Determinants of New Car Prices in the U.S.

Yavor Stoychev

Economic Forecasting
Prof. Erdogan

Contents

1. Introduction

This project studies the price of new cars in the United States from 1969 to 2011. The data set used contains 172 quarterly observations,of new car prices (y), used car prices (u) , public transportation prices (p) , insurance prices (i), and installment credit (c). No data has been seasonally adjusted. All observations were conducted in the United States.

1.1. Dependent Variable: CPI, New Cars (y)

For the purposes of this project, the price of new cars has been chosen as the dependent variable. The price index of new cars is generally expected to increase over time, as the economy grows. Data was taken from the U.S. Bureau of Labor Statistics.

Descriptive Statistics: y

```
Variable    N  N*    Mean  SE Mean  StDev  Minimum     Q1  Median      Q3
y         172   0  109.84     2.41  31.59    50.63  82.49  121.20  136.81

Variable  Maximum
y          143.51
```

Fig. 1

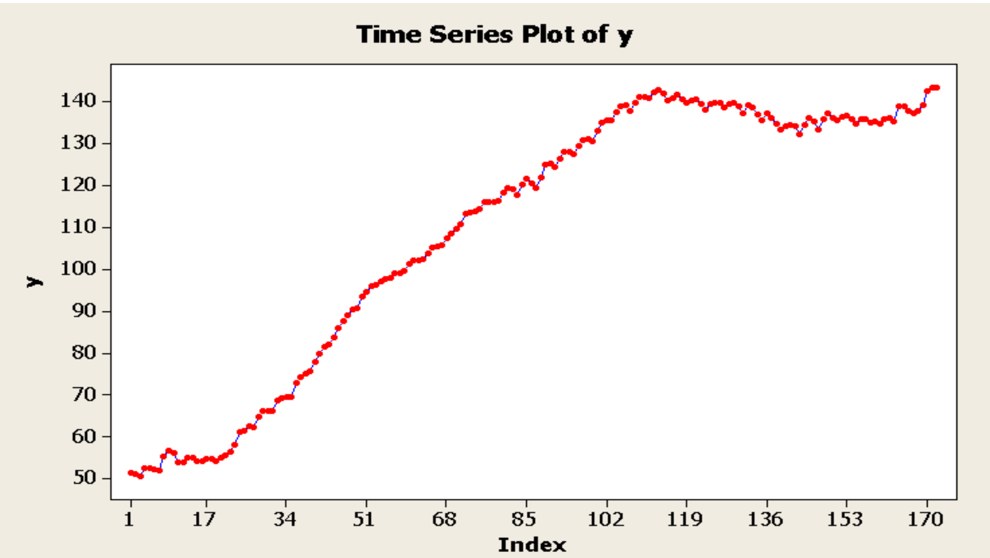

Fig. 2

Figure 2 confirms that new car prices do indeed increase over time – there is a clear upward trend. This also shows that new car prices are non-stationary.

Fig. 2A

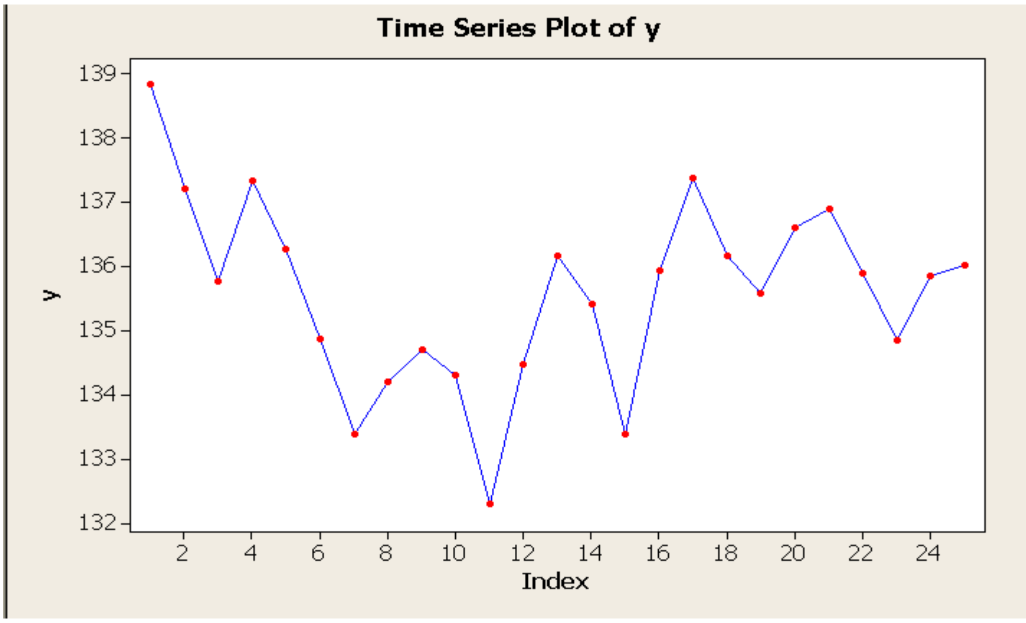

Figure 2A shows a time series plot of the price index of new cars from 2002 to 2007. I have included this plot to examine whether the price index of used cars exhibits seasonality. This figure seems to suggest that the are fluctuations in the price within each year, although they are not fully regular. After I choose the most appropriate causal and time series models. I will include dummy variables for seasonality and test them for significance.

1.2 Independent Variables

1.2.1 CPI, Public Transportation (p)

Public Transportation prices were measured by a consumer price index. Public Transportation and Private transportation are substitutes, and thus one could expect that they would be positively related – an increase in the prices of public transport would cause an increase in the price of new cars, and vice versa. Data was taken from the U.S. Bureau of Labor Statistics.

Descriptive Statistics: p

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3
p	172	0	138.81	5.58	73.20	30.30	56.21	141.70	206.82

Variable	Maximum
p	272.34

Fig. 3

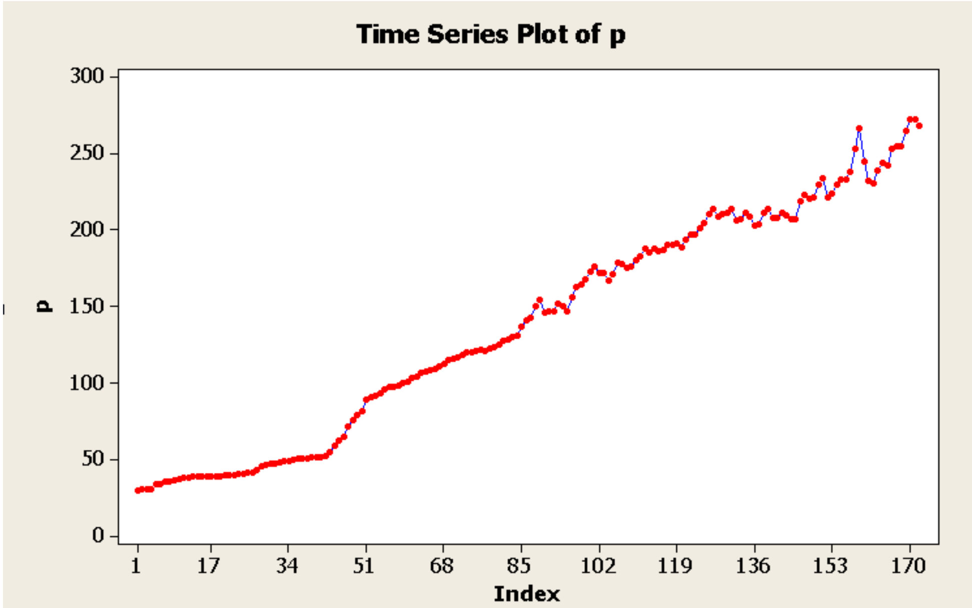

Fig. 4

The price of public transportation shows clear upward linear trend, as shown in Figure 4.

1.2.2. CPI, Used Cars & Trucks (u)

Used cars and trucks can be considered inferior goods, and, in essence, substitutes of new cars. Thus, an increase in the prices of used cars and trucks is likely to cause an increase in the prices of new cars. Used cars and trucks prices are measured by a consumer price index, and taken from the Bureau of Labor statistics.

Descriptive Statistics: u

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3
u	172	0	106.76	3.32	43.50	29.23	59.57	119.33	142.73

Variable	Maximum
u	160.23

Fig. 5

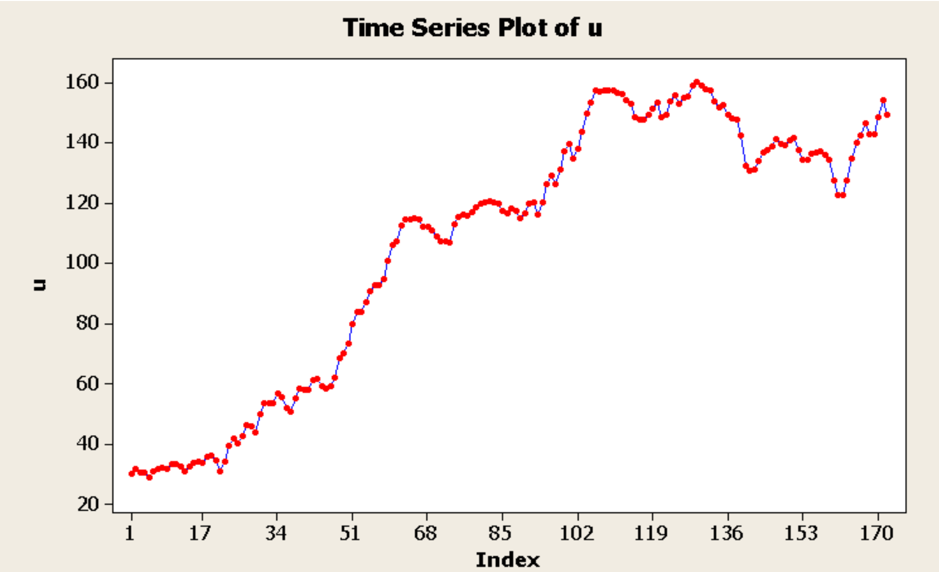

Fig. 6

Figure 6 shows that the price of used cars and trucks exhibits an upward trend, but shows more fluctuations than the previous two variables.

