ALEX VON HAFFTEN
MARCH 16, 2021

ECON 713

(1) $Q(P) = 30 - 6P$

$MC = 1$

(a) $\Rightarrow P(Q) = \dfrac{30 - Q}{6}$

$= 5 - \frac{1}{6} Q \qquad Q = \sum_i q_i$

$TR_1(q_1) = \left[5 - \frac{1}{6} \sum_i q_i\right] q_1$

$MR_1(q_1) = 5 - \frac{1}{6}[q_2 + q_3 + q_4] - \frac{1}{3} q_1$

$MC = MR_1(q_1)$

$1 = 5 - \frac{1}{6}[q_2 + q_3 + q_4] - \frac{1}{3} q_1$

Impose symmetry $\quad q_1 = q_2 = q_3 = q_4$

$1 = 5 - \frac{1}{2} q_1 - \frac{1}{3} q_1$

$1 = 5 - \frac{5}{6} q_1$

$-4 = -\frac{5}{6} q_1$

$\frac{-24}{-5} = q_1$

$\boxed{\frac{24}{5} = q_1^*}$ Same for other 3 firms

$Q^* = \boxed{\frac{96}{5}}$

$P^* = 5 - \frac{1}{6} \cdot \frac{96}{5} = 5 - \frac{16}{5} = \boxed{\frac{9}{5}}$

$\Pi^* = \frac{24}{5}\left(\frac{9}{5} - 1\right)$

$= \frac{24}{5} \cdot \frac{4}{5} = \boxed{\frac{96}{25}}$

1 (b) Firm 1 is leader and produces q_L

The total revenue of follower is

$$TR_2(q_2) = \left[5 - \frac{1}{6}(q_L + q_2 + q_3 + q_4)\right]q_2$$

$$MR_2(q_2) = 5 - \frac{1}{6}(q_L + q_3 + q_4) - \frac{1}{3}q_2$$

MC = MR

$$\Rightarrow 1 = 5 - \frac{1}{6}(q_L + q_3 + q_4) - \frac{1}{3}q_2$$

Firms 2, 3 & 4 act symmetrically.

$$1 = 5 - \frac{1}{6}(q_L + q_2 + q_2) - \frac{1}{3}q_2$$

$$4 = \frac{1}{6}q_L + \frac{2}{3}q_2$$

$$24 = q_L + 4q_2$$

$$\cancel{q_L = 24 - 4q_2}$$

$$q_2 = \frac{24 - q_L}{4}$$

$$q_2^* = 6 - \frac{q_L}{4}$$

Firm 1's ~~profit is~~ total revenue is:

$$TR_1(q_1) = \left[5 - \frac{1}{6}(q_1 + 3q_2^*)\right]q_1$$

$$= 5q_1 - \frac{q_1^2}{6} - \frac{1}{2}q_1\left(6 - \frac{q_1}{4}\right)$$

$$= 5q_1 - \frac{q_1^2}{6} - 3q_1 - \frac{q_1^2}{8}$$

$$MR_1(q_1) = 5 - \frac{q_1}{3} - 3 - \frac{q_1}{4}$$

$$= 2 - \frac{q_1}{12}$$

$MR_1(q_1) = MC$

1 b. $2 - \frac{q_1}{12} = 1$

$q_1^* = 12$

$\Rightarrow q_2^* = q_3^* = q_4^* = 6 - \frac{12}{4} = 3$

$\Rightarrow Q^* = 12 + 3 \cdot 3 = 21$

$\Rightarrow P^* = 5 - \frac{21}{6} = \frac{30-21}{6} = \frac{9}{6} = \frac{3}{2}$

Stackelberg

$CS(3/2) = \frac{1}{2}(5 - 3/2)^2 = \frac{1}{2}(7/2)^2 = \frac{49}{8}$

$PS(3/2) = 21 * (3/2 - 1) = 21/2$

Cournot

$CS(9/5) = 1/2(5 - 9/5)^2 = 1/2\left(\frac{25-9}{5}\right)^2 = 1/2\left(\frac{16}{5}\right)^2 = \frac{128}{25}$

$PS(9/5) = \frac{96}{5}(9/5 - 1) = \frac{96}{5}\frac{4}{5} = \frac{384}{25}$

Yes, Stackelberg is better for customers.

