HCMUT, VNU-HCM
Faculty of CSE

MIDTERM EXAMINATION
Course: **Discrete structures for Computing** (CO1007)
Class: **2017-1-CO1007** Group: **CC01**
Duration: **60 minutes** ***(Closed-book test)***
Examination date: **Oct. 19, 2017**

Student name: ______________

Score: _________ **Student ID:** | | | | | | | | |

Examiner: _________ **Examiner's signature:** ______________

(*There are **20** multiple-choice questions, each question is worth **0.5** point. Highlight the correct (best) answer:* ■. Cancel out to deselect: ⊠ or clear the highlight)

Question 1. Let R be the relation on the set of people consisting of pairs (a, b) where *"a is a parent of b"*. Let S be the relation on the set of people consisting of pairs (a, b) where *"a and b are siblings (brother or sisters)."* What are $S \circ R$ and $R \circ S$?

Ⓐ $S \circ R = \{(a, b) | a \text{ is a parent of } b\}$; $R \circ S = \{(a, b) | a \text{ is an aunt or uncle of } b\}$
Ⓑ $S \circ R = \{(a, b) | a \text{ is an uncle or aunt of } b\}$; $R \circ S = \{(a, b) | a \text{ is a parent of } b \text{ and b has a sibling }\}$
Ⓒ $S \circ R = \{(a, b) | a \text{ is a parent of } b \text{ and } b \text{ has a sibling }\}$; $R \circ S = \{(a, b) | a \text{ is an aunt or uncle of } b\}$
Ⓓ All of the above statements are incorrect

Question 2. In an information sercurity survey engineers asked IT companys about risk level of their websites under possible attacks by black hackers. The only three choices in the survey are *high risk*, *medium risk*, and *none*. If 31% of the responders indicated *high risk* and 49% responders indicated *medium risk*, then the percentage of respondents who chose the *none* category can be the following options:

(A) 80% (B) 70% (C) 59% (D) 20%

You then would select answer

Ⓐ A Ⓑ D Ⓒ B Ⓓ C

Question 3. The statement $p \land q \to \neg q$ is equivalent with which of the following statement

Ⓐ $q \land p$. Ⓑ 1 Ⓒ $p \lor \neg q$ Ⓓ $\neg p \lor \neg q$

Question 4. Let $F(x, y)$ be the statement "*x can fool y*", where the domain consists of all people in the world. Use quantifiers to express the statement: *"Nancy can fool exactly two people."*

Ⓐ $\forall x \forall y, (y \neq x \land F(Nancy, x) \land F(Nancy, y))$
Ⓑ $\exists x \exists y, (y \neq x \land F(Nancy, x) \land F(Nancy, y) \lor \exists z(z = x \lor z = y \lor F(Nancy, z)))$
Ⓒ $\exists x \forall y, (y \neq x \land F(Nancy, x) \land F(Nancy, y) \land \forall z(z \neq x \lor z = y \lor \neg F(Nancy, z)))$
Ⓓ $\exists x \exists y, (y \neq x \land F(Nancy, x) \land F(Nancy, y) \land \forall z(z = x \lor z = y \lor \neg F(Nancy, z)))$

Question 5. In a certain survey of a group of 200 students, 50% students indicated they can play volley ball, 65% indicated that they can play ping-pong, 15% indicate they cannot play both of them. How many student can play both of two sport games?

Ⓐ 70 Ⓑ 60 Ⓒ 50 Ⓓ 40

Question 6. Let $f : X \to Y$ be a function, and let $\{S_i : i \in I\}$ be a family subsets of X. Which of the following is incorrect?

Ⓐ $f(\bigcup_{i \in I} S_i) = \bigcup_{i \in I} f(S_i)$
Ⓑ When f is a bijection function: $f^{-1}(S_1 \cup S_2) = f^{-1}(S_1) \cup f^{-1}(S_2)$
Ⓒ $f(S_1 \cap S_2) \subseteq f(S_1) \cap f(S_2)$ Ⓓ $f(S_1) \cap f(S_2) \subset f(S_1 \cap S_2)$

Question 7. Let $P(x, y)$ denote *"x is a factor of y"* where $x \in \{1, 2, 3, \ldots\}$ and $y \in \{2, 3, 4, \ldots\}$. Let $Q(y)$ denote "$\forall x[P(x, y) \rightarrow ((x = y) \vee (x = 1))]$". When is $Q(y)$ true?

(A) $Q(y)$ always false.
(B) $Q(y)$ is an integer number.
(C) $Q(y)$ is a prime number.
(D) $Q(y)$ is a positive number.

