Bank Regulation with Uninformed Regulators

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Abstract

I analyze a bank regulator's decision problem in a simple two-period model based on Allen and Gale [3] where a bank privately chooses the risk-return characteristic of its loan portfolio. Funded by both insured deposits and equity capital, a bank chooses the quantity and the risk-return characteristic of its loans. Limited liability from deposit insurance creates an opening for regulations even without private information. From the perspective of a regulator who accounts for deposit insurance payouts, I derive allocations and regulations when the risk-return characteristic of loans is observable and unobservable. I show that regulations are tighter when the risk-return characteristic is private information; the tighter regulations limit the ability to use deposits and thus increase "skin in the game", so the bank chooses safer loans. I also show that if regulators naively neglect that the bank can underreport its loans' risk-return characteristic when it is actually private information, the bank makes the first best quantity of loans but these loans are excessively risky. These findings relate to recent empirical evidence that finds that banks underreport risk using Basel II risk weights.

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1 Introduction

Balance sheet quantities like amount of deposit funding are easily observed by regulators, but other important factors for regulation—like credit worthiness of bank borrowers—are less easily observed. This paper explores a regulator's decision problem when the risk-return characteristic of a bank's loan portfolio is observable versus when it is unobservable. I show that regulations are tighter when the risk-return characteristic is private information; the tighter regulations limit the ability to use deposits and thus increase "skin in the game", so the bank chooses safer loans.

Deposit insurance guarantees that depositors' funds are safe even in the event of a bank's failure. This insurance creates limited liability for bank managers and stockholders and it opens the door to moral hazard through making excessively risky and/or large loan portfolios (Kareken and Wallace [13]). Because their failures can hamper credit availability, financial stability, and economic activity (Romer and Romer [22]), banks are highly regulated to address the potential for this model hazard. Some of these regulations (namely, risk-based capital requirements) depend on credit risk estimates of bank loans. Naturally, banks with higher credit risks (e.g., banks with portfolios of risky corporate debt) are regulated more strictly than banks with lower credit risk (e.g., banks with loan portfolios of government debt and safe mortgages). Some regulatory regimes use credit risk estimates that regulators determine and other regulatory regimes use credit risk estimates that the banks themselves estimate and report. While the credit risk estimates from regulators may be more coarse (e.g., all mortgages get the same credit risk estimate), the estimates from banks can be highly tailored to loan-specific characteristics (e.g., mortgage credit risk estimates depend on borrower credit score). However, recent empirical evidence suggests that banks underreport credit risk in the estimates they report (Behn, Haselman, and Vig [5]). The use of credit risk estimates highlights a key trade-off that banks have better information than regulators about the credit risk of their portfolios, but banks also have the incentive to underreport credit risk to loosen requirements. How should regulators set regulation in the face of this trade-off between information and the incentive to underreport?

In this paper, I analyze a bank regulator's decision problem in a simple two-period model based on Allen and Gale [3]. Similar to Allen and Gale [3], a bank is funded by insured deposits and chooses the quantity and risk-return characteristic of loans to make. Unlike Allen and Gale [3], this risk-return characteristic is private information. Another departure from Allen and Gale [3] is that the bank is funded by both insured deposits and equity capital. The equity funding creates "skin in the game" when banks make their lending decisions. The regulator chooses regulations to address two frictions: (1) limited commitment through deposit insurance and (2) private information about the risk-return characteristic of the bank's loans.

I characterize four sets of allocations: (1) the first best allocation, (2) the allocation chosen by an unregulated bank with deposit insurance, (3) the allocation chosen by a naive regulator that misses that the risk-return characteristic of loans is private information and the bank can misreport it, and (4) the allocation that account for the bank's private choice of the risk-return characteristic. I show that the first best allocation is not incentive compatible when the risk-return characteristic is private information. The regulator chooses tighter regulations (i.e., allowing the bank to use fewer deposits) when loan risk-return is private information. With tighter regulations, the bank uses less deposit funding relative to its equity capital. Since proportionately more of its funding is equity capital instead of insured deposits, the bank has more "skin in the game," so it chooses safer loans than is incentive compatible under the first best quantity of deposits.

Since the naive regulator thinks she has full information, she naturally chooses the same regulations as the regulator with full information (i.e., the first best). Thus, the bank makes the same quantity of loans as the first best allocation because insured deposits and equity capital are both observable to the regulator. However, since the first best allocation is not incentive compatible, the bank chooses loans with a higher risk-return characteristic than in the first best allocation. These loans are riskier because the naive regulator has not tightened regulations to increase the bank's "skin in the game."

These findings relate to policy discussions and academic literature about risk-based capital requirements and specifically the determination of risk weights, which has changed dramatically since their introduction in the 1980s. Risk-weighted capital requirements ensure that a bank's equity capital exceeds a fraction of the sum of its assets weighted using risk weights. The method of determining risk weights has changed substantially across the Basel accords, which are international agreements between regulators about the broad design of bank regulations. Introduced in the mid-1980s, Basel I risk weights were relatively simple and determined by regulators (e.g., 0.5 for all residential mortgages) (Basel I [17]). Concerns arose that Basel I risk weights did not adequately account for risk sensitivity (e.g., all mortgages had the same risk weight despite different loan-to-value ratios). In the mid-2000s, the agreements changed to add granularity to buckets of assets from the Basel I system; this approach is called the "standardized approach" or "SA" (Basel II [18]). Basel II also allowed banks to develop more complex credit risk models that the banks could run in-house, which, after approval by regulators, would determine a loan-level risk weight; this approach is called "internal-rating based" or "IRB". During and after the 2008 financial crisis, critics raised concerns about the validity of IRB risk weights (Haldane [12]). In response, the post-crisis bank regulations have banks compute their risk-weighted assets under both IRB and SA and must satisfy both corresponding capital requirements (Basel III [19]). [1]

1.1 Allen and Gale [3]

My framework build on Allen and Gale [3], who focus on the trade-off between stability and competition in the banking industry. In Allen and Gale [3], oligopolistic banks Cournot compete in the insured deposit market and choose the quantity and risk-return characteristic of their loans. Allen and Gale [3] explore how the equilibrium risk-return characteristic changes as the number of banks in the banking industry changes. Here, my focus is how asymmetric information about the loan risk-return characteristic between a regulator and a bank affects regulations (specifically, capital requirements) and allocations. Thus, the three major changes between Allen and Gale [3] and what I do here are (1) the information structure about the risk-return characteristic, (2) the inclusion of equity capital as a funding source, and (3) one bank instead of n banks. I expand on

[1] Recent changes to the post-crisis bank regulations (so called "Basel IV") only includes very small adjustments to the Basel III approach to risk weights (Basel IV [20]).

the motivation for change (1) from Allen and Gale [3] in the following subsection.

