Heterogeneous Firms, Emissions, and Optimal Carbon Taxes

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Project Outline

- Prior climate-macro research focuses on optimal climate policy in representative firm framework
  - Examples include Nordhaus (2007), Golosov et al. (2014), Krusell and Smith (2022)
- Heterogeneous carbon dependence + endogenous exit → *composition effect* in Planner solution
  - Planner exits firms w/ dirty production tech. at higher rate than firms w/ green production tech.
- What do we do?
  1. Document persistent and substantial heterogeneity of emission intensity for U.S. public firms
  2. Illustrate composition effect in Planner's solution of example w/ het. carbon dependence & endo. exit
  3. Extend example to general equilibrium model w/ endogenous entry & exit and standard climate block
  4. Characterize the Planner's problem and implement in decentralized equilibrium using 2 tax wedges
  5. Decompose the Planner's solution to climate scenario from related literature [in progress]

Empirical Analysis

Empirical Approach

- How does carbon dependence vary in the firm distribution?
  - Fact #1: There exists substantial variation in emission intensity of production capital
  - Fact #2: Across-firm variation in emission intensity >>> within-firm variation in emission intensity

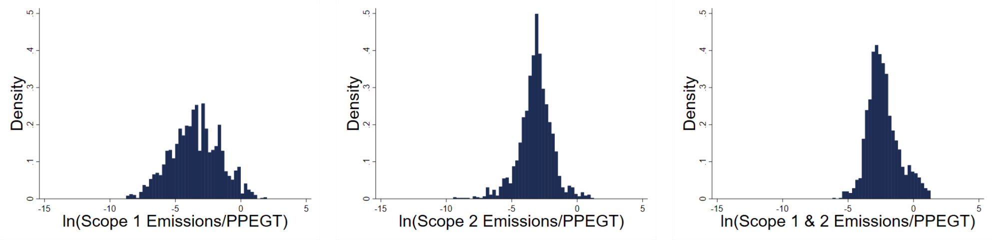
- Data: 5-year balanced panel of 389 public firms from 2016-2020 w/ annual frequency Data Availability
  - Bloomberg: Scope 1 (direct) & Scope 2 GHG emissions (from electricity use) in metric tons CO^2
  - Compustat: Gross Property, Plants, and Equipment ($PPEGT_{i,t}$), GDP-deflated to \$ thousands (2015)

- Focus on *emission intensity*: (log) ratio of emission to capital Summary Statistics

$$y_{i,t} = \log\left(\frac{Emissions_{i,t}}{PPEGT_{i,t}}\right)$$

Fact #1: There Exists Substantial Variation of Emission Intensity

- Pooled histograms show distribution of emission intensity is roughly normal Numerator



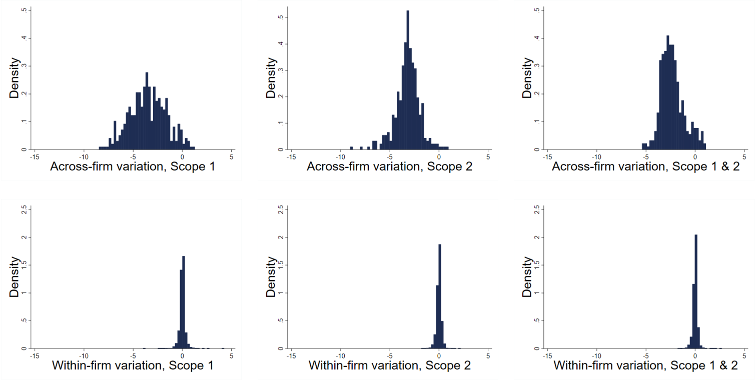
- Obs. at P90 creates 132$\times$ Scope 1 emissions per (real) dollar of capital more than obs. at P10
  - For Scope 2, obs. at P90 creates 19$\times$ more than obs. at P10
  - For Scope 1 + Scope 2, obs. at P90 creates 22$\times$ more than obs. at P10

Fact #2: Across-Firm Variation of Emission Intensity >>> Within-Firm Variation

- Regress emission intensity on firm-level fixed effects Numerator Industry FEs Firm Controls Year FEs

$$y_{i,t} = \alpha_i + \varepsilon_{i,t};\ \varepsilon_{i,t} \perp \alpha_i \Rightarrow \mathbb{V}\mathrm{ar}(y_{i,t}) = \underbrace{\mathbb{V}\mathrm{ar}(\alpha_i)}_{\text{across-firm}} + \underbrace{\mathbb{V}\mathrm{ar}(\varepsilon_{i,t})}_{\text{within-firm}}$$

- Across-firm variation ($\mathbb{V}$ar of FEs) is much larger than within-firm variation ($\mathbb{V}$ar of residuals)



- (Adjusted) R^2: (0.962) 0.970 Scope 1, (0.950) 0.960 Scope 2, (0.954) 0.964 Scope 1 + Scope 2

Illustrative Model

Illustrative 1-Period Model Economy with Climate Externality

- Two types of firms $i \in \{L, H\}$ w/ mass p_i & exo. carbon dependence a_i where $0 < a_L < a_H < 1$

- Type-i firm problem timing
  1. Observe carbon dependence a_i; invest in dirty capital k_d and clean capital k_g
  2. Draw TFP $z \sim G$ where $z > 0$; choose to exit (producing zero) or to operate and produce

$$F(z, k_g, k_d, a_i) = z[a_i k_d^{\rho} + (1 - a_i) k_g^{\rho}]^{\alpha/\rho}$$

- Climate externality:
  - Aggregate dirty capital utilized in production K^d creates environmental damage $D(K^d)Y$
  - Government pays $D(K^d)Y$ and finances it through lump-sum tax on consumer

- Consumer eats firm profits net of taxes

Model Equations

- Define investment policies $k_d(a_i)$ and $k_g(a_i)$ which solve

$$\max_{k_d, k_g} -k_d - k_g + \mathbb{E}_z \left[\max\left\{F(z, k_g, k_d, a_i), 0\right\}\right],$$

- Define exit policies $X(z, a_i)$

$$X(z, a_i) = \begin{cases} 0, & \text{if firm } (z, a_i) \text{ operates} \\ 1, & \text{if firm } (z, a_i) \text{ exits} \end{cases}$$

- Economy-wide aggregates

$$Y \equiv \sum_i p_i \int F(z, k_g(a_i), k_d(a_i), a_i)\left[1 - X(z, a_i)\right] dG(z) \qquad \text{Aggregate Output}$$

$$I \equiv \sum_i p_i \int \left[k_d(a_i) + k_g(a_i)\right] dG(z) \qquad \text{Aggregate Investment}$$

$$K^d \equiv \sum_i p_i \int k_d(a_i)\left[1 - X(z, a_i)\right] dG(z) \qquad \text{Aggregate Dirty Capital}$$

$$C = Y - I - T \qquad \text{Budget Constraint}$$

$$T = D\left(K^d\right) Y \qquad \text{GBC}$$

Business-As-Usual (BAU) Decentralized Equilibrium

- Operating generates positive value ex-post $\implies$ No firms exit $X^{BAU}(z, a_i) = 0 \; \forall (z, a_i)$
- FOCs characterize investment policies $\implies$ Capital mix is a constant given a_i

$$\frac{k_d^{BAU}(a_i)}{k_g^{BAU}(a_i)} = \left(\frac{a_i}{1 - a_i} \right)^{\frac{1}{1-\rho}} \equiv \eta(a_i)$$

- Investment policies

$$k_d^{BAU}(a_i) = \left(\alpha a_i \left[(1 - a_i)(\eta(a_i))^{\rho} + a_i \right]^{\alpha/\rho - 1} \mathbb{E}_z[z] \right)^{\frac{1}{1-\alpha}}$$

$$k_g^{BAU}(a_i) = \left(\alpha (1 - a_i) \left[a_i (\eta(a_i))^{\rho} + (1 - a_i) \right]^{\alpha/\rho - 1} \mathbb{E}_z[z] \right)^{\frac{1}{1-\alpha}} .$$

