

Hiểu tính chất của nó.

Chapter 10

Sorting

Data Structures and Algorithms

Team design phải quyết định những rủi ro để quyết định nên dùng external or internal sort (nếu đầu vào quá lớn).
Xem xét xem bài toán có quan tâm đến duy trì thứ tự input hay không
Xem xét kích thước input. Độ phức tạp giải thuật của heap sort không định. Nhược điểm: Ko duy trì được thứ tự.

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Sorting

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Sorting concepts

Insertion Sort

Straight Insertion Sort

Shell Sort

Selection Sort

Straight Selection Sort

Heap Sort

Exchange Sort

Bubble Sort

Divide-and-Conquer

Quick Sort

Merge Sort

Outcomes

- **L.O.6.1** - Depict the working steps of sorting algorithms step-by-steps.
- **L.O.6.2** - Describe sorting algorithms by using pseudocode.
- **L.O.6.3** - Implement sorting algorithms using C/C++ .
- **L.O.6.4** - Analyze the complexity and develop experiment (program) to evaluate sorting algorithms.
- **L.O.6.5** - Use sorting algorithms for problems in real-life.
- **L.O.8.4** - Develop recursive implementations for methods supplied for the following structures: list, tree, heap, searching, and graphs.
- **L.O.1.2** - Analyze algorithms and use Big-O notation to characterize the computational complexity of algorithms composed by using the following control structures: sequence, branching, and iteration (not recursion).



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Contents

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② Insertion Sort

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③ Selection Sort

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Heap Sort

④ Exchange Sort

Bubble Sort

⑤ Devide-and-Conquer

Quick Sort
Merge Sort





Sorting concepts

Sorting concepts

Insertion Sort

- Straight Insertion Sort
- Shell Sort

Selection Sort

- Straight Selection Sort
- Heap Sort

Exchange Sort

- Bubble Sort

Divide-and-Conquer

- Quick Sort
- Merge Sort

Sorting

One of the most **important concepts** and **common applications** in computing.

23	78	45	8	32	56
----	----	----	---	----	----



8	23	32	45	56	78
---	----	----	----	----	----



Sorting concepts

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Bubble Sort

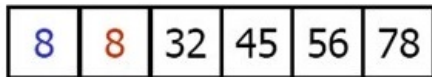
Divide-and-Conquer

Quick Sort

Merge Sort



Sort stability: data with equal keys maintain their relative input order in the output.



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Sorting

Hiệu quả giải thuật sort = số lần so sánh + số lần di chuyển.

Sort efficiency: a measure of the relative efficiency of a sort = number of **comparisons** + number of **moves**.



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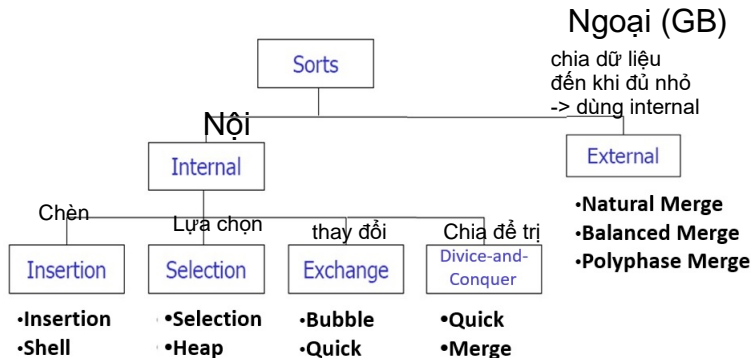
Straight Selection Sort
Heap Sort

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Divide-and-Conquer

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Merge Sort





Straight Insertion Sort
Shell Sort

Straight Selection Sort
Heap Sort

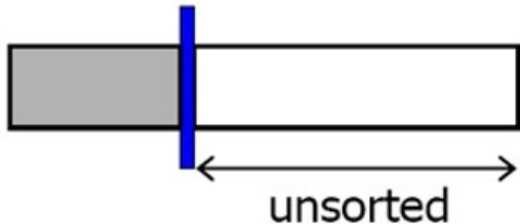
Bubble Sort

Quick Sort
Merge Sort

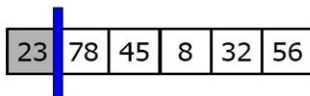
Insertion Sort

Straight Insertion Sort

- The list is divided into two parts: **sorted** and **unsorted**.
- In each pass, the first element of the unsorted sublist is **inserted** into the sorted sublist.



