correspond to 10 hurricanes that make landfall each year. Perceived risk is then captured by the belief that households assign to the likelihood that hurricanes might be more frequent in the future than in historical samples.

We focus our discussions below on household preferences with elasticity of intertemporal substitution of 1.1, i.e. larger than one so as to be consistent with following the literature on long-run risks (Bansal and Yaron (2004)). Otherwise, the other parameter values—including risk aversion and the rate of time preferences, productivity, and asset market return and volatility—are set to target various moments regarding housing stock returns. Holding fixed these parameters, we introduce a mitigation technology such that society would spend zero in the good state.

Our calibration has implications for three sets of questions in the literature. The first set of questions is on the costs and benefits of mitigating tropical storms. Existing estimates are based on reduced-form regressions of tropical storms on GDP and the counterfactual of mitigation is with respect to controlling CO_2 emissions to reduce temperatures which would impact hurricane intensities (Nordhaus (2010)). Using such an approach, the present value of tropical cyclone damage globally absent mitigation is estimated to be 10 trillion dollars (Hsiang and Jina (2014)). In contrast, our approach calibrates a structural model of disasters and growth and considers mitigation in the form of seawalls as opposed to CO_2 emissions per se.

In the competitive equilibrium without mitigation, society would experience a substantial welfare loss as pessimism rises. For moderately pessimistic beliefs of $\pi = 0.5$ and absent mitigation, the welfare loss measured in terms of certainty equivalent wealth is nearly 40% of the level households enjoy when $\pi = 0$. Welfare loss is highly non-linear in perceived risks. This non-linearity arises from risk preferences and disasters and is fundamental to an accurate cost and benefit analysis. Investment in the competitive equilibrium is not substantially impacted since there is no mitigation and Tobin's q does not decline much as a result.

In the social planner's solution, mitigation spending as a fraction of capital stock even at moderately pessimistic beliefs (i.e. $\pi = 0.5$) reaches 1.3% compared to extremely pessimistic beliefs in the bad state, when it is only 1.65%. With the optimal use of mitigation technology, the welfare loss (compared to $\pi = 0$) is only 20% as opposed to 40% in the competitive equilibrium. The difference of 25% =1-(1-40%)/(1-20%) is then the value of mitigation technology.[6] Investment is lower as a result, falling from 2.9% when $\pi = 0$ to 2.5% when $\pi = 0.5$. Similarly, Tobin's q drops from 2 to 1.863 as we move from $\pi = 0$ to $\pi = 0.5$, i.e. Tobin's average q for housing capital stock would be around 7% lower. We demonstrate that our calibration can be used to assess the costs and benefits of various seawall proposals

[6] The caveat to these calculations is that traditional willingness-to-pay calculations to avoid disasters as in our model is sensitive to modeling of multiple disasters and when disasters affect both consumption and loss of life (Martin and Pindyck (2015)).

around the world, including the \$690 billion project for Atlantic coastal property.

The second set of questions have to do with hurricane or disaster damage functions. Connected to the improved welfare from mitigation is that the conditional damage of a hurricane is far lower and the expected growth rate is higher. Without mitigation and with moderately pessimistic beliefs in the bad state, expected growth rate is close to zero. In contrast, it is around 1.65% with mitigation. Our model's implication is consistent with empirical work in the literature: countries that mitigate experience less damage per disaster arrival (see, e.g., Kahn (2005)). While existing work focuses on cross-country variation in mitigation, our model generates new time-series predictions. For instance, damages conditional on arrival are higher when an economy has few prior arrivals and long inter-arrival times since perceived risks and mitigation spending or preparedness are low as a result.

The third set of questions is on the quantitative implications of our model for tax policy and housing prices. A growing literature attempts to measure the impact of hurricanes and sea-level rise on coastal property prices. The results have been mixed — with literally no effects (Murfin and Spiegel (2020)) to modest negative price effects (Bernstein, Gustafon and Lewis (2019)). Miscalibrated beliefs of the risks are thought to perhaps play a role (see, e.g. Baldauf, Garlappi and Yannelis (2020), Bakkensen and Barrage (2017)).[7] But as far as we know there have been no model to assess what price effects one would expect in the first place using a general equilibrium model holding fixed any given level of belief regarding risks. Our model provides a framework to assess how much of a price effect one expects and the role of beliefs.

Recall the 1.3% figure for mitigation spending as a fraction of capital stock at moderate beliefs of $\pi = 0.5$. This would then be the optimal tax rate for housing capital stock to fund the mitigation spending. This tax on housing capital stock were it implemented would result in 5% lower home prices or Tobin's average q. In other words, due to mitigation externalities, Atlantic coastal property is around 5% higher in the competitive equilibrium than the first-

[7] Recent empirical work on weather disasters and climate risks also include the direct physical risks for firm cashflows such droughts (Hong, Li and Xu (2019)). See Hong, Karolyi, and Scheinkman (2020) for a review of recent findings.

best outcome. Despite the significant 1.3% tax on capital, the impact on home prices is modest because there are benefits to mitigation that protects housing capital. To see this clearly, we perform a decomposition to show that price effects would be significantly worse absent the aggregate risk reducing benefits of mitigation. Recent concerns expressed by regulators regarding such taxes on capital stock (Carney (2015)) neglects these substantial benefits.

2 Model

There is a continuum of firms with a unit measure. Time is continuous and the horizon is infinite. We will analyze both the social planner's and competitive equilibrium solutions. All firms have the same production and capital accumulation technology. Additionally, they face the same shocks. First, we present and then solve the planner's problem.

2.1 Production, Capital Dynamics, and Disasters

Aggregate Production and Resource Constraint. Let K denote the aggregate capital stock, which is the sole factor of production. Aggregate output, Y, is given by

$$Y_t = AK_t \,, \tag{1}$$

where $A > 0$ is a constant that defines productivity. K can include both physical and housing capital, while output Y can be a consumption good or housing services. This is a version of the AK model but importantly augmented with capital adjustment costs as we show later.

In each period, aggregate output is spent in one of the three possible ways—consumption, investment, and mitigation. Let C_t, I_t, and X_t denote consumption, investment, and mitigation spending, respectively.

Following the q theory of investment (Hayashi, 1982 and Abel and Eberly, 1994), we assume that when investing $I_t dt$, the firm also incurs capital adjustment costs, which we denote by $\Phi_t dt$. That is, the total cost of investment per unit of time is $(I_t + \Phi_t)$ including both capital purchase and adjustment costs. Therefore, we have the following aggregate resource constraint:

$$Y_t = C_t + (I_t + \Phi_t) + X_t \,. \tag{2}$$

The most natural interpretation of mitigation spending is seawalls or land-use zoning in the context of the climate change literature.[8] We specify the capital adjustment later in this section.

Investment and Capital Accumulation. The capital stock K evolves as:[9]

$$dK_t = I_{t-}dt + \sigma K_{t-} d\mathcal{W}_t - (1-Z)K_{t-}d\mathcal{J}_t \,. \tag{3}$$

The first term in (3) is investment I. The second term captures continuous shocks to capital, where $\mathcal{W}_t$ is a standard Brownian motion and the parameter σ is the diffusion volatility (for the capital stock growth). This diffusion shock is the source of shocks for the standard AK models in macroeconomics. To emphasize the timing of potential jumps, we use $t-$ to denote the pre-jump time so that a discrete jump may or may not arrive at t.

Arrival of Disasters. Capital stock is also subject to jump shocks that cause stochastic permanent losses of the existing capital stock. Examples include hurricanes or wildfires that destroy both physical and housing capital stock. We capture this effect via the third term, where $\mathcal{J}_t$ is a (pure) jump process with a constant but unknown arrival rate, which we denote by λ, to be described shortly.

When a jump arrives ($d\mathcal{J}_t = 1$), it permanently destroys a stochastic fraction $(1-Z)$ of the capital stock K_{t-}, as Z is the recovery fraction. Absent mitigation spending, the domain for the admissible values of Z is $(0,1)$. (For example, if a shock destroyed 15 percent of capital stock, we would have $Z = .85$.) There is no limit to the number of these jump shocks.[10] If a jump does not arrive at t, i.e., $d\mathcal{J}_t = 0$, the third term disappears.

[8] Mitigation spending X_t effectively reduces output which tightens resources constraints for consumption and investment: $Y_t - X_t = C_t + (I_t + \Phi_t)$.

[9] This capital accumulation technology has been widely used in macro and finance. For example, see Barro (2006) and Pindyck and Wang (2013).

[10] Stochastic fluctuations in the capital stock have been widely used in the growth literature with an AK

Let $\Xi(Z)$ and $\xi(Z)$ denote the cumulative distribution function (cdf) and probability density function (pdf) for the recovery fraction, Z, conditional on a jump arrival, respectively. Importantly, the conditional distribution of Z depends on aggregate mitigation spending at time $t-$. We discuss how Z depends on X_{t-} later in this section.

Next, we discuss how we model the constant but unknown arrival rate of the jump process, λ. We suppose that the arrival rate can be either low or high. If the rate is high, it is more likely that capital stock will be hit by a disaster (i.e., a negative jump shock). If the rate is low, a disaster is much less likely. We refer to the low-rate and high-rate scenarios as good state (G) and bad state (B), respectively, and use λ_G and λ_B to denote the corresponding jump arrival rate of a jump in the respective state. Naturally, $\lambda_B > \lambda_G$. While the state is constant over time, the household does not observe the state and therefore has to learn about the value of λ over time to assess the likelihood that the arrival rate is high or low. We will discuss the household's learning dynamics shortly.

We use lower-case variables to denote the corresponding upper-case variables divided by contemporaneous K. For example, $c_t = C_t/K_t$, $i_t = I_t/K_t$, $\phi_t = \Phi_t/K_t$, and $x_t = X_t/K_t$.

Homogeneity Property. To preserve our model's homogeneity, we make two economically sensible simplifying assumptions: one about capital adjustment costs and the other about the mitigation technology.

First, the capital adjustment cost function, $\Phi(I, K)$, is homogeneous with degree one in I and K and thus can be written as:

$$\Phi(I, K) = \phi(i)K \ , \tag{4}$$

where $\phi(i)$ is increasing and convex.[11] Because installing capital is costly, installed capital earns rents in equilibrium so that Tobin's q, the ratio between the value and the replacement

technology, but unlike the existing literature, we examine the economic effects of shocks to capital that involve discrete (disaster) jumps.

[11]Homogeneous adjustment cost functions are analytically tractable and have been widely used in the q theory of investment literature. Hayashi (1982) showed that with homogeneous adjustment costs and perfect capital markets, marginal and average q are equal.

cost of capital, exceeds one.[12]

Next, we specify the benefits of mitigation spending.

2.2 Mitigation Technology and Payoffs

The benefit of mitigation spending in our model derives from reduction of damages due to disasters. Our specification postulates that the distribution for the recovery fraction Z at t conditional on a jump arrival depends on the pre-jump mitigation spending X_{t-}. Otherwise, capital accumulation remains the same and is given by (3).

To preserve the homogeneity property, we assume that the distribution of the post-jump fractional recovery Z changes from $\Xi(Z)$ to $\Xi(Z; x_{t-})$ and the corresponding density function changes from $\xi(Z)$ to $\xi(Z; x_{t-})$. That is, if mitigation spending X doubles, the benefit of mitigation also doubles. Because making the distribution of Z less damaging is a public good, the private and societal interests may not line up. We show that welfare theorem does not hold due to free-rider's incentives.[13]

Finally, we complete our model description by introducing the preferences.

Preferences. We use the Duffie and Epstein (1992) continuous-time version of the recursive preferences developed by Epstein and Zin (1989) and Weil (1990), so that a representative consumer has homothetic recursive preferences given by:

$$V_t = \mathbb{E}_t \left[\int_t^{\infty} f(C_s, V_s) ds \right] , \tag{5}$$

where $f(C, V)$ is known as the normalized aggregator given by

$$f(C, V) = \frac{\rho}{1 - \psi^{-1}} \frac{C^{1-\psi^{-1}} - ((1-\gamma)V)^{\omega}}{((1-\gamma)V)^{\omega-1}} . \tag{6}$$

Here ρ is the rate of time preference, ψ the elasticity of intertemporal substitution (EIS), γ the coefficient of relative risk aversion, and we let $\omega = (1 - \psi^{-1})/(1 - \gamma)$. Unlike expected

[12]In Barro (2006), he also analyzes an endogenous AK growth model with disaster risks but without capital adjustment costs in a discrete-time setting. Therefore, Tobin's average q in his model is always one.

[13]In this paper, we are interested in a mitigation technology with externalities so as to derive implications on taxes and asset prices. For completeness, in Appendix B, we consider an alternative specification for mitigation technology where the welfare theorem holds.

utility, recursive preferences as defined by (5) and (6) disentangle risk aversion from the EIS. An important feature of these preferences is that the marginal benefit of consumption is $f_C = \rho C^{-\psi-1}/[(1-\gamma)V]^{\omega-1}$, which depends not only on current consumption but also (through V) on the expected trajectory of future consumption.

If $\gamma = \psi^{-1}$ so that $\omega = 1$, we have the standard constant-relative-risk-aversion (CRRA) expected utility, represented by the additively separable aggregator:

$$f(C,V) = \frac{\rho\, C^{1-\gamma}}{1-\gamma} - \rho V. \tag{7}$$

This more flexible recursive utility specification is widely used in asset pricing and macroeconomics for at least two important reasons: 1) conceptually, risk aversion is very distinct from the EIS, which this preference is able to capture; 2) quantitative and empirical fit with various asset pricing facts are infeasible with standard CRRA utility but attainable with this recursive utility, as shown by Bansal and Yaron (2004) and the large follow-up long-run risk literature. We show that in our model, the EIS parameter plays an important role as well.

3 Solution

The social planner maximizes the representative household's utility given in (5)-(6) subject to the production/capital accumulation technology and the aggregate resource constraint described in Section 2.

Next, we derive the representative households' or planner's Bayesian learning rule and then use dynamic programming to solve the optimal policies and value function.

Learning. The household dynamically updates her belief about the arrival rate of disasters. Let π_t denote the time-t posterior belief that $\lambda = \lambda_B$. That is,

$$\pi_t = \mathbb{P}(\lambda_t = \lambda_B | \mathcal{F}_t)\,, \tag{8}$$

where $\mathcal{F}_t$ is the household's information set up to t. At time t, the expected jump arrival rate, denoted by λ_t, is given by

$$\lambda_t = \lambda(\pi_t) = \lambda_B \pi_t + \lambda_G(1-\pi_t)\,, \tag{9}$$

which is a weighted average of λ_B and λ_G. A higher value of π_t corresponds to a belief that the economy is more likely in State B which has a high jump arrival rate.

What makes the household's belief to worsen (increasing π) is jump arrivals. What makes the household's belief to revise favorably is no jump arrivals. In this sense, no-jump news is good news. In expectation, with rational learning, belief change cannot be predicted, which means belief has to be a martingale.

Mathematically, the household updates her belief by following the Bayes rule:

$$d\pi_t = \sigma_\pi(\pi_{t-})\left(d\mathcal{J}_t - \lambda_{t-}dt\right) , \tag{10}$$

where

$$\sigma_\pi(\pi) = \frac{\pi(1-\pi)(\lambda_B - \lambda_G)}{\lambda(\pi)} = \frac{\pi(1-\pi)(\lambda_B - \lambda_G)}{\lambda_B\pi + \lambda_G(1-\pi)} > 0\,. \tag{11}$$

Here, signals come from $\mathcal{J}_t$. Because $\mathbb{E}_{t-}[d\mathcal{J}_t] = \lambda_{t-}dt$, (10) implies that the household's belief process π is a martingale. When a disaster strikes at t, the household's belief immediately increases from the pre-jump level π_{t-} to $\pi_t = \pi^{\mathcal{J}}$ by $\sigma_\pi(\pi_{t-})$, where

$$\pi^{\mathcal{J}} = \pi_{t-} + \sigma_\pi(\pi_{t-}) = \frac{\pi_{t-}\,\lambda_B}{\lambda(\pi_{t-})} > \pi_{t-}\,. \tag{12}$$

If there is no arrival over time interval dt, the household becomes more optimistic. Mathematically, if $d\mathcal{J}_t = 0$, we have $d\pi_t = \mu_\pi(\pi_{t-})dt$, where

$$\mu_\pi(\pi) = -\sigma_\pi(\pi)\lambda(\pi) = \pi(1-\pi)(\lambda_G - \lambda_B) < 0\,. \tag{13}$$

Now suppose that there is no jump during a finite time interval (s,t), i.e., $dJ_v = 0$ for $s < v \le t$. By using (13) to integrate π from s to t conditional on no jump over (s,t), we obtain the following logistic function:

$$\pi_t = \frac{\pi_s e^{-(\lambda_B-\lambda_G)(t-s)}}{1+\pi_s(e^{-(\lambda_B-\lambda_G)(t-s)}-1)}\,. \tag{14}$$

In Figure 1, we plot a simulated path for π starting from $\pi_0 = 0.1$. It shows that absent a jump arrival, belief becomes more optimistic, i.e., π_t decreases deterministically. Once a jump arrives, the belief worsens, i.e., jumps upward by a discrete amount $\sigma_\pi(\pi)$.

