ECON 712A: Macroeconomic Theory

Discussion Section Handout 1

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- Icebreaker: Names, pronouns, program, year, and field(s) of interest.

Administrative Information

- Teaching Assistants:
  - Duong Dang (dqdang@wisc.edu; 6473 Social Science; 3rd year)
  - Alex von Hafften (vonhafften@wisc.edu; 6439 Social Science; 2nd year).
- Weekly Schedule:
  - Monday: Lecture at 1:00 - 2:15 PM in 4028 Vilas.
  - Tuesday: Duong's office hours at 10:30 AM - 11:45 PM in 6473 Social Science.
  - Wednesday: Lecture at 1:00 - 2:15 PM in 4028 Vilas.
  - Thursday:
    * Discussion section handout and next problem set distributed.
    * Dean's office hours at 10:00 - 11:45 AM in 7438 Social Sciences.
    * Alex's office hours at 2:15 PM - 3:30 PM in 6439 Social Science.
    * Problem set due at 11:59 PM. Everyone needs to submit the problem set. Include a note about who you worked with.
  - Friday:
    * Discussion sections
    * Distribute discussion sections handout with solutions.
- Midterm on November 1 at 7:15 PM.
- Course Materials:
  - Everything will be posted on Canvas.
  - Lecture notes and past midterms are also available at https://sites.google.com/a/wisc.edu/deancorbae/teaching.
  - Problem sets and section handouts are also available at https://vonhafften.github.io/teaching.html.
  - Problem sets will include computational problems. Any programming language will be accepted. Use the language that you're most comfortable with.
  - Example code will be provided largely in Matlab, Python, Julia, and/or R.

- Matlab is available at https://it.wisc.edu/services/software/.
- Ljungqvist and Sargent (textbook) is available online at the UW Madison Library.

- Discussion Sections:
  - 7:45 AM section is in 6105 Social Science.
  - 8:50 AM section is in 214 Ingraham
  - 2:25 PM section is in 6109 Social Science.
  - 3:30 PM section is in 6109 Social Science.
  - Sections are 50 minutes long.
  - Duong and Alex will alternate teaching all four sections each week.
- COVID Policies:
  - Masks covering your nose and mouth are required regardless of vaccination status.
  - To keep room capacity under control, please attend the section that you're enrolled in.
- What's the point of discussion sections?
  - Solving problems using concepts from lectures.
  - Filling in material omitted from lectures due to time constraints.
  - Discussing common issues on problem sets.
- Recommendations:
  - Study in groups.
  - Engage in active learning. Do practice problems.
  - Keep a positive mindset.
  - Recommendations for us? Email or anonymous feedback form at https://vonhafften.github.io/teaching.html.

Content Review

- An **environment** is a statement of population, preferences, and technologies (e.g., production, matching, information, commitment).
- Example: An overlapping generation economy with endowments and log preferences.
  - Population: 2-period lived agents.
  - Production: Non-storable w_1 for young agents and 0 for old agents.
  - Preference: $U(c_t^t, c_{t+1}^t) = \ln(c_t^t) + \ln(c_{t+1}^t)$
- An **allocation** is a statement about how resources are distributed.
- If an allocation can be achieved given technologies, the allocation is **resource feasible**.

$$c_t^t + c_t^{t-1} \leq w_1$$

- The **planner** allocates resources optimally given feasibility.

