**BKU, VNU-HCM**
FACULTY OF CSE

**MIDTERM**
Subject: **Discrete Structures for Computing** (CO1007)

Class: **CC16KHMT** Group: **CC01**
Time: **60 minutes** ***(Closed book test)***
Test date: **March 14, 2016**

**Student's Name:** ____________ **Student's ID:** | | | | | | | |

**Final Score:** ______ **Examiner:** __________ **Examiner's Signature:** ______

(*There are* ***20*** *MCQs, each question is worth* ***0.5*** *points. Answers in bold :* ■*; cancel out to deselect:* ☒*.*)

**Question 1.** Determine the number of 5-letter words with exactly two distinct letters. (For example, AAFAFF counts, but not AAFAFX or AAAAA, since the latter two words contain three, respectively one, distinct letters.)

(A) $\binom{26}{2} \cdot 2^5$. (B) $\binom{26}{2} \cdot (2^5 - 2)$. (C) $\binom{26}{2} \cdot 3!$. (D) $\dfrac{\binom{26}{2}}{3!}$.

**Question 2.** The number of all relations on a set consisting of 2017 elements, which are reflective, is

(A) $2^{2017^2}$. (B) $2^{\frac{2017\cdot 2018}{2}}$. (C) $2^{2016\cdot 2017}$. (D) $2^{2017\cdot 2018}$.

**Question 3.** *Fibonacci* sequence is recursively determined by

$$F_{n+2} = F_{n+1} + F_n \text{ for all natural numbers } n \geq 1,$$

and by two initial conditions $F_1 = F_2 = 1$.
For any $n$, two consecutive numbers $F_n$ and $F_{n+1}$ then satisfy that

(A) both are primes.
(B) the difference $F_{n+1} - F_n$ is prime.
(C) their greatest common divisor is a natural number $d > 1$.
(D) their greatest common divisor is number 1.

**Question 4.** Let $R_1$ and $R_2$ be two relations on a set $S \neq \emptyset$. Which of the following is correct?

(A) If both $R_1$ and $R_2$ are transitive then $R_1 \circ R_2$ i also transitive.
(B) If both $R_1$ and $R_2$ are transitive then $R_1 \cup R_2$ is also transitive.
(C) The relation $R_1$ can not be both symmetric and antisymmetric.
(D) The relation $R_1$ is transitive if and only if $R_1^{-1} = \{(y,x)|(x,y) \in R_1\}$ is also transitive.

**Question 5.** Let $X$ and $Y$ be two finite set such that $|Y| = 2$ and $|X| = 2017$. Then the number of surjective function from $X$ to $Y$ is

(A) $2^{2017}$. (B) $2^{2017} - 2$. (C) $2017^2$. (D) $\binom{2017}{2}$.

**Question 6.** Consider a binary relation $R$ on the set $\mathbb{Z}$ defined by $xRy \Leftrightarrow x^2 = y^2$. Then $R$ is

(A) reflexive and symmetric. (B) an equivalence relation.
(C) a partial order relation. (D) reflexive and transitive.

**Question 7.** By assigning $p = r = 0$, and $q = 1$, the true value of the following propositions

$$(p \longrightarrow q) \wedge (q \longrightarrow r);\ p \longrightarrow q \longrightarrow r$$

are, respectively,

(A) $0; 0$. (B) $1; 1$. (C) $0; 1$. (D) $1; 0$.

**Question 8.** Let $\{U_n\}_n$ be a sequence defined by $U_n = n(-1)^n$ for $n = 1, 2, 3, ...$ and let $S$ be the sum of first $n$ items of that sequence: $S = \sum_{k=1}^{n} U_k$. Which is the correct statement?

(A) $S = n/2$ if $n$ is odd.
(B) $S = (n-1)/2 + n$ if $n$ is odd.
(C) $S = (n-1)/2 - n$ if $n$ is odd.
(D) $S = (n+1)/2 + n$ if $n$ is odd.