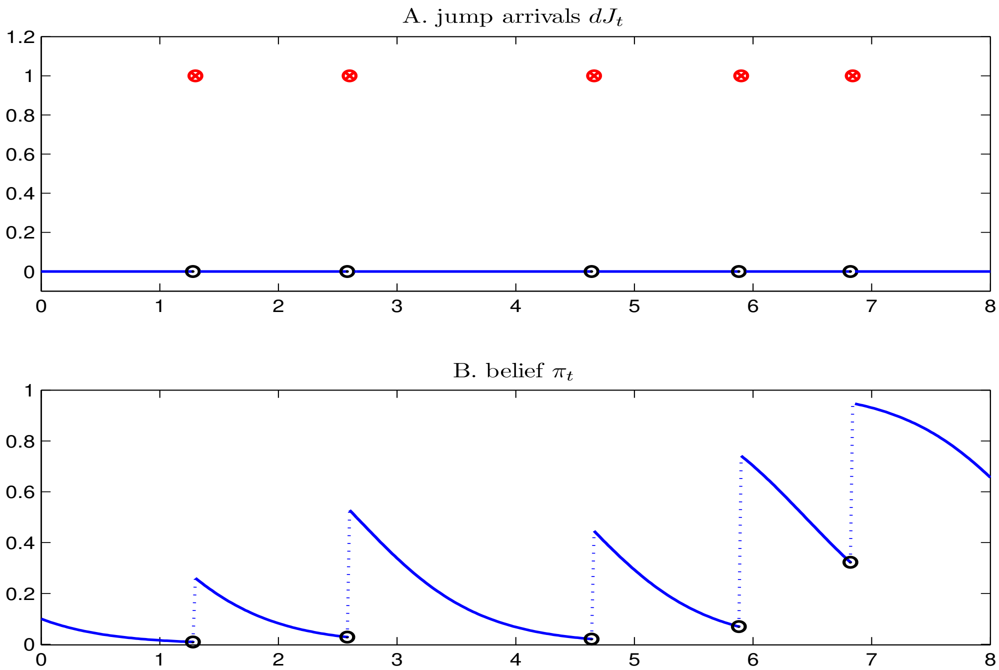

Figure 1: This figure simulates a path for jump arrival times in Panel A and plots the corresponding belief updating process in Panel B starting with $\pi_0 = 0.1$. The belief decreases deterministically in the absence of jumps but discretely increases upward upon a jump arrival.

Dynamic Programming. Let $V(K,\pi)$ denote the value function. The Hamilton-Jacobi-Bellman (HJB) equation for the planner's allocation problem is:

$$\begin{aligned} 0 \;=\; & \max_{C,\,I,\,x}\; f(C,V) + IV_K(K,\pi) + \mu_\pi(\pi)V_\pi(K,\pi) + \frac{1}{2}\sigma^2K^2V_{KK}(K,\pi) \\ & +\lambda(\pi)\mathbb{E}\left[V\left(ZK,\pi^{\mathcal{J}}\right) - V(K,\pi)\right]\;, \end{aligned} \tag{15}$$

where the expected change of belief in the absence of jumps, $\mu_\pi(\pi)$, is negative and given in (13), the expected arrival rate of a jump, $\lambda(\pi)$, is given in (9), the post-jump belief $\pi^{\mathcal{J}}$ is given in (12) as a function of the pre-jump belief π, and the expectation $\mathbb{E}[\,\cdot\,]$ is with respect to the pdf $\xi(Z;x)$ for the recovery fraction Z for a given level of scaled mitigation x.

The first term on the right side of (15) is the household's normalized aggregator; the second term captures how investment I affects $V(K,\pi)$; the third term reflects how belief updating (in the absence of jumps) impacts $V(K,\pi)$; and the fourth term captures the effect of capital-stock diffusion shocks on $V(K,\pi)$. It is worth noting that as the signals in our learning model are discrete (jump arrivals), there is no diffusion volatility induced quadratic variation term involving $V_{\pi\pi}$ in the HJB equation (15).

Direct versus Learning Effects. Finally, the last term (appearing on the second line) of (15) captures the effect of jumps on the expected change in $V(K,\pi)$. This term captures rich economic forces and warrants additional explanations. When a jump arrives at t ($d\mathcal{J}_t = 1$), capital stock falls from K_{t-} at time $t-$ to $K_t = ZK_{t-}$ at t, which also causes the household to become more pessimistic. As a result, her belief increases from the pre-jump level of π_{t-} to the post-jump level of $\pi_t = \pi^{\mathcal{J}}$, as given by (12). Therefore, the expected change of the value function conditional on a jump arrival is given by $\mathbb{E}\left[V\left(ZK_{t-},\pi^{\mathcal{J}}\right) - V(K_{t-},\pi_{t-})\right]$. To take into account that the jump arrival is uncertain, we multiply this term by the jump arrival intensity at $t-$, $\lambda(\pi_{t-})$, to obtain the last term in (15).

It is important to note that a jump triggers two effects on the value function. First, there is an direct effect: $(1-Z)$ fraction of the capital stock is permanently destroyed, which lowers the value function from $V(K_{t-},\pi_{t-})$ to $V(ZK_{t-},\pi_{t-})$. Second, there is a learning effect: the household's belief worsens to $\pi_t = \pi^{\mathcal{J}} = \pi_{t-}\lambda_B/\lambda(\pi_{t-}) > \pi_{t-}$, which further lowers the value function from $V(ZK_{t-},\pi_{t-})$ to $V(ZK_{t-},\pi^{\mathcal{J}})$. These two effects reinforce each other over time leading to potentially significant losses to the household.

The household optimally chooses consumption C, investment I, and mitigation X at all time to maximize her utility by setting the sum of all the five terms on the right side of (15) to zero, as implied by the standard argument underpinning the HJB equation generalized to the setting with recursive utility (see Duffie and Epstein, 1992). Because of the resource constraint, it is sufficient for us to focus on I and x as control variables.

First-Order Conditions for Investment and Mitigation. The first-order condition (FOC) for investment I is

$$(1+\Phi_I(I,K))f_C(C,V) = V_K(K,\pi)\ . \tag{16}$$

The right side of (16) is the marginal (utility) benefit of investment. The left side of (16) is the marginal cost of investment, which is given by the product of marginal benefit of consumption $f_C(C, V)$ and the marginal cost of investing $(1 + \Phi_I(I, K))$, the latter of which includes the marginal adjustment cost $\Phi_I(I, K)$.

The intuition for (16) is as follows. To increase the capital stock by one unit, which generates a marginal utility benefit of V_K, the household needs to give up $(1 + \Phi_I(I, K))$ units of her consumption in order to purchase one unit of capital and then install it into the firm making it productive. Therefore, the marginal cost of increasing capital stock by one unit is $(1 + \Phi_I(I, K))$ units of marginal benefit of consumption f_C. Unlike in standard expected-utility models, $f_C(C, V)$ depends on not just consumption C but also the continuation utility V, which reflects the non-separability of preferences.

The FOC with respect to mitigation is

$$f_C(C, V) = \frac{1}{K}\lambda(\pi) \int_0^1 \left[\frac{\partial \xi(Z; x)}{\partial x} V \left(ZK, \frac{\pi \lambda_B}{\lambda(\pi)} \right) \right] dZ \ , \tag{17}$$

if the solution is strictly positive, $x > 0$. Otherwise, $x = 0$ as mitigation cannot be negative. The planner optimally chooses X to equate the marginal cost of mitigation, which is the forgone marginal (utility) benefit of consumption $f_C(C, V)$ given in the left side of (17), with the marginal benefit of mitigation given in the right side of (17).[14] By doing mitigation x per unit of capital, the planner changes the pdf $\xi(Z; x)$ for the fractional capital recovery, Z, from $\xi(Z; 0)$ to $\xi(Z; x)$. We provide detailed discussions about the stochastic dominance properties of $\xi(Z; x)$ and the economic tradeoff shortly.

Using Homogeneity Property to Simplify Solution. We show that the value function $V(K, \pi)$ is homogeneous with degree $(1 - \gamma)$ in K and thus we can write $V(K, \pi)$ as follows:

$$V(K, \pi) = \frac{1}{1 - \gamma} \left(b(\pi) K \right)^{1-\gamma} , \tag{18}$$

where $b(\pi)$ is the function determined as part of the solution.

Using the FOCs (16) and (17) and substituting the value function $V(K, \pi)$ given in (18) together with the implied policy rules into the HJB equation (15), and simplifying the

[14]The second-order condition (SOC) is given by $\lambda(\pi) \int_0^1 \left[\frac{\partial^2 \xi(Z, x)}{\partial x^2} V \left(ZK, \frac{\pi \lambda_B}{\lambda(\pi)} \right) \right] dZ < 0$, which we verify.

equations, we obtain the following three-equation ODE system for $b(\pi)$, $i(\pi)$, and $x(\pi)$:

$$\begin{aligned}
0 &= \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{b(\pi)}{\rho(1+\phi'(i(\pi)))}\right)^{1-\psi}-1\right]+i(\pi)-\frac{\gamma\sigma^2}{2}+\mu_\pi(\pi)\frac{b'(\pi)}{b(\pi)} \\
&\quad+\frac{\lambda(\pi)}{1-\gamma}\left[\left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}\mathbb{E}(Z^{1-\gamma})-1\right], \qquad (19) \\
b(\pi) &= [A-i(\pi)-\phi(i(\pi))-x(\pi)]^{1/(1-\psi)}\left[\rho(1+\phi'(i(\pi)))\right]^{-\psi/(1-\psi)}, \qquad (20) \\
1 &= \frac{\lambda(\pi)(1+\phi'(i(\pi)))}{1-\gamma}\left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}\int_0^1\left[\frac{\partial\xi(Z;x(\pi))}{\partial x}Z^{1-\gamma}\right]dZ\ . \qquad (21)
\end{aligned}$$

Next, we provide the boundary conditions at $\pi = 0$ and $\pi = 1$ and discuss the intuition. As we show, the model at the two boundaries map to the model in Pindyck and Wang (2013), but generalized to allow for mitigation spending. When $\pi = 0$, the economy is permanently in state G. Therefore there is no learning and the solution boils down to solving the three unknowns, $b(0)$, investment $i(0)$, and mitigation spending $x(0)$, via the following three-equation system:

$$\begin{aligned}
0 &= \frac{\left(\frac{b(0)}{\rho(1+\phi'(i(0)))}\right)^{1-\psi}-1}{1-\psi^{-1}}\rho+i(0)-\frac{\gamma\sigma^2}{2}+\frac{\lambda_G}{1-\gamma}\left(\mathbb{E}(Z^{1-\gamma})-1\right), \qquad (22) \\
b(0) &= [A-i(0)-\phi(i(0))-x(0)]^{1/(1-\psi)}\left[\rho(1+\phi'(i(0)))\right]^{-\psi/(1-\psi)}, \qquad (23) \\
\frac{1}{1+\phi'(i(0))} &= \frac{\lambda_G}{1-\gamma}\int_0^1\left[\frac{\partial\xi(Z;x(0))}{\partial x}Z^{1-\gamma}\right]dZ\ . \qquad (24)
\end{aligned}$$

When $\pi = 0$, investment-capital ratio $i(0)$, scaled mitigation spending $x(0)$, and consumption-capital ratio $c(0)$ are all constant at all time.

By applying essentially the same analysis to the other boundary at $\pi = 1$, i.e., when the state is B, we solve for the three unknowns, $b(1)$, $i(1)$, and $x(1)$, via (A.3)-(A.5), another three-equation system in Appendix A.

The economy is on a growth path with constant investment opportunity when $\pi = 0$ or $\pi = 1$. None of these results hold obviously in our general model when $0 < \pi < 1$.

We summarize our model's solution in the following proposition.

Proposition 1 *The planner's solution is given by the triplet,* $b(\pi)$, $i(\pi)$, *and* $x(\pi)$, *where* $0 \leq \pi \leq 1$, *via the three-equation ODE system, (19)-(21) in the interior region* $0 < \pi < 1$, *together with the boundary conditions (22)-(24) for $\pi = 0$ and (A.3)-(A.5) for $\pi = 1$.*

See Appendix A for the proof.

Expected Fractional Loss, Growth Rate, and Mitigation Technology. We further assume as in Barro (2006) and Pindyck and Wang (2013) that the cdf of Z is given by the following power function defined over $(0,1)$:

$$\Xi(Z;x) = Z^{\beta(x)} , \tag{25}$$

where $\beta(x)$ is the exponent function that depends on scaled mitigation x. To ensure that our model is well defined, we require $\beta(x) > \gamma - 1$.

Conditional on a jump arrival, the expected fractional capital loss is given by

$$\ell(\pi) = 1 - \mathbb{E}(Z) = \frac{1}{\beta(x(\pi)) + 1} . \tag{26}$$

The larger the value of $\beta(\cdot)$, the smaller the expected fractional loss $\mathbb{E}(1 - Z)$. To capture the benefit of mitigation, we assume that $\beta(x)$ is increasing in x, $\beta'(x) > 0$. The benefit of mitigation is to increase the capital stock recovery (upon the arrival of a disaster) in the sense of first-order stochastic dominance, i.e., $\Xi(Z;x_1) \leq \Xi(Z;x_2)$ for $Z < 1$ if $x_1 > x_2$.

Let g_t denote the expected growth rate including the jump effect. The homogeneity property implies that $g_t = g(\pi_t)$, where

$$g(\pi) = i(\pi) - \lambda(\pi)\ell(\pi) = i(\pi) - \frac{\lambda(\pi)}{\beta(x(\pi)) + 1} . \tag{27}$$

As we show soon, while mitigation $x(\pi)$ may crowd out investment $i(\pi)$, it enhances long-run growth $g(\pi)$ by reducing the expected loss due to jumps.

For our quantitative analysis, we use the following linear specification for $\beta(x)$:

$$\beta(x) = \beta_0 + \beta_1 x , \tag{28}$$

with $\beta_0 \geq \max\{\gamma - 1, 0\}$ and $\beta_1 > 0$. The coefficient β_0 is the exponent for recovery Z in the absence of mitigation. The coefficient β_1 is a key parameter in our model and measures the efficiency of the mitigation technology.

Planner's Value Function with No Mitigation Technology. To better connect to the competitive market equilibrium solution, it is useful to summarize the planner's solution when there is no mitigation technology available, i.e., $x = 0$. By using the same argument as we have for the general case, we know that the planner's value function, $\widehat{V}(K, \pi)$, is homogeneous with degree $(1 - \gamma)$ in K:

$$\widehat{V}(K, \pi) = \frac{1}{1-\gamma} \left(\widehat{b}(\pi) K\right)^{1-\gamma}, \tag{29}$$

where $\widehat{b}(\pi)$ is a measure of welfare (proportional to the certainty equivalent wealth). By substituting $x(\pi) = 0$ into the solution for the general case and removing (17), the FOC for x, we obtain the solution for $\widehat{b}(\pi)$ together with the optimal investment-capital ratio $i(\pi)$.

In summary, $\widehat{b}(\pi)$ and $i(\pi)$ jointly solve (19)-(20) together with the boundary conditions (22)-(23) and (A.3)-(A.4) with the restriction of no mitigation spending, $x(\pi) = 0$.

4 Competitive Equilibrium and Market Failure

We analyze the decentralized market-equilibrium solution (Appendix B provides details.) Importantly, we show that the market mechanism does not implement the planner's solution in Section 3. This is because aggregate risk mitigation suffers from a free-riding problem as neither households nor firms have incentives to mitigate aggregate risk.

