ECON 712B Handout 5: McCall Search Model[1]

Instructor: Job Boerma
TAs: Duong Dang and Alex von Hafften

Setup

- An unemployed worker with discount rate $\beta \in (0, 1)$ is searching for a job.
- Each period, the worker draws offer a wage offer w from distribution F where $F(W) = P(w \leq W)$ with $F(0) = 0$ and $F(B) = 1$.
- If the worker rejects w, she receives c in unemployment compensation and draws from F next period.
- If the worker accepts w, she receives w each period forever (no firing and no quitting).

Worker's Problem

- The worker's objective is to maximize lifetime expected discounted earnings.
- The value function of a worker faced by wage offer w:

$$v(w) = \max \left\{ \underbrace{\frac{w}{1-\beta}}_{\text{accept } w}, \underbrace{c + \beta \int_0^B v(w')dF(w')}_{\text{reject } w} \right\}$$

- The value of accepting is continuously increasing in w and the value of rejecting is constant $\implies$ there exists a reservation wage $\bar{w}$ such that the worker is indifferent between accepting and rejecting w:

$$\frac{\bar{w}}{1-\beta} = c + \beta \int_0^B v(w')dF(w') \implies \bar{w} = (1-\beta)\left[c + \beta \int_0^B v(w')dF(w')\right] \tag{1}$$

- The worker accepts $w \geq \bar{w}$ and rejects $w < \bar{w}$, the value function is becomes:

$$v(w) = \begin{cases} \frac{\bar{w}}{1-\beta}, & w < \bar{w} \\ \frac{w}{1-\beta}, & w \geq \bar{w} \end{cases}$$

- Thus, the reservation wage fully characterizes the value function and the value function is increasing in the reservation wage.
- Manipulating (1), we can see that the reservation wage equates the cost of searching one more time and the expected benefit of searching one more time:

$$\underbrace{\bar{w} - c}_{\text{cost of searching one more time}} = \underbrace{\frac{\beta}{1-\beta} \int_{\bar{w}}^B (w' - \bar{w})dF(w')}_{\text{benefit of searching one more time}} \tag{2}$$

[1] See Chapter 6 of Ljungqvist and Sargent for more details.

Characterizing the Reservation Wage

- We can numerically calculate the reservation wage given parameters (see question 7 of problem set 3), but we cannot derive a closed form solution for the reservation wage.
- Here, we characterize how the reservation wage changes if the value of unemployment compensation changes and if the distribution F changes.

Unemployment Compensation

- Define $h : [0, B] \to \mathbb{R}$ as the RHS of (2) $\implies h(0) = \frac{\beta}{1-\beta}E[w]$, $h(B) = 0$, $h'(w) < 0$, and $h''(w) > 0$.
- Thus, since $\bar{w}$ satisfies $h(\bar{w}) = \bar{w} - c$, an increase in unemployment consumption c leads to an increase in the reservation wage $\bar{w}$.

Wage Offer Distribution

- Further manipulating (2), we get that

$$\bar{w} - c = \beta(E[w] - c) + \beta g(\bar{w}) \tag{3}$$

  where $g(s) := \int_0^s F(p)dp$.
- Notice that $g(0) = 0, g(s) \geq 0, g'(s) = F(s) > 0$, and $g''(s) = F'(s) > 0$ for all $s > 0$.
- Consider two wage distributions F_1 and F_2 with the same average wage but F_2 is "noisier".
- Technically, let F_2 be a mean-preserving spread of F_1:
  1. $\int_0^B [F_2(w) - F_1(w)]dw = 0$
  2. $\int_0^s [F_2(w) - F_1(s)]dw \geq 0$ for all $0 \leq s \leq B$
- Therefore, the reservation wage is higher for the noisier wage distribution by (3):
  - Property 1 of mean-preserving spreads $\implies E[w_1] = E[w_2]$ where $w_1 \sim F_1$ and $w_2 \sim F_2$.
  - Property 2 of mean-preserving spreads $\implies g_1(s) \leq g_2(s)$:

Unemployment Duration

- What is the average number of periods $\bar{N}$ before the worker accepts a job?
- Since wage offers are independent, we can show that $\bar{N} = (1 - \lambda)^{-1}$ where $\lambda = \int_0^{\bar{w}} dF(w')$ is the probability that a worker rejects in a given period.

Specializations and Extensions

- In your problem set, you will solve this problem when the wage distribution is uniform and log normal.
- There's many ways of extending this model to allow for multiple offers each period (also on your problem set), allowing workers to quit, allowing firms to fire workers (see LS Ch. 6).