

# ECON 714A - Problem Set 4

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This problem asks you to solve a model of oligopolistic competition from Atkeson and Burstein (AER 2008), which extends the Dixit-Stiglitz setup and is widely used to analyze heterogeneous markups and incomplete pass-through.

Consider a static model with a continuum of sectors  $k \in [0, 1]$  and  $i = 1, \dots, N_k$  firms in sector  $k$ , each producing a unique variety of the good. Households supply inelastically one unit of labor and have nested-CES preferences:

$$C = \left( \int C_k^{\frac{\rho-1}{\rho}} dk \right)^{\frac{\rho}{\rho-1}}, C = \left( \sum_{i=1}^{N_k} C_{ik}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}}, \theta > \rho \geq 1.$$

Production function of firm  $i$  in sector  $k$  is given by  $Y_{ik} = A_{ik}L_{ik}$ .

1. Solve household cost minimization problem for the optimal demand  $C_{ik}$ , the sectoral price index  $P_k$ , and the aggregate price index  $P$  as functions of producers' prices.

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2. Assume that firms compete à la Bertrand, i.e. choose  $P_{ik}$  taking the prices of other firms in a sector  $P_{jk}, j \neq i$  as given. Derive demand elasticity for a given firm and the optimal price.

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3. Prove that other things equal, firms with higher  $A_{ik}$  set higher markups.

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4. Assume that  $\rho = 1, \theta = 5, N_k = 20$ , and  $\log A_{ik} \sim i.i.d.N(0, 1)$ . Solve the model numerically by approximating the number of sectors with  $K = 100,000$ . You will need an efficient algorithm to compute a sectoral equilibrium (search for a fixed point, do not use "solve") nested in a general equilibrium loop solving for real wages.

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5. Compute the aggregate output  $C$  of the economy and compare it to the first-best value.

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6. Bonus task: Does the sectoral equilibrium converge to the one under Bertrand competition with homogeneous goods in the limit  $\infty \rightarrow \infty$ ?

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\*I worked on this problem set with a study group of Michael Nattinger, Andrew Smith, and Ryan Mather. I also discussed problems with Sarah Bass, Emily Case, Danny Edgel, and Katherine Kwok.