

Bank Regulation with Uninformed Regulators

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- **Question:** How should regulators deal with incentive to underreport risk?

Behn, Haselmann, and Vig (JF, 2022)

- Regulators use risk-weighted capital requirements:

$$E \geq \theta \cdot \mathbf{A} \cdot \mathbf{w}$$

where E is shareholder equity, \mathbf{A} are assets, \mathbf{w} are risk weights, θ is minimum ratio

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- BHV (2022) find evidence of **IRB banks** underreporting risks
 - Banks reported lower PD for **IRB loans** relative to **SA loans** despite **IRB loans** having higher realized losses and higher interest rates
 - Then, lending by **IRB banks** grew faster relative to **SA banks**

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- **Result:** w/ private info, regulation leads to an underprovision of bank lending and deposits relative to first best allocation
- **Why?** Fewer deposits \implies more “skin in the game” \implies safer loans

Outline

1 Introduction

2 Model

3 Appendix

- Environments with Frictions
- Proofs of Main Lemmas
- Parametric Solutions
- Private Productivity
- General Equilibrium

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- **Bank chooses s and d with s unobservable and d observable.**

First Best Problem

- First best allocation s^* and d^* solve

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Problem with Deposit Insurance Environment

- Deposit insurance \implies bank does not pay deposits back if risky tech fails

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$$\text{where } \theta^* = \frac{e}{d^* + e}$$

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- Risk-weighted capital requirement \equiv deposit limit that depends on s
- **Implementing with leverage limit only if regulator can see s and d**

What if s is unobservable to regulator?

Environment

- Regulator chooses limit on leverage that does not depend on s

$$\begin{aligned} & \max_{\theta^P} \underbrace{p(s^P) \cdot A \cdot s^P \cdot (d^P + e)}_{\text{expected risky tech output}} - \underbrace{R(d^P) \cdot d^P}_{\text{deposit return}} \\ \text{s.t. } & (s^P, d^P) \in \arg \max_{s,d} \left\{ \underbrace{p(s) \cdot A \cdot s \cdot (d + e)}_{\text{expected risky tech output}} - \underbrace{p(s) \cdot R(d) \cdot d}_{\text{expected deposit return}} \right\} \\ & \text{s.t. } e \geq \theta^P (d + e) \end{aligned}$$

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 \end{aligned}$$

- Deposit insurance \implies bank borrows until $e = \theta^P (d + e) \implies$ regulator effectively chooses d

$$\begin{aligned}
 & \max_{s^P, d^P} p(s^P) \cdot A \cdot s^P \cdot (d^P + e) - R(d^P) \cdot d^P \\
 \text{s.t. } & s^P \in \arg \max_s \left\{ p(s) \cdot A \cdot s \cdot (d^P + e) - p(s) \cdot R(d^P) \cdot d^P \right\}
 \end{aligned}$$

First best cannot be implemented

- FOC of constraint on private info problem

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- If naive regulator implements θ^* so that banks choose $d^N = d^*$, then bank chooses $s^N > s^*$

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- FOC wrt d^P

$$\underbrace{p(s^P) \cdot A \cdot s^P}_{\substack{\uparrow D \Rightarrow \uparrow \text{output} \\ \text{(MB)}}} = \underbrace{R(d^P) + R'(d^P) \cdot d^P}_{\substack{\text{but pay more for deposits} \\ \text{(MC)}}} + \underbrace{\frac{-\lambda \cdot p'(s^P)}{A \cdot (d^P + e)} \cdot \left[\frac{R(d^P) \cdot e}{d^P + e} + R'(d^P) \cdot d^P \right]}_{\substack{\text{tighten bank FOC} \\ \text{(wedge)}}$$

Comparing Problems and FOCs

$$\max_{s^*, d^*} p(s^*) \cdot A \cdot s^* \cdot (d^* + e) - R(d^*) \cdot d^* \quad [\text{First Best}]$$

$$\max_{s^U, d^U} p(s^U) \cdot A \cdot s^U \cdot (d^U + e) - p(s^U) \cdot R(d^U) \cdot d^U \quad [\text{w/ Deposit Insurance}]$$