To expand on change (2) the inclusion of equity capital, banks in Allen and Gale [3] are fully funded by insured deposits and have no equity capital. Thus, bank managers and stockholders have no "skin in the game." The policy discussion in Allen and Gale [3] is limited to deposit insurance premiums and they do not discuss capital requirements. In practice and in my environment, changing capital requirement changes the amount of "skin in the game" the bank needs to have to make loans and the moral hazard of making excessively risky loans reduces as capital requirements tighten. In my framework, the bank is born with a quantity of equity capital and then can borrow additional funds through the insured deposit market. Note that this change is not a simple relabeling of the elements in Allen and Gale [3]. The regulations imposed on a bank born with a lot of equity capital are very different from the regulations for a bank that is born with little equity capital.

My motivation for change (3) simplifying the market structure to a single bank is to simplify the exposition. I plan to show in an appendix that the major results are unaffected when there are n banks. This appendix will be a more direct extension of Allen and Gale [3].

Many papers build on Allen and Gale [3] including Boyd and De Nicolo [9], Martinez-Miera and Repullo [16], and Corbae and Levine [10]. To my knowledge, all of such papers continue in the same vein as Allen and Gale [3] with a focus on the relationship between competition and stability. For example, Corbae and Levine [10] extend Allen and Gale [3] by adding dynamics, agency conflicts, endogenous market structure, and optimal regulatory and monetary policy. I am unaware of papers that add private information to the environment from Allen and Gale [3].

1.2 Other literature

Making the bank's choice of the loan portfolio risk-return characteristic private is motivated by research that shows that lending decisions depend both on "hard" information, which often is verifiable, quantitative, easily stored, and/or easily transmitted, and "soft" information, which is often non-verifiable, qualitative, relatively complex, and/or context dependent (Liberti and Petersen [14]). Much of the empirical literature proxies for soft information by considering lenders that are physically close to borrowers or that have prior relationships with borrowers. Less physical distance and prior relationships result in better credit terms as well as better outcomes (e.g., fewer defaults) in consumer lending (Agarwal et al [1]) and in small business lending (Agarwal and Hauswald [2], Berger and Udell [7], Petersen and Rajan [21]).

Possibly due to soft information about the creditworthiness of borrowers, recent empirical literature has found that some banks underreport credit risk through IRB risk weights. Behn, Haselmann, and Vig [5] use loan-level data from Germany to study the introduction of IRB capital requirements. IRB banks run credit risk models that are approved by regulators; however, these models have many modeling choices (e.g., hundreds of estimated parameters) that may allow banks to manipulate the results.[2] They find that IRB banks systematically underreported risk compared to banks that use regulator-provided credit risk estimates associated with SA risk weights. In particular, banks with more to gain from underreporting risks (e.g., banks that are capital constrained) underreported risks more.[3] Despite these lower credit risk reports, IRB loans experience more defaults and have higher interest rates than SA loans, indicating that banks are aware that these loans are riskier. Mariathasan and Merrouche [15] find similar evidence of bank misreporting in a cross-country sample of banks using IRB risk weights.

Finally, my paper also relates to a literature that focuses on how capital requirements should be designed to account for bank misreporting. Using a theoretical model concurrently with the implementation of Basel II and the introduction of IRB risk weights, Blum [8] analyzes regulatory capital requirements that depend on the level of risk reported by banks. He argues that, if supervisors cannot detect misreporting, a risk-independent capital requirement (i.e., a leverage ratio requirement) may be necessary for truthful reporting. Following along these lines, Demirguc-Kunt

[2] Along similar lines, Berg and Koziol [6] document the considerable variability of probabilities of default internally estimated by different banks for the same borrower.

[3] Focusing on banks' trading books instead of their loan portfolios, Begley, Purnanandam, and Zheng [4] also find that banks significantly underreport risks when they have lower equity capital.

et al [11] find evidence that leverage ratios were more closely associated with higher stock returns during the 2008 financial crisis than risk-based capital ratios.

2 Model

2.1 Environment

The model has two periods. In period 1, bank is endowed with equity capital $e \geq 0$. Then the bank chooses how many deposits $d \geq 0$ to borrow from a deposit technology. In period 2, the bank will have to repay the deposit technology $R(d) \cdot d$ where R is increasing (i.e., the bank has to pay more per deposit if it pulls out more deposits). The bank then invests its equity capital and deposits $e+d$ into a risky loan technology. With its investment in the risky loan technology, the bank chooses the risk-return characteristic $s \in [0, 1]$. A higher risk-return characteristic increases the likelihood that the risky loan technology fails, but—conditional that the risky loan technology pays off—it pays off more. More specifically, with probability $p(s)$, the risky loan technology returns $A \cdot s \cdot (d+e)$ at the beginning of period 2 where p is decreasing. With probability $1 - p(s)$, the risky loan technology returns zero at the beginning of period 2. The bank's objective is to choose s and d given e to maximize expected profit at the end of period 2.