**Question 9.** Which of the following is correct for functions?

(A) If $f_1$ and $f_2$ are two functions form $A$ to $B$ and $g$ is a surjective function from $B$ to $C$ such that $g \circ f_1 = g \circ f_2$, then $f_1 = f_2$.
(B) If $f : X \longrightarrow Y$ and $g : Y \longrightarrow X$ are two functions such that $f \circ g = Id_Y$, where $Id_Y$ is the identity map on $Y$, then $f$ is an injection.
(C) If $f : X \longrightarrow Y$ and $g : Y \longrightarrow X$ are two functions such that $g \circ f = Id_X$, where $Id_X$ is the identity map on $X$, then $f$ is a surjection.
(D) If $f_1$ and $f_2$ are two functions from $A$ two $B$ and $g$ is an injective function from $B$ to $C$ such that $g \circ f_1 = g \circ f_2$, then $f_1 = f_2$..

**Question 10.** Let $A$ and $B$ be two sets. Then the difference set $A \backslash B$ is equal to

(A) $\overline{B \backslash A}$.
(B) $\overline{B} \cup A$.
(C) $B \cap A$.
(D) $\overline{\overline{A} \cup B}$

**Question 11.** There are three suspects for a murder: Adam, Brown, and Clark. It has been established beyond any reasonable doubt that exactly one of them is the killer. [So, two are innocents, one is guilty.] Your task is to figure out which one. You questioned Adam, Brown, and Clark one-by-one. Adam says "I didn't do it. The victim was old acquaintance of Brown's. But Clark hated him." Brown states "I didn't do it. I didn't know the guy. Besides I was out of town all week." Finally, Clark says "I didn't do it. I saw both Adam and Brown downtown with the victim that day; one of them must have done it." Assuming that the innocent men are telling the truth, but that the guilty man might not be, discover the killer.

(A) Adam is the killer.
(B) Brown is the killer.
(C) Clark is the killer.
(D) The given information is insufficient to discover the killer.

**Question 12.** Let $f$ and $g$ be two functions from $\mathbb{R}$ to $\mathbb{R}$. Then the negation of the formula "For each $s$ in $\mathbb{R}$, there exists $r$ in $\mathbb{R}$ such that if $f(r) > 0$, then $g(s) > 0$" is the following formula.

(A) For every $s$ in $\mathbb{R}$, there exists $r$ in $\mathbb{R}$ such that $f(r) > 0$ and $g(s) \leq 0$.
(B) For every $s$ in $\mathbb{R}$, there does not exist $r$ in $\mathbb{R}$ such that if $f(r) > 0$, then $g(s) > 0$.
(C) There exists $s$ in $\mathbb{R}$ and there exists $r$ in $\mathbb{R}$ such that $f(r) \leq 0$ and $g(s) \leq 0$.
(D) There exists $s$ in $\mathbb{R}$ such that for every $r$ in $\mathbb{R}$, $f(r) > 0$ and $g(s) \leq 0$.

**Question 13.** Let $\phi$ be a propositional formula. Consider the following statements on $\phi$.

I. $\phi$ is satisfiable or $\neg\phi$ is satisfiable.

II. $\phi$ is a tautology or $\neg\phi$ is a tautology.

Then

(A) Both I and II are correct.
(B) Both I and II are incorrect.
(C) I is correct and II is incorrect.
(D) I is incorrect and II is correct.

**Question 14.** Suppose that $A$ and $B$ play a chess match consisting of several consecutive games. The first player who win consecutively two games or win three games in total will win the match. Suppose that there is no draw in each game. How many scenarios in this match? giải này?

(A) 10.
(B) 11.
(C) 9.
(D) 8.

**Question 15.** Given the following predicates

- $Q(x)$ : $x$ is a politician,
- $P(y)$ : $y$ is a person,
- $T(z)$ : $z$ is a time,
- $F(x, y, z)$ : person $x$ fools person $y$ at time $z$.

Represent the following sentences in predicate logic:

"***Politicians can't fool all of the people all of the time.***"

(A) $\forall x[Q(x) \to \forall y \forall z((P(y) \wedge T(z)) \to \neg F(x, y, z))]$.
(B) $\forall x[Q(x) \to \exists y \exists z((P(y) \wedge T(z)) \to \neg F(x, y, z))]$.
(C) $\forall x \exists y \exists z[Q(x) \to (P(y) \wedge T(z) \wedge F(x, y, z))]$.
(D) $\forall x[Q(x) \to \exists y \exists z(P(y) \wedge T(z) \wedge \neg F(x, y, z))]$.

