Review of Propositional Logic 2/2

Online lecture

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Conditional statements

Definition:

The conditional statement $p \longrightarrow q$ is the proposition "If p, q"

The conditional statement is false when p is true and q is false

p is called the hypothesis or premise

q is called the conclusion or consequence

Ex:

p: $3<2$

q: Gold is cheap

$p \longrightarrow q$: If $3<2$, gold is cheap

Conditional statements

p	q	$p \longrightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Logical equivalence

Definition:

Compound propositions p and q are called logically equivalent if p and q have the same truth values in all possible cases

Ex: $\neg(p \rightarrow q)$ is logically equivalent to $p \wedge \neg q$

p	q	$\neg(p \rightarrow q)$	$p \wedge \neg q$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	F	F

Logical equivalence

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws

Logical equivalence

$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

Rules of Inference

Rule of Inference	*Tautology*	*Name*
p $p \rightarrow q$ $\therefore q$	$(p \wedge (p \rightarrow q)) \rightarrow q$	Modus ponens
$\neg q$ $p \rightarrow q$ $\therefore \neg p$	$(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$	Modus tollens
$p \rightarrow q$ $q \rightarrow r$ $\therefore p \rightarrow r$	$((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$	Hypothetical syllogism
$p \vee q$ $\neg p$ $\therefore q$	$((p \vee q) \wedge \neg p) \rightarrow q$	Disjunctive syllogism

Rules of Inference

Ex:

p: It's raining today

q: We will not have a barbecue today

r: We will have a barbecue tomorrow

$p \to q$: If it's raining today, we will not have a barbecue today

$q \to r$: If we will not have a barbecue today, we will have a barbecue tomorrow

Hypothetical syllogism

$p \to r$: If it's raining today, we will have a barbecue tomorrow

Rules of Inference

p $\therefore\ p \vee q$	$p \rightarrow (p \vee q)$	Addition
$p \wedge q$ $\therefore\ p$	$(p \wedge q) \rightarrow p$	Simplification
p q $\therefore\ p \wedge q$	$((p) \wedge (q)) \rightarrow (p \wedge q)$	Conjunction
$p \vee q$ $\neg p \vee r$ $\therefore\ q \vee r$	$((p \vee q) \wedge (\neg p \vee r)) \rightarrow (q \vee r)$	Resolution