HO CHI MINH CITY UNIVERSITY OF TECHNOLOGY
Faculty of Computer Science & Engineering

Student name: ______________
Student ID: ______________

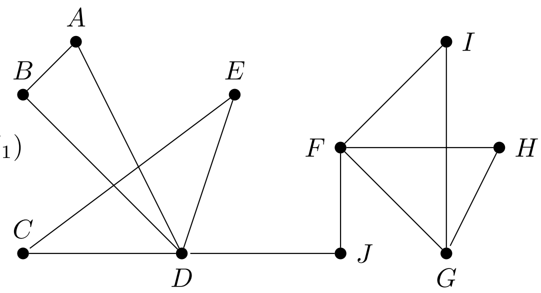
Examination on May 19th, 2017

Course: Discrete structures for Computer Science
Duration: 90 minutes Exam Code: **1712** Open book.
Choose the best answer for each multiple-choice question.

In questions 1–11, we consider undirected graph G_1 with adjacency matrix as follows:

$$
\begin{array}{cc}
 & \begin{array}{cccccccccc} A & B & C & D & E & F & G & H & I & J \end{array} \\
\begin{array}{c} A \\ B \\ C \\ D \\ E \\ F \\ G \\ H \\ I \\ J \end{array} &
\begin{bmatrix}
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{array}
$$

(G_1)



Question 1. (L.O.1.2) Is graph G_1 connected?

(A) Yes (B) No

Question 2. (L.O.1.2) Could we consider G_1 as a planar graph?

(A) Yes (B) No

Question 3. (L.O.1.2) Does there exist in G_1 any Euler path?

(A) Yes (B) No

Question 4. (L.O.1.2) Does there exist in G_1 any Euler circuit?

(A) Yes (B) No

Question 5. (L.O.1.2) Does there exist in G_1 any Hamilton path?

(A) Yes (B) No

Question 6. (L.O.1.2) How many connected components are there in graph G_1?
(A) 1 (B) 2 (C) 3 (D) 4

Question 7. (L.O.1.2) Is G_1 a bipartie graph?

(A) Yes (B) No

Question 8. (L.O.1.2) What is the chromatic number of graph G_1?
(A) 2
(B) 3
(C) 4
(D) 5

Question 9. (L.O.1.2) How many cut edges (bridges) are there in the graph G_1?
(A) 0 (B) 1 (C) 2 (D) 3

Question 10. **(L.O.1.2)** Which of the following assertions is true for the graph G_1?

- **(A)** A and C are the articulation points (cut vertices) of G_1
- **(B)** D and C and F are the articulation points (cut vertices) of G_1
- **(C)** D and G are not the articulation points (cut vertices) of G_1
- **(D)** D and F and J are the articulation points (cut vertices) of G_1

Question 11. How many vertices and how many edges does Q_7 have

- **(A)** The number of vertices is 128, the number of edges is 256
- **(B)** The number of vertices is 64, the number of edges is 448
- **(C)** The number of vertices is 256, the number of edges is 324
- **(D)** The number of vertices is 128, the number of edges is 448

Question 12. **(L.O.2.1)** Give the adjacent matrix G_2

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array} \begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Which of the following matrices is the incidence matrix of G_2?

(A)
$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array} \begin{array}{c} \begin{array}{ccccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 & e_9 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

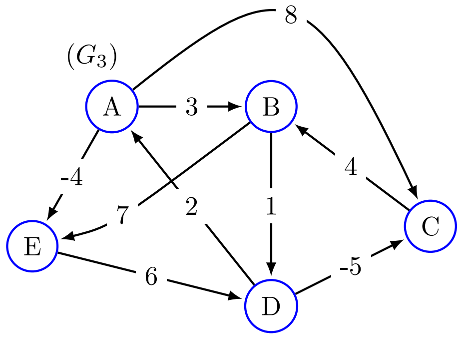
(B)
$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array} \begin{array}{c} \begin{array}{ccccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 & e_9 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

(C) None of them.