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$$\max_{s^U, d^U} p(s^U) \cdot A \cdot s^U \cdot (d^U + e) - \textcolor{red}{p(s^U)} \cdot R(d^U) \cdot d^U \quad \text{[w/ Deposit Insurance]}$$

$$\max_{s^P, d^P} p(s^P) \cdot A \cdot s^P \cdot (d^P + e) - R(d^P) \cdot d^P \quad \text{[w/ S Private]}$$

$$\text{s.t. } \textcolor{blue}{p(s^P) + \frac{-p'(s^P) \cdot R(d^P) \cdot d^P}{A \cdot (d^P + e)} = -p'(s^P) \cdot s^P}$$

$$p(s^*) = -p'(s^*) \cdot s^* \quad \text{FOC } [s^*]$$

$$p(s^U) + \frac{\textcolor{red}{-p'(s^U) \cdot R(d^U) \cdot d^U}}{\textcolor{red}{A \cdot (d^U + e)}} = -p'(s^U) \cdot s^U \quad \text{FOC } [s^U]$$

$$p(s^P) + \frac{-\lambda}{A \cdot (d^P + e)} \left[\frac{p''(s^P) \cdot [A \cdot s^P \cdot (d^P + e) - R(d^P) \cdot d^P]}{A \cdot (d^P + e)} + 2p'(s^P) \right] = -p'(s^P) \cdot s^P \quad \text{FOC } [s^P]$$

Comparing Problems and FOCs

$$\max_{s^*, d^*} p(s^*) \cdot A \cdot s^* \cdot (d^* + e) - R(d^*) \cdot d^* \quad \text{[First Best]}$$

$$\max_{s^U, d^U} p(s^U) \cdot A \cdot s^U \cdot (d^U + e) - p(s^U) \cdot R(d^U) \cdot d^U \quad \text{[w/ Deposit Insurance]}$$

$$\max_{s^P, d^P} p(s^P) \cdot A \cdot s^P \cdot (d^P + e) - R(d^P) \cdot d^P \quad \text{[w/ S Private]}$$

$$\text{s.t. } p(s^P) + \frac{-p'(s^P) \cdot R(d^P) \cdot d^P}{A \cdot (d^P + e)} = -p'(s^P) \cdot s^P$$

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$$p(s^*) \cdot A \cdot s^* = R(d^*) + R'(d^*) \cdot d^* \quad \text{FOC } [d^*]$$

$$p(s^U) \cdot A \cdot s^U = p(s^U) \cdot [R(d^U) + R'(d^U) \cdot d^U] \quad \text{FOC } [d^U]$$

$$p(s^P) \cdot A \cdot s^P = R(d^P) + R'(d^P) \cdot d^P + \frac{-\lambda \cdot p'(s^P)}{A \cdot (d^P + e)} \cdot \left[\frac{R(d^P) \cdot e}{d^P + e} + R'(d^P) \cdot d^P \right] \quad \text{FOC } [d^P]$$

Results

- W/ private s , regulation leads to underprovision of deposits

$$d^U > d^* = d^N > d^P \implies \theta^* > \theta^P$$

L2: $d^U > d^*$

L3: $d^* > d^P$

Parametric Solutions

Results

- W/ private s , regulation leads to underprovision of deposits

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L2: $d^U > d^*$

L3: $d^* > d^P$

Parametric Solutions

- Why? Fewer deposits \implies more “skin in the game” \implies safer loans

$$s^U > s^N > s^P > s^*$$

L1: $s^U > s^*$ and $s^P > s^*$

L4: $s^U > s^N > s^P$

Parametric Solutions

Conclusion

- **What do I do?** Study regulator decision problem in simple model where risk-return characteristic of loan portfolio is private information

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 - ▶ Adverse Selection: $A_H > A_L$ where A_i is private Private Productivity

Conclusion

- **What do I do?** Study regulator decision problem in simple model where risk-return characteristic of loan portfolio is private information
- **Result:** w/ private info about bank loan risk, regulation leads to underprovision of bank lending and deposits
- **Next steps**
 - ▶ Adverse Selection: $A_H > A_L$ where A_i is private Private Productivity
 - ▶ Move to general equilibrium GE Environment

