Heterogeneous Firms, Emissions, and Optimal Carbon Taxes*

Very Preliminary Draft, Work in Progress

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Abstract

The climate-macro literature largely focuses on optimal policies to address climate feedback in a representative firm framework. Empirically, we show that the emission intensity of U.S. public firms features substantial and persistent heterogeneity. In the presence of this heterogeneity, we illustrate a novel *composition effect* of the constrained Social Planner's allocation when firm exits are endogenous: the Planner exits firms with dirtier production technologies at a higher rate, making the distribution over production technologies itself greener. We extend this intuition from an illustrative model into a general-equilibrium heterogeneous-firm model, in which we characterize and solve the problem of the Planner. We derive socially optimal carbon taxes which implement the Planner's allocation in a decentralized environment, and compute the transition to the long-run equilibrium with and without these taxes.

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1 Introduction

Spearheaded by the trailblazing work of William Nordhaus and others, economists over the past 40 years have become increasingly focused on the socially optimal policy responses, such as Pigouvian carbon taxes, to the interaction between the global macroeconomy and the Earth's climate. In this paper, we highlight a novel *composition effect* of the Social Planner's allocation in the firm distribution in presence of firm heterogeneity in carbon dependence and endogenous firm exits. Put simply, the Planner exits dirty (high-emitting) firms at a higher rate than clean (lower-emitting) firms, all else equal, and hence the production technology itself of the macroeconomy becomes greener under the Planner through a change in the composition of firms operating in the economy. As the vast majority of the climate-macro literature focuses on representative-firm models, this mechanism has been ignored thus far.

We begin by documenting the vast heterogeneity of the emissions intensity of U.S. public firms. We find striking evidence that carbon emissions per unit of capital vary greatly across firms, but far less so within firms. Hence, there is strong evidence in favor of some firms having more pollutive production technologies than others. Moreover, we show that the addition of controls for firm-level profitability and capital utilization do very little to explain additional variation in the emissions intensity data. We argue that this is evidence of some firms being inherently more or less pollutive than others. We then write an illustrative model with firm heterogeneity in carbon intensity that aligns with our empirical analysis and an environmental externality. We formulate, characterize, and solve the problem of the Social Planner constrained to the firm-level technologies.

Next, we extend our illustrative model to an infinite horizon general-equilibrium firm-dynamics model with endogenous entry and exit, an off-the-shelf climate block from the literature, and otherwise standard heterogeneous-firm ingredients. We apply and extend methods recently developed by Lucas and Moll (2014), Moll and Nuno (2018), and Ottonello and Winberry (2023) to formulate, characterize, and solve the constrained Planner's problem, which features the infinite-dimensional firm distribution as a state variable. We prove that the constrained Planner's allocation can be implemented in a competitive equilibrium through simple distortionary taxes which we derive, and demonstrate robustness of the implementability result to differences in assumptions on the observability of carbon-related firm-level objects by the taxation authority. Finally, we compute the BAU and Social Planner transitions in response to a standard climate-macro scenario, and decompose the difference between the allocations to measure the quantitative magnitude of the composition effect.

Our paper is organized as follows. We briefly review related literature in Section 2.

Then, we document evidence of heterogeneity in the carbon dependence of production in Section 3 We then write down a simple model, consistent with our empirical analysis, in Section 4 to illustrate the composition effect. Then, we extend the example into a dynamic GE heterogeneous-firm model in Section 5. In Section 6, we solve the problem of the social planner and derive the taxes that implement the socially optimal allocation. In Section 7, we compute the transition path for the evolution of the economy and climate under a scenario from the climate-macro literature. Finally, we conclude in Section 8.

2 Related Literature

Our analysis relates most closely to Integrated Assessment Models (IAMs) in the climate-macro literature, such as Golosov et al. (2014), Hassler et al. (2020), and Krusell and Smith (2022). These models bring the approach pioneered by Nordhaus (2007) into a modern macroeconomic framework. IAMs are based on a simple extension of the neoclassical growth model, whereby the production of final goods relies on energy as an input as well as aggregate capital and labor. In this framework, energy use is treated as a static decision, similar to labor. The production of energy involves the combination of various energy inputs, such as solar and coal, using a CES aggregator. This is an effective and generally accepted modelling approach for questions involving the transition of the macroeconomy across energy sources and the taxation of these sources to achieve an optimal energy mix. However, the limited ability to explain within-firm heterogeneity in carbon emissions intensity using controls that the IAM production function would predict as being particularly relevent is suggestive evidence that the IAM production function is inconsistent with the firm-level emissions generation process. Specifically, we show using data from Bloomberg on the carbon emissions of public firms, merged with Compustat, that the variation in the emissions intensity of firms is driven almost entirely by variation across firms as opposed to variation within firms. Moreover, the inclusion of within-firm controls for capital profitability and labor utilization of capital explain very little of the variation of the emissions intensity data.

Motivated by our empirical findings, we instead approach our formulation of firm-level production and heterogeneity of carbon intensity from a different angle. We allow final goods to be produced using two types of capital: polluting dirty capital k_d and non-polluting green capital k_g. The relative mixture between the two types of capital that a given firm produces with in our model then determines that firm's carbon intensity of capital. Emissions intensity of capital is mechanically disentangled from capital profitability and labor utilization within-firm as the capital allocations are fixed in the short-run, yet varies greatly across-firms due to varying carbon dependencies of each firms' production function. The result is a form of persistent heterogeneity of carbon intensity of production that matches what we document in the data.

The model we develop extends the class of endogenous entry and exit firm-dynamics models a la Hopenhayn and Rogerson (1993) to a climate-macro setting. While models of firm investment in this environment such as Gomes (2001), Khan and Thomas (2013), and Clementi and Palazzo (2015) typically involve firms heterogenous in productivity z and one type of physical capital, we split capital into two forms, clean k_g and dirty k_d. This split allows us to match the form of carbon emissions heterogeneity that we document in the firm-level data. It also results in the addition of a continuous endogenous state variable. Moreover, we add an exogenous state variable, firm-level carbon dependence of production a, which allows us to match the large across-firm heterogeneity from the data. The result is a relatively large firm-level state-space: $s = [z, k_g, k_d, a]' \in \mathcal{Z} \times \mathcal{K}_g \times \mathcal{K}_d \times \mathcal{A}$. We further consider transition paths of the economy over very long time horizons, $T = 1000$ periods, reflecting the high persistence of atmospheric carbon emissions present in the calibration of our computed climate scenario. Despite these computational challenges, we are able to solve the model globally in general equilibrium due to a form of conditional block recursivity provided by the firm entry condition originating from our use of endogenous entry within the model, as Hopenhayn and Rogerson (1993) and Gomes (2001) use in the computation of their steady-state models.

An additional challenge we face is to solve the problem of a planner, constrained to the firm-level technologies, in a heterogenous-agent environment. In particular, the challenge is that a state variable for the constrained Planner is the infinite-dimensional firm distribution. We look to techniques recently developed by Lucas and Moll (2014) and Moll and Nuno (2018), and brought into discrete time Ottonello and Winberry (2023) as our starting point. In particular, we derive the marginal social value function and show that the policies induced by the equivalent augmented Bellman equation result in the planner's firm-level policies. We extend the technique in two ways: 1) we allow for firms to endogenously exit (and, in general, make a discrete choice) and 2) we allow for endogenous firm entry.

After solving the Planner's problem, we prove that the constrained first-best allocation can be implemented in a competitive equilibrium through simple taxes, constant cross-sectionally across firms and varying over time. This finding relates to Golosov et al. (2014) and Krusell and Smith (2022) who derive the solutions to Planner's problems in related (but representative-firm) environmental-macro models of the IAM flavor, and find similar results in that a cross-sectionally constant taxation scheme can implement the planner's solution. The intuition of such findings is fairly general - distortions in the laissez-faire equilibrium arise due to externalities from the perspective of a decision-maker. So long as the externality is constant cross-sectionally, i.e. so long as the **per-emission environmental damage** is the same for emissions emitted by each agent, then the **per-emission Pigouvian tax** which corrects the externality will be the same across all agents in the economy. Hence, although firms may emit a different intensity of emissions from their production process, taxing different firms the same on a per-emission level is sufficient to implement the constrained first-best allocation.

3 Persistent Heterogeneity of Carbon Intensity of US Firms

We begin with an empirical analysis of the distribution of carbon emissions across U.S. public firms. In this section, we document two stylized facts. First, we document that there exists substantial cross-sectional heterogeneity in the carbon emission intensity, the ratio of emissions to capital.[1] Second, we show that a substantial portion of the variation in the carbon intensity in the data originates from across-firm variation, rather than within-firm. These facts together imply that firms are persistently heterogeneous in their emissions intensity to a degree that would influence the relative social costs from carbon emissions that a Social Planner would internalize. This social cost enters the Planner's decision-making when deciding whether-or-not to exit firms, and hence the Planner is more likely to exit a green firm than a pollutive firm, all else equal. As we show in our illustrative model, this causes a change in the composition of the production technology in use in the economy—the so-called composition effect that we highlight.

To this aim, we construct a balanced panel of firm-level data over a five-year period from 2016 to 2020 using two primary data sources. Firstly, we utilize the Scope 1 and Scope 2 greenhouse gas emissions data obtained from firms' annual reports, which are collected by Bloomberg (referred to as GHG_SCOPE_1 and GHG_SCOPE_2). Scope 1 emissions are thousand metric tonnes of carbon-equivalent emissions (COe) generated by sources under the control or ownership of the firm. This category includes carbon emissions resulting from the combustion of fossil fuels in boilers, furnaces, or vehicles, for instance. Scope 2 emissions are indirectly caused by the firm's activities, such as the emissions associated with the firm's purchase and use of energy.[2] In our variable of interest, we use Scope 1, Scope 2, and the sum

[1] In the appendix, we show that all of our stylized facts hold equally if we consider the level of carbon emissions, as opposed to the intensity of emissions. We focus on the intensity as it maps more directly into our model and in order to adjust for the scale of the firms. In unreported results, we have found that our results hold robustly to a slew of alternative denominators available from Compustat.

[2] Greenhouse gas emissions are also available at Scope 3, which are emissions that are produced upstream or downstream within the firm's value chain. To avoid double-counting emissions up and down stream, we

of Scope 1 and Scope 2 emissions. Compustat provides us with standard variables including gross property, plants, and equipment (PPEGT). We adjust nominal variables for inflation using a GDP deflator to express these in 2015 dollars. For industry classification, we also rely on Compustat for Standard Industrial Classification (SIC) divison codes and 2-digit SIC industry codes.[3]

We construct our variable of interest, emission intensity of capital, as metric tons of COe emitted per thousand 2015 dollars of gross property, plants, and equipment $y = \log\left(\frac{\text{GHG}}{\text{PPEGT}}\right)$, where $GHG \in \{\text{GHG\_SCOPE\_1, GHG\_SCOPE\_2, GHG\_SCOPE\_1 + GHG\_SCOPE\_2}\}$. The balanced panel with Scope 1 has 389 firms, the balanced panel with Scope 2 emissions has 365 firms, and the balanced panel with Scope 1 plus Scope 2 has 362 firms. As shown in Appendix A, the firms with available emissions data are on average four-to-five times larger than the average firm in the full Compustat sample in terms of number of employees, earnings, and capital. These differences may arise due to more scrutiny from ESG-focused investors demanding that larger firms release information about their emissions. These firms are similarly more capital intensive with a lower employees to capital ratio. In terms of sectoral representation, firms providing services are underrepresented compared to full Compustat sample and mining, manufacturing, and transportation firms are overrepresented. The profitability of firms with emissions data and the full Compustat sample are not statistically different. Figure 1 shows that the unconditional distribution of Scope 1 and Scope 2 emission intensity is roughly log normal centered around 0.15 metric tonnes per thousand real dollars of capital and 0.09 metric tonnes per thousand real dollars of capital, respectively (see Table 1). The sum of Scope 1 and Scope 2 emission intensity is more right skewed and with fatter tails as seen in Table 1.

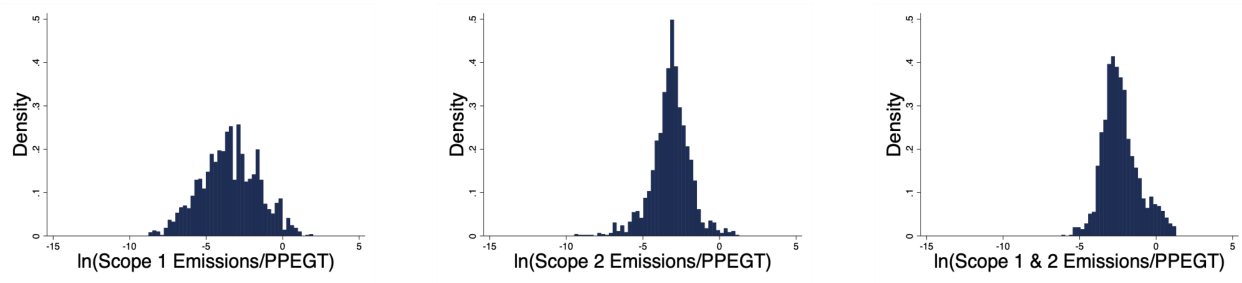
Table 1: Summary Statistics

Variable	Mean	SD	Skewness	Kurtosis	N
Scope 1 + 2 Emissions / PPEGT (real)	0.23	0.44	3.81	16.91	1810
Scope 1 Emissions / PPEGT (real)	0.15	0.40	7.01	74.76	1965
Scope 2 Emissions / PPEGT (real)	0.09	0.22	7.21	63.89	1830

do not use Scope 3 emissions.

[3]As is standard practice in corporate finance, we filter out financial firms (1-digit SIC code of 6), utilities (2-digit SIC code of 49), and unclassified firms (1-digit SIC of 9).

Figure 1: Distribution of Carbon Emission Intensity



(a) Scope 1 Emissions (b) Scope 2 Emissions (c) Scope 1 & 2 Emissions

Notes: This figure shows the unconditional pooled distribution of carbon emission intensity—defined as metric tonnes of CO2e per thousands of 2015 dollar of property, plants, and equipment for Compustat firms in a five-year balanced panel between 2016-2020. Panel (a) shows Scope 1 emissions, which are those created by the operations directly controlled by the firm including subsidiaries and affiliates. Panel (b) shows Scope 2 emissions, which are those resulting indirectly from the firm's activity in particular energy generation. Panel c shows Scope 1 emissions plus Scope 2 emission.

As is clear from Figure 1, there is a noticeably large dispersion in carbon emissions intensity among US public firms.[4] In order for our mechanism to be supported by the data, not only do we need dispersion in emission intensity, but also some persistence of the emission intensity at the firm level. Hence, we need to answer the question of whether the variation in this data is across-firm or within-firm.

To decompose variation across- and within-firm of carbon emission intensity, we regress emission intensity on firm-level fixed effects (see Equation 1). Following the empirical corporate finance literature on the leverage persistence puzzle (e.g., Lemmon, Roberts, and Zender (2008)), we interpret firm-level fixed effects as statistical "stand-ins" for the permanent component of carbon emission intensity associated with a particular firm.

$$y_{i,t} = \alpha_i + \varepsilon_{i,t};\ \varepsilon_{i,t} \perp \alpha_i \tag{1}$$

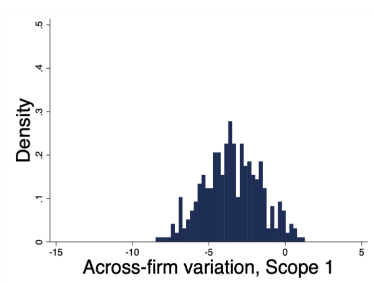
After estimating equation (1) via OLS, we decompose the variance of emissions intensity into the across- and within-firm components. Given our assumption of independence of the residuals from equation (1), the variance can be easily decomposed into the variance of the fixed effects themselves (across-firm variation) and the variance of the residuals (within-firm variation):

$$\underbrace{\mathbb{V}\mathrm{ar}(y_{i,t})}_{\text{total}} = \underbrace{\mathbb{V}\mathrm{ar}(\alpha_i)}_{\text{across-firm}} + \underbrace{\mathbb{V}\mathrm{ar}(\varepsilon_{i,t})}_{\text{within-firm}}.$$

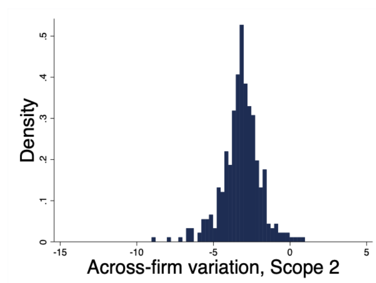
[4]As is shown in Appendix A, the dispersion also holds in the level of emissions produced.

Note that the ratio of the across-firm variation to total variation $\frac{\mathbb{V}\text{ar}(\alpha_i)}{\mathbb{V}\text{ar}(y_{i,t})}$ is equal to the R^2 of the OLS regression. Hence, a high R^2 indicates that the variance of the emissions intensity data is primarily driven by across-firm variation, as opposed to within-firm variation. We report the R^2 of regression (1) in Table 3 and visualize the empirical distributions of each component through histograms in Figure 2.

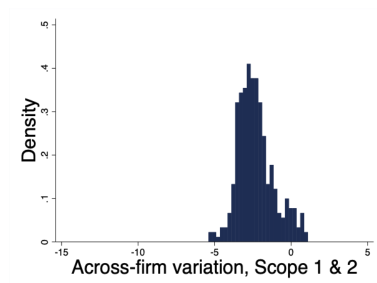
Figure 2: Across-Firm vs Within-Firm Variation



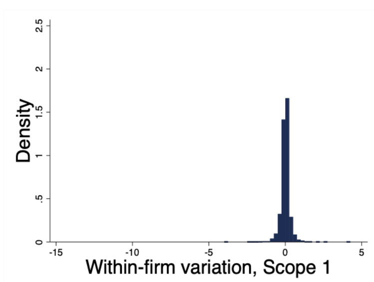
(a) Scope 1 Emissions



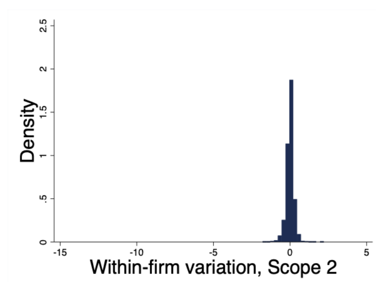
(b) Scope 2 Emissions



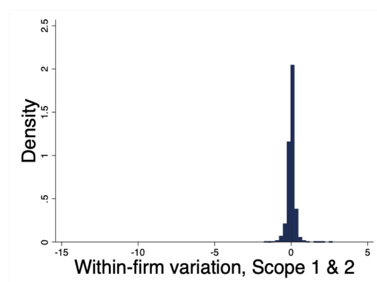
(c) Scope 1 & 2 Emissions



(d) Scope 1 Emissions



(e) Scope 2 Emissions



(f) Scope 1 & 2 Emissions

Notes: This figure shows the distribution of α_i OLS estimates (top row) and residuals (bottom row). The distribution of α_i is visually very similar to the distribution of the raw data as reported in Figure 1. By comparison, the distribution of residuals is much tighter, concentrated closely around zero.