Straight Insertion Sort



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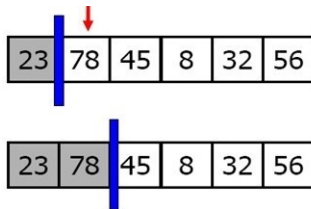
Bubble Sort

Divide-and-Conquer

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Straight Insertion Sort



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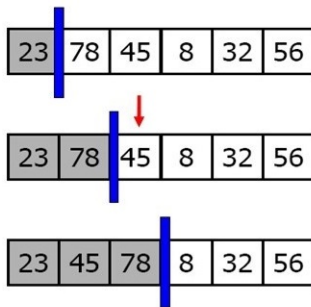
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Sorting

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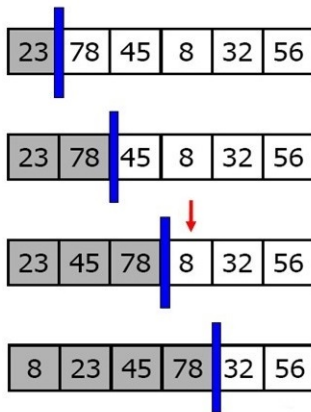
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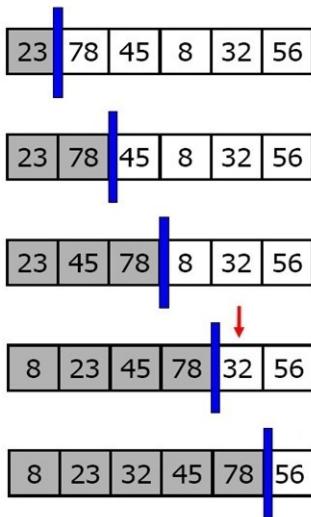
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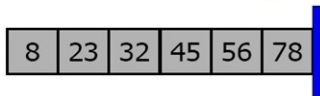
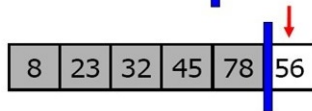
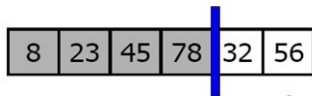
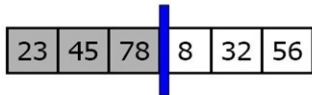
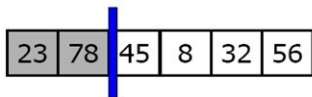
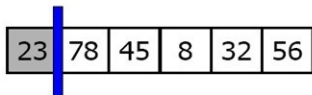
Bubble Sort

Divide-and-Conquer

Quick Sort

Merge Sort

Straight Insertion Sort



Straight Insertion Sort

Algorithm InsertionSort()

Sorts the contiguous list using straight insertion sort.

```
if count > 1 then
    current = 1
    while current < count do
        temp = data[current]
        walker = current - 1
        while walker >= 0 AND temp.key <
            data[walker].key do
            data[walker+1] = data[walker]
            walker = walker - 1
        end
        data[walker+1] = temp
        current = current + 1
    end
end
End InsertionSort
```