No, Cournot is better for firms.

① (a) Total revenue for followers: #1:

$$TR_F(q_F) = \left[5 - \frac{1}{6}(q_{L,1} + q_{L,2} + q_{F,1} + q_{F,2})\right] q_{F,1}$$

$$MR_{F,1}(q_{F,1}) = 5 - \frac{1}{6} q_{L,1} - \frac{1}{6} q_{F,2} - \frac{1}{3} q_{F,1} - \frac{1}{6} q_{L,2}$$

$$MC = MR_F(q_F)$$

$$1 = 5 - \frac{1}{6} q_{L,1} - \frac{1}{3} q_{F,1} - \frac{1}{6} q_{F,2} - \frac{1}{6} q_{L,2}$$

Impose symmetry $q_{F,1} = q_{F,2} = q_F$

$$1 = 5 - \frac{1}{6} q_{L,1} - \frac{1}{2} q_F - \frac{1}{6} q_{L,2}$$

$$\frac{1}{2} q_F = 4 - \frac{1}{6} q_{L,1} - \frac{1}{6} q_{L,2}$$

$$q_F = 8 - \frac{1}{3} q_{L,1} - \frac{1}{3} q_{L,2}$$

Leader #1 TR is

$$TR_{L,1}(q_{L,1}) = \left[5 - \frac{1}{6}(q_{L,1} + q_{L,2} + 2q_F)\right] q_{L,1}$$

$$= 5q_{L,1} - \frac{1}{6} q_{L,1}^2 - \frac{1}{6} q_{L,1} q_{L,2} - \frac{1}{3} q_{L,1} \cdot \left(8 - \frac{1}{3} q_{L,1} - \frac{1}{3} q_{L,2}\right)$$

$$MR_{L,1}(q_{L,1}) = 5 - \frac{1}{3} q_{L,1} - \frac{1}{6} q_{L,2} - \frac{8}{3} + \frac{2}{9} q_{L,1} + \frac{1}{9} q_{L,2}$$

Impose symmetry $q_{L,1} = q_{L,2} = q_L$ and $MR = MC$

$$1 = 5 - \frac{1}{3} q_L - \frac{1}{6} q_L - \frac{8}{3} + \frac{2}{9} q_L + \frac{1}{9} q_L$$

① (c) con't

$$1 = \frac{7}{3} - \frac{1}{2}q_L + \frac{1}{3}q_L$$

$$\frac{4}{3} = \frac{1}{6}q_L$$

$$\frac{24}{3} = q_L^*$$

$$8 = q_L^*$$

$$\Rightarrow q_F^* = 8 - \frac{8}{3} - \frac{8}{3} = 8 - \frac{16}{3} = \frac{8}{3}$$

$$= Q^* = 8 + 8 + \frac{8}{3} + \frac{8}{3} = \frac{48}{3} + \frac{16}{3} = \frac{64}{3}$$

$$\Rightarrow P^* = 5 - \frac{64}{3}\frac{1}{6}$$

$$= 5 - \frac{32}{9}$$

$$= \frac{45 - 32}{9}$$

$$= \frac{13}{9}$$

$$PS(13/9) = \frac{64}{3}\left(\frac{13}{9} - 1\right) = \frac{64}{3}\left(\frac{4}{9}\right) = \frac{256}{27}$$