1.2.3. Revolving Consumer Credit (c)

An increase in revolving consumer credit would likely mean an increase in consumption. And an increase in consumption would eventually lead to an increase in prices. Thus, it is expected that as the amount of revolving credit increases, prices of new cars will also increase. In this data set, revolving credit data is measured in billions of U.S. Dollars, and taken from the U.S. Board of Governors of the Federal Reserve System.

Descriptive Statistics: c

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3	Maximum
c	172	0	350.6	25.1	329.7	2.4	51.6	228.6	704.3	989.1

Fig. 7

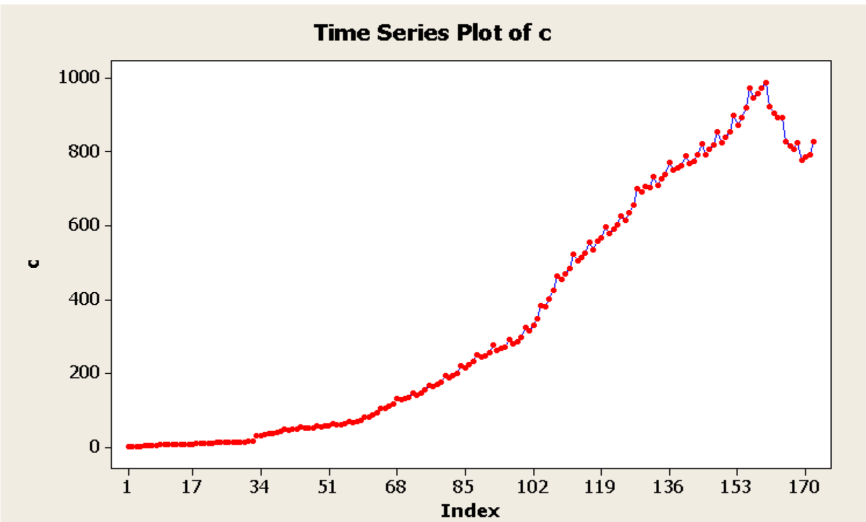

Fig. 8

Figure 8 shows that the amount of revolving consumer credit increases in a polynomial fashion. A sudden drop can be noticed around the time of the 2008 financial crisis.

1.2.4. CPI, Motor Vehicle Insurance (i)

Vehicle insurance incurs an extra cost of driving a car, and thus an increase in this cost would discourage people from buying cars, and would likely lead to a drop in the price of new cars. The cost of insurance was measured as a price index and taken from the U.S. Bureau of Labor Statistics.

Descriptive Statistics: i

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3
i	172	0	184.86	8.42	110.46	35.87	77.02	176.96	264.62

Variable	Maximum
i	394.53

Fig. 9

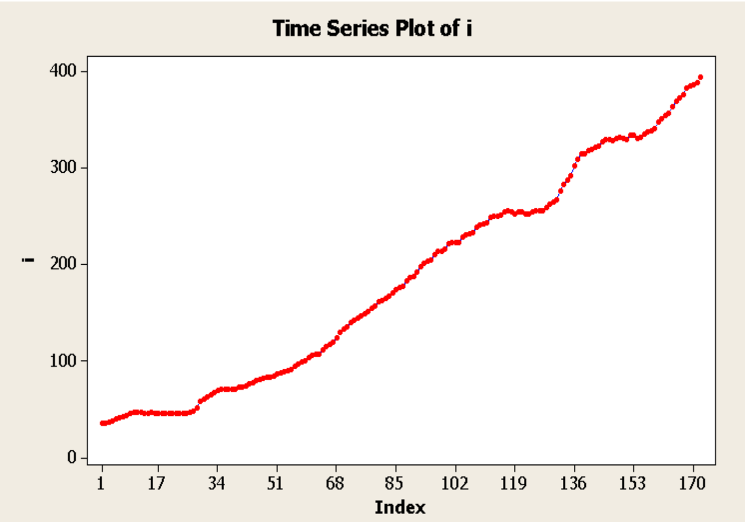

Fig. 10

Figure 10 clearly shows an upward linear trend – the price of vehicle insurance has been growing at a constant rate for the past 40 years.

2. Models

2.1. Causal Model

Before building a causal model, I have tested my dependent variables for multicolinearity.

```
Correlations: p, u, c, i

       p      u      c
u  0.917
   0.000

c  0.948  0.785
   0.000  0.000

i  0.987  0.866  0.970
   0.000  0.000  0.000


Cell Contents: Pearson correlation
               P-Value
```

The correlation matrix (on the left) shows that the dependent variables I had chosen are highly correlated. To correct this, I have excluded the price of public transport (p) and the price of vehicle insurance (i) from the model.

To obtain a causal model that is significant, I have applied the step-wise regression method, and retained all variables that are significant at the 5% level. A summary of the results at each step can be seen in figure 12.

```
Stepwise Regression: y versus u, c

  Alpha-to-Enter: 0.15  Alpha-to-Remove: 0.15


Response is y on 2 predictors, with N = 172


Step             1       2
Constant     33.41   36.09

u           0.7159  0.6598
T-Value      76.75   46.86
P-Value      0.000   0.000

c                   0.0094
T-Value               5.07
P-Value              0.000

S             5.31    4.96
R-Sq         97.20   97.57
R-Sq(adj)    97.18   97.54
Mallows Cp    26.7     3.0
```

The step-wise regression shows that all the remaining dependent variables (after we excluded insurance price and public transportation price due to multicolinearity) are significant at the 5% level. We can also draw the following conclusions from the values of the estimated coefficients:

(i) As expected, the price of used cars and trucks is positively related to the price of new cars.

(ii) As expected, the amount of revolving consumer credit is positively related to the price of new cars, although the relationship is weaker than I expected.

Fig. 12

To verify that our regression is adequate, the four assumptions of OLS estimation must hold. In order to test that, I use the equation **y-pred = 36.09 + 0.6598u + 0.0094** to estimate y and calculate the residuals.

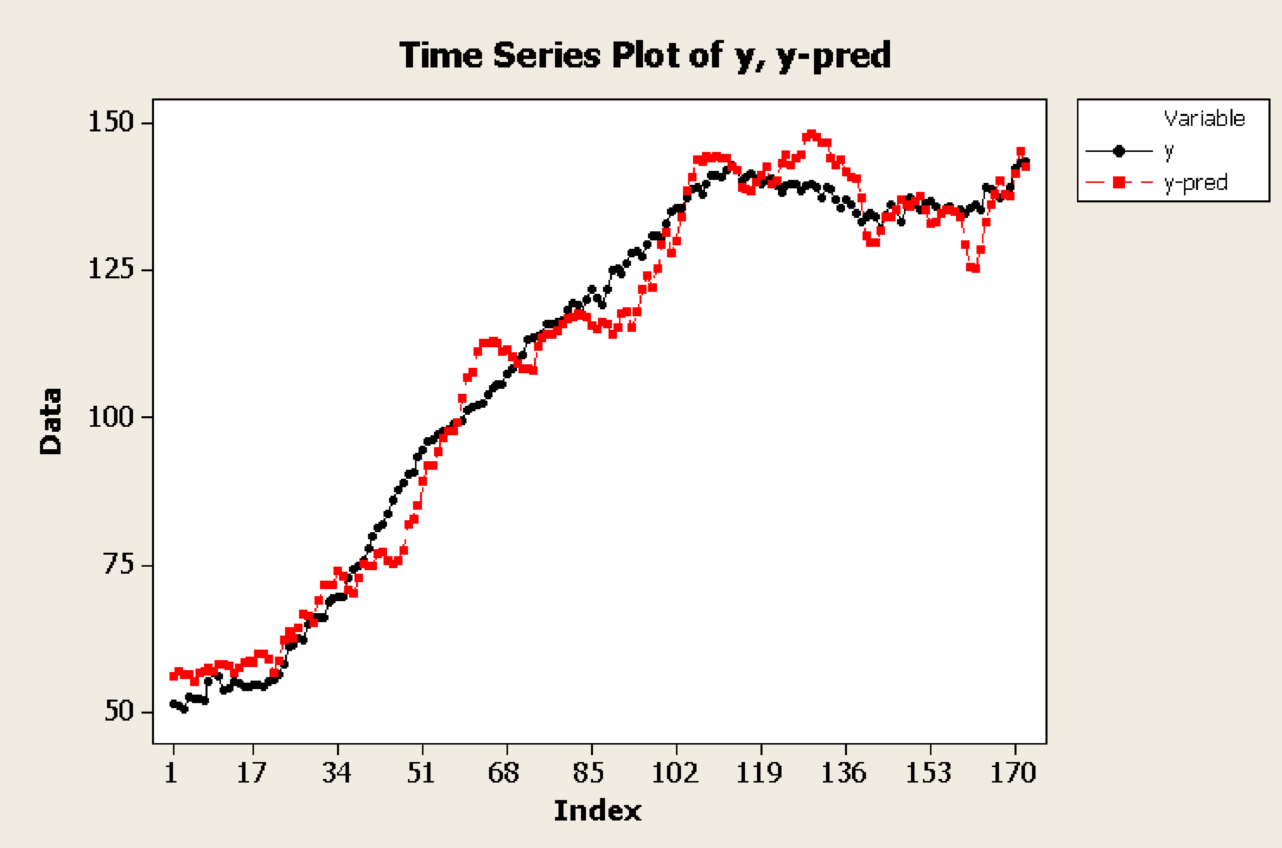

Fig. 13

Figure 13 shows a time plot of both y and y-pred. The predictions seem reasonably accurate.