(D) Both of them.

Question 13. **(L.O.4.1)** We consider the following graph G_3 to find shortest paths from any couple of vertices.

(G_3)



$$L^{(0)} = \begin{pmatrix} 0_0 & 3_0 & 8_0 & \infty_0 & -4_0 \\ \infty_0 & 0_0 & \infty_0 & 1_0 & 7_0 \\ \infty_0 & 4_0 & 0_0 & \infty_0 & \infty_0 \\ 2_0 & \infty_0 & -5_0 & 0_0 & \infty_0 \\ \infty_0 & \infty_0 & \infty_0 & 6_0 & 0_0 \end{pmatrix}$$

$$L^{(1)} = \begin{pmatrix} 0_0 & 3_0 & 8_0 & \infty_0 & -4_0 \\ \infty_0 & 0_0 & \infty_0 & 1_0 & 7_0 \\ \infty_0 & 4_0 & 0_0 & \infty_0 & \infty_0 \\ 2_0 & 5_1 & -5_0 & 0_0 & -2_1 \\ \infty_0 & \infty_0 & \infty_0 & 6_0 & 0_0 \end{pmatrix} \quad L^{(2)} = \begin{pmatrix} 0_0 & 3_0 & 8_0 & 4_2 & -4_0 \\ \infty_0 & 0_0 & \infty_0 & 1_0 & 7_0 \\ \infty_0 & 4_0 & 0_0 & 5_2 & 11_2 \\ 2_0 & 5_1 & -5_0 & 0_0 & -2_1 \\ \infty_0 & \infty_0 & \infty_0 & 6_0 & 0_0 \end{pmatrix}$$

$$L^{(3)} = \begin{pmatrix} 0_0 & 3_0 & 8_0 & 4_2 & -4_0 \\ \infty_0 & 0_0 & \infty_0 & 1_0 & 7_0 \\ \infty_0 & 4_0 & 0_0 & 5_2 & 11_2 \\ 2_0 & -1_3 & -5_0 & 0_0 & -2_1 \\ \infty_0 & \infty_0 & \infty_0 & 6_0 & 0_0 \end{pmatrix} \quad L^{(4)} = \begin{pmatrix} 0_0 & 3_0 & -1_4 & 4_2 & -4_0 \\ 3_4 & 0_0 & -4_4 & 1_0 & -1_4 \\ 7_4 & 4_0 & 0_0 & 5_2 & 3_4 \\ 2_0 & -1_3 & -5_0 & 0_0 & -2_1 \\ 8_4 & 5_4 & 1_4 & 6_0 & 0_0 \end{pmatrix}$$

$$L^{(5)} = \begin{pmatrix} 0_0 & 1_5 & -3_5 & 2_5 & -4_0 \\ 3_4 & 0_0 & -4_4 & 1_0 & -1_4 \\ 7_4 & 4_0 & 0_0 & 5_2 & 3_4 \\ 2_0 & -1_3 & -5_0 & 0_0 & -2_1 \\ 8_4 & 5_4 & 1_4 & 6_0 & 0_0 \end{pmatrix}$$

Using Floyd-Warshall algorithm, determine the matrix $L^{(3)}$.

(A) $L^{(3)} = \begin{pmatrix} 0_0 & 3_0 & 8_0 & 4_2 & -4_0 \\ \infty_0 & 0_0 & \infty_0 & 1_0 & 7_0 \\ \infty_0 & 4_0 & 0_0 & 5_2 & 11_2 \\ 2_0 & -1_3 & -5_0 & 0_0 & -2_1 \\ \infty_0 & \infty_0 & \infty_0 & 6_0 & 0_0 \end{pmatrix}$

