UNIVERSSITY OF TECHNOLOGY FACULTY OF CSE	FINAL EXAM	Semester / Academic year	2	2020-2021
		Date		empty
	Course title	Discrete Structure for Computing		
	Course ID	CO1007		
	Duration	90 mins	Question sheet code	2024

Notes: - Student does not use course materials except one A4 (single side) hand-writing document.
- Submit the question sheet together with the answer sheet.
- Choose the best answer (only 1) for each question.

I. Multiple choices (10.0p):

1. [L.O.1.2] *[Definition]: A r-regular graph G is an undirected graph in which every vertex of G has degree r.* Which one of the following is TRUE?

   (a) Any r-regular graph where r is an even number has a Eulerian circuit

   (b) The cube Q_n is regular for all values of $n \geq 0$

   (c) The wheel W_n is regular if and only if $n > 3$

   (d) None of these answers is correct.

2. [L.O.4.1] Given a simple graph G. Which of the following statements is correct:

   (a) If G is a connected graph, then it is possible to remove vertices to disconnect G iff G is a complete graph.

   (b) A simple graph G is bipartite if and only if it has no circuits with an odd number of edges.

   (c) A simple graph G with n vertices is connected if it has more than $(n-1)(n-2)/3$ edges.

   (d) If a connected simple graph G is the union of the graphs G_1 and G_2, then G_1 and G_2 have at least two common vertex.

3. [L.O.4.1] Let G be an arbitrary graph with n nodes and k components. If a vertex is removed from G, the number of components in the resultant graph must necessarily lie down between

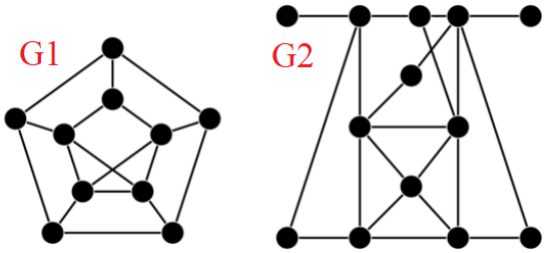
   (a) k and n

   (b) $k-1$ and $n-1$

   (c) $k-1$ and $n+1$

   (d) $k+1$ and $n-k$

4. [L.O.4.1] Given two graphs $G1$ and $G2$:



   Which of the following statements is TRUE?

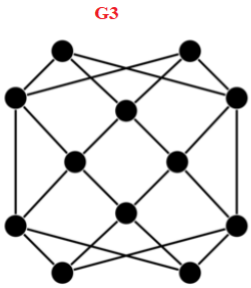
   (a) $G1$: Not planar, $G2$: Planar

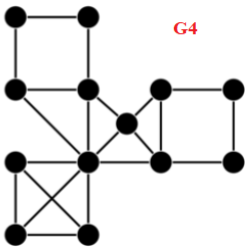
   (b) $G1$: Not planar, $G2$: Not planar

   (c) $G1$: Planar, $G2$: Planar

   (d) $G1$: Planar, $G2$: Not planar

5. [L.O.4.1] Given two graphs $G3$ and $G4$:





Which of the following statements is TRUE?

(a) $G3$: Exists Hamilton circuit, $G4$: Exists Hamilton circuit

(b) $G3$: Exists Hamilton circuit, $G4$: No Hamilton circuit

(c) $G3$: Exist Euler circuit, $G4$: Exist Euler circuit

(d) $G3$: No Hamilton circuit, $G4$: No Euler path

6. [L.O.4.1] What is the number of vertices in an undirected connected graph with 27 edges, 6 vertices of degree 2, 3 vertices of degree 4 and remaining of degree 3?

