Notes on Hennessy and Whited (2007)

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October 17, 2022

- Notation is a bit confusing in the original paper, so trying to clear up my confusion here.
- Notation: Current productivity is z, yesterday productivity is z^-, and tomorrow productivity is z'
- Cash flow from operations:

$$z\pi(k) \equiv zk^\alpha$$

- Corporate tax income bill:

$$T^C(k, b, z^-, z) \equiv [\tau_c^+ \chi + \tau_c^-(1-\chi)] \cdot [z\pi(k) - \delta k - r(k, b, z^-)b]$$

  where $\chi \equiv \mathbb{1}[z\pi(k) - \delta k - r(k, b, z^-)b > 0]$

- Individual tax rate $\tau_i \implies$ firms discounts using $\frac{1}{1+r(1-\tau_i)}$
- Cash distribution taxes

$$
\begin{aligned}
T^d(X) &\equiv \int_0^X \tau_d(x)dx \\
\text{where } \tau_d(x) &\equiv \bar{\tau}_d \times [1 - e^{-\phi x}] \\
\implies T^d(X) &\equiv \begin{cases} 0, & X \leq 0 \\ \frac{\bar{\tau}_d}{\phi}(\phi X + e^{-\phi X} - 1), & X > 0 \end{cases}
\end{aligned}
$$

- Costly external equity financing

$$
\Lambda(x) = \lambda_0 + \lambda_1 x + \lambda_2 x^2
$$

where $\lambda_0 \geq 0$, $\lambda_1 \geq 0$, $\lambda_2 \geq 0$

- State variable is net worth w and choice variables are debt and capital for next period
- Equity value function

$$
\begin{aligned}
V(w,z) = \max_{k',b'} \Bigg\{ \quad & \underbrace{\Phi[w + b' - k' - T^d(w + b' - k')]}_{\text{cash payment to equity holders}} \\
& \underbrace{-(1-\Phi)[k' - w - b' + \Lambda(k' - w - b')]}_{\text{equity issuance}} \\
& + \left[\frac{1}{1 + r(1 - \tau_i)}\right] E\left[\left(V(w(k', b', z, z'), z') \right)^+ \Bigg| z \right] \Bigg\}
\end{aligned}
$$

where

$$\Phi \equiv \mathbb{1}(w + b' > k')$$

$$w(k', b', z, z') \equiv z'\pi(k') + (1-\delta)k' - T^C(k', b', z, z') - (1 + r(k', b', z))b'$$

- Naive value function:

$$
\begin{aligned}
V(k,b,z,z^-) = \max_{(k',b')} \Bigg\{ & \underbrace{w + b' - k'}_{\text{cash dividend if } (+) \text{ or equity issuance if } (-)} \\
& - \underbrace{T^d(w + b' - k')}_{\text{taxes on cash dividend}} \\
& - \underbrace{\Lambda(-(w + b' - k'))}_{\text{equity issuance cost}} \\
& + \frac{1}{1 + r(1-\tau_i)} E\Big[\underbrace{\max\{V(k',b',z',z),0\}}_{\text{if } V \text{ is } (-) \text{ can default}}\Big]\Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
\text{where} \quad \underbrace{y}_{\text{taxable corporate income}} &\equiv \underbrace{zk^{\alpha}}_{\text{operating profits}} - \underbrace{\delta k}_{\text{depreciation}} - \underbrace{\tilde{r}(k,b,z^-)b}_{\text{interest on debt}} \\
\underbrace{T^C(x)}_{\text{corporate income tax bill}} &\equiv \begin{cases} \tau_c^+ x, & \text{if } x > 0 \\ \tau_c^- x, & \text{if } x \leq 0 \end{cases} \\
\underbrace{w}_{\text{realized net worth}} &\equiv \underbrace{y - T^C(y)}_{\text{after-tax corporate income}} + \underbrace{k}_{\text{capital}} - \underbrace{b}_{\text{debt principal}} \\
\underbrace{T^d(x)}_{\text{taxes on cash dividend}} &= \begin{cases} \frac{\bar{\tau}_d}{\phi}(\phi x + e^{-\phi x} - 1), & x > 0 \\ 0, & x \leq 0 \end{cases} \\
\underbrace{\Lambda(x)}_{\text{equity issuance cost}} &= \begin{cases} \lambda_0 + \lambda_1 x + \lambda_2 x^2, & \text{if } x > 0 \\ 0, & \text{if } x \leq 0 \end{cases}
\end{aligned}
$$