The timing is below:

- In period 1,
  - A bank is endowed with $e \geq 0$
  - The bank chooses $d \geq 0$ and $s \in [0, 1]$
  - The bank pulls d out of deposit technology
  - The bank inputs $d + e$ into risky loan technology at s
- In period 2 with probability $p(s)$,
  - Risky loan technology outputs $A \cdot s \cdot (d + e)$
  - Bank pays $R(d) \cdot d$ to deposit technology

- Profit is $A \cdot s \cdot (d+e) - R(d) \cdot d$

- In period 2 with probability $1 - p(s)$,
  - Risky loan technology outputs zero
  - Bank pays $R(d) \cdot d$ to deposit technology
  - Profit is $-R(d) \cdot d$

Assumptions 1 and 2 stipulate conditions on p and R. Differentiability assumptions are required for analytical solutions to exist; monotonicity and concavity/convexity assumptions are important for all major results; and boundary values ensure interior solutions.

Assumption 1. *p is twice differentiable, strictly decreasing, and weakly concave with $p(0) = 1$ and $p(1) = 0$.*

Assumption 2. *R is twice differentiable, strictly increasing, and weakly convex with $R(0) = 0$.*

I introduce two frictions to this environment: deposit insurance and private choice of s. With deposit insurance, the bank does not pay back the deposit technology if the risky loan technology fails. Thus, if the risky loan technology fails, the bank's profit with deposit insurance is zero. The second friction is the bank's choice of the risk-return characteristic of the risky loan technology is private. In addition, the output and profit in period 2 are assumed to be private because an outsider could backing out s from either output and profit using d and e, which are both observable.

2.2 First Best Allocation

In this section, I solve for the first best allocation denoted (s^*, d^*). Characterizing this allocation will provide a benchmark from which to evaluate the effects of adding frictions and regulations. The first best allocation maximizes the expected risky loan technology output minus the deposit return:

$$
\begin{aligned}
&\max_{s^*,d^*} p(s^*)\cdot \underbrace{[A\cdot s^*\cdot(d^*+e)-R(d^*)\cdot d^*]}_{\text{profit if risky tech succeeds}} + (1-p(s^*))\cdot \underbrace{[-R(d^*)\cdot d^*]}_{\text{profit if risky tech fails}} \\
\implies &\max_{s^*,d^*} \underbrace{p(s^*)\cdot A\cdot s^*\cdot(d^*+e)}_{\text{expected output}} - \underbrace{R(d^*)\cdot d^*}_{\text{deposit return}} \qquad (1)
\end{aligned}
$$

The marginal benefit of higher s^* is more output if the risky loan technology is successful and the marginal cost is that failure is more likely. The first order condition with respect to s^* equates the marginal benefit and cost:

$$
\underbrace{p(s^*)\cdot A\cdot(d^*+e)}_{\substack{\text{higher } s^* \implies \text{more output if success} \\ \text{(MB)}}} = \underbrace{-p'(s^*)\cdot A\cdot s^*\cdot(d^*+e)}_{\substack{\text{but failure more likely} \\ \text{(MC)}}} \qquad (2)
$$

Dividing both sides by $A\cdot(d^*+e)$, the first order condition simplifies to the following expression:

$$
p(s^*) = -p'(s^*)\cdot s^* \qquad (3)
$$

Notice that s^* only depends on p and it is neither a function of d^* nor e. The first best risk-return characteristic is only determined by the characteristics of the risky loan technology itself. This feature of s^* does not hold for allocations after frictions are added.

Now, let us turn to the first best allocation of deposits. The marginal benefit of more deposits is more expected output and the marginal cost is that the marginal deposits must be acquired plus more must be paid for the inframarginal deposits. The first order condition with respect to d^* equates the marginal benefit of more deposits against the marginal costs:

$$\underbrace{p(s^*) \cdot A \cdot s^*}_{\substack{\text{higher } d^* \implies \text{more output} \\ \text{(MB)}}} = \underbrace{R(d^*)}_{\substack{\text{but pay for marginal deposits} \\ \text{(MC)}}} + \underbrace{R'(d^*) \cdot d^*}_{\substack{\text{and pay inframarginal deposits more} \\ \text{(MC)}}} \tag{4}$$

Notice that d^*, similar to the choice of s^*, does not depend on the level of equity financing e. Again similar to s^*, this feature does not extend to allocations with frictions.

2.3 Deposit Insurance

In this section, I solve for the allocation denoted (s^U, d^U) chosen by an unregulated bank with deposit insurance. With deposit insurance, the bank does not pay back deposits if the risky loan technology fails. The unregulated allocation maximizes the expected risky loan technology output minus the expected deposit return:

$$\begin{aligned} &\max_{s^U, d^U} p(s^U) \cdot \underbrace{[A \cdot s^U \cdot (d^U + e) - R(d^U) \cdot d^U]}_{\text{profit if risky tech succeeds}} + (1 - p(s^U)) \cdot \underbrace{0}_{\text{profit if risky tech fails}} \\ \implies &\max_{s^U, d^U} \underbrace{p(s^U) \cdot A \cdot s^U \cdot (d^U + e)}_{\text{expected output}} - \underbrace{p(s^U) \cdot R(d^U) \cdot d^U}_{\text{expected deposit return}} \end{aligned} \tag{5}$$

Notice that the difference between (1) and (5) is that the second term in (5) includes the probability that the risky loan technology succeeds. The first best allocation accounts for the deposit return when the risky loan technology fails and the allocation chosen by an unregulated bank with deposit insurance does not.

Similar to the first best, the marginal benefit of higher s^U is more output if the risky loan technology is successful and the marginal cost is that failure is more likely, but there is an additional marginal benefit that a higher choice of s^U reduces the likelihood of paying back the deposits. The first order condition with respect to s^U equates the marginal benefit and cost:

$$\underbrace{p(s^U)\cdot A\cdot(d^U+E)}_{\substack{\uparrow s^U \implies \uparrow \text{ output if success} \\ \text{(MB)}}} + \underbrace{-p'(s^U)\cdot R(d^U)\cdot d^U}_{\substack{\text{and less likely to pay back deposits} \\ \text{(MB)}}} = \underbrace{-p'(s^U)\cdot s^U\cdot A\cdot(d^U+E)}_{\substack{\text{but failure more likely} \\ \text{(MC)}}} \tag{6}$$

Dividing both sides by $A\cdot(d^U+e)$, the first order condition simplifies to the following expression:

$$p(s^U) + \frac{-p'(s^U)\cdot R(d^U)\cdot d^U}{A\cdot(d^U+e)} = -p'(s^U)\cdot s^U \tag{7}$$

With the additional marginal benefit, an unregulated bank with deposit insurance chooses more risk and return than in the first best allocation, $s^U > s^*$. Furthermore, notice now that, unlike s^*, s^U depends on d^U and e; that is, the choice of the risk-return characteristic depends on the funding mix of deposits and equity with deposit insurance. We can show that as the bank becomes fully funded by equity, it chooses the first best risk-return characteristic, $s^U \to s^*$ as $e \to \infty$.