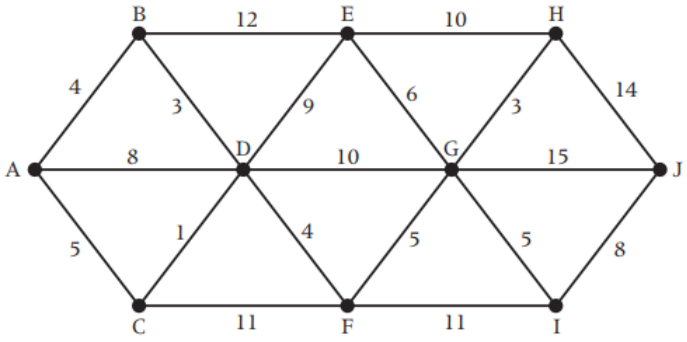
(a) 18

(b) 19

(c) 20

(d) 21

7. [L.O.2.3] Find the shortest distance from A to J on the network below.



(a) The shortest distance is 26, and the path is ACDFGIJ

(b) None of these answers is correct.

(c) The shortest distance is 28, and the path is ACDFGHJ

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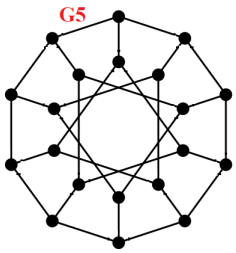
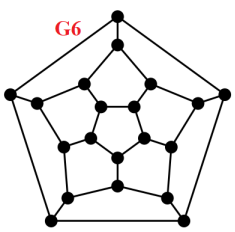
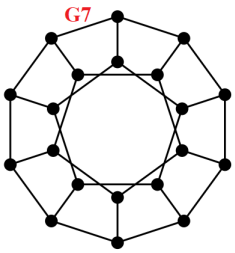
8. [L.O.1.2] Are the following graphs isomorphic?







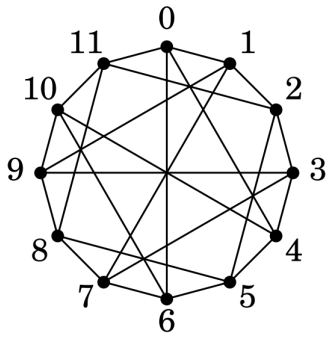
(a) G5 and G6 are isomorphic

(b) G6 and G7 are isomorphic

(c) All of thess graphs are isomorphic

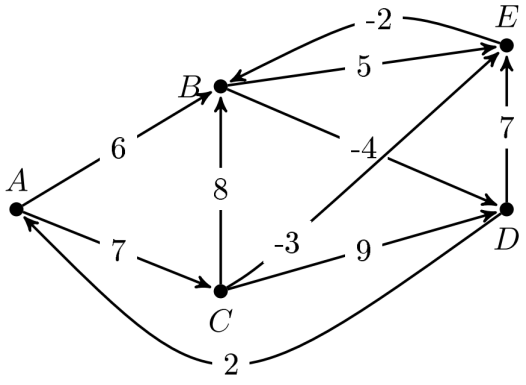
(d) All of these graphs are not isomorphic

9. [L.O.1.2] What is the chromatic number of the following graph?



(a) 3
(b) 4
(c) 5
(d) 6

10. [L.O.4.1] What is the stopping matrix of the Floyd-Warshall algorithm when we apply it to the following graph?



(a) L^3
(b) L^5
(c) L^4
(d) L^2

11. [L.O.1.2] Which of the following statement(s)is/are correct regarding Bellman-Ford shortest path algorithm?

1. Always finds a negative weighted cycle, if one exists.
2. Finds whether any negative weighted cycle is reachable from the source.

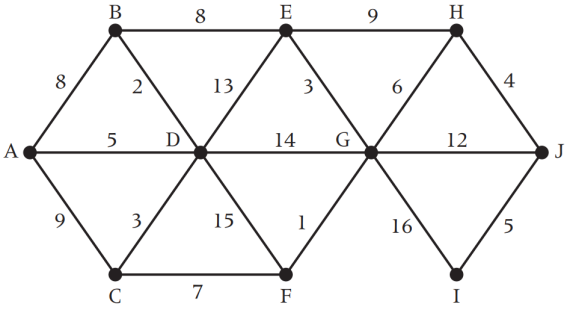
(a) 1 only
(b) 2 only
(c) Both 1 and 2
(d) Neither 1 nor 2

12. [L.O.1.2] Let $T = (V, E)$ be a tree and let $d(v)$ be the degree of a vertex. Which of the following statement(s)is/are correct?