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Divide-and-Conquer

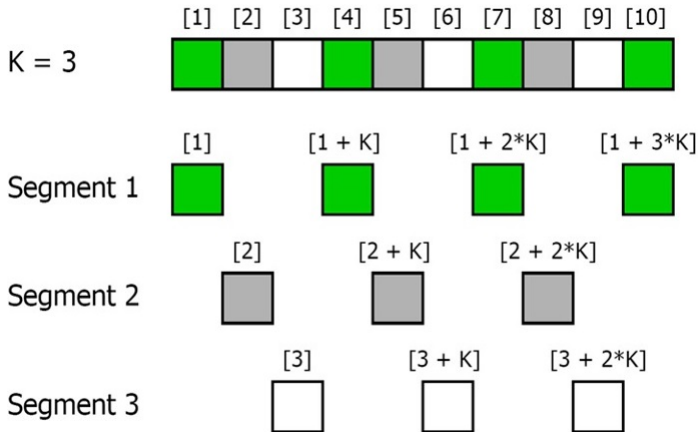
Quick Sort

Merge Sort

- Named after its creator Donald L. Shell (1959).
- Given a list of N elements, the list is divided into K **segments** (K is called the **increment**).
- Each segment contains N/K or more elements.
- Segments are dispersed throughout the list.
- Also is called **diminishing-increment sort**.



Shell Sort



Shell Sort



- For the value of K in each iteration, sort the K segments.
- After each iteration, K is reduced until it is 1 in the final iteration.



Example of Shell Sort

Unsorted

Tim
Dot
Eva
Roy
Tom
Kim
Guy
Amy
Jon
Ann
Jim
Kay
Ron
Jan

Sublists incr. 5

Tim Dot
↑ ↑
Eva
↑ ↑
Roy Tom
↑ ↑
Kim Guy
↑ ↑
Amy
↑ ↑
Jon Ann
↑ ↑
Jim Kay
↑ ↑
Ron Jan

5-Sorted

Jim Dot
↓ ↓
Amy
↓ ↓
Jan Ann
↓ ↓
Kim Guy
↓ ↓
Eva
↓ ↓
Jon Tom
↓ ↓
Tim Kay
↓ ↓
Ron Roy

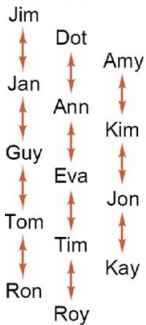
Recombined

Jim
Dot
Amy
Jan
Ann
Kim
Guy
Eva
Jon
Tom
Tim
Kay
Ron
Roy

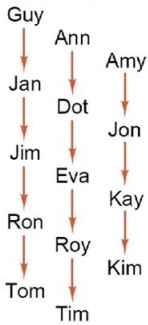


Example of Shell Sort

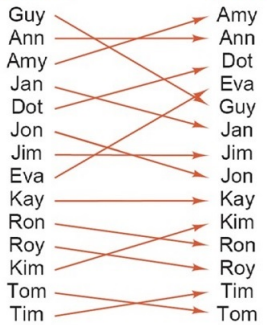
Sublists incr. 3



3-Sorted



List incr. 1



Sorted



Choosing incremental values

- From more of the comparisons, it is better when we can receive more new information.
- Incremental values should not be multiples of each other, other wise, the same keys compared on one pass would be compared again at the next.
- The final incremental value must be 1.



Choosing incremental values

- Incremental values may be:

1, 4, 13, 40, 121, ...

$$k_t = 1$$

$$k_{i-1} = 3 * k_i + 1$$

$$t = \lceil \log_3 n \rceil - 1$$

- or:

1, 3, 7, 15, 31, ...

$$k_t = 1$$

$$k_{i-1} = 2 * k_i + 1$$

$$t = \lceil \log_2 n \rceil - 1$$



Shell Sort

Algorithm ShellSort()

Sorts the contiguous list using Shell sort.

```
k = first_incremental_value
while k >= 1 do
    segment = 1
    while segment <= k do
        SortSegment(segment)
        segment = segment + 1
    end
    k = next_incremental_value
end
End ShellSort
```



Shell Sort

Algorithm SortSegment(val segment <int>, val k <int>)

Sorts the segment beginning at segment using insertion sort, step between elements in the segment is k.

current = segment + k

while *current* < *count* **do**

temp = data[current]

walker = current - k

while *walker* ≥ 0 AND *temp.key* < *data[walker].key* **do**

data[walker + k] = data[walker]

walker = walker - k

end

data[walker + k] = temp

current = current + k

end

End SortSegment



Insertion Sort Efficiency

- Straight insertion sort:
$$f(n) = n(n + 1)/2 = O(n^2)$$
- Shell sort:
 $O(n^{1.25})$ (Empirical study)



Selection Sort



Sorting concepts

Insertion Sort

Straight Insertion Sort
Shell Sort

Selection Sort

Straight Selection Sort
Heap Sort

Exchange Sort

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Divide-and-Conquer

Quick Sort
Merge Sort

Selection Sort

In each pass, the smallest/largest item is **selected** and placed in a sorted list.