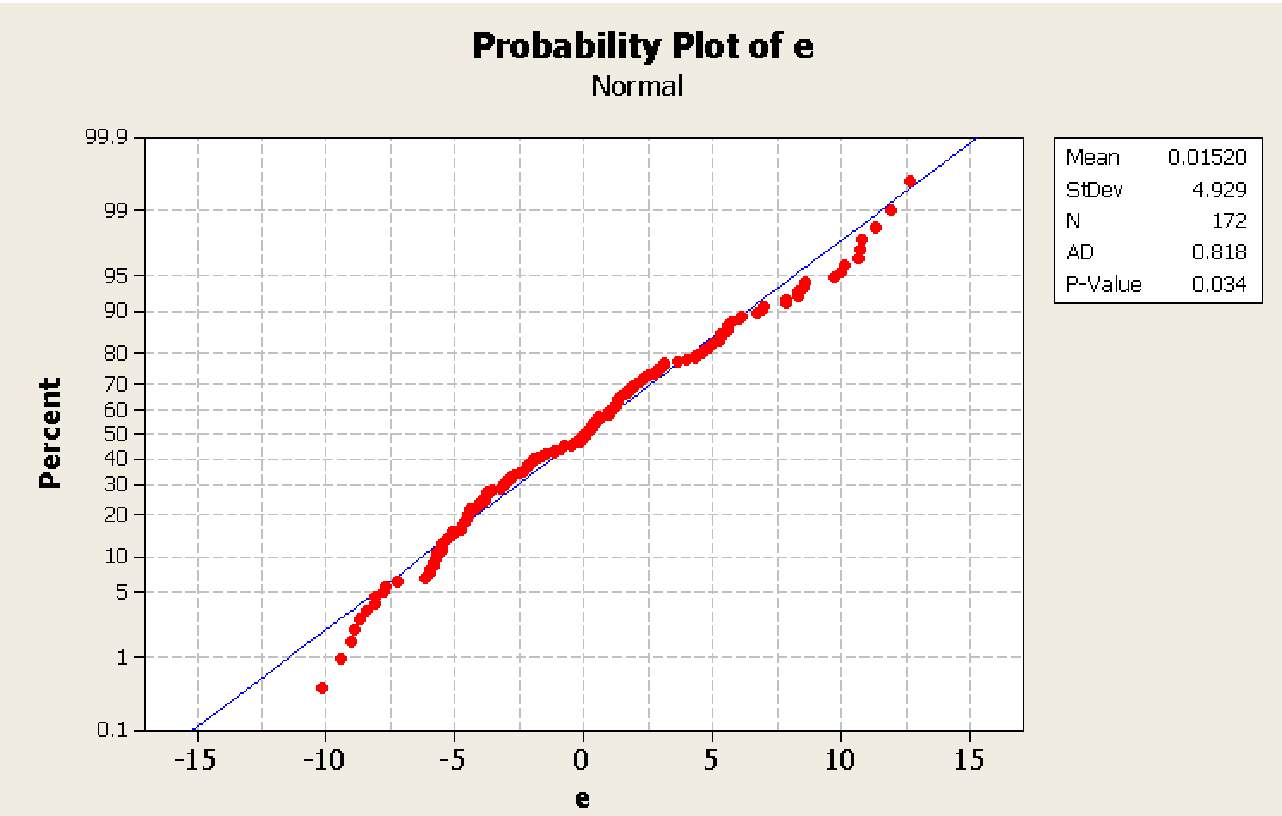

Fig. 14

Figure 14 shows that the error terms follow a normal distribution with mean 0 at the 1%

significance level. The residuals also exhibit constant variance.

```
Correlations: e, u, c

       e      u
u  0.002
   0.983

c  0.002  0.785
   0.980  0.000


Cell Contents: Pearson correlation
               P-Value
```

Fig. 15

Figure 15 shows that the residuals (e) are not correlated with any of the independent variables. Both prices of used cars and trucks (u) and revolving consumer credit (c) relate to the residuals with very small correlation coefficients and p-values of more than 98%

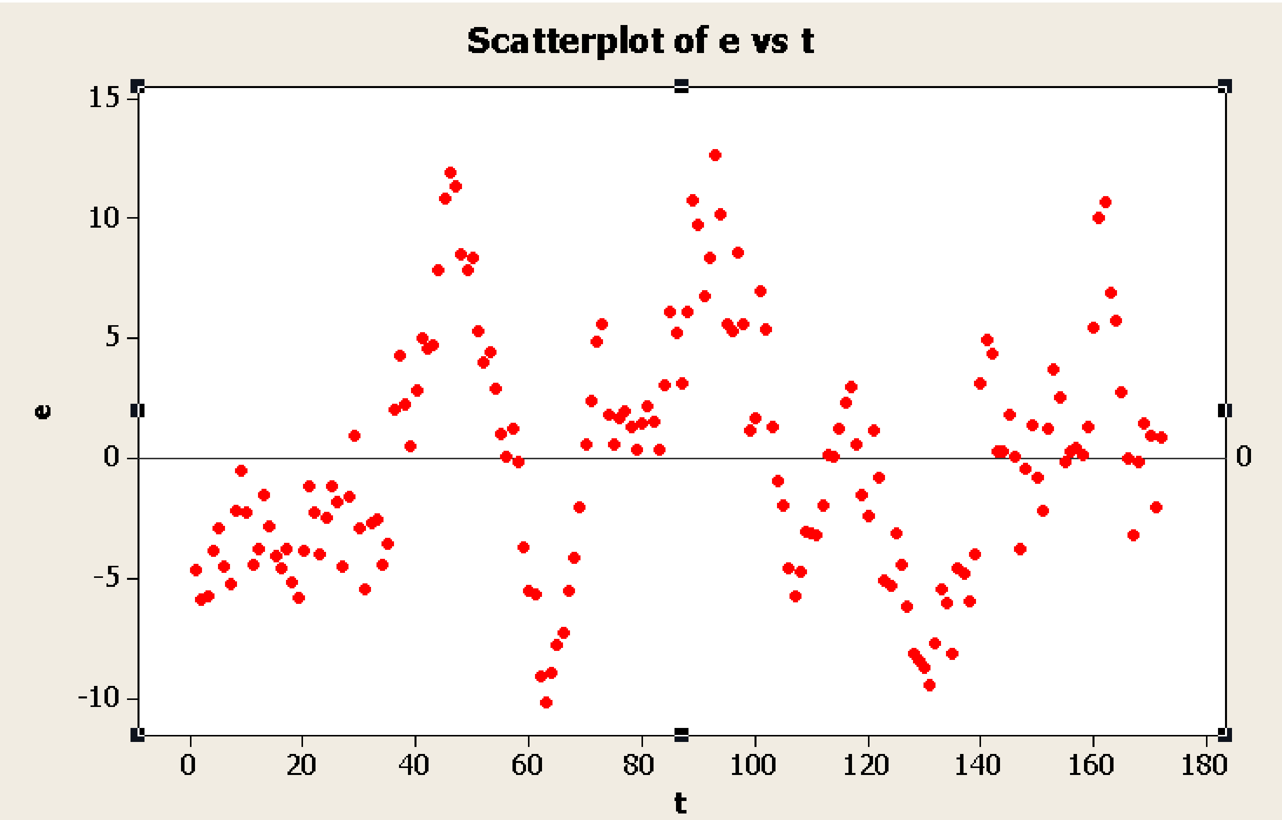

Fig. 16

Figure 16 suggests that the residuals exhibit positive autocorrelation. Thus, I have conducted the Durbin-Watson test to test for positive autocorrelation. I ran a regression of *y* on *c* and *u*, and calculated the Durbin-Watson statistic.

```
Regression Analysis: y versus u, c

The regression equation is
y = 36.1 + 0.660 u + 0.00942 c

Predictor       Coef   SE Coef      T      P
Constant      36.092     1.135  31.79  0.000
u            0.65985   0.01408  46.86  0.000
c           0.009423  0.001858   5.07  0.000

S = 4.95797   R-Sq = 97.6%   R-Sq(adj) = 97.5%

Analysis of Variance

Source           DF      SS     MS        F      P
Regression        2  166508  83254  3386.85  0.000
Residual Error  169    4154     25
Total           171  170662

Source  DF  Seq SS
u        1  165876
c        1     632

Unusual Observations

Obs    u        y      Fit  SE Fit  Residual  St Resid
 45   59   86.130   75.335   0.565    10.795     2.19R
 46   60   87.800   75.897   0.560    11.903     2.42R
 47   62   89.070   77.727   0.544    11.343     2.30R
 63  115  102.600  112.774   0.685   -10.174    -2.07R
 89  115  125.100  114.405   0.483    10.695     2.17R
 93  117  128.100  115.469   0.472    12.631     2.56R
 94  121  128.270  118.127   0.501    10.143     2.06R
161  123  135.860  125.901   0.973     9.959     2.05R
162  123  136.230  125.605   0.944    10.625     2.18R

R denotes an observation with a large standardized residual.

Durbin-Watson statistic = 0.207655
```

Fig. 17

The Durbin-Watson statistics for the current model is **0.207655** .

I have compared it to the Durbin-Watson critical values for 150 observations (that is the closest I could find) , 2 independent variables and 5% significance level. The values used were the following:

D_L = 1.64
D_U = 1.83

The Durbin-Watson statistic of the current model is smaller than the lower critical value, and thus the test shows that the residuals do exhibit positive autocorrelation.

Thus, the independence assumption of the Linear Regression Model is violated, and the current model is invalid.

2.2. Simple Time Series Regression

To obtain a simple time series model, I ran a regression of the price index of new cars (y) on the time index.

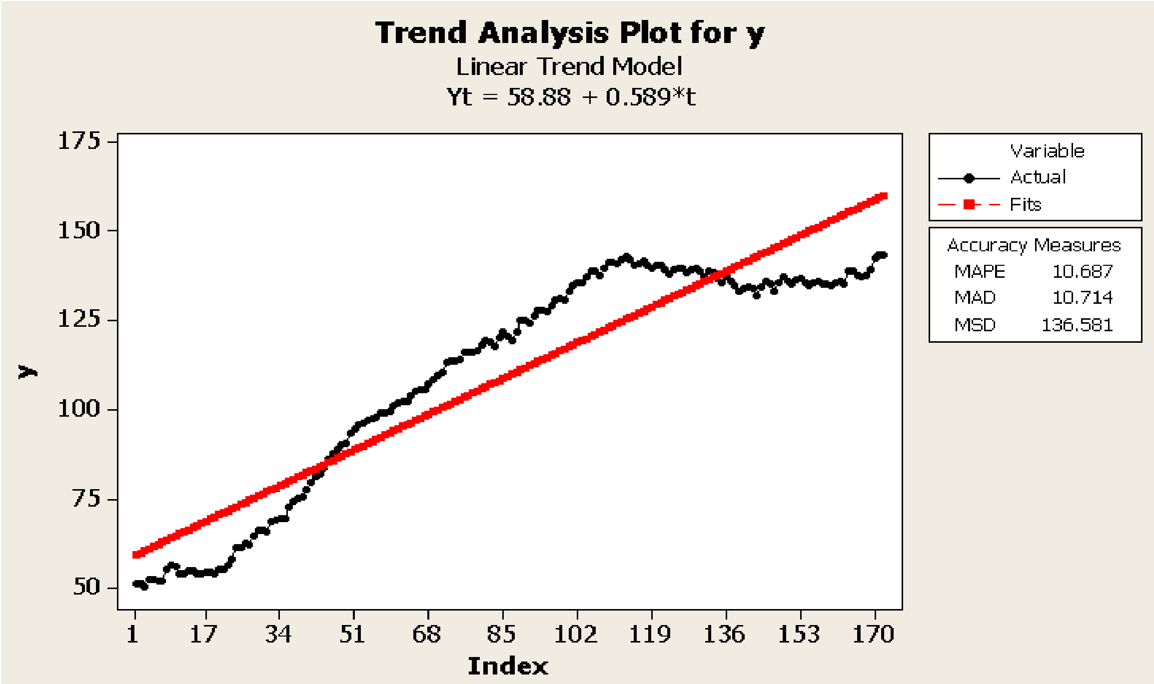

Fig. 18

Figure 18 shows that the simple time series is not a good fit for the data.