4.1 Market Structure and Problem Formulation

Consider a decentralized competitive equilibrium with (dynamically) complete markets. That is, the following securities can be traded at each point in time: (i) a risk-free asset, (ii) the aggregate asset market (a claim on the value of capital of the representative firm), and (iii) insurance claims for disaster with every possible recovery fraction Z.

Disaster Risk Insurance (DIS). We define DIS as follows: a DIS for the survival fraction in the interval $(Z, Z + dZ)$ is a swap contract in which the buyer makes insurance payments $p(Z; x^*)dZ$, where x^* is the aggregate (scaled) mitigation spending, to the seller and in exchange receives a lump-sum payoff if and only if a shock with survival fraction in $(Z, Z+dZ)$

occurs. That is, the buyer stops paying the seller if and only if the defined disaster event occurs and then collects one unit of the consumption good as a payoff from the seller. The DIS contracts, e.g., the insurance premium payment $p(Z; x^*)$, are priced at actuarially fairly so that investors earn zero profits. $p(Z; x^*)$ depends on not only Z but also x^*. This is because the aggregate mitigation spending x^* changes the distribution for $\Xi(Z)$.

Let $X_{c,t} \geq 0$ and $X_{f,t} \geq 0$ denote the mitigation spending at t by households and firms, respectively. Let H_t denote the household's wealth allocated to the market portfolio at t. For disaster with recovery fraction in $(Z, Z + dZ)$, $\delta_t(Z)W_t dt$ gives the total demand for the DIS over time period $(t, t + dt)$. Let W_t denote the representative household's wealth.

We define the recursive competitive equilibrium as follows: (1) The representative household chooses consumption C, allocation to the asset market H, various DIS claims $\delta(Z)$, and mitigation spending X_c to maximize utility as given by (5)-(6). (2) The representative firm (operating the same technology as the one that the social planner does in Section 2) chooses investment I and mitigation spending X_f to maximize its market value, which is the present discounted value of future cash flows. Private agents take the equilibrium prices of all goods and financial assets including the risk-free rate $r(\pi)$ and the stock-market price process as given. (3) All markets clear.

Next we solve for the resource allocation in the decentralized market setting.

It is useful to differentiate variables at the micro and macro levels. We use superscript * to denote the equilibrium variables. For example, X_c^* and X_f^* denote the equilibrium mitigation spending by households and firms, and $x_c^* = X_c^*/K$ and $x_f^* = X_f^*/K$. Let $x^* = x_c^* + x_f^*$.

The representative firm solves the following value maximization problem:[15]

$$\max_{I, X_f} \mathbb{E}\left[\int_0^\infty \frac{\mathbb{M}_s}{\mathbb{M}_0} \left(AK_s - I_s - \Phi_s - X_{f,s}\right) ds\right], \tag{30}$$

where $\mathbb{M}$ is the equilibrium stochastic discount factor that the firm takes as given. Let Q_t denote the solution for (30), the firm's market value. Using (30) and the homogeneity

[15]Financial markets are perfectly competitive and complete. While the firm can hold financial positions (e.g., DIS contracts), these financial hedging transactions generate zero NPV for the firm. Therefore, financial hedging policies are indeterminate, essentially a version of the Modigliani-Miller result. The firm can thus ignore financial contracts without loss of generality.

property, e.g., $Q(K_t, \pi_t) = q(\pi_t)K_t$, we obtain the following HJB equation:

$$
\begin{aligned}
0 = \max_{i, x_f} \; & A - i - \phi(i) - x_f - (r(\pi) - i(\pi))q(\pi) + \mu_\pi(\pi)q'(\pi) \\
& - \left[\gamma\sigma^2 + \lambda(\pi)\left(\left(\frac{u^*(\pi^{\mathcal{J}})}{u^*(\pi)}\right)^{1-\gamma}\left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{-\gamma}\int_0^1 Z^{-\gamma}\xi(Z;x^*)dZ - 1\right)\right] q(\pi) \\
& + \lambda(\pi)\left[\left(\frac{u^*(\pi^{\mathcal{J}})}{u^*(\pi)}\right)^{1-\gamma}\left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{-\gamma}\frac{q(\pi^{\mathcal{J}})}{q(\pi)}\int_0^1 Z^{1-\gamma}\xi(Z;x^*)dZ - 1\right] q(\pi)\,. \quad (31)
\end{aligned}
$$

The FOC for investment implied by (31) is

$$q(\pi) = 1 + \phi'(i(\pi))\,, \quad (32)$$

which equates the marginal q to the marginal cost of investing $1 + \phi'(i)$.

Let $J_t = J(W_t, \pi_t)$ denote the household's value function. We show that

$$J(W, \pi) = \frac{1}{1-\gamma}(u(\pi)W)^{1-\gamma}\,, \quad (33)$$

where $u(\pi)$ is to be determined. The household solves the following problem:

$$
\begin{aligned}
0 = \max_{c,h,\delta,x_c} \; & \frac{\rho\left(\frac{u(\pi)}{\rho}\right)^{1-\psi} - \rho}{1-\psi^{-1}} + \left[r(\pi) - \int_0^1 \delta(Z)p(Z;x^*)dZ + \frac{(\mu_Q(\pi) - r(\pi))h - c - x_c}{w}\right] \\
& + \mu_\pi(\pi)\frac{u'(\pi)}{u(\pi)} - \frac{\gamma\sigma^2}{2} + \lambda(\pi)\left[\left(\frac{u(\pi^{\mathcal{J}})}{u(\pi)}\right)^{1-\gamma}\int_0^1\left(\frac{w^{\mathcal{J}}}{w}\right)^{1-\gamma}\xi(Z;x^*)dZ - 1\right], \quad (34)
\end{aligned}
$$

where $\mu_Q(\pi)$ is given in (B.26).

The consumption FOC implied by (34) yields the following consumption rule:

$$c(\pi) = \rho^\psi u(\pi)^{1-\psi} w\,. \quad (35)$$

Consumption is linear in W with a marginal propensity to consume that depends on π. Neither households nor firms have incentives to spend on mitigation:

$$x_c = x_f = 0\,. \quad (36)$$

This is because the benefit of mitigation spending is thinning the fat tail for the disaster damage by increasing the $\beta(x)$ function. As no individual agent influences the distribution

$\Xi(Z)$ for the recovery fraction, Z, at the margin, they have no incentives to spend on mitigation spending. In essence, mitigating disaster damages is providing a public good. As a result, market equilibrium features no aggregate mitigation spending: $x^* = x_c^* + x_f^* = 0$.

We thus cannot use the planner's solution given in Section 3 to infer the equilibrium resource allocation and prices as the welfare theorem does not hold in our model.[16] Instead, our market-equilibrium solution is equivalent to the planner's solution when the planner has no access to the mitigation technology. We summarize the competitive-market solution in the following proposition.

Proposition 2 *There is no mitigation in competitive equilibrium. The competitive equilibrium solution corresponds to the social planner's solution only when there is no mitigation technology (i.e.* $\beta_1 = 0$*):* $J(W_t, \pi_t) = \widehat{V}(K_t, \pi_t)$*, where* $W_t = q(\pi_t)K_t$*.*

5 Capital Taxation, Mitigation Subsidies, and Welfare in First-Best versus Competitive Equilibrium

In this section, we first resurrect the planner's first-best solution in Section 3 by using optimal capital taxation and subsidies and then propose a metric to measure the welfare gains of mitigation spending.

Capital Taxes and Mitigation Subsidies Restore First-Best. The government imposes a time-varying proportional tax on each firm's capital stock (or equivalently sales as $Y = AK$ at the firm level) and then fully subsidizes the firm's mitigation spending at market price. Let ν denote the tax rate on an individual firm's capital stock K. By setting $\nu_t = x_t$, where x_t is the socially optimal mitigation spending obtained in Section 3 and implementing 100% reimbursement of all the firm's mitigation spending, we show that the planner's solution is attained in the competitive market equilibrium.

[16] In contrast, the planner's solution in Pindyck and Wang (2013) can be achieved via market decentralization, as there is no mitigation spending and hence welfare theorem holds in their model.

Given the government taxation and subsidy policy, each firm solves the following problem:

$$\max_{I, X_f} \mathbb{E}\left[\int_0^\infty \left(\frac{\mathbb{M}_t}{\mathbb{M}_0}\left(AK_t - I_t - \Phi_t - X_{f,t} - \nu_t K_t\right) + p_{0,s}X_{f,t}\right) dt\right], \tag{37}$$

where $p_{0,t}$ is the time-0 value of the government subsidy to the firm for a unit of its mitigation spending at time t for each sample path (e.g., state). The firm makes a tax payment $\nu_t K_t = x_t K_t$ and receives a subsidy $p_{0,t}$ for each unit of mitigation spending. Because markets are complete, we know that in equilibrium $p_{0,t} = \mathbb{M}_t/\mathbb{M}_0$ holds. Therefore, the firm breaks even with probability one for any level of mitigation spending that it chooses, as the firm is fully reimbursed for every unit of spending it incurs on mitigation. One solution is for firms to choose the socially optimal level of mitigation spending, X_t, prescribed by the planner's problem. As we show, this is the level of X that is consistent with equilibrium market clearing.

The firm's HJB equation is then

$$\begin{aligned}
0 = \max_i\ & (A - \nu(\pi)) - i - \phi(i) - (r(\pi) - i(\pi))q(\pi) + \mu_\pi(\pi)q'(\pi) \\
& - \left[\gamma\sigma^2 + \lambda(\pi)\left(\left(\frac{u^*(\pi^{\mathcal{J}})}{u^*(\pi)}\right)^{1-\gamma}\left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{-\gamma}\int_0^1 Z^{-\gamma}\xi(Z;x^*)dZ - 1\right)\right] q(\pi) \\
& + \lambda(\pi)\left[\left(\frac{u^*(\pi^{\mathcal{J}})}{u^*(\pi)}\right)^{1-\gamma}\left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{-\gamma}\frac{q(\pi^{\mathcal{J}})}{q(\pi)}\int_0^1 Z^{1-\gamma}\xi(Z;x^*)dZ - 1\right] q(\pi)\,.
\end{aligned} \tag{38}$$

Intuitively, capital taxation effectively lowers the firm's productivity from A to $A - \nu_t = A - x(\pi_t)$, which in turns decreases its investment i_t. The firm's investment FOC is still given by (32). Since taxes lower the firm's productivity, which in turn lowers Tobin's average q, the firm also lowers investment. Hence, taxation fixes the firm's over-investment. The household optimization is essentially the same as that discussed in Section 4. For brevity, we leave the details out.

The next proposition summarizes the key results.

Proposition 3 *In a competitive (and complete) market economy, household consumption and corporate investment attain the first-best solution as the planner does in Section 3, provided that the government chooses the capital taxation (by setting the rate to the planner's* *chosen x_t for all firms) and then subsidies 100% of all private mitigation spending. The government balances its budget period by period.*

Alternatively, the government taxes all firms' capital stocks also at the rate ν_t (chosen by the planner's) and spend all the proceeds collected from firms to mitigate aggregate risk by itself, i.e., by choosing $X_t = \nu_t K_t$, where $\nu_t = x_t = x(\pi_t)$. In response to taxation, a firm voluntarily lowers its investment from its competitive-market level (absent any government intervention) to the first-best investment level as the firm perceives that its productivity is lowered from A to $A - \nu_t$. As both investment and mitigation spending are now at the first-best levels, we attain the first-best outcome as in the planner's problem.[17]

Measuring the Welfare Gain of Mitigation Spending. How much are we worse off if the economy is completely laissez faire? To answer this question, we introduce the following willingness to pay (WTP) metric as in Pindyck and Wang (2013).

Let ζ denote the fraction of capital stock that the society is willing to pay to go from the competitive market economy with no mitigation spending to an economy where the government either chooses the optimal regulation or directly the optimal level of mitigation spending, as discussed in Section 5. To make the society indifferent between the two options, the following condition has to hold:

$$V((1 - \zeta(\pi))K, \pi) = \widehat{V}(K, \pi) \ . \tag{39}$$

The left side of (39) is the value function under optimal government mitigation mandate or spending in an otherwise market economy with a lower level of capital stock (as a ζ fraction of K is deducted) and the right side is the value function in a completely laissez-faire economy given in (29).

By substituting the value functions given in (18) and (29) into the household's indifference condition (39), we obtain the following equation for $\zeta(\pi)$:

$$\zeta(\pi) = 1 - \frac{\widehat{b}(\pi)}{b(\pi)} > 0 \ . \tag{40}$$

[17] In Internet Appendix A, we verify that together with household optimization and market clearing, the first-best planner's solution is attainable in equilibrium given the government's policies discussed above.

The WTP $\zeta(\pi)$ measures the value creation by government mitigation regulation/spending measured by the percentage increase in the society's certainty-equivalent wealth.

Comments. In our competitive equilibrium model, households can hedge using DIS contracts. While an individual may have hedging demand (given by $\delta_t(Z)$), in equilibrium the aggregate demand for these DIS contracts is zero. The financial hedging demand does not change the distribution of Z and the aggregate risk is not mitigated at all in representative-agent models. In contrast, aggregate mitigation spending is a form of real hedging.

We choose a representative sector to keep our model analytically tractable. With say two sectors to invest in, not only the households' belief about the disaster matters, the relative size distribution between the two sectors also influences the optimal mitigation, investment, and welfare. In this economy, the planner may want to subsidize households to move away from the coastal areas and spend less on mitigation (e.g., building seawalls).[18]

6 Atlantic Hurricanes, Seawalls, and Coastal Property

6.1 Calibration and Parameter Choices

Our calibration exercise is intended to highlight the importance of mitigation for welfare analysis. To this end, we start with a disaster calibration of an Atlantic States regional economy—both with and without mitigation technology in the form of seawalls to guard against fat-tailed damages from hurricane arrivals.

EIS. Estimates of the EIS ψ in the literature vary considerably, ranging from a low value near zero (e.g., Hall, 1988) to values as high as two.[19] Bansal and Yaron (2004) argue that an EIS larger than one is necessary for equilibrium asset pricing predictions. Attanasio and Vissing-Jørgensen (2003) estimate the elasticity to be above unity for stockholders, while Hall (1988), using aggregate consumption data, obtains an estimate near zero. We choose a value that is around the middle between the two ends of the EIS estimates in the literature,

[18]Eberly and Wang (2011) develop a tractable two-sector AK model with capital adjustment costs.
[19]Appendix to Hall (2009) provides a brief survey of estimates in the literature.

Table 1: Parameter Values

Parameters	Symbol	Value
elasticity of intertemporal substitution	ψ	1.1
power law exponent with no mitigation	β_0	249
jump arrival rate if State is G	λ_G	1
jump arrival rate if State is B	λ_B	10
time rate of preference	ρ	4.83%
productivity	A	14.2%
quadratic adjustment cost parameter	θ	34.48
coefficient of relative risk aversion	γ	3.27
capital diffusion volatility	σ	14.19%
mitigation technology parameter	β_1	3.0×10^4
Targeted observables without mitigation (State G)		
(real) risk-free rate		0.8%
housing return risk premium		6.6%
housing market return volatility		14.2%
expected growth rate		2.5%
Tobin's q		2
mitigation level (State G)		$x(0) = 0$

All parameter values, whenever applicable, are continuously compounded and annualized.

$\psi = 1.1$, slightly above one so as to be consistent with following the literature on long-run risks (Bansal and Yaron (2004)).

Hurricane arrival and conditional damage. We calibrate the arrival rate λ_G in state G by using historical data: Atlantic States are exposed to about one to two major landfall hurricanes per year. We could set λ_G to be one or two per year. For our calculations, we set $\lambda_G = 1$ so as to maximize the difference from the bad state B. A conservative ballpark estimate of Atlantic coastal housing stock at risk from hurricanes and flooding is \$5 trillion in 2018 dollars.[20] We can then extrapolate the conditional damage from Atlantic

[20] US housing stock in 2018 is \$30 trillion. Atlantic housing stock is around 35-40% of this figure or around \$10 trillion. Of course not all the Atlantic housing stock is coastal property at risk from hurricanes and

hurricanes which Nordhaus (2010) estimates as 0.1% of US GDP (which in 2018 dollars is \$20.54 trillion) to a conditional damage of 0.4% of coastal housing stock in the Atlantic region. We then use the expression for damage to capital stock conditional on a disaster arrival $\ell(\pi) = 1/(\beta_0 + 1) = 0.4\%$ in the absence of mitigation as implied by equation (26) to obtain the calibrated value of $\beta_0 = 249$.