Table 2: R^2 Values from Regression 1

Scope	R^2	95 % CI
Scope 1	0.970	(0.960, 0.982)
Scope 2	0.954	(0.943, 0.969)
Scope 1 & 2	0.956	(0.945, 0.970)

Notes: This table reports the R^2 values from OLS estimates of Regression 1, along with 95% confidence intervals. The confidence intervals are constructed as block bootstrap pivot intervals following Efron (1981).

As is verified by Table 3, the R^2 of our regressions are very high, with values across all specifications statistically significantly above 0.9.[5,6] The visualization in Figure 2 shows the same fact visually, the fixed effect values are highly dispersed whereas the residuals are much more tightly distributed.

A fair question to ask is how much of the variation in our data is driven by the industries that the firms operate in. To answer this question, we report adjusted and unadjusted R^2 for regressions with fixed effects at the SIC division and 2-digit SIC industry level in Appendix A. The values are not nearly as extreme as those of the firm-level fixed effects, with SIC divisions explaining about 10 percent of the total variation, and industry explaining about 30-60 percent of the variation depending on the specification. While these results show that there is certainly correlation of carbon emission intensity of firms within industries, there also remains substantial heterogeneity within-industry.

To summarize, we document that there exists substantial heterogeneity in the carbon emission intensity of US public firms. Furthermore, the variation appears to be driven in a substantial magnitude by across-firm variation. From the perspective of a planner that internalizes social costs from emissions, all else equal a more heavily polluting firm will be more costly than a less pollutive firm. As we demonstrate through our simple example, this form of heterogeneity impacts the socially optimal response to climate feedback, and can lead the planner to exit more pollutive firms at a higher rate than lower firms. If this occurs, the composition of the production technology of the economy becomes cleaner.

4 Illustrative Model

Motivated by the heterogeneity in emission intensity of capital in the data, we develop an illustrative model to highlight the composition effect in the socially optimal allocation given a climate externality.

In the illustrative model, there is a unit mass of two types of atomistic expected-profit-maximizing firms $i \in \{L, H\}$ each with mass p_i, and there is one period broken into two subperiods. In the first subperiod, each type-i firm observes its exogenous carbon dependence a_i where $0 < a_L < a_H < 1$ and chooses dirty capital investment $k_d \geq 0$ and clean capital investment $k_g \geq 0$.[7] In the second subperiod, each firm draws idiosyncratic total factor productivity $z \sim_{iid} G$ where $z > 0$ and G is continuous. Then, a type-i firm with productivity

[5] While there is some concern that the R^2 is elevated by the short duration of the panel, we report adjusted R^2 values in Appendix A which punish our short panel length. These adjusted R^2 values are also very high, which is suggestive that the panel length is not driving the result.

[6] Confidence intervals for the R^2 are computed as block bootstrap pivot intervals, following Efron (1981).

[7] Throughout we will drop the nonnegativity constraint on both types of capital, as our assumed functional form for $F(z, k_d, k_g, a)$ satisfies Inada conditions such that the constraints will not bind.

z chooses either to exit and produce zero or to operate and produce

$$\underbrace{[1 - D(K^d)]}_{\text{climate damage}} \times F(z, k_d, k_g, a_i)$$

where aggregate *utilized* dirty capital (i.e., the dirty capital of firms that choose to operate) creates climate damage $D(K^d) \in (0, 1)$ with $D'(K^d) > 0$.[8] Production gross of climate damage F is

$$F(z, k_d, k_g, a_i) = \underbrace{z}_{\text{TFP}} \times \underbrace{\left[a_i k_d^\rho + (1 - a_i) k_g^\rho\right]^{\alpha/\rho}}_{\text{CES aggregator nested in DRS}}$$

with $\alpha < 1$ captures decreasing returns to scale of production and $\rho \geq -1$ is the substitution parameter. Type-H firms have high carbon dependence in the sense that these firms have a higher weight on dirty capital in their production function than type-L firms, thus they are relatively more productive with dirty capital. The externality in this model is that individual firms take K^d as given and do not account that their investment and exit choices change K^d and thus cause climate damage for other firms. At the end of the period, all capital depreciate fully whether or not it is used in production. Households consume firm profits with preferences $U(C)$ with $U'(C) > 0$.

Putting this together, the problem for a firm with carbon dependence a_i is to maximize expected profit:

$$\pi(a_i) = \max_{k_d, k_g} -k_d - k_g + \mathbb{E}_z \left[\max\{[1 - D(K^d)] F(z, k_d, k_g, a_i), 0\}\right]. \tag{2}$$

Let $k_d(a_i)$ and $k_g(a_i)$ be the investment policies for a firm with carbon dependence a_i, and let $X(z, a_i)$ be the exit policy a firm with carbon dependence a_i and productivity z. The exit policy is an indicator function that equals one if the firm chooses to exit (i.e., chooses the 0 in the max operator) and equals zero if the firm chooses to operate. Given firm-level policies, we can define economy-wide aggregates including gross aggregate output Y, aggregate investment I, and aggregate dirty capital of operating firms K^d as

$$Y \equiv \sum_i p_i \int F(z, k_d(a_i), k_g(a_i), a_i) \left[1 - X(z, a_i)\right] dG(z) \tag{3}$$

[8]In the quantitative model, utilizing dirty capital in production creates emissions. We follow the standard approach in climate-macro literature (e.g., Golosov et al, 2014) where emissions then create future climate damages. In the illustrative model, we assume a form of environmental externality similar to that used in our full model.

$$I \equiv \sum_i p_i \int [k_d(a_i) + k_g(a_i)]\, dG(z) \tag{4}$$

$$K^d \equiv \sum_i p_i \int k_d(a_i)\, [1 - X(z, a_i)]\, dG(z) \tag{5}$$

Finally, the household budget constraint is

$$C = \sum_i p_i \pi(a_i) = [1 - D(K^d)]Y - I. \tag{6}$$

A competitive equilibrium is defined an aggregate allocation $\{C, Y, I, K^d\}$, firm-level investment policies $\{k_g(a_i), k_d(a_i)\}_i$, and firm-level exit policies $\{X(a_i, z)\}_{i,z}$ such that firm-level investment and exit policies maximize their problem (2), the definitions of aggregates are satisfied (3), (4), (5), and the household budget constraint is satisfied (6).

We can now characterize the laissez-faire decentralized economy, which we follow the macro-climate literature and call "Business-As-Usual" (BAU). We work backward starting with the exit choice in the second subperiod and then investment choice in the first subperiod. When deciding whether to exit, the capital investment is sunk and operating generates positive profit ex-post for all firms, so no firms exit

$$\underbrace{[1 - D(K^d)]F(z, k_d(a_i), k_g(a_i), a_i)}_{\text{operating}} > \underbrace{0}_{\text{exiting}}$$

Thus, $X(z, a_i) = 0$ for all (z, a_i).[9] The investment is characterized by two first order conditions that equate the marginal cost of capital with the expected marginal product of capital:

$$\underbrace{1}_{\text{MC}} = \underbrace{[1 - D(K^d)]\mathbb{E}_z[F_2(z, k_d(a_i), k_g(a_i), a_i)]}_{\text{expected MPK}}$$
$$\underbrace{1}_{\text{MC}} = \underbrace{[1 - D(K^d)]\mathbb{E}_z[F_3(z, k_d(a_i), k_g(a_i), a_i)]}_{\text{expected MPK}}$$

where F_2 and F_3 denote the partial derivatives with respect to the second and third arguments of F, respectively. The ratio of the first order conditions implies each firm chooses a constant capital ratio depending on its carbon dependence

$$\frac{k_d(a_i)}{k_g(a_i)} = \left(\frac{a_i}{1 - a_i}\right)^{\frac{1}{1-\rho}} \equiv \eta(a_i)$$

[9]The quantitative model features fixed cost of production, so firms with exit in the Business-As-Usual case.

Thus, firms with higher carbon dependence invest relatively more of its total capital in dirty capital.

We compare the Business-As-Usual economy to the economy operated by a Social Planner who internalizes the climate externality but is constrained to operate the firm-level technologies. That is, the Planner runs each firm, making investment decisions in subperiod one after observing its a_i and exit decisions in subperiod two after observing its z. Thus, the Planner's problem is to choose firm-level policies and aggregate dirty capital to maximize household welfare subject to an adding-up constraint:

$$\max_{\{k_d(a_i),k_g(a_i)\}_i,\{X(a_i,z)\}_{i,z},K^d} U(C)$$

$$\text{s.t. } K^d = \sum_i p_i \int k_d(a_i)\,[1 - X(z, a_i)]\,dG(z)$$

$$\text{where } C = [1 - D(K^d)] \underbrace{\sum_i p_i \int F(z, k_d(a_i), k_g(a_i), a_i)][1 - X(z, a_i)]dG(z)}_{\equiv Y}$$

$$- \underbrace{\sum_i p_i \int [k_d(a_i) + k_g(a_i)]\,dG(z)}_{\equiv I}$$

We follow a four-step procedure to solve the Planner problem that mirrors how we approach solving the quantitative model in Section 6 and Appendix C:

1) Write the Planner Lagrangian.

2) Characterize Planner aggregate choices.

3) Characterize Planner firm-level policies.

4) Guess taxation scheme based on augmented firm problem and verify it induces both the Planner's policies and satisfy conditions of a competitive equilibrium.

Step 1: The Planner Lagrangian is consumer utility given climate and resource constraints and dirty capital adding-up constraint attached with multiplier λ_k:

$$\mathcal{L} = U\Bigg([1 - D(K^d)] \underbrace{\sum_i p_i \int F(z, k_d(a_i), k_g(a_i), a_i)][1 - X(z, a_i)]dG(z)}_{\equiv Y}$$

$$
\underbrace{- \sum_i p_i \int [k_d(a_i) + k_g(a_i)]\, dG(z)}_{\equiv I} \Bigg)
$$

$$
+ \lambda^k \left(K^d - \sum_i p_i \int k_d(a_i)\, [1 - X(z, a_i)]\, dG(z) \right).
$$

Step 2: The first order condition of the Planner Lagrangian with respect to aggregate utilized dirty capital equates the shadow cost of utilizing dirty capital to marginal climate damage

$$
\underbrace{\lambda^k}_{\text{shadow cost of utilizing dirty capital}} = \underbrace{U'(C) D'(K^d) Y}_{\text{marginal climate damage}} .
$$

Step 3: The Planner chooses exit policies for each firm (z, a_i) and investment policies for each firm type i. Starting with the exit policies, the Planner operates firm (z, a_i) iff

$$
\underbrace{[1 - D(K^d)] F(z, k_d(a_i), k_g(a_i), a_i)}_{\text{operating}} > \underbrace{\frac{\lambda^k}{U'(C)} k_d(a_i)}_{\text{shadow cost of firm's dirty capital}} > 0 \tag{7}
$$

Thus, unlike in the Business-As-Usual economy, the Planner exits firms. In particular, there is a threshold $\bar{z}(a_i)$ such that the Planner is indifferent between operating and exiting firm $(\bar{z}(a_i), a_i)$. As in the Business-As-Usual economy, first order conditions character Planner's investment policies. The planner chooses investment to equate marginal cost of capital with expected marginal product of capital (accounting for exit policies) minus the expected marginal shadow cost of dirty investment:

$$
\underbrace{1}_{\text{MC}} = \underbrace{[1 - D(K^d)] \mathbb{E}_z \left[F_2(z, k_d(a_i), k_g(a_i), a_i) [1 - X(z, a_i)] \right]}_{\text{expected MPK}} - \underbrace{\frac{\lambda^k}{U'(C)} \mathbb{E}_z [1 - X(z, a_i)]}_{\text{expected marginal shadow cost}} \tag{8}
$$

$$
\underbrace{1}_{\text{MC}} = \underbrace{[1 - D(K^d)] \mathbb{E}_z \left[F_3(z, k_d(a_i), k_g(a_i), a_i) [1 - X(z, a_i)] \right]}_{\text{expected MPK}} \tag{9}
$$

Step 4: Now introduce the optimization problem of a firm facing a tax τ on utilized dirty capital k_d. The firm solves:

$$
\max_{k_d, k_g} -k_d - k_g + \mathbb{E}_z \left[\max\{ [1 - D(K^d)] F(z, k_d, k_g, a_i) - \tau k_d, 0 \} \right]. \tag{10}
$$

Consider the firm-level policies associated with the solution to this optimization problem.

Firm (z, a_i) operates iff

$$\underbrace{[1 - D(K^d)]F(z, k_d, k_g, a_i) - \tau k_d}_{\text{operating}} > \underbrace{0}_{\text{exiting}} \tag{11}$$

Here, τk_d acts as an avoidable cost of production, so, for sufficiently low z, the LHS of (11) can be negative and some firm choose to exit. Investment by firms with carbon dependence a_i is characterized by the following first order conditions:

$$\underbrace{1}_{\text{MC}} = \underbrace{[1 - D(K^d)]\mathbb{E}_z \left[F_2(z, k_d(a_i), k_g(a_i), a_i)[1 - X(z, a_i)]\right]}_{\text{expected MPK}} - \tau \mathbb{E}_z[1 - X(z, a_i)] \tag{12}$$

$$\underbrace{1}_{\text{MC}} = \underbrace{[1 - D(K^d)]\mathbb{E}_z \left[F_3(z, k_d(a_i), k_g(a_i), a_i)[1 - X(z, a_i)]\right]}_{\text{expected MPK}} \tag{13}$$

Consider a particular tax $\tau = \frac{\lambda^k}{U'(C)}$. We will show that Planner's allocation holds as a competitive equilibrium under this tax scheme. When K^d is at the level implied by the Planner's allocation, it is immediate that the exit condition for the Planner (7) matches the exit condition for the firm (11) at the given tax rate, so the exit policies $X(z, a_i)$ coincide. Thus, the investment first order conditions of the Planner (8) and (9) also match the investment first order conditions of the (12) and (13), respectively, so the investment policies $k_d(a_i)$ and $k_g(a_i)$ coincide with that of the Planner. We assume the government redistributes the new tax income lump-sum to the households, so the government budget constraint continues to hold. Finally, definitions of aggregates (3), (4), (5) hold, as they held under the Planner's firm-level allocation, and the household budget constraint holds (6) due to the resource constraint holding under the Planner's allocation. Hence, the Planner's allocation is induced in a competitive equilibrium with the guessed taxation scheme.

4.1 Analyzing the Planner's Solution

The Planner's solution differs from the Business-As-Usual decentralized equilibrium in three key ways. First, there is a **composition effect** that changes the utilized production technologies of the economy. While no firms exit in the decentralized Business-As-Usual equilibrium, the planner internalizes an environmental cost of production for each firm which is avoidable through exit. Hence, the planner instructs the most unproductive firms in the economy to exit. The environmental cost of production for each firm differs according to the dirty capital level of the firms. Since high carbon dependence firms invest in more dirty capital, the planner makes more such firms exit than low carbon dependence firms.

$$X^{Planner}(z, a_i) = 1 \text{ for all } z < \bar{z}(a_i) \text{ where } \bar{z}(a_H) > \bar{z}(a_L) > 0$$

Second, there is a **substitution effect** that shifts the capital ratio of all firms toward green capital. Internalized environmental damage from dirty capital adds a wedge $M(a_i)$ into the optimal dirty capital ratio for each firm:

$$\frac{k_d^{Planner}(a_i)}{k_g^{Planner}(a_i)} = M(a_i)\eta_i^{BAU} < \eta_i^{BAU} = \frac{k_d^{BAU}(a_i)}{k_g^{BAU}(a_i)}$$

where $M(a_i) \equiv \left(\frac{1}{1+\frac{\lambda^k}{U'(C)}\mathbb{E}_z[1-X(z,a)]}\right)^{\frac{1}{1-\rho}} < 1$.

Third, there is a **scale effect** that changes total investment for all firms. The sign of this effect is ambiguous and depends on the carbon dependence a_i of the firm in question. This is due to two opposing forces. First, the internalized climate damage increases the cost of investment from the perspective of the Planner relative to the BAU firm. This force pushes investment down. Secondly, the Planner's improvement over the BAU equilibrium implies that the total dirty capital level in the economy strictly decreases, which strictly reduces the damage to production $D(K^d)$ from the climate feedback, which in-turn increases the marginal benefit from investment. Due to the presence of these opposing forces, the total effect is ambiguous and in general can even vary within an equilibrium between the L and H firms. In the limit as $a_H \to 1$, the former effect dominates for the H firm and the Planner reduces their scale, while in the limit as $a_L \to 0$ the latter effect dominates for the L-firm and the Planner increases their scale.

While comparing the BAU allocation to the Planner's is useful for understanding the mechanisms through which the Planner responds to climate externalities, we wish also to investigate the impact of firm heterogeneity on the Planner's response. To investigate this, we define a related economic environment which is the most comparable representative-firm economy to our baseline illustrative model. In particular, we assume that the entire production technology from the illustrative model is owned by a unit mass of identical representative firms. They choose an investment amount for each a_i and each type of capital, and can exit or produce. The BAU outcome of this economy is identical to the BAU outcome of the baseline illustrative model, but the Planner loses the ability to change the composition of the production technology used in the economy. Hence, the Planner's allocation differs in the representative-firm economy, and in particular the Planner is able to achieve less of a welfare gain in moving from the BAU to the optimal allocation, as the Planner has lost an instrument in the representative firm economy. In Table 3, we report welfare gains associated with moving from the BAU allocation to the Planner allocation for an example parameterization of the illustrative model.[10]

Table 3: Welfare Gains (CE) of Planner rel. to BAU

Illustrative Model	Het-firm	Rep-firm
CE: Planner rel to BAU (%)	1.954	1.875

Notes: This table reports the welfare gains (measured by consumption equivalence, reported as %) of moving to the Planner's allocation from the BAU allocation.

While the illustrative model is not quantitatively realistic, Table 3 verifies that the firm-level heterogeneity, through the composition effect, has a noticeably large impact on the response to climate feedback. We ultimately aim to measure the magnitude of the composition effect in the more realistic quantitative model outlined in Section 5, however this currently remains work-in-progress.

5 Heterogeneous-Firm Climate-Cycle Model

In this section, we extend the illustrative model from Section 4 into a Hopenhayn-esque heterogeneous-firm dynamic GE model where dirty capital creates carbon emissions. We use the model to quantify the evolution of the macroeconomy through the green transition, under both a decentralized BAU scenario and under the derived socially optimal allocation. We aim to retain comparability to the climate-macro literature by modeling the climate block using ingredients from Nordhaus (2007), Golosov et al. (2014), and Krusell and Smith (2022). Moreover, we add particular economic features common to these related papers: an infinite horizon and exogenous growth. We also add features common to the heterogeneous-firm literature - a log-AR(1) TFP process for firm-level productivity and adjustment costs to smooth the evolution of capital. We further add general equilibrium forces through a general-equilibrium wage and risk-free interest rate. Finally, we add endogenous firm entry and allow for both exogenous and endogenous firm exits.