Sorting

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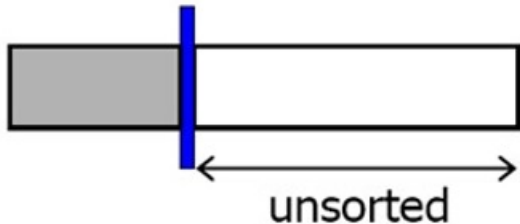
Divide-and-Conquer

Quick Sort

Merge Sort

Straight Selection Sort

- The list is divided into two parts: **sorted** and **unsorted**.
- In each pass, in the unsorted sublist, the smallest element is **selected** and **exchanged** with the first element.



Straight Selection Sort

	23	78	45	8	32	56
--	----	----	----	---	----	----

Sorting

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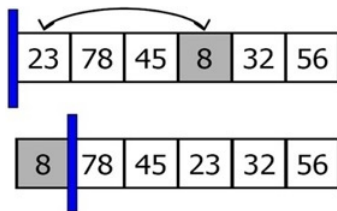
Bubble Sort

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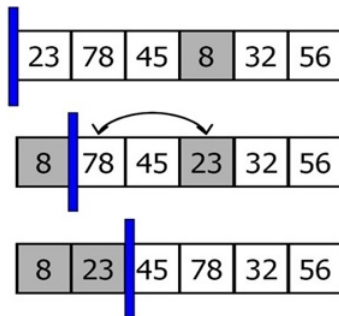
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Sorting

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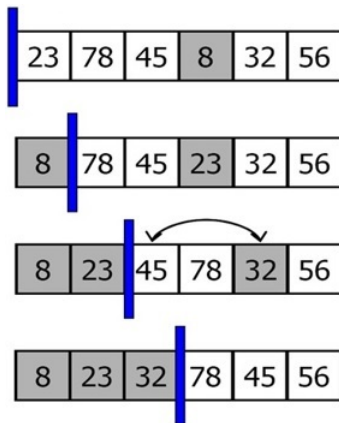
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Sorting

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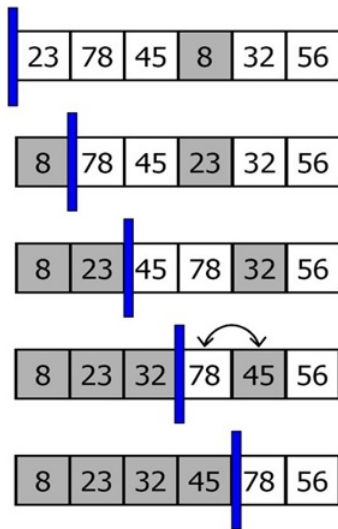
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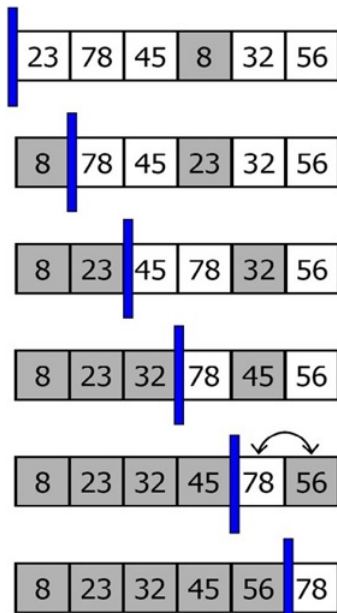
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Merge Sort

Straight Selection Sort

Algorithm SelectionSort()

Sorts the contiguous list using straight selection sort.