$$CS(13/9) = 1/2\left(5 - 13/9\right)^2 = 1/2\left(\frac{45-13}{9}\right)^2$$

$$= 1/2\left(\frac{32}{9}\right)^2$$

$$= \frac{1024}{2 \cdot 81} = \frac{512}{81}$$

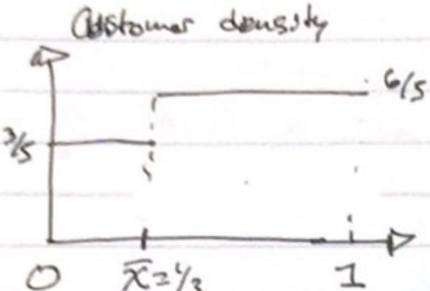
① (c) con't

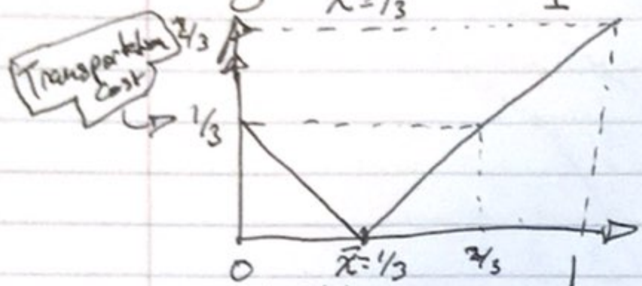
	CS	PS
a	128/25 $\approx$ 5.12	384/25 $\approx$ 15
b	49/8 $\approx$ 6.125	21/2 $= 10.5$
c	512/81 $\approx$ 6.3	$\frac{256}{27}$ $\approx$ 9.48

For consumers, $c \succsim b \succsim a$

For producers, $a \succsim b \succsim c$.

(2)





If a is the height of the to the left of the barbers

$$a\frac{1}{3} + 2a\frac{2}{3} = 1$$

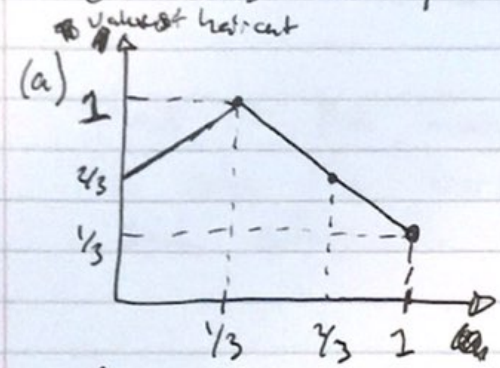
$$\frac{a}{3} + \frac{4a}{3} = 1$$

$$\frac{5a}{3} = 1$$

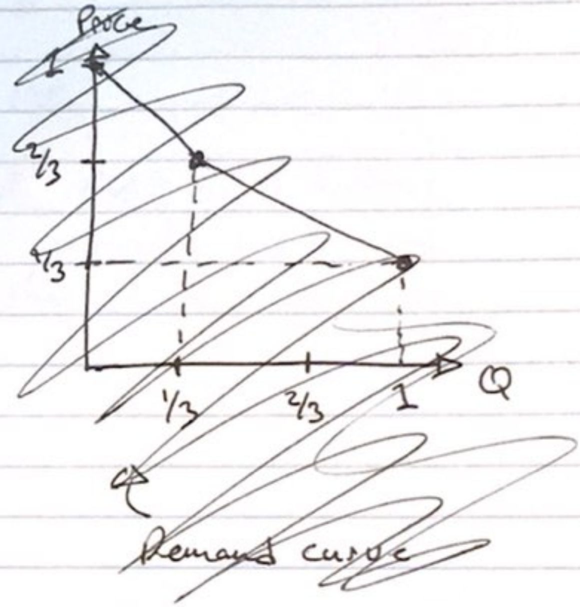
$$a = \frac{3}{5}$$

$$\frac{3}{5}\frac{1}{3} + 2\frac{3}{5}\frac{2}{3} = 1$$

(a)