Firms vary heterogeneously across four dimensions in our model. Firm i operating in date t has exogenously evolving TFP $z_{i,t}$, constant carbon dependence of production a_i, and endogenously evolving green capital $k_{g,i,t}$ and dirty capital $k_{d,i,t}$. Dirty capital is pollutive and

[10] $D(K^d) = 1 - \exp(-\gamma K^d)$, $\gamma = 2.5$, $U(C) = C$, $a_L = 0.3$, $a_H = 0.7$, $p_L = p_H = 0.5$, $z \sim U[0,1]$, $\alpha = 0.3$, $\rho = 0.5$.

creates carbon emissions $\xi_{i,t} = \gamma k_{d,i,t}$ where $\gamma \in \mathbb{R}^+$ is the constant of proportionality.[11] We use the vector $s_{i,t} = [z_{i,t}, a_i, k_{g,i,t}, k_{d,i,t}]'$ to summarize a given firm's idiosyncratic state. As in the illustrative model, firms produce with the Dixit-Stiglitz aggregate of green and dirty capital with substitution parameter ρ and weight on dirty capital a_i. Given a market-clearing wage w_t, firm i also employs labor $L_{i,t}$ and produces decreasing return-to-scale Cobb-Douglas production of labor and effective capital with coefficients ν and α, respectively. The current total stock of carbon emissions S_t in the atmosphere destroys fraction $D(S_t)$ of output of all firms; the climate damage and the carbon cycle are discussed further in subsection 5.1. Firm s generates cash flows $\pi_t(s)$ that is production net of climate damage, wages paid to workers, and an unavoidable fixed cost of c_f units of labor required to operate

$$\pi_t(s) = \max_L [1 - D(S_t)] \exp(z) A_t^{1-\alpha} [a k_d^\rho + (1-a) k_g^\rho]^{\frac{\alpha}{\rho}} L^\nu - w_t L - w_t c_f,$$

where $A_{t+1} = (1+\iota)A_t$ is an economy-wide component of TFP that grows at rate ι. Since $\pi_t(s)$ can become negative, firm endogenously exit in equilibrium.

We allow for both exogenous and endogenous exits after production for each firm. Firms are forced to exit with probability $\lambda \in [0,1)$ and with complementary probability $1-\lambda$ firm i is allowed to choose whether or not to endogenously exit. A nonexiting firm chooses investments in clean and dirty capital ($x_{g,i,t}$ and $x_{d,i,t}$, respectively) subject to adjustment costs $\psi(x_{j,i,t}, k_{j,i,t})$ for $j \in \{g, d\}$ and depreciation δ. An exiting firm eats its nondepreciated capital net of adjustment costs to drive its capital stocks to zero $\psi^X(k) \equiv \psi[-(1-\delta)k, k]$. Given the continuation value functions from exiting $V^X(s)$ and nonexiting $V^C(s)$, the ex-ante value function of the firm entering period t with state vector s is

$$V_t(s) = \pi_t(s) + \lambda V^X(k_g, k_d) + (1-\lambda) \max\{V^X(k_g, k_d), V_t^C(s)\}, \tag{14}$$

where the exiting value is

$$V^X(k_g, k_d) = (1-\delta)(k_d + k_g) - \psi^X(k_d) - \psi^X(k_g),$$

and nonexiting value is defined by the solution to the following optimization problem:

$$V_t^C(s) = \max_{x_d, x_g} -x_d - x_g - \psi(x_d, k_d) - \psi(x_g, k_g) + \frac{1}{R_t}\mathbb{E}[V_{t+1}(s')], \tag{15}$$

$$\text{s.t. } k_j' = (1-\delta)k_j + x_j, \quad \text{for } j \in \{d, g\} \tag{16}$$

[11] In Subsection 6.2, we relax the assumption that carbon emissions are deterministic and show that the Planner's solution is unchanged and remains implementable through simple taxes in a competitive equilibrium.

A large mass of potential firms can enter competitively with entry cost κ denominated in units of labor to begin a firm; and an endogenous measure B_t chooses to do so. An entrant then draws their permanent carbon dependence $a_i \sim Q_a$. Then they choose whether to exit before setting up their firm or invest $k_{g,i,t}$ and $k_{d,i,t}$ in clean and dirty capital. The firm then draws an initial $z_{i,t} \sim Q_z$ and becomes an operating firm with value as defined in equation (14). Mathematically, the entrant problem is free entry condition

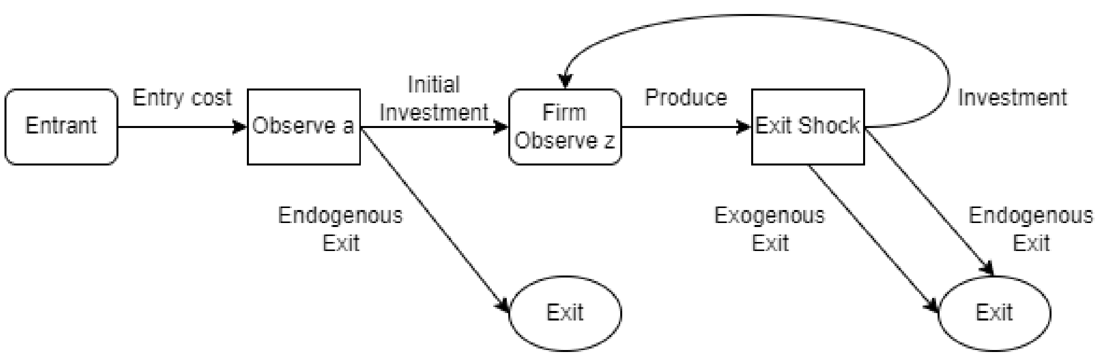
$$w_t \kappa \geq \mathbb{E}[V_t^E(a)],$$

where an entrant's value $V_t^E(a)$ after observing carbon dependence a is given by:

$$V_t^E(a) = \max\{0, \max_{k_d, k_g} -k_d - k_g + \mathbb{E}[V_t(s)]\}.$$

Figure 3 summarizes the timing associated with firms' decisions.

Figure 3: Timing of Firm Problem



Notes: This figure describes the timing of a model period for the detailed model as described in Section 5.

A unit mass of representative consumers inelastically supplies a unit of labor $L^S = 1$ and invests in a risk-free asset b_t in zero net supply with gross interest rate R_t. They receive all flow profits from firms Π_t. Their optimization problem at t is given by the following:

$$\max_{\{C_s, b_s\}_{s=t}^{\infty}} \sum_{s=t}^{\infty} \beta^s U(C_s)$$
$$\text{s.t. } C_s + \frac{1}{R_s} b_{s+1} + T_t = b_s + w_s L^S + \Pi_s \; \forall s \geq t.$$

A standard Euler equation prices the risk-free bond:

$$U'(C_t) = \beta R_t U'(C_{t+1}).$$

This interest rate is also used by firms discount dividend streams.

Finally, market clearing in the labor and risk-free asset markets imply:

$$\int L^d(s)\Phi_t(\mu_t)(s)ds = 1,$$
$$b_t = 0,$$

where $\Phi_t(\mu_t)$ is the distribution of productive firms as determined by the endogenous mass B_t and entrance policies $X_t^E(a), k_{g,t}^E(a), k_{d,t}^E(a)$ at date t given incumbent firm measure μ_t, an infinite-dimensional state variable for the economy.[12]

5.1 Carbon Cycle

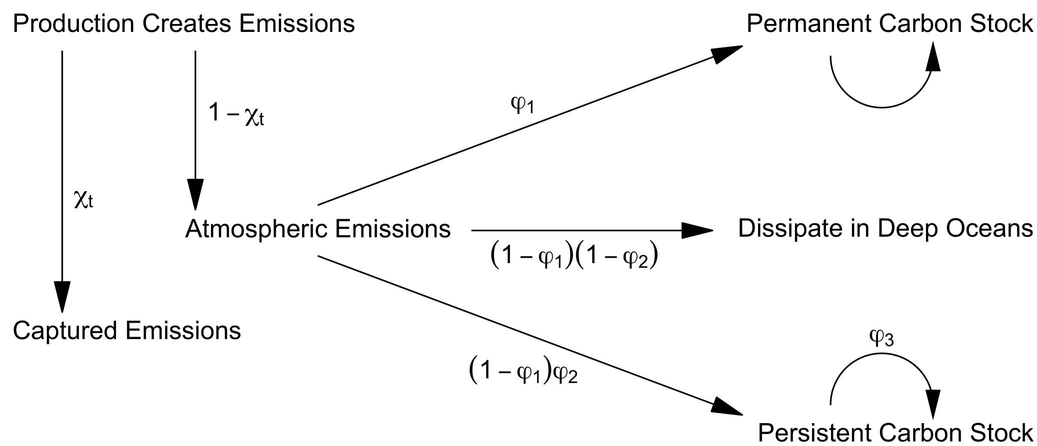
We broadly follow in the spirit of Nordhaus (2007), Golosov et al. (2014), and Krusell and Smith (2022) in defining and calibrating the carbon cycle in the model economy. Figure 4 graphically represents the carbon cycle within our economy. We track two carbon stocks as state variables for the economy: permanent emissions S_t^1 and persistent emissions S_t^2. The total emissions in the atmosphere are the sum of these stocks, $S_t = S_t^1 + S_t^2$. A fraction φ_1 of the flow of carbon emissions remain in the atmosphere permanently and do not ever dissipate adding to S_{t-1}^1, while a fraction $(1-\varphi_1)\varphi_2$ of emissions eventually dissipate away with decay rate φ_3 adding to S_{t-1}^2. Finally, the remaining fraction $(1-\varphi_1)(1-\varphi_2)$ of emissions are immediately captured by environmental reservoirs like the deep oceans and do not create damage. We further follow Krusell and Smith (2022) in modeling expected improvements in emissions reduction technologies through an exogenously evolving carbon-absorption parameter χ_t. The explicit law-of-motion for the carbon stocks are given by the following:

$$S_t^1 = S_{t-1}^1 + (1-\chi_t)\varphi_1\gamma K_t^d$$
$$S_t^2 = \varphi_3 S_{t-1}^2 + (1-\chi_t)(1-\varphi_1)\varphi_2\gamma K_t^d$$

The total carbon stock $S_t = S_t^1 + S_t^2$ is then assumed to have a damaging effect on the environment. Specifically, as mentioned earlier in Section 5, we assume that the carbon stock causes damages to output, $D(S_t^1 + S_t^2)Y_t$ where $D(S_t^1 + S_t^2)$ is an increasing function, locally convex over the relevant regions of the state-space.

[12] $\Phi_t(\mu)(s) \equiv \mu(s) + B_t Q_z(z) Q_a(a) \mathbb{1}_{k_d = k_{d,t}^E(a)} \mathbb{1}_{k_g = k_{g,t}^E(a)} \mathbb{1}_{X_t^E(a)=0}$.

Figure 4: The Carbon Cycle



Notes: This figure details the carbon cycle as described in Subsection 5.1. Atmospheric emissions enter two carbon stocks, one permanent and the other persistent but not permanent. Some atmospheric emissions dissipate immediately and do not enter either stock.

Definition 1. *A* ***Carbon-Cycle Competitive Equilibrium*** *is a set of allocations* $\{k^E_{d,t}(a), k^E_{g,t}(a), x_{d,t}(s), x_{g,t}(s), b_t, L^d_t(s), S^1_t, S^2_t, \mu_t\}_{t=0}^{\infty}$*, prices* $\{w_t, R_t\}_{t=0}^{\infty}$*, taxes* $\{T_t\}_{t=0}^{\infty}$*, continuation rules* $\{X_t(s), X^E_t(s)\}_{t=0}^{\infty}$*, and mass of entrants* $\{B_t\}_{t=0}^{\infty}$ *such that firm decisions solve their problems, household decisions solve their problem, the free entry condition holds, the law of motion of environmental carbon stocks hold, and the labor nad risk-free asset markets clear in each period.*

6 Socially Optimal Firm Policies

A natural point of comparison from the Business-As-Usual economy described in Section 5 is the economy as run optimally by a constrained Social Planner. The Social Planner is constrained to the same technologies as the agents in the decentralized economy including the firm-level adjustment cost functions, the exogenous supply of labor to the economy, the law-of-motion for the environment as described by Subsection 5.1, and resource feasibility constraints. The planner's objective is to maximize consumer utility subject to these constraints. The planner's problem can be defined using the following recursive formulation:

$$\begin{aligned}\mathcal{W}_t(\mu, S^1, S^2) = & \max_{x_d(\cdot), x_g(\cdot), X(\cdot), x^E_d(\cdot), x^E_g(\cdot), X^E(\cdot), L^d(\cdot), B} U(C_t) \\ & + \beta \mathcal{W}_{t+1}(T^*(\mu, x_d(\cdot), x_g(\cdot), X(\cdot), x^E_d(\cdot), x^E_g(\cdot), X^E(\cdot), B), S^{1\prime}, S^{2\prime})\end{aligned} \tag{17}$$

subject to

$$
\begin{aligned}
C_t &= (1 - D(S^{1\prime} + S^{2\prime}))Y_t - I_t - \Psi_t \\
1 &= \int (L^d(s) + c_f)\Phi(\mu, x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(s)ds + B\kappa \\
K^d &= \int k_d \Phi(\mu, x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(s)ds \\
S^{1\prime} &= S^1 + (1 - \chi_t)\varphi_1 \gamma K^d \\
S^{2\prime} &= \varphi_3 S^2 + (1 - \chi_t)(1 - \varphi_1)\varphi_2 \gamma K^d \\
B &\geq 0
\end{aligned}
$$

where

$$
\begin{aligned}
Y_t &= \int \exp(z) A_t^{1-\alpha} [a k_d^\rho + (1-a) k_g^\rho]^{\frac{\alpha}{\rho}} (L^d(s))^\mu \Phi(\mu, x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(s)ds \\
I_t &= \int (x_g(s) + x_d(s))(1 - \lambda)\Phi(\mu, x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(s)ds \\
&+ \int (-(1-\delta)k_d - (1-\delta)k_g)\lambda\Phi(\mu, x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(s)ds \\
&+ \int (x_g^E(a) + x_d^E(a)) \mathbb{1}_{X^E(a)=0} B Q_a(a) da \\
\Psi_t &= \int (\psi[x_g(s), k_g] + \psi[x_d(s), k_d])(1 - \lambda)\Phi(\mu, x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(s)ds \\
&+ \int [\psi^X(k_g) + \psi^X(k_d)]\lambda\Phi(\mu, x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(s)ds,
\end{aligned}
$$

and where $T^*(\cdot)$ is the measure operator from one period to the next and $\Phi(\cdot)$ is the measure of productive firms, both are formally defined in Appendix B.

As is notated by the dynamic program (17), the planner takes as given as state variables the infinite-dimensional distribution of incumbent firms μ, as well as the permanent and persistent carbon stocks from period $t-1$, S^1 and S^2 respectively. The planner chooses the firm-level policies for entrants and operating firms, including the continuation decisions, and also chooses the mass of entrant firms.

In Lagrangian form, this problem can be written:

$$
\begin{aligned}
\mathcal{L}_t = U(C_t) + \lambda_t^L \underbrace{(1 - \int (L^d(s) + c_f)\Phi(\mu, x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(s)ds - B\kappa)}_{\text{Labor Supply Constraint}} \\
+ \lambda_t^k \underbrace{(K^d - \int k_d \Phi(\mu, x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(s)ds)}_{\text{Definition of total dirty capital}} + \lambda_t^1 \underbrace{(S^{1\prime} - S^1 - (1 - \chi_t)\varphi_1 \gamma K^d)}_{\text{Law of motion of permanent stock}}
\end{aligned}
$$

$$+ \lambda_t^2 \underbrace{(S^{2\prime} - \varphi_3 S^2 - (1-\chi_t)(1-\varphi_1)\varphi_2\gamma K^d)}_{\text{Law of motion of persistent stock}} + \lambda_t^B \underbrace{(B - 0)}_{\text{Nonnegative entrant mass}}$$

$$+ \beta \mathcal{W}_{t+1}(T^*(\mu, x_d(\cdot), x_g(\cdot), X(\cdot), x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B), S^{1\prime}, S^{2\prime})$$

where C_t, T^*, Φ follow their previous definitions. Proposition 2 provides the relationship between the Lagrange multiplier on aggregate dirty capital λ_t^k to the Lagrange multipliers on the permanent and persistent carbon stocks.

Proposition 2. *The Lagrange multiplier on aggregate dirty capital, λ_t^k, can be written:*

$$\lambda_t^k = (1-\chi_t)\gamma\varphi_1\lambda_t^1 + (1-\chi_t)\gamma(1-\varphi_1)\varphi_2\lambda_t^2,$$

where the Lagrange multipliers on the permanent and persistent carbon stocks are given by:

$$\lambda_t^1 = \sum_{s=t}^{\infty} \beta^{s-t} U'(C_s) D'(S_s^1 + S_s^2) Y_s$$

$$\lambda_t^2 = \sum_{s=t}^{\infty} (\varphi_3 \beta)^{s-t} U'(C_s) D'(S_s^1 + S_s^2) Y_s.$$

Proof. See Appendix B.2. □

We then proceed as in Lucas and Moll (2014), Moll and Nuno (2018), and Ottonello and Winberry (2023). Specifically, we define the *augmented Bellman equation* and *augmented entry problem* in Definition 3.

Definition 3. *Let:*

$$\tau_t^k \equiv \frac{\lambda_t^k}{U'(C_t)},$$

$$\hat{w}_t \equiv \frac{\lambda_t^L}{U'(C_t)},$$

$$\frac{1}{\hat{R}_t} \equiv \beta \frac{U'(C_{t+1})}{U'(C_t)}.$$

Then, the augmented Bellman equation is defined as:

$$\hat{\omega}_t(s, \mu, S^1, S^2) = \max_{L, x_d, x_g, X} (1 - D(S^{1\prime} + S^{2\prime})) \exp(z) A_t^{1-\alpha} [a k_d^\rho + (1-a) k_g^\rho]^{\frac{\alpha}{\rho}} L^\nu \tag{18}$$

$$+ (1-\lambda)\left(-x_g - x_d - \psi(x_g, k_g) - \psi(x_d, k_d)\right)$$

$$+ \lambda \left((1-\delta)k_g + (1-\delta)k_d - \psi^X(k_g) - \psi^X(k_d)\right)$$

$$- \hat{w}_t(L + c_f) - \tau_t^k k_d$$
$$+ (1 - X)(1 - \lambda)\frac{1}{\hat{R}_t}\mathbb{E}[\hat{\omega}_{t+1}(s', \mu', S^{1\prime}, S^{2\prime})]$$

subject to:

$$x_i = -(1 - \delta)k_i \text{ if } X = 1$$
$$k_i' = (1 - \delta)k_i + x_i,$$

Then, we define the augmented entry problem:

$$\hat{\omega}_t^E(a, \mu, S^1, S^2) = \max_{\hat{X}^E(a), \hat{x}_d^E(a), \hat{x}_g^E(a)} (1 - \hat{X}^E(a))[-\hat{x}_d^E(a) - \hat{x}_g^E(a) + \mathbb{E}[\hat{\omega}_t(s, \mu, S^1, S^2)]] \quad (19)$$

subject to:

$$\hat{x}_i^E = -(1 - \delta)k_i \text{ if } \hat{X}(s) = 1 \; \forall i \in \{d, g\}$$

.