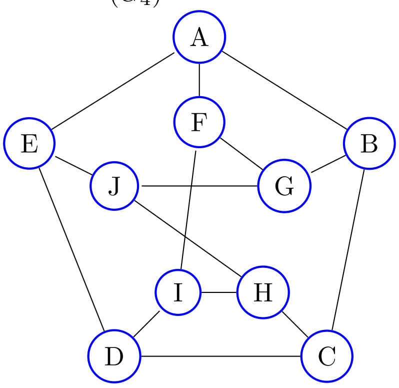
(B) $L^{(2)} = \begin{pmatrix} 0_0 & 3_0 & 8_0 & 4_2 & -4_0 \\ \infty_0 & 0_0 & \infty_0 & 1_0 & 7_0 \\ \infty_0 & 4_0 & 0_0 & 5_2 & 11_2 \\ 2_0 & 5_1 & -5_0 & 0_0 & -2_1 \\ \infty_0 & \infty_0 & \infty_0 & 6_0 & 0_0 \end{pmatrix}$

(C) $L^{(4)} = \begin{pmatrix} 0_0 & 3_0 & -1_4 & 4_2 & -4_0 \\ 3_4 & 0_0 & -4_4 & 1_0 & -1_4 \\ 7_4 & 4_0 & 0_0 & 5_2 & 3_4 \\ 2_0 & -1_3 & -5_0 & 0_0 & -2_1 \\ 8_4 & 5_4 & 1_4 & 6_0 & 0_0 \end{pmatrix}$

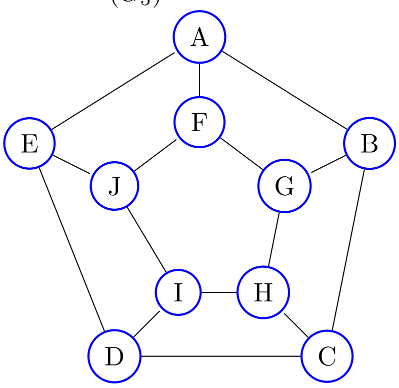
(D) $L^{(3)} = \begin{pmatrix} 0_0 & 3_0 & 8_0 & 4_2 & -4_0 \\ \infty_0 & 0_0 & \infty_0 & 1_0 & 7_0 \\ \infty_0 & 4_0 & 0_0 & 5_3 & 11_2 \\ 2_0 & -1_2 & -5_0 & 0_0 & -2_1 \\ \infty_0 & \infty_0 & \infty_0 & 6_0 & 0_0 \end{pmatrix}$

Question 14. Are the following graphs G_4 and G_5 isomorphic to each other ?

(G_4)



(G_5)



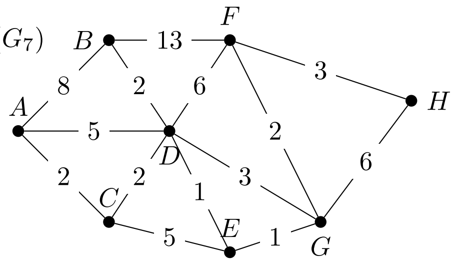
(A) Yes

(B) G_4 and G_5 are not due to the number of circuits having length of 4.

(C) G_4 and G_5 are not due to the number of vertices with degree of 3.

Questions from 15–16, consider the following graph G_7 in order to find the shortest paths from vertex A to the others by **Dijkstra** algorithm.

(G_7)



S	A	B	C	D	E	F	G	H
$\emptyset$	0	∞	∞	∞	∞	∞	∞	∞
A		8	2	5	∞	∞	∞	∞
C		8	2	4	7	∞	∞	∞
D		6		4	5	10	7	∞
E		6	2	4	5	10	6	∞
B		6					10	6
G						8	6	12
F						8		11

Using Dijkstra algorithm, we build a tracing table containing labels for corresponding vertices. Note that columns are ordered (from left to right) in alphabetical order (i.e., $A \to B \to \ldots$).
Suppose that the initialization row is numbered as row 1 (corresponding to $S = \emptyset$) and includes 0 ; ∞ ; ∞ ; ∞ ; ∞ ; ∞ ; ∞ ; ∞.