2.3. Non-linear Time Series

2.3.1 Quadratic Trend Model

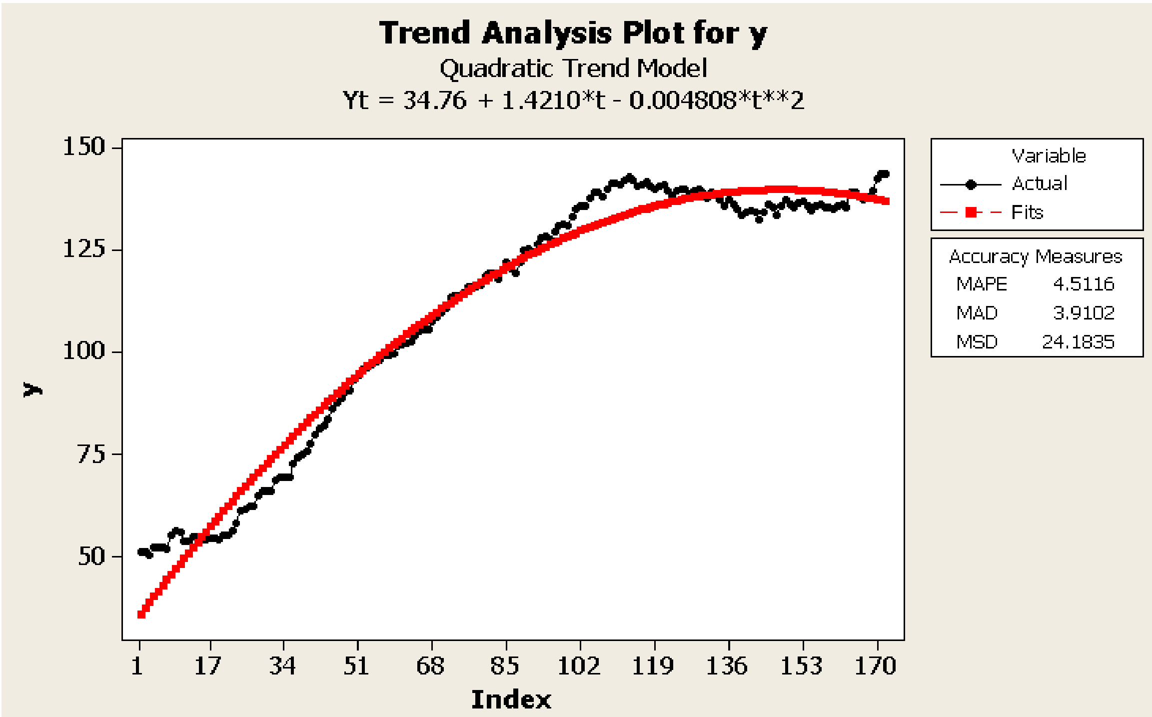

Fig. 19

Figure 19 shows that the quadratic trend model is better suited for the data. The Mean Absolute Deviation equals **3.9102** , as opposed to **10.714** for the linear trend model.

2.3.2 Growth Curve Model

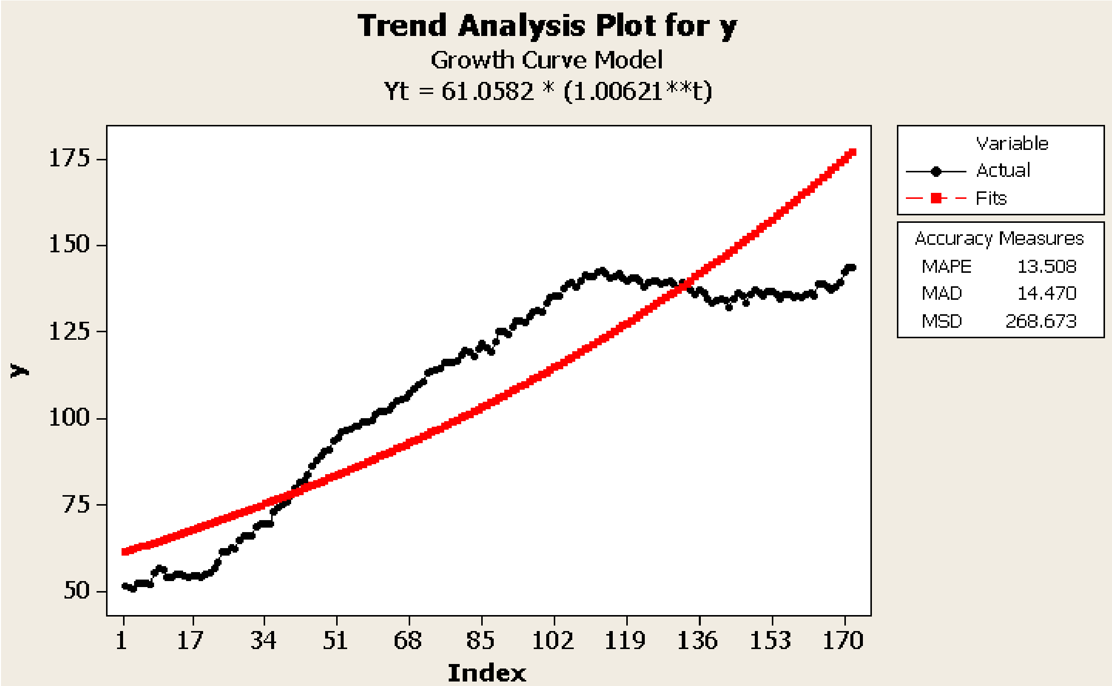

Fig. 20

Figure 20 shows that the growth curve model does not fit the data. The Mean Average Deviation of this model is 14.470 which is higher than the Mean Average Deviation of the linear trend model.

2.3.3 S-Curve Trend Model

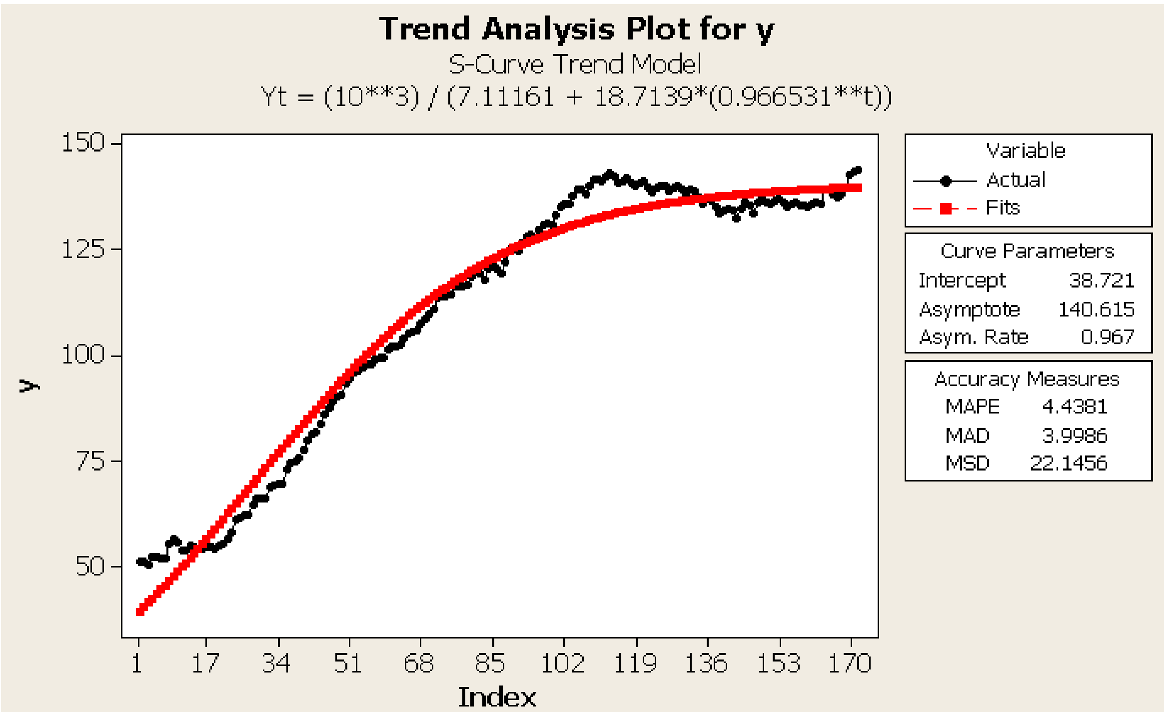

Fig. 21

Figure 21 Shows that the S-Curve Trend Model fits the data very well. The MAD is 3.9986, which is slightly higher than the MAD of the Quadratic Trend Model. Thus, the quadratic model seems the best choice for the data.

2.3.4 Seasonality

To test for seasonality with the causal model, I created the dummies q1, q2, q3. Each of them is defined as 1 if we are in quarter 1, 2 or 3 respectively, and 0 otherwise. I then chose q4 as my base case and ran a regression of **y** on **t^2**, **q1, q2**, and **q3** and test the significance of q1, q2, and q3. I chose to include t^2 in this regression, since the Quadratic Trend model provided the best fit (as measured by MAD) for the data.

