# Bank Regulation with Uninformed Regulators

Alex von Hafften

UW-Madison

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# Motivation

- Deposit insurance creates moral hazard through limited liability
- Some regulatory regimes use tailored credit risk estimates created by banks
- Other regimes only use standardized credit risk estimates from regulators
- **Key tradeoffs:**
  - Banks have better information about their credit risk
  - Banks have incentive to underreport credit risk to loosen requirements
- Behn, Haselmann, and Vig (2022) find evidence of banks underreporting risk
- **Question:** How should regulators deal with incentive to underreport risk?

# Behn, Haselmann, and Vig (JF, 2022)

- Regulators use risk-weighted capital requirements:

$$E \geq \theta \cdot \mathbf{A} \cdot \mathbf{w}$$

  where $E$ is shareholder equity, $\mathbf{A}$ are assets, $\mathbf{w}$ are risk weights, $\theta$ is minimum ratio

- $\mathbf{w}$ is determined differently across regulatory regimes
  - *Standardized approach* (SA): Regulators determine set of risk weights
  - *Internal ratings based approach* (IRB): Banks report credit risk estimates from models they develop and regulators approved

- BHV (2022) find evidence of IRB banks underreporting risks
  - Banks reported lower PD for IRB loans relative to SA loans despite IRB loans having higher realized losses and higher interest rates
  - Then, lending by IRB banks grew faster relative to SA banks

Related Literature

# What do I do?

- Regulator decision problem w/ 2-period model à la Allen and Gale (2004)
- Bank is funded by equity and insured deposits and invests in risky loans
- Bank privately chooses risk-return characteristic of its loan portfolio
- How does private information change allocations and regulation?
- **Result:** w/ private info, regulation leads to an underprovision of bank lending and deposits relative to first best allocation
- **Why?** Fewer deposits $\implies$ more "skin in the game" $\implies$ safer loans

# Outline

# Environment

- Bank endowed with equity financing $e \geq 0$
- Bank obtains $d \geq 0$ from **deposit technology** where bank has to pay back $R(d) \cdot d$ where $R$ is twice differentiable with $R' > 0$, $R'' \geq 0$, and $R(0) = 0$
- Bank invests $d + e$ into **risky loan technology** with risk-return $s \in [0, 1]$
  - With prob. $p(s)$, risky technology outputs $A \cdot s \cdot (d + e)$ where $A > 0$
  - With prob. $1 - p(s)$, risky loan technology outputs zero
  - $p$ is twice differentiable with $p' < 0$, $p'' \leq 0, p(0) = 1$, $p(1) = 0$
  - Risk-return trade-off: $\uparrow s \implies \uparrow$ return and $\uparrow$ probability of failure
- **Bank chooses $s$ and $d$ with $s$ unobservable and $d$ observable.**

# First Best Problem

- First best allocation $s^*$ and $d^*$ solve

$$\max_{s^*,d^*} \underbrace{p(s^*) \cdot A \cdot s^* \cdot (d^* + e)}_{\text{expected risky tech output}} - \underbrace{R(d^*) \cdot d^*}_{\text{deposit return}}$$

- FOC wrt $s^*$

$$\underbrace{p(s^*)}_{\substack{\uparrow s^* \implies \uparrow \text{ output if success} \\ \text{(MB)}}} = \underbrace{-p'(s^*) \cdot s^*}_{\substack{\text{but failure more likely} \\ \text{(MC)}}}$$

- FOC wrt $d^*$

$$\underbrace{p(s^*) \cdot A \cdot s^*}_{\substack{\uparrow d^* \implies \uparrow \text{ expected output} \\ \text{(MB)}}} = \underbrace{R(d^*) + R'(d^*) \cdot d^*}_{\substack{\text{but pay more for deposits} \\ \text{(MC)}}}$$

# Problem with Deposit Insurance Environment

- Deposit insurance $\implies$ bank does not pay deposits back if risky tech fails

$$\max_{s^U, d^U} \underbrace{p(s^U) \cdot A \cdot s^U \cdot (d^U + e)}_{\text{expected risky tech output}} - \underbrace{p(s^U) \cdot R(d^U) \cdot d^U}_{\text{expected deposit return}}$$

