ECON 712A: Macroeconomic Theory

Discussion Section Handout 1

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- Icebreaker: Names, pronouns, program, year, and field(s) of interest.

Administrative Information

- Teaching Assistants:
  - Duong Dang (dqdang@wisc.edu; 6473 Social Science; 3rd year)
  - Alex von Hafften (vonhafften@wisc.edu; 6439 Social Science; 2nd year).
- Weekly Schedule:
  - Monday: Lecture at 1:00 - 2:15 PM in 4028 Vilas.
  - Tuesday: Duong's office hours at 10:30 AM - 11:45 PM in 6473 Social Science.
  - Wednesday: Lecture at 1:00 - 2:15 PM in 4028 Vilas.
  - Thursday:
    * Discussion section handout and next problem set distributed.
    * Dean's office hours at 10:00 - 11:45 AM in 7438 Social Sciences.
    * Alex's office hours at 2:15 PM - 3:30 PM in 6439 Social Science.
    * Problem set due at 11:59 PM. Everyone needs to submit the problem set. Include a note about who you worked with.
  - Friday:
    * Discussion sections
    * Distribute discussion sections handout with solutions.
- Midterm on November 1 at 7:15 PM.
- Course Materials:
  - Everything will be posted on Canvas.
  - Lecture notes and past midterms are also available at https://sites.google.com/a/wisc.edu/deancorbae/teaching.
  - Problem sets and section handouts are also available at https://vonhafften.github.io/teaching.html.
  - Problem sets will include computational problems. Any programming language will be accepted. Use the language that you're most comfortable with.
  - Example code will be provided largely in Matlab, Python, Julia, and/or R.

  - Matlab is available at https://it.wisc.edu/services/software/.
  - Ljungqvist and Sargent (textbook) is available online at the UW Madison Library.
- Discussion Sections:
  - 7:45 AM section is in 6105 Social Science.
  - 8:50 AM section is in 214 Ingraham
  - 2:25 PM section is in 6109 Social Science.
  - 3:30 PM section is in 6109 Social Science.
  - Sections are 50 minutes long.
  - Duong and Alex will alternate teaching all four sections each week.
- COVID Policies:
  - Masks covering your nose and mouth are required regardless of vaccination status.
  - To keep room capacity under control, please attend the section that you're enrolled in.
- What's the point of discussion sections?
  - Solving problems using concepts from lectures.
  - Filling in material omitted from lectures due to time constraints.
  - Discussing common issues on problem sets.
- Recommendations:
  - Study in groups.
  - Engage in active learning. Do practice problems.
  - Keep a positive mindset.
  - Recommendations for us? Email or anonymous feedback form at https://vonhafften.github.io/teaching.html.

Content Review

- An **environment** is a statement of population, preferences, and technologies (e.g., production, matching, information, commitment).
- Example: An overlapping generation economy with endowments and log preferences.
  - Population: 2-period lived agents.
  - Production: Non-storable w_1 for young agents and 0 for old agents.
  - Preference: $U(c_t^t, c_{t+1}^t) = \ln(c_t^t) + \ln(c_{t+1}^t)$
- An **allocation** is a statement about how resources are distributed.
- If an allocation can be achieved given technologies, the allocation is **resource feasible**.

$$c_t^t + c_t^{t-1} \leq w_1$$

- The **planner** allocates resources optimally given feasibility.

- What does optimally mean? So far, the planner has equally weighed the utility of each generation alive at period t:[1]

$$
\begin{aligned}
\max_{c_t^t, c_t^{t-1} \in \mathbb{R}_+^2} \ & \ln(c_t^t) + \ln(c_t^{t-1}) \\
\text{s.t. } & c_t^t + c_t^{t-1} \leq w_1
\end{aligned}
$$

- If the resource constraint holds with equality, we can use **substitution** to modify the planner problem into an unconstrained optimization problem.

$$
\max_{c_t^t \in \mathbb{R}_+} \ln(c_t^t) + \ln(w_1 - c_t^t)
$$

- The first order condition implies:

$$
\implies c_t^t = c_t^{t-1} = \frac{w_1}{2}
$$

- Planner problem is much better than **autarky**, in which agents only consume their endowments.
- How to **decentralize** the planner solution?
- One way is with **fiat currency**. The government issues M units of currency to the initial old.
- Taking p_1 as given, the problem of the initial old agents:

$$
\begin{aligned}
\max_{c_1^0 \in R_+} \ & \ln(c_1^0) \\
\text{s.t. } & p_1 c_1^0 \leq M
\end{aligned}
$$

$$
\implies c_1^0 = M/p_1
$$

- Let M_{t+1}^t be the fiat currency holding of generation t to period $t+1$.
- Taking p_t, p_{t+1} as given, the problem of agents in all generations born in $t \geq 1$:

$$
\begin{aligned}
\max_{(c_t^t, c_{t+1}^t, M_{t+1}^t) \in \mathbb{R}_+^3} \ & \ln(c_t^t) + \ln(c_{t+1}^t) \\
\text{s.t. } & p_t c_t^t + M_{t+1}^t = p_t w_1 \\
& p_{t+1} c_{t+1}^t = M_{t+1}^t
\end{aligned}
$$

$$
\begin{aligned}
\implies M_{t+1}^t &= \frac{p_t w_1}{2} \\
c_t^t &= \frac{w_1}{2} \\
c_{t+1}^t &= \frac{p_t}{p_{t+1}} \frac{w_1}{2}
\end{aligned}
$$

- Any questions?

[1] To be specific, this planner problem is a *period-by-period utilitarian planner problem*

Growing and Shrinking Populations

Consider the baseline 2-period overlapping-generation model outlined in lecture but the population changes each generation. In particular, if there is N_t measure of generation t, then there is $N_{t+1} = nN_t$ of generation $t+1$ where $n \in \mathbb{R}_+$. Notice that population could be shrinking ($0 < n < 1$), staying the same ($n = 1$), or growing ($n > 1$).

1. What is the resource constraint with the changing population?
2. Is the planners allocation without population growth (i.e., $c_t^t = c_t^{t-1} = \frac{w_1}{2}$) resource feasible for a growing population? For a shrinking population?
3. The planner cares equally about all agents alive at period t. What is the planners problem?
4. What is the planners allocation?
5. Consider decentralizing the planner solution. How should we design a lump-sum tax and transfer system would achieve the planner solution?
6. Would agents prefer to live in an economy with a growing population or a shrinking population?

Pareto Weights

So far, we've consider a planner than weights agents equally, but we can consider optimal allocations where the planner applies different weights to different agents. These weights are referred to as "Pareto weights."

1. Consider the baseline overlapping generations model with 2-period lived agents without population growth. Setup a planner problem where young agents have weight λ_1 and old agents have weight λ_2.
2. Solve for the optimal allocation a function of w_1, λ_1, and λ_2.
3. Set $\lambda_2 = 1 - \lambda_1$. How does the optimal allocation change for $\lambda_1 \in [0, 1]$?