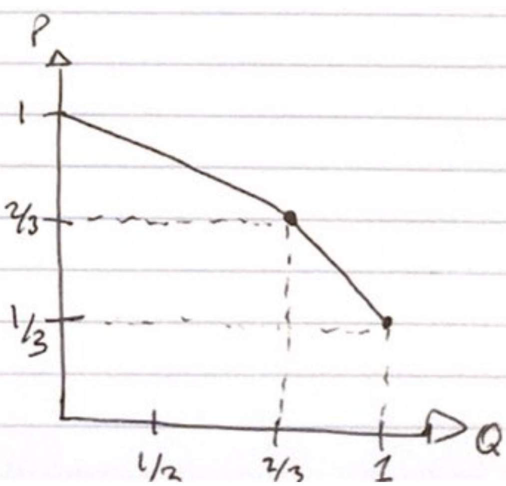
Value of haircut minus transportation cost



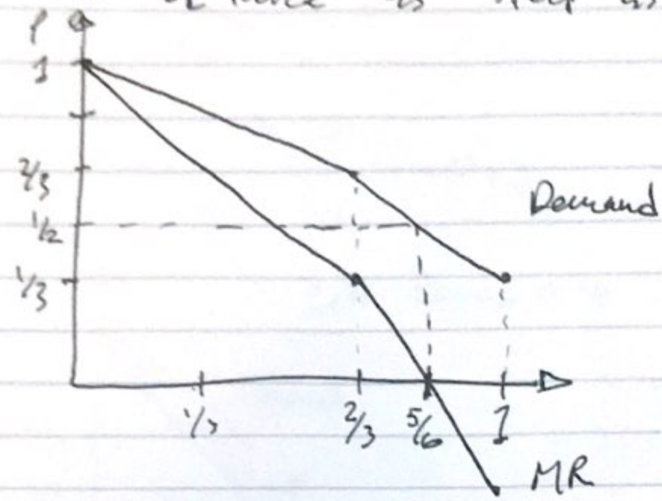
At $p = 1/3$, everyone gets a haircut

A $p = 2/3$, $2/3$ of the people get haircuts over one third left and one third right.

At $p=1$, no one get a haircut.

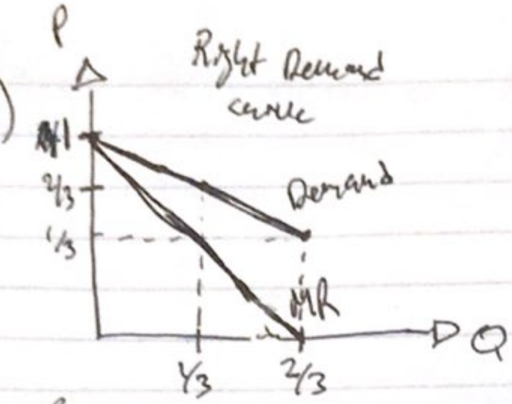


(3) (a) The marginal revenue for a monopolist is ~~th~~ twice as steep as marginal cost



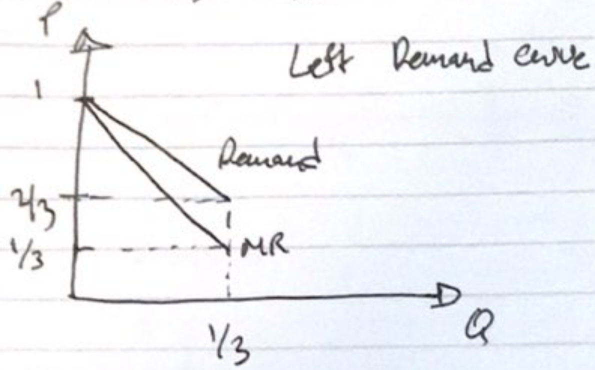
Thus if the marginal cost of a haircut is zero, the barber should charge 1/2 and offer 5/6 of the [0,1] haircuts.