- FOC wrt $s^U$

$$\underbrace{p(s^U)}_{\substack{\uparrow s^U \implies \uparrow \text{output if success} \\ \text{(MB)}}} + \underbrace{\frac{-p'(s^U) \cdot R(d^U) \cdot d^U}{A \cdot (d^U + e)}}_{\substack{\text{and less likely to pay back deposits} \\ \text{(MB)}}} = \underbrace{-p'(s^U) \cdot s^U}_{\substack{\text{but failure more likely} \\ \text{(MC)}}}$$

- **Deposit insurance introduces additional MB** $\implies s^U > s^*$
- FOC wrt $d^U$

$$\underbrace{p(s^U) \cdot A \cdot s^U}_{\substack{\uparrow d^U \implies \uparrow \text{expected output} \\ \text{(MB)}}} = \underbrace{p(s^U) \cdot [R(d^U) + R'(d^U) \cdot d^U]}_{\substack{\text{but might pay more for deposits} \\ \text{(MC)}}}$$

- **Deposit insurance reduces MC** $\implies d^U > d^*$

# Can regulation implement the first best?

- Implement with limit on leverage that depends on $s$
- For example, risk-weighted capital requirement

$$e \geq \underbrace{\theta^*}_{\text{minimum ratio}} \cdot \underbrace{w(s)}_{\text{risk weight}} \cdot \underbrace{(d+e)}_{\text{loans}}$$

$$\text{where } \theta^* = \frac{e}{d^* + e}$$

$$\text{and } w(s) = \begin{cases} 1, & \text{if } s = s^* \\ K, & \text{if } s \neq s^* \end{cases}$$

- Risk-weighted capital requirement $\equiv$ deposit limit that depends on $s$
- **Implementing with leverage limit only if regulator can see $s$ and $d$**

# What if $s$ is unobservable to regulator? Environment

- Regulator chooses limit on leverage that does not depend on $s$

$$\max_{\theta^P} \underbrace{p(s^P) \cdot A \cdot s^P \cdot (d^P + e)}_{\text{expected risky tech output}} - \underbrace{R(d^P) \cdot d^P}_{\text{deposit return}}$$

$$\text{s.t.} \quad (s^P, d^P) \in \arg\max_{s,d} \left\{ \underbrace{p(s) \cdot A \cdot s \cdot (d + e)}_{\text{expected risky tech output}} - \underbrace{{\color{red}p(s)} \cdot R(d) \cdot d}_{{\color{red}\text{expected}} \text{ deposit return}} \right\}$$

$$\text{s.t.} \;\; e \geq \theta^P (d + e)$$

- Deposit insurance $\implies$ bank borrows until $e = \theta^P (d + e)$ $\implies$ regulator effectively chooses $d$

$$\max_{s^P, d^P} p(s^P) \cdot A \cdot s^P \cdot (d^P + e) - R(d^P) \cdot d^P$$

$$\text{s.t.} \quad s^P \in \arg\max_{s} \left\{ p(s) \cdot A \cdot s \cdot (d^P + e) - {\color{red}p(s)} \cdot R(d^P) \cdot d^P \right\}$$

# First best cannot be implemented

- FOC of constraint on private info problem

$$\underbrace{p(s^P)}_{\substack{\uparrow s^P \implies \uparrow \text{ output if success} \\ \text{(MB)}}} + \underbrace{\frac{-p'(s^P)\cdot R(d^P)\cdot d^P}{A\cdot(d^P+e)}}_{\substack{\text{so less likely to pay back deposits} \\ \text{(MB)}}} = \underbrace{-p'(s^P)\cdot s^P}_{\substack{\text{but failure more likely} \\ \text{(MC)}}}$$

- At $d^*$, marginal benefit of higher $s^* >$ marginal cost of higher $s^*$

$$\underbrace{p(s^*) + \frac{-p'(s^*)\cdot R(d^*)\cdot d^*}{A\cdot(d^*+e)}}_{\text{MB}} > \underbrace{-p'(s^*)\cdot s^*}_{\text{MC}}$$