Turning to the choice of deposits by the unregulated bank, similar to the first best allocation, the marginal benefit of more deposits is more expected output. The marginal cost, however, is multiplied by the probability that the risky loan technology succeeds because the marginal deposits must be acquired and the inframarginal deposits must be paid more only if the risky loan technology succeeds. The first order condition with respect to d^U equates the marginal benefit of more deposits against the marginal costs:

$$\underbrace{p(s^U)\cdot A\cdot s^U}_{\substack{\uparrow d^U \implies \uparrow \text{ expected output} \\ \text{(MB)}}} = \underbrace{p(s^U)\cdot[R(d^U)+R'(d^U)\cdot d^U]}_{\substack{\text{but might pay more for deposits} \\ \text{(MC)}}} \tag{8}$$

The difference between (4) and (8) is that the probability of the risky loan technology succeeding is included on the RHS of (8). Unsurprisingly, since the marginal cost of more deposits is lower, the unregulated bank chooses more deposits, $d^U > d^*$. Furthermore, as the amount of equity financing becomes large, the bank still benefits from deposit insurance, so it will still want to take out more deposits than in the first best.

2.4 Implementing the First Best

In the previous section, I discussed how that $s^U > s^*$ and $d^U > d^*$. In this section, I show that a regulator can implement the first best allocation with limit on leverage that importantly depends on s. In particular, we can consider the model equivalent of a risk-weight capital requirement. Such regulations are in practice used by the Federal Reserve System, the Federal Deposit Insurance Corporation, and the Office of the Comptroller to limit banks' ability to take on leverage.

In particular, a risk-weighted capital requirement ensures that a bank's equity capital is larger than a weighted sum of its loans. In this model, there is a single asset so the requirement becomes:

$$e \geq \underbrace{\theta^*}_{\text{minimum ratio}} \cdot \underbrace{w(s)}_{\text{risk weight}} \cdot \underbrace{(d+e)}_{\text{loans}}$$

$$\text{where } \theta^* = \frac{e}{d^* + e}$$

$$\text{and } w(s) = \begin{cases} 1, & \text{if } s = s^* \\ K, & \text{if } s \neq s^* \end{cases}$$

With such a risk-weighted capital requirement, a bank chooses s^* for a K that is sufficiently high. Then, setting the risk weight at s^* to one and the minimum leverage ratio θ^* at the leverage ratio associated with the first best allocation, the bank can use up to d^* deposits. With deposit insurance, the bank wants to use as many deposits as possible, this constraint binds. Thus, they choose (s^*, d^*). Suppose not and the bank chooses a different risk-return characteristic $\hat{s} \neq s^*$, then bank can only borrow up to $\frac{d^*+(1-K)e}{K}$, which converges to zero as $K \to \infty$. So for sufficiently large K, it will be optimal for bank to choose s^* over other value of s.

Importantly, implementing the first best with deposit insurance using limits on leverage like a risk-weighted capital requirement requires that the regulator know s. In the next section, I consider how the regulator show set a leverage limit if she cannot observe s.

In this section, I focus on risk-weight capital requirements because they most closely correspond to regulation used in practice, but I plan to add an appendix section where I evaluate other policies (e.g., output taxes, deposit taxes) to achieve the first best allocation in the presence of deposit insurance.

2.5 Regulator Problem with Private s

In the previous section, I discussed how the first best allocation could be implemented in a world with deposit insurance using a risk-weighted capital requirement that depended on s. In this section, I consider s being unobservable to outsiders. Making s unobservable is motivated by empirical evidence that creditworthiness depends not just on hard, verifiable, easily-communicated information but also soft, non-verifiable, complex information as well as recent empirical evidence that banks underreport risk. In particular, I consider the problem facing a regulator who chooses a limit on leverage θ^P that does not depend on s:

$$\begin{aligned}
&\max_{\theta^P} \underbrace{p(s^P)\cdot A\cdot s^P\cdot(d^P+e)}_{\text{expected risky tech output}} - \underbrace{R(d^P)\cdot d^P}_{\text{deposit return}} \qquad (9)\\
&\text{s.t.}\quad (s^P,d^P)\in\arg\max_{s,d}\left\{\underbrace{p(s)\cdot A\cdot s\cdot(d+e)}_{\text{expected risky tech output}} - \underbrace{p(s)\cdot R(d)\cdot d}_{\text{expected deposit return}}\right\}\\
&\qquad\qquad\qquad\qquad\qquad\qquad\text{s.t. } e\geq\theta^P(d+e)
\end{aligned}$$

The regulator chooses θ^P to maximize the frictionless objective function, which importantly accounts for the deposit return after the risky loan technology fails, subject to the allocation satisfying the bank's maximization problem, which importantly has an objective function that does not account for the deposit return after the risky loan technology fails, subject to a leverage requirement. Following along similar lines of a Ramsay planner, problem (9) is equivalent to problem (10) in which the regulator chooses the allocation directly. This simplification works in this context because, with deposit insurance, the banks wants to borrow as many deposits as possible making the leverage requirement binding. Thus, the regulator effectively gets to choose d subject to s satisfying the bank problem at that chosen level of d.

$$\begin{aligned} &\max_{s^P, d^P} p(s^P) \cdot A \cdot s^P \cdot (d^P + e) - R(d^P) \cdot d^P \\ &\text{s.t.} \quad s^P \in \arg\max_s \left\{ p(s) \cdot A \cdot s \cdot (d^P + e) - p(s) \cdot R(d^P) \cdot d^P \right\} \end{aligned} \tag{10}$$