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Related Literature

- Banking models with risk-return choice
 - ▶ Allen and Gale (2004), Boyd and DeNicolo (2005), Martinez-Miera and Repullo (2010), Corbae and Levine (2022)
- Hard and soft information in lending
 - ▶ Liberti and Peterson (2018), Agarwal et al (2018), Agarwal and Hauswald (2010), Petersen and Rajan (2002), Berger and Udell (1995)
- Banks underreporting risk
 - ▶ Behn, Haselmann, and Vig (2022), Mariathasan and Merrouche (2014), Berg and Koziol (2017), Begley, Purnanandam, Zheng (2017), Demirguc-Kunt et al (2010), Blum (2007)

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Environment with Deposit Insurance

- Period 1
 - ▶ Bank born with $e \geq 0$ and chooses $d \geq 0$ and $s \in [0, 1]$
 - ▶ Bank pulls d out of deposit technology
 - ▶ Bank inputs $d + e$ into risky technology at s
- Period 2 with probability $p(s)$
 - ▶ Risky technology outputs $A \cdot s \cdot (d + e)$
 - ▶ Bank pays back $R(d) \cdot d$
 - ▶ Profit is $A \cdot s \cdot (d + e) - R(d) \cdot d$
- Period 2 with probability $1 - p(s)$
 - ▶ Risky technology outputs 0
 - ▶ ~~Bank pays back $R(d) \cdot d$~~
 - ▶ Profit is 0

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Environment with Deposit Insurance and s Private

- Period 1
 - ▶ Bank born with $e \geq 0$ and chooses $d \geq 0$ and $s \in [0, 1]$ (*hidden*)
 - ▶ Bank pulls d out of deposit technology
 - ▶ Bank inputs $d + e$ into risky technology at s (*hidden*)
- Period 2 with probability $p(s)$
 - ▶ Risky technology outputs $A \cdot s \cdot (d + e)$ (*hidden*)
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Lemma 1: $s^U > s^*$ and $s^P > s^*$

- FOC wrt s^* , FOC wrt s^U , and constraint on regulator problem

$$p(s^*) + p'(s^*) \cdot s^* = 0 \quad (1)$$

$$p(s^U) + p'(s^U) \cdot s^U = \frac{p'(s^U) \cdot R(d^U) \cdot d^U}{A \cdot (d^U + e)} \quad (2)$$

$$p(s^P) + p'(s^P) \cdot s^P = \frac{p'(s^P) \cdot R(d^P) \cdot d^P}{A \cdot (d^P + e)} \quad (3)$$

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- LHS of (1), (2), and (3) are strictly decreasing in s

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Lemma 1: $s^U > s^*$ and $s^P > s^*$

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- LHS of (1), (2), and (3) are strictly decreasing in s
- RHS of (2) and (3) are negative because $p' < 0$

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Lemma 2: $d^U > d^*$

- Since $R' > 0$ and $R'' \geq 0$,

$$d^U > d^* \iff R(d^U) + R'(d^U) \cdot d^U > R(d^*) + R'(d^*) \cdot d^*$$

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- FOC wrt d^* and FOC wrt d^U hold

$$p(s^*) \cdot A \cdot s^* = R(d^*) + R'(d^*) \cdot d^* \quad (4)$$

$$p(s^U) \cdot A \cdot s^U = p(s^U) \cdot [R(d^U) + R'(d^U) \cdot d^U] \quad (5)$$

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- Substituting in (4) and (5):

$$\begin{aligned} R(d^U) + R'(d^U) \cdot d^U > R(d^*) + R'(d^*) \cdot d^* &\iff A \cdot s^U > p(s^*) \cdot A \cdot s^* \\ &\iff \frac{s^U}{s^*} > 1 \geq p(s^*) \end{aligned}$$

because $s^U > s^*$ by Lemma 1 and $p(s^*) \in [0, 1]$

Lemma 3: $d^* > d^P$

- Constraint on private info problem and FOC wrt s^P hold

$$p(s^P) + p'(s^P) \cdot s^P = \frac{p'(s^P) \cdot R(d^P) \cdot d^P}{A \cdot (d^P + e)} \quad (6)$$

$$p(s^P) + p'(s^P) \cdot s^P = \frac{\lambda}{A \cdot (d^P + e)} \cdot \left[\frac{p''(s^P) \cdot [A \cdot s^P \cdot (d^P + e) - R(d^P) \cdot d^P]}{A \cdot (d^P + e)} + 2p'(s^P) \right] \quad (7)$$