We set the arrival rate in the bad state $\lambda_B = 10$ per year. That is, in the bad state, a landfall hurricane on average arrives 10 times a year, which means that state B is about ten times more damaging per year than state G. Our choice of $\lambda_B = 10$ is based on extremely pessimistic states laid out in recent climate research.

Other parameters. We choose a widely used quadratic function (e.g., Hayashi (1982)):

$$\phi(i) = \frac{\theta i^2}{2}\ , \tag{41}$$

to model capital adjustment costs. The parameter θ measures how costly it is to adjust capital and will be chosen with other parameters to target certain moments, as we describe next.

We calibrate the adjustment cost parameter θ along with the following four parameters—time rate of preference ρ, risk aversion γ, diffusion volatility σ, and productivity A—by targeting the five key moments for state G. These include the annual (real) risk-free rate of 0.8%, the expected annual housing market risk premium of 6.6%, the annual housing market return volatility of $\sqrt{0.0211} = 14.2\%$, the expected growth rate of 2.5%, and Tobin's q of 2, i.e., $q(0) = 2$ (e.g., Flavin and Yamashita, 2002). Doing so yields the following parameter values: $\sigma = 14.19\%$, $\theta = 34.48$, $\gamma = 3.27$, $A = 14.2\%$, and $\rho = 4.83\%$. These parameter values are broadly in line with those used in the literature.[21]

Mitigation technology β_1. Finally, we calibrate the parameter β_1 for the mitigation technology by targeting the optimal mitigation at zero for State G, i.e., $x(0) = 0$. That is,

flooding. Assuming half of it is at risk, we then arrive at \$5 trillion as a ballpark estimate.

[21] As an example, while using a different calibration strategy (for example, they do not target the capital adjustment costs), Barro and Jin (2011) also report the calibrated coefficient of relative risk aversion is their paper is about three. Our estimates are also close to those in Pindyck and Wang (2013), even though they use a different set of moments for the disaster arrival rate and the damage function.

the optimal usage of the mitigation technology by the planner under the most optimistic belief, $\pi = 0$ is zero. The implied value of β_1 is 1.5×10^4. That is, there is no value-add from the mitigation technology in State G and hence $b(0) = \widehat{b}(0)$.

6.2 Undermitigated Competitive Equilibrium

The solutions to our model in Figures 2 and 3 emphasize how key variables of interest depend on households' belief about arrival rates of disasters. The dashed red lines correspond to the competitive market solution. In Figure 2, we plot optimal mitigation spending (Panel A), investment (Panel B), consumption (Panel C) and the value of capital (Tobin's average q) (Panel D) as functions of belief π, where $\pi = 0$ means the economy is in State G while $\pi = 1$ means the economy is in State B.

Due to free-riders' incentives that we described in Section 4, there is no mitigation in equilibrium (Panel A). Additionally, investment (Panel B) and Tobin's average q (Panel C) move in sync as the standard investment optimality condition, $1 + \theta i(\pi) = q(\pi)$, holds. Both i and q decrease mildly as beliefs worsen.[22] Since no mitigation implies that $c_t + i_t + \phi_t = A$ has to hold at all t and for all levels of π_t, consumption c_t thus has to increase with π_t.

In Figure 3, we examine how key outcomes vary with π as a result of these policies. In Panel A, we plot the the (scaled) certainty-equivalent wealth for the competitive-market economy, $\widehat{\tau}(\pi) = \widehat{b}(\pi)/\widehat{b}(0)$. The curve starts at one when $\pi = 0$ by definition. In competitive-market economy (with no mitigation), $\widehat{\tau}(\pi)$ declines non-linearly with beliefs. The welfare loss is significant. At $\pi = 0.5$, welfare loss is nearly 40%. This welfare loss is connected to lower growth rates as π increases, i.e. unsustainable growth.

In Panel C of Figure 3, since there is no mitigation spending, the expected damage conditional on the arrival of a disaster is $\ell(\pi) = 1/(\beta_0 + 1) = 0.4\%$, which is independent of π, as if the mitigation technology were unavailable. In Panel D, we report the growth rate in the competitive economy, $g(\pi) = i(\pi) - \lambda(\pi)/(\beta_0 + 1)$, is declining with π. The reason is that as π increases, $i(\pi)$ decreases and also the frequency of hurricanes hitting landfall,

[22]This is primarily due to the assumption that the EIS is slightly above one ($\psi = 1.1$). If we increase the EIS to values at the higher end of estimates in the literature, there would be a more pronounced decline with π. We know that if we set $\psi = 1$, both i and q are independent of π in competitive equilibrium.

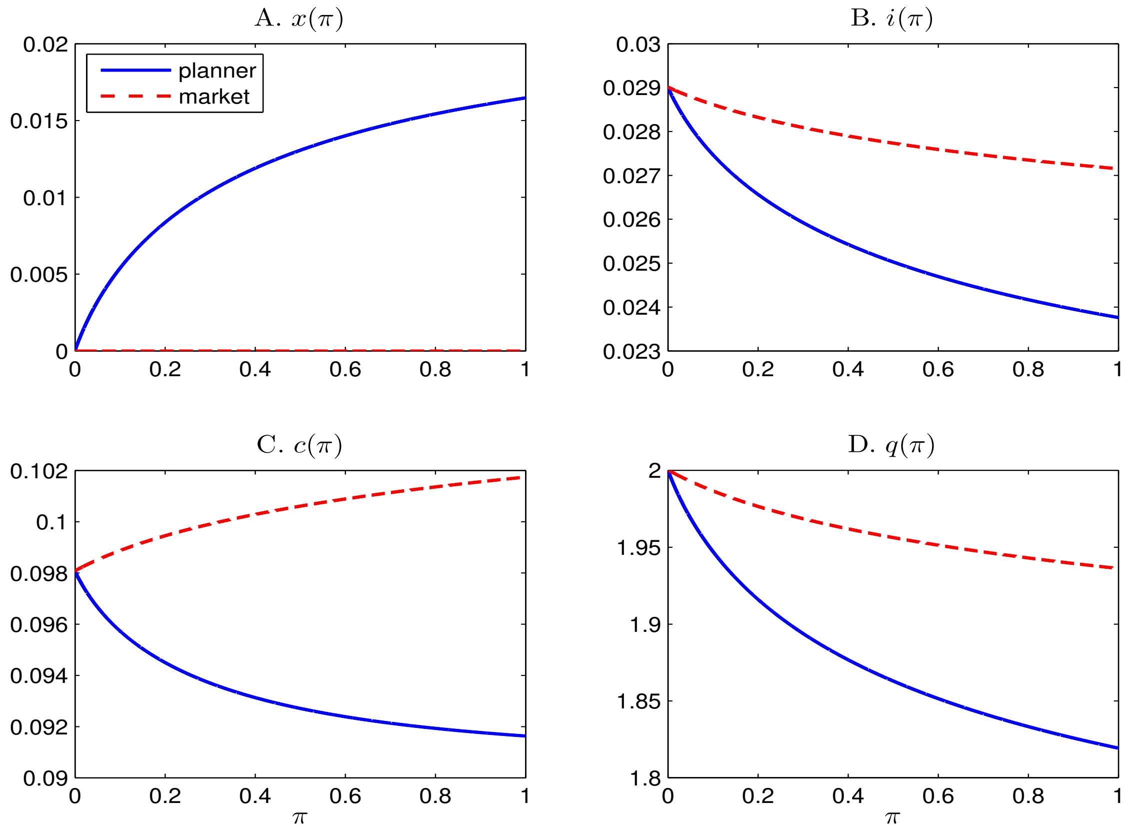

Figure 2: This figure plots (scaled) mitigation $x(\pi)$ (Panel A), investment-to-capital ratio $i(\pi)$ (Panel B), consumption-to-capital $c(\pi)$ (Panel C), and the value of capital (Panel D) as functions of π, belief regarding disaster arrival rates. $\pi = 0$ is the most optimistic belief in a low arrival rate and $\pi = 1$ is the most pessimistic belief in a high arrival rate. The parameters values are given in Table 1.

$\lambda(\pi)$, increases. Hence, expected total damage increases with π since conditional damage $\ell(\pi) = 1/(\beta_0 + 1) = 0.4\%$, invariant with π. The growth rate when $\pi = 0$ is close to 2.5% per annum. But at $\pi = 0.5$, the growth rate is close to zero due to adverse effects of disasters on capital stock.

6.3 Optimal Mitigation Spending on Seawalls

Now we turn to the planner's solution (solid blue lines) in Figure 2. Panel A shows that mitigation ramps up from zero to 1.3% of the capital stock and consumption decreases by 5% from 0.098 to 0.093 as we increase belief π from near zero to 0.5 (Panels A and C).

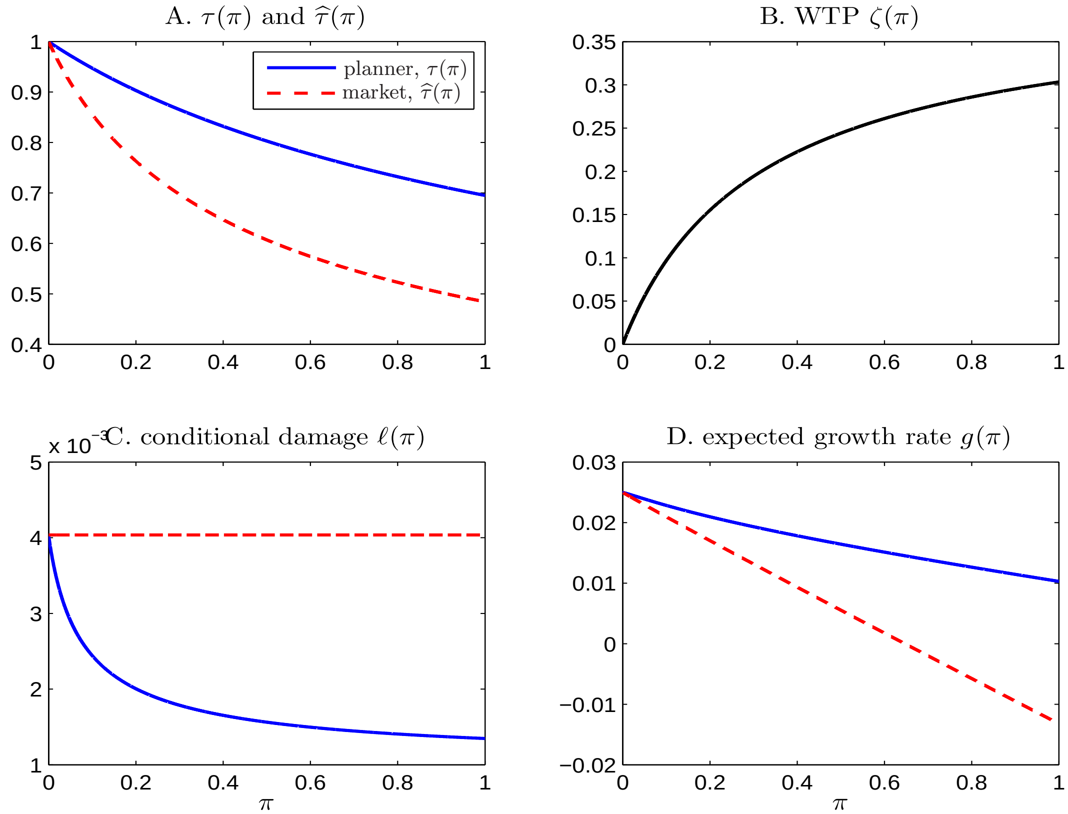

Figure 3: This figure plots the welfare and growth implications of optimal mitigation strategy. Panel A reports a welfare measure proportional to the household's certainty equivalent wealth. Panel B reports the household's willingness to pay (specifically, certainty equivalent wealth) for government mitigation spending (in percentage terms). Panels C and D report the conditional damage $\ell(\pi)$ and the expected growth $g(\pi)$, respectively. The parameters values are given in Table 1.

This is in contrast to the competitive equilibrium where there is no mitigation spending and consumption is rising with π. The additional impact of increasing π diminishes. Even when the household believes entirely in the bad scenario ($\pi = 1$), the mitigation spending increases only to 1.65%.

We can relate our calibration to existing seawall proposals. There are several proposals for seawalls under discussion. In particular, a non-profit group representing homeowners of coastal property have produced a *High Tide Tax* report for the Atlantic Region. Their estimate is around \$15 million per mile of seawall. With 46,000 miles long Atlantic coastline, this translates to roughly \$690 billion. There are other estimates based on proposals of Army Corp of Engineers for New York City that are significantly more expensive per mile. We can relate these estimates to our model's mitigation spending by amortizing these figures into an annual payment (a risk-free rate of 1% plus 8% maintenance cost per annum). In short, this proposal would entail spending \$62.1 billion dollars. The housing capital stock for is around \$15 trillion dollars for Atlantic region per year for seawall.[23] Realistically, not all \$15 trillion dollars of housing capital is under threat from hurricanes. A reasonable estimate is that say 50% of it is. This would then correspond to spending around 0.80% of per year on seawall mitigation. This 0.80% figure is less than our model's optimal mitigation spending of 1.3% corresponding to $\pi = 0.5$ but it does not account for improved drainage measures that we consider as part of seawall spending.

6.4 Investment, Tobin's q of Coastal Property, and Seawall Tax

Next we report the effect of mitigation on behavior of investment and Tobin's q for housing capital in planner's equilibrium. Going from $\pi = 0$ to $\pi = 0.5$, mitigation lowers investment i by 14% (from 0.029 to 0.025) and Tobin's average q by 7% from 2 at $\pi = 0$ to 1.863 (the value of $q(0.5)$ in the planner's problem). It is also informative to compare the distance in Tobin's q between the red dashed line corresponding to the competitive economy to the blue solid line corresponding to the first-best solution. Tobin's average q falls by 5% from 1.956 (the value of $q(0.5)$ in the competitive market with no mitigation) to 1.863 (the value of $q(0.5)$ in the planner's problem.)

In this vein, it is interesting to reflect on the quantitative implications of our model for tax policy and housing prices. The 1.3% figure for mitigation spending x as a fraction of capital stock from our calibration of Atlantic seawalls would then be the optimal annual tax rate for housing capital stock to fund the seawall spending, i.e. an annual seawall tax. Moreover, Tobin's q for housing capital stock would be around 5% lower as a result with optimal mitigation and moderate beliefs in the bad state ($\pi = 0.5$). In other words, due to

[23] We take 45% of the total housing capital stock in the US, which is around 30 trillion dollars, as an approximate value for the total capital stock in the Atlantic region.

mitigation externalities, Atlantic coastal property is around 5% higher in the competitive equilibrium than the first-best outcome. Moreover, in the competitive equilibrium, housing prices are only mildly sensitive to household beliefs to begin with. The change in prices moving from a competitive equilibrium to the first-best outcome where there are taxes is much larger quantitatively.

6.5 Social Welfare and WTP for Mitigation Technology

Despite the reduction of the asset market valuation, the society is better off. To see this, consider the blue solid line in Panel A of Figure 3, which captures the (scaled) certainty-equivalent wealth for the social-planner economy, $\tau(\pi) = b(\pi)/b(0)$. In contrast to the competitive equilibrium, $\tau(\pi)$ (the solid blue line) drops much less and stays above $\widehat{\tau}(\pi) = \widehat{b}(\pi)/\widehat{b}(0)$ (the dashed red line) because government mitigation generates substantial downside protection (curtailment or loosely speaking hedging benefits) which leads to higher social welfare.

In Panel B, we plot the WTP $\zeta(\pi)$ given in (40), which is equal to the difference between one and $\widehat{\tau}(\pi)/\tau(\pi)$, the ratio between the two lines in Panel A.[24] For example, even with $\pi = 0.5$, the household is willing to give up $\zeta(0.5) = 1 - \widehat{\tau}(0.5)/\tau(0.5) = 25\%$ of the existing capital stock for the government mitigation spending, as $\tau(0.5) = 0.80$ for the planner's problem and $\widehat{\tau}(0.5) = 0.60$ for the competitive market solution. And the WTP reaches the maximum of 30.4% at $\pi = 1$, i.e. $\zeta(1) = 30.4\%$.