The first order condition of the constraint on (10) is the same as the first order condition with respect to s^U from the previous section. Thus, problem (10) can be further simplified to problem (11).

$$\begin{aligned} &\max_{s^P, d^P} \underbrace{p(s^P) \cdot A \cdot s^P \cdot (d^P + e)}_{\text{expected risky tech output}} - \underbrace{R(d^P) \cdot d^P}_{\text{deposit return}} \\ &\text{s.t.} \quad p(s^P) + \frac{-p'(s^P) \cdot R(d^P) \cdot d^P}{A \cdot (d^P + e)} = -p'(s^P) \cdot s^P \qquad \text{[Bank FOC]} \end{aligned} \tag{11}$$

2.6 Naive Regulator

In this section, I consider whether the first best allocation can be implemented with a leverage limit that does not depend on s. At (s^*, d^*), from the bank's perspective, the marginal benefit of higher s is larger than the marginal cost, so the constraint on (11) does not hold:

$$\underbrace{p(s^*) + \frac{-p'(s^*) \cdot R(d^*) \cdot d^*}{A \cdot (d^* + e)}}_{\text{MB}} > \underbrace{-p'(s^*) \cdot s^*}_{\text{MC}}$$

Thus, if a naive regulator ignores the private information and implements a leverage requirement that gets the bank to choose $d^N = d^*$, then the bank will choose $s^N > s^*$ to equate its first order condition:

$$p(s^N) + \frac{-p'(s^N) \cdot R(d^*) \cdot d^*}{A \cdot (d^* + e)} = -p'(s^N) \cdot s^N$$

2.7 Sophisticated Regulator

Unlike the naive regulator, a sophisticated regulator accounts for the bank's private choice of s and solves problem (11). Let λ be the multipler on the bank first order condition. Similar to the first best allocation, the marginal benefit of higher s^P is more output if the risky loan technology is successful and the marginal cost is failure is more likely, but there is an additional wedge term that a higher s^P eases the bank first order condition.

$$\begin{aligned}
&\underbrace{p(s^P)}_{\substack{\uparrow S \implies \uparrow \text{ output if success} \\ \text{(MB)}}} + \underbrace{\frac{-\lambda}{A \cdot (d^P + e)} \left[\frac{p''(s^P) \cdot [A s^P (d^P + e) - R(d^P) \cdot d^P]}{A \cdot (d^P + e)} + 2p'(s^P) \right]}_{\substack{\text{eases bank FOC} \\ \text{(wedge)}}} \\
&= \underbrace{p'(s^P) \cdot s^P}_{\substack{\text{but failure more likely} \\ \text{(MC)}}} \qquad (12)
\end{aligned}$$

Since higher s^P eases the bank's first order condition, it is an additional marginal benefit. The additional marginal benefit means that the regulator chooses a higher risk-return characteristic in the presence of private information relative to the first best, $s^P > s^*$. Similar to s^U, as $e \to \infty$, this wedge term converges to zero, so $s^P \to s^*$.

Turning to the choice of deposits, similar to d^*, the marginal benefit of more deposits is higher expected output and the marginal cost is the increased cost of deposits. Similar to s^P, there is an additional wedge term that encompasses the impact on the bank first order condition:

$$\underbrace{p(s^P) \cdot A \cdot s^P}_{\substack{\uparrow D \implies \uparrow \text{output} \\ \text{(MB)}}} = \underbrace{R(d^P) + R'(d^P) \cdot d^P}_{\substack{\text{but pay more for deposits} \\ \text{(MC)}}} + \underbrace{\frac{-\lambda \cdot p'(s^P)}{A \cdot (d^P + e)} \cdot \left[\frac{R(d^P) \cdot e}{d^P + e} + R'(d^P) \cdot d^P \right]}_{\substack{\text{tighten bank FOC} \\ \text{(wedge)}}}$$

Since higher d^P makes the bank want to choose higher s, it tightens the bank first order condition. Thus, this wedge term represents an additional marginal cost. With the additional marginal cost, the regulator chooses a lower level of deposits relative to the first best, $d^P < d^*$.

3 Results

3.1 Lemmas

In previous section, I have described and derived equations that characterize four allocations: (1) the first best allocation (s^*, d^*), (2) the allocation chosen by an unregulated bank with deposit insurance (s^U, d^U), (3) the allocation chosen by a naive regulator (s^N, d^N) facing private information about s, and (4) the allocation chosen by a sophisticated regulator (s^P, d^P) facing private information about s. In this section, I formally prove that (1) $d^U > d^* = d^N > d^P$ and (2) $s^U > s^N > s^P > s^*$. The unregulated bank chooses the highest d and s because they are not forced to internalize any effects of deposit insurance. The naive regulator chooses regulation to implement the same level of deposits as the first best, but the sophisticated regulator chooses to tighten regulations $d^N = d^* > d^P$, so that with fewer deposits the bank has more "skin in the game" and chooses safer loans, $s^N > s^P > s^*$.

Proofs of existence, uniqueness, and interiority for all allocations are relegated to section 5.1 in the appendix. While these results are required for the main results of the paper to hold, they are not my primary focus. All proofs in the main text assume the allocations exist, are unique, and are interior. Section 5.1 also includes proofs that describe how allocations change as $e \to \infty$.