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- Using (6) and (7),

$$\lambda = \frac{p'(s^P) \cdot R(d^P) \cdot d^P}{2p'(s^P) + p''(s^P) \cdot \frac{A \cdot s^P \cdot (d^P + e) - R(d^P) \cdot d^P}{A \cdot (d^P + e)}} > 0$$

because both numerator and denominator are negative with $p' < 0$, $p'' \leq 0$, and $A \cdot s \cdot (d + e) > R(d) \cdot d$

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- FOC wrt d^* and FOC wrt d^P holds

$$R(d^*) + R'(d^*) \cdot d^* = p(s^*) \cdot A \cdot s^* \quad (8)$$

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- Second term of (9) is negative because $\lambda > 0$, $p'' < 0$ and $R' > 0$
- Since $R' > 0$, $R(d^*) + R'(d^*) \cdot d^* > R(d^P) + R'(d^P) \cdot d^P \implies d^* > d^P$

Lemma 4: $s^U > s^N > s^P$

- By Lemma 2 and 3, $d^U > d^* > d^P$ with $d^N = d^*$ by assumption

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- By Lemma 2 and 3, $d^U > d^* > d^P$ with $d^N = d^*$ by assumption
- FOC wrt s^U holds both at (s^U, d^U) and (s^P, d^P)

$$p(s) + p'(s) \cdot s = \frac{p'(s) \cdot R(d) \cdot d}{A \cdot (d + e)} \quad (10)$$

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- Implicitly differentiating (10)

$$\frac{\partial s}{\partial d} = \frac{(p'(s))^2 \cdot [R'(d) \cdot d \cdot (d + e) + R(d) \cdot e]}{A \cdot (d + e)^2 \cdot [2(p'(s))^2 - p(s)p''(s)]} > 0$$

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Lemma 4: $s^U > s^N > s^P$

- By Lemma 2 and 3, $d^U > d^* > d^P$ with $d^N = d^*$ by assumption
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- Numerator is positive because $p' < 0$ and $R' > 0$
- Denominator is positive because $p' < 0$ and $p'' \leq 0$
- Since $d^U > d^N > d^P$ and $\frac{\partial s}{\partial d} > 0 \implies s^U > s^N > s^P$

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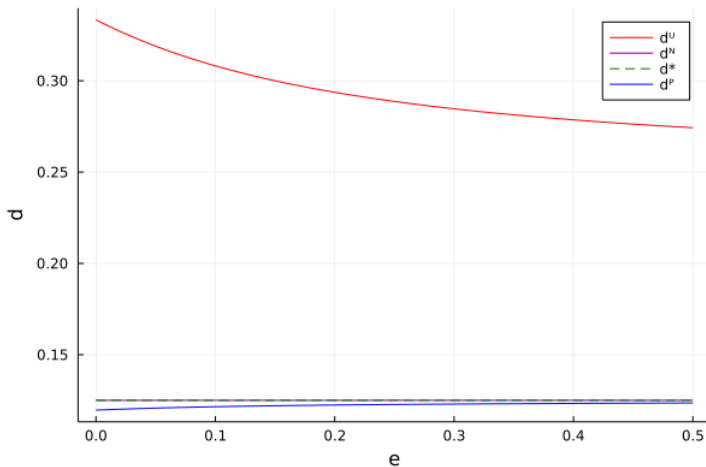
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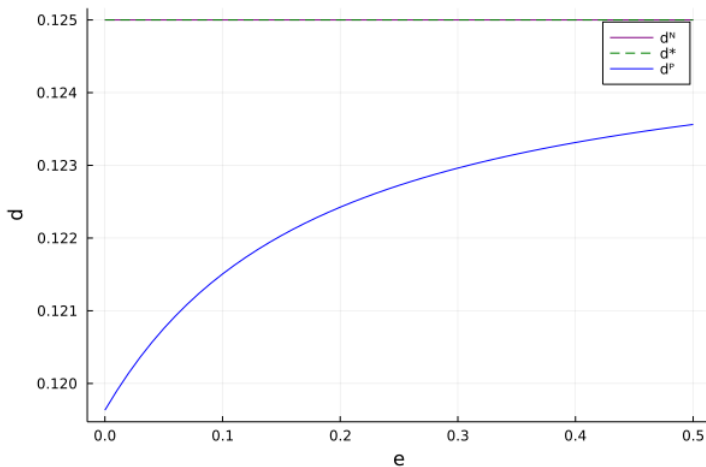
$$d^U > d^* = d^N > d^P$$



where $A = 1$, $p(s) = 1 - s$, and $R(d) = d$.