In summary, the planner economy is willing to pay about 25% of the capital stock to move from the market solution to the planner's economy: $\zeta(0.5) = 25\%$. That the WTP for the mitigation technology (25% of capital stock) is significantly larger than the asset market value reduction (5%) reflects the general equilibrium effect (endogenous change of the stochastic discount factor as the economy switches from the market economy to the planner's economy). We elaborate on this point below.

[24] As we noted earlier, $b(0) = \widehat{b}(0)$. This is because the mitigation technology parameter so that the household optimally chooses no mitigation even with access to the technology in the most optimistic scenario.

6.6 Damage Functions and Growth Rates

In Panel C, we corroborate the benefit of using the mitigation technology by showing that the conditional damage $\ell(\pi)$ is lower as a result of mitigation. The more pessimistic the society, the greater the benefit of curtailing disaster risks and hence the lower the conditional damage $\ell(\pi)$, explaining the decreasing relation of $\ell(\pi)$ in π (see the solid blue line.)

Notice that the decline is highly non-linear in π. The reason is the following. Frequency of arrivals and inter-arrival times entirely drive perceived risks and hence mitigation in our model. As a result, damage of a disaster conditional on an arrival is much higher when perceived risks and mitigation are low, i.e., less preparedness. The reduced-form implication is that a disaster that strikes when society has a higher perceived risk will lead to lower conditional damage since society is more prepared with mitigation spending. In other words, a disaster that strikes after a long absence of disasters (i.e. low π) leads to much larger conditional damages than a disaster that strikes following a recent cluster of disasters (i.e. high π) due to time-varying preparedness.

As the society becomes more pessimistic (i.e., more weight on the bad scenario), the government mitigation spending increases and the conditional damage decreases (as we just discussed), which in turn significantly buffers growth slowdown by reducing the expected disaster damages.[25] This buffering is captures in Panel D. Whereas the expected grow rate $g(\pi)$ falls quickly with π in the competitive equilibrium, the decline in the expected growth rate corresponding to the social planner economy is milder. At $\pi = 0.5$, whereas the expected growth rate in the competitive economy is close to 0, it is nearly 1.6% in the planner economy.

6.7 Decomposing Effects of Seawall Tax on Tobin's Average q

Finally, we take a deeper look at the impact of the 1.3% seawall tax on Tobin's q of housing capital, which was only 5%. The impact on home prices is quite modest and contrary to discussions in regulatory circles on how the large impact of climate change taxes might affect

[25]As $i(\pi)$ decreases at a faster rate with π with mitigation, there is an opposing force that may cause expected growth $g(\pi)$ with mitigation to be higher than without mitigation. Quantitatively, we do not see this possibility in the figure with our parameter values.

asset values (Carney (2015)). One of the main messages of our model is that the mitigation tax comes with significant aggregate risk reduction benefits that are not emphasized enough. When valuing capital stock, the competitive market equilibrium solution and the solution for the planner's first-best outcome differ in two key aspects: one is that the cash flows differ as mitigation and investment are different in the two settings; the other is that the equilibrium SDF in the two economies is different as the equilibrium consumption for the representative agent is different.

To make this point more transparent, we consider a counterfactual exercise where the 1.3% tax is not spend on mitigation. That is, we decompose the total change of value of capital, Tobin's average q, as we move from competitive market equilibrium to the competitive equilibrium injected with optimal capital taxation and government optimal mitigation policy. In the latter economy, the first-best outcome is attained as (government and/or private) mitigation spending funded by stage-contingent capital taxation/subsidies fix the under-provision of aggregate risk mitigation achieving the first best.

We separate out the cash flow effect of taxation from the equilibrium SDF effect by conducting the following decomposition. First, we hold the SDF channel the same as we introduce optimal capital taxation into the economy. That is, we use the same SDF determined in the competitive market equilibrium with no capital taxation to value a firm's cash flows. But, in terms of cash flows, we fix I_t^* and X_t^* to the (first-best) levels for the planner's problem, and then interpret $x_t^* = X_t^*/K_t^*$ as the stochastic tax rate on capital to support the first-best outcome in the competitive equilibrium as discussed earlier.

We then ask the following counterfactual: If the government did not spend its tax proceeds to optimally mitigate and the firm pretends that it is still in the same market equilibrium (i.e., taking the SDF determined in the competitive market equilibrium), what is the value of capital stock, which we denote by $\overline{Q}_t$? We call this effect the "partial equilibrium" effect of taxation on the value of capital. Next, we quantify this "partial equilibrium" effect of taxation.

By using the standard asset-pricing theory, we obtain the following expression for $\overline{Q}_t$:

$$\overline{Q}_t = \mathbb{E}\left[\int_0^{\infty} \frac{\mathbb{M}_t}{\mathbb{M}_0}\left(AK_t - I_t^* - \Phi(I_t^*, K_t) - X_t^*\right) dt\right], \tag{42}$$

where $\mathbb{M}_t$ is the SDF for the competitive market equilibrium with no mitigation and is given by (B.22). As there is no mitigation spending despite capital taxation, the cumulative distribution function for Z is $\Xi(Z) = Z^{\beta_0}$.

By applying Ito's Lemma to $\mathbb{M}_{t-}(AK_{t-} - I_{t-}^* - \Phi(I_{t-}^*, K_{t-}) - X_{t-}^*)dt + d\left(\mathbb{M}_t\overline{Q}_t\right)$, which is a martingale (Duffie, 2001), we obtain the following ODE for $\overline{q}_t = \overline{q}_t/K_t = \overline{q}(\pi_t)$:

$$(r(\pi) + rp(\pi) - i^*(\pi))\overline{q}(\pi) = A - i^* - \phi(i^*) - x^* + \mu_\pi(\pi)\overline{q}'(\pi) + \lambda(\pi)(\overline{q}(\pi^{\mathcal{J}})\mathbb{E}(Z) - \overline{q}(\pi)), \tag{43}$$

where the risk-free rate, $r(\pi)$, and the risk premium, $rp(\pi)$, are given by (B.32) and (B.33), respectively.

Figure 4 decomposes the change of Tobin's average q as we introduce optimal capital taxation into the competitive equilibrium and use the tax proceeds to fund government mitigation. The red dashed line depicts the average q in the competitive equilibrium with no mitigation and the blue solid line describes the competitive equilibrium with optimal capital taxation and government mitigation. The dotted black line shows the counterfactual: the value of Tobin's average q if we value the firm's cash flow (obtained from the competitive equilibrium with optimal capital taxation and government mitigation) and value this cash flow using the SDF (obtained from the competitive equilibrium with no capital taxation and hence no mitigation.)

We show that the average q drops much more (for sufficiently large and empirically relevant range of π) in this "partial equilibrium" counter-factual calculation as the benefit of aggregate risk mitigation is ignored in this counterfactual exercise. Instead of a 4.8% drop (the gap between the dashed red and solid blue lines in Figure 4 at $\pi = 0.5$), property prices would then be lower by 7.7% with taxes but no benefits of mitigation (the gap between the dashed red and dotted black lines in Figure 4.) This effect is even more significant when we consider $\pi = 1$. In this instance, the optimal capital tax is 1.65% (see Figure 2) and the property value would be lower by 6.0% with optimal mitigation (the gap between dashed red and solid blue lines in 4) and by 15.4% without mitigation (the gap between dashed red and dotted black lines in 4.)

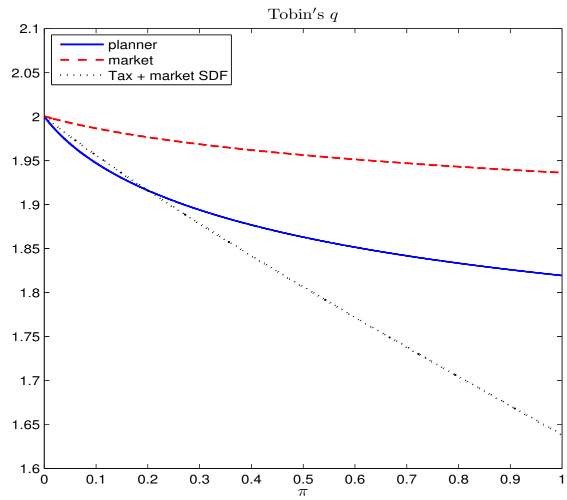

Figure 4: This figure plots the value of capital, i.e., Tobin's average q, in competitive equilibrium featuring no mitigation (dashed red line) and the planner's problem (solid blue line). The dotted black line depicts the average q, if the firm assumes the government does not spend on mitigation and investors take the SDF in the competitive market equilibrium to value the firm. The parameters values are given in Table 1.

7 Conclusion

We provide the planner's solution to a model where households learn from exogenous natural disaster arrivals about arrival rates and spend to mitigate potential future damages. Mitigation—by curtailing aggregate risk and insuring sustainable growth—is undersupplied relative to the first-best planner's solution in competitive markets due to externalities. The planner's solution can be implemented via a capital tax and mitigation subsidy scheme. Our model provides an integrated assessment of the cost and benefit of mitigation efforts such as seawalls via an aggregate risk management rationale. Our model also delivers a number of testable implications pertaining to damage functions and regulatory risks.

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Appendices

A Proof for Proposition 1 in Section 3

A.1 Planner's Resource Allocation

Substituting the value function (18) into the FOC (16) for investment and the FOC (17) for mitigation spending, we obtain:

$$b(\pi) = c(\pi)^{1/(1-\psi)} \left[\rho(1+\phi'(i(\pi)))\right]^{-\psi/(1-\psi)} , \tag{A.1}$$

$$\rho c(\pi)^{-\psi^{-1}} b(\pi)^{\psi^{-1}-1} = \frac{\lambda(\pi)}{1-\gamma} \left(\frac{b(\pi^{\mathcal{J}})}{b(\pi)}\right)^{1-\gamma} \int_0^1 \left[\frac{\partial \xi(Z;x)}{\partial x} Z^{1-\gamma}\right] dZ \, , \tag{A.2}$$

where the post-jump belief $\pi^{\mathcal{J}}$ is given in (12) as a function of the pre-jump belief π. Then substituting the resource constraint, $c(\pi) = A - i(\pi) - \phi(i(\pi)) - x(\pi)$, into (A.1), we obtain (20). By substituting (A.1) into (A.2), we obtain (21). Finally, substituting the value function, (18), and (20) and (21) into the HJB equation (15) and simplifying, we obtain the ODE given in (19).

By applying essentially the same argument to the right boundary, $\pi = 1$, we obtain the solution for $b(1)$, $i(1)$, and $x(1)$ by jointly solving the following three equations:

$$\begin{aligned} 0 &= \frac{\left(\frac{b(1)}{\rho(1+\phi'(i(1)))}\right)^{1-\psi} - 1}{1-\psi^{-1}} \rho + i(1) - \frac{\gamma\sigma^2}{2} + \frac{\lambda_B}{1-\gamma}\left(\mathbb{E}(Z^{1-\gamma}) - 1\right) , & \text{(A.3)} \\ b(1) &= \left(A - i(1) - \phi(i(1)) - x(1)\right)^{1/(1-\psi)} \left(\rho(1+\phi'(i(1)))\right)^{-\psi/(1-\psi)} , & \text{(A.4)} \\ \frac{1}{1+\phi'(i(1))} &= \frac{\lambda_B}{1-\gamma} \int_0^1 \left[\frac{\partial \xi(Z;x)}{\partial x} Z^{1-\gamma}\right] dZ \, . & \text{(A.5)} \end{aligned}$$

Using the same argument, we obtain the three equations, (22)-(24), for the left boundary, $\pi = 0$. Solving these three equations yields $b(0)$, $i(0)$, and $x(0)$.

A.2 Asset Pricing Implications of Planner's Problem

By using the results in Duffie and Epstein (1992), we obtain the following stochastic discount factor (SDF), $\{\mathbb{M}_t : t \geq 0\}$, implied by the planner's solution:

$$\mathbb{M}_t = \exp\left[\int_0^t f_V(C_s, V_s)\, ds\right] f_C(C_t, V_t) \, . \tag{A.6}$$

Using the FOC for investment (16), the value function (18), and the resource constraint, we obtain:

$$f_C(C, V) = \frac{1}{1+\phi'(i(\pi))} b(\pi)^{1-\gamma} K^{-\gamma} , \tag{A.7}$$

and

$$f_V(C,V) = \frac{\rho}{1-\psi^{-1}}\left[\frac{(1-\omega)C^{1-\psi^{-1}}}{((1-\gamma))^{\omega-1}}V^{-\omega} - (1-\gamma)\right] = -\epsilon(\pi)\,, \tag{A.8}$$

where

$$\epsilon(\pi) = -\frac{\rho(1-\gamma)}{1-\psi^{-1}}\left[\left(\frac{c(\pi)}{b(\pi)}\right)^{1-\psi^{-1}}\left(\frac{\psi^{-1}-\gamma}{1-\gamma}\right) - 1\right]. \tag{A.9}$$

Using the equilibrium relation between $b(\pi)$ and $c(\pi)$, we simplify (A.9) as:

$$\epsilon(\pi) = \rho + \left(\psi^{-1}-\gamma\right)\left[i(\pi) - \frac{\gamma\sigma^2}{2} + \mu_\pi(\pi)\frac{b'(\pi)}{b(\pi)} + \frac{\lambda(\pi)}{1-\gamma}\left(\left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}\mathbb{E}(Z^{1-\gamma}) - 1\right)\right], \tag{A.10}$$

where the post-jump belief $\pi^{\mathcal{J}}$ is given in (12) as a function of the pre-jump belief π.

Using Ito's Lemma and the optimal allocation, we have

$$\begin{aligned}\frac{d\mathbb{M}_t}{\mathbb{M}_{t-}} = & -\epsilon(\pi)dt - \gamma\left[i(\pi)dt + \sigma d\mathcal{W}_t\right] + \frac{\gamma(\gamma+1)}{2}\sigma^2 dt + \left((1-\gamma)\frac{b'(\pi)}{b(\pi)} - \frac{i'(\pi)\phi''(i(\pi))}{1+\phi'(i(\pi))}\right)\mu_\pi(\pi)dt \\ & + \left[\frac{1+\phi'\left(i(\pi)\right)}{1+\phi'\left(i(\pi^{\mathcal{J}})\right)}\left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma} Z^{-\gamma} - 1\right] d\mathcal{J}_t\,.\end{aligned} \tag{A.11}$$

As the expected rate of percentage change of $\mathbb{M}_t$ equals $-r_t$ (Duffie, 2001), we obtain the following expression for the interest rate:

$$\begin{aligned} r(\pi) \; = & \; \rho + \psi^{-1}i(\pi) - \frac{\gamma(\psi^{-1}+1)\sigma^2}{2} - \left[(1-\psi^{-1})\frac{b'(\pi)}{b(\pi)} - \frac{i'(\pi)\phi''(i(\pi))}{1+\phi'(i(\pi))}\right]\mu_\pi(\pi) \\ & - \lambda(\pi)\left[\frac{1+\phi'\left(i(\pi)\right)}{1+\phi'\left(i(\pi^{\mathcal{J}})\right)}\left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}\mathbb{E}(Z^{-\gamma}) - 1\right] \\ & - \lambda(\pi)\left[\frac{\psi^{-1}-\gamma}{1-\gamma}\left(1 - \left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}\mathbb{E}(Z^{1-\gamma})\right)\right]\,. \end{aligned} \tag{A.12}$$

Since the dividend D_t is equal to C_t in equilibrium and $\mathbb{M}_{t-}D_{t-}dt + d(\mathbb{M}_tQ_t)$ is a martingale under the physical measure (Duffie, 2001), using Ito's Lemma and setting its drift to zero, we obtain

$$\begin{aligned}\frac{c(\pi)}{q(\pi)} = & \; r(\pi) + \gamma\sigma^2 + \lambda(\pi)\left[\frac{1+\phi'\left(i(\pi)\right)}{1+\phi'\left(i(\pi^{\mathcal{J}})\right)}\left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}\left(\mathbb{E}(Z^{-\gamma}) - \frac{q(\pi^{\mathcal{J}})}{q(\pi)}\mathbb{E}(Z^{1-\gamma})\right)\right] \\ & - i(\pi) - \mu_\pi(\pi)\frac{q'(\pi)}{q(\pi)} \\ = & \; \rho - (1-\psi^{-1})\left[i(\pi) - \frac{\gamma\sigma^2}{2} + \mu_\pi(\pi)\frac{b'(\pi)}{b(\pi)}\right] + \lambda(\pi)\omega\left[1 - \left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}\mathbb{E}(Z^{1-\gamma})\right],\end{aligned} \tag{A.13}$$

where $\omega = (1-\psi^{-1})/(1-\gamma)$. We can calculate Tobin's average q from (A.13). For the special case with $\psi = 1$, $c(\pi)/q(\pi) = \rho$.

