


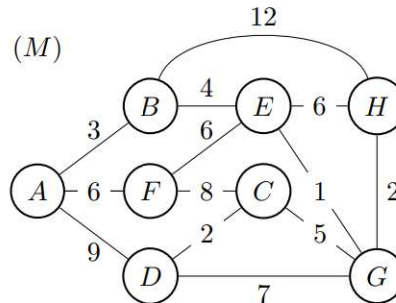
Lecturer(s):

Approved by:

Head of Department of CS

 <b>UNIVERSITY OF TECHNOLOGY</b> <b>FACULTY OF CSE</b>	<b>FINAL EXAM</b>		Semester/Academic year	3	2021-2022	
			Date	28/08/2022		
	Course title	Discrete Structures for Computing				
	Course ID	CO1007				
	Duration	80 minutes	Question sheet code	2881		
<b><u>Notes:</u></b> - One single sheet (both sides) of A4 paper of necessary hand-written notes is allowed. - Stu. ID and Stu. Fullname fields at the bottom of the question sheet and at the top of the answer sheet must be filled in. - Mark the correct answers on the answer sheet. Submit the answer sheet together with the question sheet when finishing the test. - The test consists of 30 multiple-choice questions, each of which has the score of $\frac{1}{3}$ .						

Use the following graph for questions from 1 to 3.



The Dijkstra algorithm is considered to find the shortest paths from the source node  $A$  to the others. Assume that the algorithm starts with the following table and for each iteration step, a new row is added to the table.

$S$	$A$	$B$	$C$	$D$	$E$	$F$	$G$	$H$
$\emptyset$	0	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$

If more than one vertices can be chosen to add to the set  $S$  at an iteration step, we choose the vertex that has the letter appearing before the other in the English alphabet.

**Question 1.** (L.O.3.2) What are the weights updated after the first two iteration steps excluding the initial step given in the table?

- ☐ (A) 0, 3, 10, 9, 9, 6,  $\infty$ , 13  
☐ (B) 0, 3,  $\infty$ , 9, 7, 6,  $\infty$ , 15  
☐ (C) 0, 3,  $\infty$ , 9, 9, 11, 8, 10  
☐ (D) None of the other choices is correct.

**Question 2.** (L.O.3.2) What are the weights updated if another two iteration steps are performed?

- ☐ (A) 0, 3, 13, 9, 7, 6,  $\infty$ , 15  
☐ (B) 0, 3, 14, 9, 7, 6, 12, 15  
☐ (C) 0, 3, 14, 9, 7, 6, 8, 13  
☐ (D) None of the other choices is correct.

**Question 3.** (L.O.3.2) What is the total weight of the minimum spanning tree that we obtain by means of the Prim algorithm starting from  $E$ ?

- ☐ (A) 31  
☐ (B) 15  
☐ (C) 23  
☐ (D) 19

**Question 4.** (L.O.3.1) A fruit package has 4 Fuji apples and 6 Gala apples. Two people  $A$  and  $B$  take the apples from the package one by one following one after the other and  $A$  first until they get a Fuji apple. What is the probability  $A$  takes the Fuji apple?

- ☐ (A) 0.57  
☐ (B) 0.25  
☐ (C) 0.41  
☐ (D) 0.62

**Question 5.** (L.O.3.1) A palindrome is a string whose reversal is identical to the string. How many bit strings of length 11 are palindromes?

- ☐ (A) 16  
☐ (B) 128  
☐ (C) 32  
☐ (D) 64

**Question 6.** (L.O.3.1) A connected simple undirected graph has 10 edges. Five of the vertices have degree 3 and the degrees of the others are at most 4. How many vertices does the graph have at most?

- ☐ (A) 20  
☐ (B) 6  
☐ (C) 8  
☐ (D) 10

**Question 7.** (L.O.3.1) The inorder and preorder traversal of a binary tree are  $DBEAFCG$  and  $ABDECFG$  respectively. The postorder traversal of the binary tree is

- (A)  $DEBFGCA$  (B)  $DEFGBCA$  (C)  $EDBFGCA$  (D)  $EDBGFCA$

**Question 8.** (L.O.3.1) The basic rule in Fleury's algorithm is?

- (A) never travel across a bridge of the original graph.  
 (B) only travel across a bridge of the untraveled part of the graph if there is no other alternative.  
 (C) only travel across a bridge on the original graph if there is no other alternative.  
 (D) never travel across a bridge of the untraveled part of the graph.