Regression Analysis: y versus t^2, q1, q2, q3

Fig. 22

```
The regression equation is
y = 81.3 + 0.00290 t^2 + 0.23 q1 - 0.22 q2 - 1.13 q3


Predictor        Coef     SE Coef      T      P
Constant       81.295       3.252  25.00  0.000
t^2         0.0028981   0.0001589  18.23  0.000
q1              0.227       3.987   0.06  0.955
q2             -0.223       3.986  -0.06  0.955
q3             -1.131       3.986  -0.28  0.777


S = 18.4812   R-Sq = 66.6%   R-Sq(adj) = 65.8%


Analysis of Variance

Source           DF      SS     MS      F      P
Regression        4  113623  28406  83.17  0.000
Residual Error  167   57039    342
Total           171  170662
```

As figure 22 shows, all dummies are largely insignificant, which confirms the suspicion that the data has no clear seasonality.

2.4Multiplicative Time Series Decomposition

Multiplicative Time Series Decomposition has not been performed. Multiplicative decomposition is suited for estimating data that exhibits increasing or decreasing seasonal variation. The time plot of the price index of used cars exhibits no seasonal variation, as proven in section 2.3.4.

2.5Additive Time Series Decomposition

Additive Time Series Decomposition has not been performed. Additive decomposition is suited for estimating data that exhibits constant seasonal variation. The time plot of the price index of used cars exhibits no seasonal variation, as proven in in section 2.3.4.

2.6Smoothing Models

2.6.1 Single Exponential Smoothing

Single Exponential Smoothing is useful for data that has no clear trend or seasonal pattern. The price index of new cars has a clear upward trend, but exhibits no seasonality. Although this model is not the perfect match for the data. I have decided to perform it.

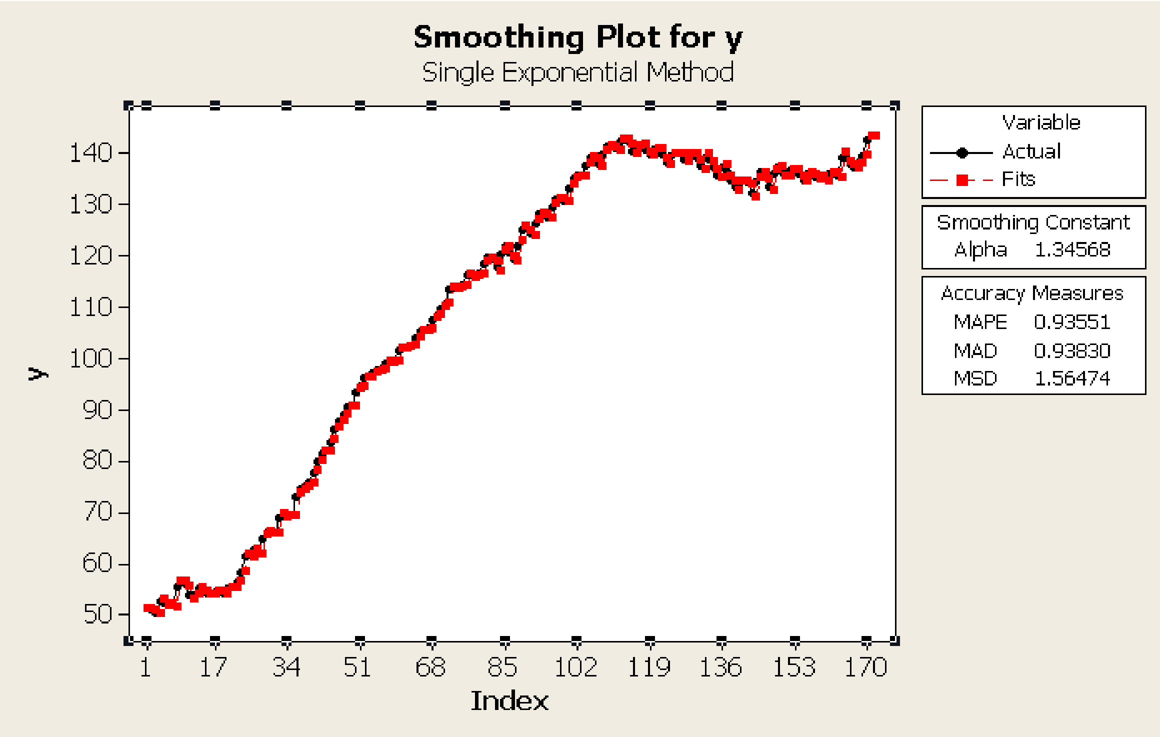

Fig. 23

As seen in figure 23, the simple exponential smoothing model offers a very good fit of the data. The Mean Absolute Deviation equals 0.9383, which confirms the visual observation that this model fits the data very well.

2.6.2 Holt's Trend Corrected Exponential Smoothing

This method is best used with time series with linear trend with changing level and growth rate, and no seasonal pattern. Thus, it is expected that Holt's Trend Corrected Exponential Smoothing will be an adequate model for the data.

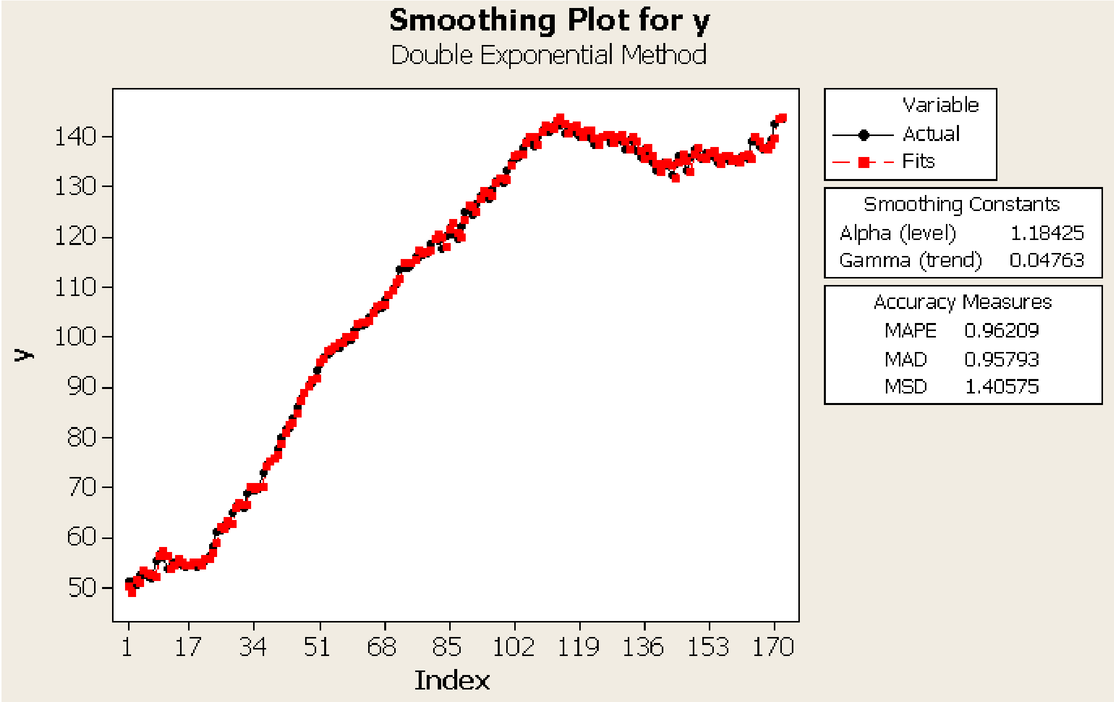

Fig. 24

Holt's Trend Corrected Exponential Smoothing fits the data well. The MAD is 0.95793, which is a slightly higher than the MAD of the Single Exponential Smoothing model.

2.6.3 Additive Holt-Winters Exponential Smoothing

Additive Holt-Winters Exponential Smoothing has not been performed. This model is best used with data that has a constant seasonal variation. However, the price index of new cars has no seasonal variation.

2.6.4 Multiplicative Holt-Winters Exponential Smoothing

Multiplicative Holt-Winters Exponential Smoothing has not been performed. This model is best used with data that has increasing seasonal variation. However, the price index of new cars has no seasonal variation.

2.7 ARIMA

Since the price index of new cars exhibits no seasonality, Nonseasonal Box-Jenkins models would be most appropriate. For this purpose, I have examined the Sample Autocorrelation of the price index of new cars.

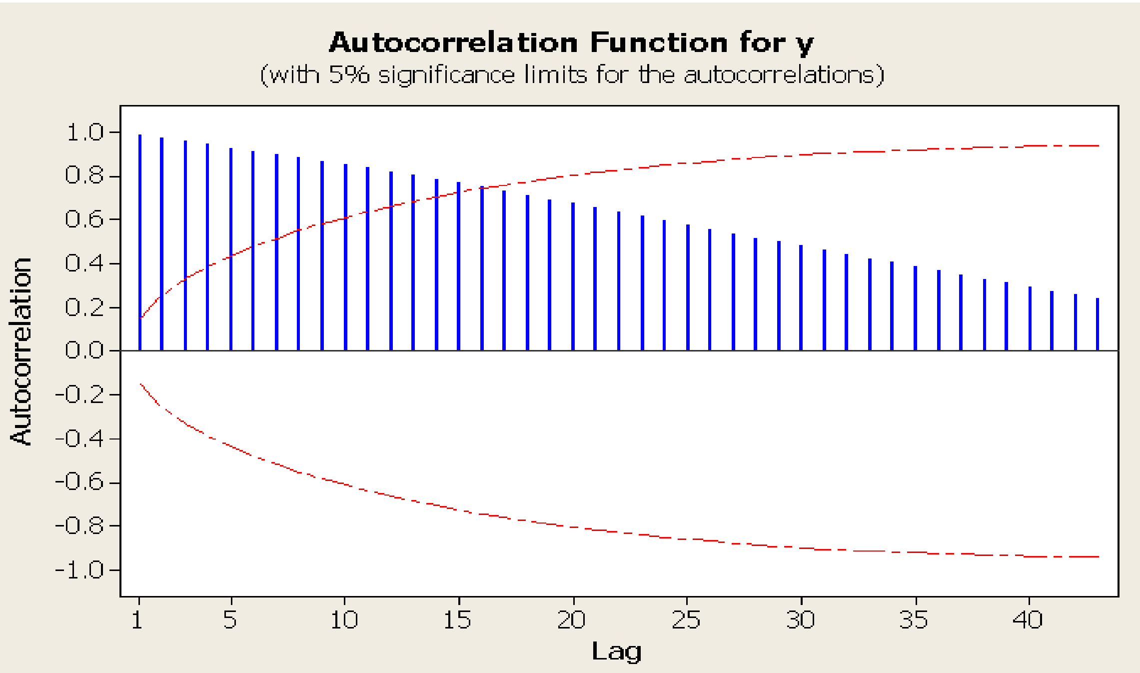

Fig. 25

Figure 25 shows that the SAC of the price index of new cars dies down quite slowly. This, I can conclude that the price index of new cars is not stationary.