Planner Problem

- Planner internalizes climate externality while constrained to using firm-level technologies

$$W \equiv \max_{\{k_d(a_i),k_g(a_i)\}_{\forall i},\{X(z,a_i)\}_{\forall(i,z)},K^d} U(C)$$

$$\text{s.t. } K^d = \sum_i p_i \int k_d(a_i)[1 - X(z,a_i)]dG(z)$$

$$\text{where } Y = \sum_i p_i \int F(z, k_g(a_i), k_d(a_i), a_i)[1 - X(z,a_i)]dG(z)$$

$$I = \sum_i p_i \int [k_d(a_i) + k_g(a_i)]dG(z)$$

$$C = \left[1 - D\left(K^d\right)\right] Y - I$$

"Recipe" to Solve and Decentralize Planner Problem

1) Write Planner Lagrangian (max consumer utility s.t. climate and resource constraint)

2) Take FOCs of Planner Lagrangian with respect to aggregate choice variable $\{K_d\}$

3) Characterize Planner firm-level policies $\{X(z, a_i), k_g(a_i), k_d(a_i)\}$

4) Guess taxation scheme & verify it induces Planner firm-level policies

"Recipe" to Solve and Decentralize Planner Problem: Steps 1 and 2

1) Write Planner Lagrangian (max consumer utility s.t. climate and resource constraint)

$$\mathcal{L} = U\Bigg([1 - D(K^d)] \underbrace{\sum_i p_i \int F(z, k_g(a_i), k_d(a_i), a_i)[1 - X(z, a_i)] dG(z)}_{\equiv Y}$$

$$- \underbrace{\sum_i p_i \int [k_d(a_i) + k_g(a_i)] dG(z)}_{\equiv I} \Bigg)$$

$$+ \lambda^k \Bigg(K^d - \underbrace{\sum_i p_i \int k_d(a_i)[1 - X(z, a_i)] dG(z)}_{\equiv K^d} \Bigg)$$

2) Take FOCs of Planner Lagrangian with respect to aggregate choice variable $\{K_d\}$

$$\underbrace{\lambda^k}_{\text{shadow cost of aggregate dirty capital}} = \underbrace{U'(C) D'(K^d) Y}_{\text{change in environmental damage (in marginal utility)}}$$

3) Characterize Planner firm-level policies $\{X(z, a_i), k_g(a_i), k_d(a_i)\}$

- Planner exits firm (z, a_i) iff

$$\underbrace{[1 - D(K^d)]F(z, k_g(a_i), k_d(a_i), a_i)}_{\text{output (after environmental damage)}} < \underbrace{\frac{\lambda^k}{U'(C)} k_d(a_i)}_{\text{shadow cost of firm's dirty capital}}$$

- FOC wrt $k_g(a_i)$

$$\underbrace{1}_{\text{MC of investment}} = \underbrace{[1 - D(K^d)]\mathbb{E}_z\left[F_2(z, k_g(a_i), k_d(a_i), a_i)[1 - X(z, a_i)]\right]}_{\text{expected MPK (after environmental damage)}}$$

- FOC wrt $k_d(a_i)$

$$\underbrace{1}_{\text{MC of investment}} = \underbrace{[1 - D(K^d)]\mathbb{E}_z\left[F_3(z, k_g(a_i), k_d(a_i), a_i)[1 - X(z, a_i)]\right]}_{\text{expected MPK (after environmental damage)}}$$

$$\underbrace{-\frac{\lambda^k}{U'(C)}\mathbb{E}_z[1 - X(z, a_i)]}_{\text{expected marginal shadow cost of firm's dirty capital}}$$

4) Guess taxation scheme & verify its induces Planner firm-level policies

- Conjecture taxation scheme with output tax $\tau^d = D(K^d)$ and dirty capital tax $\tau^k = D'(K^d)Y$

- Firm problem w/ output tax and dirty capital tax

$$\max_{k_d, k_g} -k_d - k_g + \mathbb{E}_z \left[\max\{(1-\tau^D)F(z, k_d, k_g, a_i) - \tau^k k_d, 0\} \right]$$

- Firm-level policies under taxation scheme match Planner firm-level policies

How does the Planner allocation differ from the BAU allocation?

1. *Composition Effect*: Planner exits firms with low z especially among a_H firms

$$X^{Planner}(z, a_i) = 1 \text{ for all } z < \bar{z}(a_i) \text{ where } \bar{z}(a_H) > \bar{z}(a_L) > 0$$

2. *Substitution Effect*: Planner chooses greener capital mix for all firms

$$\frac{k_d^{Planner}(a_i)}{k_g^{Planner}(a_i)} = M(a_i)\eta(a_i) < \eta(a_i) = \frac{k_d^{BAU}(a_i)}{k_g^{BAU}(a_i)}$$

where $M(a_i) \equiv (1 + \tau^k \mathbb{E}_z[1 - X(z, a_i)])^{\frac{1}{\rho-1}} < 1$

3. *Scale Effect*: Planner lowers total investment by all firms, lower marginal benefit of investing

$$k_d^{Planner}(a_i) + k_g^{Planner}(a_i) < k_d^{BAU}(a_i) + k_g^{BAU}(a_i)$$

Parameterized Example[1]

- Planner raises consumption by lowering dirty capital and environmental costs despite lower output

	C	Y	I	$D(K^d)$
BAU	0.112	0.176	0.052	0.064
Planner	0.115	0.162	0.041	0.036

- *Composition* (low z firms exit especially H), *substitution* (greener capital mix), *scale* (less capital

i		$\bar{z}(a_i)$	$k_d(a_i)/k_g(a_i)$	$k_d(a_i)+k_g(a_i)$
L	BAU	0	0.183	0.053
	Planner	0.005	0.095	0.047
H	BAU	0	5.47	0.053
	Planner	0.034	2.86	0.035

- Decompose welfare change into *composition* (7%), + *substitution* (14%), + *scale* (79%) Details

[1] $D(K^d) = 1 - \exp(-\gamma K^d)$, $\gamma = 2.5$, $U(C) = C$, $a_L = 0.3$, $a_H = 0.7$, $p_L = p_H = 0.5$, $z \sim U[0,1]$, $\alpha = 0.3$, $\rho = 0.5$

Quantitative Model

Extension to Quantitative Model

- We ultimately aim to show magnitude of effect in full dynamic GE model comparable to literature
- Climate-macro literature: infinite-horizon, exogenous growth, carbon cycle feedback
- Standard firm-dynamics ingredients: AR(1) log-TFP process, adjustment costs, fixed prod. cost
- GE effects: labor as production input with GE wage & endogenous RF rate
- *Composition effect*: endogenous & exogenous firm exits, endogenous entry w/ fixed entry cost

Firm Problem Timing Firm Bellman Firm Entry Government and Consumers Equilibrium Definition

Carbon Cycle in Dynamic GE Model

- Law of motion of carbon stocks (environmental state variables), given exo. emissions capturing χ_t:

$$\underbrace{S_t^1}_{\textbf{permanent}\text{ carbon stock}} = S_{t-1}^1 + \varphi_1 \underbrace{(1-\chi_t)}_{\text{uncaptured}} \underbrace{\gamma K_t^d}_{\text{emissions}}$$

$$\underbrace{S_t^2}_{\textbf{persistent}\text{ carbon stock}} = \varphi_3 S_{t-1}^2 + (1-\varphi_1)\varphi_2 \underbrace{(1-\chi_t)}_{\text{uncaptured}} \underbrace{\gamma K_t^d}_{\text{emissions}}$$

- Environmental damage fraction $D(S_t^1 + S_t^2)$ increasing function of total carbon stock $S_t^1 + S_t^2$
- Total damage is scaled by output of economy $D(S_t^1 + S_t^2)Y_t$ and paid by government
- Ultimately paid by consumers through lump-sum taxes to balance government budget constraint

"Recipe" to Solve and Decentralize the Planner Problem

in words in math

1) Write Planner Lagrangian (max consumer utility stream st climate, resource constraints) Lagrangian