Next, we prove that the policies induced by the social planner problem match the policies induced by the augmented firm problem (Proposition 4) and that the augmented firm problem can be interpreted as the marginal social value of firms at s (Proposition 5).

Proposition 4. *The augmented Bellman equation and augmented entry problem defined in (18) and (19) induces firm-level and entrant-level policies that match those of the social planner, whose problem is defined by (17).*

Proof. See Appendix B. □

Proposition 5. *The augmented Bellman equation defined in (18) induces values $\hat{\omega}_t(s, \mu, S^1, S^2)$ equal to the Gateaux derivative of social welfare with respect to the measure of firms in a state s, denoted in units of the consumption good. That is,*

$$\hat{\omega}_t(s, \mu, S^1, S^2) = \frac{\partial \mathcal{W}_t(\mu, S^1, S^2)}{\partial \mu(s)} \frac{1}{U'(C_t)}$$

Proof. See Appendix B. □

Finally, we can characterize the first order condition with respect to the mass of entering firms using the augmented Bellman equation (Proposition 6).

Proposition 6. *The planner's first order condition with respect to B can be written as the following:*

$$\mathbb{E}[\hat{\omega}_t^E(a, \mu, S^1, S^2)] + \frac{\lambda_t^B}{U'(C_t)} = \hat{w}_t \kappa, \tag{20}$$

Proof. See Appendix B.1. □

The authors note that, while the objects $\mathcal{P}_t \equiv \{\tau_s^k, \hat{w}_s, \frac{1}{\hat{R}_s}\}_{t=s}^{\infty}$ are written notationally to be reminiscent of taxes and prices, in the planner's problem they are more concretely defined as internalized environmental damage, internalized disutility of emissions, the marginal value of labor in units of the consumption good given the exogenous supply, and the SDF of the household under the optimal policy.[13]

6.1 Decentralization through Tax on Dirty Capital

We consider the addition of a distortionary tax $\{\tau_s^k\}_{s=t}^{\infty}$, to our competitive equilibrium. We show that the definition of an equilibrium is satisfied under the set of taxes $\{\tau_s^k\}_{s=t}^{\infty}$ and prices $\{\hat{w}_s, \hat{R}_s\}_{s=t}^{\infty}$ given by the planner's problem, and that the equilibrium policies and mass of entrants match those of the social planner (Proposition 7).

Proposition 7. *Under the set of taxes $\{\tau_s^k\}_{s=t}^{\infty}$ matching those defined in Definition 3, the set of wages and risk free interest rates given by $\{\hat{w}_t, \hat{R}_t\}_{s=t}^{\infty}$, also as defined in Definition 3, induces a competitive equilibrium. Moreover, the firm-level policies and mass of entrants within the induced competitive equilibrium match that of the social planner.*

Proof. See Appendix C. □

The intuition is simple: When the distortionary taxes are levied on the firms in the economy, their dynamic program at the appropriate prices is identical to the dynamic program that characterizes the planner's solution, and hence the solutions are identical. The firm-level policies and values match exactly those of the planner. Then, equation (20) ensures the competitive entry condition is satisfied, the labor market clearing is ensured by the labor resource constraint of the planner under the planner's chosen mass of entrants B_t, and Walras' law ensures market clearing in the risk-free asset market. Hence, a competitive equilibrium can be constructed from the appropriate objects within the planner's problem, contained within $\mathcal{P}_t$.

[13] To foreshadow our results, in Subsection 6.1 we formalize this relationship by showing that $\mathcal{P}_t$, interpreted as taxes and prices, implements the Planner's solution in a competitive equilibrium.

6.2 Decentralization through Tax on Noisy Emissions

Here, we consider implementation of the planner's solution under a model extension. In this subsection, we consider a case where the level of dirty capital each firm owns $k_{d,i,t}$ is not observed and hence is not taxable. However, the taxation authority is allowed to observe the level of emissions $\xi_{i,t}$. Of course, if emissions are still deterministic then the taxation authority trivially observes dirty capital by inverting the linear relationship between the observed emissions and dirty capital level. To make the case more interesting, we hence further assume that emissions are no longer deterministic for a given firm, but instead allow for a mean-preserving spread, making emissions a noisy signal of unobserved dirty capital:

$$\xi_{j,t} = \eta_{j,t}\gamma k_{d,j,t};\ \eta_{j,t} \sim_{iid} F; \mathbb{E}[\eta_{j,t}] = 1\ \eta_{j,t} \geq 0\ a.s. \tag{21}$$

We prove that the constrained social planner's solution does not change (Proposition 8) and that it can decentralized with a tax on emissions instead of a tax on dirty capital (Proposition 9).

Proposition 8. *The constrained social planner's solution is unchanged under the noisy emissions as given by equation 21.*

Proof. See Appendix D. □

Proposition 9. *The planner's solution remains implementable, with a tax on emissions replacing a tax on dirty capital.*

Proof. See Appendix D. □

The intuition behind these proofs is straightforward. We first show that the Planner's solution is unchanged. The uncertainty over firm-level emissions washes out in the Planner's problem when integrating across the firm measure during aggregation. Next, we show that the Planner's solution remains implementable through a linear tax on (noisy) emissions. The intuition here is that the risk-neutral firms integrate out the mean-preserving noise in their future profits coming from the realization of their emissions (since the tax is linear) when making their investment decisions, and after realization of the emissions shock the carbon tax is sunk from the perspective of the firm and hence does not impact the chosen policies. Hence the policies of the firm remain identical to those of the Planner.

7 Transition Paths

7.1 Calibration

Table 4 presents the calibrated parameters used in computing the transition paths and determining the optimal taxes within our framework. The selected parameter values are based on standard functional forms and commonly used parameters from the existing literature. These choices ensure consistency and comparability with prior studies in the literature. We calibrate the model to an annual frequency with a discount factor of 0.971 and a growth rate of 1 percent following Krusell and Smith (2022). We initiate transition paths in 1990 where the assume the economy is on its balance growth path with zero dirty capital taxes.

In the firm block of our model, we take standard parameters for the capital and labor share as well as capital depreciation. Idiosyncratic productivity follows a log AR(1) process. We use estimates from Khan and Thomas (2013) for the mean and volatility at 0.65 and 0.15, respectively. We assume quadratic adjustment costs

$$\psi(x,k) = \hat{\gamma}\left(\frac{x}{k}\right)^2 k$$

and set curvature of the capital adjustment costs $\hat{\gamma} = 0.297$ following Corbae and D'Erasmo (2021). For the exogenous exit rate, we use 0.08 from Ottonello and Winberry (2023). At this rate, firms are expected to receive an exit shock every 12.5 years. The value for the fixed cost of production c_f and the substitutability of capital ρ are somewhat arbitrary and require further work to discipline. We have two types of firms L and H with carbon dependence $a_L = 0.3$ and $a_H = 0.7$ with equal mass. In future versions of the paper, we aim to discipline the distribution of carbon dependence within the model to our empirical analysis from Section 3. Finally, we start each transition path on the balanced growth path with neither taxes nor damages from the environment at a unit wage, then back out the implied entry cost from the entry condition normalizing the wage in the BAU case to 1.[14]

Regarding the climate block of the model, we follow Golosov. et al (2014) in using the following damage function:

$$D(S) = 1 - \exp(-\Delta(S - \bar{S}))$$

where $\bar{S}$ is the pre-industrial stock of carbon emissions. Furthermore, we follow Krusell

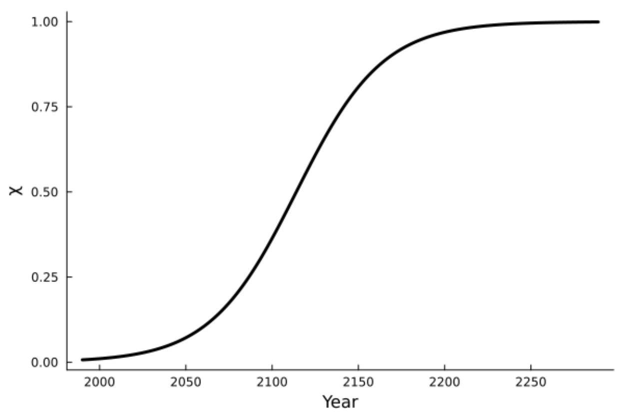
[14] This class of models is well-known to feature a one-to-one relationship between the equilibrium wage and entry cost κ. Hence different calibrations of the entry cost κ result in different wages, all else equal. It is more convenient in the computation of the model to calibrate the wage than the entry cost itself, see for example Corbae and D'Erasmo (2021).

and Smith (2022) in how we model the rest of the carbon cycle. In particular, the stock of permanent and transitory carbon emissions in 1999, how emissions toward permanent and transitory stocks of carbon emissions, and the persistence of the transitory component of carbon emissions. We assume the same functional form for expected exogenous improvements in emission capture technologies as Krusell and Smith (2022):

$$\chi_t = \begin{cases} 1 - (1 + \exp(\log(\frac{0.01}{0.99}))^{\frac{t - t_{\chi=0.5}}{t_{\chi=0.01} - t_{\chi=0.5}}})^{-1}, & \text{if } t < t_{\chi=1} \\ 1, & \text{if } t \geq t_{\chi=1} \end{cases} \tag{22}$$

This functional forms results in one-percent of emissions are captured at $t_{\chi=0.01}$, one-half of emissions are captured at $t_{\chi=0.5}$, and all emissions after $t_{\chi=1}$. Finally, we use amount of emissions in 1999 reported by Krusell and Smith (2022) to determine the linear production parameter of utilized dirty capital to emissions.

Figure 5: Captured Fraction of Emissions



Notes: Figure 5 shows the fraction of emissions that are captured before they enter the atmosphere and the carbon cycle. We follow Krusell and Smith (2022) in assuming that one percent of emissions are captured before entering the atmosphere by 2000, half of the emissions are captured by 2115, and all emissions are captured by 2291. This functional form is given by equation (22).

Table 4: Parameters

(a) Preferences

Parameter	Value	Source	Description
β	0.971	$R^{SS} = 1.03$	Time preference
ι	0.01	Krusell and Smith (2022)	Economy growth rate

(b) Production

Parameter	Value	Source	Description
a_L	0.3	-	Low carbon dependence
a_H	0.7	-	High carbon dependence
$Q_a(a_i)$	0.5	-	Entrant probability of drawing a_i
α	0.3	Standard parameter	Capital share
ν	0.65	Standard parameter	Labor share
$\hat{\gamma}$	0.297	Corbae and D'Erasmo (2021)	Capital adjustment cost
λ	0.08	Ottonello and Winberry (2023)	Exogenous Exit Rate
c_f	0.0001	-	Fixed cost of production
δ	0.12	Standard parameter	Depreciation rate
ρ	0.5	-	Substitutability between capital types
ρ_z	0.659	Khan and Thomas (2013)	Idiosyncratic productivity persistence
σ_z	0.118	Khan and Thomas (2013)	Idiosyncratic productivity volatility

(c) Climate

Parameter	Value	Source	Description
Δ	0.000053	Golosov et al. (2014)	Damage function parameter
φ_1	0.2	Krusell and Smith (2022)	Fraction of permanent emissions
φ_2	0.398	Krusell and Smith (2022)	Fraction of dissipated persistent emissions
φ_3	0.998	Krusell and Smith (2022)	Persistence of persistent emission stock
$\bar{S}$	581	Golosov et al. (2014)	Pre-industrial level of emissions
$S_{9,1}$	684	Krusell and Smith (2022)	Stock of persistent emission in 1999
$S_{9,2}$	118	Krusell and Smith (2022)	Stock of permanent emission in 1999
E_9	8.741	Krusell and Smith (2022)	Emissions in 1999
$t_{\chi=0.01}$	10	Krusell and Smith (2022)	Years until 1% of emissions are captured
$t_{\chi=0.5}$	125	Krusell and Smith (2022)	Years until half emissions are captured
$t_{\chi=1}$	301	Krusell and Smith (2022)	Years until all emissions are captured

Notes: This table outlines the parameterization for the quantitative model, as described in Subsection 7.1 and Subsection 7.2.

7.2 Results

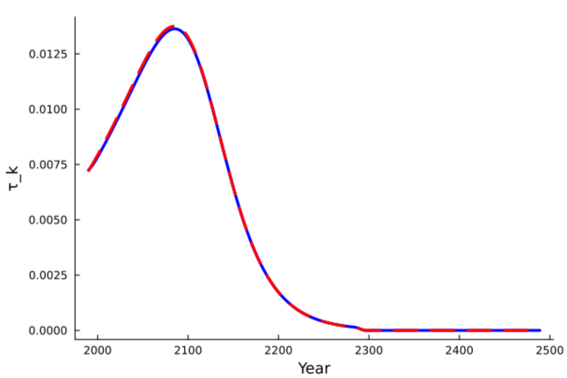
We initiate transition paths in 1990, assuming that the economy is on its balanced growth path with neither dirty capital nor output taxes. Throughout the figures in this section, the blue lines represent periods when taxes are set to zero, while the orange lines represent the optimal tax levels.

In Figure 6, we present the planner multiplier τ derived from our calibrated heterogeneous agent carbon-cycle model evaluated at both the Business-As-Usual equilibrium allocation, defined as no taxes on firms, and the decentralized Planner equilibrium allocation, defined as taxes on dirty capital taxes equal to these multipliers. The left panel illustrates the optimal tax on dirty capital, whereas the right panel displays environmental damage. Taxes on dirty capital are forward-looking and serves two purposes. Firstly, it directly incentivizes all firms to shift their capital allocation toward clean capital. Secondly, it encourages firms with higher carbon intensities (high values of a) to endogenously exit. To prevent the accumulation of permanent emissions and mitigate the long-term damage to output, dirty capital taxes are high during the early stages of the transition path. As the technology to capture emissions improves and all emissions are captured $\chi_t \to 1$, dirty capital taxes no longer prevent future climate damage, so dirty capital taxes drop to zero $\tau \to 0$.

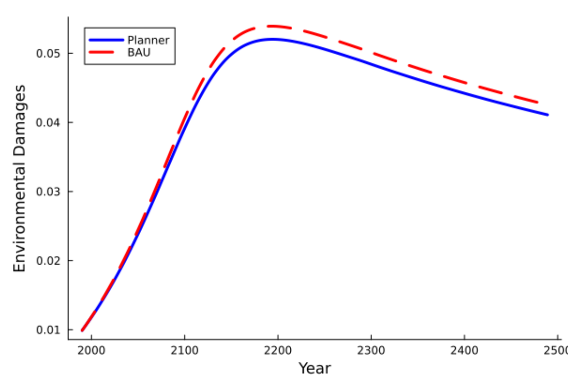
Environmental damages begin at relatively lower levels and gradually increase as carbon emissions accumulate in the atmosphere. Once all new carbon emissions are captured (χ_t reaches unity), climate damages experience a gradual decline as the persistent component of the stock of carbon emissions dissipates. Ultimately, climate damages stabilize at a level solely caused by the permanent stock of carbon emissions. The optimal taxes result in lower climate damage throughout the entire transition path.

Figure 6: Social Cost of Dirty Capital and Environmental Damage

(a) Social Cost of Dirty Capital



(b) Environmental Damage

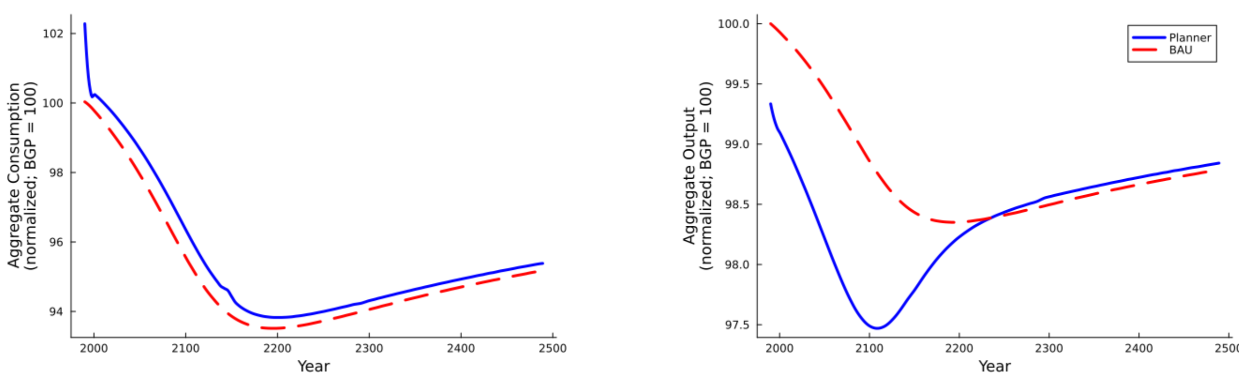


Notes: Figure 6 describes the planner multipliers computed at two computed equilibrium allocations. The first equilibrium is the Business-As-Usual equilibrium, defined by zero taxes on firms. The second equilibrium is the decentralized planner's solution, which using taxes equal to the planner multipliers. The multipliers are as described in Definition 3.

Figure 7 presents detrended aggregate consumption and output under both zero taxes and optimal taxes. In the absence of taxes, aggregate consumption and output experiences a decline due to climate damages from accumulated carbon emissions. Optimal dirty capital taxes are positive in the initial period. This sudden increase in taxes is a negative MIT shock to the point of view of firms and leads to a decline in firm value, which prompts firms to disinvest and exit. As a consequence of this disinvestment, firms pay larger dividends boosting aggregate consumption. Turning to aggregate output, this disinvestment leads to a drop in aggregate output as the measure of operating firms is lower. Aggregate consumption with optimal taxes is higher throughout the transition path despite lower aggregate output due to lower climate damages. Once all emissions are capture ($\chi_t = 1$), aggregate output under optimal taxes starts to exceed aggregate output under zero taxes because climate damages with optimal taxes are lower, so firms are in a sense more productive.