- If naive regulator implements $\theta^*$ so that banks choose $d^N = d^*$, then bank chooses $s^N > s^*$

# What if $s$ is unobservable to regulator?

- Sophisticated regulator accounts for bank's choice of $s$

$$\max_{s^P, d^P} \underbrace{p(s^P) \cdot A \cdot s^P \cdot (d^P + e)}_{\text{expected risky tech output}} - \underbrace{R(d^P) \cdot d^P}_{\text{deposit return}}$$

$$\text{s.t.} \quad p(s^P) + \frac{-p'(s^P) \cdot R(d^P) \cdot d^P}{A \cdot (d^P + e)} = -p'(s^P) \cdot s^P \qquad \text{[Bank FOC]}$$

- FOC wrt $s^P$

$$\underbrace{p(s^P)}_{\substack{\uparrow S \implies \uparrow \text{ output if success} \\ \text{(MB)}}} + \underbrace{\frac{-\lambda}{A \cdot (d^P + e)} \left[ \frac{p''(s^P) \cdot [A s^P (d^P + e) - R(d^P) \cdot d^P]}{A \cdot (d^P + e)} + 2p'(s^P) \right]}_{\substack{\text{ease bank FOC} \\ \text{(wedge)}}} = \underbrace{p'(s^P) \cdot s^P}_{\substack{\text{but failure more likely} \\ \text{(MC)}}}$$

- FOC wrt $d^P$

$$\underbrace{p(s^P) \cdot A \cdot s^P}_{\substack{\uparrow D \implies \uparrow \text{ output} \\ \text{(MB)}}} = \underbrace{R(d^P) + R'(d^P) \cdot d^P}_{\substack{\text{but pay more for deposits} \\ \text{(MC)}}} + \underbrace{\frac{-\lambda \cdot p'(s^P)}{A \cdot (d^P + e)} \cdot \left[ \frac{R(d^P) \cdot e}{d^P + e} + R'(d^P) \cdot d^P \right]}_{\substack{\text{tighten bank FOC} \\ \text{(wedge)}}}$$

# Comparing Problems and FOCs

$$\max_{s^*,d^*} p(s^*) \cdot A \cdot s^* \cdot (d^* + e) - R(d^*) \cdot d^* \qquad \text{[First Best]}$$

$$\max_{s^U,d^U} p(s^U) \cdot A \cdot s^U \cdot (d^U + e) - p(s^U) \cdot R(d^U) \cdot d^U \qquad \text{[w/ Deposit Insurance]}$$

$$\max_{s^P,d^P} p(s^P) \cdot A \cdot s^P \cdot (d^P + e) - R(d^P) \cdot d^P \qquad \text{[w/ } S \text{ Private]}$$

$$\text{s.t. } p(s^P) + \frac{-p'(s^P) \cdot R(d^P) \cdot d^P}{A \cdot (d^P + e)} = -p'(s^P) \cdot s^P$$

$$p(s^*) = -p'(s^*) \cdot s^* \qquad \text{FOC } [s^*]$$

$$p(s^U) + \frac{-p'(s^U) \cdot R(d^U) \cdot d^U}{A \cdot (d^U + e)} = -p'(s^U) \cdot s^U \qquad \text{FOC } [s^U]$$

$$p(s^P) + \frac{-\lambda}{A \cdot (d^P + e)} \left[ \frac{p''(s^P) \cdot [A \cdot s^P \cdot (d^P + e) - R(d^P) \cdot d^P]}{A \cdot (d^P + e)} + 2p'(s^P) \right] = -p'(s^P) \cdot s^P \qquad \text{FOC } [s^P]$$

$$p(s^*) \cdot A \cdot s^* = R(d^*) + R'(d^*) \cdot d^* \qquad \text{FOC } [d^*]$$

$$p(s^U) \cdot A \cdot s^U = p(s^U) \cdot [R(d^U) + R'(d^U) \cdot d^U] \qquad \text{FOC } [d^U]$$

$$p(s^P) \cdot A \cdot s^P = R(d^P) + R'(d^P) \cdot d^P + \frac{-\lambda \cdot p'(s^P)}{A \cdot (d^P + e)} \cdot \left[ \frac{R(d^P) \cdot e}{d^P + e} + R'(d^P) \cdot d^P \right] \qquad \text{FOC } [d^P]$$