- What does optimally mean? So far, the planner has equally weighed the utility of each generation alive at period t:[1]

$$\max_{c_t^t, c_t^{t-1} \in \mathbb{R}_+^2} \ln(c_t^t) + \ln(c_t^{t-1})$$
$$\text{s.t. } c_t^t + c_t^{t-1} \leq w_1$$

- If the resource constraint holds with equality, we can use **substitution** to modify the planner problem into an unconstrained optimization problem.

$$\max_{c_t^t \in \mathbb{R}_+} \ln(c_t^t) + \ln(w_1 - c_t^t)$$

- The first order condition implies:

$$\implies c_t^t = c_t^{t-1} = \frac{w_1}{2}$$

- Planner problem is much better than **autarky**, in which agents only consume their endowments.
- How to **decentralize** the planner solution?
- One way is with **fiat currency**. The government issues M units of currency to the initial old.
- Taking p_1 as given, the problem of the initial old agents:

$$\max_{c_1^0 \in R_+} \ln(c_1^0)$$
$$\text{s.t. } p_1 c_1^0 \leq M$$

$$\implies c_1^0 = M/p_1$$

- Let M_{t+1}^t be the fiat currency holding of generation t to period $t+1$.
- Taking p_t, p_{t+1} as given, the problem of agents in all generations born in $t \geq 1$:

$$\max_{(c_t^t, c_{t+1}^t, M_{t+1}^t) \in \mathbb{R}_+^3} \ln(c_t^t) + \ln(c_{t+1}^t)$$
$$\text{s.t. } p_t c_t^t + M_{t+1}^t = p_t w_1$$
$$p_{t+1} c_{t+1}^t = M_{t+1}^t$$

$$\implies M_{t+1}^t = \frac{p_t w_1}{2}$$
$$c_t^t = \frac{w_1}{2}$$
$$c_{t+1}^t = \frac{p_t}{p_{t+1}} \frac{w_1}{2}$$

- Any questions?

[1] To be specific, this planner problem is a *period-by-period utilitarian planner problem*

Growing and Shrinking Populations

Consider the baseline 2-period overlapping-generation model outlined in lecture but the population changes each generation. In particular, if there is N_t measure of generation t, then there is $N_{t+1} = nN_t$ of generation $t+1$ where $n \in \mathbb{R}_+$. Notice that population could be shrinking ($0 < n < 1$), staying the same ($n = 1$), or growing ($n > 1$).

1. What is the resource constraint with the changing population?

The resource constraint:

$$N_t c_t^t + N_{t-1} c_t^{t-1} \leq N_t w_1$$

2. Is the planners allocation without population growth (i.e., $c_t^t = c_t^{t-1} = \frac{w_1}{2}$) resource feasible for a growing population? For a shrinking population?

Substituting in population growth in the resource constraint:

$$nN_{t-1} c_t^t + N_{t-1} c_t^{t-1} \leq nN_{t-1} w_1$$

$$\implies nc_t^t + c_t^{t-1} \leq nw_1$$

Substituting in the planners allocation without population growth:

$$n\frac{w_1}{2} + \frac{w_1}{2} \leq nw_1$$
$$\implies 1 \leq n$$

The resource constraint holds only if the population is staying the same or growing.

3. The planner cares equally about all agents alive at period t. What is the planners problem?

The planners problem:

$$\max_{c_t^t, c_t^{t-1} \in \mathbb{R}_+^2} N_t \ln(c_t^t) + N_{t-1} \ln(c_t^{t-1})$$
$$\text{s.t. } N_t c_t^t + N_{t-1} c_t^{t-1} \leq N_t w_1$$

4. What is the planners allocation?

Notice that the constraint will hold with equality and substituting in population growth:

$$\max_{c_t^t, c_t^{t-1} \in \mathbb{R}_+^2} nN_{t-1} \ln(c_t^t) + N_{t-1} \ln(c_t^{t-1})$$
$$\text{s.t. } nN_{t-1} c_t^t + N_{t-1} c_t^{t-1} = nN_{t-1} w_1$$

We can divide both the resource constraint and the objective function by N_{t-1}.

$$\implies \max_{c_t^t, c_t^{t-1} \in \mathbb{R}_+^2} n \ln(c_t^t) + \ln(c_t^{t-1})$$
$$\text{s.t. } nc_t^t + c_t^{t-1} = nw_1$$

Substituting in the constraint for c_t^{t-1}:

$$\implies \max_{c_t^t \in \mathbb{R}_+} n \ln(c_t^t) + \ln(nw_1 - nc_t^t)$$

The first order condition:

$$\frac{n}{c_t^t} = \frac{n}{nw_1 - nc_t^t} \implies c_t^t = \frac{n}{1+n} w_1$$

Substituting into the resource constraint:

$$c_t^{t-1} = nw_1 - n\left[\frac{n}{1+n} w_1\right] = \frac{n}{1+n} w_1$$

5. Consider decentralizing the planner solution. How should we design a lump-sum tax and transfer system would achieve the planner solution?