Question 15. **(L.O.4.1)** Running the algorithm, which is obtained for row 5 ?

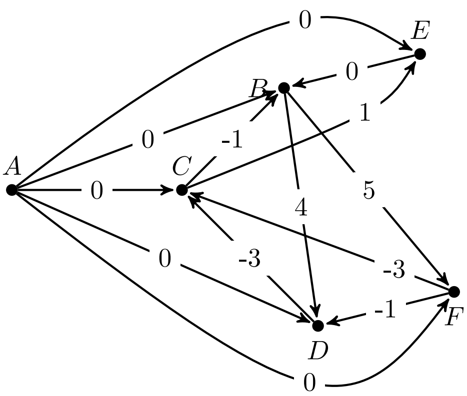
(A) $0; 6; 2; 4; 7; 10; 6; \infty$
(B) $0; 8; 2; 5; 5; 8; 6; \infty$
(C) $0; 6; 2; 4; 5; 10; 6; \infty$
(D) $0; 6; 2; 4; 5; 10; 7; 14$

Question 16. **(L.O.4.1)** The shortest path from A to H is (path; value) ?

(A) $A \to C \to D \to E \to G \to H; 12$
(B) $A \to B \to D \to G \to H; 10$
(C) $A \to D \to C \to E \to G \to F \to H; 13$
(D) $A \to C \to D \to E \to G \to F \to H; 11$

Question 17. **(L.O.4.1)** Consider the following graph G_8 for finding shortest paths from vertex A to the others.

(G_8)



Step	A	B	C	D	E	F
0	0	∞	∞	∞	∞	∞
1		0A	0A	0A	0A	0A
2		-1C	-3D	-1F	0A	0A
3		-4C	-4D	-1F	-2C	0A
4		-5C	-4D	-1F	-3C	0A
5						

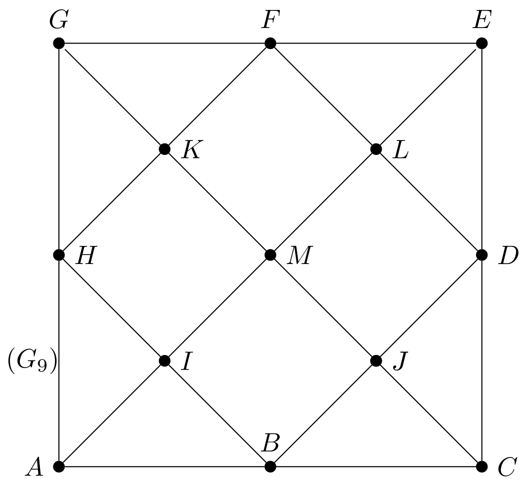
Suppose that columns in the tracing table are ordered (from left to right) in alphabetical order (i.e., $A \to B \to \ldots$).
Suppose that the initialization row corresponds to step 1.
Which are the weights of the shortest paths from A to all other vertices of G_8? (in order: A to B,C,D,E,F)

(A) $-4, -5, 1, -1, 0$
(B) $-4, -4, -1, -2, 0$
(C) There exists a circle of negative length
(D) $-5, -4, -1, -3, 0$

Question 18. Which of the following is a sufficient condition to exist an Euler path on an undirected graph ?

(A) The graph is connected and there is less than 2 odd-nodes exist in the graph.
(B) There are 2 or no odd-degree nodes exist in the graph.
(C) The graph is connected and there are minimum 2 odd-nodes exist in the graph.
(D) The graph is connected and there are 2 or no odd-degree nodes exist in the graph.

Question 19. Does exists in G_9 any Hamilton circuit/ Euler path?



(A) No / Yes. (B) Yes / No. (C) Yes / Yes. (D) No / No.

Question 20. Does exists in G_9 any Hamilton circuit/ Euler path? How many edges does a graph have if it has vertices of degree 5,4,4,4,3. Can it be a planar graph?

(A) 9 / Yes. (B) 11 / Yes. (C) 10 / No. (D) 10 / Yes.