Figure 7: Aggregate Quantities

(a) Consumption (b) Output Net of Environmental Damage



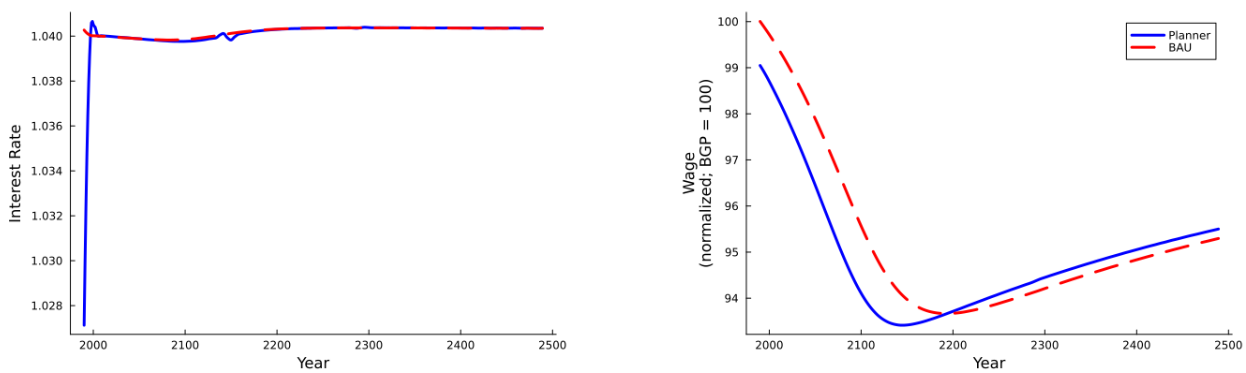
Notes: Figure 7 describes the evolution of macroeconomic aggregates in the two computed equilibria. The first equilibrium is the Business-As-Usual equilibrium, defined by zero taxes on firms. The second equilibrium is the decentralized planner's solution.

In Figure 8, we illustrate the endogenous movements of interest rates and wages over the transition path. Interest rates are determined by the consumption Euler equation of the representative household. With zero taxes, as climate damages reduce aggregate output, interest rates experience a slight decline due to the corresponding decrease in aggregate consumption. With the introduction of optimal taxes, there is an initial jump in aggregate consumption as firms respond to the higher tax burden by disinvesting causing the interest rate to drop. As dirty capital taxes gradually decrease and climate damage converge to a constant, the interest rate returns to its balanced growth path level.

Wages are determined by the entrant problem, so wages without taxes fall as climate damages rise to satisfy the free entry condition. With lower wages, both the entry cost, which is denominated in units of labor, decreases and the value of being a firm increases. The initial jump in optimal taxes and subsequent drop in firm value exert downward pressure on wages through the entry condition. While dirty capital taxes are positive, the wage with optimal dirty capital taxes is lower than without taxes from the lower firm value. But once dirty capital taxes are zero, firms are more productive in the sense that less of their output is destroyed, so wages in the economy with optimal taxes exceed wages in the economy without taxes.

Figure 8: Prices

(a) Interest Rates (b) Wages



Notes: Figure 8 describes the equilibrium prices in the two computed equilibria. The first equilibrium is the Business-As-Usual equilibrium, defined by zero taxes on firms. The second equilibrium is the decentralized planner's solution. The equilibrium interest rate falls immediately under the optimal allocation as the planner optimally invests less in dirty capital and less in capital overall in response to the internalized environmental externalities. As the emissions capturing technology improves, the planner increases investment and the interest rate rises. The marginal product of labor falls in the planner's equilibrium due to the reduction in investment in capital. Hence, the wage falls. Wages eventually increase as climate damages fall.

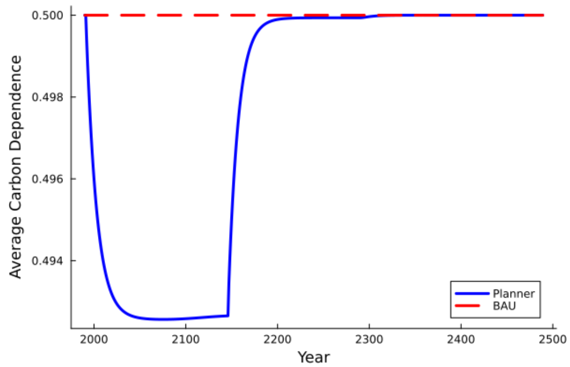
In Figure 9, we examine the composition, substitution, and scale effects in the Planner allocation. We can examine the composition effect of dirty capital taxes on the measure of firms, as demonstrated by the average carbon dependence of firms in the economy. Without taxes, the average carbon dependence remains unchanged. However, with the implementation of carbon taxes, we observe a higher rate of endogenous exits among firms with highly carbon-intensive production processes. As a result, the average carbon dependence decreases, indicating a shift towards cleaner production methods. As emission capture technology progresses and eventually captures all emissions, leading to the phase-out of dirty capital taxes, the average carbon dependence returns to its initial level.

We can examine the substitution effect through the ratio of aggregate dirty capital to aggregate clean capital. In the Business-As-Usual economy, this ratio stays at unity. In the Planner's allocation, the capital ratio drops, as the Planner pushes firms to use more clean capital relative to dirty capital. Once $\chi_t \to 1$, the capital ratio returns to unity.

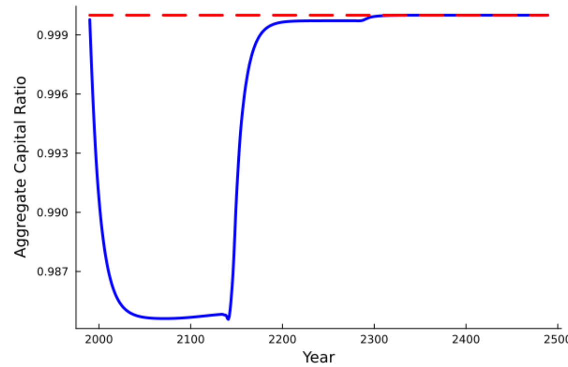
We can examine the scale effect through the aggregate total capital. As discussed with the illustrative model, the Planner's choice of total capital at the firm-level is ambiguous relative to Business-As-Usual. Holding climate damages fixed, the Planner lowers total investment because the Planner internalizes the capital choices on climate damage (analogous to a "partial equilibrium" change). However, the Planner also lowers climate damage, so firms are effectively more productive, so there's a force increasing the Planner's total capital choice (analogous to a "general equilibrium" change). Total capital in the quantitative scenarios shows that the partial equilibrium effect dominates before $\chi_t \to 1$ and the Planner lowers total capital relative to Business-As-Usual. The general equilibrium effect dominates after $\chi_t \to 1$ and the Planner raises total capital relative to Business-As-Usual.

Figure 9: Composition, Substitution, and Scale Effects

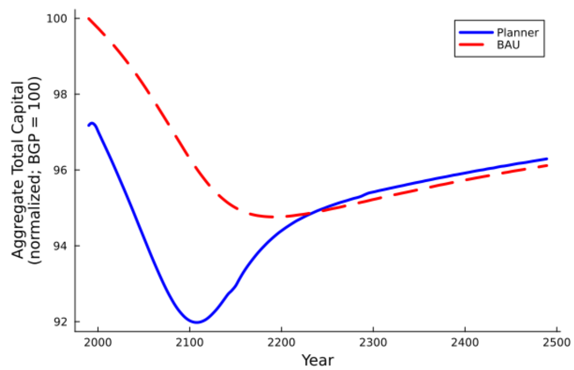
(a) Composition Effect



(b) Substitution Effect



(c) Scale Effect

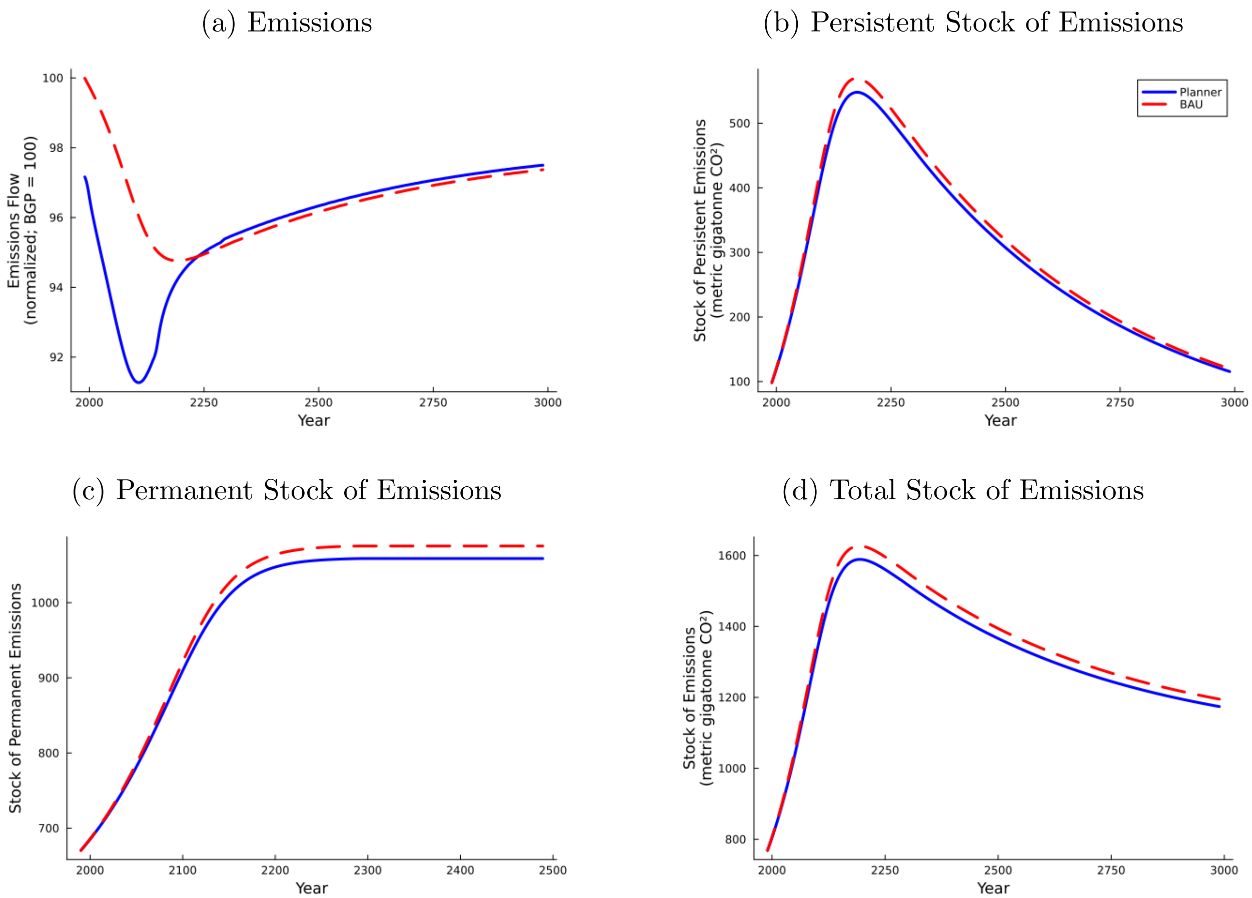


Notes: Figure 9 describes the composition, substitution, and scale effects by comparing the average a level, the aggregate capital ratio, and aggregate total capital in the two computed equilibria. The first equilibrium is the Business-As-Usual equilibrium, defined by zero taxes on firms. The second equilibrium is the decentralized Planner's solution. The average a in the economy falls under the planners' allocation because the planner exits more dirty firms than clean firms, whereas the firms are symmetric in the Business-As-Usual economy. The aggregate capital ratio in the Planner's allocation drops (more green capital relative to dirty capital), while it stays at one in the Business-As-Usual economy. Aggregate capital drops, initially more for the planner relative to Business-As-Usual and then Planner exceeds Business-As-Usual.

Figure 10 shows the climate outcomes under both transition paths. When zero taxes are implemented, aggregate dirty capital remains unchanged, resulting in a consistent flow of emissions. These emissions contribute to both the permanent and persistent stock of emissions, which in turn cause environmental damage to output.

In contrast, with optimal taxes, the level of dirty capital decreases, leading to a reduction in the flow of emissions. This decline in emissions has two significant effects: it lowers the level of the permanent stock of emissions in the atmosphere and reduces the peak of the persistent stock of emissions. Consequently, environmental damages are mitigated both in the short run, when the persistent stock of emissions causes damages, and in the long run, after the persistent stock of emissions has fully dissipated.

Figure 10: Climate



Notes: Figure 10 describes the climate variables in the two computed equilibria. The first equilibrium is the Business-As-Usual equilibrium, defined by zero taxes on firms. The second equilibrium is the decentralized planner's solution. Emissions are substantially lower in the short run in the planner's allocation than in the Business-As-Usual allocation. This results in a lower permanent and persistent stock of emissions over time, and a substantially lower environmental damage under the planner's allocation.

8 Conclusion

In conclusion, this paper introduces a general equilibrium framework to examine the implications of persistent heterogeneity in the firm-level carbon dependence of production. This model features the externality of environmental costs and the lack of internalization in the Business-as-Usual case. We tackle this issue by formulating the problem from the perspective of a social planner who must work within the technological constraints defined at the firm level.

Novel to our paper is that the optimal allocation derived from the social planner's perspective exhibits a composition effect the distribution of production technologies utilized in the economy. Specifically, the Social Planner exits more polluting firms than in the BAU equilibrium, and these exits contribute to a reduction in aggregate emissions.

Furthermore, we have decentralized the social planner's allocation by introducing a time-varying Pigouvian tax wedge that is applied uniformly across firms. The robustness of this decentralization finding is explored through extensions of the baseline model. This analysis enhances our understanding of conditions under which the first best can be achieved in our and similar frameworks with hetergeneous agents.

Empirically, we have provided suggestive evidence supporting our model's assumption of persistent heterogeneity in carbon dependence at the firm level. Our findings demonstrate that the variation in emissions intensity observed between U.S. public firms is predominantly explained by across-firm variation rather than within-firm variation.

Overall, this research contributes to the existing literature by offering a comprehensive framework to analyze the interplay between firm-level carbon dependence, environmental costs, and optimal policy design.

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A Data Appendix

Table A1: Scope 1 Sample Selection

Variable	Mean (all)	SD (all)	N (all)	Mean (Scope 1)	SD (Scope 1)	N (Scope 1)	T-Statistic
EBITDA (real)	845.36	3477.96	19279	4722.73	8229.06	1965	-20.70
EBITDA/PPEGT	-1.65	135.51	19279	0.39	0.82	1965	-2.10
Employment	15.27	60.00	19279	61.93	137.52	1965	-14.90
Employment/PPEGT (real)	0.04	0.46	19279	0.01	0.03	1965	8.64
PPEGT (real)	4776.78	24032.17	19279	25724.36	55777.58	1965	-16.49

Table A2: Scope 2 Sample Selection

Variable	Mean (all)	SD (all)	N (all)	Mean (Scope 2)	SD (Scope 2)	N (Scope 2)	T-Statistic
EBITDA (real)	845.36	3477.96	19279	4891.34	8481.78	1830	-20.25
EBITDA/PPEGT	-1.65	135.51	19279	0.42	0.88	1830	-2.12
Employment	15.27	60.00	19279	64.35	141.67	1830	-14.70
Employment/PPEGT (real)	0.04	0.46	19279	0.01	0.03	1830	8.41
PPEGT (real)	4776.78	24032.17	19279	25605.26	55276.59	1830	-15.98

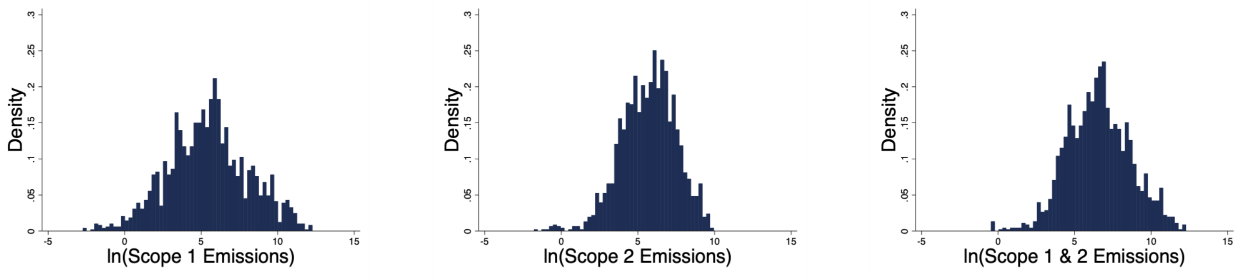
Table A3: Scope 1 + Scope 2 Sample Selection

Variable	Mean (all)	SD (all)	N (all)	Mean (Scope 1+2)	SD (Scope 1+2)	N (Scope 1+2)	T-Statistic
EBITDA (real)	845.36	3477.96	19279	4863.87	8426.99	1810	-20.13
EBITDA/PPEGT	-1.65	135.51	19279	0.40	0.84	1810	-2.11
Employment	15.27	60.00	19279	64.48	142.25	1810	-14.60
Employment/PPEGT (real)	0.04	0.46	19279	0.01	0.03	1810	8.51
PPEGT (real)	4776.78	24032.17	19279	25631.08	55421.38	1810	-15.87

Table A4: Sample Selection by Sector

SIC Division	Percent (all)	Percent (Scope 1)	Percent (Scope 2)	Percent (Scope 1+2)
A: Agricultural, Forestry, and Fishing	0.35	0.00	0.00	0.00
B: Mining	7.38	11.09	9.23	9.34
C: Construction	1.30	0.25	0.55	0.28
D: Manufacturing	50.19	59.08	59.07	59.72
E: Transportion, Communications, Electric, and Gas	7.75	11.20	11.48	11.33
F: Wholesale Trade	3.44	2.09	2.19	2.21
G: Retail Trade	6.44	4.83	4.92	4.97
I: Services	23.13	11.45	12.57	12.15

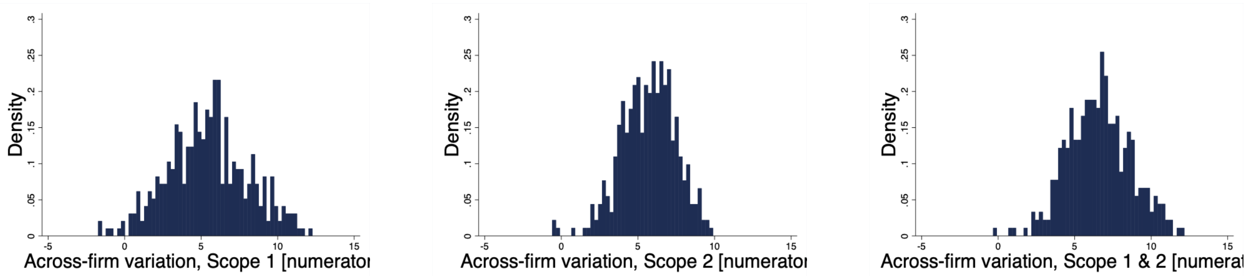
Figure 11: Distribution of Emissions



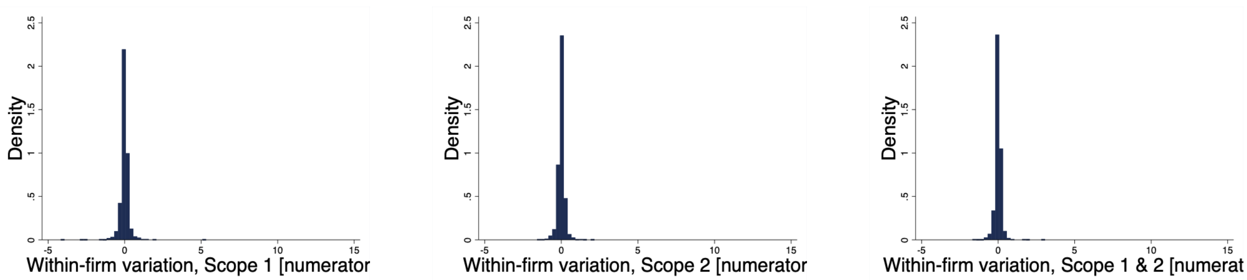
(a) Scope 1 Emissions (b) Scope 2 Emissions (c) Scope 1 & 2 Emissions

Notes: This figure shows the distribution of the log of carbon emissions across scopes.