# Results

- W/ private $s$, regulation leads to underprovision of deposits

$$d^U > d^* = d^N > d^P \implies \theta^* > \theta^P$$

L2: $d^U > d^*$ L3: $d^* > d^P$ Parametric Solutions

- Why? Fewer deposits $\implies$ more "skin in the game" $\implies$ safer loans

$$s^U > s^N > s^P > s^*$$

L1: $s^U > s^*$ and $s^P > s^*$ L4: $s^U > s^N > s^P$ Parametric Solutions

# Conclusion

- **What do I do?** Study regulator decision problem in simple model where risk-return characteristic of loan portfolio is private information
- **Result:** w/ private info about bank loan risk, regulation leads to underprovision of bank lending and deposits
- **Next steps**
  - Adverse Selection: $A_H > A_L$ where $A_i$ is private Private Productivity
  - Move to general equilibrium GE Environment

# Outline

# Related Literature

- Banking models with risk-return choice
  - Allen and Gale (2004), Boyd and DeNicolo (2005), Martinez-Miera and Repullo (2010), Corbae and Levine (2022)
- Hard and soft information in lending
  - Liberti and Peterson (2018), Agarwal et al (2018), Agarwal and Hauswald (2010), Petersen and Rajan (2002), Berger and Udell (1995)
- Banks underreporting risk
  - Behn, Haselmann, and Vig (2022), Mariathasan and Merrouche (2014), Berg and Koziol (2017), Begley, Purnanandam, Zheng (2017), Demirguc-Kunt et al (2010), Blum (2007)

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# Outline

1. Introduction
2. Model
3. Appendix
   - Environments with Frictions
   - Proofs of Main Lemmas
   - Parametric Solutions
   - Private Productivity
   - General Equilibrium

# Environment with Deposit Insurance

- Period 1
  - Bank born with $e \geq 0$ and chooses $d \geq 0$ and $s \in [0, 1]$
  - Bank pulls $d$ out of deposit technology
  - Bank inputs $d + e$ into risky technology at $s$
- Period 2 with probability $p(s)$
  - Risky technology outputs $A \cdot s \cdot (d + e)$
  - Bank pays back $R(d) \cdot d$
  - Profit is $A \cdot s \cdot (d + e) - R(d) \cdot d$
- Period 2 with probability $1 - p(s)$
  - Risky technology outputs 0
  - ~~Bank pays back $R(d) \cdot d$~~
  - Profit is 0

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# Environment with Deposit Insurance and $s$ Private

- Period 1
  - Bank born with $e \geq 0$ and chooses $d \geq 0$ and $s \in [0, 1]$ (*hidden*)
  - Bank pulls $d$ out of deposit technology
  - Bank inputs $d + e$ into risky technology at $s$ (*hidden*)
- Period 2 with probability $p(s)$
  - Risky technology outputs $A \cdot s \cdot (d + e)$ (*hidden*)
  - Bank pays back $R(d) \cdot d$
  - Profit is $A \cdot s \cdot (d + e) - R(d) \cdot d$ (*hidden*)
- Period 2 with probability $1 - p(s)$
  - Risky technology outputs 0
  - ~~Bank pays back $R(d) \cdot d$~~
  - Profit is 0

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# Outline

# Lemma 1: $s^U > s^*$ and $s^P > s^*$

- FOC wrt $s^*$, FOC wrt $s^U$, and constraint on regulator problem

$$p(s^*) + p'(s^*) \cdot s^* = 0 \tag{1}$$

$$p(s^U) + p'(s^U) \cdot s^U = \frac{p'(s^U) \cdot R(d^U) \cdot d^U}{A \cdot (d^U + e)} \tag{2}$$

$$p(s^P) + p'(s^P) \cdot s^P = \frac{p'(s^P) \cdot R(d^P) \cdot d^P}{A \cdot (d^P + e)} \tag{3}$$