B Proof for Proposition 2 in Section 4

B.1 Household's Optimization Problem

The equilibrium aggregate stock value is $Q_t^* = q^*(\pi_t)K_t$. We conjecture and later verify that the cum-dividend return of the aggregate asset market is given by

$$\frac{dQ_t^* + D_{t-}dt}{Q_{t-}^*} = \mu_Q(\pi_{t-})dt + \sigma d\mathcal{W}_t - \left(1 - Z\frac{q^*(\pi_t^{\mathcal{J}})}{q^*(\pi_{t-})}\right) d\mathcal{J}_t\,, \tag{B.14}$$

where $\pi_t^{\mathcal{J}} = \lambda_B \pi_{t-}/\lambda(\pi_{t-})$ is the post-jump belief and $\mu_Q(\pi)$ is the expected cum-dividend return (ignoring the jump effect). In (B.14), the diffusion volatility is equal to σ, the same parameter as in (3), which we verify later. Also, by using the homogeneity property, we have conjectured a specific form for the change of the cum-dividend return should a jump occur, which we also verify later.

When a disaster occurs at time t, wealth changes discretely from W_{t-} to $W_t^{\mathcal{J}}$, where

$$W_t^{\mathcal{J}} = W_{t-} - \left(1 - Z\frac{q^*(\pi_t^{\mathcal{J}})}{q^*(\pi_{t-})}\right) H_{t-} + \delta_{t-}(Z)W_{t-}\,. \tag{B.15}$$

The second term is the loss of the portfolio's market value upon the arrival of a disaster and the last term is the repayment from the DIS contract entered at $t-$. While the mitigation spending X_c makes disasters less damaging for the society, it does not generate any direct benefit for the household upon a jump arrival. This is at the core of the market failure.

The household accumulates wealth as:

$$\begin{aligned} dW_t \;=\;& r(\pi_{t-})W_{t-}dt + (\mu_Q(\pi_{t-}) - r)H_{t-}dt + \sigma H_{t-}d\mathcal{W}_t - C_{t-}dt - X_{c,t-}dt \\ & - \left(\int_0^1 \delta_{t-}(Z)p(Z; x_{t-}^*)dZ\right) W_{t-}dt + \delta_{t-}(Z)W_{t-}d\mathcal{J}_t - \left(1 - Z\frac{q^*(\pi_t^{\mathcal{J}})}{q^*(\pi_{t-})}\right) H_{t-}d\mathcal{J}_t\,. \end{aligned} \tag{B.16}$$

The first four terms in (B.16) are standard in the classic portfolio-choice problem with no insurance or disasters (Merton, 1971). The fifth term $X_{c,t-}dt$ is the cost of the household's mitigation spending. The sixth term is the total DIS premium paid by the households before the arrival of disasters. Note that this term captures the financial hedging cost. The seventh term describes the DIS payments by the DIS seller to the household when a disaster occurs. The last term is the loss of the household's wealth from her portfolio's exposure to the asset market.

The HJB equation for the household in our decentralized market setting is given by

$$\begin{aligned} 0 = \max_{C,H,\delta,X_c}\; & f(C,J) + \mu_\pi(\pi)J_\pi + \frac{\sigma^2 H^2 J_{WW}}{2} + \lambda(\pi)\int_0^1 \left[J\left(W^{\mathcal{J}}, \pi^{\mathcal{J}}\right) - J(W,\pi)\right]\xi(Z; x^*)dZ \\ & + \left[r(\pi)W + (\mu_Q(\pi) - r(\pi))H - \left(\int_0^1 \delta(Z)p(Z;x^*)dZ\right) W - C - X_c\right] J_W\,, \end{aligned} \tag{B.17}$$

where $\pi^{\mathcal{J}}$ is the post-jump belief given in (12) and $W^{\mathcal{J}}$ is the post-jump wealth given in (B.15). The term, $\left(\int_0^1 \delta_t(Z)p(Z;x^*)dZ\right) W_t dt$, is the total DIS premium payment in the time interval $(t, t+dt)$.

The FOCs for consumption C and the market portfolio allocation H are given by

$$f_C(C,J) = J_W(W,\pi) \tag{B.18}$$

$$\sigma^2 H J_{WW}(W,\pi) = -(\mu_Q(\pi) - r(\pi))J_W(W,\pi) + \lambda(\pi)\mathbb{E}\left[\left(1 - Z\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right) J_W\left(W^{\mathcal{J}}, \pi^{\mathcal{J}}\right)\right]. \tag{B.19}$$

The second term in (B.19) captures the the jump effect on the household's portfolio choice. The DIS demand $\delta(Z)$ for each Z is given by

$$p(Z;x^*)J_W(W,\pi) = \lambda(\pi)J_W\left(W^{\mathcal{J}},\pi^{\mathcal{J}}\right)\xi(Z;x^*). \tag{B.20}$$

The left side of (B.20) is the marginal (utility) cost when the household purchases a unit of DIS contract and the right side of (B.20) is the marginal (utility) benefit. This FOC follows from the point-by-point optimization in (B.17) for the DIS demand and hence it holds for all levels of Z.

Substituting (33) into the FOC (B.18) yields the following consumption rule:

$$C(W,\pi) = \rho^{\psi} u(\pi)^{1-\psi} W, \tag{B.21}$$

which is linear in W but nonlinear in belief π in general.

B.2 Firm Value Maximization

Using (33) and (B.21), we conjecture and then verify later that the SDF is given by

$$\frac{d\mathbb{M}_t}{\mathbb{M}_{t-}} = -r(\pi_{t-})dt - \gamma\sigma d\mathcal{W}_t + \left[\left(\frac{u^*(\pi_t^{\mathcal{J}})}{u^*(\pi_{t-})}\right)^{1-\gamma}\left(\frac{q^*(\pi_t^{\mathcal{J}})}{q^*(\pi_{t-})}\right)^{-\gamma} Z^{-\gamma} - 1\right](d\mathcal{J}_t - \lambda(\pi_{t-})dt). \tag{B.22}$$

The equilibrium drift of $d\mathbb{M}_t/\mathbb{M}_{t-}$ is $-r(\pi_{t-})$ (Duffie, 2001). The last term is a jump martingale and the terms inside the square bracket follow from (A.6), (33), and (B.21).

By using Ito's Lemma, we obtain the following dynamics for $Q_t = Q(K_t, \pi_t)$:

$$dQ = \left(IQ_K + \frac{1}{2}\sigma^2 K^2 Q_{KK} + \mu_\pi(\pi)Q_\pi\right)dt + \sigma K Q_K d\mathcal{W}_t + \left(Q(ZK,\pi^{\mathcal{J}}) - Q(K,\pi)\right)d\mathcal{J}_t. \tag{B.23}$$

No arbitrage implies the drift of $\mathbb{M}_{t-}(AK_{t-} - I_{t-} - \Phi(I_{t-},K_{t-}) - X_{f,t-})dt + d\left(\mathbb{M}_t Q_t\right)$ is zero. By applying Ito's Lemma to this martingale, we obtain

$$\begin{aligned}
0 = \max_{I, X_f}\ & \mathbb{M}_{t-}(AK - I - \Phi(I,K) - X_f)dt + \mathbb{M}_{t-}\left(Q_K + \frac{1}{2}\sigma^2 K^2 Q_{KK} + \mu_\pi(\pi)Q_\pi\right)dt \\
& + Q\left[-r(\pi) - \lambda(\pi)\mathbb{E}\left(\left(\frac{u^*(\pi^{\mathcal{J}})}{u^*(\pi)}\right)^{1-\gamma}\left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{-\gamma} Z^{-\gamma} - 1\right)\right]\mathbb{M}_{t-}dt - \mathbb{M}_{t-}\gamma\sigma^2 K Q_K dt \\
& + \lambda(\pi)\mathbb{E}\left[\left(\frac{u^*(\pi^{\mathcal{J}})}{u^*(\pi)}\right)^{1-\gamma}\left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{-\gamma} Z^{-\gamma}Q(ZK,\pi^{\mathcal{J}}) - Q(K,\pi)\right]\mathbb{M}_{t-}dt.
\end{aligned} \tag{B.24}$$

And then by using $Q(K,\pi) = q(\pi)K$, we obtain (31).

B.3 Market Equilibrium

First, mitigation spending for both households and firms is zero: $x_c = x_f = 0$. Second, in equilibrium, the household (1) invests all wealth in the asset market and holds no risk-free asset, $H = W$ and $W = Q^*$; (2) has zero disaster hedging position, $\delta(Z) = 0$ for all Z. Simplifying the FOCs, (B.18), (B.19), and (B.20), and using the preceding equilibrium conditions, we obtain we obtain:

$$c^*(\pi) = \rho^{\psi} u(\pi)^{1-\psi} q^*(\pi)\,, \tag{B.25}$$

$$\mu_Q(\pi) = r(\pi) + \gamma\sigma^2 + \lambda(\pi)\left(\frac{u(\pi^{\mathcal{J}})}{u(\pi)}\right)^{1-\gamma}\left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{-\gamma}\left[\mathbb{E}(Z^{-\gamma}) - \frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\mathbb{E}(Z^{1-\gamma})\right], \tag{B.26}$$

$$p(Z;0) = \lambda(\pi)\left(\frac{u(\pi^{\mathcal{J}})}{u(\pi)}\right)^{1-\gamma}\left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{-\gamma} Z^{-\gamma}\xi(Z;0)\,. \tag{B.27}$$

Using these equilibrium conditions, we simplify the HJB equation (B.17) as follows:

$$\begin{aligned} 0 \quad = \quad & \frac{1}{1-\psi^{-1}}\left(\frac{c^*(\pi)}{q^*(\pi)} - \rho\right) + \left(\mu_Q(\pi) - \frac{c^*(\pi)}{q^*(\pi)}\right) - \frac{\gamma\sigma^2}{2} + \mu_\pi(\pi)\frac{u'(\pi)}{u(\pi)} \\ & + \frac{\lambda(\pi)}{1-\gamma}\left[\left(\frac{u(\pi^{\mathcal{J}})q^*(\pi^{\mathcal{J}})}{u(\pi)q^*(\pi)}\right)^{1-\gamma}\mathbb{E}(Z^{1-\gamma}) - 1\right]. \end{aligned} \tag{B.28}$$

Third, by substituting $c^*(\pi) = A - i^*(\pi) - \phi(i^*(\pi))$ into (31), we obtain

$$\begin{aligned} 0 = {} & \frac{c^*(\pi)}{q^*(\pi)} - r(\pi) + i(\pi) + \mu_\pi(\pi)\frac{q^*_\pi(\pi)}{q^*(\pi)} - \gamma\sigma^2 \\ & - \lambda(\pi)\left(\frac{u^*(\pi^{\mathcal{J}})}{u^*(\pi)}\right)^{1-\gamma}\left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{-\gamma}\left[\mathbb{E}(Z^{-\gamma}) - \frac{q(\pi^{\mathcal{J}})}{q(\pi)}\mathbb{E}(Z^{1-\gamma})\right]. \end{aligned} \tag{B.29}$$

By using the homogeneity property and comparing (B.14) and (B.23), we have

$$\mu_Q(\pi) = \frac{c^*(\pi)}{q^*(\pi)} + i^*(\pi) + \mu_\pi(\pi)\frac{(q^*(\pi))'}{q^*(\pi)}\,. \tag{B.30}$$

And then substituting (B.30) into (B.28), we obtain

$$\begin{aligned} \frac{c^*(\pi)}{q^*(\pi)} \quad = \quad & \rho - (1-\psi^{-1})\left[i^*(\pi) - \frac{\gamma\sigma^2}{2} + \mu_\pi(\pi)\left(\frac{u'(\pi)}{u(\pi)} + \frac{(q^*(\pi))'}{q^*(\pi)}\right)\right] \\ & + \lambda(\pi)\left(\frac{1-\psi^{-1}}{1-\gamma}\right)\left[1 - \left(\frac{u(\pi^{\mathcal{J}})q^*(\pi^{\mathcal{J}})}{u(\pi)q^*(\pi)}\right)^{1-\gamma}\mathbb{E}(Z^{1-\gamma})\right]. \end{aligned} \tag{B.31}$$

Finally, substituting (B.31) into (B.29), we obtain the following equilibrium interest rate:

$$
\begin{aligned}
r(\pi) &= \rho + \psi^{-1} i^*(\pi) - \frac{\gamma(\psi^{-1}+1)\sigma^2}{2} - \left[(1-\psi^{-1})\left(\frac{u'(\pi)}{u(\pi)} + \frac{(q^*(\pi))'}{q^*(\pi)}\right) - \frac{(q^*(\pi))'}{q^*(\pi)}\right]\mu_\pi(\pi) \\
&\quad - \lambda(\pi)\left[\left(\frac{u^*(\pi^{\mathcal{J}})}{u^*(\pi)}\right)^{1-\gamma}\left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{-\gamma}\mathbb{E}(Z^{-\gamma}) - 1\right] \\
&\quad - \lambda(\pi)\left[\frac{\psi^{-1}-\gamma}{1-\gamma}\left(1 - \left(\frac{u(\pi^{\mathcal{J}})q^*(\pi^{\mathcal{J}})}{u(\pi)q^*(\pi)}\right)^{1-\gamma}\mathbb{E}(Z^{1-\gamma})\right)\right] .
\end{aligned}
\tag{B.32}
$$

Finally, we obtain the following expression for the market risk premium, which we denote by $rp(\pi)$:

$$
rp(\pi) = \gamma\sigma^2 + \lambda(\pi)\left[\left(\frac{u(\pi^{\mathcal{J}})}{u(\pi)}\right)^{1-\gamma}\left(\frac{q(\pi^{\mathcal{J}})}{q(\pi)}\right)^{-\gamma}\left(\mathbb{E}(Z^{-\gamma}) - \frac{q(\pi^{\mathcal{J}})}{q(\pi)}\mathbb{E}[Z^{1-\gamma}]\right) + \frac{q\left(\pi^{\mathcal{J}}\right)}{q(\pi)}\mathbb{E}(Z) - 1\right] \tag{B.33}
$$

B.4 Equivalence between Market Solution and Planner's Problem with No Mitigation Technology

The value function for the planner's problem $V(K,\pi)$ with no mitigation technology (i.e., $x=0$), is equal to the household's value function under competitive equilibrium, $J(W,\pi)$. As the household's wealth is equal to the total asset market capitalization, i.e., $W = q(\pi)K$ in equilibrium, $b(\pi)$ in the planner's problem is equal to $u(\pi)q(\pi)$ in the decentralization formulation. The optimal consumption in the planner's problem (20) with no mitigation is the same as (B.25) in the decentralized market formulation. The resource constraints $A = i(\pi) + \phi(i(\pi)) + c(\pi)$ then implies that investment is also the same in the two formulations.

By substituting $b(\pi) = u(\pi)q(\pi)$ into ODE (19) for the planner's problem, we have

$$
\begin{aligned}
0 \quad = \quad & \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{u(\pi)}{\rho}\right)^{1-\psi} - 1\right] + i(\pi) - \frac{\gamma\sigma^2}{2} + \mu_\pi(\pi)\left(\frac{u'(\pi)}{u(\pi)} + \frac{q'(\pi)}{q(\pi)}\right) \\
& + \frac{\lambda(\pi)}{1-\gamma}\left[\left(\frac{u(\pi^{\mathcal{J}})}{u(\pi)q(\pi)}\right)^{1-\gamma}\mathbb{E}(Z^{1-\gamma}) - 1\right] ,
\end{aligned}
\tag{B.34}
$$

which is consistent with the market solution given in (B.31). By substituting $b(\pi) = u(\pi)q(\pi)$ and $q(\pi) = 1 + \phi'(i(\pi))$ into (B.57), we verify that the interest rate process is the same in the two formulations, e.g., (B.32) and (B.57) are the same in equilibrium.

In sum, we have verified that the resource allocation in the decentralized market formulation features no mitigation in equilibrium due (a free-rider's problem) and hence is the same as in the social planner's problem with no mitigation spending.

