ECON 717B: PS 1

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1 Part 1: Analytic Exercises

1. Returns to schoolings
   (a) ATE
   Marginal treatment effect is
   $$MTE(A) = Y_1(A) - Y_0(A) = 1 + 0.5A - A = 1 - 0.5A$$
   Average treatment effect is
   $$E[MTE(A)] = E[1 - 0.5A] = 1 - 0.5E[A] = 1 - 0.5 * 0.5 = 0.75$$
   (b) Fraction of treated population
   $$Pr\{D = 1\} = Pr\{-0.5 + A > 0\} = Pr\{A > 0.5\} = 0.5$$
   (c) Maximum and minimum treatment effect
   $$\max_{A\in[0,1]} MTE(A) = \max_{A\in[0,1]} [1 - 0.5A] = 1$$
   at $A = 0$.
   $$\min_{A\in[0,1]} MTE(A) = \min_{A\in[0,1]} [1 - 0.5A] = 0.5$$
   at $A = 1$.
   (d) $A \sim N(0, 1)$
   $$\sup_{A\in(-\infty,\infty)} MTE(A) = \sup_{A\in(-\infty,\infty)} [1 - 0.5A] = \infty$$
   as $A \to -\infty$.
   $$\inf_{A\in(-\infty,\infty)} MTE(A) = \inf_{A\in(-\infty,\infty)} [1 - 0.5A] = -\infty$$
   as $A \to \infty$.
   (e) ATET and ATEU
   $$ATET = E[MTE(A)|D = 1] = E[1-0.5A|A > 0.5] = 1-0.5E[A|A > 0.5] = 1-0.5*0.75 = 0.625$$
   $$ATEU = E[MTE(A)|D = 0] = E[1-0.5A|A < 0.5] = 1-0.5E[A|A < 0.5] = 1-0.5*0.25 = 0.875$$

(f) Why is ATEU > ATET?

ATEU > ATET because the marginal treatment effect is decreasing in A, but selection into treatment is increasing in A.

(g) OLS estimand

$$\beta(OLS) = E[Y|D=1]-E[Y|D=0] = E[1+0.5A|A>0.5]-E[A|A<0.5] = 1+0.5*0.75-0.25 = 1.125$$

(h) Why is OLS biased upward for ATE?

Because conditional independence fails due to selection effects. If treatment was random, then OLS would be unbiased.

2. Monotonicity

(a) Prove monotonicity holds.

For each observation i, define $V_{0,i} \equiv \delta_0 + U_{V,i}$ as the outcome without treatment and $V_{1,i} \equiv \delta_0 + \delta_1 + U_{V,i}$ as the outcome with treatment.

Case 1: $\delta_1 > 0 \implies \delta_0 + \delta_1 + U_{V,i} > \delta_0 + U_{V,i} \implies V_{1,i} > V_{0,i}$ for all i. Monotonicity holds.

Case 2: $\delta_1 < 0 \implies \delta_0 + \delta_1 + U_{V,i} < \delta_0 + U_{V,i} \implies V_{1,i} < V_{0,i}$ for all i. Monotonicity holds.

Case 3: $\delta_1 = 0 \implies \delta_0 + \delta_1 + U_{V,i} = \delta_0 + U_{V,i} \implies V_{1,i} = V_{0,i}$ for all i. Monotonicity holds.

(b) Define V such that monotonicity fails.