**Question 9.** (L.O.3.1) Suppose that 8% of all bicycle racers use steroids, that a bicyclist who uses steroids tests positive for steroids 96% of the time, and that a bicyclist who does not use steroids tests positive for steroids 9% of the time. What is the probability that a randomly selected bicyclist who tests positive for steroids actually uses steroids? Choose the most correct answer.

- (A) 0.15 (B) 1.0 (C) 0.48 (D) 0.56

**Question 10.** (L.O.3.2) In a connected simple graph  $G$ , the standard length  $l(P)$  of a path  $P$  between a pair of nodes  $u$  and  $v$  is the number of edges of  $P$ . A shortest path  $P_0$  between two distinct nodes  $u$  and  $v$  in  $G$  is the path that the value  $l(P_0)$  is minimum in the set of all lengths of simple paths connecting  $u$  and  $v$ . In the complete bipartite graph  $K_{m,2b}$  ( $m, b > 1$ ), the standard length of a shortest path between any two distinct nodes then is

- (A) at most  $2b$  (B)  $\min(m, n)$  (C) at most 2 (D)  $m$

**Question 11.** (L.O.3.1) How many ways to divide 32 identical blackboards among 4 schools if each school must receive at least one?

- (A) 3146 (B) 1456 (C) 4495 (D) 2389

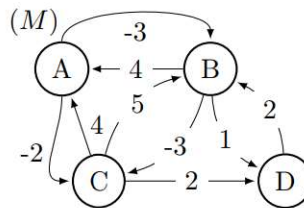
**Question 12.** (L.O.3.1) A die is rolled until the face numbered 3 appears. What is the probability the first time the face numbered 3 appears is the 6th roll.

- (A) 0.067 (B) 0.042 (C) 0.141 (D) 0.0125

**Question 13.** (L.O.3.1) Minh has 3 cats and at least one of them is female. What is the (approximate) probability that all the 3 cats are female, provided the probability that a cat is female is 0.42?

- (A) 0.074 (B) 0.092 (C) 0.125 (D) 0.5

Use the following graph for questions from 14 to 15.



The Floyd-Warshall algorithm is considered to find the shortest paths from the source node  $A$  to the others.

**Question 14.** (L.O.3.2) What is  $L^{(2)}$ ?

- (A)  $\begin{pmatrix} 0_0 & -3_0 & -2_0 & -2_2 \\ 4_0 & 0_0 & -3_0 & 1_0 \\ 4_0 & 1_1 & 0_0 & 2_0 \\ 3_2 & 2_0 & -1_2 & 0_0 \end{pmatrix}$  (B)  $\begin{pmatrix} 0_0 & -3_0 & -6_2 & -4_2 \\ 4_0 & 0_0 & -3_0 & 1_0 \\ 4_0 & 5_0 & 0_0 & 2_0 \\ 6_2 & 2_0 & -1_2 & 0_0 \end{pmatrix}$   
 (C)  $\begin{pmatrix} 0_0 & -3_0 & -6_2 & -2_2 \\ 4_0 & 0_0 & -3_0 & 1_0 \\ 4_0 & 5_0 & 0_0 & 2_0 \\ 6_2 & 2_0 & -1_2 & 0_0 \end{pmatrix}$  (D)  $\begin{pmatrix} 0_0 & -3_0 & -6_2 & -2_2 \\ 4_0 & 0_0 & -3_0 & 1_0 \\ 4_0 & 1_1 & 0_0 & 2_0 \\ 6_2 & 2_0 & -1_2 & 0_0 \end{pmatrix}$

**Question 15.** (L.O.3.2) After the algorithm is executed, how many different matrices occur?

- (A) 4 (B) 3 (C) 2 (D) 5



**Question 16.** (L.O.3.1) There are 7 balls of the same size, volume, and shape in a box. All of them are numbered from one to seven. Two of them are then picked from the box randomly without putting them back. Let  $X$  be the difference of the two numbers of the balls that are picked. What is the expectation of the random variable  $X$ ?

- (A) 2.97 (B) 2.67 (C) 2.37 (D) 2.47

**Question 17.** (L.O.3.1) A full  $n$ -ary tree is a tree in which each node has  $n$  children or it is a leaf. Let  $\ell$  be the number of internal nodes and  $L$  be the number of leaves in a full  $n$ -ary tree. If  $L = 41$  and  $\ell = 10$ , what is the value of  $n$ ?

- (A) 3 (B) 6 (C) 4 (D) 5

**Question 18.** (L.O.3.1) How many strings with six or more characters can be formed from the letters in “NO-COPYP”?

- (A) 2520 (B) 3060 (C) 1024 (D) 3688

**Question 19.** (L.O.3.1) Consider a wheel graph  $W_n = (V, E)$ , with a center  $v_0$ , and with adjacency matrix  $A$ . In your opinion, which of the following statements is correct?