1. if T has a vertex of degree $m \geq 2$, then it has at least m vertices of degree 1.
2. $\sum_{v \in V}(2 - d(v)) = 2$

(a) Neither 1 nor 2
(b) Both 1 and 2
(c) 1
(d) 2

13. [L.O.4.1] Use Kruskal's algorithm to find a minimal spanning tree for the graph below. What is the number of edge (N) of the minimum spanning tree and its total weight (W)?



(a) N=10 and W=36

(b) N=9 and W=36

(c) N=10 and W=37

(d) N=9 and W=38

14. [L.O.4.1] Given a graph $G = (V, E)$ (loops allowed), and two properties:

    1. If $n \geq 0$, G is connected, and G has v vertices and $v + n$ edges, then G has at least $n + 1$ cycles.
    2. If G has v vertices, e edges and c components, then G has at least $c + e - v$ cycles.

    Which of the following statements is correct?

(a) 1 is correct, 2 is incorrect

(b) Both of 1 and 2 are correct

(c) Neither 1 nor 2

(d) 1 is incorrect, 2 is correct

15. [L.O.4.1] Pre-order and In-order traversal of a given binary search tree, T produces the following sequence of keys $\{1, 2, 4, 8, 9, 5, 3, 6, 7\}$ and $\{8, 4, 9, 2, 5, 1, 6, 3, 7\}$. Which of the following sequences of keys can be the result of a Post-order traversal of the tree T?

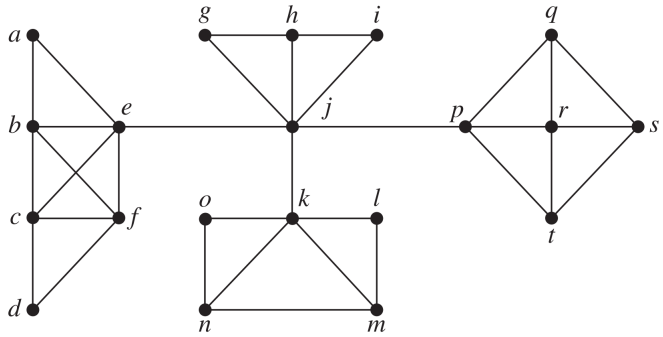
(a) {8, 4, 2, 9, 1, 5, 6, 3, 7}

(b) {8, 9, 4, 5, 2, 6, 7, 3, 1}

(c) {8, 4, 9, 1, 2, 5, 6, 3, 7}

(d) {7, 4, 9, 2, 1, 5, 6, 3, 8}

16. [L.O.1.2] Choose as the root, what is the depth-first (start from a) search of the given the graph G: (assume that the vertices are ordered alphabetically)



(a) a b c d e f g h i j k l m n o p q r s t

(b) a b c d f e j g h i k l m n o p q r s t

(c) a b c d f e g h i j k l m n o p q r s t

(d) None of these answers is correct

17. [L.O.4.1] What is the correct Post-fix expression for the following expression ($\uparrow$ means power)?

$$A + B \times C/(E - F) - 3 \uparrow (C - D) - (2 \times G + H) + 1$$

(a) $ABC \times EF - / + 3CD - \uparrow -2 \times GH + -1+$

(b) $ABC \times EF - / + 3CD - \uparrow -2G \times H + -1+$

(c) $ABCEF \times -/ + 3CD - \uparrow -2G \times H + -1+$

(d) Another answer

18. [L.O.3.2] There are 100 people attending an environmental conference. Each member can use one of three languages English, Russian, French. Based on a survey, there are 60 members who can only speak one kind of language, 180 members can only speak two languages English and French, 150 members can only speak two languages English and Russian, 170 members can only speak two languages Russian and French. How many members can speak all three languages?