Supplementary Internet Appendices

A Proof for Proposition 3 in Section 5

As in Section 4.1, we include the following securities (traded at each point in time): (i) a risk-free asset, (ii) a claim on the value of firm's capital, and (iii) insurance claims for disasters with every possible recovery fraction Z.

We define the economy as follows: (a) The representative household dynamically chooses consumption C_t, investments in the risk-free asset and risky equity, and various DIS claims to maximize utility as given by (5) and (6); (b) The representative firm chooses the level of investment I_t to maximize its market value taking the equilibrium SDF as given; (c) The government chooses mitigation spending X_t to maximize the representative household's utility as given by (5) and (6); (d) All markets clear. We use superscript * to denote the equilibrium variables and/or processes.

In this section, we verify that the market mechanism delivers the first-best consumption and investment policies provided that the mitigation spending is chosen by a benevolent government.

A.1 Household Optimization

As in Section B, the household accumulates wealth as:

$$\begin{aligned} dW_t &= r(\pi_{t-})W_{t-}dt + (\mu_Q(\pi_t) - r)H_{t-}dt + \sigma H_{t-}d\mathcal{W}_t - C_{t-}dt \\ &\quad - \left(\int_0^1 \delta_{t-}(Z)p(Z;x^*_{t-})dZ\right) W_{t-}dt + \delta_{t-}(Z)W_{t-}d\mathcal{J}_t - \left(1 - Z\frac{q^*(\pi^{\mathcal{J}}_{t-})}{q^*(\pi_{t-})}\right) H_{t-}d\mathcal{J}_t\,, \end{aligned} \tag{A.35}$$

where x^* is chosen by the government. As it is in the household's interest to choose no mitigation spending, we leave this term out of (A.35). The HJB equation for the household in this setting is:

$$\begin{aligned} 0 = \max_{C,H}\ & f(C,J) + \left[r(\pi)W + (\mu_Q(\pi) - r(\pi))H - \left(\int_0^1 \delta(Z)p(Z;x^*)dZ\right) W - C\right] J_W \\ & + \mu_\pi(\pi)J_\pi + \frac{\sigma^2 H^2 J_{WW}}{2} + \lambda(\pi)\int_0^1 \left[J\left(W^{\mathcal{J}}, \pi^{\mathcal{J}}\right) - J(W,\pi)\right]\xi(Z;x^*)dZ\,, \end{aligned} \tag{A.36}$$

Additionally, the FOCs for consumption, market portfolio allocation, and DIS demand are the same as (B.18)-(B.20). Let $J(W,\pi)$ denote the household's value function, given in (33).

Imposing the equilibrium outcome on the households' side, we obtain (B.25), (B.26), and (B.27). Using (33) and these conditions to simplify (A.36), we obtain the following ODE for $u(\pi)$:

$$\begin{aligned} 0 \;=\;& \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{u(\pi)}{\rho}\right)^{1-\psi} - 1\right] + \left(\mu_Q(\pi) - \rho^\psi u(\pi)^{1-\psi}\right) - \frac{\gamma\sigma^2}{2} \\ & + \mu_\pi(\pi)\frac{u'(\pi)}{u(\pi)} + \frac{\lambda(\pi)}{1-\gamma}\left[\left(\frac{u(\pi^{\mathcal{J}})q^*(\pi^{\mathcal{J}})}{u(\pi)q^*(\pi)}\right)^{1-\gamma}\int_0^1 Z^{1-\gamma}\xi(Z;x^*)dZ - 1\right]. \end{aligned} \tag{A.37}$$

A.2 Firm Value Maximization

Taking the SDF in (B.22) as given, the firm chooses investment I to solve:

$$\max_{I,X_f} \mathbb{E}\left[\int_0^\infty \frac{\mathbb{M}_t}{\mathbb{M}_0}\left(AK_t - I_t - \Phi_t - X_{f,t} - X_t^*\right)dt\right], \tag{A.38}$$

where X^* is chosen by the government and hence exogenous to the firm. By applying Ito's Lemma to $\mathbb{M}_{t-}(AK_{t-} - I_{t-} - \Phi(I_{t-}, K_{t-}) - X_{t-}^*)dt + d\left(\mathbb{M}_t Q_t\right)$, which is a martingale due to no arbitrage (Duffie, 2001), we obtain the following ODE for $q_t = Q_t/K_t = q(\pi_t)$:

$$\begin{aligned}
r(\pi)q(\pi) = {} & A - i - \phi(i) - x^* + i(\pi)q(\pi) + \mu_\pi(\pi)q'(\pi) \\
& - \left[\gamma\sigma^2 + \lambda(\pi)\left(\left(\frac{u^*(\pi\mathcal{J})}{u^*(\pi)}\right)^{1-\gamma}\left(\frac{q^*(\pi\mathcal{J})}{q^*(\pi)}\right)^{-\gamma}\int_0^1 Z^{-\gamma}\xi(Z;x^*)dZ - 1\right)\right]q(\pi) \\
& + \lambda(\pi)\left[\left(\frac{u^*(\pi\mathcal{J})}{u^*(\pi)}\right)^{1-\gamma}\left(\frac{q^*(\pi\mathcal{J})}{q^*(\pi)}\right)^{-\gamma}\frac{q(\pi\mathcal{J})}{q(\pi)}\int_0^1 Z^{1-\gamma}\xi(Z;x^*)dZ - 1\right]q(\pi)\,.
\end{aligned} \tag{A.39}$$

By differentiating (A.39) with respect to i, we obtain the investment FOC:

$$q(\pi) = 1 + \phi'(i)\,. \tag{A.40}$$

By using the aggregate resource constraint, $c^*(\pi) = A - i^*(\pi) - \phi(i^*(\pi)) - x^*$, we obtain the following expression for the equilibrium expected return of the aggregate asset market:

$$\mu_Q(\pi) = \frac{A - i^*(\pi) - \phi(i^*(\pi)) - x^*}{q^*(\pi)} + i^*(\pi) + \mu_\pi(\pi)\frac{(q^*(\pi))'}{q^*(\pi)}\,. \tag{A.41}$$

By substituting (A.41) into (A.37), we obtain,

$$\begin{aligned}
0 \quad = \quad & \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{u(\pi)}{\rho}\right)^{1-\psi} - 1\right] + \left(\frac{A - i^*(\pi) - \phi(i^*(\pi)) - x^*}{q^*(\pi)} + i^*(\pi) + \mu_\pi(\pi)\frac{(q^*(\pi))'}{q^*(\pi)} - \rho^\psi u(\pi)^{1-\psi}\right) \\
& -\frac{\gamma\sigma^2}{2} + \mu_\pi(\pi)\frac{u'(\pi)}{u(\pi)} + \frac{\lambda(\pi)}{1-\gamma}\left[\left(\frac{u(\pi\mathcal{J})q^*(\pi\mathcal{J})}{u(\pi)q^*(\pi)}\right)^{1-\gamma}\int_0^1 Z^{1-\gamma}\xi(Z;x^*)dZ - 1\right],
\end{aligned} \tag{A.42}$$

which is the simplified HJB equation for the government. Using the FOC for x, we obtain

$$1 = \frac{\lambda(\pi)q^*(\pi)}{1-\gamma}\left(\frac{u(\pi\mathcal{J})q^*(\pi\mathcal{J})}{u(\pi)q^*(\pi)}\right)^{1-\gamma}\int_0^1 Z^{1-\gamma}\frac{\partial\xi(Z;x^*)}{\partial x^*}dZ\,, \tag{A.43}$$

which implies (21) by using $b(\pi) = u(\pi)q(\pi)$ and $q(\pi) = 1 + \phi'(i)$, which also hold in Section B.

Using the same analysis as in Section B, we can show that the equilibrium dividend yield, $c^*(\pi)/q^*(\pi)$, is given by (B.31) and the equilibrium interest rate is given by (B.32).

Finally, we require all the variables at the micro level to equal the corresponding variables at the macro level, e.g., $c(\pi) = c^*(\pi)$, $i(\pi) = i^*(\pi)$, and $u(\pi) = u^*(\pi)$.

B Mitigation Spending Benefits: Alternative Model

In this section, we consider an alternative specification where by spending on mitigation, a private agent reduces the damage of a disaster upon its arrival. This specification is different from the baseline model in Section 2, which assumes that mitigation spending influences the distribution of the post-jump recovery fraction Z.

Specifically, we assume that for a given pre-jump mitigation spending X_{t-}, the post-jump fractional loss changes from $(1-Z)$ to $N(x_{t-})(1-Z)$, where $0 \leq N(x) \leq 1$, $N'(x) \leq 0$, and $N''(x) \leq 0$. Note that as in our baseline model, this specification also has the homogeneity property in K. Capital stock K evolves as:

$$dK_t = I_{t-}dt + \sigma K_{t-}d\mathcal{W}_t - N(x_{t-})(1-Z)K_{t-}d\mathcal{J}_t \ . \tag{B.44}$$

All the other parts of the model remain unchanged. We show that the welfare theorem holds in this setting as no private agent has incentive to free ride on others. This is because the private agent's and the societal FOCs are the same regarding mitigation spending and other choices.

Next, we show planner's solution and then competitive market solution.

B.1 Planner's Solution

The HJB equation for the planner's allocation problem is:

$$\begin{aligned} 0 \ &= \ \max_{C,I,x} \ f(C,V) + IV_K(K,\pi) + \mu_\pi(\pi)V_\pi(K,\pi) + \frac{1}{2}\sigma^2K^2V_{KK}(K,\pi) \\ &\quad + \lambda(\pi)\mathbb{E}\left[V\left((1-N(x)(1-Z))K,\pi^{\mathcal{J}}\right) - V(K,\pi)\right] \ , \end{aligned} \tag{B.45}$$

And the the first-order condition (FOC) for investment I is

$$(1+\Phi_I(I,K))f_C(C,V) = V_K(K,\pi) \ . \tag{B.46}$$

The FOC with respect to mitigation spending X is

$$Kf_C(C,V) = \lambda(\pi)N'(x)\mathbb{E}\left[(Z-1)V_K\left((1-N(x)(1-Z))K, \frac{\pi\lambda_B}{\lambda(\pi)}\right)\right] \ . \tag{B.47}$$

Using the FOCs (B.46) and (B.47) and substituting the value function $V(K,\pi)$ given in (18) together with the implied policy rules into the HJB equation (B.45), and simplifying the equations, we obtain the following three-equation ODE system for $b(\pi)$, $i(\pi)$, and $x(\pi)$:

$$\begin{aligned} 0 \ &= \ \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{b(\pi)}{\rho(1+\phi'(i(\pi)))}\right)^{1-\psi} - 1\right] + i(\pi) - \frac{\gamma\sigma^2}{2} + \mu_\pi(\pi)\frac{b'(\pi)}{b(\pi)} \\ &\quad + \frac{\lambda(\pi)}{1-\gamma}\left[\left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}\mathbb{E}\left[(1-N(x)(1-Z))^{1-\gamma}\right] - 1\right] \ , \end{aligned} \tag{B.48}$$

$$b(\pi) \ = \ [A - i(\pi) - \phi(i(\pi)) - x(\pi)]^{1/(1-\psi)}\left[\rho(1+\phi'(i(\pi)))\right]^{-\psi/(1-\psi)} \ , \tag{B.49}$$

$$1 \ = \ \lambda(\pi)(1+\phi'(i(\pi)))\left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}N'(x)\mathbb{E}\left[(Z-1)(1-N(x)(1-Z))^{-\gamma}\right] \ . \tag{B.50}$$

And by setting $\pi = 0$ and $\pi = 1$, we obtain the corresponding boundary conditions.

B.2 Asset Pricing Implications of Planner's Problem

By using the results in Duffie and Epstein (1992), we obtain the following stochastic discount factor (SDF), $\{\mathbb{M}_t : t \geq 0\}$, implied by the planner's solution:

$$\mathbb{M}_t = \exp\left[\int_0^t f_V(C_s, V_s)\, ds\right] f_C(C_t, V_t)\ . \tag{B.51}$$

Using the investment FOC (B.46), the value function (18), and the resource constraint, we obtain:

$$f_C(C, V) = \frac{1}{1 + \phi'(i(\pi))} b(\pi)^{1-\gamma} K^{-\gamma}\,, \tag{B.52}$$

and

$$f_V(C, V) = \frac{\rho}{1 - \psi^{-1}} \left[\frac{(1-\omega)C^{1-\psi^{-1}}}{((1-\gamma))^{\omega-1}} V^{-\omega} - (1-\gamma)\right] = -\epsilon(\pi)\,, \tag{B.53}$$

where

$$\epsilon(\pi) = -\frac{\rho(1-\gamma)}{1-\psi^{-1}} \left[\left(\frac{c(\pi)}{b(\pi)}\right)^{1-\psi^{-1}} \left(\frac{\psi^{-1} - \gamma}{1-\gamma}\right) - 1\right]. \tag{B.54}$$

Using the equilibrium relation between $b(\pi)$ and $c(\pi)$, we simplify (B.54) as:

$$\begin{aligned}\epsilon(\pi) = \rho &+ \left(\psi^{-1} - \gamma\right) \left[i(\pi) - \frac{\gamma\sigma^2}{2} + \mu_\pi(\pi)\frac{b'(\pi)}{b(\pi)}\right] \\ &+ \left(\psi^{-1} - \gamma\right) \left[\frac{\lambda(\pi)}{1-\gamma} \left(\left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma} \mathbb{E}\left[(1 - N(x)(1-Z))^{1-\gamma}\right] - 1\right)\right]\,.\end{aligned} \tag{B.55}$$

Using Ito's Lemma and the optimal allocation, we have

$$\begin{aligned}\frac{d\mathbb{M}_t}{\mathbb{M}_{t-}} = &-\epsilon(\pi)dt - \gamma\left[i(\pi)dt + \sigma d\mathcal{W}_t\right] + \frac{\gamma(\gamma+1)}{2}\sigma^2 dt + \left((1-\gamma)\frac{b'(\pi)}{b(\pi)} - \frac{i'(\pi)\phi''(i(\pi))}{1+\phi'(i(\pi))}\right)\mu_\pi(\pi)dt \\ &+ \left[\frac{1+\phi'\left(i(\pi)\right)}{1+\phi'\left(i(\pi^{\mathcal{J}})\right)} \left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma} (1 - N(x)(1-Z))^{-\gamma} - 1\right] d\mathcal{J}_t\,.\end{aligned} \tag{B.56}$$

As the expected rate of percentage change of $\mathbb{M}_t$ equals $-r_t$ (Duffie, 2001), we obtain the following expression for the interest rate:

$$\begin{aligned}r(\pi) \ = \rho &+ \psi^{-1} i(\pi) - \frac{\gamma(\psi^{-1}+1)\sigma^2}{2} - \left[(1-\psi^{-1})\frac{b'(\pi)}{b(\pi)} - \frac{i'(\pi)\phi''(i(\pi))}{1+\phi'(i(\pi))}\right]\mu_\pi(\pi) \\ &- \lambda(\pi)\left[\frac{1+\phi'\left(i(\pi)\right)}{1+\phi'\left(i(\pi^{\mathcal{J}})\right)} \left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma} \mathbb{E}((1 - N(x)(1-Z))^{-\gamma}) - 1\right] \\ &- \lambda(\pi)\left[\frac{\psi^{-1}-\gamma}{1-\gamma}\left(1 - \left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma} \mathbb{E}((1 - N(x)(1-Z))^{1-\gamma})\right)\right]\,.\end{aligned} \tag{B.57}$$

Since the dividend D_t is equal to C_t in equilibrium and $\mathbb{M}_{t-}D_{t-}dt + d(\mathbb{M}_t Q_t)$ is a martingale (Duffie, 2001), by using Ito's Lemma and setting its drift to zero, we obtain

$$\frac{c(\pi)}{q(\pi)} = \rho - (1-\psi^{-1})\left[i(\pi) - \frac{\gamma\sigma^2}{2} + \mu_\pi(\pi)\frac{b'(\pi)}{b(\pi)}\right] + \lambda(\pi)\omega\left[1 - \left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}\mathbb{E}((1-N(x)(1-Z))^{1-\gamma})\right], \tag{B.58}$$

where $\omega = (1-\psi^{-1})/(1-\gamma)$.