- Smarter value function:

$$V(w,z) = \max_{(k',b')} \Bigg\{ \underbrace{w+b'-k'}_{\text{cash dividend if }(+)\text{ or equity issuance if }(-)} - \underbrace{T^d(w+b'-k')}_{\text{taxes on cash dividend}} - \underbrace{\Lambda(-(w+b'-k'))}_{\text{equity issuance cost}} + \frac{1}{1+r(1-\tau_i)} E\Big[\underbrace{\max\{V(w',z'),0\}}_{\text{if } V \text{ is } (-) \text{ can default}} \Big] \Bigg\}$$

where

$$\underbrace{y'}_{\text{taxable corporate income}} \equiv \underbrace{z'(k')^{\alpha}}_{\text{operating profits}} - \underbrace{\delta k'}_{\text{depreciation}} - \underbrace{\tilde{r}(k',b',z)b'}_{\text{interest on debt}}$$

$$\underbrace{T^C(x)}_{\text{corporate income tax bill}} \equiv \begin{cases} \tau_c^{+} x, & \text{if } x > 0 \\ \tau_c^{-} x, & \text{if } x \le 0 \end{cases}$$

$$\underbrace{w'}_{\text{realized net worth}} \equiv \underbrace{y' - T^C(y')}_{\text{after-tax corporate income}} + \underbrace{k'}_{\text{capital}} - \underbrace{b'}_{\text{debt principal}}$$

$$\underbrace{T^d(x)}_{\text{taxes on cash dividend}} = \begin{cases} \frac{\bar{\tau}_d}{\phi}(\phi x + e^{-\phi x} - 1), & x > 0 \\ 0, & x \le 0 \end{cases}$$

$$\underbrace{\Lambda(x)}_{\text{equity issuance cost}} = \begin{cases} \lambda_0 + \lambda_1 x + \lambda_2 x^2, & \text{if } x > 0 \\ 0, & \text{if } x \le 0 \end{cases}$$

- Smarter value function with bond prices instead of interest rates:

$$
\begin{aligned}
V(w,z) = \max_{(k',b')} \Bigg\{ & \underbrace{w + b'q(k',b',z) - k'}_{\text{cash dividend if } (+) \text{ or equity issuance if } (-)} \\
& - \underbrace{T^d(w + b'q(k',b',z) - k')}_{\text{taxes on cash dividend}} \\
& - \underbrace{\Lambda(-(w + b'q(k',b',z) - k'))}_{\text{equity issuance cost}} \\
& + \frac{1}{1 + r(1-\tau_i)} E\Big[\underbrace{\max\{V(w',z'),0\}}_{\text{if } V \text{ is } (-)\text{, firm can default}} \; |z \Big] \Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
\text{where} \quad \underbrace{y'}_{\text{taxable corporate income}} & \equiv \underbrace{z'(k')^\alpha}_{\text{operating profits}} - \underbrace{\delta k'}_{\text{depreciation}} - \underbrace{(1 - q(k',b',z))b'}_{\text{interest on debt}} \\
\underbrace{T^C(x)}_{\text{corporate income tax bill}} & \equiv \begin{cases} \tau_c^+ x, & \text{if } x > 0 \\ \tau_c^- x, & \text{if } x \le 0 \end{cases} \\
\underbrace{w'}_{\text{realized net worth}} & \equiv \underbrace{y' - T^C(y')}_{\text{after-tax corporate income}} + \underbrace{k'}_{\text{capital}} - \underbrace{q(k',b',z)b'}_{\text{debt principal}} \\
\underbrace{T^d(x)}_{\text{taxes on cash dividend}} & = \begin{cases} \frac{\bar{\tau}_d}{\phi}(\phi x + e^{-\phi x} - 1), & x > 0 \\ 0, & x \le 0 \end{cases} \\
\underbrace{\Lambda(x)}_{\text{equity issuance cost}} & = \begin{cases} \lambda_0 + \lambda_1 x + \lambda_2 x^2, & \text{if } x > 0 \\ 0, & \text{if } x \le 0 \end{cases}
\end{aligned}
$$

- Zero profit condition:

$$b'(1 + r(1 - \tau_i)) = (1 + (1 - \tau_i)\tilde{r}(k', b', z))b' \int_{z_d(k',b',z)}^{\bar{z}} Q(z, dz') + \int_{\underline{z}}^{z_d(k',b',z)} R(k', z')Q(z, dz')$$