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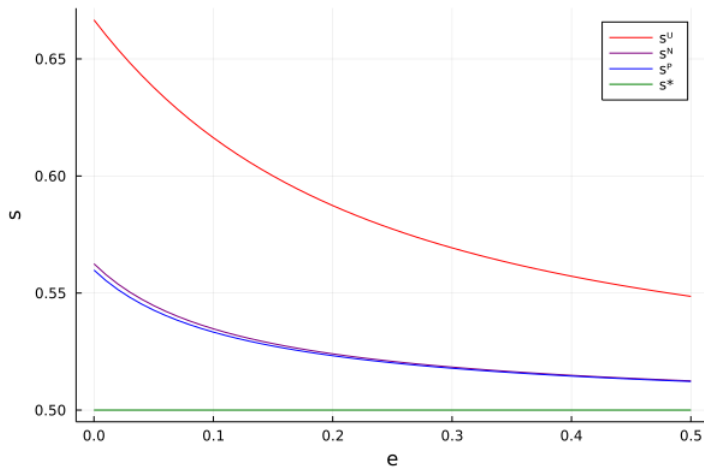
$$d^* = d^N > d^P$$



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$$s^U > s^N > s^P > s^*$$



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- Parametric Solutions
- **Private Productivity**
- General Equilibrium

Environment w/ Private Productivity

- Period 1

- ▶ Bank born w/ $e \geq 0$ and A_H w/ prob. π or A_L w/ prob. $1 - \pi$ (*hidden*)
- ▶ Bank chooses $d \geq 0$ and $s \in [0, 1]$
- ▶ Bank pulls d out of deposit technology
- ▶ Bank inputs $d + e$ into risky technology at s

- Period 2 with probability $p(s)$

- ▶ Risky technology outputs $A_i \cdot s \cdot (d + e)$ (*hidden*)
- ▶ Bank pays back $R(d) \cdot d$
- ▶ Profit is $A_i \cdot s \cdot (d + e) - R(d) \cdot d$ (*hidden*)

- Period 2 with probability $1 - p(s)$

- ▶ Risky technology outputs 0
- ▶ ~~Bank pays back $R(d) \cdot d$~~
- ▶ Profit is 0

First Best with Multiple Types

- If A_i and s are observable to outsiders

$$\max_{s_i^*, d_i^*} p(s_i^*) \cdot A_i \cdot s_i^* \cdot (d_i^* + e) - R(d_i^*) \cdot d_i^*$$

Back

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$$\max_{s_i^*, d_i^*} p(s_i^*) \cdot A_i \cdot s_i^* \cdot (d_i^* + e) - R(d_i^*) \cdot d_i^*$$

- FOC wrt s_i^*

$$p(s_i^*) + p'(s_i^*) \cdot s_i^* = 0 \implies s_H^* = s_L^* \equiv s^*$$

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$$p(s_i^*) \cdot A_i \cdot s_i^* = R(d_i^*) + R'(d_i^*) \cdot d_i^* \implies d_H^* > d_L^*$$

Back

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- Both types invest at same s , but high type gets more d

Back

Regulator Problem with Private A_i and Deposit Insurance

$$\max_{s_H^P, d_H^P, s_L^P, d_L^P} \pi \cdot [p(s_H^P) \cdot A_H \cdot s_H^P \cdot (d_H^P + e) - R(d_H^P) \cdot d_H^P] \\ + (1 - \pi) \cdot [p(s_L^P) \cdot A_L \cdot s_L^P \cdot (d_L^P + e) - R(d_L^P) \cdot d_L^P]$$

$$\text{s.t. } \underbrace{p(s_H^P)A_Hs_H^P(d_H^P + e) - \textcolor{red}{p}(s_H^P)R(d_H^P)d_H^P}_{\text{Profit from truthfully reporting } H} \geq \underbrace{p(s_L^P)A_Hs_L^P(d_L^P + e) - \textcolor{red}{p}(s_L^P)R(d_L^P)d_L^P}_{\text{Profit from falsely reporting } L} \quad [IC_H]$$

$$\underbrace{p(s_L^P)A_Ls_L^P(d_L^P + e) - \textcolor{red}{p}(s_L^P)R(d_L^P)d_L^P}_{\text{Profit from truthfully reporting } L} \geq \underbrace{p(s_H^P)A_Ls_H^P(d_H^P + e) - \textcolor{red}{p}(s_H^P)R(d_H^P)d_H^P}_{\text{Profit from falsely reporting } H} \quad [IC_L]$$