ARMIA models, however, requite stationary data. For this purpose, I define the **z1** variable as the first differences of **y**. Then, I examine the SAC and the SPAC of **z1**.

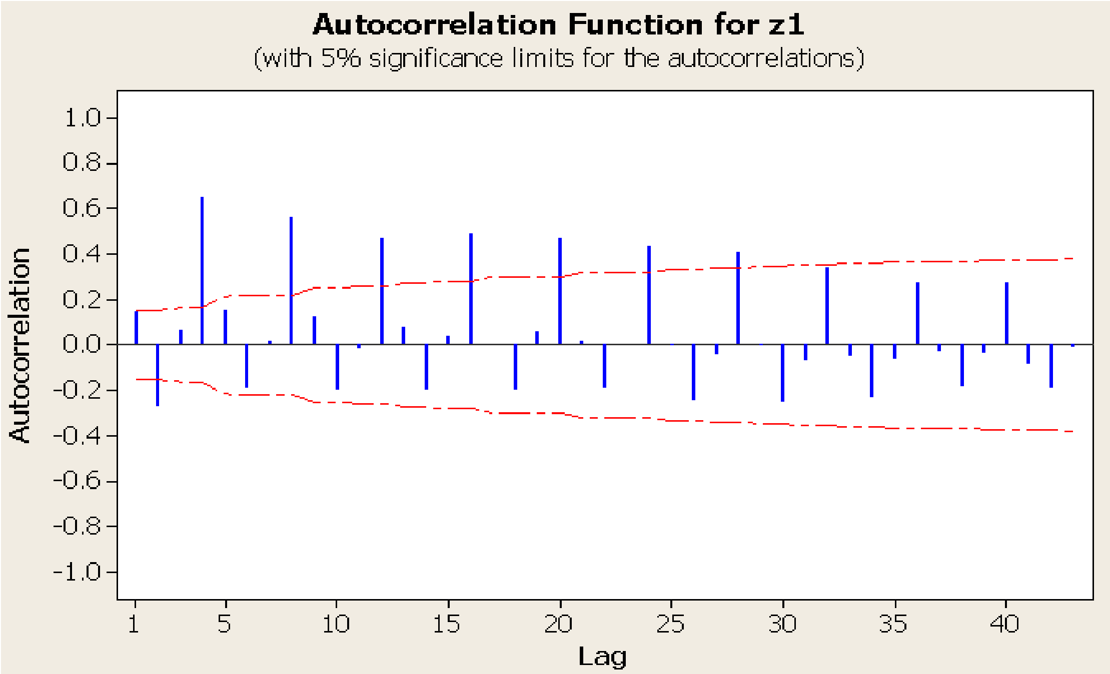

Fig. 26: The SAC of **z1** cuts off after lag 2.

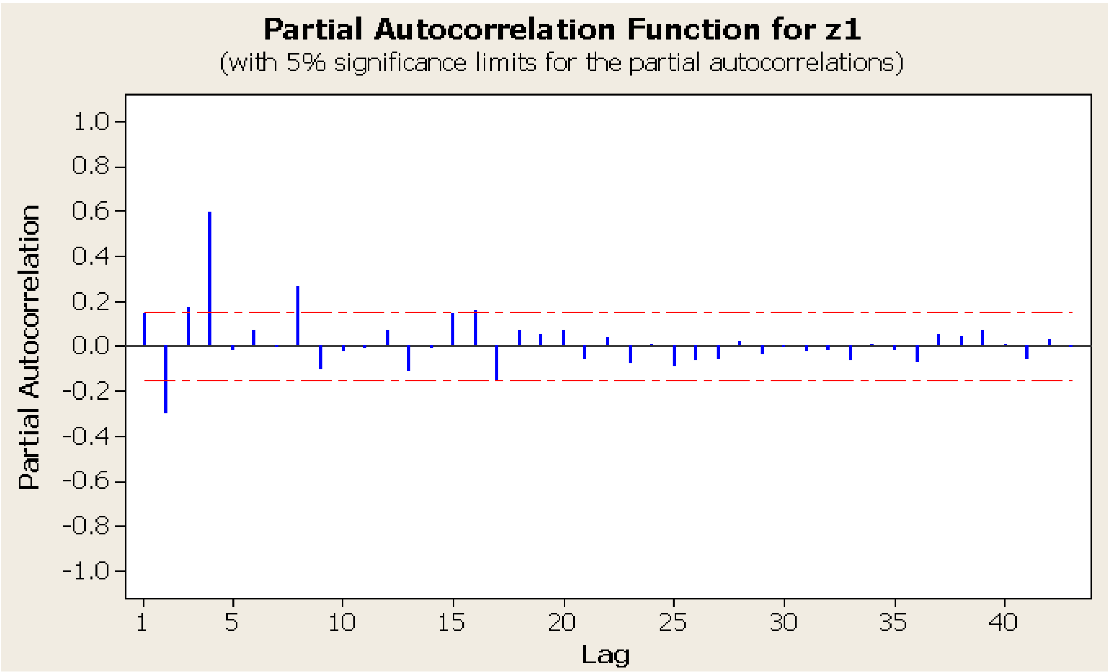

Fig. 27: The SPAC of **z1** cuts off after lag 4.

By observing the SAC and SPAC of z1 (figures 26 and 27), I can see that the SAC of **z1** cuts faster than its SPAC. Thus, the SPAC is treated as dying down. Since SAC cuts off after lag 2, I conclude that the MA(2) model would be an appropriate tentative model.

The next step is to estimate the MA(2) model and perform diagnostic checking.

```
Final Estimates of Parameters

Type          Coef  SE Coef      T      P
MA   1     -0.1777   0.0766  -2.32  0.022
MA   2      0.1172   0.0766   1.53  0.128
Constant   0.53931  0.09588   5.62  0.000
Mean       0.53931  0.09588


Number of observations:  171
Residuals:    SS =  234.781 (backforecasts excluded)
              MS =  1.398  DF = 168


Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag            12     24     36     48
Chi-Square  175.4  311.6  414.2  497.3
DF              9     21     33     45
P-Value     0.000  0.000  0.000  0.000
```

Figure 28 (on the left) shows that MA 2 is insignificant at the 5% significance level. Thus, we exclude it from the model, and switch to an MA(1) model.

Figure 28

```
Final Estimates of Parameters

Type          Coef  SE Coef      T      P
MA   1     -0.2722   0.0740  -3.68  0.000
Constant    0.5373   0.1160   4.63  0.000
Mean        0.5373   0.1160

Number of observations:  171
Residuals:    SS =  240.405 (backforecasts excluded)
              MS =  1.423  DF = 169

Modified Box-Pierce (Ljung-Box) Chi-Square statistic

Lag            12     24     36     48
Chi-Square  195.9  353.2  479.0  579.8
DF             10     22     34     46
P-Value     0.000  0.000  0.000  0.000
```

Figure 29 (on the left) shows that both MA 1 and the constant term are significant at the 5% significance level.

However, the P-values fort the Ljung-Box statistic are smaller than the 5% significance level, and thus provide grounds for rejecting the adequacy of this model.

Figure 29

Based on the results obtained from the diagnostic checking, I conclude that the Nonseasonal Box-Jenkins is not an adequate model for the data.

3. Model Choice Tables and Discussion

The following three models has been chosen as most adequate for predicting the price index of used cars:

Model	MAD
Single Exponential Smoothing	0.9383
Holt's Trend Corrected Mode	0.95793
Quadratic Trend Model	3.9102

Since the Single Exponential Smoothing model provides a fit with the smallest Mean Absolute Deviation, it has been chosen as the best model for forecasting future values of the price index of new cars.

4. Forecasts

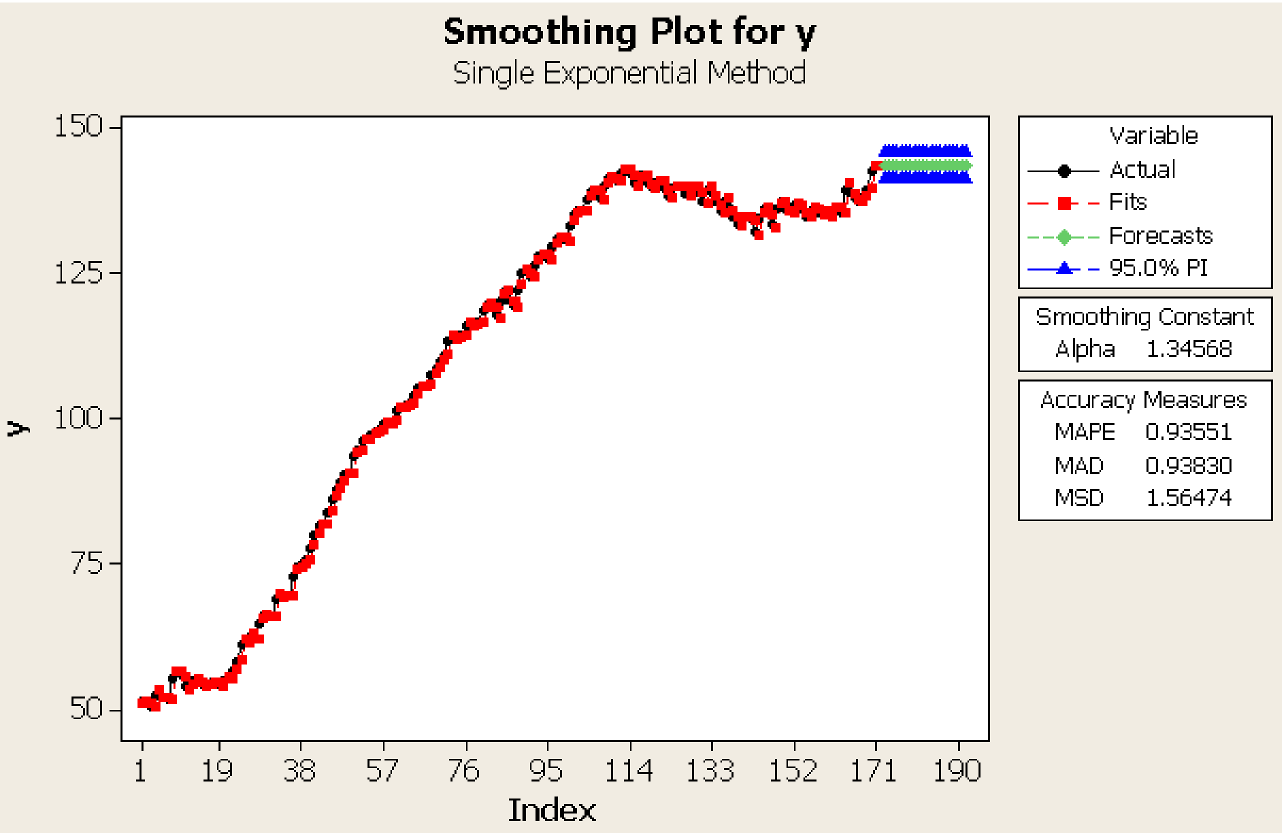

Figure 30: Single Exponential Smoothing plot of y (including forecasts and prediction intervals)