Figure 12: Across-Firm Variation vs Within-Firm Variation, of log(emissions)



(a) Scope 1 Emissions (b) Scope 2 Emissions (c) Scope 1 & 2 Emissions



(d) Scope 1 Emissions (e) Scope 2 Emissions (f) Scope 1 & 2 Emissions

Notes: This figure shows the distribution of α_i OLS estimates (top row) and residuals (bottom row) from the regressions ran with log(emissions) as the dependent variable.

Table A5: R^2 Values from Regression 1 with log(emissions) as dependent variable.

Scope	R^2	Adjusted R^2
Scope 1	0.986	0.982
Scope 2	0.984	0.980
Scope 1 & 2	0.989	0.986

Notes: This table reports the R^2 values from OLS estimates of Regression 1, with log(emissions) as the dependent variable. R^2 values remain very high.

Table A6: SIC Division Fixed Effects

Scope	R^2	Adjusted R^2
Scope 1	0.160	0.157
Scope 2	0.137	0.134
Scope 1 & 2	0.070	0.067

Notes: This table reports unadjusted and adjusted R^2 values with seven SIC division fixed effects instead of firm-level fixed effects.

Table A7: SIC 2-Digit Fixed Effects

Scope	R^2	Adjusted R^2
Scope 1	0.657	0.649
Scope 2	0.383	0.368
Scope 1 & 2	0.537	0.526

Notes: This table reports unadjusted and adjusted R^2 values with 45 2-digit SIC code fixed effects instead of firm-level fixed effects.

B Characterization of Planner's Solution

In this appendix, we characterize the Planner's solution.[15] We start by characterizing the full Planner's problem as well as its Lagrangian. Then we derive the marginal social value

[15]We use a similar approach to Ottonello and Winberry (2023). The major departure is firms in our framework exit both endogenously and exogenously while all exits in Ottonello and Winberry (2023) are exogenous.

of firms at particular location in the state-space. We derive optimally conditions from the full Planner's problem including first order conditions for continuous choices, like labor and investment, and inequalities (or equivalently thresholds) for discrete choices, like exit. Then we show that marginal social value is equal to the value function induced by an augmented firm Bellman equation written similar to the marginal social value.

B.0.1 Planner's Problem

Let (S^1, S^2) be last period's atmospheric carbon stock and μ the measure of incumbent firms continuing from the prior period. The planner's objective is

$$\begin{aligned}\mathcal{W}_t(\mu, S^1, S^2) = &\max_{x_d(\cdot), x_g(\cdot), X(\cdot), x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), L^d(\cdot), B} U(C_t) \\ &+ \beta \mathcal{W}_{t+1}(T^*(\mu, x_d(\cdot), x_g(\cdot), X(\cdot), x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B), S^{1\prime}, S^{2\prime})\end{aligned}$$

where $(S^{1\prime}, S^{2\prime})$ follow environmental laws of motion

$$\begin{aligned}S^{1\prime} &= S^1 + (1-\chi_t)\varphi_1 \gamma K^d \\ S^{2\prime} &= \varphi_3 S^2 + (1-\chi_t)(1-\varphi_1)\varphi_2 \gamma K^d,\end{aligned}$$

K^d is the aggregate capital of operating firms

$$K^d = \int k_d \Phi(\mu, x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(s) ds,$$

Φ is the measure of operating firms

$$\begin{aligned}\Phi(\mu, x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(z', k_d', k_g', a') = &\underbrace{\mu(z', k_d', k_g', a')}_{\text{incumbents}} \\ &+ \underbrace{\mathbb{1}_{k_d'=x_d^E(a')} \mathbb{1}_{k_g'=x_g^E(a')} \mathbb{1}_{X^E(a)=0} Q_z(z') Q_a(a') B}_{\text{entrants}}, \quad (23)\end{aligned}$$

and T^* operator defines how the measure of firms evolves

$$\begin{aligned}T^*(\mu, x_d(\cdot), x_g(\cdot), &X(\cdot), x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(z', k_d', k_g', a') \\ &= \int \mathbb{1}_{k_d'=(1-\delta)k_d(s)+x_d(s)} \mathbb{1}_{k_g'=(1-\delta)k_g(s)+x_g(s)} \mathbb{1}_{z'=\rho z+\epsilon} \mathbb{1}_{a'=a} \mathbb{1}_{X(s)=0} \\ &\quad \times p(\epsilon)(1-\lambda)\Phi(\mu, x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(s) ds \quad (24)\end{aligned}$$

where firms that the Planner exits disinvest capital $x_i(s) = (1-\delta)k_i$.

The planner's problem is subject to three constraints. First, the planner's problem is subject to the resource constraint of the economy:

$$C_t = (1 - D(S^{1\prime} + S^{2\prime}))Y_t - I_t - \Psi_t$$

where aggregate output (net of environmental damage) Y_t, aggregate investment I_t, and aggregate capital adjustment costs Ψ_t are

$$
\begin{aligned}
Y_t = & \int \exp(z) A_t^{1-\alpha}[a k_d^\rho + (1-a)k_g^\rho]^{\frac{\alpha}{\rho}} (L^d(s))^\nu \Phi(\mu, x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(s) ds \\
I_t = & \underbrace{\int (x_g(z, k_g, k_d, a) + x_d(z, k_g, k_d, a))(1-\lambda)\Phi(\mu, x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(s) ds}_{\text{(dis)investment for operating firms}} \\
& + \underbrace{\int (-(1-\delta)k_d - (1-\delta)k_g)\lambda\Phi(\mu, x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(s) ds}_{\text{disinvestment from exogenously exiting firms}} \\
& + \underbrace{\int (x_g^E(a) + x_d^E(a)) \mathbb{1}_{X^E(a)=0} B Q_a(a) da}_{\text{investment for entrants}} \\
\Psi_t = & \underbrace{\int (\psi[x_g(z, k_g, k_d, a), k_g] + \psi[x_d(z, k_g, k_d, a), k_d])(1-\lambda)\Phi(\mu, x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(s) ds}_{\text{capital adjustment costs from firms w/o exogenous exit shock}} \\
& + \underbrace{\int [\psi^X(k_g) + \psi^X(k_d)]\lambda\Phi(\mu, x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(s) ds}_{\text{capital adjustment costs from exogenously exiting firms}}.
\end{aligned}
$$

Plugging in these definitions, we can rewrite the resource constraint as

$$
\begin{aligned}
C_t = & (1 - D(S^{1\prime} + S^{2\prime})) \int \exp(z) A_t^{1-\alpha}[a k_d^\rho + (1-a)k_g^\rho]^{\frac{\alpha}{\rho}} (L^d(s))^\nu \Phi(\mu, x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(s) ds \\
& - \int (x_g(z, k_g, k_d, a) + x_d(z, k_g, k_d, a))(1-\lambda)\Phi(\mu, x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(s) ds \\
& - \int (-(1-\delta)k_d - (1-\delta)k_g)\lambda\Phi(\mu, x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(s) ds \\
& - \int (x_g^E(a) + x_d^E(a)) \mathbb{1}_{X^E(a)=0} B Q_a(a) da \\
& - \int (\psi[x_g(z, k_g, k_d, a), k_g] + \psi[x_d(z, k_g, k_d, a), k_d])(1-\lambda)\Phi(\mu, x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(s) ds \\
& - \int [\psi^X(k_g) + \psi^X(k_d)]\lambda\Phi(\mu, x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(s) ds \qquad (25)
\end{aligned}
$$

Second, the planner's problem is subject to the labor supply constraint:

$$1 = \underbrace{\int (L^d(s) + c_f)\Phi(\mu, x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(s)ds}_{\text{labor used by operating firms}} + \underbrace{B\kappa}_{\text{labor to cover entry costs}}$$

Third, the planner must choose a nonnegative mass of entrants $B \geq 0$.

B.0.2 Lagrangian of Planner's Problem

We can write the planner's Lagrangian as

$$\begin{aligned}
\mathcal{L}_t = {} & U(C_t) + \lambda_t^1[S^{1\prime} - S^1 - (1-\chi_t)\varphi_1\gamma K^d)] + \lambda_t^2[S^{2\prime} - \varphi_3 S^2 - (1-\chi_t)(1-\varphi_1)\varphi_2\gamma K^d] + \lambda_t^B B \\
& + \lambda_t^L\left[1 - \int (L^d(s) + c_f)\Phi(\mu, x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(s)ds - B\kappa\right] \\
& + \lambda_t^k\left[K^d - \int k_d\Phi(\mu, x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(s)ds\right] \\
& + \beta\mathcal{W}_{t+1}(T^*(\mu, x_d(\cdot), x_g(\cdot), X(\cdot), x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B), S^{1\prime}, S^{2\prime})
\end{aligned}$$

where Φ, T^*, and C_t are defined by (23), (24), and (25), respectively.

B.1 Marginal Social Value

We start by computing the Gateaux derivative of the social welfare function with respect to the mass of firms at a particular location $(z, k_d, k_g, a) = s$ in the state-space at time t, which we follow Moll and Nuno (2018) in assuming exists:

$$\begin{aligned}
\frac{\partial \mathcal{W}_t(\mu, S^1, S^2)}{\partial \mu(s)} = {} & U'(C_t)\Big[(1 - D(S^{1\prime} + S^{2\prime}))\exp(z)A_t^{1-\alpha}[ak_d^\rho + (1-a)k_g^\rho]^{\frac{\alpha}{\rho}}(L^d(s))^\nu \\
& \quad + (1-\lambda)\left(-x_g(s) - x_d(s) - \psi[x_g(s), k_g] - \psi[x_d(s), k_d]\right) \\
& \quad + \lambda\left((1-\delta)k_g + (1-\delta)k_d - \psi^X(k_g) - \psi^X(k_d)\right)\Big] \\
& - \lambda_t^L(L^d(s) + c_f) - \lambda_t^k k_d \\
& + (1 - X(s))\beta\int \frac{\partial \mathcal{W}_{t+1}(\mu', S^{1\prime}, S^{2\prime})}{\partial \mu'(s')}\frac{\partial T^*(s')}{\partial \mu(s)}ds' \\
= {} & U'(C_t)\Big[(1 - D(S^{1\prime} + S^{2\prime}))\exp(z)A_t^{1-\alpha}[ak_d^\rho + (1-a)k_g^\rho]^{\frac{\alpha}{\rho}}(L^d(s))^\nu \qquad (26) \\
& \quad + (1-\lambda)\left(-x_g(s) - x_d(s) - \psi[x_g(s), k_g] - \psi[x_d(s), k_d]\right)
\end{aligned}$$

$$+ \lambda \Big((1-\delta)k_g + (1-\delta)k_d - \psi^X(k_g) - \psi^X(k_d)\Big) \Big]$$

$$- \lambda_t^L(L^d(s) + c_f) - \lambda_t^k k_d$$

$$+ (1 - X(s))(1-\lambda)\beta \int \frac{\partial \mathcal{W}_{t+1}(\mu', S^{1*}, S^{2*})}{\partial \mu'(\rho z + \epsilon, k_d', k_g', a)} p(\epsilon) d\epsilon$$

B.2 Planner's Optimality Conditions

B.2.1 Environmental Aggregate Variables

The Planner's first order conditions with respect to tomorrow's carbon stock $S^{1\prime}$ and $S^{2\prime}$

$$\frac{\partial \mathcal{L}_t}{\partial S^{1\prime}} = 0 \implies \lambda_t^1 = U'(C_t)D'(S^{1\prime} + S^{2\prime})Y_t - \beta \frac{\partial}{\partial S^{1\prime}} \mathcal{W}_{t+1}(T^*(\mu, \cdot), S^{1\prime}, S^{2\prime})$$
$$\frac{\partial \mathcal{L}_t}{\partial S^{2\prime}} = 0 \implies \lambda_t^2 = U'(C_t)D'(S^{1\prime} + S^{2\prime})Y_t - \beta \frac{\partial}{\partial S^{2\prime}} \mathcal{W}_{t+1}(T^*(\mu, \cdot), S^{1\prime}, S^{2\prime}).$$

Taking the envelope conditions with respect to S^1 and S^2

$$\frac{\partial}{\partial S^1} \mathcal{W}_t(\mu, S^1, S^2) = -\lambda_{t+1}^1$$
$$\frac{\partial}{\partial S^2} \mathcal{W}_t(\mu, S^1, S^2) = -\varphi_3 \lambda_{t+1}^2$$

and iterating forward, we can write λ_t^1 and λ_t^2 as

$$\lambda_t^1 = \sum_{s=t}^{\infty} \beta^{s-t} U'(C_s) D'(S_s^1 + S_s^2) Y_s$$
$$\lambda_t^2 = \sum_{s=t}^{\infty} (\varphi_3 \beta)^{s-t} U'(C_s) D'(S_s^1 + S_s^2) Y_s$$

assuming appropriate transversality condition.[16] The Planner's first order condition with respect to aggregate dirty capital utilized by operating firms is

$$\frac{\partial \mathcal{L}_t}{\partial K^d} = 0 \implies \lambda_t^k = \lambda_t^1 (1 - \chi_t)\varphi_1 \gamma + \lambda_t^2 (1 - \chi_t)(1 - \varphi_1)\varphi_2 \gamma.$$

This proves Proposition 2 in the main text.

[16] We can explicitly assume $\lim_{t\to\infty} \beta^t U'(C_t)D'(S_t^1 + S_t^2)Y_t = 0$. Because $\varphi_3 < 1$, this condition implies $\lim_{t\to\infty} (\varphi_3\beta)^t U'(C_t)D'(S_t^1 + S_t^2)Y_t = 0$ by the squeeze theorem.

B.2.2 Operating Firm Labor Allocation

Next, the Planner's first order condition with respect to firm-level labor allocation $L^d(s)$ at particular location in the state-space $(z, k_g, k_d, a) = s$ is

$$\begin{aligned}
&\frac{\partial \mathcal{L}_t}{\partial L^d(s)} = 0 \\
\implies &\lambda_t^L \Phi(\cdot)(s) = \Phi(\cdot)(s) U'(C_t)(1 - D(S^{1\prime} + S^{2\prime})) \\
&\qquad \times \nu \exp(z) A_t^{1-\alpha} [a k_d^\rho + (1-a) k_g^\rho]^{\frac{\alpha}{\rho}} (L^d(s))^{\nu - 1} \\
\implies &\frac{\lambda_t^L}{U'(C_t)} = (1 - D(S^{1\prime} + S^{2\prime})) \nu \exp(z) A_t^{1-\alpha} [a k_d^\rho + (1-a) k_g^\rho]^{\frac{\alpha}{\rho}} (L^d(s))^{\nu - 1}.
\end{aligned} \tag{27}$$

B.2.3 Incumbent Firm Investment

Next, we turn to the Planner's first order condition with respect to firm-level investment policies $x_g(s), x_d(s)$ at a particular location $(z, k_g, k_d, a) = s$ conditional on not exiting. This first order condition for firm-level investment in clean capital is

$$\begin{aligned}
&\frac{\partial \mathcal{L}_t}{\partial x_g(s)} = 0 \\
\implies &U'(C_t)(1-\lambda)(1 + \psi'[x_g(s), k_g]) \Phi(\mu, \cdot)(s) \\
&\qquad = \beta \int \frac{\partial W(\mu', S^{1*}, S^{2*})}{\partial \mu'(s')} \frac{\partial T^*(\mu, \cdot)(s')}{\partial k_g'} \underbrace{\frac{\partial k_g'(s)}{\partial x_g(s)}}_{=1} ds' \\
&\qquad = \beta \int \frac{\partial}{\partial k_g'(s)} \frac{\partial W(\mu', S^{1*}, S^{2*})}{\partial \mu'(s')} T^*(\mu, \cdot)(s') ds' \\
&\qquad = \beta (1-\lambda) \Phi(\mu, \cdot)(s) \\
&\qquad \times \int \frac{\partial}{\partial k_g'(s)} \frac{\partial W(\mu', S^{1*}, S^{2*})}{\partial \mu'(\rho z + \epsilon, k_d'(z), k_g'(s), a)} p(\epsilon) d\epsilon \\
\implies &U'(C_t)(1 + \psi'[x_g(s), k_g]) = \beta \frac{\partial}{\partial k_g'(s)} \int \frac{\partial W(\mu', S^{1*}, S^{2*})}{\partial \mu'(\rho z + \epsilon, k_d'(z), k_g'(s), a)} p(\epsilon) d\epsilon
\end{aligned} \tag{28}$$

assuming the order of integration can be swapped. Similarly, the first order condition with respect to firm-level investment in dirty capital can be written as

$$\begin{aligned}
&\frac{\partial \mathcal{L}_t}{\partial x_d(s)} = 0 \\
\implies &U'(C_t)(1 + \psi'[x_d(s), k_d]) = \beta \frac{\partial}{\partial k_d'(s)} \int \frac{\partial W(\mu', S^{1*}, S^{2*})}{\partial \mu'(\rho z + \epsilon, k_d'(z), k_g'(s), a)} p(\epsilon) d\epsilon
\end{aligned} \tag{29}$$

B.2.4 Incumbent Firm Continuation

The Planner strictly prefers not to exit the firm in state s if the marginal welfare benefit of firm s continuing is greater than the marginal welfare benefit of firm s exiting,

$$\underbrace{U'(C_t)\Big[(1-\delta)[k_g+k_d]-\psi^X(k_g)-\psi^X(k_d)\Big]}_{\text{exit}} \tag{30}$$
$$< \underbrace{U'(C_t)\Big[-x_g(s)-\psi[x_g(s),k_g]-x_d(s)-\psi[x_d(s),k_d]\Big]+\beta\int\frac{\partial\mathcal{W}_{t+1}(\mu',S^{1\prime},S^{2\prime})}{\partial\mu'(\rho z+\epsilon,k_d'(s),k_g'(s),a)}p(\epsilon)d\epsilon}_{\text{operate}},$$

and in this case the Planner chooses for the firm not to exit. Relatedly, if the exiting welfare benefit strictly exceeds the operating welfare benefit then the Planner will exit the firm. In general, it is also possible for the Planner to be indifferent. In our case, this will hold for a measure zero of firms, due to the absolute continuity of the distribution over a in our state space. However, more generally there may be a point mass at s for which the Planner optimally chooses for some positive measure to exit and the rest of the mass to not exit. If this is the case, the optimality condition of the Planner's choice of measure of firms within s to exit is indifference between the exit and operating value of the firms in s.

