HCMUT
FACULTY OF CS AND ENGINEERING

MIDTERM
Subject: **Discrete Structures** (CO1007)
Classes: **CT14QUEE + CT14KTMT + CT14KHMT**
Time: **60 minutes** *(Closed book test)*
Test date: **July 09, 2015**

Student's Full Name: _____ **Student's ID:** | | | | | | | | |

Final Score: _____ **Examiner:** _____ **Examiner's Signature:** _____

(*There are **20** MCQs, each question is worth **0.5** points. Answers in bold :* ■; *cancel out to deselect:* ▣.)

Question 1. Suppose that $P(x, y)$ means "x is a parent of y" and $M(x)$ means "x is male." If $F(v, w)$ equals

$$M(v) \wedge \exists x \exists y (P(x, y) \wedge P(x, v) \wedge (y \neq v) \wedge P(y, w)),$$

then what is the meaning of the expression $F(v, w)$?

A v is a brother of w.
C v is an uncle of w.
B v is a nephew of w.
D v is a grandfather of w.

Question 2. In this question, assume the following predicate and constant symbols:

$W(x, y) : x$ wrote y | h : Hardy | p : Pride and Predjudice.
$L(x, y) : x$ is longer than y | a : Austen
$N(x) : x$ is a novel | j : Jude the Obscure

Given these specifications, which of the predicate logic formulas below represent the sentence, "*Hardy wrote a novel which is longer than any of Austen's*" in predicate logic?

A $\forall x (W(h, x) \rightarrow L(x, a))$.
C $\forall x \forall y (W(h, x) \wedge W(a, y) \rightarrow L(x, y)))$.
B $\forall x \exists y (L(x, y) \rightarrow W(h, y) \wedge W(a, x))$.
D $\exists x (N(x) \wedge W(h, x) \wedge \forall y (N(y) \wedge W(a, y) \rightarrow L(x, y)))$.

Question 3. Which of the following predicate calculus formulas must be true under all interpretations?

I. $(\forall x P(x) \vee \forall x Q(x)) \longrightarrow \forall x (P(x) \vee Q(x))$.
II. $\forall x (P(x) \vee \forall x Q(x)) \longrightarrow (\forall x P(x) \vee \forall x Q(x))$.
III. $(\exists x P(x) \vee \exists x Q(x)) \longrightarrow \exists x (P(x) \vee Q(x))$.

A I only.
B III only.
C I and II.
D I and III.

Question 4. Which of these is NOT a valid inference rule, where A, B and C are any propositional formula?

A From $\neg B$ and $A \longrightarrow B$ infer $\neg A$.
C From A and $A \longrightarrow B$ infer B.
B From A infer $A \wedge B$.
D From A infer $\neg\neg A$.

Question 5. Which formula captures the following statement most accurately?

"*When the next large bank gets into trouble (t), the financial system collapses (c) unless the Central Bank buys the bank (b).*"

A $(\neg c \rightarrow b) \rightarrow t$
B $(c \wedge \neg b) \rightarrow \neg t$
C $(c \wedge \neg b) \rightarrow t$
D $t \rightarrow (\neg c \rightarrow b)$

Question 6. Which of the following propositional formulas has value T by assigning $p \mapsto T$, and $q, r \mapsto F$?

A $\neg p \vee \neg r \longrightarrow q$
B $\neg r \longrightarrow (p \wedge q)$
C $\neg(\neg r \longrightarrow (p \wedge q))$
D $\neg p \vee q \vee r$

Question 7. Of all the students at HCMUT, 55% were born in HCMC. Of those born in HCMC, 85% speak English well. Of those not born in HCMC, 32% speak English well. A student was selected at random. What is the probability that this student was born in HCMC **or** speaks English well?

A $0.55 \times 0.85 \times 0.32$. B $0.55 + 0.45 \times 0.32$. C $0.55 + 0.32$. D $0.55 \times 0.85 + 0.32$.

Question 8. A newspaper sports reporter has a 58% accuracy for predicting the winners in V-league 2015. A radio sports reporter has a 65% accuracy for predicting the winners. For a particular match, what is the probability that **at least one of these** reporters will make a correct prediction?