Consider heterogenous $\delta_{1,i} \in [-A, A]$:

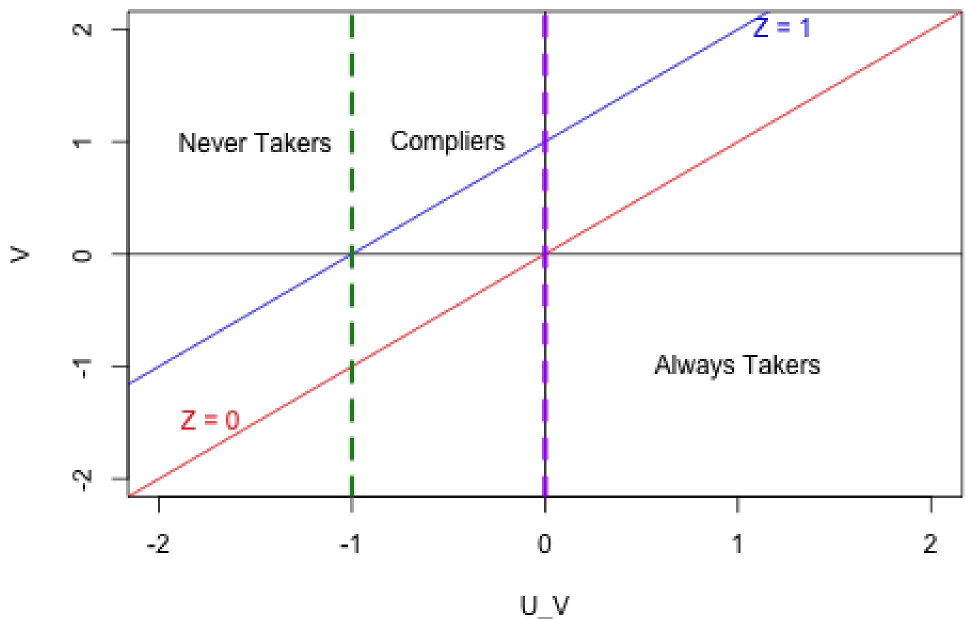
$$V_i = \delta_0 + \delta_{1,i} Z_i + U_{V,i}$$

Since $\delta_{1,i}$ can be positive or negative, defier will not choose the treatment even if they are exposed to the instrument.

3. Potential outcomes with uniform instrument

(a) Show range of always takers, compliers, defiers, and never takers.

Monotonicity holds, so there are no defiers. Always takers have $V > 0$ for both $Z = 0$ and $Z = 1$, so $U_V \in [0, 2]$. Compliers $V > 0$ for $Z = 1$, but $V < 0$ for $Z = 0$, so $U_V \in [-1, 0)$. Never takers have $V < 0$ for both $Z = 0$ and $Z = 1$, so $U_V \in [-2, -1)$. The figure below summarizes these ranges:



(b) Compute fraction of population in each group.
Using the uniform distribution, defiers are 0 percent, always takers are 50 percent, compliers are 25 percent, and never takers are 25 percent.

4. Two types

(a) Compute ATE

$$\begin{aligned} ATE &= E[\Delta] \\ &= Pr(Type1)E[\Delta|Type1] + Pr(Type2)E[\Delta|Type2] \\ &= (0.3)(2) + (0.7)(-1) \\ &= -0.1 \end{aligned}$$

(b) Compute $Pr(D = 1|Z = 1)$ and $Pr(D = 1|Z = 0)$

$$\begin{aligned} Pr(D = 1|Z = 1) &= Pr(D = 1|Z = 1, Type1)Pr(Type1) + Pr(D = 1|Z = 1, Type2)Pr(Type2) \\ &= P(1 + U_V > 0)(0.3) + P(2 + U_V > 0)(0.7) \\ &= (1.0)(0.3) + (1.0)(0.7) \\ &= 1.0 \end{aligned}$$

$$\begin{aligned} Pr(D = 1|Z = 0) &= Pr(D = 1|Z = 0, Type1)Pr(Type1) + Pr(D = 1|Z = 0, Type2)Pr(Type2) \\ &= P(U_V > 0)(0.3) + P(U_V > 0)(0.7) \\ &= (0.5)(0.3) + (0.5)(0.7) \\ &= 0.5 \end{aligned}$$