- LHS of (1), (2), and (3) are strictly decreasing in $s$
- RHS of (2) and (3) are negative because $p' < 0$

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# Lemma 2: $d^U > d^*$

- Since $R' > 0$ and $R'' \geq 0$,

$$d^U > d^* \iff R(d^U) + R'(d^U) \cdot d^U > R(d^*) + R'(d^*) \cdot d^*$$

- FOC wrt $d^*$ and FOC wrt $d^U$ hold

$$p(s^*) \cdot A \cdot s^* = R(d^*) + R'(d^*) \cdot d^* \tag{4}$$
$$p(s^U) \cdot A \cdot s^U = p(s^U) \cdot [R(d^U) + R'(d^U) \cdot d^U] \tag{5}$$

- Substituting in (4) and (5):

$$R(d^U) + R'(d^U) \cdot d^U > R(d^*) + R'(d^*) \cdot d^* \iff A \cdot s^U > p(s^*) \cdot A \cdot s^*$$
$$\iff \frac{s^U}{s^*} > 1 \geq p(s^*)$$

because $s^U > s^*$ by Lemma 1 and $p(s^*) \in [0, 1]$

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# Lemma 3: $d^* > d^P$

- Constraint on private info problem and FOC wrt $s^P$ hold

$$p(s^P) + p'(s^P) \cdot s^P = \frac{p'(s^P) \cdot R(d^P) \cdot d^P}{A \cdot (d^P + e)} \tag{6}$$

$$p(s^P) + p'(s^P) \cdot s^P = \frac{\lambda}{A \cdot (d^P + e)} \cdot \left[ \frac{p''(s^P) \cdot [A \cdot s^P \cdot (d^P + e) - R(d^P) \cdot d^P]}{A \cdot (d^P + e)} + 2p'(s^P) \right] \tag{7}$$

- Using (6) and (7),

$$\lambda = \frac{p'(s^P) \cdot R(d^P) \cdot d^P}{2p'(s^P) + p''(s^P) \cdot \frac{A \cdot s^P \cdot (d^P + e) - R(d^P) \cdot d^P}{A \cdot (d^P + e)}} > 0$$

  because both numerator and denominator are negative with $p' < 0$, $p'' \leq 0$, and $A \cdot s \cdot (d + e) > R(d) \cdot d$

- FOC wrt $d^*$ and FOC wrt $d^P$ holds

$$R(d^*) + R'(d^*) \cdot d^* = p(s^*) \cdot A \cdot s^* \tag{8}$$

$$R(d^P) + R'(d^P) \cdot d^P = p(s^P) \cdot A \cdot s^P + \frac{\lambda \cdot p'(s^P)}{A \cdot (d^P + e)} \cdot \left[ \frac{R(d^P) \cdot e}{d^P + e} + R'(d^P) \cdot d^P \right] \tag{9}$$

- $s^*$ maximizes $p(S) \cdot S$ and $s^* \neq s^P$ by Lemma 1 $\implies p(s^*) \cdot A \cdot s^* > p(s^P) \cdot A \cdot s^P$
- Second term of (9) is negative because $\lambda > 0$, $p'' < 0$ and $R' > 0$
- Since $R' > 0$, $R(d^*) + R'(d^*) \cdot d^* > R(d^P) + R'(d^P) \cdot d^P \implies d^* > d^P$

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# Lemma 4: $s^U > s^N > s^P$

- By Lemma 2 and 3, $d^U > d^* > d^P$ with $d^N = d^*$ by assumption
- FOC wrt $s^U$ holds both at $(s^U, d^U)$ and $(s^P, d^P)$

$$p(s) + p'(s) \cdot s = \frac{p'(s) \cdot R(d) \cdot d}{A \cdot (d+e)} \tag{10}$$

- Implicitly differentiating (10)

$$\frac{\partial s}{\partial d} = \frac{(p'(s))^2 \cdot [R'(d) \cdot d \cdot (d+e) + R(d) \cdot e]}{A \cdot (d+e)^2 \cdot [2(p'(s))^2 - p(s)p''(s)]} > 0$$