current = 0

while *current* < *count* - 1 **do**

 smallest = current

 walker = current + 1

while *walker* < *count* **do**

if *data* [*walker*].key < *data* [*smallest*].key **then**

 smallest = walker

end

 walker = walker + 1

end

 swap(current, smallest)

 current = current + 1

end

End SelectionSort

Sorting

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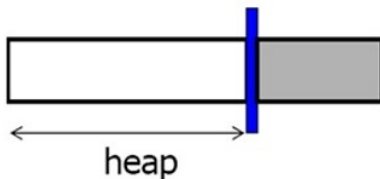
Devide-and-Conquer

Quick Sort

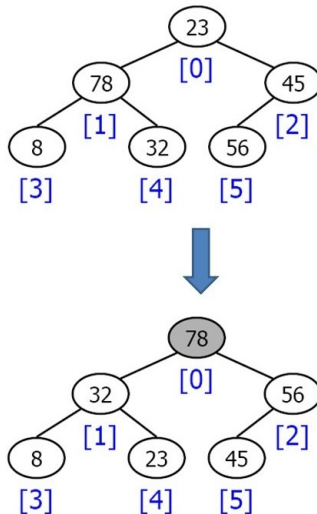
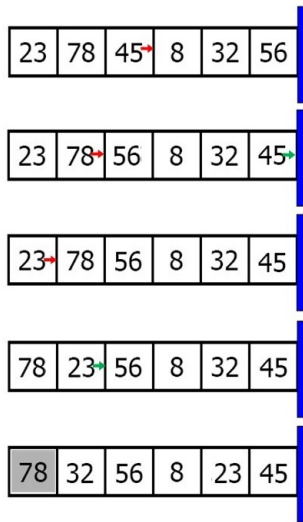
Merge Sort

Heap Sort

- The unsorted sublist is organized into a **heap**.
- In each pass, in the unsorted sublist, the largest element is **selected** and **exchanged** with the last element.
- The the heap is **reheaped**.



Heap Sort



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Sorting concepts

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Straight Insertion Sort
Shell Sort

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Straight Selection Sort

Heap Sort

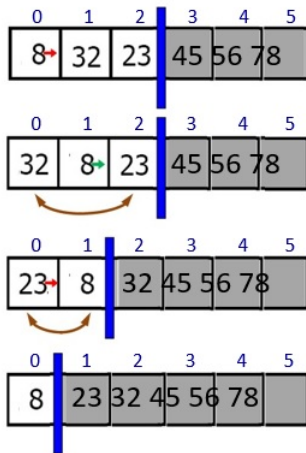
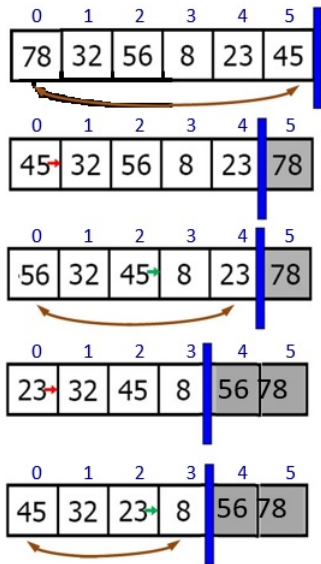
Exchange Sort

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Quick Sort

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Heap Sort

Algorithm HeapSort()

Sorts the contiguous list using heap sort.

```
position = count/2 - 1
while position >= 0 do
    | ReheapDown(position, count - 1)
    | position = position - 1
end
last = count - 1
while last > 0 do
    | swap(0, last)
    | last = last - 1
    | ReheapDown(0, last - 1)
end
End HeapSort
```



Selection Sort Efficiency

- Straight selection sort:
 $O(n^2)$
- Heap sort:
 $O(n \log_2 n)$

Heap sort is not stability sort
because in reHeapDown() function, we can't be sure that the order of input
is choose the same. e



Exchange Sort



Sorting concepts

Insertion Sort

Straight Insertion Sort

Shell Sort

Selection Sort

Straight Selection Sort

Heap Sort

Exchange Sort

Bubble Sort

Divide-and-Conquer

Quick Sort

Merge Sort

Exchange Sort

- In each pass, elements that are out of order are **exchanged**, until the entire list is sorted.
- **Exchange** is extensively used.

