ECON 712 - PS 6

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(Non-) Commitment in a black-box example with discrete choice set

Let there be a continuum of identical households in the economy, taking actions $\xi \in X$.[1] Let the economy wide average (the aggregate) of these actions be x. The benevolent government takes action $y \in Y$. The payoff to households is $u(\xi, x, y)$. Let the optimal choice of households, as a function of aggregates, be $f(x, y) := \arg\max_{\xi \in X} u(\xi, x, y)$.

In a competitive equilibrium, household action is consistent with the aggregate, i.e. $x = f(x, y)$. Now for each y, let $x = h(y)$ be such that $h(y) = f(h(y), y)$. That is, $(x = h(y), y)$ is a competitive equilibrium.

Let $X = \{x_H, x_l\}, Y = \{y_H, y_L\}$. For the one-period economy, with $\xi_i = x_i$, the payoffs $u(x_i, x_i, y_j)$ is given by the following table of one-period payoffs:

	x_L	x_H
y_L	12*	25
y_H	0	24*

Here the values $u(\xi_k, x_i, y_j)$ not reported are such that the outcomes with * are competitive equilibria. For example, $u(\xi_k, x_i, y_j) = -1$ for $k \neq i$ and $i = j$, and $u(\xi_k, x_i, y_j) = 30$ for $k \neq i$ and $i \neq j$. You should convince yourself that this is the case.

1. Find the Ramsey outcome, that is when the government has commitment/moves first. Find the outcome when the government cannot commit/moves second (in pure strategies). We will refer to this case as the Nash equilibrium in pure strategies (NE).
2. Suppose the economy is repeated 5 times. Can the Ramsey outcome be supported in any period?

Now consider the expanded version of the previous economy. The payoffs $u(x_i, x_i, y_j)$ is given by the following the following table of one-period payoffs:

	$x_L L$	x_L	x_H
$y_L L$	2*	6	10
y_L	1	12*	25
y_H	-1	0	24*

3. What are the NEs? Suppose the economy is repeated 3 times, with agents discounting future utilities by $\beta = 0.9$. Can the Ramsey outcome be supported in any period?

*I worked on this problem set with a study group of Michael Nattinger, Andrew Smith, and Ryan Mather. I also discussed problems with Emily Case, Sarah Bass, and Danny Edgel.

[1]These problems draw extensively from Ljungqvist and Sargent's Recursive Macroeconomic Theory.

Static taxation

Let there be a unit measure of households with preferences over leisure, (private) consumption, and public goods (l, c, g), defined by the utility

$$u(l, c, g) = \ln l \ln(\alpha + c) + \ln(\alpha + g), \alpha \in (0, 0.5)$$

Each household is endowed with 1 unit of time, which can be spent on leisure or labor. Production is linear in labor, i.e. the economy resource constraint is $\bar{l} + g + \bar{c} = 1$ where $\bar{l}, \bar{c}$ are aggregate leisure and consumption. To provide the public good, the government can levy a flat proportional tax τ on labor. That is $g = \tau(1 - l)$.

1. Set up and solve the Planner's problem.
2. Set up and solve for the Ramsey outcome.
3. Set up and solve for the NE outcome.
4. Comment on the differences between the above 3 outcomes, and the reason as to why they are different.
5. Suppose the economy is repeated for infinite periods, with discount factor $\beta < 1$. For high enough β, can the Ramsey outcome be sustained?