Question 8. Which of the following answers is the negation of the statement:

$$\exists C > 0, \exists d \in \mathbb{N}, \exists m \in \mathbb{N}, \forall n \in \mathbb{N}(n \geq m \implies |T(n)| < C \times n^d)$$

(A) $\forall C > 0, \exists d \in \mathbb{N}, \forall m \in \mathbb{N}, \exists n \in \mathbb{N}(n \geq m \wedge |T(n)| > C \times n^d)$
(B) $\forall C > 0, \forall d \in \mathbb{N}, \forall m \in \mathbb{N}, \exists n \in \mathbb{N}(n \geq m \implies |T(n)| > C \times n^d)$
(C) $\forall C > 0, \forall d \in \mathbb{N}, \forall m \in \mathbb{N}, \exists n \in \mathbb{N}(n \geq m \wedge |T(n)| \geq C \times n^d)$
(D) $\forall C > 0, \forall d \in \mathbb{N}, \forall m \in \mathbb{N}, \exists n \in \mathbb{N}(n < m \wedge |T(n)| \geq C \times n^d)$

Question 9. Let R be the relation on $\mathbb{N} \times \mathbb{N}$ defined by $(a, b)R(c, d)$ if and only if $ad = bc$. Which of the following answer is the most accurately?

(A) R is not a equivalence relation on $\mathbb{N} \times \mathbb{N}$.
(B) R is a equivalence relation on $\mathbb{N} \times \mathbb{N}$.
(C) R is not a symmetric and ir-reflexive relation on $\mathbb{N} \times \mathbb{N}$.
(D) R is not a symmetric and transitive relation on $\mathbb{N} \times \mathbb{N}$.

Question 10. Let x be any integer. To prove the statement $x^2 + x$ is even, we follow these steps: First, because an arbitrary integer is either even or odd, we setup for proof-by-cases inference p: x is even; q : x is odd; r : $x^2 + x$ is even.

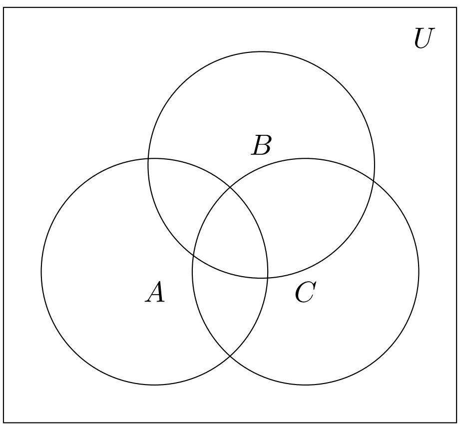
Verify premise 1. If x is even, then $x = 2n$, for some integer n. Hence, $x^2 + x = (2n)2 + 2n = 4n^2 + 2n$, which is even.

Verify premise 2. If x is odd, then $x = 2n+1$, for some n. Hence, $x^2 + x = (2n+1)2 + (2n+1) = (4n^2 + 4n + 1) + (2n + 1) = 4n^2 + 6n + 2$, which is even.

What is the proving method used above?

(A) Contradiction
(B) Contraposition
(C) Direct
(D) Induction

Question 11. Given the Venn diagram of 3 sets A, B, C and the universal set U



Which of the following assertions is correct?

(A) $A \cap B \cap C = A - B - C$
(B) $(A \cup (B - C)) \cap \overline{B} = A - C$
(C) $A \cap B \cap C = (A \cap C) - (C - B)$
(D) $\overline{A - B} = A \cap C$

Question 12. Express the mathematical statements *"The difference of two negative integers is not necessarily negative."* using predicates, quantifiers, logical connectives, and mathematical operators, where the domain consists of all integers.

(A) $\forall m \forall n(m < 0 \wedge n < 0 \wedge \neg(m - n < 0))$
(B) $\exists m \forall n(m < 0 \wedge n < 0 \wedge \neg(m - n < 0))$
(C) $\exists m \exists n(m < 0 \wedge n < 0 \wedge \neg(m - n < 0))$
(D) $\exists m \forall n(m \wedge n \wedge \neg(m - n < 0))$

Question 13. From a deck of cards we take 12 cards.

- Hearts 1, 2 and 3
- Clubs 1, 2, 3 and 4
- Diamond 1, 2, 3, 4 and 5

Take 5 cards (from 12 cards) such that there is at least one card of each type. In how many ways is that possible? The order of these five cards is irrelevant.