(a) 50

(b) 60

(c) 70

(d) 80

19. [L.O.3.1] Lets a, b, c be the number of elements in three sets A, B, C $(a, b, c > 0)$, respectively. We know that $a = 2b$, and $b > c$. The number of the subset of A is greater than the number of the subset of B (the difference is x). The number of the subset of B is greater than the number of the subset of C (the difference is 15). Find x

(a) 230

(b) 240

(c) 250

(d) 260

20. [L.O.4.2] In a residential area, the probability that a person smokes is 30%. Know the rate of sore throat among smokers is 60%, among non-smokers is 30%. Random examination of 1 person and found that person has a sore throat. Find the probability that the person is a smoker.

(a) 36.2%

(b) 46.2%

(c) 56.2%

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21. [L.O.4.2] Suppose A and B are finite sets with cardinalities $|A| = n$ and $|B| = m$. How many functions $f : A \to B$ are there?

(a) $m! \times n!$

(b) m^n

(c) $m!/n$

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22. [L.O.4.2] There are n bags each containing $n + 1$ balls such that the i^{th} bag contains i white balls and $(n + 1 - i)$ red balls. Let u_i be the event of selecting an i^{th} bag, $i = 1, 2, \ldots n$; $P(u_i) = \frac{1}{n}$. w denote the event of getting a white ball. If n is even and E denotes the event of choosing an even-numbered bag, then the value of $P(w|E)$ is:

(a) $\frac{n+2}{2n+1}$

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23. [L.O.3.1] According to statistics, it is shown that 85% of the products of the COMP company meet the requirements of the clients. Assume this factory's productivity is 20 pieces per hour. What is the probability that there will be 8 or 9 satisfactory products in 30 minutes?

    (a) 61.33%

    (b) 62.33%

    (c) 63.33%

    (d) 64.33%

24. [L.O.4.2] There is a game, such that people have to pay x USD to play. In this game the player will roll a normal dice (6 faces: 1 to 6) three times. If all three times the number of the face is 6, the player can get back 6 USD. If two times the number of the face is 6, the player can get back 4 USD. If only one time the number of the face is 6, the player can get back 2 USD. Otherwise, the player will be lost x USD. Calculate the value of x so that the game is fair.

    (a) $x \approx 0.8$

    (b) $x \approx 1.0$

    (c) $x \approx 1.2$

    (d) $x \approx 1.4$

25. [L.O.4.2] If $P(A|C) > P(B|C)$ and $P(A|\overline{C}) > P(B|\overline{C})$ then:

    (a) $P(A) = P(B)$

    (b) $P(A) > P(B)$

    (c) $P(A) \leq P(B)$

    (d) $P(A) \geq P(B)$

Solution 2021

I. Multiple choices (10.0p):

1. [L.O.1.2] *[Definition]: A r-regular graph G is an undirected graph in which every vertex of G has degree r.*
Which one of the following is TRUE?

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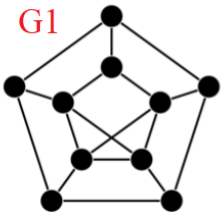
4. [L.O.4.1] Given two graphs $G1$ and $G2$:



Which of the following statements is TRUE?

   (a) $G1$: Not planar, $G2$: Planar

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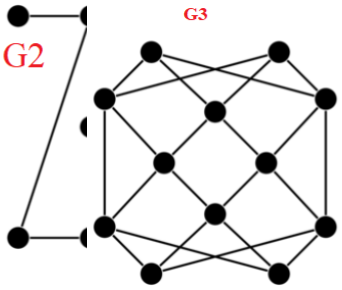

Which of the following statements is TRUE?