(c) Compute LATE

Notice that compliers of both Type 1 and Type 2 have $U_V \in [-1, 0]$

$$\begin{aligned}
LATE &= E[\Delta | U_V < 0] \\
&= Pr(Type1)E[\Delta | U_V < 0, Type1] + Pr(Type2)E[\Delta | U_V < 0, Type2] \\
&= (0.3)(2.0) + (0.7)(-1.0) \\
&= -0.1
\end{aligned}$$

2 Monte Carlo Exercises

2.1 Question 1

1. See `p2_q1.do`.

2. See table below.

3. z_1 and z_2 are valid instruments because they affect schooling s (i.e. relevance) but they do not affect log wages except through schooling (i.e. exogeneity). z_3 is not a valid instrument because it does not affect schooling (i.e., it fails relevance). z_1 is likely a weak instrument because its variance is relatively small.

4. See table below.

VARIABLES	(1) 2SLS	(2) 2SLS	(3) 2SLS	(4) 2SLS	(5) 2SLS	(6) 2SLS	(7) 2SLS
s	0.0589***	0.0501***	-0.0202	0.0501***	0.0501***	0.0531***	0.0501***
	(0.0170)	(0.000573)	(0.222)	(0.000573)	(0.000573)	(0.0164)	(0.000573)
Constant	1.000***	1.004***	1.036***	1.004***	1.004***	1.002***	1.004***
	(0.0158)	(0.0142)	(0.114)	(0.0142)	(0.0142)	(0.0158)	(0.0142)
Observations	2,000	2,000	2,000	2,000	2,000	2,000	2,000
R-squared	0.866	0.857		0.857	0.857	0.864	0.857
Instruments	z_1	z_2	z_3	z_1 z_2	z_2 z_3	z_1 z_3	z_1 z_2 z_3
F-Statistic	1.711	8154	0.130	4075	4075	0.926	2716

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

5. The OLS estimate is biased upward for β_2. Including just z_1 exacerbated the bias likely due it being a weak instrument. Including just z_2 reduces the bias and boosts the first stage F-statistic. Including just z_3, the results are way off which makes sense because it is not a relevant instrument. Any of the combinations that include z_2 do well, but without z_2 does poorly. Including all three instruments lowers the F-statistics. This table suggest that we would want to include just z_2 or both z_1 and z_2.

6. See table below for $N = 500,000$.

VARIABLES	(1) 2SLS	(2) 2SLS	(3) 2SLS	(4) 2SLS	(5) 2SLS	(6) 2SLS	(7) 2SLS
s	0.0477***	0.0500***	0.0557***	0.0500***	0.0500***	0.0498***	0.0500***
	(0.0124)	(3.62e-05)	(0.0194)	(3.62e-05)	(3.62e-05)	(0.0103)	(3.62e-05)
Constant	0.999***	0.999***	0.999***	0.999***	0.999***	0.999***	0.999***
	(0.00108)	(0.000905)	(0.00122)	(0.000905)	(0.000905)	(0.00102)	(0.000905)
Observations	500,000	500,000	500,000	500,000	500,000	500,000	500,000
R-squared	0.845	0.854	0.865	0.854	0.854	0.854	0.854
Instruments	z_1	z_2	z_3	z_1 z_2	z_2 z_3	z_1 z_3	z_1 z_2 z_3
F-Statistic	3.688	2.151e+06	1.317	1.075e+06	1.075e+06	2.504	716915

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

2.2 Question 2

1. See `p2_q2.jl`
2. ATE is 2.992, ATET is 3.179, ATEU is 2.869, OLS/Naive estimator is 3.397, direct/reduced form/ITT estimator is 2.359, and IV estimator is 2.873.
3. Fraction of compliers is 0.8216.
4. ATE is 2.995, ATET is 3.275, ATEU is 2.837, OLS/Naive estimator is 3.568, direct/reduced form/ITT estimator is 2.029, and IV estimator is 2.974. Fraction of compliers is 0.6863.