(2)(b)



$MR_R = MC$ at $Q_R = 2/3$

$P_R = 1/3$

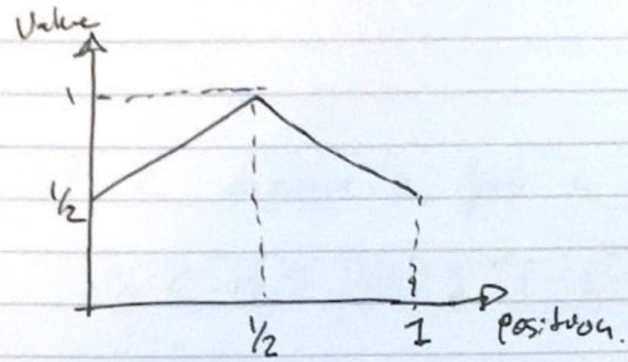


MR is always higher than MC.

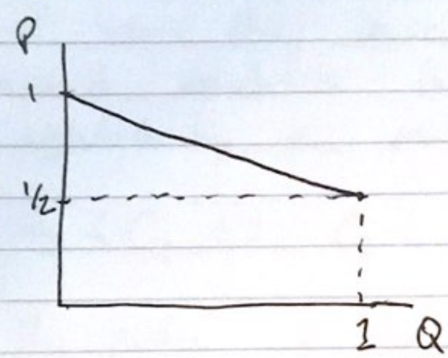
$\Rightarrow \quad Q_L = 1/3$

$P_L = 2/3$

2(c) No, because the demand from each side would be the same:

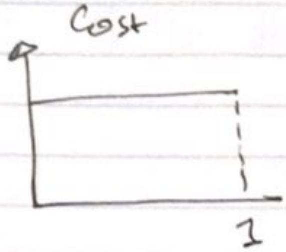


=> Demand



There's no benefit to the monopolist using price discrimination for identical demand curves

Problem 3



(a) If c_1 decides to get a shot then,

$$c_1 < 0.5\left[0.25\,(1-p)\right]$$

Namely the cost of getting the vaccine is smaller than the expected benefit of having the vaccine (ie, the avoid desutility from getting sick weighted by the probability of getting sick).

If $c_2 < c_1 \Rightarrow c_2 < 0.5\left[0.25\,(1-p)\right]$.

So, the person w/ lower cost should get vaccinated

(b) To get vaccinated, benefit needs to outweigh cost,

$$c < 0.5\left[0.25(1-p)\right]$$

So $c \in \left(0, \frac{1}{8} - \frac{p}{8}\right)$.

Since each person is atomless their action doesn't affect the aggregate p.

~~If $p = 0 \Rightarrow c \in (0, 1/8)$ get vaccinated~~

~~$\Rightarrow p = 1/8$~~

~~$\Rightarrow c \in (0, 7/64)$~~

~~$\Rightarrow p = 7/64$~~

This implies that

$$\Rightarrow \frac{1}{8} - \frac{p}{8} = p$$

$$\frac{1}{8} = \frac{9p}{8}$$

$$\frac{1}{9} = p$$

Thus $c \in (0, 1/9)$ will get vaccinated.

Total cost of vaccinately p people $\Rightarrow \int_0^p c\,dc = \left[\frac{c^2}{2}\right]_0^p = \frac{p^2}{2}$

(3)(c) The social optimal level

$$\max_{p} \underbrace{0.5(0.25(1-p))}_{\text{TB of } p \text{ people being vaccinated}} - \underbrace{p^2/2}_{\text{Total cost of } p \text{ people being vaccinated}}$$

$$\Rightarrow \max_{p} \frac{1}{8} - \frac{1}{8}p - \frac{p^2}{2}$$

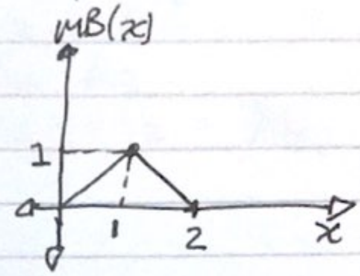
FOC[p] $\quad -\frac{1}{8} - p = 0$

$\Rightarrow p^* = \frac{1}{8}$

SOC [p] $\quad -1 < 0 \Rightarrow p^*$ is max.