2) Take FOCs of Planner Lagrangian wrt aggregate choice variables $\{K_t^d, S_t^1, S_t^2, B_t\}_{t=0}^{\infty}$ Climate Multipliers

3) Characterize Planner firm-level policies $\{k_{d,t}(s), k_{g,t}(s), X_t(s), L_t^D(s), k_{d,t}^E(a), k_{g,t}^E(a), X^E(a)\}_{t=0}^{\infty}$ Continuation

4a) Conjecture optimization problem and verify its optimal policies match Planner's policies Conjecture

4b) Prove optimization problem (evaluated at optimal policies) achieves marginal social value of firm
$\Longrightarrow$ Derive **Augmented Firm Bellman Equation** Augmented Firm Bellman Augmented Entrant Bellman

- As in Lucas and Moll (2014), Moll and Nuno (2018), Ottonello and Winberry (2023)
- We extend methodology to handle discrete firm choice (endogenous exit) and endogenous entry

4c) Use **Augmented Firm Bellman Equation** to guess taxation scheme that induces Planner policies

4d) Construct CE to verify taxation scheme implements Planner allocation Implementation

Robustness

Optimal Carbon Taxes

We prove by construction that the Planner's solution is implementable in a CE through 2 taxes:

1. Output Tax

$$\tau_t^D = \underbrace{D(S_t^1 + S_t^2)}_{\text{Environmental Damage}}$$

2. Carbon Tax

$$\tau_t^k = \frac{\lambda_t^k}{U'(C_t)}$$

where

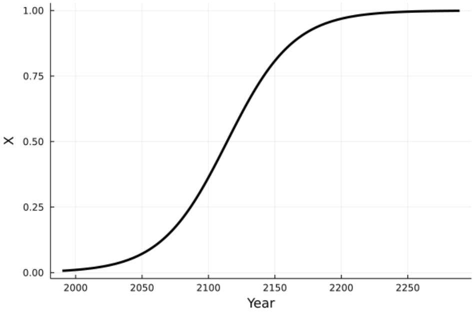
$$\underbrace{\lambda_t^k}_{\text{Shadow Cost of Dirty Capital}} = \underbrace{(1-\chi_t)\gamma}_{\text{Uncaptured Emissions per } K^d} \times \underbrace{\left[\sum_{s=t}^{\infty} \beta^{s-t} U'(C_s) D'(S_s^1 + S_s^2) Y_s [\varphi_1 + (1-\varphi_1)\varphi_2\varphi_3^{s-t}]\right]}_{\text{Future Marginal Cost of Uncaptured Carbon Emissions}}$$

Quantitative Climate Scenario

Quantitative Climate Scenario

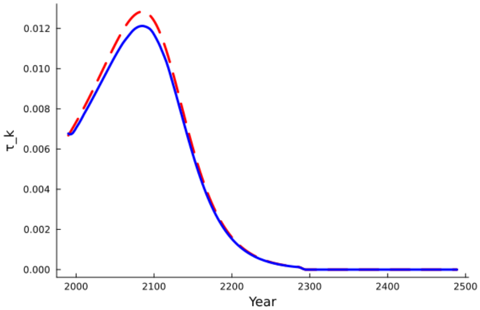
- Start firm distribution on BGP (i.e., $R^{SS} = \frac{1}{\beta}$, BAU with constant environmental damage)
- Initialize world in 1990 at true carbon stock, and calibrate to match global CO^2 emissions in 1990
- Assume exo. emissions capturing improvement from Krusell and Smith (2022)

Figure: Exogenous Emissions Capturing Improvement

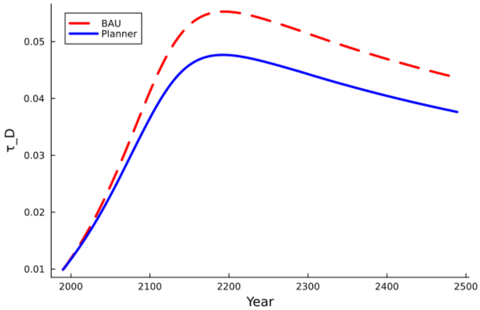


Functional Forms Full Calibration

Figure: Planner Multipliers



(a) Shadow Cost of Dirty Capital to Marginal Utility

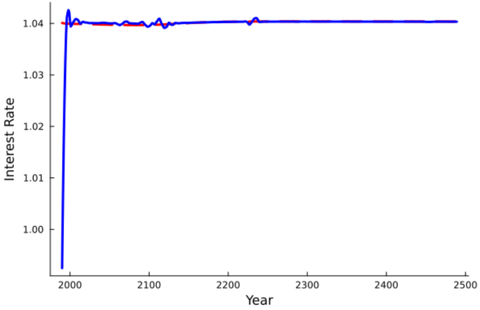


(b) Environmental Damage

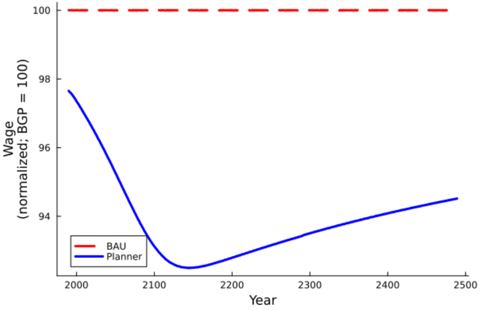
- Left: $\tau_t^k = \frac{\lambda_t^k}{U'(C_t)}$ evaluated under the BAU and Planner scenarios
- Right: $\tau_t^D = D(S_t^1 + S_t^2)$ evaluated under the BAU and Planner scenarios

Long Horizon

Figure: Equilibrium Prices



(a) Interest Rate

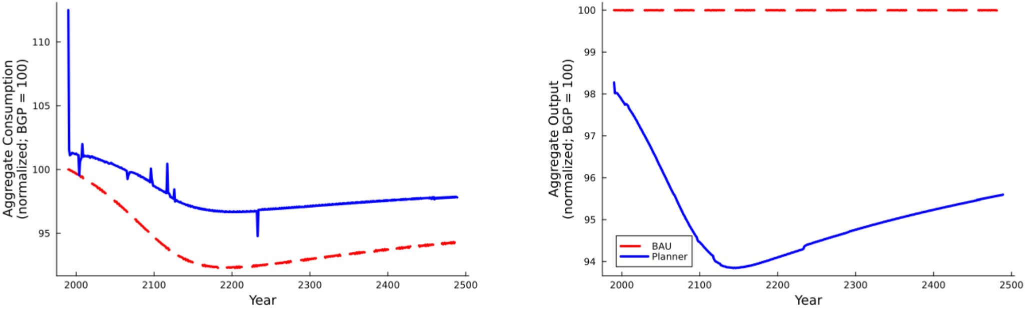


(b) Wage

- Left: Initial disinvestment $\implies$ consumption $\uparrow$ $\implies$ interest rate $\downarrow$
- Right: Capital level $\downarrow \Rightarrow$ marginal product of labor $\downarrow$

Long Horizon Short Horizon

Figure: Aggregates (1/2)

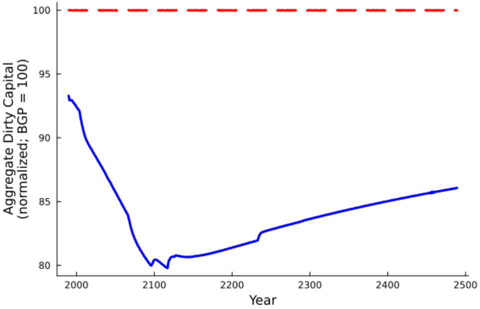


(a) Aggregate Consumption (b) Aggregate Output

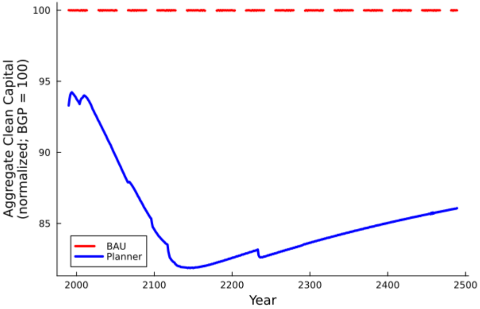
- Left: Initial spike from sudden disinvestment. Planner solution ensures higher consumption path
- Right: Gross production Y_t is smaller under planner, despite the higher consumption path