- Both types want more deposits $\implies IC_H$ is slack and IC_L binds

Back

FOCs with Multiple Types

$$p'(s_H^P) \cdot s_H^P + p(s_H^P) = \frac{\gamma}{\pi - \gamma} \left[- \frac{p'(s_H^P) R(d_H^P) d_H^P}{A_H(d_H^P + e)} \right] \quad [s_H^P]$$

$$p'(s_L^P) \cdot s_L^P + p(s_L^P) = \frac{\gamma}{1 - \pi - \gamma} \left[- \frac{p'(s_L^P) R(d_L^P) d_L^P}{A_L \cdot (d_L^P + e)} \right] \quad [s_L^P]$$

$$p(s_H^P) \cdot A_H \cdot s_H^P - R(d_H^P) - R'(d_H^P) d_H^P = \frac{\gamma p(s_H^P)}{\pi} [A_L s_H^P - R'(d_H^P) d_H^P - R(d_H^P)] \quad [d_H^P]$$

$$p(s_L^P) \cdot A_L \cdot s_L^P - R'(d_L^P) \cdot d_L^P - R(d_L^P) = - \frac{\gamma p(s_L^P)}{(1 - \pi)} [A_L s_L^P - R'(d_L^P) d_L^P - R(d_L^P)] \quad [d_L^P]$$

$$p(s_L^P) A_L s_L^P (d_L^P + e) - p(s_L^P) R(d_L^P) d_L^P = p(s_H^P) A_L s_H^P (d_H^P + e) - p(s_H^P) R(d_H^P) d_H^P \quad [\gamma]$$

Back

Preliminary Numerical Results

- Use $p(s) = 1 - s$, $R(d) = d$, $A_H = 2.0$, $A_L = 1.0$, $\pi = 0.5$, $e = 0.5$

Allocation	First Best	Private Productivity
s_H	0.5	0.487
s_L	0.5	0.491
d_H	0.25	0.252
d_L	0.125	0.149
γ	-	0.248

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- But give more deposits to low type w/ $d_L^P > d_L^*$ so they truthfully reveal
- Interesting preliminary result:
 - ▶ With hidden s , deposits and lending is lower: $d^* < d^P$
 - ▶ With hidden A , deposits and lending might be higher:

$$\pi d_H^* + (1 - \pi) d_L^* > \pi d_H^P + (1 - \pi) d_L^P$$

Outline

1 Introduction

2 Model

3 Appendix

- Environments with Frictions
- Proofs of Main Lemmas
- Parametric Solutions
- Private Productivity
- General Equilibrium

GE Environment - HH Problems

- **Depositor problem:**

Given (y^D, R, T^D) , solves

$$\begin{aligned} \max_{c_1^D, c_2^D, d} \quad & u(c_1^D) + \beta E[u(c_2^D)] \\ \text{s.t.} \quad & c_1^D + d = y^D \\ & c_2^D = R \cdot d + T^D \end{aligned}$$

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- **Bank owner problem:**

Given (y^O, T^O, π) , solves

$$\begin{aligned} \max_{c_1^O, c_2^O, e} \quad & u(c_1^O) + \beta E[u(c_2^O)] \\ \text{s.t.} \quad & c_1^O + e = y^O \\ & c_2^O = \pi + T^O \end{aligned}$$

Contracting Problem Setup

- Principal is regulator and agent is competitive bank

Back

Contracting Problem Setup

- Principal is regulator and agent is competitive bank
- Period 1

Back

Contracting Problem Setup

- Principal is regulator and agent is competitive bank
- Period 1
 - ▶ Principal meets agent with $e \geq 0$

