MIDTERM
Subject: **Discrete Structures** (CO1007)
Class: **CC15KTM2+QT15CNT2** Group: **CC01**
Time: **60 minutes** ***(Closed book test)***
Test date: **June 27 2016**

Student's Name: ____________________ **Student's ID:** | | | | | | | |

Final Score: ________ **Examiner:** ________________ **Examiner's Signature:** ________

(*There are* ***20*** *MCQs, each question is worth* ***0.5*** *points. Answers in bold* : ■; *cancel out to deselect:* ■.)

Question 1. Represent the sentence "All Vietnamese speak the same languages" in predicate calculus.

(A) $\forall x, \forall y, \exists l\ Vietnamese(x) \wedge Vietnamese(y) \wedge Speak(x,l) \longrightarrow Speak(y,l)$.
(B) $\forall x, \exists l\ Vietnamese(x) \wedge Speak(x,l)$.
(C) $\forall x, \exists y, \exists l\ Vietnamese(x) \wedge Vietnamese(y) \wedge Speak(x,l) \longrightarrow Speak(y,l)$.
(D) $\forall x, \forall y, \forall l\ Vietnamese(x) \wedge Vietnamese(y) \wedge Speak(x,l) \longrightarrow Speak(y,l)$.

Question 2. If a student randomly guesses at 20 multiple choice questions, find the probability that the student gets exactly four right. Each answer has four possible choices.

(A) $\frac{1}{\binom{20}{4}}$. (B) $\frac{1}{5}$. (C) 3.5692×10^{-7}. (D) 0.1897.

Question 3. Translate the specification $\exists p(F(p) \wedge B(p)) \rightarrow \exists j L(j)$ into English where

- $F(p)$ is "Printer p is out of service,"
- $B(p)$ is "Printer p is busy,"
- $L(j)$ is "Print job j is lost."

(A) If there is a printer that is both out of service and busy, then some job has been lost.
(B) If there is a printer that is both out of service and busy, then all jobs have been lost.
(C) If all printer that are both out of service and busy, then some job has been lost.
(D) If some job has been lost, then there is a printer that is both out of service and busy.

Question 4. How many sequences of bits of length 100 have as many 0's as 1's?

(A) 2^{100}. (B) $\binom{100}{50}$. (C) 2^{50}. (D) $2^{100} - 2^{50}$.

Question 5. Let p, q, and r be the propositions, where:

- p: Grizzly bears have been seen in the area.
- q: Hiking is safe on the trail.
- r: Berries are ripe along the trail.

Write the following proposition using p, q, and r and logical connectives.

(A) $(p \wedge r) \rightarrow q$. (B) $(p \wedge r) \rightarrow \neg q$. (C) $\neg q \rightarrow (p \wedge r)$. (D) $(p \vee r) \rightarrow \neg q$.

Question 6. How many ways of coloring 5 objects a_1, a_2, a_3, a_4, a_5 with 3 colors, such that each color must be used at least once?

(A) 15. (B) 60. (C) 10. (D) 150.

Question 7. Find the flaw(s) in the following inductive"proof" of a RIDICULOUS CLAIM that ***"All tables are the same height."***

"*Proof*". To prove this by induction, we let $P(n)$ be the statement "For any set of n tables, all n tables are the same height." If we prove this true for all n, it will certainly be true for $n =$ the number of tables that exist.

Now we proceed by induction on the number of tables.

Step 1: The base case is the case in which there is one table. Since this table is the same height as itself, the base case is true.

Step 2: Now assume that the statement holds for ANY set of n tables, and consider a set of $n+1$ tables.

Step 3: Put the tables in a line. If we remove the first table, we are left with a set of n tables. Then by the inductive hypothesis, these n tables must all be the same height. If, instead, we had removed the last table, we would again have n tables, which would now include the first one, and again by inductive hypothesis all n tables would be the same height. Therefore, all of the tables must be the same height as, for instance, the second table from the front, and consequently must be the same height as one another.

The result then follows from induction. QED.

(A) NO. The "proof" above has no flaw.
(B) Step 3 is not correct when $n = 1$.
(C) We can't say that if we prove $P(n)$ true for all n, it will certainly be true for $n =$ the number of tables that exist, and therefore "All tables are the same height."
(D) All arguments in the "proof" above is invalid because the statement "All tables are the same height" is a fallacy.

