ECON 899B - PS1

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Problem 1: Likelihood, score, and hessian

The log-likelihood is -6942.8049. The score is below. And the hessian is on the following page.

-2605.91
-556.32
-1156.86
-222.82
-933.04
-1215.13
-2109.63
-948.07
-5049.88
-4534.79
-19401.90
-19164.66
-918.86
-351.75
-466.69
-582.47
-546.41

-3225	-880	-1428	-388	-1306	-1547	-2619	-1211	-6305	-5761	-23783	-23599	-1405	-664	-682	-674	-583
-880	-880	0	-10	-404	-421	-686	-332	-1721	-1655	-6608	-6563	-390	-164	-189	-212	-170
-1428	0	-1428	-165	-560	-676	-1192	-545	-2796	-2551	-10513	-10429	-587	-284	-309	-299	-267
-388	-10	-165	-716	-186	-188	-325	-152	-784	-694	-2739	-2721	44	-60	-105	-92	-77
-1306	-404	-560	-186	-1306	-694	-973	-501	-2556	-2593	-9554	-9587	-502	-215	-291	-299	-193
-1547	-421	-676	-188	-694	-806	-1232	-585	-3025	-2841	-11435	-11359	-660	-312	-326	-326	-283
-2619	-686	-1192	-325	-973	-1232	-2225	-992	-5126	-4621	-19219	-19051	-1228	-545	-551	-540	-477
-1211	-332	-545	-152	-501	-585	-992	-529	-2370	-2169	-8870	-8799	-557	-248	-257	-253	-220
-6305	-1721	-2796	-784	-2556	-3025	-5126	-2370	-12464	-11265	-46483	-46118	-2719	-1298	-1333	-1319	-1135
-5761	-1655	-2551	-694	-2593	-2841	-4621	-2169	-11265	-10835	-42612	-42303	-2420	-1170	-1224	-1214	-1034
-23783	-6608	-10513	-2739	-9554	-11435	-19219	-8870	-46483	-42612	-176722	-175071	-10062	-4908	-5018	-4970	-4272
-23599	-6563	-10429	-2721	-9587	-11359	-19051	-8799	-46118	-42303	-175071	-174101	-9979	-4851	-4979	-4945	-4247
-1405	-390	-587	44	-502	-660	-1228	-557	-2719	-2420	-10062	-9979	-1405	-293	-307	-306	-268
-664	-164	-284	-60	-215	-312	-545	-248	-1298	-1170	-4908	-4851	-293	-664	0	0	0
-682	-189	-309	-105	-291	-326	-551	-257	-1333	-1224	-5018	-4979	-307	0	-682	0	0
-674	-212	-299	-92	-299	-326	-540	-253	-1319	-1214	-4970	-4945	-306	0	0	-674	0
-583	-170	-267	-77	-193	-283	-477	-220	-1135	-1034	-4272	-4247	-268	0	0	0	-583

Problem 2: Numerical Gradient and Hessian

I use a central approximation to numerically derive the score and hessian. My hessian approximation seems very wrong.

-2628.66
-563.39
-1167.15
-222.67
-941.82
-1215.13
-2109.09
-948.98
-5036.85
-4528.09
-19401.78
-19164.96
-929.22
-349.72
-467.20
-595.62
-557.88

-17803358787	-4843059287	-7730704965	-227373675	-7185008144	1205080480	-682121026	909494702	1682565198	-1591615728	-34106
-4843059287	-4843059287	0	-659383659	-2228262019	386535248	204636308	227373675	432009983	136424205	27284
-7730704965	0	-7707967598	-318323146	-2955857781	659383659	90949470	341060513	727595761	-704858394	-8867
-227373675	-659383659	-318323146	-12596501620	-613908924	-1455191523	1000444172	-1386979420	-1773514668	-1114131010	-7730
-7185008144	-2228262019	-2955857781	-613908924	-7185008144	113686838	136424205	341060513	1068656275	1159605745	-5456
1205080480	386535248	659383659	-1455191523	113686838	-14210854715	-954969437	-2341948857	-454747351	341060513	1136
-682121026	204636308	90949470	1000444172	136424205	-954969437	-8958522812	-1045918907	9049472283	6707523426	-2046
909494702	227373675	341060513	-1386979420	341060513	-2341948857	-1045918907	-2910383046	20713741833	17234924599	4320
1682565198	432009983	727595761	-1773514668	1068656275	-454747351	9049472283	20713741833	-1591615728	-363797881	-6366
-1591615728	136424205	-704858394	-1114131010	1159605745	341060513	6707523426	17234924599	-363797881	-4365574569	-5684
-341060513	272848411	-886757334	-773070497	-545696821	113686838	-204636308	432009983	-636646291	-568434189	-1225
113686838	90949470	-45474735	227373675	272848411	432009983	-500222086	1000444172	-1068656275	-522959454	-1068
-8344613889	-2410160960	-3524291969	-272848411	-3228706191	636646291	-2091837814	0	363797881	-1455191523	-909
-10277290130	-2569322533	-4342837201	90949470	-3547029337	68212103	-1318767318	-113686838	-500222086	-2319211490	682
-4592948244	-1591615728	-2069100447	-727595761	-2478373062	136424205	-1091393642	-454747351	-113686838	-568434189	-2273
-45474735	-22737368	0	591171556	-22737368	545696821	1296029950	-22737368	-22737368	-22737368	-909
0	0	0	-272848411	0	704858394	477484718	0	0	0	-1818