Long Horizon

Figure: Aggregates (2/2)



(a) Aggregate Dirty Capital

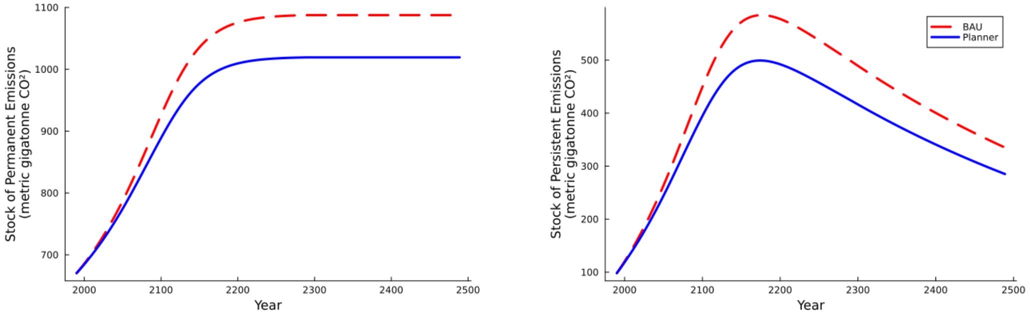


(b) Aggregate Clean Capital

- *Scale effect*: Both types of capital are lower
- *Substitution effect*: Dirty capital drops more than clean capital

Long Horizon

Figure: Climate



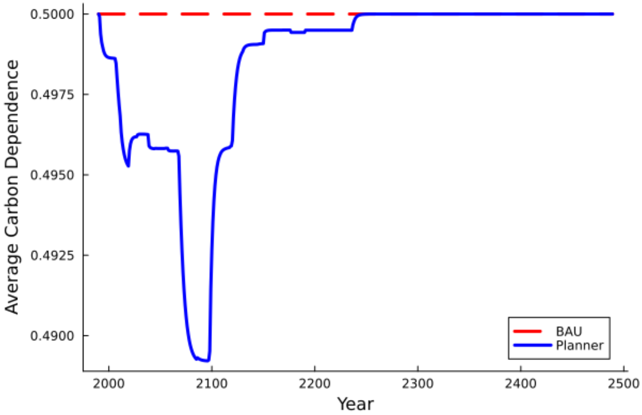
(a) Permanent Stock of Emissions

(b) Transitory Stock of Emissions

- Lower dirty capital $\implies$ fewer emissions $\implies$ lower stocks of carbon emissions

Long Horizon

Figure: Average Carbon Dependence



- *Composition effect*: Planner firm-level policies $\implies$ lower average carbon intensity of firms

Long Horizon

Conclusion

Conclusion

- We find persistent heterogeneity in the carbon emissions intensity of U.S. public firms
- We illustrate the composition effect that is a result of this heterogeneity
- We develop a GE environmental-macro model with heterogeneous firm carbon dependence
- We characterize and solve the Planner's problem, and prove implementability via simple taxes
- We decompose the Planner's impact on economy to measure the composition effect [in progress]

Appendix

Data Availability

Table: Scope 1 Sample Selection

Variable	Mean (all)	SD (all)	N (all)	Mean (Scope 1)	SD (Scope 1)	N (Scope 1)	T-Statistic
EBITDA (real)	845.36	3477.96	19279	4767.99	8259.01	1945	-20.76
EBITDA/PPEGT	-1.65	135.51	19279	0.41	0.80	1945	-2.11
Employment	15.27	60.00	19279	62.54	138.09	1945	-14.95
Employment/PPEGT (real)	0.04	0.46	19279	0.01	0.03	1945	8.63
PPEGT (real)	4776.78	24032.17	19279	25960.71	56013.92	1945	-16.53

Table: Scope 2 Sample Selection

Variable	Mean (all)	SD (all)	N (all)	Mean (Scope 2)	SD (Scope 2)	N (Scope 2)	T-Statistic
EBITDA (real)	845.36	3477.96	19279	4904.51	8489.66	1825	-20.27
EBITDA/PPEGT	-1.65	135.51	19279	0.42	0.89	1825	-2.12
Employment	15.27	60.00	19279	64.50	141.83	1825	-14.71
Employment/PPEGT (real)	0.04	0.46	19279	0.01	0.03	1825	8.46
PPEGT (real)	4776.78	24032.17	19279	25675.04	55336.19	1825	-15.99

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Data Availability cont.

Table: Scope 1 + Scope 2 Sample Selection

Variable	Mean (all)	SD (all)	N (all)	Mean (Scope 1+2)	SD (Scope 1+2)	N (Scope 1+2)	T-Statistic
EBITDA (real)	845.36	3477.96	19279	4863.87	8426.99	1810	-20.13
EBITDA/PPEGT	-1.65	135.51	19279	0.40	0.84	1810	-2.11
Employment	15.27	60.00	19279	64.48	142.25	1810	-14.60
Employment/PPEGT (real)	0.04	0.46	19279	0.01	0.03	1810	8.51
PPEGT (real)	4776.78	24032.17	19279	25631.08	55421.38	1810	-15.87

Table: Sample Selection by Sector

SIC Division	Percent (all)	Percent (Scope 1)	Percent (Scope 2)	Percent (Scope 1+2)
A: Agricultural, Forestry, and Fishing	0.35	0.00	0.00	0.00
B: Mining	7.38	10.95	9.26	9.34
C: Construction	1.30	0.26	0.55	0.28
D: Manufacturing	50.19	59.18	59.23	59.72
E: Transportion, Communications, Electric, and Gas	7.75	11.31	11.51	11.33
F: Wholesale Trade	3.44	2.11	2.19	2.21
G: Retail Trade	6.44	4.88	4.93	4.97
I: Services	23.13	11.31	12.33	12.15

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Summary Statistics

Variable	Mean	SD	P10	P25	Median	P75	P90	N
ln(Scope 1 + 2 Emissions / PPEGT (real))	-2.34	1.32	-3.66	-3.11	-2.50	-1.69	-0.55	1810
ln(Scope 1 Emissions / PPEGT (real))	-3.48	1.88	-5.93	-4.74	-3.49	-2.15	-1.04	1945
ln(Scope 2 Emissions / PPEGT (real))	-3.22	1.37	-4.70	-3.81	-3.13	-2.47	-1.77	1825

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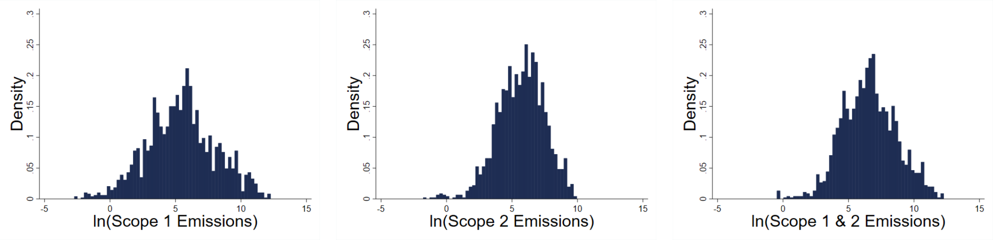
Fact #1: There Exists Substantial Variation of Log Emission [numerator]

- Pooled histograms show distribution of log emissions is roughly normal



- Obs. at P90 creates 2340× Scope 1 emissions per (real) dollar of capital more than obs. at P10
  - For Scope 2, obs. at P90 creates 248× more than obs. at P10
  - For Scope 1 + Scope 2, obs. at P90 creates 333× more than obs. at P10