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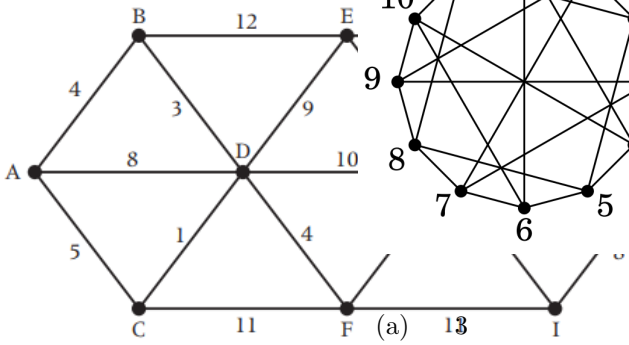

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9. [L.O.1.2] What is the chromatic number of the following graph?

(a) 13

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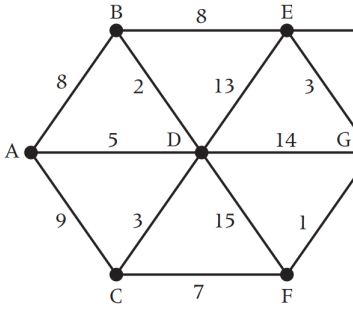

(a) N=10 and W=36

[(b)] N=9 and W=36

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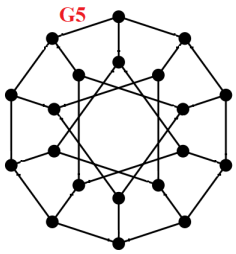
8. [L.O.1.2] Are the following graphs isomorphic?



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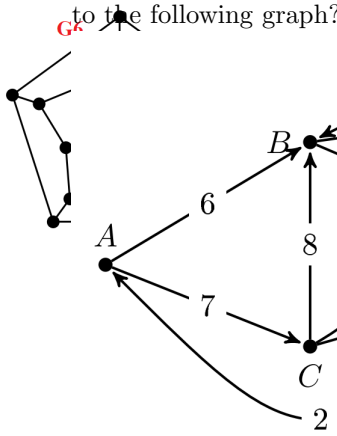

(a) L^3

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12. [L.O.1.2] Let $T = (V, E)$ be a tree and let $d(v)$ be the degree of a vertex. Which of the following statement(s)is/are correct?

1. if T has a vertex of degree $m \geq 2$, then it has at least m vertices of degree 1.
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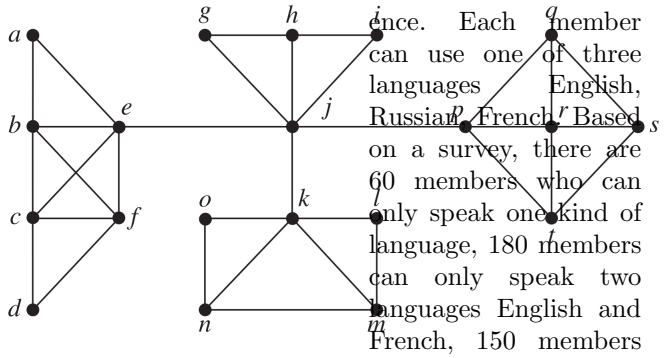

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$$A+B\times C/(E-F)-3\uparrow(C-D)-(2\times G+H)+1$$

(a) $ABC\times EF-/+3CD-\uparrow -2\times GH+-1+$

(b) $ABC\times EF-/+3CD-\uparrow -2G\times H+-1+$

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(c) 63.33%

(d) 64.33%

24. [L.O.4.2] There is a game, such that people have to pay x USD to play. In this game the player will roll a normal dice (6 faces: 1 to 6) three times. If all three times the number of the face is 6, the player can get back 6 USD. If two times the number of the face is 6, the player can get back 4 USD. If only one time the number of the face is 6, the player can get back 2 USD. Otherwise, the player will be lost x USD. Calculate the value of x so that the game is fair.

(a) $x \approx 0.8$
[(b)] $x \approx 1.0$
(c) $x \approx 1.2$
(d) $x \approx 1.4$

25. [L.O.4.2] If $P(A|C) > P(B|C)$ and $P(A|\overline{C}) > P(B|\overline{C})$ then:

(a) $P(A) = P(B)$
[(b)] $P(A) > P(B)$
(c) $P(A) \leq P(B)$
(d) $P(A) \geq P(B)$