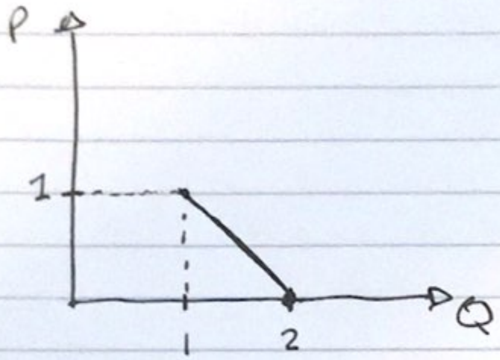
This p^* is larger than the p I found in (b) because agents don't internalize the effect of their vaccination on others' ~~costs~~ reduction of risk of getting the disease.

(4) (a) $MB(x) = \begin{cases} x & \text{if } x \leq 1 \\ 2-x & \text{if } x \geq 1 \end{cases}$



At $p=0$, JB demands 2 units of information.

At $p=1$, JB demands 1 unit of information



← This is JB's demand for information

(b) Informatic is a monopolist

He can maximize his producer surplus by charging $P = 1/2$ and JB demands $Q = 1\frac{1}{2}$. So his producer surplus is $1/2 \cdot 1\frac{1}{2} = 3/4$.

(5)

(a) The social planner's problem [Firm produces at frontier]

$$\max_{\{h,x,y\}} \log(h^a x^a y^a) \quad s.t \quad x^2+y^2 = \frac{(12-h)^2}{2}$$

$$\Rightarrow \max_{\{h,x,y\}} a\left[\log(h)+\log(x)+\log(y)\right]$$

$$s.t. \quad x^2+y^2 = \frac{(12-h)^2}{2}$$

$$\Rightarrow \max_{\{h,x,y\}} \log(h)+\log(x)+\log(y)$$

$$s.t \quad x^2+y^2 = \frac{(12-h)^2}{2}$$

$$\mathcal{L} = \log(h)+\log(x)+\log(y) - \lambda\left[x^2+y^2-\frac{(12-h)^2}{2}\right]$$

FOC[h]: $\frac{1}{h} = -\lambda \frac{\cancel{2}(12-h)}{\cancel{2}}$

$$\frac{1}{h} = -\lambda 12 + \lambda h$$

$$0 = \lambda h^2 - \lambda 12 h - 1 \qquad (I)$$

FOC[x]: $\frac{1}{x} = 2\lambda x$

$$\frac{1}{2\lambda} = x^2$$

$$x^* = \sqrt{1/2\lambda} \qquad \lambda = \frac{1}{2x^2}$$

FOC[y]: $\frac{1}{y} = 2\lambda y \qquad \Rightarrow x^* = y^*$

$$y^* = \sqrt{\frac{1}{2\lambda}}$$

(a) con't

The feasibility constraint becomes

$$2x^2 = \frac{(12-h)^2}{2}$$
$$4x^2 = (12-h)^2$$
$$2x = 12-h$$
$$x = 6-h/2$$
$$h = 12-2x$$

$$(I) \Rightarrow 0 = \frac{1}{2x^2}(12-2x)^2 - \frac{1}{2x^2}(12-2x)12 - 1$$

$$0 = \frac{144-48x+4x^2}{2x^2} - \frac{\cancel{144}-\cancel{24x}\,24x}{2x^2} - 1$$

$$0 = \frac{-24x}{2x^2} + 2 - 1$$

$$0 = \frac{-12}{x} + 1$$

$$x = 12$$

$$\Rightarrow x^* = 12 \qquad y^* = 12$$

$$12^2 + 12^2 = \frac{(12-h)^2}{2}$$
$$2(144+144) = (12-h)^2$$
$$\sqrt{576} = 12-h \Rightarrow 24 = 12-h \Rightarrow h = -12$$

(6) (a) Buyers $j = 1, \ldots, 100$
Sellers $k = 1, \ldots, 200$

Seller k values house at 100K

Buyer j values house from seller k at
$500K - 2jK$

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