Homework 5: Intergenerational Mobility

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Abstract

In this homework you will solve a simplified version of the model in Lee and Seshadri (2019).

1 Model

1.1 Model Overview

Timeline

- Let $j \in \{0, 2, .., T\}$ denote model age. Each period in the model corresponds to 6-years as in Lee and Seshadri (2019).
  - i.e. $j = 0$ corresponds to $0 - 5$.
- At age $j = 4$, the child becomes an independent individual and makes their own decisions.
  - When making their own decisions the individual simply solves a consumption savings decision in the Bewley-Huggett-Aiyagari tradition.
- The agent works independently in the labor market in periods $j = 4$, and then in period 5 has a child. We refer to the years where an agent is an adult but not a parent, as the newly independent adult phase.
  - When an agent has a child, in addition to their consumption savings decisions, the agent also decides how much to invest in the child's human capital.
- The agent is responsible for the child for 4-periods (periods $5, 6, 7, 8$), and then makes a transfer to the child immediately before the child becomes an independent agent. We refer to this stage as the *parenting* stage.
- The agent then works for an additional 4 periods $(9, 10, 11, 12)$ and then exits the model.
  - During this period the agent simply makes consumption savings decisions in the Bewley-Huggett-Aiyagari tradition.

Notation

- Let h denote the human capital of an agent.
- Let h^c denote the human capital of an agents child.
- Let b denote the asset position of an agent.

1.2 Bellman Equations

1.2.1 Newly Independent Adults ($j = 4$)

Agents with credit. Let $V_4(b, h)$ denote the value function for an age 4 adult, who has assets b and human capital h. In the current period, the agent makes a consumption savings decision. At the start of the next period (when the agent is age 5), shocks to human capital are revealed. Additionally, the agent takes into account that in the next stage they will become a parent and take expectations over the value of becoming a parent. The decision problem for an age j agent with credit access, assets b, and human capital h is given by,

$$V_4(b, h) = \max_{b'} u(c) + \beta \mathbb{E}\left[V_5(b', h', h^{c'})\right]$$

subject to a budget constraint:

$$c + b' \leq w_j(h) + b(1 + r)$$

where $w_j(h)$ maps human capital into earnings following:

$$w_j(h) = \exp(\kappa_j + h) \tag{1}$$

where κ_j is the value of an age earnings profile for an age j agent. Human capital of agent evolves according to the following law of motion

$$h' = \rho_h h + \eta \tag{2}$$

where shocks to human capital η are defined such that $\eta \sim N(0, \sigma_\eta^2)$, and ρ_h governs the persistence of shocks.

1.2.2 Parenting Stage ($j = 5, 6, 7, 8$)

Let $V_j(b, h, h^c)$ denote the value function for an age j parent with credit access with assets b, human capital h, and whose child has human capital h^c.[1] In the current period, the agent makes a consumption savings decision, as well as a decision for how much to invest in their child's human capital. Investing in the child's human capital (i) has the potential to increase the child's human capital, which will subsequently impact their earnings.[2] At the start of the next period (when the parent is age $j + 1$), shocks to human capital are revealed (for both the parent and child). The decision problem for an age j parent with credit access with assets b, human capital h, and whose child has human capital h^c is given by,

[1] Note that because of the life-cycle structure of the model, we only need to keep track of the age of the parent.
[2] For simplicity, just allowing the parent to make a monetary choice. Need to revisit this decision.

$$V_j(b,h,h^c) = \max_{b',i\geq 0} u(c) + \beta \mathbb{E}\left[V_{j+1}(b^{'},h^{'},h^{c'})\right] \tag{3}$$

subject to a budget constraint:

$$c + b^{'} + i \leq w_j(h) + b(1+r)$$

The wage process for adults is governed by equation 1 and the parents human capital is governed by the law of motion in 2. Finally, the human capital of the child evolves according to,

$$h^{c'} = (1-\omega_c)h^c + \kappa\omega_c i \tag{4}$$

1.2.3 Post Child Working Stage ($j = 9, 10, 11, 12$)

Agents begin their post child working stage ($j = 9$) by making a one-time transfer $\tau > 0$ to the child when making their consumption savings decision. The transfer to the child (τ) governs the amount of assets the child begins their newly independent adult stage with. The parent receives utility from this transfer to the child, which is governed by an altruism parameter θ. At the start of the next period, shocks to human capital are realized. The parents solves the following maximization problem,

$$\begin{aligned}
V_9(b,h,h^c) &= \max_{b',\tau\geq 0} u(c) + \theta V_4(\tau,h^c) + \beta\mathbb{E}[V_{10}(b',h')] \\
V_j(b,h) &= \max_{b',\tau\geq 0} u(c) + \beta\mathbb{E}[V^C_{j+1}(b',h')] \text{ for } j = 10,11,12 \\
V^C_{13}(b,h) &= 0
\end{aligned}$$

subject to the budget constraint

$$\begin{aligned}
c + \tau + b' &= y_j(h) + b(1+r) \text{ for } j = 9 \\
c + b' &= y_j(h) + b(1+r) \text{ for } j = 10,11,12
\end{aligned}$$

and the wage equation (equation 1), as well as the laws of motion for the parent's human capital (equation 2).

2 Estimation

In this section we discuss the estimation and calibration of the model.

Demographics and preferences.

- Given the 6-year timing of the model, and annual risk-free rate of 4%, set the discount factor for agents $\beta = \left(\frac{1}{1.04}\right)^6$

Credit markets.

- Given the 6-year timing of the model, set the risk-free rate $r = (1.04)^6$.

Income process

- Use your estimates of the age-earnings profile from homework 1 to set the values of κ_j by taking the average values for the age range in period j.
- For simplicity, use the value of σ^2 for non-college graduates from Lee and Seshadri (2019) to set the variance of η. Set $\rho_h = (.97)^6$.

Child's human capital process

- Set $\omega_c = 0.5$ and set $\kappa = \frac{1}{10}$.
- For simplicity, assume children's human capital is drawn randomly from a uniform distribution over the bottom half of the human capital distribution (grid).

3 Assignment

To complete the assignment, please do the following:

1. Show if the child's human capital function exhibits dynamic complementarity. How do you think this will impact your results?
2. Solve and simulate data from the model using the suggested parameters above.
3. Plot policy functions for investment in the child's human capital.
4. Plot policy functions for the transfer that parents give to their children.
5. Plot the distribution of human capital in the economy when agents enter the labor market.
6. Plot the variance of earnings as a function of age in the economy.

4 Suggested solution method

- This model will require you to iterate to convergence.
- You will need to guess a value of being a newly independent adult $V_4(h, b)$. I would start with a guess of zeros.
- Make an initial guess and then solve the model starting from the terminal period and work backwards. This will produce a new estimate of the value of being a newly independent adult, lets call this $\hat{V}_4(h, b)$. Check if $V_4(h, b)$ and $\hat{V}_4(h, b)$ are the same.
- If not, set $V_4(h, b) = \hat{V}_4(h, b)$ and repeat until these align.

References

Lee, S. Y. and A. Seshadri (2019). On the intergenerational transmission of economic status. *Journal of Political Economy 127*(2), 000–000.