Lemma 1. $s^U > s^*$ *and* $s^P > s^*$

Proof. The first order condition with respect to s^*, first order condition with respect to s^U, and constraint on sophisticated regulator problem hold:

$$p(s^*) + p'(s^*) \cdot s^* = 0 \tag{13}$$

$$p(s^U) + p'(s^U) \cdot s^U = \frac{p'(s^U) \cdot R(d^U) \cdot d^U}{A \cdot (d^U + e)} \tag{14}$$

$$p(s^P) + p'(s^P) \cdot s^P = \frac{p'(s^P) \cdot R(d^P) \cdot d^P}{A \cdot (d^P + e)} \tag{15}$$

LHS of (13), (14), and (15) are strictly decreasing in s. RHS of (14) and (15) are negative because $p' < 0$. Therefore, $s^U > s^*$ and $s^P > s^*$. □

Lemma 2. $d^U > d^*$

Proof. Since $R' > 0$ and $R'' \geq 0$,

$$d^U > d^* \iff R(d^U) + R'(d^U) \cdot d^U > R(d^*) + R'(d^*) \cdot d^*$$

First order condition with respect to d^* and first order condition with respect to d^U hold:

$$p(s^*) \cdot A \cdot s^* = R(d^*) + R'(d^*) \cdot d^* \tag{16}$$

$$p(s^U) \cdot A \cdot s^U = p(s^U) \cdot [R(d^U) + R'(d^U) \cdot d^U] \tag{17}$$

Substituting in (16) and (17),

$$\begin{aligned} R(d^U) + R'(d^U) \cdot d^U > R(d^*) + R'(d^*) \cdot d^* &\iff A \cdot s^U > p(s^*) \cdot A \cdot s^* \\ &\iff \frac{s^U}{s^*} > 1 \geq p(s^*) \end{aligned}$$

because $s^U > s^*$ by Lemma 1 and $p(s^*) \in [0, 1]$. □

Lemma 3. $d^* > d^P$

Proof. Constraint on sophisticated regulator problem and first order condition with respect to s^P hold:

$$p(s^P) + p'(s^P) \cdot s^P = \frac{p'(s^P) \cdot R(d^P) \cdot d^P}{A \cdot (d^P + e)} \tag{18}$$

$$p(s^P) + p'(s^P) \cdot s^P = \frac{\lambda}{A \cdot (d^P + e)} \cdot \left[\frac{p''(s^P) \cdot [A \cdot s^P \cdot (d^P + e) - R(d^P) \cdot d^P]}{A \cdot (d^P + e)} + 2p'(s^P) \right] \tag{19}$$

Using (18) and (19),

$$\lambda = \frac{p'(s^P) \cdot R(d^P) \cdot d^P}{2p'(s^P) + p''(s^P) \cdot \frac{A \cdot s^P \cdot (d^P + e) - R(d^P) \cdot d^P}{A \cdot (d^P + e)}} > 0$$

because both numerator and denominator are negative with $p' < 0$, $p'' \leq 0$, and $A \cdot s^P \cdot (d^P + e) > R(d^P) \cdot d^P$. FOC wrt d^* and FOC wrt d^P holds:

$$R(d^*) + R'(d^*) \cdot d^* = p(s^*) \cdot A \cdot s^* \tag{20}$$

$$R(d^P) + R'(d^P) \cdot d^P = p(s^P) \cdot A \cdot s^P + \frac{\lambda \cdot p'(s^P)}{A \cdot (d^P + e)} \cdot \left[\frac{R(d^P) \cdot e}{d^P + e} + R'(d^P) \cdot d^P \right] \tag{21}$$

By definition, s^* maximizes $p(S) \cdot S$ and $s^* \neq s^P$ by Lemma 1 $\implies p(s^*) \cdot A \cdot s^* > p(s^P) \cdot A \cdot s^P$. Second term of (21) is negative because $\lambda > 0$, $p'' < 0$ and $R' > 0$. Since $R' > 0$, $R(d^*) + R'(d^*) \cdot d^* > R(d^P) + R'(d^P) \cdot d^P \implies d^* > d^P$. $\square$

Lemma 4. $s^U > d^N > d^P$

Proof. By Lemma 2 and 3, $d^U > d^* > d^P$ with $d^N = d^*$ by assumption. The first order condition with respect to s^U holds both at (s^U, d^U) and (s^P, d^P):

$$p(s) + p'(s) \cdot s = \frac{p'(s) \cdot R(d) \cdot d}{A \cdot (d + e)} \tag{22}$$

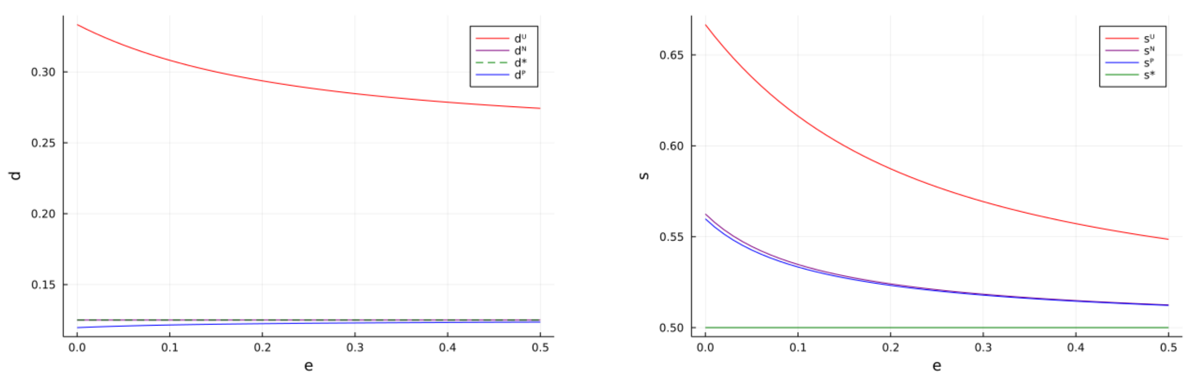
Implicitly differentiating (22),

$$\frac{\partial s}{\partial d} = \frac{(p'(s))^2 \cdot [R'(d) \cdot d \cdot (d+e) + R(d) \cdot e]}{A \cdot (d+e)^2 \cdot [2(p'(s))^2 - p(s)p''(s)]} > 0$$

Numerator is positive because $p' < 0$ and $R' > 0$. Denominator is positive because $p' < 0$ and $p'' \leq 0$. Since $d^U > d^N > d^P$ and $\frac{\partial s}{\partial d} > 0 \implies s^U > s^N > s^P$ $\square$

3.2 With Functional Form and Parameters

In this section, I show the allocations from assuming functional forms and parameters for p and R. Specifically, I assume that $p(s) = 1 - s$, $R(d) = d$, and $A = 1$. Figure 1a shows the quantity of deposits. The ordinal ranking of $d^U > d^* = d^N > d^P$ from the theoretical results is apparent as well as the result from the appendix that d^U does not converge to d^* as $e \to \infty$. Figure 1b shows the risk-return characteristic of the loan portfolio over the four allocations. The ordinal ranking $s^U > s^N > d^P > d^*$ from the theoretical results is also apparent. The reduction in deposits from d^N to d^P by the regulator leads the bank to choosing to reduce s^N to s^P.