We know that we will be taxing the young agents T_1 and transferring resources to the old agents T_2.

Tax each young agent, so that they end up with the planner allocation:

$$\frac{n}{1+n} w_1 = w_1 - T_1 \implies T_1 = \frac{1}{1+n} w_1$$

Transfer to each old agent, so that they end up with the planner allocation:

$$\frac{n}{1+n} w_1 = 0 + T_2 \implies T_2 = \frac{n}{1+n} w_1$$

The government budget balances:

$$N_t T_1 = N_{t-1} T_2$$

Substituting in population change:

$$\implies nN_{t-1} T_1 = N_{t-1} T_2$$

Substituting in T_1 and T_2:

$$\implies n \frac{1}{1+n} w_1 = \frac{n}{1+n} w_1$$

6. Would agents prefer to live in an economy with a growing population or a shrinking population?

c_t^t and c_t^{t-1} is increasing in n and agents like consumption, so they would prefer to live in an economy with a growing population.

The value of of an agent is:

$$V(n) = U\left(\frac{n}{1+n} w_1, \frac{n}{1+n} w_1\right) = 2 \ln\left(\frac{n}{1+n} w_1\right)$$

V_n is increasing in n.

Pareto Weights

So far, we've consider a planner than weights agents equally, but we can consider optimal allocations where the planner applies different weights to different agents. These weights are referred to as "Pareto weights."

1. Consider the baseline overlapping generations model with 2-period lived agents without population growth. Setup a planner problem where young agents have weight λ_1 and old agents have weight λ_2.

$$\max_{c_t^t, c_t^{t-1} \in \mathbb{R}_+^2} \lambda_1 \ln(c_t^t) + \lambda_2 \ln(c_t^{t-1})$$
$$\text{s.t. } c_t^t + c_t^{t-1} \leq w_1$$

2. Solve for the optimal allocation a function of w_1, λ_1, and λ_2.

Substituting in the resource constraint:

$$\max_{c_t^t \in \mathbb{R}_+} \lambda_1 \ln(c_t^t) + \lambda_2 \ln(w_1 - c_t^t)$$

First order condition:

$$\frac{\lambda_1}{c_t^t} = \frac{\lambda_2}{w_1 - c_t^t} \implies c_t^t = \frac{\lambda_1}{\lambda_1 + \lambda_2} w_1$$

Substituting into the resource constraint:

$$c_t^{t-1} = w_1 - \frac{\lambda_1}{\lambda_1 + \lambda_2} w_1 = \frac{\lambda_2}{\lambda_1 + \lambda_2} w_1$$

3. Set $\lambda_2 = 1 - \lambda_1$. How does the optimal allocation change for $\lambda_1 \in [0, 1]$?

Substituting $\lambda_2 = 1 - \lambda_1$:

$$c_t^t = \lambda_1 w_1$$

$$c_t^{t-1} = (1 - \lambda_1) w_1$$

- At $\lambda_1 = 0$ and $\lambda_2 = 1$, the planner puts no weight on the utility of young agents, so the entire endowment is allocated to the old.
- At $\lambda_1 = 1$ and $\lambda_2 = 0$, the planner puts no weight on the utility of old agents, so the entire endowment is allocated to the young.
- At $\lambda_1 = \lambda_2 = 1/2$, the planner puts equal weights on the utility of both agents, so we get the same solution as in lecture.