Single Exponential Smoothing for y

Data y
Length 172

Forecasts

Period	Forecast	Lower	Upper
173	143.531	141.233	145.830
174	143.531	141.233	145.830
175	143.531	141.233	145.830
176	143.531	141.233	145.830
177	143.531	141.233	145.830
178	143.531	141.233	145.830
179	143.531	141.233	145.830
180	143.531	141.233	145.830
181	143.531	141.233	145.830
182	143.531	141.233	145.830
183	143.531	141.233	145.830
184	143.531	141.233	145.830
185	143.531	141.233	145.830
186	143.531	141.233	145.830
187	143.531	141.233	145.830
188	143.531	141.233	145.830
189	143.531	141.233	145.830
190	143.531	141.233	145.830
191	143.531	141.233	145.830
192	143.531	141.233	145.830

Fig. 31: Forecast values and prediction intervals of the price index of new cars for the next 5 years

As seen in figures 30 and 31 , the Single Exponential Smoothing method can only predict values for 1 period in the future, I have also performed forecasting using Holt's Trend Corrected Exponential Smoothing Method.

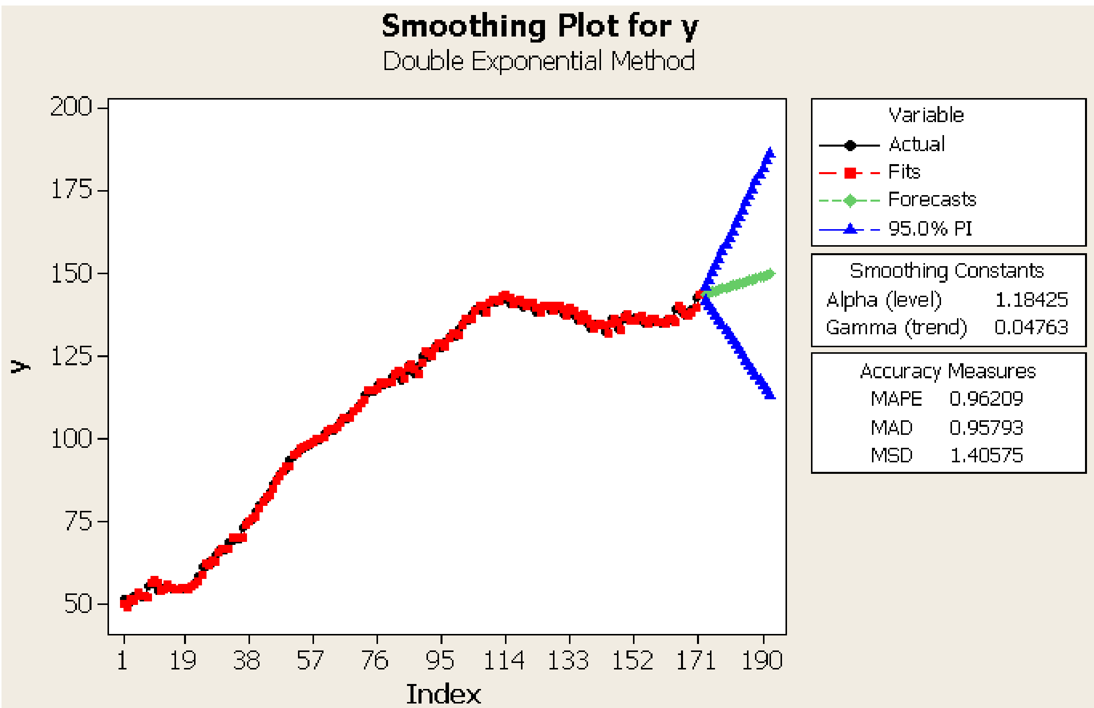

Figure 32: Holt's Trend Corrected Exponential Smoothing plot of y

Double Exponential Smoothing for y

Forecasts

Period	Forecast	Lower	Upper
173	143.768	141.422	146.115
174	144.083	140.023	148.142
175	144.397	138.570	150.224
176	144.711	137.101	152.322
177	145.025	135.624	154.427
178	145.340	134.144	156.536
179	145.654	132.661	158.647
180	145.968	131.177	160.759
181	146.283	129.693	162.872
182	146.597	128.207	164.986
183	146.911	126.721	167.101
184	147.225	125.235	169.216
185	147.540	123.748	171.331
186	147.854	122.262	173.446
187	148.168	120.775	175.562
188	148.482	119.287	177.677
189	148.797	117.800	179.793
190	149.111	116.313	181.909
191	149.425	114.825	184.025
192	149.739	113.338	186.141

Figure 33: Forecast values and confidence intervals obtained using Holt's Trend Corrected Exponential Smoothing.

5. Conclusion

The prices of new cars will continue to rise at a steady pace. This outcome is expected since economic growth causes inflation, and inflation causes a gradual rise in the price level. This forecast, however, cannot take into account any technological breakthroughs or shocks in the market.

6. Appendix I: Data Used

y	p	u	c	i	t	t^2	q	q1	q2	q3	q4	z1
51,43	30,3	30,3	2,35	35,87	1	1	1	1	0	0	0	*
51,2	30,8	31,73	2,7	36,4	2	4	2	0	1	0	0	-0,23
50,63	31,1	30,7	3,08	37,1	3	9	3	0	0	1	0	-0,57
52,6	31,47	30,77	3,72	38,43	4	16	4	0	0	0	1	1,97
52,5	34,33	29,23	3,97	40,43	5	25	1	1	0	0	0	-0,1
52,27	34,6	31,27	4,23	41,5	6	36	2	0	1	0	0	-0,23
51,97	35,67	31,9	4,6	42,57	7	49	3	0	0	1	0	-0,3
55,4	36,3	32,5	5,13	43,57	8	64	4	0	0	0	1	3,43
56,67	36,97	31,87	7,36	45,9	9	81	1	1	0	0	0	1,27
56,1	37,63	33,63	7,64	47,03	10	100	2	0	1	0	0	-0,57
54	38,17	33,7	8,15	47,27	11	121	3	0	0	1	0	-2,1
54,07	38,23	32,83	8,53	46,9	12	144	4	0	0	0	1	0,07
55,2	39,23	31,13	8,26	46,67	13	169	1	1	0	0	0	1,13
54,97	39,13	32,77	8,67	46,63	14	196	2	0	1	0	0	-0,23
54,4	39,37	33,8	9,27	46,73	15	225	3	0	0	1	0	-0,57
54,37	39,53	34,53	9,7	46,33	16	256	4	0	0	0	1	-0,03
54,7	39,6	33,83	9,63	45,83	17	289	1	1	0	0	0	0,33
54,77	39,57	35,93	10,22	45,87	18	324	2	0	1	0	0	0,07
54,33	39,77	36,33	10,91	45,8	19	361	3	0	0	1	0	-0,44
55,23	39,9	34,67	11,71	45,6	20	400	4	0	0	0	1	0,9
55,57	40,1	31,13	11,35	45,73	21	441	1	1	0	0	0	0,34
56,57	40,33	34,3	12,14	45,93	22	484	2	0	1	0	0	1
58,27	40,8	39,5	13,18	45,83	23	529	3	0	0	1	0	1,7
61,33	41,17	41,83	13,68	45,7	24	576	4	0	0	0	1	3,06
61,63	41,77	40,3	13,21	45,93	25	625	1	1	0	0	0	0,3
62,63	41,97	42,73	13,43	47,2	26	676	2	0	1	0	0	1
62,43	43,83	46,5	14,31	48,1	27	729	3	0	0	1	0	-0,2
64,9	46,43	45,9	15,02	52,23	28	784	4	0	0	0	1	2,47
66,2	46,9	43,97	14,46	58,83	29	841	1	1	0	0	0	1,3
66,27	47,4	49,93	14,89	61,23	30	900	2	0	1	0	0	0,07
66,17	48,07	53,63	16,03	63,47	31	961	3	0	0	1	0	-0,1
68,9	48,73	53,6	17,19	65,73	32	1024	4	0	0	0	1	2,73
69,4	49,2	53,83	31,44	67,37	33	1089	1	1	0	0	0	0,5
69,6	49,83	57,03	32,94	69,7	34	1156	2	0	1	0	0	0,2
69,7	50,37	55,83	34,58	71	35	1225	3	0	0	1	0	0,1
73	50,73	52,27	39,27	71,17	36	1296	4	0	0	0	1	3,3
74,47	51,23	51,1	38,03	71,53	37	1369	1	1	0	0	0	1,47
75,13	51,37	55,23	39,88	71,2	38	1444	2	0	1	0	0	0,66
75,8	51,53	58,77	42,59	71,53	39	1521	3	0	0	1	0	0,67
77,83	51,93	58,3	48,31	73,13	40	1600	4	0	0	0	1	2,03
79,9	52,3	58,13	46,77	73,77	41	1681	1	1	0	0	0	2,07
81,57	53	61,33	48,54	75	42	1764	2	0	1	0	0	1,67