- Numerator is positive because $p' < 0$ and $R' > 0$
- Denominator is positive because $p' < 0$ and $p'' \leq 0$
- Since $d^U > d^N > d^P$ and $\frac{\partial s}{\partial d} > 0 \implies s^U > s^N > s^P$

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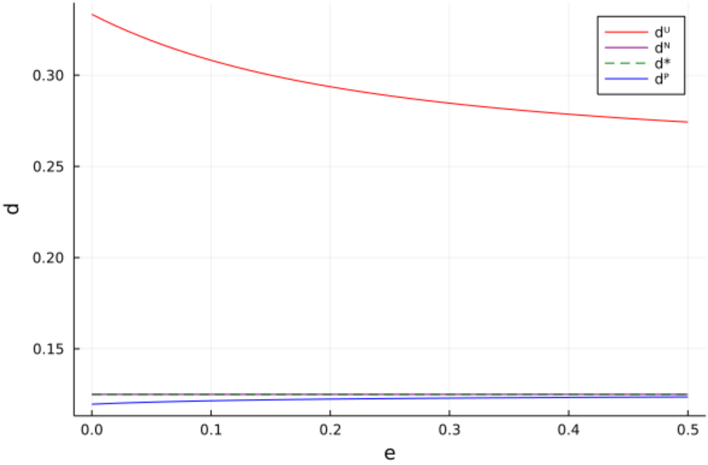
# Outline

# $d^U > d^* = d^N > d^P$




where $A = 1$, $p(s) = 1 - s$, and $R(d) = d$.

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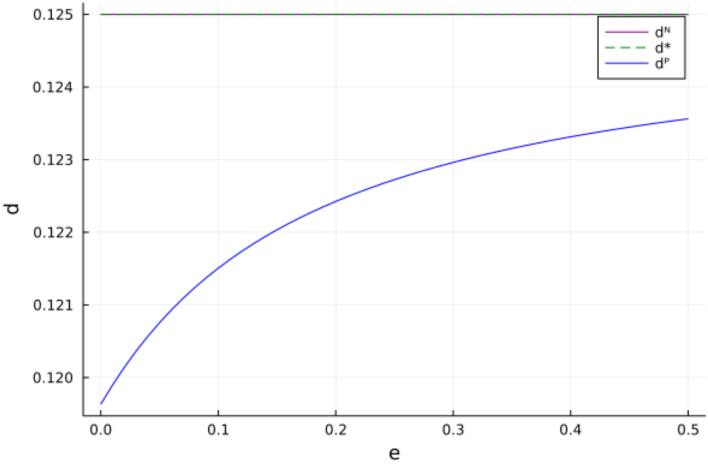
# $d^* = d^N > d^P$




where $A = 1$, $p(s) = 1 - s$, and $R(d) = d$.

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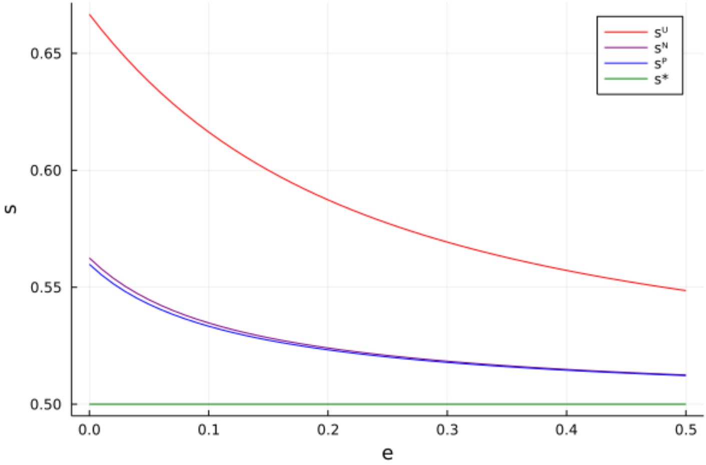
# $s^U > s^N > s^P > s^*$




where $A = 1$, $p(S) = 1 - s$, and $R(d) = d$.