B.3 Market Equilibrium Solution

Competitive Equilibrium. (i) the net supply of the risk-free asset is zero; (ii) the demand for the unlevered equity claim to the representative firm is equal to unity, the normalized aggregate supply; (iii) the net demand for the DIS of each possible recovery fraction Z is zero; and (iv) the goods market clears, i.e., $Y_t = C_t + I_t + \Phi_t + X_{c,t} + X_{f,t}$ at all $t \geq 0$.

Household's Optimization Problem. The equilibrium aggregate stock value is $Q_t^* = q^*(\pi_t)K_t$. We later verify that the cum-dividend return of the aggregate asset market is given by

$$\frac{dQ_t^* + D_{t-}dt}{Q_{t-}^*} = \mu_Q(\pi_{t-})dt + \sigma d\mathcal{W}_t - \left(1 - (1-N(x_f^*)(1-Z))\frac{q^*(\pi_t^{\mathcal{J}})}{q^*(\pi_{t-})}\right)d\mathcal{J}_t\,, \tag{B.59}$$

where $\mu_Q(\pi_{t-})$ is the expected cum-dividend return absent jumps. When a disaster occurs at time t, wealth changes discretely from W_{t-} to $W_t^{\mathcal{J}}$, where

$$W_t^{\mathcal{J}} = W_{t-} - \left(1 - (1-N(x_f^*)(1-Z))\frac{q^*(\pi_t^{\mathcal{J}})}{q^*(\pi_{t-})}\right)H_{t-} + \delta_{t-}(Z)W_{t-}\,. \tag{B.60}$$

And the household accumulates wealth as:

$$dW_t = r(\pi_{t-})W_{t-}dt + (\mu_Q(\pi_{t-}) - r)H_{t-}dt + \sigma H_{t-}d\mathcal{W}_t - C_{t-}dt - X_{c,t-}dt + \delta_{t-}(Z)W_{t-}d\mathcal{J}_t - \left(\int_0^1 \delta_{t-}(Z)p(Z)dZ\right)W_{t-}dt - \left(1 - (1-N(x_f^*)(1-Z))\frac{q^*(\pi_t^{\mathcal{J}})}{q^*(\pi_{t-})}\right)H_{t-}d\mathcal{J}_t\,. \tag{B.61}$$

The HJB equation for the household in our decentralized market setting is given by

$$0 = \max_{C,H,\delta,X_c}\; f(C,J) + \mu_\pi(\pi)J_\pi + \frac{\sigma^2 H^2 J_{WW}}{2} + \lambda(\pi)\mathbb{E}\left[J\left(W^{\mathcal{J}},\pi^{\mathcal{J}}\right) - J(W,\pi)\right] + \left[r(\pi)W + (\mu_Q(\pi) - r(\pi))H - \left(\int_0^1 \delta(Z)p(Z)dZ\right)W - C - X_c\right]J_W\,. \tag{B.62}$$

The FOCs for consumption C and the market portfolio allocation H are given by

$$f_C(C,J) = J_W(W,\pi) \tag{B.63}$$

$$\sigma^2 H J_{WW}(W,\pi) = -(\mu_Q(\pi) - r(\pi)) J_W(W,\pi) + \lambda(\pi)\mathbb{E}\left[\left(1-(1-N(x_f^*)(1-Z))\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right) J_W\left(W^{\mathcal{J}},\pi^{\mathcal{J}}\right)\right]. \tag{B.64}$$

The DIS demand $\delta(Z)$ for each Z is given by

$$p(Z)J_W(W,\pi) = \lambda(\pi)J_W\left(W^{\mathcal{J}},\pi^{\mathcal{J}}\right)\xi(Z)\,. \tag{B.65}$$

Since there is not benefit for the household to spend on mitigation, we have

$$x_c(\pi) = 0\,. \tag{B.66}$$

Substituting (33) into the FOC (B.63) yields the following consumption rule:

$$C(W,\pi) = \rho^{\psi} u(\pi)^{1-\psi} W\,. \tag{B.67}$$

Firm Value Maximization. Using (33) and (B.67), we verify that the SDF is given by

$$\frac{d\mathbb{M}_t}{\mathbb{M}_{t-}} = -r(\pi_{t-})dt - \gamma\sigma d\mathcal{W}_t + \left[\left(\frac{u^*(\pi_t^{\mathcal{J}})}{u^*(\pi_{t-})}\right)^{1-\gamma}\left(\frac{q^*(\pi_t^{\mathcal{J}})}{q^*(\pi_{t-})}\right)^{-\gamma}(1-N(x_f^*)(1-Z))^{-\gamma} - 1\right](d\mathcal{J}_t - \lambda(\pi_{t-})dt)\,. \tag{B.68}$$

And then by using Ito's Lemma, we obtain the following dynamics for $Q_t = Q(K_t,\pi_t)$:

$$dQ = \left(IQ_K + \frac{1}{2}\sigma^2 K^2 Q_{KK} + \mu_\pi(\pi)Q_\pi\right)dt + \sigma K Q_K d\mathcal{W}_t + \left(Q((1-N(x_f)(1-Z))K,\pi^{\mathcal{J}}) - Q(K,\pi)\right)d\mathcal{J}_t\,. \tag{B.69}$$

No arbitrage implies the drift of $\mathbb{M}_{t-}(AK_{t-} - I_{t-} - \Phi(I_{t-},K_{t-}) - X_{f,t-})dt + d\left(\mathbb{M}_t Q_t\right)$ is zero. By applying Ito's Lemma to this martingale and then by using $Q(K,\pi) = q(\pi)K$, we obtain

$$\begin{aligned} 0 = \max_{i,\,x_f}\; & A - i - \phi(i) - x_f - (r(\pi) - i(\pi))q(\pi) + \mu_\pi(\pi)q'(\pi) \\ & - \left[\gamma\sigma^2 + \lambda(\pi)\left(\left(\frac{u^*(\pi^{\mathcal{J}})}{u^*(\pi)}\right)^{1-\gamma}\left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{-\gamma}\mathbb{E}\left[(1-N(x_f^*)(1-Z))^{-\gamma}\right] - 1\right)\right]q(\pi) \\ & + \lambda(\pi)\left[\left(\frac{u^*(\pi^{\mathcal{J}})}{u^*(\pi)}\right)^{1-\gamma}\left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{-\gamma}\frac{q(\pi^{\mathcal{J}})}{q(\pi)}\mathbb{E}\left[(1-N(x_f^*)(1-Z))^{-\gamma}(1-N(x_f)(1-Z))\right] - 1\right]q(\pi)\,. \end{aligned} \tag{B.70}$$

The FOC for investment implied by (B.70) is

$$q(\pi) = 1 + \phi'(i(\pi))\,. \tag{B.71}$$

And the FOC for mitigation implied by (B.70) is

$$1 = \lambda(\pi)\left(\frac{u^*(\pi^{\mathcal{J}})}{u^*(\pi)}\right)^{1-\gamma}\left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{-\gamma} q(\pi^{\mathcal{J}})N'(x_f)\mathbb{E}\left[(Z-1)(1-N(x_f^*)(1-Z))^{-\gamma}\right]. \tag{B.72}$$

Market Equilibrium. First, the mitigation spending for households is zero, $x_c = 0$. Second, a firm's mitigation spending is $x_f = x_f^*$ and rewriting the FOC for mitigation given in (B.72) yields

$$1 = \lambda(\pi) q^*(\pi) \left(\frac{u^*(\pi^{\mathcal{J}})}{u^*(\pi)}\right)^{1-\gamma} \left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{1-\gamma} N'(x_f^*)\mathbb{E}\left[(Z-1)(1-N(x_f^*)(1-Z))^{-\gamma}\right] , \quad \text{(B.73)}$$

where $q^*(\pi)$ solves

$$\begin{aligned}
0 &= A - i^* - \phi(i^*) - x_f^* - (r(\pi) - i(\pi))q^*(\pi) + \mu_\pi(\pi) q_\pi^*(\pi) \\
&\quad - \left[\gamma\sigma^2 + \lambda(\pi)\left(\left(\frac{u^*(\pi^{\mathcal{J}})}{u^*(\pi)}\right)^{1-\gamma} \left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{-\gamma} \mathbb{E}\left[(1-N(x_f^*)(1-Z))^{-\gamma}\right] - 1\right)\right] q^*(\pi) \\
&\quad + \lambda(\pi)\left[\left(\frac{u^*(\pi^{\mathcal{J}})}{u^*(\pi)}\right)^{1-\gamma} \left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{1-\gamma} \mathbb{E}\left[(1-N(x_f^*)(1-Z))^{1-\gamma}\right] - 1\right] q^*(\pi) \,. \qquad \text{(B.74)}
\end{aligned}$$

Third, in equilibrium, the household invests all wealth in the asset market and holds no risk-free asset, $H = W = Q^*$, and has zero disaster hedging position, $\delta(Z) = 0$ for all Z. Simplifying the FOCs (B.63), (B.64), and (B.65), and using the equilibrium conditions, we obtain

$$c^*(\pi) = \rho^\psi u(\pi)^{1-\psi} q^*(\pi) \,, \qquad \text{(B.75)}$$

$$\begin{aligned}
\mu_Q(\pi) &= r(\pi) + \gamma\sigma^2 \qquad \text{(B.76)} \\
&\quad + \lambda(\pi)\left(\frac{u(\pi^{\mathcal{J}})}{u(\pi)}\right)^{1-\gamma} \left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{-\gamma} \left[\mathbb{E}((1-N(x_f^*)(1-Z))^{-\gamma}) - \frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\mathbb{E}((1-N(x_f^*)(1-Z))^{1-\gamma})\right] ,
\end{aligned}$$

$$p(Z) = \lambda(\pi)\left(\frac{u(\pi^{\mathcal{J}})}{u(\pi)}\right)^{1-\gamma} \left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{-\gamma} (1-N(x_f^*)(1-Z))^{-\gamma}\xi(Z) \,. \qquad \text{(B.77)}$$

Using these equilibrium conditions, we simplify the HJB equation (B.62) as follows:

$$\begin{aligned}
0 \;=\; & \frac{1}{1-\psi^{-1}}\left(\frac{c^*(\pi)}{q^*(\pi)} - \rho\right) + \left(\mu_Q(\pi) - \frac{c^*(\pi)}{q^*(\pi)}\right) - \frac{\gamma\sigma^2}{2} + \mu_\pi(\pi)\frac{u'(\pi)}{u(\pi)} \\
& + \frac{\lambda(\pi)}{1-\gamma}\left[\left(\frac{u(\pi^{\mathcal{J}})q^*(\pi^{\mathcal{J}})}{u(\pi)q^*(\pi)}\right)^{1-\gamma} \mathbb{E}((1-N(x_f^*)(1-Z))^{1-\gamma}) - 1\right] . \qquad \text{(B.78)}
\end{aligned}$$

Fourth, by substituting $c^*(\pi) = A - i^*(\pi) - \phi(i^*(\pi)) - x_f^*$ into (B.74), we obtain

$$\begin{aligned}
0 &= \frac{c^*(\pi)}{q^*(\pi)} - r(\pi) + i(\pi) + \mu_\pi(\pi)\frac{q_\pi^*(\pi)}{q^*(\pi)} - \gamma\sigma^2 \qquad \text{(B.79)} \\
&\quad - \lambda(\pi)\left(\frac{u^*(\pi^{\mathcal{J}})}{u^*(\pi)}\right)^{1-\gamma} \left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{-\gamma} \left[\mathbb{E}((1-N(x_f^*)(1-Z))^{-\gamma}) - \frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\mathbb{E}((1-N(x_f^*)(1-Z))^{1-\gamma})\right] .
\end{aligned}$$

By using the homogeneity property and comparing (B.59) and (B.69), we have

$$\mu_Q(\pi) = \frac{c^*(\pi)}{q^*(\pi)} + i^*(\pi) + \mu_\pi(\pi)\frac{(q^*(\pi))'}{q^*(\pi)} \,. \qquad \text{(B.80)}$$

And then substituting (B.80) into (B.78), we obtain

$$\begin{aligned}\frac{c^*(\pi)}{q^*(\pi)} &= \rho-(1-\psi^{-1})\left[i^*(\pi)-\frac{\gamma\sigma^2}{2}+\mu_\pi(\pi)\left(\frac{u'(\pi)}{u(\pi)}+\frac{(q^*(\pi))'}{q^*(\pi)}\right)\right]\\ &\quad+\lambda(\pi)\left(\frac{1-\psi^{-1}}{1-\gamma}\right)\left[1-\left(\frac{u(\pi^{\mathcal{J}})q^*(\pi^{\mathcal{J}})}{u(\pi)q^*(\pi)}\right)^{1-\gamma}\mathbb{E}((1-N(x_f^*)(1-Z))^{1-\gamma})\right]. \end{aligned} \tag{B.81}$$

Finally, substituting (B.81) into (B.79), we obtain the following equilibrium interest rate:

$$\begin{aligned} r(\pi) &= \rho+\psi^{-1}i^*(\pi)-\frac{\gamma(\psi^{-1}+1)\sigma^2}{2}-\left[(1-\psi^{-1})\left(\frac{u'(\pi)}{u(\pi)}+\frac{(q^*(\pi))'}{q^*(\pi)}\right)-\frac{(q^*(\pi))'}{q^*(\pi)}\right]\mu_\pi(\pi)\\ &\quad-\lambda(\pi)\left[\left(\frac{u^*(\pi^{\mathcal{J}})}{u^*(\pi)}\right)^{1-\gamma}\left(\frac{q^*(\pi^{\mathcal{J}})}{q^*(\pi)}\right)^{-\gamma}\mathbb{E}((1-N(x_f^*)(1-Z))^{-\gamma})-1\right]\\ &\quad-\lambda(\pi)\left[\frac{\psi^{-1}-\gamma}{1-\gamma}\left(1-\left(\frac{u(\pi^{\mathcal{J}})q^*(\pi^{\mathcal{J}})}{u(\pi)q^*(\pi)}\right)^{1-\gamma}\mathbb{E}((1-N(x_f^*)(1-Z))^{1-\gamma})\right)\right]. \end{aligned} \tag{B.82}$$

B.4 Equivalence between Market Solution and Planner's Problem

The equivalence between market solution and planner's problem implies $u(\pi)q(\pi)=b(\pi)$. First, substituting $u(\pi)q(\pi)=b(\pi)$ and (B.71) into (B.72), we obtain the following equation for the optimal mitigation in the market solution:

$$1=\lambda(\pi)(1+\phi'(i))\left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}N'(x)\mathbb{E}\left[(Z-1)(1-N(x)(1-Z))^{-\gamma}\right], \tag{B.83}$$

which is the same as (B.50) for the planner's problem. Substituting $u(\pi)q(\pi)=b(\pi)$ and (B.71) into (B.75), we obtain the following expression for optimal investment in market solution:

$$b(\pi)=[A-i(\pi)-\phi(i(\pi))-x(\pi)]^{1/(1-\psi)}\left[\rho(1+\phi'(i(\pi)))\right]^{-\psi/(1-\psi)}, \tag{B.84}$$

which is the same as (B.49) for the planner's problem. Substituting (B.80) into (B.78), and combining $c(\pi)=A-i(\pi)-\phi(i(\pi))-x$ (the equilibrium condition) with (B.71), we obtain:

$$\begin{aligned} 0 &= \frac{\rho}{1-\psi^{-1}}\left[\left(\frac{b(\pi)}{\rho(1+\phi'(i(\pi)))}\right)^{1-\psi}-1\right]+i(\pi)-\frac{\gamma\sigma^2}{2}+\mu_\pi(\pi)\frac{b'(\pi)}{b(\pi)}\\ &\quad+\frac{\lambda(\pi)}{1-\gamma}\left[\left(\frac{b\left(\pi^{\mathcal{J}}\right)}{b(\pi)}\right)^{1-\gamma}\mathbb{E}\left[(1-N(x)(1-Z))^{1-\gamma}\right]-1\right], \end{aligned} \tag{B.85}$$

which is the same as (B.48) for the planner's problem.

By using $u(\pi)q(\pi)=b(\pi)$, we also verify that the equilibrium dividend yield $c(\pi)/q(\pi)$ given in (B.81) for the market solution is the same as (B.58) for the planner's problem.

Finally, by using $u(\pi)q(\pi)=b(\pi)$ and (B.71), we verify that the interest rate $r(\pi)$ given in (B.57) for the market solution is the same as (B.82) for the planner's problem.