A $1 - 0.58 \times 0.65$.

B $0.58 + 0.65$.

C $0.58 + 0.65 - 0.58 \times 0.65$.

D 0.58×0.65.

Question 9. Suppose the monthly demand for tomatoes (a perishable good) in a small town is random. With probability 1/2, demand is 50; with probability 1/2, demand is 100. You are the only producer of tomatoes in this town. Tomatoes sell for a fixed price of 1 USD, cost 0.50 USD to produce, and can only be sold in the local market. If you produce 60 tomatoes, your expected profit is:

A 15 USD. B 25 USD. C 45 USD D 50 USD

Question 10. Let S be the collection of all sets with at most 5 elements. Then

A An element of S is a set with 1, 2, 3, 4, or 5 elements.

B An element of S is a number which is not greater than 5.

C An element of S is a set with 25 elements.

D An element of S is a set with an arbitrary number of elements.

Question 11. Let $A = \{\text{all diet soda pops}\}$, $B = \{\text{all cola soda pops}\}$, and $D = \{\text{all caffeine-free soda pops}\}$. Describe the set $(A - D) \cap B$ in words.

A All diet soda pops that contain caffeine and all cola soda pops

B All non-diet, caffeine-free cola soda pops

C All diet caffeine-free cola soda pops

D All diet cola soda pops that contain caffeine

Question 12. Say that two real numbers a and b are **related** if the sum of their squares is 2015. Then this relation is

A A function.

B An equivalence relation.

C Symmetric.

D Anti-symmetric.

Question 13. Say that two functions f and g, with domain $\mathbb{R}$, are **related** if $f(x) \leq g(x)$ for every $x \in \mathbb{R}$. Then

A This is an equivalence relation.

B This is an order relation.

C This is a function.

D This does not allow us to compare any two functions.

Question 14. Exactly which of the relations R_1, R_2, and R_3 on $\{1, 2, 3, 4\}$ that are given below are anti-symmetric?

$$R_1 = \{(1,1), (1,2), (2,2), (2,3), (3,3), (3,4), (4,4), (4,1)\};$$
$$R_2 = \{(1,1), (1,2), (2,3), (3,3), (3,2), (4,2)\};$$
$$R_3 = \emptyset.$$

A R_1, R_2 B R_1, R_3 C R_2, R_3 D R_2

Question 15. The number of functions $f : \{1, 2, ..., 2015\} \longrightarrow \{1, 2, ..., 100\}$ is:

A 100^{2015}. B 2015^{100}. C 2015×100. D $2015! + 100!$.

Question 16. The number of **relations** from $\{1, 2, ..., 5\}$ to $\{1, 2, ..., 100\}$ is:

A 100^5. B 5^{100}. C 2^{105}. D 2^{500}.

Question 17. The number of **increasing** functions $f : \{1, 2, ..., 15\} \longrightarrow \{1, 2, ..., 2015\}$ is:

A 15^{2015}. B 2015^{15}. C $\frac{2015!}{15!2000!}$. D $\frac{2015!}{15!}$.

Question 18. The number of **surjective (onto)** functions $f : \{1, 2, ..., 5\} \longrightarrow \{1, 2, ..., 100\}$ is:

A 0. B 5^{100}. C 100^5. D $\frac{100!}{5!}$.

Question 19. Let $f : A \longrightarrow B$ and $g : B \longrightarrow C$ be functions. Which of the following statements is incorrect?

A If f and g are one-to-one, then $g \circ f$ is one-to-one.

B If $g \circ f$ is onto, then f is onto.

C If $g \circ f$ is one-to-one, then f is one-to-one.

D If f is onto and $g \circ f$ is one-to-one, then g is one-to-one.

Question 20. How many solutions are there of the equation $x_1 + x_2 + x_3 = 12$ with x_1, x_2, x_3 positive integers?

A $\binom{14}{2}$. B $\binom{13}{2}$. C $\binom{12}{2}$. D $\binom{11}{2}$.

THE END

Test Approved by:	**Test Prepared by:**
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