Sorting

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Sorting concepts

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Bubble Sort

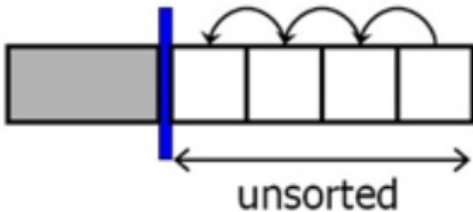
Divide-and-Conquer

Quick Sort

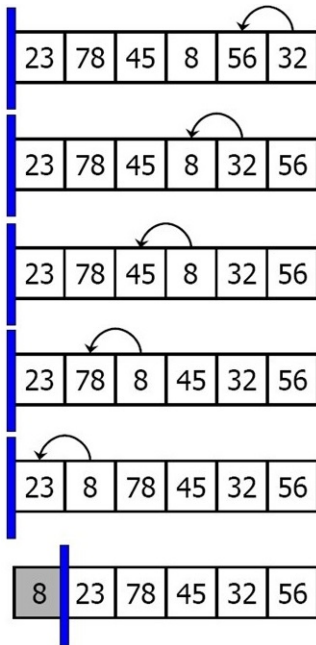
Merge Sort

Bubble Sort

- The list is divided into two parts: **sorted** and **unsorted**.
- In each pass, the smallest element is **bubbled** from the unsorted sublist and moved to the sorted sublist.



Bubble Sort



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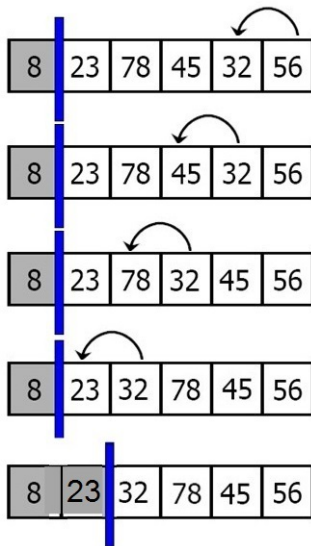
Bubble Sort

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Sorting

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Bubble Sort

Algorithm BubbleSort()

Sorts the contiguous list using bubble sort.

current = 0

flag = False

while *current* < *count* **AND** *flag* = *False* **do**

 walker = *count* - 1

 flag = True

while *walker* > *current* **do**

if *data* [*walker*].key < *data* [*walker*-1].key **then**

 flag = False

 swap(*walker*, *walker* - 1)

end

walker = *walker* - 1

end

current = *current* + 1

end

End BubbleSort



Exchange Sort Efficiency

- Bubble sort:

$$f(n) = n(n + 1)/2 = O(n^2)$$

Sorting

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Devide-and-Conquer Sort

Algorithm DevideAndConquer()

if *the list has length* > 1 **then**

 partition the list into lowlist and highlist

 lowlist.DevideAndConquer()

 highlist.DevideAndConquer()

 combine(lowlist, highlist)

end

End DevideAndConquer

Sorting

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Devide-and-Conquer Sort

	Partition	Combine
Merge Sort	easy	hard
Quick Sort	hard	easy

Sorting

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Quick Sort
Merge Sort

Quick Sort

not stable sort

Algorithm QuickSort()

Sorts the contiguous list using quick sort.

recursiveQuickSort(0, count - 1)

End QuickSort

Sorting

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Divide-and-Conquer

Quick Sort

Merge Sort

Quick Sort

Algorithm recursiveQuickSort(val left <int>, val right <int>)

Sorts the contiguous list using quick sort.