B.2.5 Mass of Entrants

Next, the Planner's first order condition with respect to the mass of entrants B is

$$\begin{aligned}\frac{\partial\mathcal{L}_t}{\partial B}&=0\\ \implies U'(C_t)\frac{\partial C_t}{\partial B}+\lambda_t^B+\beta\frac{\partial\mathcal{W}_{t+1}(\cdot)}{\partial B}&=\lambda_t^L\left(\int(L^d(s)+c_f)\frac{\partial\Phi(\cdot)(s)}{\partial B}ds+\kappa\right)\\ &\quad+\lambda_t^k\int k_d\frac{\partial\Phi(\cdot)(s)}{\partial B}ds.\end{aligned} \tag{31}$$

We evaluate derivatives of the measure of operating firms Φ, consumption C_t, and welfare tomorrow $\mathcal{W}_{t+1}$ with respect to the mass of entrants B, separately. First, the derivative of the measure of operating firms with respect to B is

$$\frac{\partial\Phi(\cdot)(s)}{\partial B}=\mathbb{1}_{k_d=x_d^E(a)}\mathbb{1}_{k_g=x_g^E(a)}\mathbb{1}_{X^E(a)=0}Q_z(z)Q_a(a). \tag{32}$$

Second, using (25), the derivative of consumption with respect to B is

$$\frac{\partial C_t}{\partial B}=(1-D(S^{1\prime}+S^{2\prime}))\int\exp(z)A_t^{1-\alpha}[ak_d^\rho+(1-a)k_g^\rho]^{\frac{\alpha}{\rho}}(L^d(s))^\nu\frac{\partial\Phi(\cdot)(s)}{\partial B}ds \tag{33}$$

$$
\begin{aligned}
&- \int (x_g(z,k_g,k_d,a) + x_d(z,k_g,k_d,a))(1-\lambda)\frac{\partial \Phi(\cdot)(s)}{\partial B} ds \\
&- \int (-(1-\delta)k_d - (1-\delta)k_g)\lambda \frac{\partial \Phi(\cdot)(s)}{\partial B} ds \\
&- \int (x_g^E(a) + x_d^E(a))\mathbb{1}_{X^E(a)=0} Q_a(a) da \\
&- \int (\psi[x_g(z,k_g,k_d,a),k_g] + \psi[x_d(z,k_g,k_d,a),k_d])(1-\lambda)\frac{\partial \Phi(\cdot)(s)}{\partial B} ds \\
&- \int (\psi[-(1-\delta)k_g,k_g] + \psi[-(1-\delta)k_d,k_d])\lambda \frac{\partial \Phi(\cdot)(s)}{\partial B} ds
\end{aligned}
$$

where (32) defines $\frac{\partial \Phi(\cdot)(s)}{\partial B}$. Finally, the derivative of welfare tomorrow with respect to B is

$$
\begin{aligned}
\frac{\partial \mathcal{W}_{t+1}(\mu',\cdot)}{\partial B} &= \int \frac{\partial \mathcal{W}_{t+1}(\mu',\cdot)}{\partial \mu'(s')} \frac{\partial \mu'(s')}{\partial B} ds' \\
&= \int \frac{\partial \mathcal{W}_{t+1}(\mu',\cdot)}{\partial \mu'(s')} \frac{\partial T^*(\mu,\cdot)(s')}{\partial B} ds' \\
&= \int \frac{\partial \mathcal{W}_{t+1}(\mu',\cdot)}{\partial \mu'(s')} \Bigg[\int \mathbb{1}_{k_d'=(1-\delta)k_d(s)+x_d(s)} \mathbb{1}_{k_g'=(1-\delta)k_g(s)+x_g(s)} \mathbb{1}_{z'=\rho z+\epsilon} \mathbb{1}_{a'=a} \mathbb{1}_{X(s)=0} p(\epsilon) \\
&\qquad \times (1-\lambda)\frac{\partial \Phi(\cdot)(s)}{\partial B} ds \Bigg] ds' \\
&= \int \int \frac{\partial \mathcal{W}_{t+1}(\mu',\cdot)}{\partial \mu'(s')} \mathbb{1}_{k_d'=(1-\delta)k_d(s)+x_d(s)} \mathbb{1}_{k_g'=(1-\delta)k_g(s)+x_g(s)} \mathbb{1}_{z'=\rho z+\epsilon} \mathbb{1}_{a'=a} \mathbb{1}_{X(s)=0} p(\epsilon) \\
&\qquad \times (1-\lambda)\frac{\partial \Phi(\cdot)(s)}{\partial B} ds' ds \\
&= \int (1-\lambda)\mathbb{1}_{X(s)=0} \frac{\partial \Phi(\cdot)(s)}{\partial B} \int \frac{\partial \mathcal{W}_{t+1}(\mu',\cdot)}{\partial \mu'(s')} \mathbb{1}_{k_d'=(1-\delta)k_d(s)+x_d(s)} \mathbb{1}_{k_g'=(1-\delta)k_g(s)+x_g(s)} \\
&\qquad \times \mathbb{1}_{z'=\rho z+\epsilon} \mathbb{1}_{a'=a} p(\epsilon) ds' ds \\
&= \int (1-\lambda)\mathbb{1}_{X(s)=0} \frac{\partial \Phi(\cdot)(s)}{\partial B} \\
&\qquad \times \left(\int \frac{\partial \mathcal{W}_{t+1}(\mu',\cdot)}{\partial \mu'(\rho z+\epsilon,(1-\delta)k_d + x_d(s),(1-\delta)k_g + x_g(s),a)} p(\epsilon) d\epsilon \right) ds \\
&= \int (1-\lambda)\mathbb{1}_{X(s)=0} \frac{\partial \Phi(\cdot)(s)}{\partial B} \left(\int \frac{\partial \mathcal{W}_{t+1}(\mu',\cdot)}{\partial \mu'(\rho z+\epsilon,k_d'(s),k_g'(s),a)} p(\epsilon) d\epsilon \right) ds, \qquad (34)
\end{aligned}
$$

where the first equality holds by the law of total differentiation (in integral form), the last equality holds with $k_i'(s) = (1-\delta)k_i + x_i(s)$, and $\frac{\partial \Phi(\cdot)(s)}{\partial B}$ is defined in (32). Substituting (32), (33), and (34) into (31) and rearranging terms, we get

$$
\begin{aligned}
\lambda_t^L \kappa = \lambda_t^B + \int \Bigg(& U'(C_t) \Big[(1 - D(S^{1\prime} + S^{2\prime})) \exp(z) A_t^{1-\alpha} [a k_d^\rho + (1-a) k_g^\rho]^{\frac{\alpha}{\rho}} (L^d(s))^\nu \\
& + (1-\lambda)\left(-x_g(s) - x_d(s) - \psi[x_g(s), k_g] - \psi[x_d(s), k_d]\right) \\
& + \lambda \left((1-\delta) k_g + (1-\delta) k_d - \psi[-(1-\delta) k_g, k_g] - \psi[-(1-\delta) k_d, k_d]\right) \Big] \\
& - \lambda_t^k k_d - \lambda_t^L L^d(s) + \mathbb{1}_{X(s)=0} (1-\lambda) \beta \int \frac{\partial \mathcal{W}_{t+1}(\mu', S^{1*}, S^{2*})}{\partial \mu'(\rho z + \epsilon, k_d', k_g', a)} p(\epsilon) d\epsilon \\
& - U'(C_t)(x_g^E(a) + x_d^E(a)) \Bigg) \mathbb{1}_{k_d = x_d^E(a)} \mathbb{1}_{k_g = x_g^E(a)} \mathbb{1}_{X^E(a)=0} Q_z(z) Q_a(a) ds
\end{aligned}
\tag{35}
$$

Substituting in the marginal social value (26), we can simplify to

$$
\begin{aligned}
\lambda_t^L \kappa = {} & \lambda_t^B \\
& + \int \int \left[\frac{\partial \mathcal{W}_t}{\partial \mu(z, x_d^E(a), x_g^E(a), a)} - U'(C_t)(x_g^E(a) + x_d^E(a)) \right] Q_z(z) \mathbb{1}_{X^E(a)=0} Q_a(a) dz da,
\end{aligned}
\tag{36}
$$

B.2.6 Entrant Firm Investment

As a preliminary, note that for any Lebesgue-integrable function $\mathcal{G}$ on the state space where its derivative with respect to entrant investment $\frac{\partial \mathcal{G}(z, x_d^E(a), x_g^E(a), a)}{\partial x_d^E(a)}$ exists and is bounded,

$$
\begin{aligned}
\frac{\partial}{\partial x_d^E(a)} \int \mathcal{G}(s) \Phi(s) ds &= \frac{\partial}{\partial x_d^E(a)} \int \mathcal{G}(s) \left[\mu(z, k_d, k_g, a) + \mathbb{1}_{k_d = x_d^E(a)} \mathbb{1}_{k_g = x_g^E(a)} \mathbb{1}_{X^E(a)=0} Q_z(z) Q_a(a) B \right] ds \\
&= \frac{\partial}{\partial x_d^E(a)} \int \mathcal{G}(s) \mathbb{1}_{k_d = x_d^E(a)} \mathbb{1}_{k_g = x_g^E(a)} \mathbb{1}_{X^E(a)=0} Q_z(z) Q_a(a) B ds \\
&= (1 - X^E(a)) Q_a(a) B \frac{\partial}{\partial x_d^E(a)} \int \mathcal{G}(z, x_d^E(a), x_g^E(a), a) Q_z(z) dz \\
&= (1 - X^E(a)) Q_a(a) B \int \frac{\partial \mathcal{G}(z, x_d^E(a), x_g^E(a), a)}{\partial x_d^E(a)} Q_z(z) dz,
\end{aligned}
$$

where the fourth inequality applies Leibniz integral rule and requires the conditions on $\mathcal{G}$. We assume all equilibrium functions are Lebesgue-integrable with bounded partial derivatives.

The Planner's first order condition with respect to investment in dirty capital by entrant firm at given carbon dependence $x_d^E(a)$ is

$$
\frac{\partial \mathcal{L}_t}{\partial x_d^E(a)} = 0
$$

$$\implies U'(C_t)\frac{\partial C_t}{\partial x_d^E(a)} + \beta\frac{\partial W_{t+1}(\cdot)}{\partial x_d^E(a)} = \lambda_t^L \frac{\partial}{\partial x_d^E(a)}\int (L^d(s)+c_f)\Phi(\cdot)(s)ds$$
$$+ \lambda_t^k \frac{\partial}{\partial x_d^E(a)}\int k_d\Phi(\cdot)ds,$$

where

$$\frac{\partial C_t}{\partial x_d^E(a)} = (1-D(S^{1\prime}+S^{2\prime}))\frac{\partial}{\partial x_d^E(a)}\int \exp(z)A_t^{1-\alpha}[ak_d^\rho+(1-a)k_g^\rho]^{\frac{\alpha}{\rho}}(L^d(s))^\nu\Phi(\cdot)(s)ds$$
$$-\int (x_g(z,k_g,k_d,a)+x_d(z,k_g,k_d,a))(1-\lambda)\frac{\partial\Phi(\cdot)(s)}{\partial x_d^E(a)}ds$$
$$-\frac{\partial}{\partial x_d^E(a)}\int (-(1-\delta)k_d-(1-\delta)k_g)\lambda\Phi(\cdot)(s)ds$$
$$-BQ_a(a)$$
$$-\frac{\partial}{\partial x_d^E(a)}\int (\psi[x_g(z,k_g,k_d,a),k_g]+\psi[x_d(z,k_g,k_d,a),k_d])(1-\lambda)\Phi(\cdot)(s)ds$$
$$-\frac{\partial}{\partial x_d^E(a)}\int (\psi[-(1-\delta)k_g,k_g]+\psi[-(1-\delta)k_d,k_d])\lambda\Phi(\cdot)(s)ds.$$

Applying Leibniz integral rule simplifies the first order condition to

$$\begin{aligned}
&U'(C_t)\\
&=\int \frac{\partial}{\partial x_d^E(a)}\Bigg(U'(C_t)\Bigg[(1-D(S^{1\prime}+S^{2\prime}))\exp(z)A_t^{1-\alpha}[ax_d^E(a)^\rho+(1-a)x_g^E(a)^\rho]^{\frac{\alpha}{\rho}}\\
&\qquad\times(L^d(z,x_g^E(a),x_d^E(a),a))^\nu\\
&\qquad-(x_g(z,x_g^E(a),x_d^E(a),a)+x_d(z,x_g^E(a),x_d^E(a),a))(1-\lambda)\\
&\qquad+(1-\delta)(k_d+k_g)\lambda\\
&\qquad-(\psi[x_g(z,x_g^E(a),x_d^E(a),a),x_g^E(a)]+\psi[x_d(z,x_g^E(a),x_d^E(a),a),x_d^E(a)])(1-\lambda)\\
&\qquad-[\psi^X(x_g^E(a))+\psi^X(x_d^E(a))]\lambda\Bigg]\\
&\qquad-\lambda_t^L(L^d(z,x_g^E(a),x_d^E(a),a)+c_f)-\lambda_t^k x_d^E(a)\Bigg)Q_z(z)dz\\
&+\beta\frac{1}{BQ_a(a)}\frac{\partial W_{t+1}(\cdot)}{\partial x_d^E(a)}.
\end{aligned}$$

Now, consider the derivative of tomorrow's welfare with respect to entrant investment

$$\frac{\partial W_{t+1}(\cdot)}{\partial x_d^E(a)} = \int \frac{\partial W(\mu',S^{1*},S^{2*})}{\partial\mu'(s')}\frac{\partial\mu'(s')}{\partial x_d^E(a)}ds'$$

$$
\begin{aligned}
&= \int \frac{\partial W(\mu', S^{1*}, S^{2*})}{\partial \mu'(s')} \frac{\partial T^*(\mu, \cdot)(s')}{\partial x_d^E(a)} ds' \\
&= BQ_a(a) \int \frac{\partial}{\partial x_d^E(a)} (1 - X(z, x_g^E(a), x_d^E(a), a))(1-\lambda) Q_z(z) \\
&\qquad \times \int \frac{\partial W(\mu', S^{1*}, S^{2*})}{\partial \mu'(\rho z + \epsilon, k_g'(z, x_g^E(a), x_d^E(a), a), k_d'(z, x_g^E(a), x_d^E(a), a), a)} p(\epsilon) d\epsilon dz
\end{aligned}
$$

by switching the order of differentiation, plugging in the definition of T^*, and switching the order of integration. Plugging in the marginal social value (26), the first order condition simplifies to

$$
U'(C_t) = \frac{\partial}{\partial x_d^E(a)} \int \frac{\partial W_t(\mu, S^{1\prime}, S^{2\prime})}{\partial \mu(z, x_d^E(a), x_g^E(a), a)} Q_z(z) dz, \tag{37}
$$

Following an identical process, the first order condition with respect to $x_g^E(a)$ is

$$
U'(C_t) = \frac{\partial}{\partial x_g^E(a)} \int \frac{\partial W_t(\mu, S^{1\prime}, S^{2\prime})}{\partial \mu(z, x_d^E(a), x_g^E(a), a)} Q_z(z) dz. \tag{38}
$$

B.2.7 Entrant Firm Continuation

Finally, the planner's decision of $X^E(a)$ is similar to the incumbent firm continuation decision. The Planner chooses $X^E(a) = 1$ iff the marginal value of exiting an entrant firm (zero) is greater than the marginal value of the entrant operating:

$$
\begin{aligned}
&\underbrace{0}_{\text{exit}} > \underbrace{BQ_a(a) \left[U'(C_t) \left(-x_g^E(a) - x_d^E(a) \right) + \int \frac{\partial \mathcal{W}_t(\mu, S^{1\prime}, S^{2\prime})}{\partial \mu(z, x_d^E, x_g^E, a)} Q_z(z) dz \right]}_{\text{operate}} \\
&\implies 0 > U'(C_t) \left(-x_g^E(a) - x_d^E(a) \right) + \int \frac{\partial \mathcal{W}_t(\mu, S^{1\prime}, S^{2\prime})}{\partial \mu(z, x_d^E, x_g^E, a)} Q_z(z) dz
\end{aligned} \tag{39}
$$

B.3 Augmented Firm Bellman Equation

Now define $\omega_t(s, \mu, S^1, S^2) \equiv \frac{\partial \mathcal{W}_t(\mu, S^1, S^2)}{\partial \mu(s)}$ as the marginal social value of firms at a particular location in state-space s at time t,

$$
\begin{aligned}
\omega_t(s, \mu, S^1, S^2) = U'(C_t) \Big[&(1 - D(S^{1\prime} + S^{2\prime})) \exp(z) A_t^{1-\alpha} [a k_d^\rho + (1-a) k_g^\rho]^{\frac{\alpha}{\rho}} (L^d(s))^\nu \\
&+ (1-\lambda) \left(-x_g(s) - x_d(s) - \psi[x_g(s), k_g] - \psi[x_d(s), k_d] \right) \\
&+ \lambda \left((1-\delta)(k_g + k_d) - \psi^X(k_g) - \psi^X(k_d) \right) \Big]
\end{aligned}
$$

$$
\begin{aligned}
&- \lambda_t^L(L^d(s)+c_f) - \lambda_t^k k_d \\
&+ (1-X(s))(1-\lambda)\beta\mathbb{E}[\omega_{t+1}(s',\mu,S^1,S^2)] \\
\implies \bar{\omega}_t(s,\mu,S^1,S^2) &= (1-D(S^{1\prime}+S^{2\prime}))\exp(z)A_t^{1-\alpha}[ak_d^\rho+(1-a)k_g^\rho]^{\frac{\alpha}{\rho}}(L^d(s))^\mu \\
&+ (1-\lambda)\left(-x_g(s)-x_d(s)-\psi[x_g(s),k_g]-\psi[x_d(s),k_d]\right) \\
&+ \lambda\left((1-\delta)k_g+(1-\delta)k_d-\psi^X(k_g)-\psi^X(k_d)\right) \\
&- \hat{\lambda}_t^L(L^d(s)+c_f)-\hat{\lambda}_t^k k_d \\
&+ (1-X(s))(1-\lambda)\beta\frac{U'(C_{t+1})}{U'(C_t)}\mathbb{E}[\bar{\omega}_{t+1}(s',\mu',S^{1\prime},S^{2\prime})]
\end{aligned}
$$

where $\bar{\omega}_t(s,\mu,S^1,S^2) \equiv \frac{\omega_t(s,\mu,S^1,S^2)}{U'(C_t)}$, $\hat{\lambda}_t^k \equiv \frac{\lambda_t^k}{U'(C_t)}$, and $\hat{\lambda}_t^L \equiv \frac{\lambda_t^L}{U'(C_t)}$.