Problem 3: Newton's Method

Starting from the value of β from problem as the initial guess, my Newton's method code resulted in the following estimates:

constant	-6.06
i_large_loan	0.87
i_medium_loan	0.53
rate_spread	0.60
i_refinance	0.16
age_r	0.87
cltv	-0.06
dti	0.22
cu	1.01
first_mort_r	0.34
score_0	-0.28
score_1	0.19
i_FHA	0.76
i_open_year2	1.15
i_open_year3	0.77
i_open_year4	0.38
i_open_year5	0.24

Problem 4: Built-In method comparison

Starting from the initial guess in problem 1, I ran Julia's built-in Nelder-Mead, LBFGS, and Newton to optimize. The Nelder-Mead from that initial guess was unable to converge (first column). The other method resulted in very similar estimates.

variable	Nelder-Mead 1	Nelder-Mead 2	LBFGS	Newton
constant	-1.95	-6.06	-6.06	-6.06
i_large_loan	0.69	0.87	0.87	0.87
i_medium_loan	0.32	0.53	0.53	0.53
rate_spread	0.41	0.60	0.60	0.60
i_refinance	0.01	0.16	0.16	0.16
age_r	-0.59	0.87	0.87	0.87
cltv	-0.07	-0.06	-0.06	-0.06
dti	-0.38	0.22	0.22	0.22
cu	0.28	1.01	1.01	1.01
first_mort_r	0.52	0.34	0.34	0.34
score_0	-0.48	-0.28	-0.28	-0.28
score_1	0.23	0.19	0.19	0.19
i_FHA	0.68	0.76	0.76	0.76
i_open_year2	0.34	1.15	1.15	1.15
i_open_year3	0.06	0.77	0.77	0.77
i_open_year4	-0.41	0.38	0.38	0.38
i_open_year5	-0.49	0.24	0.24	0.24

Appendix

To verify my functions are correct, I ran the logit regression using R's built-in function (based on the do file):

```
r_logit <- glm(i_close_first_year ~ i_large_loan + i_medium_loan + rate_spread +
                 i_refinance + age_r + cltv + dti + cu + first_mort_r +
                 score_0 + score_1 + i_FHA + i_open_year2 + i_open_year3 +
                 i_open_year4 + i_open_year5,
    data = data, family = binomial(link = "logit"))
```

Table 1: R Output

	Dependent variable:
	i_close_first_year
Constant	-6.056^{***} (0.578)
i_large_loan	0.868^{***} (0.075)
i_medium_loan	0.527^{***} (0.070)
rate_spread	0.596^{***} (0.061)
i_refinance	0.163^{***} (0.060)
age_r	0.871^{***} (0.207)
cltv	-0.057 (0.179)
dti	0.215 (0.169)
cu	1.008^{***} (0.176)
first_mort_r	0.336^{***} (0.081)
score_0	-0.284^{***} (0.060)
score_1	0.189^{***} (0.060)
i_FHA	0.759^{***} (0.059)
i_open_year2	1.153^{***} (0.090)
i_open_year3	0.770^{***} (0.091)
i_open_year4	0.379^{***} (0.096)
i_open_year5	0.240^{**} (0.102)
Observations	16,401
Log Likelihood	$-5,311.495$
Note:	*p<0.1; **p<0.05; ***p<0.01