Lecturer:	*(Date)*	**Approved by:**	(Date)
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UNIVERSITY OF TECHNOLOGY **FACULTY OF CSE**	**MIDTERM EXAM**	Semester / Academic year	1	2023-2024
		Date		30/05/2023
	Course title	Discrete Structure for Computing		
	Course ID	CO1007		
	Duration	60 mins	Question sheet code	2311

Notes: - Students do not use course materials except one A4 hand-writing document.
- Submit the question sheet together with the answer sheet.
- Choose the best answer (only 1) for each question.

1. (L.O.3.2) Determine the value of n and k (integers) satisfy

$$C_{n+1}^{k} : C_{n}^{k+1} : C_{n}^{k-1} = 6 : 5 : 2$$

A. $n = 8, k = 3$ B. $n = 10, k = 4$
C. $n = 9, k = 2$ D. Another answer.

2. (L.O.3.2) How many bit strings of length 12 don't contain a '11' substring?

A. 366 B. 367 C. 377 D. 387

3. (L.O.3.1) Suppose that Smartphone A has 256MB RAM and 32GB ROM, and the resolution of its camera is 8 MP; Smartphone B has 288MB RAM and 64GB ROM, and the resolution of its camera is 4 MP; and Smartphone C has 128MB RAM and 32GB ROM, and the resolution of its camera is 5MP. Determine the truth value of each of these propositions.

- (I) Smartphone C has more ROM or a higher resolution camera than Smartphone B.
- (II) Smartphone B has more RAM, more ROM, and a higher resolution camera than Smartphone A.
- (III) If Smartphone B has more RAM and more ROM than Smartphone C, then it also has a higher resolution camera.

A. (I) True, (II) True, (III) True B. (I) True, (II) False, (III) True
C. (I) True, (II) True, (III) False D. (I) True, (II) False, (III) False

4. (L.O.3.1) Which of the following propositions is a tautology?

- (I) $(p \vee q) \bigoplus (p \wedge q)$
- (II) $(p \leftrightarrow q) \bigoplus (\neg p \leftrightarrow q)$
- (III) $(p \bigoplus q) \wedge (p \bigoplus \neg q)$

A. (I) B. (II) and (III) C. (I) and (II) D. Another answer.

5. (L.O.3.1) Let $C(x, y)$ mean that student x is enrolled in class y, where the domain for x consists of all students in your school and the domain for y consists of all classes being given at your school. Use quantifiers to express the statement: *"There exist two distinct people enrolled in exactly the same courses."*

A. $\exists x \exists y \exists z((x \neq y) \wedge (C(x, z) \leftrightarrow C(y, z)))$ B. $\exists x \exists y \forall z((x \neq y) \wedge (C(x, z) \leftrightarrow C(y, z)))$
C. $\exists x \exists y \exists z((x \neq y) \wedge (C(x, z) \rightarrow C(y, z)))$ D. $\exists x \exists y \forall z((x \neq y) \wedge (C(x, z) \vee C(y, z)))$

6. (L.O.2.2) Let $T(x, y)$ mean that student x likes cuisine y, where the domain for x consists of all students at your school and the domain for y consists of all cuisines. Express the following statement in a simple English sentence: $\forall x \forall z \exists y((x \neq z) \to \neg(T(x, y) \land T(z, y)))$

A. For every pair of distinct students at your school, there is some cuisine that at least one does not like.
B. There are some cuisines that all the students at your school do not like.
C. For every pair of students at your school, there is some cuisine about which they have the same opinion.
D. Two students at your school have exactly the same tastes (either they both like it or they both do not like it).

7. (L.O.2.2) Is this reasoning for finding the solutions to the equation $\sqrt{2x^2 - 1} = x$ correct?
(1) $\sqrt{2x^2 - 1} = x$ is given,
(2) $2x^2 - 1 = x^2$ obtained by squaring both sides of (1);
(3) $x^2 - 1 = 0$, obtained by subtracting x^2 from both sides of (2);
(4) $(x - 1)(x + 1) = 0$, obtained by factoring the left-hand side of (3);
(5) $x = 1$ or $x = -1$, which follows because $ab = 0$ implies that $a = 0$ or $b = 0$.

A. Yes   B. No

8. (L.O.2.2) Let S be an equivalence relation on A, let B be a subset of A, and suppose that T is an equivalence relation on B. Defining $R = S \cup T$. It's given that there exists $x \in A$ such that B is the equivalence class of x to S ($[x]_S = B$). Which of the following assessments is true?

A. R is also an equivalence relation.
B. R is not an equivalence relation.
C. R is just symmetric
D. R is only reflexive

9. (L.O.1.2) Let S be the equivalence relation defined on $\{x : x \in R, 0 \leq x \leq 2\}$. We define xSy if and only if $\lceil x \rceil = \lceil y \rceil$. Which of the following is the equivalence classes of S ?

A. $[S] = \{[0, 1), [1, 2)\}$ B. $[S] = \{0, (0, 1), (1, 2]\}$ C. $[S] = \{0, (0, 1], (1, 2]\}$ D. $[S] = \{[0, 1], [1, 2]\}$

10. (L.O.1.2) Let $A = \{x^2 \mid 0 < x < 1\}$, and $B = \{x^3 \ \ 1 < x < 2\}$. Which of the statements is true?

A. Exists one-to-one function from A to B.
B. Exists on-to function from A to B.
C. Exists bijective function from A to B.
D. There is neither a one-to-one nor an on-to function from A to B.