B.3.1 Incumbent Firm

We now show that the marginal social value $\bar{\omega}_t(s,\mu,S^1,S^2)$ achieves a value function $\hat{V}_t(s,\mu,S^1,S^2)$ induced by an augmented Bellman equation written similarly. In particular, consider value function

$$
\begin{aligned}
\hat{V}_t(s,\mu,S^1,S^2) := \max_{\hat{L}^d(s),\hat{x}_d(s),\hat{x}_g(s),\hat{X}(s)} &(1-D(S^{1\prime}+S^{2\prime}))\exp(z)A_t^{1-\alpha}[ak_d^\rho+(1-a)k_g^\rho]^{\frac{\alpha}{\rho}}(\hat{L}^d(s))^\nu \\
&+ (1-\lambda)\left(-\hat{x}_g(s)-\hat{x}_d(s)-\psi[\hat{x}_g(s),k_g]-\psi[\hat{x}_d(s),k_d]\right) \\
&+ \lambda\left[(1-\delta)(k_g+k_d)-\psi^X(k_g)-\psi^X(k_d)\right] \\
&- \hat{\lambda}_t^L(\hat{L}^d(s)+c_f)-\hat{\lambda}_t^k k_d \\
&+ (1-\hat{X}(s))(1-\lambda)\beta\frac{U'(C_{t+1})}{U'(C_t)}\mathbb{E}[\bar{\omega}_{t+1}(s',\mu',S^{1\prime},S^{2\prime})]
\end{aligned}
\tag{40}
$$

subject to $\hat{x}_g = -(1-\delta)k_g$ and $\hat{x}_d = -(1-\delta)k_d$ if $\hat{X}(s) = 1$. The first order condition with respect to $\hat{L}^d(s)$

$$
\hat{\lambda}_t^L = (1-D(S^{1\prime}+S^{2\prime}))\nu\exp(z)A_t^{1-\alpha}[ak_d^\rho+(1-a)k_g^\rho]^{\frac{\alpha}{\rho}}(\hat{L}^d(s))^{\nu-1}
$$

matches labor allocation optimality condition from full Planner's problem (27). The first order condition with respect to $\hat{x}_g(s)$ conditional on choosing $\hat{X}(s) = 0$

$$
1+\psi'[x_g(s),k_g] = \beta\frac{U'(C_{t+1})}{U'(C_t)}\frac{\partial}{\partial k_g'(s)}\mathbb{E}[\bar{\omega}_{t+1}(s',\mu',S^{1\prime},S^{2\prime})]
$$

matches dirty investment optimality condition from full Planner's problem (28). The first order condition with respect to $\hat{x}_d(s)$ conditional on choosing $\hat{X}(s) = 0$ is

$$1 + \psi'[x_d(s), k_d] = \beta \frac{U'(C_{t+1})}{U'(C_t)} \frac{\partial}{\partial k_d'(s)} \mathbb{E}[\bar{\omega}_{t+1}(s', \mu', S^{1\prime}, S^{2\prime})]).$$

matches clean investment optimality condition from full Planner's problem (29). Finally, firm s exits $\hat{X}(s) = 1$ if

$$\begin{aligned} &\underbrace{(1-\delta)(k_g + k_d) - \psi^X(k_g) - \psi^X(k_d)}_{\text{exit}} \\ > &\underbrace{-\hat{x}_g(s) - \psi[\hat{x}_g(s), k_g] - \hat{x}_d(s) - \psi[\hat{x}_d(s), k_d] + \beta \frac{U'(C_{t+1})}{U'(C_t)} \mathbb{E}[\bar{\omega}_{t+1}(s', \mu', S^{1\prime}, S^{2\prime})]}_{\text{operate}}, \end{aligned}$$

continues if operating value strictly exceeds exiting, and is indifferent if the values are equal. We assume that an indifferent firm breaks the tie by following the exit policy assigned by the Planner. As a reminder, this holds for a measure zero of firms in our model. The exit decision rules thus matches those of the Planner in (30).

Hence, the policies induced by the optimization problem given by equation (40) satisfy the optimality conditions from the full Planner's problem. Furthermore, $\bar{\omega}_t(s, \mu, S^1, S^2) = \hat{V}_t(s, \mu, S^1, S^2)$, so we can now write $\bar{\omega}_t(s, \mu, S^1, S^2)$ as an augmented Bellman equation:

$$\begin{aligned} \bar{\omega}_t(s, \mu, S^1, S^2) = &\max_{\hat{L}^d(s), \hat{x}_d(s), \hat{x}_g(s), \hat{X}(s)} (1 - D(S^{1\prime} + S^{2\prime})) \exp(z) A_t^{1-\alpha} [a k_d^\rho + (1-a) k_g^\rho]^{\frac{\alpha}{\rho}} (\hat{L}^d(s))^\nu \\ &+ (1-\lambda)\left(-\hat{x}_g(s) - \hat{x}_d(s) - \psi[\hat{x}_g(s), k_g] - \psi[\hat{x}_d(s), k_d]\right) \\ &+ \lambda \left[(1-\delta)(k_g + k_d) - \psi^X(k_g) - \psi^X(k_d)\right] \\ &- \hat{\lambda}_t^L (\hat{L}^d(s) + c_f) - \hat{\lambda}_t^k k_d \\ &+ (1 - \hat{X}(s))(1-\lambda) \beta \frac{U'(C_{t+1})}{U'(C_t)} \mathbb{E}[\bar{\omega}_{t+1}(s', \mu', S^{1\prime}, S^{2\prime})] \end{aligned} \tag{41}$$

subject to $\hat{k}_i' = (1-\delta)k_i + \hat{x}_i$ and $\hat{x}_i = -(1-\delta)k_i$ if $\hat{X}(s) = 1$.

B.3.2 Entrant Firm

Consider the following optimization problem for entrants:

$$\bar{\omega}_t^E(a, \mu, S^1, S^2) = \max_{\hat{X}^E(a), \hat{x}_d^E(a), \hat{x}_g^E(a)} (1 - \hat{X}^E(a)) \left[-\hat{x}_d^E(a) - \hat{x}_g^E(a) + \mathbb{E}[\bar{\omega}_t(s, \mu, S^1, S^2)]\right] \tag{42}$$

subject to $\hat{x}_g^E = 0$ and $\hat{x}_d^E = 0$ if $\hat{X}(s) = 1$. Conditional on not exiting, the first order condition with respect to $\hat{x}_d^E$ is

$$1 = \frac{\partial}{\partial \hat{x}_d^E(a)} \mathbb{E}[\bar{\omega}_t(s, \mu, S^1, S^2)],$$

which matches the Planner's first order condition (**??**). Similarly, the first order condition with respect to $\hat{x}_g^E(a)$

$$1 = \frac{\partial}{\partial \hat{x}_g^E(a)} \mathbb{E}[\bar{\omega}_t(s, \mu, S^1, S^2)]$$

matches the Planner's first order condition (38). Finally, entrant a exits $\hat{X}^E(a) = 1$ iff

$$\underbrace{0}_{\text{exit}} > \underbrace{-x_g^E(a) - x_d^E(a) + \mathbb{E}[\bar{\omega}_t(s, \mu, S^1, S^2)]}_{\text{operate}},$$

which matches Planner's optimality condition (39).

Let $\tau_t^k \equiv \frac{\lambda_t^k}{U'(C_t)}$, $\hat{w} \equiv \frac{\lambda_t^L}{U'(C_t)}$, and $\frac{1}{\hat{R}_t} \equiv \beta \frac{U'(C_{t+1})}{U'(C_t)}$. Since the dynamic program given by (41) matches that of (18), it must be that $\bar{\omega}_t(s, \mu, S^1, S^2) = \hat{\omega}_t(s, \mu, S^1, S^2)$, and then (19) is an identical dynamic program to (42), so $\bar{\omega}_t^E(a, \mu, S^1, S^2) = \hat{\omega}_t^E(a, \mu, S^1, S^2)$, and the policies induced by the programs are also equal. Hence, Propositions 4 and 5 hold.

We finally note that equation (36) can be written:

$$\mathbb{E}[\hat{\omega}_t^E(a, \mu, S^1, S^2)] + \frac{\lambda_t^B}{U'(C_t)} = \hat{w}_t \kappa,$$

proving Proposition 6.

C Decentralization of the Planner's Solution

Let us rewrite the firm's Bellman equations in the decentralized economy with the appropriate tax wedges added:

$$\mathcal{V}_t(s) = \pi_t^D(s) - \tau_t^k k_{d,t} + \lambda \mathcal{V}^X(k_g, k_d) + (1 - \lambda) \max\{\mathcal{V}^X(k_g, k_d), V_t^C(s)\}, \tag{43}$$

where $\pi_t(s)$ is the cash flow function:

$$\pi_t(s) = \max_L (1 - D(S^{1\prime} + S^{2\prime})) \exp(z) A_t^{1-\alpha} [a k_d^\rho + (1 - a) k_g^\rho]^{\frac{\alpha}{\rho}} L^\nu - w_t L - w_t c_f,$$

and where the exiting value is given by:

$$\mathcal{V}^x(k_g, k_d) = (1-\delta)(k_d + k_g) - \psi^X(k_d) - \psi^X(k_g),$$

and nonexiting value defined by the solution to the following optimization problem:

$$\mathcal{V}_t^C(s) = \max_{x_d, x_g} -x_d - x_g - \psi(x_d, k_d) - \psi(x_g, k_g) + \frac{1}{R_t}\mathbb{E}[\mathcal{V}_{t+1}(s')], \tag{44}$$

subject to:

$$k_j' = (1-\delta)k_j + x_j, \quad \text{for } j \in \{d, g\}.$$

Free entry implies:

$$w_t \kappa \geq \mathbb{E}[\mathcal{V}_t^E(a)],$$

where an entrant's value after observing carbon dependence a is given by:

$$\mathcal{V}_t^E(a) = \max\{0, \max_{k_d, k_g} -k_d - k_g + \mathbb{E}[\mathcal{V}_t(s)]\}. \tag{45}$$

Now, for the set of taxes and prices as defined by the set $\mathcal{P}_t$, the dynamic program given by (43) and (45) in the competitive economy are identical to the analogous dynamic programs from the augmented Bellman equation and augmented entry problem given by (18) and (19). Hence, it follows that the values and induced policies are the same, as the dynamic programs are the same. That is, $\mathcal{V}_t(s) = \hat{\omega}_t(s, \mu_t, S^1_{t-1}, S^2_{t-1})$ and $\mathcal{V}_t^E(a) = \hat{\omega}_t^E(a, \mu_t, S^1_{t-1}, S^2_{t-1})$.

Now, note that by definition $\frac{1}{\hat{R}_t} = \beta\frac{U'(C_{t+1})}{U'(C_t)}$ so the Euler of the representative household holds at that risk-free rate, and market clearing of the risk-free asset is achieved.

Next, note that equation (19) implies that the free entry condition, (45) is satisfied. Moreover, the labor market clears under B_t from the planner's solution,[17] and as the firm-level labor demands are equal under both problems and the supply of labor is exhausted in the planner's problem with mass of entrants B_t.

Finally, note that the firm's policies solve their problems, and thus a competitive equilibrium that implements the planner's problem is achieved under a path of taxes $\{\tau_t^k\}_{t=t_0}^{\infty}$, prices $\{\hat{w}_t, \hat{R}_t\}_{t=t_0}^{\infty}$, and mass of entering firms $\{B_t\}_{t=t_0}^{\infty}$.

[17] Note that the labor market clearing condition must trivially bind for the planner, else consumption can be strictly increased by providing the remaining labor.

D Alternative Decentralization of the Planner's Solution

Suppose the taxation authority cannot observe k_d, and hence implementing the tax on dirty capital such as in section 6.1 is infeasible. In this section, we will show that the socially efficient allocation is achievable through a tax on emissions so long as emissions are observable to the taxation authority, even if the emissions are noisy.

Suppose firm-level emissions are given by equation (21). From the perspective of the social planner, since the law of large numbers washes out the idiosyncratic uncertainty over $\eta_{i,t}$, the problem remains the same as in 17, and hence the socially optimal policies are identical. We will now show that a decentralized equilibrium that implements the planner's policies can be achieved using a tax on emissions at the firm-level rather than dirty capital.

Consider the following decentralized problem with emissions taxes $\tau_t^E = \frac{\tau_t^k}{\gamma}$, for a realization ξ of emissions:

$$
\begin{aligned}
\hat{\mathcal{V}}_t(s,\xi,\mu,S^1,S^2) = \max_{x_d(s,\xi),x_g(s,\xi),L^d(s,\xi),X(s,\xi)} & (1-D(S^{1\prime}+S^{2\prime}))\exp(z)A_t^{1-\alpha}[ak_d^\rho+(1-a)k_g^\rho]^{\frac{\alpha}{\rho}}(L^d(s,\xi))^\mu \qquad (46)\\
&+(1-\lambda)\left(-x_g(s,\xi)-x_d(s,\xi)-\psi[x_g(s,\xi),k_g]-\psi[x_d(s,\xi),k_d]\right)\\
&+\lambda\left((1-\delta)k_g+(1-\delta)k_d-\psi^X(k_g)-\psi^X(k_d)\right)\\
&-\tau_t^E\xi-\hat{w}(L^d(s,\xi)+c_f)\\
&+(1-X(s,\xi))(1-\lambda)\frac{1}{\hat{R}_t}\mathbb{E}[\hat{\mathcal{V}}_{t+1}(s',\mu',S^{1\prime},S^{2\prime})]\\
&=-\tau_t^E\xi\\
&+\max_{x_d(s,\xi),x_g(s,\xi),L^d(s,\xi),X(s,\xi)}(1-D(S^{1\prime}+S^{2\prime}))\exp(z)A_t^{1-\alpha}[ak_d^\rho+(1-a)k_g^\rho]^{\frac{\alpha}{\rho}}(L^d(s,\xi))^\mu\\
&+(1-\lambda)\left(-x_g(s,\xi)-x_d(s,\xi)-\psi[x_g(s,\xi),k_g]-\psi[x_d(s,\xi),k_d]\right)\\
&+\lambda\left((1-\delta)k_g+(1-\delta)k_d-\psi^X(k_g)-\psi^X(k_d)\right)\\
&-\hat{w}(L^d(s,\xi)+c_f)\\
&+(1-X(s,\xi))(1-\lambda)\frac{1}{\hat{R}_t}\mathbb{E}[\hat{\mathcal{V}}_{t+1}(s',\xi',\mu',S^{1\prime},S^{2\prime})].
\end{aligned}
$$

Hence, the policies are independent of the stochastic ξ, and are only a function of s. Moreover, since $\mathbb{E}_t[\tau_t^E\xi] = \mathbb{E}[\frac{\tau_t^k}{\gamma}\gamma k_{d,t}]$,

$$
\mathbb{E}_{t-1}[\hat{\mathcal{V}}_t(s,\xi,\mu,S^1,S^2)] = \mathbb{E}_{t-1}\Bigg[\max_{x_d(s),x_g(s),L^d(s),X(s)}(1-D(S^{1\prime}+S^{2\prime}))\exp(z)A_t^{1-\alpha}[ak_d^\rho+(1-a)k_g^\rho]^{\frac{\alpha}{\rho}}(L^d(s))^\mu
$$

$$
\begin{aligned}
&+ (1-\lambda)\left(-x_g(s) - x_d(s) - \psi[x_g(s), k_g] - \psi[x_d(s), k_d]\right) \\
&+ \lambda\big((1-\delta)k_g + (1-\delta)k_d - \psi^X(k_g) - \psi^X(k_d)\big) \\
&- \tau_t^E \xi - \hat{w}(L^d(s) + c_f) \\
&+ (1 - X(s))(1-\lambda)\frac{1}{\hat{R}_t}\mathbb{E}[\hat{\mathcal{V}}_{t+1}(s', \mu', S^{1\prime}, S^{2\prime})]\Bigg], \\
&= \mathbb{E}_{t-1}\Bigg[\max_{x_d(s), x_g(s), L^d(s), X(s)} (1 - D(S^{1\prime} + S^{2\prime}))\exp(z) A_t^{1-\alpha}[a k_d^\rho + (1-a)k_g^\rho]^{\frac{\alpha}{\rho}}(L^d(s))^\mu \\
&+ (1-\lambda)\left(-x_g(s) - x_d(s) - \psi[x_g(s), k_g] - \psi[x_d(s), k_d]\right) \\
&+ \lambda\big((1-\delta)k_g + (1-\delta)k_d - \psi^X(k_g) - \psi^X(k_d)\big) \\
&- \tau_t^k k_d - \hat{w}(L^d(s) + c_f) \\
&+ (1 - X(s))(1-\lambda)\frac{1}{\hat{R}_t}\mathbb{E}[\hat{\mathcal{V}}_{t+1}(s', \mu', S^{1\prime}, S^{2\prime})]\Bigg].
\end{aligned}
$$

i.e. the ex-ante values and ex-post policies induced by (46) equal those of the augmented Bellman (18), and hence the planner's allocation can be decentralized by taxation of noisy, but observed, emissions in lieu of a direct tax on dirty capital.

E Computation of Detailed Model

We solve the problem numerically using value function iteration. We first detrend the firm problems by dividing the Bellman equations and augmented Bellman equations by A_t. We then discretize the state-space: we approximate the z process following Tauchen (1986), discretize both k_d/A_t and k_g/A_t linearly in logs.

To compute a transition path, we first solve for the firm distribution associated with the balanced growth path with no climate damages nor taxes. At this point, the interest rate is at the balanced growth path level. In this balanced growth path, we set the wage to one and solve for the implied entry cost κ, which is held constant for the rest of the transition path while wages move to satisfy the entry condition. We take as the initial firm distribution μ the invariant distribution over the detrended balanced growth path. We then guess the path of interest rates, output taxes, and dirty capital taxes over the transition path. Given these guesses, we move backward along the transition path solving for value and policy function. The value functions imply the value of entry, which pins down the wage along the transition path. We then move forward along the transition path updating the measure of firms consistent with the policy functions. We solve the full transition path over 5,000 years, which allows for the convergence of the firm distribution. This extended timeframe is largely driven by the strong persistence of the transitory component of the stock of carbon emissions. The stock of emissions determines damages from climate change which then pin down optimal output taxes. We provide sufficient time for the temporary carbon stock to dissipate for the optimal output taxes to converge and then the firm distribution converge. After 2,000 years, we extend the aggregate variable for an additional 5,000 years assuming that are on the balanced growth path and grow by exogenous growth rate of TFP. We then update the guess of the interest rate based on the consumption Euler equation of the representative household and the guesses of the optimal output and dirty capital taxes based on the solutions to the Planner's problem.