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NOTES AND COMMENTS

ON THE GREATEST LOWER BOUND TO RELIABILITY

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Certain ambiguities in a recent paper on the computation and statistics of the greatest lower bound are clarified.

Following the publication of our paper (Bentler & Woodward, 1980) on the greatest lower bound ρ_+ to reliability, we continued to work on certain computational and statistical issues, namely, a Kuhn-Tucker based theory with an accompanying proof of convergence for an iterative algorithm, and the asymptotic variance of $\hat{\rho}_+$ under both normal and arbitrary distribution theories. The statistical work was reported at a meeting (Woodward, Browne, & Bentler, 1980), while both results were incorporated in an overview presented at a conference to honor Fred Lord's contributions to psychometrics that was later published (Bentler & Woodward, 1983). Although Lord had not worked on this particular reliability problem, we were able to utilize his contributions through our approach to the asymptotic variance using the delta method and his computer program AUTEST.

Several statements in our chapter contain ambiguities or inaccuracies that require clarification. First, in discussing the algorithm we stated "A proof for convergence under [step] (b) was not previously available anywhere." ten Berge (personal communication) recently noted that a proof was given by ten Berge, Snijders and Zegers (1981). Although we cited these authors, we failed to note that they did report a convergence proof which we should have cited with the explanation that we were interested in providing an explicit verification of the function decrease $g^{(i+1)} \leq g^{(i)}$. Second, in discussing the variance of $\hat{\rho}_+$ we stated that "Since $\hat{\rho}_+$ is not a closed form expression, but is obtained by an iterative procedure represented in equation (2), an exact analytic expression for h (a vector of derivatives) has not been found. In our initial work, the derivatives are calculated numerically using a secant approximation developed by Lord (1975)." Shapiro (1985) correctly noted that, given two matries ψ^2 and T computed with our iterative algorithm, his expression for h is closed form ("exact analytic" to use his phrase). Shapiro's clarification of that sentence is similar to our own elaboration in the Lord book chapter, where we stated: "Recently, Shapiro (1982) provided a quasi-analytic expression for var ($\hat{\rho}_+$) under the assumption of normality. It is quasi-analytical in the sense that it provides an *analytic* expression involving ψ^2 and T; however, T is unknown and it must be solved by an algorithm such as (2) (emphasis added). Shapiro's estimator for var ($\hat{\rho}_+$) is based upon the Taylor approximation as is the method presented above; however, the computations required by the Shapiro estimator are simpler. Currently we are working on the extension of the Shapiro estimator to the asymptotic distribution free case." (p. 248).

We hope these comments remove any ambiguities and show the inadvertent nature of any inaccuracies.

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