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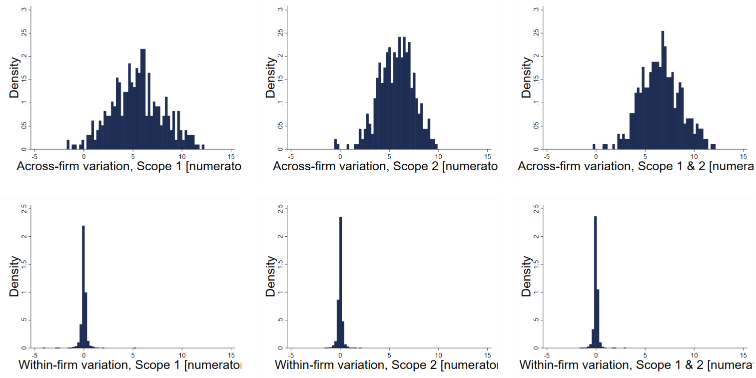
Fact #2: Across-Firm Variation of Log Emission >>> Within-Firm Variation [numerator]

- Regress log emissions on firm-level fixed effects

$$y_{i,t} = \alpha_i + \varepsilon_{i,t};\ \varepsilon_{i,t} \perp \alpha_i \Rightarrow \mathbb{V}\mathrm{ar}(y_{i,t}) = \underbrace{\mathbb{V}\mathrm{ar}(\alpha_i)}_{\text{across-firm}} + \underbrace{\mathbb{V}\mathrm{ar}(\varepsilon_{i,t})}_{\text{within-firm}}$$

- Across-firm variation (Var of FE) is much larger than within-firm variation (Var of residuals)



- (Adjusted) R^2 is (0.982) 0.986 Scope 1, (0.980) 0.984 Scope 2, (0.986) 0.989 Scope 1 + Scope 2

Empirical Results with Industry FEs

Table: Adjusted R-Squares of OLS Regressions

	Firm FEs	2-digit SIC FEs	4-digit SIC FEs
Scope 1 Emissions	0.962	0.649	0.801
Scope 2 Emissions	0.943	0.368	0.569
Scope 1 + Scope 2 Emissions	0.945	0.526	0.734

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Empirical Results with Within-Firm Controls

Table: Adjusted R-Squares of OLS Regressions with Included Within-Firm Controls

	No Controls	Profitability	Profitability, Capital Intensity
Scope 1 Emissions	0.962	0.967	0.970
Scope 2 Emissions	0.943	0.950	0.962
Scope 1 + Scope 2 Emissions	0.945	0.957	0.968

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Addition of Year FEs

- Regress emission intensity on firm-level fixed effects + year fixed effects

$$y_{i,t} = \alpha_i + \gamma_t + \varepsilon_{i,t}$$

Table: Adjusted R-Squares of OLS Regressions with Year FE's added

	No Year FE	With Year FE
Scope 1 Emissions	0.962	0.964
Scope 2 Emissions	0.943	0.956
Scope 1 + Scope 2 Emissions	0.945	0.956

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Decomposition Between BAU and Planner Allocations

What changes between BAU and Planner allocations?

1. *Composition*: Planner makes low productivity—especially high carbon dependence—firms exit
2. *Substitution*: Planner chooses greener ratios of capital for all firms
3. *Scale*: Planner lowers total investment for all firms

Composition	$X(z,a)$	BAU	Planner	Planner	Planner
Substitution	$k_d(a)/k_g(a)$	BAU	BAU	Planner	Planner
Scale	$k_d(a)+k_g(a)$	BAU	BAU	BAU	Planner
Contribution (cumulative, %)		0	7	21	100
Contribution (%)		0	7	14	79

Notes: Cumulative contribution measured as $(C - C_{BAU})/(C_{Planner} - C_{BAU})$.

Composition effect alone accounts for 7 percent of change in welfare from BAU to planner allocation

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Firm Problem

- Firms max dividend stream discounted at RF rate; face fixed costs of production c_f [units of labor]
- Firms heterogeneous in: TFP z, carbon dependence a, dirty capital k_d, green capital k_g
  - For convenience, define $s \equiv [z, a, k_d, k_g]'$
- Production Cobb-Douglas over capital and labor L, Dixit-Stiglitz aggregator over capital k_d, k_g:

$$\pi_t(s) = \max_L \exp(z) A_t^{1-\alpha} [a k_d^\rho + (1-a) k_g^\rho]^{\frac{\alpha}{\rho}} L^\nu - w_t L - w_t c_f$$

- Exogenous exit probability $\lambda \in [0, 1)$, otherwise can choose to exit or continue
- Adjustment costs $\phi(x_i, k_i)$ for each capital; dirty capital produces emissions linearly $\xi = \gamma k_g$
- New firms enter competitively with entry cost κ [units of labor], endogenous mass $B_t \geq 0$
  - Firms observe carbon dependence a after entry cost paid, then choose initial investment in capital

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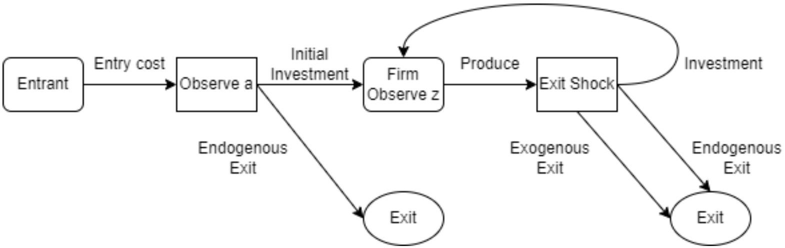
Timing



1. Entrants pay entry cost $w_t \kappa$
2. Entrants observe a, decide whether to continue entering, and set up initial capital k_d, k_g
3. Firms observe z, employ labor L, generate cash flows $\pi(s)$
4. Exogenous exit shock realizes
5. Remaining firms make endogenous exit choice, continuing firms make investment decisions x_d, x_g

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Firm Bellman

Ex-ante value of firm in state s:

$$V_t(s) = \pi_t(s) + \lambda V_t^X(k_d, k_g) + (1-\lambda)\max\{V_t^C(s), V_t^X(k_d, k_g)\}$$

Exiting firm eats nondepreciated capital net of adjustments costs of driving capital to zero:

$$V_t^X(k_d, k_g) = (1-\delta)(k_d + k_g) - \phi[-(1-\delta)k_d, k_d] - \phi[-(1-\delta)k_g, k_g]$$

A continuing firm chooses investment in clean and dirty capital, subject to adjustment costs:

$$V_t^C(s) = \max_{x_d, x_g} -x_d - x_g - \phi[x_d, k_d] - \phi[x_g, k_g] + \frac{1}{R_t}\mathbb{E}[V_{t+1}(s')],$$
$$\text{s.t. } k_i' = (1-\delta)k_i + x_i, \quad \text{for } i \in \{d, g\}$$

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Firm Entry

Competitive entry of entrants before observing information:

$$w_t \kappa \geq \mathbb{E}[V_t^E(a)]$$

After observing their signals, an entrant's value is the following:

$$V_t^E(a) = \max\{0, \max_{k_d, k_g} -k_d - k_g + \mathbb{E}[V_t(s)]\}$$

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Government and Consumers

- Government finances environmental damage through lump-sum to clear GBC: $D(S_t^1 + S_t^2)Y_t = T_t$
- Consumers pay tax T_t, supply unit labor, receive firm profits, invest in RF asset, discount at β

$$\max \sum_{t=0}^{\infty} \beta^t U(C_t)$$

$$\text{s.t. } C_t + A_{t+1} + T_t = w_t L_t + \Pi_t + R_t A_t$$

- Consumer FOC for risk-free asset implies standard Euler condition $U'(C_t) = \beta U'(C_{t+1}) R_t$

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Equilibrium Definition

A **Carbon-Cycle Competitive Equilibrium** is a set of allocations $\{k^E_{d,t}(a), k^E_{g,t}(a), x_{d,t}(s), x_{g,t}(s), b_t, L^d_t(s), S^1_t, S^2_t, \mu_t\}_{t=0}^{\infty}$, prices $\{w_t, R_t\}_{t=0}^{\infty}$, taxes $\{T_t\}_{t=0}^{\infty}$, continuation rules $\{X_t(s), X^E_t(a)\}_{t=0}^{\infty}$, and mass of entrants $\{B_t\}_{t=0}^{\infty}$ such that