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# Outline

# Environment w/ Private Productivity

- Period 1
  - Bank born w/ $e \geq 0$ and $A_H$ w/ prob. $\pi$ or $A_L$ w/ prob. $1 - \pi$ (*hidden*)
  - Bank chooses $d \geq 0$ and $s \in [0, 1]$
  - Bank pulls $d$ out of deposit technology
  - Bank inputs $d + e$ into risky technology at $s$
- Period 2 with probability $p(s)$
  - Risky technology outputs $A_i \cdot s \cdot (d + e)$ (*hidden*)
  - Bank pays back $R(d) \cdot d$
  - Profit is $A_i \cdot s \cdot (d + e) - R(d) \cdot d$ (*hidden*)
- Period 2 with probability $1 - p(s)$
  - Risky technology outputs 0
  - ~~Bank pays back $R(d) \cdot d$~~
  - Profit is 0

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# First Best with Multiple Types

- If $A_i$ and $s$ are observable to outsiders

$$\max_{s_i^*, d_i^*} p(s_i^*) \cdot A_i \cdot s_i^* \cdot (d_i^* + e) - R(d_i^*) \cdot d_i^*$$

- FOC wrt $s_i^*$

$$p(s_i^*) + p'(s_i^*) \cdot s_i^* = 0 \implies s_H^* = s_L^* \equiv s^*$$

- FOC wrt $d_i^*$

$$p(s_i^*) \cdot A_i \cdot s_i^* = R(d_i^*) + R'(d_i^*) \cdot d_i^* \implies d_H^* > d_L^*$$

- Both types invest at same $s$, but high type gets more $d$

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# Regulator Problem with Private $A_i$ and Deposit Insurance

$$\max_{s_H^P, d_H^P, s_L^P, d_L^P} \pi \cdot [p(s_H^P) \cdot A_H \cdot s_H^P \cdot (d_H^P + e) - R(d_H^P) \cdot d_H^P]$$
$$+(1-\pi) \cdot [p(s_L^P) \cdot A_L \cdot s_L^P \cdot (d_L^P + e) - R(d_L^P) \cdot d_L^P]$$

$$\text{s.t. } \underbrace{p(s_H^P)A_H s_H^P(d_H^P + e) - p(s_H^P)R(d_H^P)d_H^P}_{\text{Profit from truthfully reporting } H} \geq \underbrace{p(s_L^P)A_H s_L^P(d_L^P + e) - p(s_L^P)R(d_L^P)d_L^P}_{\text{Profit from falsely reporting } L} \quad [IC_H]$$

$$\underbrace{p(s_L^P)A_L s_L^P(d_L^P + e) - p(s_L^P)R(d_L^P)d_L^P}_{\text{Profit from truthfully reporting } L} \geq \underbrace{p(s_H^P)A_L s_H^P(d_H^P + e) - p(s_H^P)R(d_H^P)d_H^P}_{\text{Profit from falsely reporting } H} \quad [IC_L]$$

- Both types want more deposits $\implies$ $IC_H$ is slack and $IC_L$ binds

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# FOCs with Multiple Types

$$p'(s_H^P) \cdot s_H^P + p(s_H^P) = \frac{\gamma}{\pi - \gamma}\left[ - \frac{p'(s_H^P)R(d_H^P)d_H^P}{A_H(d_H^P + e)} \right] \qquad [s_H^P]$$

$$p'(s_L^P) \cdot s_L^P + p(s_L^P) = \frac{\gamma}{1 - \pi - \gamma}\left[ - \frac{p'(s_L^P)R(d_L^P)d_L^P}{A_L \cdot (d_L^P + e)} \right] \qquad [s_L^P]$$

$$p(s_H^P) \cdot A_H \cdot s_H^P - R(d_H^P) - R'(d_H^P)d_H^P = \frac{\gamma p(s_H^P)}{\pi}[A_L s_H^P - R'(d_H^P)d_H^P - R(d_H^P)] \qquad [d_H^P]$$

$$p(s_L^P) \cdot A_L \cdot s_L^P - R'(d_L^P) \cdot d_L^P - R(d_L^P) = -\frac{\gamma p(s_L^P)}{(1 - \pi)}[A_L s_L^P - R'(d_L^P)d_L^P - R(d_L^P)] \qquad [d_L^P]$$

$$p(s_L^P)A_L s_L^P(d_L^P + e) - p(s_L^P)R(d_L^P)d_L^P = p(s_H^P)A_L s_H^P(d_H^P + e) - p(s_H^P)R(d_H^P)d_H^P \qquad [\gamma]$$