11. Consider the following statement: `''A female athlete won every sports prize and her brother also won some mathematics prize''`. We define some predicates below to symbolize the given statement

- $Girl(a) =$ "a is a girl", $Boy(b) =$ "b is a boy",
- $SportPrize(x) =$ "x is a prize in Sport tournament",
- $MathPrize(y) =$ "y is a prize in Mathematics competition",
- $Sibling(a, b) =$ "a and b are siblings in a family",
- $WinSport(w, s) =$ "w won a sport prize s",
- $WinMath(u, m) =$ "u won a mathematics prize m".

Which of the following options best represents the given statement?

(I) $\forall x\ [SportPrize(x) \longrightarrow \exists y\ (Girl(y) \wedge WinSport(y, x))]$

(II) $[\ (\exists f, Girl(f) \wedge\ \forall x, SportPrize(x)) \longrightarrow WinSport(f, x)\ ]$
$\wedge\ [\ (\exists b, Sibling(f, b) \wedge Boy(b) \wedge\ \exists y, MathPrize(y)) \longrightarrow WinMath(b, y)]$

(III) $\exists f\ [\ (Girl(f) \wedge\ \forall x, SportPrize(x)) \longrightarrow WinSport(f, x)\ ]$
$\wedge\ \exists m\ [\ (Sibling(f, m) \wedge\ \exists y, MathPrize(y)) \longrightarrow WinMath(m, y)\ ]$

(IV) $\exists f\ [\ (Girl(f) \wedge\ \forall x, SportPrize(x)) \longrightarrow WinSport(f, x)\ ]$
$\wedge\ \exists m\ [\ (Sibling(f, m) \wedge\ Boy(m) \wedge\ \exists y, MathPrize(y)) \longrightarrow WinMath(m, y)\ ]$

A. (IV) B. (III) C. (II) D. (I)

12. (L.O.1.1) Let P and Q be binary predicates and S be nullary predicates. Which formulas are valid?
(I) $\forall x \forall y (P(x) \rightarrow P(y)) \wedge (P(y) \rightarrow P(x))$ (II) $\exists y((\forall x P(x)) \rightarrow P(y))$
(III) $(\forall x P(x) \rightarrow S) \rightarrow \exists x(P(x) \rightarrow S)$ (IV) $\forall x(P(x) \vee Q(x)) \rightarrow (\forall x P(x)) \vee (\exists x Q(x))$

A. Only (II), (III), and (IV) B. Only (II) and (III)
C. Only (I), (II), and (IV) D. Only (III) and (IV)

13. (L.O.2.2) The statement "x is a minimal element A with respect to the partial order relation $\preccurlyeq$" is equivalent to

A. $\forall y \in A((x = y) \vee (x \not\succcurlyeq y))$. B. $\forall y \in A((x = y) \vee (y \not\succcurlyeq x))$.
C. $\forall y \in A((x = y) \vee (x \not\preccurlyeq y))$. D. $\exists y \in A((x = y) \vee (x \not\preccurlyeq y))$.

14. (L.O.3.2) How many positive integer solutions (x_1, x_2, x_3) does the equation $x_1 + x_2 + x_3 = 2023$ have?

A. $\binom{2021}{2}$. B. $\binom{2022}{2}$. C. $\binom{2023}{2}$. D. $\binom{2024}{2}$.

15. (L.O.3.2) A *Boolean vector function* of m components, n variables is a function from $\{0,1\}^n$ to $\{0,1\}^m$, for m, n positive integers and $\{0,1\}^k$ is Cartesian product of k copies of $\{0,1\}$. Then the number of vector Boolean functions from $\{0,1\}^2$ to $\{0,1\}^3$ is

A. 256. B. 65536. C. 4096. D. 64.

16. (L.O.3.2) How many ways to color five objects a_1, a_2, a_3, a_4, a_5 using three colors, such that each color is used at least once?

A. 15. B. 60. C. 10. D. 150.

17. (L.O.3.2) The number of reflexive relations over a set of 2 elements is

A. 5. B. 3. C. 15. D. 4.

18. (L.O.2.2) Which of the following is correct?

A. Every relation R over a set A has to satisfy at least one of the properties: reflexive, symmetric, anti-symmetric, transitive.

B. If the relation R over A having the property that R^2 is reflexive then R is also reflexive.

C. There is no relation R over a set A satisfying all 4 properties: reflexive, symmetric, anti-symmetric, transitive at the same time.

D. If the two relations R_1 and R_2 over the set A are transitive then it is not necessary that $R_1 \cup R_2$ is transitive.

19. (L.O.2.2) Suppose that R is a relation over the set of integers such that xRy if and only if $y = x + 1$. Which of the following is the transitive closure of R?

A. $R^* = \{(x, y)|x \leq y\}$.

B. $R^* = \{(x, y)|x \geq y\}$.

C. $R^* = \{(x, y)|x < y\}$.

D. $R^* = \{(x, y)|x, y \in \mathbb{Z}\}$.

20. (L.O.1.2) Let A, B, C, D be the four subsets in the universe S. Which of the following is not correct?

A. $(A \cup B)\backslash(A \cap B) = (A\backslash B) \cup (B\backslash A)$.

B. $A \supseteq B \Longleftrightarrow \overline{A} \subseteq \overline{B}$.

C. $(A\backslash B)\backslash C = (A\backslash C)\backslash(B\backslash C)$.

D. $(A \cap C) \cup (B \cap D) = (A \cup B) \cap (C \cup D)$.

Solution 2311

1. A.	6. A.	11. A.	16. D.
2. C.	7. B.	12. A.	17. D.
3. D.	8. A.	13. C.	18. D.
4. D.	9. C.	14. B.	19. C.
5. B.	10. C.	15. C.	20. D.