- (A)  $W_n$  has an even number of edges, and every row of  $A$  consists of the same number of 1.  
 (B)  $W_n$  has  $(n + 1)$  vertices and does not contain a spanning tree.  
 (C)  $W_n$  has  $(n + 1)$  vertices and contains an Euler circuit.  
 (D)  $W_n$  contains a spanning tree and the number of edges of  $W_n$  is even.

**Question 20.** (L.O.3.1) A tree  $T = (V, E)$  has six nodes of degree 1, three nodes of degree 2, two nodes of degree 3, and the other nodes all have degree 4. The number of edges  $m$  of tree  $T$  is

- (A) 10 (B) 11 (C) 13 (D) 19

**Question 21.** (L.O.3.1) How many functions  $f : \{1, 2, 3, 4, 5\} \rightarrow \{1, 2, 3, 4\}$  satisfying  $|f(\{1, 2, 3, 4, 5\})| = 3$ ? The notation  $|X|$  denotes the cardinality of the set  $X$ .

- (A) 25 (B) 10 (C) 21 (D) 38

**Question 22.** (L.O.3.1) A  $r$ -regular graph  $G$  is an undirected graph in which each vertex of  $G$  has degree  $r$ . Let  $G = (V, E)$  be a 9-regular graph with 14 vertices. The number of edges of  $G$  is

- (A) another answer (B) 53 (C) 58 (D) 126

**Question 23.** (L.O.3.1) What is the value of the prefix expression  $E$ ?

$$E = + + 3 + 2 \uparrow 3 - 5 2 3.$$

The symbol  $\uparrow$  denotes the power of a number, e.g.  $5 \uparrow 2 = 25$ .

- (A) 121 (B) 46 (C) 35 (D) 12

**Question 24.** (L.O.3.1) How many positive integers not exceeding 2022 are divisible either by 4 or by 6?

- (A) 432 (B) 674 (C) 525 (D) 981

Use the following information for questions from **25** to **29**. The adjacency matrix of an undirected graph  $M$  is given as follows.

	$A$	$B$	$C$	$D$	$E$	$F$	$G$	$H$	$I$
$A$	0	1	1	0	1	0	0	1	1
$B$	1	0	0	1	0	1	1	1	0
$C$	1	0	0	1	1	1	0	0	0
$D$	0	1	1	0	1	0	0	0	0
$E$	1	0	1	1	0	1	0	1	0
$F$	0	1	1	0	1	0	0	1	0
$G$	0	1	0	0	0	0	0	1	1
$H$	1	1	0	0	1	1	1	0	1
$I$	1	0	0	0	0	0	1	1	0

**Question 25.** (L.O.2.2) Which of the following statements is correct?

- ☐ (A)  $M$  is planar but not connected. ☐ (B)  $M$  is connected and planar.  
☐ (C)  $M$  is connected but not planar. ☐ (D)  $M$  is neither connected nor planar.

**Question 26.** (L.O.3.1) Which of the following statements is correct?

- ☐ (A)  $M$  has neither an Euler circuit nor a Hamiltonian circuit.  
☐ (B)  $M$  has an Euler circuit and a Hamiltonian circuit.  
☐ (C)  $M$  has an Euler path but not an Euler circuit.  
☐ (D)  $M$  has neither an Euler path nor an Euler circuit.

**Question 27.** (L.O.3.1) The chromatic number  $\chi(M)$  is

- ☐ (A) 4 ☐ (B) 3 ☐ (C) 5 ☐ (D) 2

**Question 28.** (L.O.2.3) What is the cut edge of  $M$ ?

- ☐ (A)  $\{A, C\}$  ☐ (B) No cut edge exists in the graph.  
☐ (C)  $\{G, I\}$  ☐ (D) None of the other choices is correct.

**Question 29.** (L.O.2.3) What is the vertex cut of  $M$ ?

- ☐ (A)  $\{G, I\}$  ☐ (B)  $\{A\}$   
☐ (C) The vertex cut is empty. ☐ (D)  $\{G\}$

**Question 30.** (L.O.2.2) Which of the following statements is correct for a simple undirected graph that has more than 2 vertices?

- ☐ (A) The graph has 2 vertices that have the same degrees.  
☐ (B) The graph does not have any isolated vertex.  
☐ (C) The degrees of the vertices of the graph are less than the number of vertices minus 2.  
☐ (D) The graph has at least one pendant vertex.

–End–