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# Preliminary Numerical Results

- Use $p(s) = 1 - s$, $R(d) = d$, $A_H = 2.0$, $A_L = 1.0$, $\pi = 0.5$, $e = 0.5$

| Allocation | First Best | Private Productivity |
|---|---|---|
| $s_H$ | 0.5 | 0.487 |
| $s_L$ | 0.5 | 0.491 |
| $d_H$ | 0.25 | 0.252 |
| $d_L$ | 0.125 | 0.149 |
| $\gamma$ | - | 0.248 |

- Suggests $s_L^P \approx s^*$ and "no distortion at the top" with $s_H^P \approx s^*$ and $d_H^P \approx d_H^*$
- But give more deposits to low type w/ $d_L^P > d_L^*$ so they truthfully reveal
- Interesting preliminary result:
  - With hidden $s$, deposits and lending is lower: $d^* < d^P$
  - With hidden $A$, deposits and lending might be higher:

$$\pi d_H^* + (1 - \pi) d_L^* > \pi d_H^P + (1 - \pi) d_L^P$$

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# Outline

# GE Environment - HH Problems

- **Depositor problem:**

  Given $(y^D, R, T^D)$, solves

$$\begin{aligned} \max_{c_1^D, c_2^D, d} \; & u(c_1^D) + \beta E[u(c_2^D)] \\ \text{s.t. } \; & c_1^D + d = y^D \\ & c_2^D = R \cdot d + T^D \end{aligned}$$

- **Bank owner problem:**

  Given $(y^O, T^O, \pi)$, solves

$$\begin{aligned} \max_{c_1^O, c_2^O, e} \; & u(c_1^O) + \beta E[u(c_2^O)] \\ \text{s.t. } \; & c_1^O + e = y^O \\ & c_2^O = \pi + T^O \end{aligned}$$

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# Contracting Problem Setup

- Principal is regulator and agent is competitive bank
- Period 1
  - Principal meets agent with $e \geq 0$
  - Principal and agent agree on contract $(b, s, d)$
  - Principal borrows $d \geq 0$ at interest rate $R$ from HHs
  - Agent puts $d + e$ into risky tech at $s$
- Period 2 with probability $p(s)$
  - Agent collects $A \cdot s \cdot (d + e)$ from risky tech
  - Agent gives $b$ to principal
  - Principal pays $R \cdot d$ and lump-sum transfer $T^D + T^O = b - R \cdot d$ to HHs
  - Agent profit is $\pi = A \cdot s \cdot (d + e) - b$
- Period 2 with probability $1 - p(s)$
  - Agent collects zero from risky tech
  - Principal lump-sum taxes HHs $T^D + T^O = -R \cdot d$
  - Principal pays depositors $R \cdot d$
- Agent outside option is to run risky technology only with $e$

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# Contracting Problems

- Contracting problem maximizes principal objective s.t. agent participation

$$\max_{s,d,b} p(s) \cdot [b - Rd] + (1 - p(s)) \cdot [-Rd]$$

$$p(s) \cdot [As(d+e) - b] \geq \max_{\hat{s}} \{p(\hat{s}) \cdot A \cdot \hat{s} \cdot e\} \qquad \text{[Agent PC]}$$

- Hidden $s$ introduces agent incentive compatibility constraint

$$\max_{s,d,b} p(s) \cdot [b - Rd] + (1 - p(s)) \cdot [-Rd]$$

$$p(s) \cdot [As(d+e) - b] \geq \max_{\hat{s}} \{p(\hat{s}) \cdot A \cdot \hat{s} \cdot e\} \qquad \text{[Agent PC]}$$

$$s \in \arg\max_{\hat{s}} \{p(\hat{s}) \cdot A \cdot \hat{s} \cdot (d+e) - p(\hat{s}) \cdot b\} \qquad \text{[Agent IC]}$$

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