- Firm decisions solve their problems
- Household decisions solve their problem
- Government budget constraint holds
- Free entry condition holds
- Markets clear in the labor and risk-free asset market

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Calibration

Parameter	Value	Source	Description
		Preferences	
β	0.971	$R^{SS} = 1.03$	Time preference
ι	0.01	Krusell and Smith (2022)	Economy growth rate
		Production	
α	0.3	Standard parameter	Capital share
ν	0.65	Standard parameter	Labor share
λ	0.08	Ottonello and Winberry (2023)	Exogenous Exit Rate
$\hat{\gamma}$	0.297	Corbae and D'Erasmo (2021)	Capital adjustment cost
c_f	0.0003	-	Fixed cost of production
δ	0.12	Standard parameter	Depreciation rate
ρ	0.5	-	Substitutability between capital types
ρ_z	0.659	Khan and Thomas (2013)	Idiosyncratic productivity persistence
σ_z	0.118	Khan and Thomas (2013)	Idiosyncratic productivity volatility
		Climate	
Δ	0.000053	Golosov et al. (2014)	Damage function parameter
φ_1	0.2	Krusell and Smith (2022)	Fraction of permanent emissions
φ_2	0.398	Krusell and Smith (2022)	Fraction of dissipated persistent emissions
φ_3	0.998	Krusell and Smith (2022)	Persistence of persistent emission stock
$\bar{S}$	581	Golosov et al. (2014)	Pre-industrial level of emissions
$S_{9,1}$	684	Krusell and Smith (2022)	Stock of persistent emission in 1999
$S_{9,2}$	118	Krusell and Smith (2022)	Stock of permanent emission in 1999
E_9	8.741	Krusell and Smith (2022)	Emissions in 1999
$t_{\chi=0.01}$	10	Krusell and Smith (2022)	Years until one percent of emissions are captured
$t_{\chi=0.5}$	125	Krusell and Smith (2022)	Years until half emissions are captured
$t_{\chi=1}$	301	Krusell and Smith (2022)	Years until all emissions are captured

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Description of Planner Problem

- Planner operates firm-level tech. & internalizes future environmental damage from utilizing dirty capital in production
- Choose labor demand, continuation decision, investment decisions of incumbent firms
- Choose mass of entrants, entry/investment decision after observing each a for entrants
- Faces environmental damage, law of motion of carbon stocks, and resource constraint
- Why hard to solve? State variable of Planner Problem is firm distr. (infinite-dimensional object)

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Planner Problem

The planner solves

$$\mathcal{W}_t(\mu, S^1, S^2) = \max_{x_d(\cdot), x_g(\cdot), X(\cdot), x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), L^d(\cdot), B, K^d, S^{1\prime}, S^{2\prime}} U(C_t)$$
$$+ \beta \mathcal{W}_{t+1}(T^*(\mu, x_d(\cdot), x_g(\cdot), X(\cdot), x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B), S^{1\prime}, S^{2\prime})$$

subject to

$$C_t = (1 - D(S^{1\prime} + S^{2\prime}))Y_t - I_t - G_t$$
$$1 = \int (L^d(s) + c_f)\Phi(\mu, x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(s)ds + B\kappa$$
$$K^d = \int k_d \Phi(\mu, x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(s)ds$$
$$S^{1\prime} = S^1 + (1 - \chi_t)\varphi_1 \gamma K^d$$
$$S^{2\prime} = \varphi_3 S^2 + (1 - \chi_t)(1 - \varphi_1)\varphi_2 \gamma K^d$$
$$B \geq 0$$

where

$$Y_t = \int \exp(z) A_t^{1-\alpha} [a k_d^\rho + (1 - a) k_g^\rho]^{\frac{\alpha}{\rho}} (L^d(s))^\mu \Phi(\mu, x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(s)ds$$
$$I_t = \int (x_g(s) + x_d(s))(1 - \lambda)\Phi(\mu, x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(s)ds$$
$$+ \int (-(1 - \delta)k_d - (1 - \delta)k_g)\lambda \Phi(\mu, x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(s)ds$$
$$+ \int (x_g^E(a) + x_d^E(a))\mathbb{1}_{X^E(a)=0} B Q_a(a)da$$
$$G_t = \int (\phi[x_g(s), k_g] + \phi[x_d(s), k_d])(1 - \lambda)\Phi(\mu, x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(s)ds$$
$$+ \int (\phi[-(1 - \delta)k_g, k_g] + \phi[-(1 - \delta)k_d, k_d])\lambda \Phi(\mu, x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(s)ds$$

and measure of productive firms $\Phi(\cdot)$ and measure operator $T^*(\cdot)$ follow definitions on next slide

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Planner Problem: Definitions of $\Phi(\cdot)$ and $T^*(\cdot)$

- $\Phi(\cdot)$ is the measure of productive firms in economy

$$\Phi(\mu, x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(z', k_d', k_g', a') = \mu(z', k_d', k_g', a') + \mathbb{1}_{k_d' = x_d^E(a')} \mathbb{1}_{k_g' = x_g^E(a')} \mathbb{1}_{X^E(a') = 0} Q_z(z') Q_a(a') B$$

- $T^*(\cdot)$ is the next period's measure of incumbent firms

$$T^*(\mu, x_d(\cdot), x_g(\cdot), X(\cdot), x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(z', k_d', k_g', a') = \int \mathbb{1}_{k_d' = (1-\delta) k_d(s) + x_d(s)} \mathbb{1}_{k_g' = (1-\delta) k_g(s) + x_g(s)} \mathbb{1}_{z' = \rho z + \epsilon} \mathbb{1}_{a' = a} \mathbb{1}_{X(s) = 0} p(\epsilon) \times (1-\lambda) \Phi(\mu, x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(s) ds$$

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Step 1: Write Planner Lagrangian

Planner Lagrangian

$$\mathcal{L}_t = U(C_t) + \lambda_t^L \underbrace{\left(1 - \int (L^d(s) + c_f)\Phi(\mu, x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(s)ds - B\kappa\right)}_{\text{Labor Supply Constraint}}$$

$$+ \lambda_t^k \underbrace{\left(K^d - \int k_d \Phi(\mu, x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B)(s)ds\right)}_{\text{Definition of total dirty capital}} + \lambda_t^1 \underbrace{\left(S^{1\prime} - S^1 - (1-\chi_t)\varphi_1\gamma K^d\right)}_{\text{Law of motion of permanent stock}}$$

$$+ \lambda_t^2 \underbrace{\left(S^{2\prime} - \varphi_3 S^2 - (1-\chi_t)(1-\varphi_1)\varphi_2\gamma K^d\right)}_{\text{Law of motion of persistent stock}} + \lambda_t^B \underbrace{(B - 0)}_{\text{Nonnegative entrant mass}}$$

$$+ \beta \mathcal{W}_{t+1}\left(T^*(\mu, x_d(\cdot), x_g(\cdot), X(\cdot), x_d^E(\cdot), x_g^E(\cdot), X^E(\cdot), B), S^{1\prime}, S^{2\prime}\right)$$

where C_t, T^*, Φ are defined on previous slide Back

Step 2 and 3: FOCs of Planner Lagrangian wrt aggregates and firm-level policies

2) Take FOCs of Planner Lagrangian wrt aggregate choice variables
   - Assuming transversality conditions, FOC w.r.t. $[K^d, S^{1\prime}, S^{2\prime}]$ imply

$$\lambda_t^k = (1-\chi_t)\gamma\varphi_1\lambda_t^1 + (1-\chi_t)\gamma(1-\varphi_1)\varphi_2\lambda_t^2$$

$$\lambda_t^1 = \sum_{s=t}^{\infty} \beta^{s-t} U'(C_s) D'(S_s^1 + S_s^2) Y_s$$

$$\lambda_t^2 = \sum_{s=t}^{\infty} (\varphi_3\beta)^{s-t} U'(C_s) D'(S_s^1 + S_s^2) Y_s$$

3) Characterize Planner firm-level policies
   - Marginal social value of firm at s' shows up in continuation value for investment FOCs

$$\frac{\partial \mathcal{W}_{t+1}(\mu', S^{1\prime}, S^{2\prime})}{\partial \mu'(s')}$$