Back

Contracting Problem Setup

- Principal is regulator and agent is competitive bank
- Period 1
 - ▶ Principal meets agent with $e \geq 0$
 - ▶ Principal and agent agree on contract (b, s, d)

Back

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 - ▶ Principal borrows $d \geq 0$ at interest rate R from HHs

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 - ▶ Agent puts $d + e$ into risky tech at s

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- Period 2 with probability $p(s)$

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- Period 2 with probability $p(s)$
 - ▶ Agent collects $A \cdot s \cdot (d + e)$ from risky tech

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 - ▶ Agent gives b to principal

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 - ▶ Agent puts $d + e$ into risky tech at s
- Period 2 with probability $p(s)$
 - ▶ Agent collects $A \cdot s \cdot (d + e)$ from risky tech
 - ▶ Agent gives b to principal
 - ▶ Principal pays $R \cdot d$ and lump-sum transfer $T^D + T^O = b - R \cdot d$ to HHs

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 - ▶ Principal pays $R \cdot d$ and lump-sum transfer $T^D + T^O = b - R \cdot d$ to HHs
 - ▶ Agent profit is $\pi = A \cdot s \cdot (d + e) - b$

Back

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- Period 2 with probability $1 - p(s)$

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- Period 2 with probability $1 - p(s)$
 - ▶ Agent collects zero from risky tech

Back

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- Period 2 with probability $p(s)$
 - ▶ Agent collects $A \cdot s \cdot (d + e)$ from risky tech
 - ▶ Agent gives b to principal
 - ▶ Principal pays $R \cdot d$ and lump-sum transfer $T^D + T^O = b - R \cdot d$ to HHs
 - ▶ Agent profit is $\pi = A \cdot s \cdot (d + e) - b$
- Period 2 with probability $1 - p(s)$
 - ▶ Agent collects zero from risky tech
 - ▶ Principal lump-sum taxes HHs $T^D + T^O = -R \cdot d$

Back

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 - ▶ Agent gives b to principal
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 - ▶ Agent profit is $\pi = A \cdot s \cdot (d + e) - b$
- Period 2 with probability $1 - p(s)$
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 - ▶ Principal lump-sum taxes HHs $T^D + T^O = -R \cdot d$
 - ▶ Principal pays depositors $R \cdot d$

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 - ▶ Agent gives b to principal
 - ▶ Principal pays $R \cdot d$ and lump-sum transfer $T^D + T^O = b - R \cdot d$ to HHs
 - ▶ Agent profit is $\pi = A \cdot s \cdot (d + e) - b$
- Period 2 with probability $1 - p(s)$
 - ▶ Agent collects zero from risky tech
 - ▶ Principal lump-sum taxes HHs $T^D + T^O = -R \cdot d$
 - ▶ Principal pays depositors $R \cdot d$
- Agent outside option is to run risky technology only with e

Back

Contracting Problems

- Contracting problem maximizes principal objective s.t. agent participation

$$\max_{s,d,b} p(s) \cdot [b - Rd] + (1 - p(s)) \cdot [-Rd]$$
$$p(s) \cdot [As(d + e) - b] \geq \max_{\hat{s}} \{p(\hat{s}) \cdot A \cdot \hat{s} \cdot e\} \quad [\text{Agent PC}]$$

Contracting Problems

- Contracting problem maximizes principal objective s.t. agent participation

$$\begin{aligned} \max_{s,d,b} & p(s) \cdot [b - Rd] + (1 - p(s)) \cdot [-Rd] \\ p(s) \cdot [As(d + e) - b] & \geq \max_{\hat{s}} \{p(\hat{s}) \cdot A \cdot \hat{s} \cdot e\} \quad [\text{Agent PC}] \end{aligned}$$

- Hidden s introduces agent incentive compatibility constraint

$$\begin{aligned} \max_{s,d,b} & p(s) \cdot [b - Rd] + (1 - p(s)) \cdot [-Rd] \\ p(s) \cdot [As(d + e) - b] & \geq \max_{\hat{s}} \{p(\hat{s}) \cdot A \cdot \hat{s} \cdot e\} \quad [\text{Agent PC}] \\ s & \in \arg \max_{\hat{s}} \{p(\hat{s}) \cdot A \cdot \hat{s} \cdot (d + e) - p(\hat{s}) \cdot b\} \quad [\text{Agent IC}] \end{aligned}$$