**Question 16.** Determine the number of points $(x, y, z)$ with integer coordinates in the first octant (i.e., with $x, y, z \geq 0$) for which the sum of all three coordinates is at most 13. (For example, $(2, 1, 3)$ or $(0, 3, 10)$ count, but $(1, 3, 10)$ or $(1, 2, 6)$ do not count.)

(A) 1365. (B) 455. (C) 560. (D) 680.

**Question 17.** Which of the following is correct about power sets?

(A) $\mathcal{P}(A \cap B) = \mathcal{P}(A) \cap \mathcal{P}(B)$.
(B) $\mathcal{P}(A \cup B) = \mathcal{P}(A) \cup \mathcal{P}(B)$.
(C) $\mathcal{P}(A \times B) = \mathcal{P}(A) \times \mathcal{P}(B)$.
(D) $\mathcal{P}(A \backslash B) = \mathcal{P}(A) \backslash \mathcal{P}(B)$.

**Question 18.** Let $A = \{1, 2\}$ and $B = \{1\}$. Then,

(A) $\mathcal{P}(A \backslash B) \subseteq \mathcal{P}(A) \backslash \mathcal{P}(B)$.
(B) $\mathcal{P}(A) \backslash \mathcal{P}(B) \subseteq \mathcal{P}(A \backslash B)$.
(C) $\emptyset \in \mathcal{P}(A) \backslash \mathcal{P}(B)$.
(D) $|\mathcal{P}(A \backslash B)| = |\mathcal{P}(A) \backslash \mathcal{P}(B)|$.

**Question 19.** Consider the following statement: "**Suppose $\mathcal{F}$ and $\mathcal{G}$ are families of sets. If $\cup\mathcal{F}$ and $\cup\mathcal{G}$ are disjoint, then so are $\mathcal{F}$ and $\mathcal{G}$.**"

And consider a proof for that statement as follows.

"Suppose $\cup\mathcal{F}$ and $\cup\mathcal{G}$ are disjoint. Suppose $\mathcal{F}$ and $\mathcal{G}$ are not disjoint. Then we can choose some set $A$ such that $A \in \mathcal{F}$ and $A \in \mathcal{G}$. Since $A \in \mathcal{F}$, it is clear that $A \subseteq \cup\mathcal{F}$, so every element of $A$ is in $\cup\mathcal{F}$. Similarly, since $A \in \mathcal{G}$ every element of $A$ is in $\cup\mathcal{G}$. But then every element of $A$ is in both $\cup\mathcal{F}$ and $\cup\mathcal{G}$, and this is impossible since $\cup\mathcal{F}$ and $\cup\mathcal{G}$ are disjoint. Thus, we have reached a contradiction, so $\mathcal{F}$ and $\mathcal{G}$ must be disjoint. QED."

Then,

(A) The statement is correct and its proof if also correct.
(B) The statement is incorrect and the proof is also incorrect, since the claim that all elements of $S$ are also in $\cup\mathcal{F}$ and $\cup\mathcal{G}$ does not contradict to the fact that $\cup\mathcal{F}$ and $\cup\mathcal{G}$ are disjoint.
(C) The statement is incorrect. The proof is correct, but that is a proof for different statement.
(D) The proof is incorrect since the statement is incorrect, as we can take a counterexample with $\mathcal{F} = \{\{1\}, \emptyset\}$ and $\mathcal{G} = \{\{2\}, \emptyset\}$.

**Question 20.** Let a set $S \subset \mathbb{N}$ that the cardinality $|S| = 12$. Then

(A) $S$ must contain two distinct number $s_1$, $s_2$ such that $s_1 - s_2$ is a multiple of 10.
(B) $S$ must contain two distinct number $s_1$, $s_2$ such that $s_1 - s_2$ is a multiple of 11.
(C) $S$ must contain two distinct number $s_1$, $s_2$ such that $s_1 - s_2$ is a multiple of 12.
(D) $S$ must contain two distinct number $s_1$, $s_2$ such that $s_1 - s_2$ is a multiple of 13.