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Step 4a: Conjecture optimization problem and verify optimal policies match Planner's policies

- Define $\hat{\omega}_t(s, \mu, S^1, S^2)$ as marginal social value of firm over marginal utility

$$\hat{\omega}_t(s, \mu, S^1, S^2) \equiv \frac{\partial \mathcal{W}_t(\mu, S^1, S^2)}{\partial \mu(s)} \frac{1}{U'(C_t)}$$

- Conjecture optimization problem $V_t(s, \mu, S^1, S^2)$

$$\begin{aligned} V_t(s, \mu, S^1, S^2) \equiv \max_{L, x_d, x_g, X} & (1 - \tau_t^D) \exp(z) A_t^{1-\alpha} [a k_d^\rho + (1-a) k_g^\rho]^{\frac{\alpha}{\rho}} L^\nu \\ & + (1-\lambda)(-x_g - x_d - \phi[x_g, k_g] - \phi[x_d, k_d]) \\ & + \lambda((1-\delta)k_g + (1-\delta)k_d - \phi[-(1-\delta)k_g, k_g] - \phi[-(1-\delta)k_d, k_d]) \\ & - \hat{w}_t(L + c_f) - \tau_t^k k_d + (1-X)(1-\lambda) \frac{1}{\hat{R}_t} \mathbb{E}[\hat{\omega}_{t+1}(s', \mu', S^{1\prime}, S^{2\prime})] \end{aligned}$$

subject to $x_i = -(1-\delta)k_i$ if $X = 1$ and $k_i' = (1-\delta)k_i + x_i$,

where $\tau_t^k \equiv \frac{\lambda_t^k}{U'(C_t)}, \tau_t^D \equiv D(S^{1\prime} + S^{2\prime}), \hat{w}_t \equiv \frac{\lambda_t^L}{U'(C_t)}, \frac{1}{\hat{R}_t} \equiv \beta \frac{U'(C_{t+1})}{U'(C_t)}$

- Optimal policies from $V_t(s, \mu, S^1, S^2)$ match Planner policies $\Longrightarrow$ problems induce same solution

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Step 4a: Derive Augmented Firm Bellman Equation

- Problem $V_t(s, \mu, S^1, S^2)$ evaluated at its optimal policies $\implies$ $V_t(s, \mu, S^1, S^2)$ achieves marginal social value of firm

$$V_t(s, \mu, S^1, S^2) = \hat{\omega}_t(s, \mu, S^1, S^2) \underbrace{\equiv \frac{\partial \mathcal{W}_t(\mu, S^1, S^2)}{\partial \mu(s)} \frac{1}{U'(C_t)}}_{\text{by definition}}$$

- Therefore, $\hat{\omega}_t(s, \mu, S^1, S^2)$ is a Bellman equation

$$\begin{aligned} \hat{\omega}_t(s, \mu, S^1, S^2) = \max_{L, x_d, x_g, X} & (1 - \tau_t^D) \exp(z) A_t^{1-\alpha} [a k_d^\rho + (1-a) k_g^\rho]^{\frac{\alpha}{\rho}} L^\nu \\ & + (1-\lambda)\left(-x_g - x_d - \phi[x_g, k_g] - \phi[x_d, k_d]\right) \\ & + \lambda\left((1-\delta)k_g + (1-\delta)k_d - \phi[-(1-\delta)k_g, k_g] - \phi[-(1-\delta)k_d, k_d]\right) \\ & - \hat{w}_t(L + c_f) - \tau_t^k k_d + (1-X)(1-\lambda)\frac{1}{\hat{R}_t}\mathbb{E}[\hat{\omega}_{t+1}(s', \mu', S^{1\prime}, S^{2\prime})] \end{aligned} \tag{1}$$

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Step 4a: Derive Augmented Entrant Bellman Equation

Planner's FOC wrt B reads:

$$\mathbb{E}[\hat{\omega}_t^E(a,\mu,S^1,S^2)] + \frac{\lambda_t^B}{U'(C_t)} = \hat{w}_t\kappa, \tag{2}$$

where:

$$\hat{\omega}_t^E(a,\mu,S^1,S^2) = \max_{\hat{X}^E(a),\hat{x}_d^E(a),\hat{x}_g^E(a)} (1-\hat{X}^E(a))[-\hat{x}_d^E(a) - \hat{x}_g^E(a) + \mathbb{E}[\hat{\omega}_t(s,\mu,S^1,S^2)]]$$

subject to $\hat{x}_i^E = -(1-\delta)k_i$ if $\hat{X}(s) = 1\ \forall i \in \{d,g\}$

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Step 4d) Implementation and Robustness

- Under taxes $\{\tau_s^D, \tau_s^k\}_{s=t}^{\infty}$ and prices $\{w_s, R_s\}_{s=t}^{\infty} = \{\tau_s^L / U'(C_s), (\beta U'(C_{s+1})/U'(C_s))^{-1}\}_{s=t}^{\infty}$, decentralized Bellman is identical to augmented Bellman

- Planner FOC's then allow us to construct a CE under these taxes and prices

- Implementation is fairly robust, can allow for: Emissions Details
    - Unobservability of k_d by taxation authority, where instead implementation is through tax on emissions
    - Such emissions can have noise, and/or can only be noisily observed (due to risk-neutrality of firms)
    - Most other changes to the firm problem

- What breaks implementation? Financial frictions
    - Taxes affect balance sheet in decentralized world, but not equivalent terms from Planner's problem
    - Implementation in CE through taxes affected, not our ability to characterize Planner's solution

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Decentralization under Emissions Tax

- Suppose k_d is unobserved by the taxation authority, but noisy emissions are observable:

$$\xi_{j,t} = \eta_{j,t}\gamma k_{d,j,t};\ \eta_{j,t} \sim_{iid} F; \mathbb{E}[\eta_{j,t}] = 1\ \eta_{j,t} \geq 0\ a.s.$$

[e.g. $\log(\eta_{j,t}) \sim_{iid} N\left(-\frac{\sigma_E^2}{2}, \sigma_E^2\right)$]

- From the Planner's perspective, mean preserving spread integrates out; first-best unchanged
- Consider new emissions tax, $-\tau_t^E \xi_{j,t}$ in firm's problem in place of tax on dirty capital
  - Specifically, consider $\tau_t^E = \frac{\tau_t^k}{\gamma}$
- We show that both ex-ante value & ex-post policies match that of original decentralized problem
- Hence, the set of prices and taxes $\{\tau_t^E, \tau_t^D, \hat{w}_t, \frac{1}{\hat{R}_t}\}_{t=0}^{\infty}$ implements the Planner solution in a CE

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Functional Forms for Computation

- $D(S) = 1 - \exp(-\Delta(S - \bar{S}))$
- $\chi_t = \begin{cases} 1 - (1 + \exp(\log(\frac{0.01}{0.99}))^{\frac{t - t_{\chi=0.5}}{t_{\chi=0.01} - t_{\chi=0.5}}})^{-1}, & \text{if } t < t_{\chi=1} \\ 1, & \text{if } t \geq t_{\chi=1} \end{cases}$
- $\phi(x, k) = \hat{\gamma}(x/k)^2 k$

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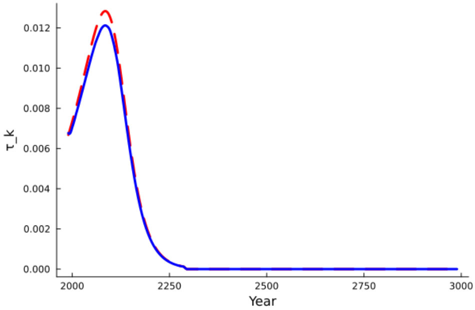
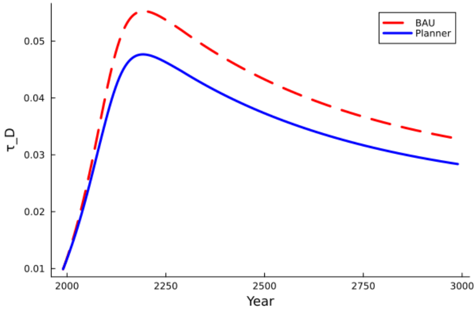
Figure: Planner Multipliers