Question 8. Let $\mathbb{Z}_8 = \{\overline{0}, \overline{1}, \overline{2}, \overline{3}, \overline{4}, \overline{5}, \overline{6}, \overline{7}\}$ be the set of all residue classes in modulo 8, i.e., $\mathbb{Z}_8$ is the set of all equivalency classes of the following equivalent relation

$$a \equiv b \Longleftrightarrow 8|(a-b),\ \forall a, b \in \mathbb{Z}.$$

Then the map

$$\begin{aligned} f : &\mathbb{Z}_8 \longrightarrow \mathbb{Z}_8 \\ &\overline{x} \longmapsto \overline{2x} \end{aligned}$$

(A) is a surjection but not an injection. (B) is a bijection.
(C) is neither surjective nor injective. (D) is an injection but not a surjectiion.

Question 9. How many of the natural numbers between 100 and 1000 are either multiples of 3 or multiples of 7?

(A) 300. (B) 128. (C) 428. (D) 385.

Question 10. Let X, Y be two nonempty sets and let $f : X \longrightarrow Y$ be a function. Consider $A, B \subseteq X$, and $C, D \subseteq Y$. Which of the following is correct?

(A) $f^{-1}(\overline{C}) = \overline{f^{-1}(C)}$. (B) $f(A \cup B) = f(A) \cup f(B)$.
(C) $f(\overline{A}) = \overline{f(A)}$. (D) $f(f^{-1}(D)) = D$.

Question 11. Which of the following predicate calculus formulas is not a tautology?

I. $\forall x(P(x) \to A) \longleftrightarrow \exists x P(x) \to A$, where x does not occur as a free variable in A.

II. $\exists x(P(x) \to A) \longleftrightarrow \forall x P(x) \to A$, where x does not occur as a free variable in A.

III. $\forall x(P(x) \to Q(x)) \longrightarrow (\forall x P(x) \to \forall x Q(x))$.

IV. $(\forall x(P(x) \leftrightarrow Q(x))) \longleftrightarrow (\forall x P(x) \leftrightarrow \forall x Q(x))$.

(A) Formula I. (B) Formula II.
(C) Formula III. (D) Formula IV.

Question 12. Which of the following statements is not true?
(A) $\varnothing \in \varnothing$. (B) $\varnothing \in \{\varnothing\}$. (C) $\varnothing \subseteq \varnothing$. (D) $\varnothing \subseteq \{\varnothing,\ \{\varnothing\}\}$.

Question 13. A sequence is defined by the recurrence relation $U_{n+1} = 2U_n - 5$ and $U_0 = 10$. Which term of this sequence is the first to exceed 90?
(A) U_5. (B) U_3. (C) U_4. (D) U_6.

Question 14. How many terms of the sequence -5, -1, 3, ... must be added to give a sum of 400?
(A) 15. (B) 16. (C) 17. (D) 18.

Question 15. The statement "x is a minimal of the poset $(A, \preccurlyeq)$" is equivalent to
(A) $\forall y \in A((x = y) \vee (x \succcurlyeq y))$. (B) $\forall y \in A((x = y) \vee (y \succcurlyeq x))$.
(C) $\forall y \in A((x = y) \vee (x \not\succcurlyeq y))$. (D) $\exists y \in A((x = y) \vee (x \not\succcurlyeq y))$.

Question 16. How many ways are there to distribute 40 identical jelly beans among 7 CSE-HCMUT students, if each student gets at least 1 bean?
(A) $\binom{39}{33}$. (B) $\binom{40}{7}$. (C) $\binom{46}{6}$. (D) $\binom{46}{7}$.

Question 17. Suppose that $A, B, C, D \subset S$. Which of the following is not correct?
(A) $(A \cup B)\backslash(A \cap B) = (A\backslash B) \cup (B\backslash A)$. (B) $A \supseteq B \Longleftrightarrow \overline{A} \subseteq \overline{B}$.
(C) $(A\backslash B)\backslash C = (A\backslash C)\backslash(B\backslash C)$. (D) $(A \cup B) \cap (C \cup D) \subset (A \cap C) \cup (B \cap D)$.

Question 18. How many equivalence relations on a set with 3 elements?
(A) 3. (B) 4. (C) 5. (D) 6.

Question 19. The contrapositive of the propositional formula $p \longrightarrow (q \vee r)$ is
(A) $\neg q \wedge \neg r \longrightarrow p$. (B) $\neg p \longrightarrow \neg(q \vee r)$. (C) $\neg q \vee \neg r \longrightarrow p$. (D) $q \vee r \longrightarrow p$.

Question 20. Let S be the relation defined on $\mathbb{R}$ by $aSb \Longleftrightarrow |a - b| \leq 5$. Then S is
(A) reflexive and transitive. (B) symmetric and anti-symmetric.
(C) symmetric but not transitive. (D) neither reflexive nor transitive.