(a) Deposits (b) Risk-Return Characteristic

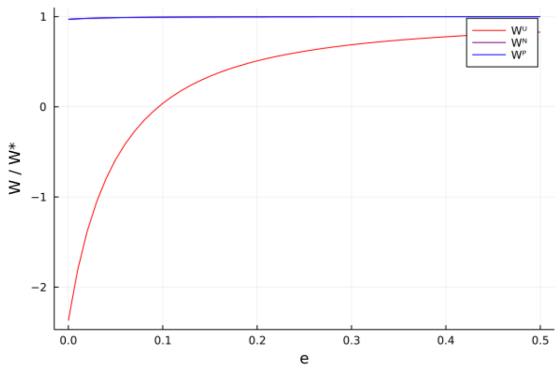
Figure 1: Allocations

The next figure plots the objective functions across the allocations. To help with the scale, these figures are relative to the first best allocation. Figure 2a plots the objective function of the first best allocation: $W \equiv p(s) \cdot A \cdot (d+e) - R(d) \cdot d$. The change from the unregulated to regulated outcomes is so large that it is difficult to see any differences between the naive and sophisticated regulation. Figure 2b plots the objective function of the problem with deposit insurance: $\pi \equiv p(s) \cdot A \cdot (d+e) - p(s) \cdot R(d) \cdot d$. By construction, the expected bank profits in the unregulated allocation is much larger, while the expected bank profits stemming from sophisticated regulation is slightly lower relative to that of the allocation stemming from naive regulation. Figure 2c plots the difference, which can be interpreted as the expected deposit insurance payout $\phi = (1-p(s)) \cdot R(d) \cdot d$. The unregulated allocation results in higher expected deposit insurance payouts, whereas the lower expected bank profits with the sophisticated regulation are more than fully offset by the reduction in the expected deposit insurance payout relative to the allocation associated with naive regulation.

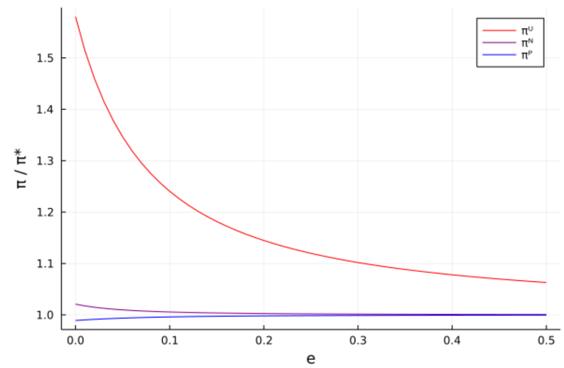
4 Conclusion

In this paper, I analyze a bank regulator's decision problem in a simple two-period model based on Allen and Gale [3]. Similar to Allen and Gale [3], the bank is funded by insured deposits and choose the quantity and risk-return characteristic of its loan portfolio. Unlike Allen and Gale [3], the risk-return characteristic of the loan portfolio is private information. Another departure from Allen and Gale [3] is that the bank is funded by both insured deposits and equity capital. The equity funding creates "skin in the game" when the bank makes its lending decisions. The regulator chooses allocations and regulations to address two frictions: limited commitment through deposit insurance and private information about the loan portfolio risk-return characteristic. The private information about risk-return characteristic of the loan portfolio is the key departure from Allen and Gale [3].

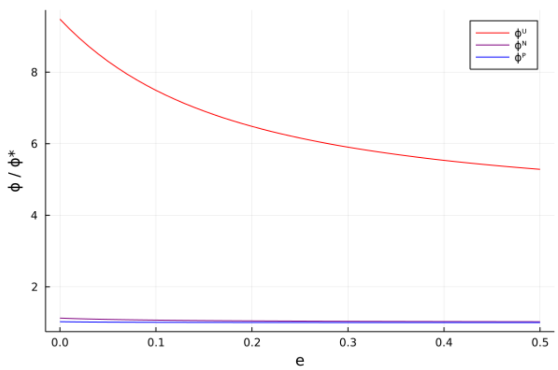
I characterize four allocations: (1) the first best allocation, (2) the allocation chosen by an unregulated bank with deposit insurance, (3) the allocation resulting from the leverage requirement set by a naive regulator who does not account for the bank's private choice of the risk-return



(a) First Best Objective



(b) Objective with Deposit Insurance



(c) Expected Deposit Insurance Payout

Figure 2: Objective Functions

characteristic of the loan portfolio, and (4) the allocation from the leverage requirement set by a sophisticated regulator who does account for the bank's private choice. I show that the first best allocation is not implementable by a leverage requirement when the risk-return characteristic is private information. Thus, the sophisticated regulator chooses tighter regulations. With tighter regulations, the bank uses less deposit funding relative to their equity capital. Since proportionately more of their funding is equity capital instead of insured deposits, the bank has more "skin in the game," so it chooses safer loans than would be chosen at the first best deposit allocation. The naive regulator does not account for the bank's private choice so she chooses the regulations that lead to same amount of deposits as in the first best allocation. Thus, the bank makes the same quantities of loans as the first best allocation, but the bank chooses loans with more risk and more return than the loans in the first best allocation. These loans have higher risk and return because the naive regulator has not tightened regulations to increase the banks' "skin in the game" like the sophisticated regulator.

References

[1] Sumit Agarwal, Souphala Chomsisengphet, Chunlin Liu, Changcheng Song, and Nicholas S. Souleles. Benefits of relationship banking: Evidence from consumer credit markets. *Journal of Monetary Economics*, 96, 2018.