(A) 590 (B) 690 (C) 790 (D) 490

Question 14. How many sequences contain 6 numbers from $1, 2, 3, 4, 5, 6$ that meet the conditions: 6 numbers in this sequence are different, and the sum of three first numbers less than the sum of three last number a (1) unit?

(A) 12 (B) 36 (C) 72 (D) 108

Question 15. Suppose that R and S are reflexive relations on a set A. Which of the following statement is correct?

(A) $R \cup S$ is reflexive, $R \cap S$ is ir-reflexive, $R - S$ is reflexive,$R \circ S$ is ir-reflexive .
(B) $R \cup S$ is reflexive, $R \cap S$ is reflexive, $R - S$ is ir-reflexive,$R \circ S$ is reflexive .
(C) $R \cup S$ is ir-reflexive, $R \cap S$ is reflexive, $R - S$ is reflexive,$R \circ S$ is ir-reflexive .
(D) $R \cup S$ is reflexive, $R \cap S$ is ir-reflexive, $R - S$ is ir-reflexive,$R \circ S$ is ir-reflexive .

Question 16. Given the sequence of statements (n is an integer)
p_1: *"n is an even number"*
p_2: *"n+1 is an odd number"*
p_3: *"3n+1 is an odd number"*
p_4: *"3n is an even number."*
Find the inference rule to prove four above statements are equivalent?

(A) $(p_1 \leftrightarrow p_2) \wedge (p_3 \leftrightarrow p_4)$ (B) $(p_1 \rightarrow p_4) \wedge (p_4 \leftrightarrow p_3) \vee (p_3 \rightarrow p_2)$
(C) $(p_1 \leftrightarrow p_4) \vee (p_2 \rightarrow p_3) \wedge (p_3 \leftrightarrow p_4)$ (D) $(p_1 \rightarrow p_2) \wedge (p_2 \rightarrow p_3) \wedge (p_3 \rightarrow p_4)$

Question 17. Translate the following statement into English expressions using predicates, quantifiers and logical connectives. Let $C(x)$ denote the predicate *"x is in the correct place"*, $E(x)$ denote the predicate *"x is in excellent condition"*. Suppose that the domain consists of all tools.

$$(\exists x(\neg C(x) \wedge E(x))) \wedge \forall y((\neg C(y) \wedge E(y)) \implies (x = y))$$

(A) One of your tools is in the correct place, and is in excellent condition.
(B) One of your tools is not in the correct place, but is in excellent condition.
(C) There are some of your tools is in the correct place, but is in excellent condition.
(D) One of your tools is not in the correct place, and is not in excellent condition.

Question 18. A computer manufacturing firm **W** in HCMC have ordered batches in type I from Thailand and type II from China, each batch contains 10 parts; and some parts of those batches will be used to build up new *computer main-boards.*

- We know that each type I batch has 8 good parts and 2 bad (malfunctioning) parts; and each type II batch has 7 good parts and 3 bad parts. Also this year the firm **W** imported 80 type I batches, 20 type II batches and stored all in a warehouse.
- Each new computer mainboard need precisely 3 part to build, hence engineers go to the warehouse and randomly choose a batch (either type I or type II batch) then get randomly 3 parts out of that batch to make the mainboard.

The probability that engineers rightly choose 3 good parts for making a new main-board is

(A) 3/10 (B) 6/70 (C) 26/70 (D) 27/70

Question 19. Assume there are five different kinds of scholarships. How many students (at least) are needed to ensure that at least 6 students receive a same kind of scholarship?

(A) 20 (B) 22 (C) 24 (D) 26

Question 20. Which of the following assertions is true for the function $f(x) = |x+3| - |x-3|$

(A) *Domain* - $D(f) = (-\infty; +\infty)$, *Range* - $R(f) = [-6; 6]$, f is not one-to-one function, intersect with x-axis and y-axis at $(0,0)$ and $(0,0)$.

(B) $D(f) = (0; +\infty)$, $R(f) = [-\infty; \infty]$, f is not one-to-one function, intersect with x-axis and y-axis at $(3,0)$ and $(0,3)$.

(C) $D(f) = (-\infty; +\infty)$, $R(f) = [-3; 3]$, f is not one-to-one function, intersect with x-axis and y-axis at $(0,0)$ and $(0,0)$.

(D) $D(f) = (-\infty; +\infty)$, $R(f) = [0; 3]$, f is not one-to-one function, intersect with x-axis and y-axis at $(0,3)$ and $(3,0)$.