Pre: left and right are valid positions in the list

Post: list sorted

if *left* < *right* **then**

 pivot_position = Partition(left, right)

 recursiveQuickSort(left, pivot_position - 1)

 recursiveQuickSort(pivot_position + 1, right)

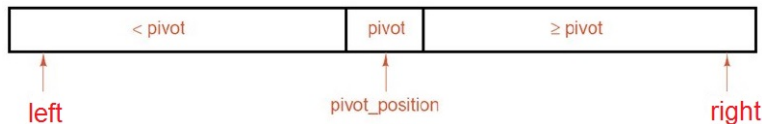
end

End recursiveQuickSort



Quick Sort

Given a pivot value, the partition rearranges the entries in the list as the following figure:



Sorting

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Sorting concepts

Insertion Sort

Straight Insertion Sort

Shell Sort

Selection Sort

Straight Selection Sort

Heap Sort

Exchange Sort

Bubble Sort

Divide-and-Conquer

Quick Sort

Merge Sort

Quick Sort Efficiency

- Quick sort:
 $O(n \log_2 n)$

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Exchange Sort

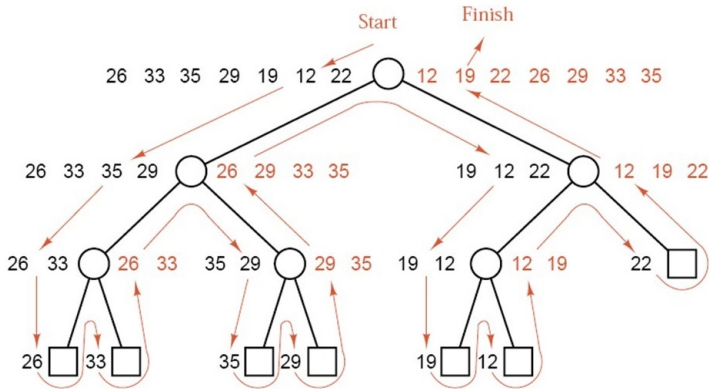
Bubble Sort

Divide-and-Conquer

Quick Sort

Merge Sort

Merge Sort



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Merge Sort

Algorithm MergeSort()

Sorts the linked list using merge sort.

recursiveMergeSort(head)

End MergeSort

Sorting

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Merge Sort

Algorithm recursiveMergeSort(ref sublist
<pointer>)

Sorts the linked list using recursive merge sort.

if *sublist is not NULL AND sublist->link is not NULL* **then**

 Divide(sublist, second_list)
 recursiveMergeSort(sublist)
 recursiveMergeSort(second_list)
 Merge(sublist, second_list)

end

End recursiveMergeSort



Merge Sort

Algorithm Divide(val sublist <pointer>, ref second_list <pointer>)

Divides the list into two halves.

midpoint = sublist

position = sublist->link

while *position is not NULL* **do**

 position = position->link

if *position is not NULL* **then**

 midpoint = midpoint->link

 position = position->link

end

end

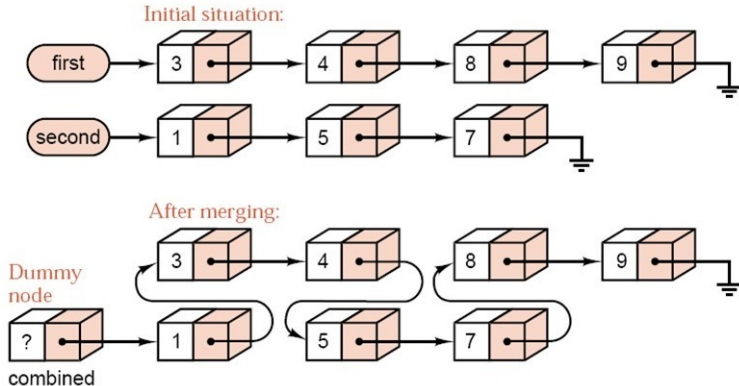
second_list = midpoint->link

midpoint->link = NULL

End Divide



Merge two sublists



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Merge Sort

Merge two sublists

Algorithm Merge(ref first <pointer>, ref second <pointer>)

Merges two sorted lists into a sorted list.

lastSorted = address of combined

while *first is not NULL AND second is not NULL* **do**

if *first->data.key <= second->data.key* **then**

 lastSorted->link = first

 lastSorted = first

 first = first->link

else

 lastSorted->link = second

 lastSorted = second

 second = second->link

end

end

// ...



Merge two sublists

```
// ...
```

```
if first is NULL then  
    lastSorted->link = second  
    second = NULL  
else  
    lastSorted->link = first  
end  
first = combined.link  
End Merge
```

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