AoC 2021 Day 17

Von Vic Cayas

July 2022

1 Introduction

Instead of creating a massively slow brute force solution for Day 17, I recognized that I can view the problem using equations. With this, we can achieve a solution with $O(m^2n)$ time complexity for Part One.

2 Calculation

Let's observe the x-position function: $p_x(t, v_x)$

$$p_x(t, v_x) = \sum_{i=0}^{t} d_x(t, v_x)$$

$$d_x(t, v_x) = \begin{cases} v_x - (\frac{v_x}{|v_x|})t & t < |v_x| \\ 0 & t \geq |v_x| \end{cases}$$

We can convert p_x into closed form. Let's consider when $t < |v_x|$:

$$p_x(t, v_x) = \sum_{i=0}^{t} d_x(t, v_x)$$

$$p_x(t, v_x) = \sum_{i=0}^{t} [v_x - (\frac{v_x}{|v_x|})i]$$

$$p_x(t, v_x) = \sum_{i=0}^{t} v_x - \sum_{i=0}^{t} \frac{v_x}{|v_x|} i$$

$$p_x(t, v_x) = v_x \sum_{i=0}^{t} 1 - \frac{v_x}{|v_x|} \sum_{i=0}^{t} i$$

$$p_x(t, v_x) = v_x t - \frac{v_x}{|v_x|} (\frac{t(t+1)}{2})$$

$$p_x(t, v_x) = v_x t - \frac{v_x}{2|v_x|} (t^2 + t)$$

$$p_x(t, v_x) = v_x t - \frac{v_x}{2|v_x|}t^2 + \frac{v_x}{2|v_x|}t$$
$$p_x(t, v_x) = (v_x + \frac{v_x}{2|v_x|})t - \frac{v_x}{2|v_x|}t^2$$
$$p_x(t, v_x) = (-\frac{v_x}{2|v_x|})t^2 + (v_x + \frac{v_x}{2|v_x|})t$$

When $t \leq |v_x|$, we can simply replace t with $|v_x|$.

$$p_x(t, v_x) = (-\frac{v_x^3}{2|v_x|}) + (1 + \frac{1}{2|v_x|})v_x^2$$

Therefore, we can form p_x into a piece-wise function.

$$p_x(t, v_x) = \begin{cases} (-\frac{v_x}{2|v_x|})t^2 + (v_x + \frac{v_x}{2|v_x|}) & t < |v_x| \\ (-\frac{v_x^3}{2|v_x|}) + (1 + \frac{1}{2|v_x|})v_x^2 & t \geq |v_x| \end{cases}$$

Let's also observe the y-position function: $p_y(t, v_y)$

$$p_y(t, v_y) = \sum_{i=0}^{t} d_y(i, v_y)$$
$$d_y(t, v_y) = v_y - t - 1$$

We can convert p_y into closed form:

$$p_y(t, v_y) = \sum_{i=1}^{t} d_y(i, v_y)$$
$$p_y(t, v_y) = \sum_{i=1}^{t} v_y - i + 1$$
$$p_y(t, v_y) = \sum_{i=1}^{t} v_y - \sum_{i=1}^{t} i + \sum_{i=1}^{t} 1$$
$$p_y(t, v_y) = v_y \sum_{i=1}^{t} 1 - \sum_{i=1}^{t} i + \sum_{i=1}^{t} 1$$
$$p_y(t, v_y) = (v_y + 1) \sum_{i=1}^{t} 1 - \sum_{i=1}^{t} i$$
$$p_y(t, v_y) = (v_y + 1)t - \frac{t(t+1)}{2}$$
$$p_y(t, v_y) = (v_y + 1)t - \frac{t^2 + t}{2}$$
$$p_y(t, v_y) = (v_y + 1)t - \frac{1}{2}t^2 - \frac{1}{2}t$$
$$p_y(t, v_y) = (v_y + 0.5)t - 0.5t^2$$
$$p_y(t, v_y) = (-0.5)t^2 + (v_y + 0.5)t$$