(a) Shadow Cost of Dirty Capital to Marginal Utility of Consumption



(b) Environmental Damage

- Left: $\tau_t^k = \frac{\lambda_t^k}{U'(C_t)}$ evaluated under the BAU and Planner scenarios
- Right: $(1 - \tau_t^D) = (1 - D(S_t^1 + S_t^2))$ evaluated under the BAU and Planner scenarios

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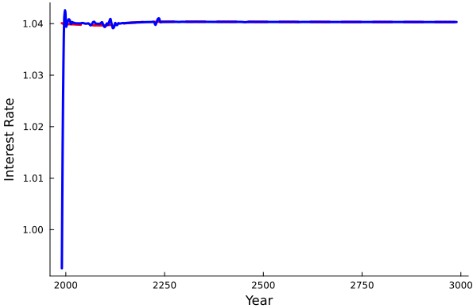
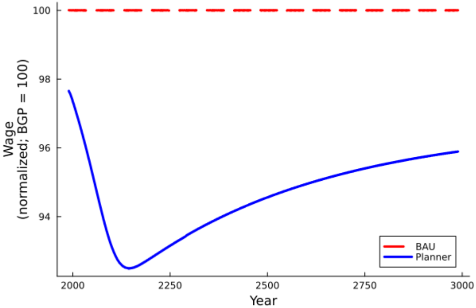
Figure: Equilibrium Prices



(a) Interest Rate

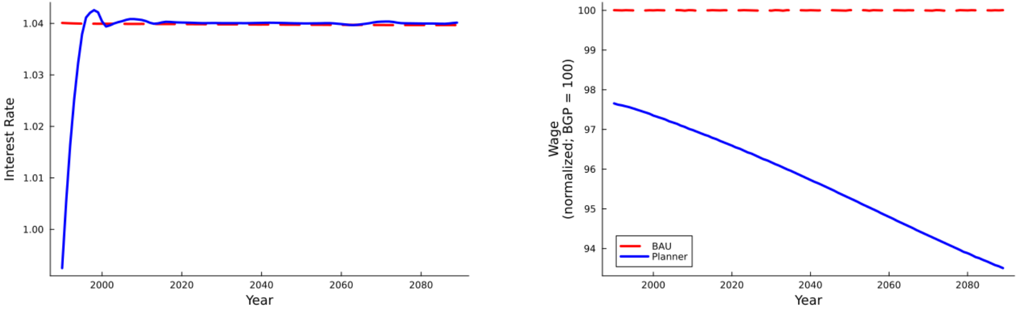


(b) Wage

- Left: Initial disinvestment $\implies$ consumption $\uparrow$ $\implies$ interest rate $\downarrow$
- Right: Capital level $\downarrow$ $\Rightarrow$ marginal product of labor $\downarrow$

Back Short Horizon

Figure: Equilibrium Prices



(a) Interest Rate

(b) Wage

- Left: Initial disinvestment $\implies$ consumption $\uparrow$ $\implies$ interest rate $\downarrow$
- Right: Capital level $\downarrow \Rightarrow$ marginal product of labor $\downarrow$

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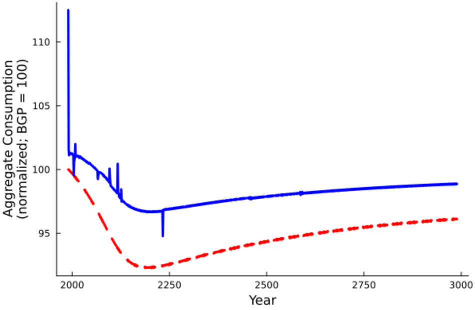
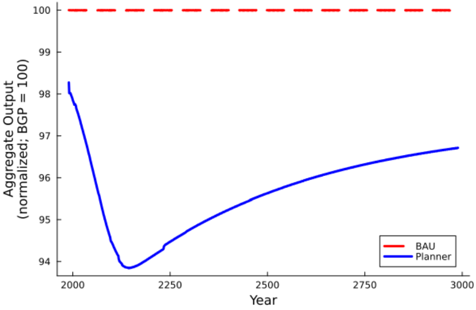
Figure: Aggregates (1/2)



(a) Aggregate Consumption



(b) Aggregate Output

- Left: Initial spike from sudden disinvestment. Planner solution ensures higher consumption path
- Right: Gross production Y_t is smaller under planner, despite the higher consumption path

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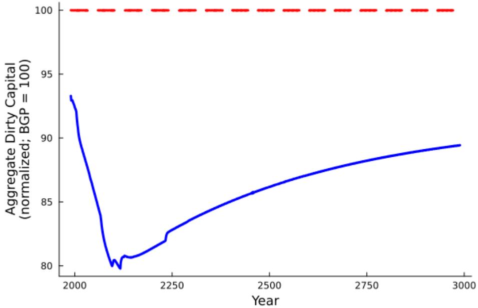
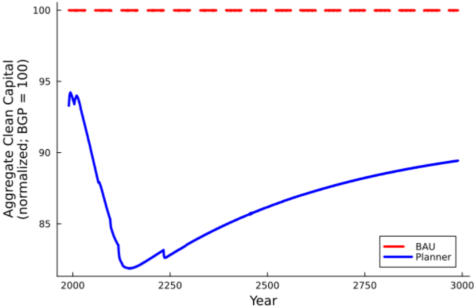
Figure: Aggregates (2/2)



(a) Aggregate Dirty Capital



(b) Aggregate Clean Capital

- *Scale effect*: Both types of capital are lower
- *Substitution effect*: Dirty capital drops more than clean capital

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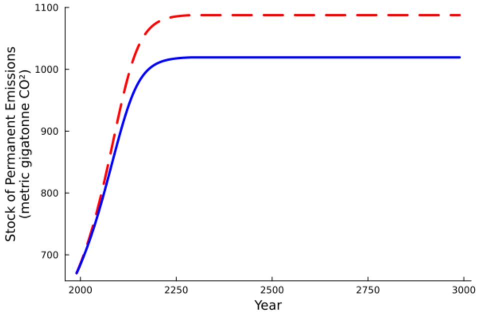
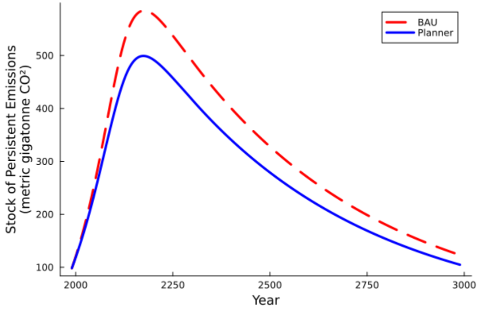
Figure: Climate



(a) Permanent Stock of Emissions



(b) Transitory Stock of Emissions

- Lower dirty capital $\implies$ fewer emissions $\implies$ lower stocks of carbon emissions

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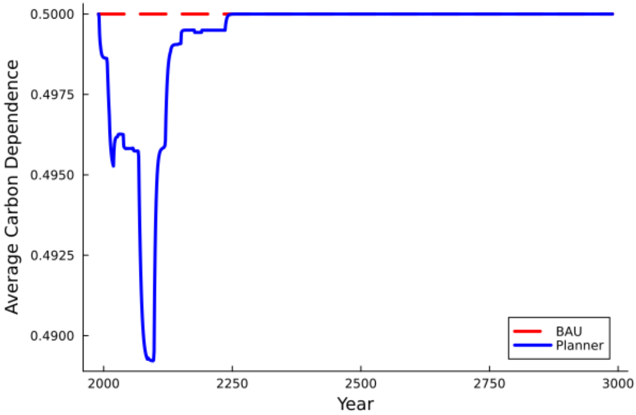
Figure: Average Carbon Dependence



- *Composition effect*: Average carbon intensity of firms is less

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