[2] Sumit Agarwal and Robert Hauswald. Distance and private information in lending. *The Review of Financial Studies*, 23(7), 2010.

[3] Franklin Allen and Douglas Gale. Competition and financial stability. *Journal of Money, Credit, and Banking*, 36(3), 2004.

[4] Taylor A. Begley, Amiyatosh Purnanandam, and Kuncheng Zheng. The strategic underreporting of bank risk. *The Review of Financial Studies*, 30(10):3376–3415, 2017.

[5] Markus Behn, Rainer Haselmann, and Vikrant Vig. The limits of model-based regulation. *The Journal of Finance*, 77(3):1635–1684, 2022.

[6] Tobias Berg and Philipp Koziol. An analysis of the consistency of banks' internal ratings. *Journal of Banking and Finance*, 78:27–41, 2017.

[7] Allen N. Berger and Gregory F. Udell. Relationship lending and lines of credit in small firm finance. *The Journal of Business*, 68(3):351–81, 1995.

[8] Jurg M. Blum. Why "Basel II" may need a leverage ratio restriction. *Journal of Banking and Finance*, 32:1699–1707, 2008.

[9] John H. Boyd and Gianni De Nicolo. The theory of bank risk taking and competition revisited. *The Journal of Finance*, 60:1329–42, 2005.

[10] Dean Corbae and Ross Levine. Competition, stability, and efficiency in the banking industry. *Working paper*, 2022.

[11] Asli Demirguc-Kunt, Enrica Detragiache, and Ouarda Merrouche. Bank capital: Lessons from the financial crisis. *IMF Working Paper*, (286), 2010.

[12] Andrew Haldane. Constraining discretion in bank regulation, April 2013.

[13] John H. Kareken and Neil Wallace. Deposit insurance and bank regulation: A partial-equilibrium exposition. *The Journal of Business*, 51(3):413–438, July 1978.

[14] Jose Maria Liberti and Mitchell A. Petersen. Information: Hard and soft. *Review of Corporate Finance Studies*, 8(1):1–41, 2019.

[15] Mike Mariathasan and Ouarda Merrouche. The manipulation of basel risk-weights. *Journal of Financial Intermediation*, 23:300–321, 2014.

[16] David Martinez-Miera and Rafael Repullo. Does competition reduce the risk of bank failure? *The Review of Financial Studies*, 23(10):3638–3664, October 2010.

[17] Basel Committee on Banking Supervision. International convergence of capital measurement and capital standards, July 1988.

[18] Basel Committee on Banking Supervision. International convergence of capital measurement and capital standards: A revised framework, June 2004.

[19] Basel Committee on Banking Supervision. Basel iii: A global regulatory framework for more resilient banks and banking systems, December 2010.

[20] Basel Committee on Banking Supervision. High-level summary of basel iii reforms, December 2017.

[21] Mitchell A. Petersen and Raghuram G. Rajan. Does distance still matter? the information revolution in small business lending. *The Journal of Finance*, 57(6), 2002.

[22] Christina D. Romer and David H. Romer. New evidence on the aftermath of financial crises in advanced countries. *American Economic Review*, 107(10):3072–118, 2017.

5 Appendix

5.1 Proofs of Existence, Uniqueness, Interiority, and Limits

Proposition 1. *$s^* \in (0,1)$ is unique.*

Proof. The LHS of (3) is strictly decreasing in s^* and the RHS is strictly increasing in s^* because p is strictly decreasing. At $s^* = 0$, the LHS equals 1 and the RHS equals 0; at $s^* = 1$, the LHS equals 0 and the RHS equals $-p'(1) > 0$. Therefore, (3) does not hold at corners, so p^* is unique and interior. The second order condition is $2p'(s^*) + s^*p''(s^*)$, which is negative because p is strictly decreasing and weakly concave, so s^* is a maximum. Therefore, s^* uniquely exists on $(0,1)$. $\square$

Proposition 2. *$d^* > 0$ is unique and does not depend on e.*

Proof. Since s^* is interior, the LHS of (4) is positive and constant with respect to d^*. The RHS equals zero at $d^* = 0$. Therefore, $d^* > 0$. The RHS of (4) is strictly increasing, so d^* is unique. The second order condition is $-2R'(d^*) - R''(d^*)$, which is nonpositive, so d^* is a maximum. $\square$

Proposition 3. *$s^U \in (0,1)$ is unique and $s^U \to s^*$ as $e \to \infty$.*

Proof. The LHS of (7) is strictly decreasing in s^U because p is strictly decreasing and weakly concave; the RHS is strictly increasing in s^U because p is strictly decreasing. At $s^U = 0$, the LHS is larger than 1 because p is strictly decreasing and the RHS equals 0. At $s^U = 1$, (7) becomes $R(d^U) \cdot d^U = A \cdot (d^U + e) \implies$ bank makes zero profit, which can be ruled out by contradiction. Suppose bank makes zero profit at $d^U > 0$, then bank could make positive profit as $\tilde{d}^U = d^U - \varepsilon$ because risky loan technology is linear and deposit technology is convex $\Rightarrow\Leftarrow$. So, $p^U \in (0,1]$ is unique.

Taking limits as $e \to \infty$, the second term on the LHS of (7) converges to zero, so the resulting condition is identical to (3) $\implies s^U \to s^*$ as $e \to \infty$. $\square$

Proposition 4. *$d^U > 0$ is unique and $d^U \to \bar{d}^U > d^*$ as $e \to \infty$.*

Proof. Since s^U is interior, the LHS of (8) is positive and constant with respect to d^U. The RHS of (8) is strictly increasing, so $d^U > 0$ is unique.

Taking limit as $e \to \infty$, $s^U \to s^*$ by Proposition 3, so (8) becomes $p(s^*) \cdot A \cdot s^* = p(s^*) \cdot [R(d^U) + R'(d^U) \cdot d^U]$. Comparing to (4), we can that d^U converges to deposit level higher than d^*. $\square$