82,07	55,17	61,8	51,23	76,8	43	1849	3	0	0	1	0	0,5
83,77	59,33	59,57	56,94	77,7	44	1936	4	0	0	0	1	1,7
86,13	62,93	58,7	54,15	79,73	45	2025	1	1	0	0	0	2,36
87,8	65,6	59,57	52,85	81,87	46	2116	2	0	1	0	0	1,67
89,07	71,57	62,33	53,81	82,97	47	2209	3	0	0	1	0	1,27
90,47	75,93	68,67	58,51	83,53	48	2304	4	0	0	0	1	1,4
90,83	79,4	70,27	55,85	84,07	49	2401	1	1	0	0	0	0,36
93,5	82,2	73,57	57,8	85,13	50	2500	2	0	1	0	0	2,67
94,6	89,5	79,83	59,81	86,47	51	2601	3	0	0	1	0	1,1
96,1	91,23	84	64,81	87,97	52	2704	4	0	0	0	1	1,5
96,5	92,2	83,97	60,95	88,97	53	2809	1	1	0	0	0	0,4
97,17	93,9	87,33	62,58	90,23	54	2916	2	0	1	0	0	0,67
97,73	95,87	91	64,83	91,53	55	3025	3	0	0	1	0	0,56
98,03	97,6	92,8	70,46	95	56	3136	4	0	0	0	1	0,3
99,17	97,57	92,77	66,74	97,77	57	3249	1	1	0	0	0	1,14
99,3	98,87	95,07	69,39	99,6	58	3364	2	0	1	0	0	0,13
99,6	100,1	100,87	73,2	100,47	59	3481	3	0	0	1	0	0,3
101,43	101,27	106,2	83,8	103,6	60	3600	4	0	0	0	1	1,83
102,1	103,57	107,5	82,08	106,2	61	3721	1	1	0	0	0	0,67
102,3	104,6	112,8	88,39	107,17	62	3844	2	0	1	0	0	0,2
102,6	106,93	114,87	93,91	107,97	63	3969	3	0	0	1	0	0,3
104	107,5	114,9	106,26	111,43	64	4096	4	0	0	0	1	1,4
105,3	108,47	115,13	106,69	114,9	65	4225	1	1	0	0	0	1,3
105,67	109,33	114,87	111,19	117,43	66	4356	2	0	1	0	0	0,37
105,77	111	112,3	117,52	119,83	67	4489	3	0	0	1	0	0,1
107,43	113,1	112,53	131,55	124,67	68	4624	4	0	0	0	1	1,66
108,53	115,47	111	128,65	129,93	69	4761	1	1	0	0	0	1,1
109,8	116,23	108,9	133,08	133,3	70	4900	2	0	1	0	0	1,27
110,77	117,43	107,6	136,26	136,23	71	5041	3	0	0	1	0	0,97
113,37	118,67	107,63	148,9	140,7	72	5184	4	0	0	0	1	2,6
113,8	120,7	107,27	141,33	143,1	73	5329	1	1	0	0	0	0,43
113,97	120,57	113,13	148,42	145,23	74	5476	2	0	1	0	0	0,17
114,4	121,27	115,63	155,36	147,3	75	5625	3	0	0	1	0	0,43
116,13	121,77	116,33	169,64	149,3	76	5776	4	0	0	0	1	1,73
116,13	121,33	116,03	165,38	152,17	77	5929	1	1	0	0	0	0
116,23	122,67	117,07	171,37	154,83	78	6084	2	0	1	0	0	0,1
116,53	123,8	118,83	176,37	157,8	79	6241	3	0	0	1	0	0,3
118,5	125,33	119,93	194,46	161,57	80	6400	4	0	0	0	1	1,97
119,57	127,93	120,5	188,47	163,1	81	6561	1	1	0	0	0	1,07
119,33	128,97	121	195,89	165,37	82	6724	2	0	1	0	0	-0,24
117,77	129,97	120,4	202,08	167,07	83	6889	3	0	0	1	0	-1,56
120,3	131,2	119,83	222,3	170,67	84	7056	4	0	0	0	1	2,53
121,83	136,67	117,63	214,92	174,23	85	7225	1	1	0	0	0	1,53

120,57	140,9	116,9	223,52	176,23	86	7396	2	0	1	0	0	-1,26
119,43	142,5	118,27	233,65	177,7	87	7569	3	0	0	1	0	-1,14
122,03	150,43	117,47	250,91	183,3	88	7744	4	0	0	0	1	2,6
125,1	154,97	115,2	243,91	186,43	89	7921	1	1	0	0	0	3,07
125,33	146,57	116,93	249,68	188,5	90	8100	2	0	1	0	0	0,23
124,47	146,97	120,07	257,75	192,83	91	8281	3	0	0	1	0	-0,86
126,4	147,23	120,3	277,09	198,3	92	8464	4	0	0	0	1	1,93
128,1	151,9	116,53	263,74	201,87	93	8649	1	1	0	0	0	1,7
128,27	150,53	120,5	267,82	204,33	94	8836	2	0	1	0	0	0,17
127,6	146,87	126,3	272,16	204,83	95	9025	3	0	0	1	0	-0,67
129,47	156,17	129,33	292,26	210,87	96	9216	4	0	0	0	1	1,87
130,9	163,07	126,67	281,77	213,57	97	9409	1	1	0	0	0	1,43
131,13	164,27	131,5	286,86	214,63	98	9604	2	0	1	0	0	0,23
130,77	168,07	137,43	298,29	216,87	99	9801	3	0	0	1	0	-0,36
133,17	172,57	139,93	325,01	221,67	100	10000	4	0	0	0	1	2,4
135	176,57	134,83	316,4	223,1	101	10201	1	1	0	0	0	1,83
135,63	172,1	138,03	331,25	223,23	102	10404	2	0	1	0	0	0,63
135,7	172,1	144	348,62	223,57	103	10609	3	0	0	1	0	0,07
137,6	167,07	149,77	383,19	229,33	104	10816	4	0	0	0	1	1,9
139,03	170,93	153,5	381,57	231,2	105	11025	1	1	0	0	0	1,43
139,23	178,63	157,57	401,75	232,67	106	11236	2	0	1	0	0	0,2
138	178,33	157	427,09	233,93	107	11449	3	0	0	1	0	-1,23
139,8	175,63	157,73	464,95	239,37	108	11664	4	0	0	0	1	1,8
141,3	175,97	157,57	456,19	240,87	109	11881	1	1	0	0	0	1,5
141,27	180,57	157,4	471,73	242,17	110	12100	2	0	1	0	0	-0,03
140,9	182,9	156,83	484,73	243,6	111	12321	3	0	0	1	0	-0,37
142,27	188,13	156,37	524,42	248,77	112	12544	4	0	0	0	1	1,37
142,93	185,43	154,5	505,7	249,87	113	12769	1	1	0	0	0	0,66
142,13	188,17	153,33	513,92	250,67	114	12996	2	0	1	0	0	-0,8
140,5	186,27	148,87	527,7	251,07	115	13225	3	0	0	1	0	-1,63
141,13	187,03	147,8	555,53	254,8	116	13456	4	0	0	0	1	0,63
141,73	190,67	147,93	535,47	255,53	117	13689	1	1	0	0	0	0,6
140,7	190,67	149,7	559,89	254,87	118	13924	2	0	1	0	0	-1,03
139,83	191,47	151,43	567,1	252,2	119	14161	3	0	0	1	0	-0,87
140,53	188,57	153,37	597,66	254,53	120	14400	4	0	0	0	1	0,7
140,83	194,1	148,77	579,34	254,47	121	14641	1	1	0	0	0	0,3
139,6	197,47	149,6	593,01	253,1	122	14884	2	0	1	0	0	-1,23
138,27	197,53	153,93	604,13	252,63	123	15129	3	0	0	1	0	-1,33
139,5	201,63	155,83	627,47	255,13	124	15376	4	0	0	0	1	1,23
139,93	204,5	153,3	616,81	255,83	125	15625	1	1	0	0	0	0,43
139,93	210,73	155,03	636,95	256,13	126	15876	2	0	1	0	0	0
138,77	214,13	155,57	657,91	255,53	127	16129	3	0	0	1	0	-1,16
139,57	208,87	159,13	702,25	259,4	128	16384	4	0	0	0	1	0,8
139,93	210,77	160,23	694,17	262,77	129	16641	1	1	0	0	0	0,36

139,1	211,3	159,23	709,03	265,23	130	16900	2	0	1	0	0	-0,83
137,47	214,17	157,87	705,03	267,93	131	17161	3	0	0	1	0	-1,63
139,2	206,33	157,47	735,09	276,63	132	17424	4	0	0	0	1	1,73
138,83	207	153,87	711,47	283,2	133	17689	1	1	0	0	0	-0,37
137,2	210,97	151,93	729,22	288,17	134	17956	2	0	1	0	0	-1,63
135,77	208,53	152,77	741,4	292,9	135	18225	3	0	0	1	0	-1,43
137,33	202,9	149,33	772,12	302,07	136	18496	4	0	0	0	1	1,56
136,27	203,97	148,4	750,64	308,83	137	18769	1	1	0	0	0	-1,06
134,87	211,07	147,9	759,33	314,67	138	19044	2	0	1	0	0	-1,4
133,4	213,9	142,67	764,7	315,23	139	19321	3	0	0	1	0	-1,47
134,2	208,27	132,7	790,39	319,03	140	19600	4	0	0	0	1	0,8
134,7	208,1	131	768,59	319,9	141	19881	1	1	0	0	0	0,5
134,3	211,5	131,23	774,31	321,97	142	20164	2	0	1	0	0	-0,4
132,3	209,8	134,13	792,72	322,87	143	20449	3	0	0	1	0	-2
134,47	206,83	136,93	824,42	328,03	144	20736	4	0	0	0	1	2,17
136,17	206,8	137,6	793,9	330,27	145	21025	1	1	0	0	0	1,7
135,4	218,47	138,93	807,87	329,67	146	21316	2	0	1	0	0	-0,77
133,4	223,37	141,5	818,7	328,57	147	21609	3	0	0	1	0	-2
135,93	220,37	139,73	856,68	330,9	148	21904	4	0	0	0	1	2,53
137,37	221,27	139,6	825,59	331,63	149	22201	1	1	0	0	0	1,44
136,17	229,6	140,93	840,48	330,67	150	22500	2	0	1	0	0	-1,2
135,57	233,73	141,83	855,83	330,03	151	22801	3	0	0	1	0	-0,6
136,6	221,7	137,6	900,15	334,77	152	23104	4	0	0	0	1	1,03
136,88	223,79	134,75	873,41	333,86	153	23409	1	1	0	0	0	0,28
135,89	229,74	134,64	895,55	331,09	154	23716	2	0	1	0	0	-0,99
134,84	233,19	136,77	920,45	331,92	155	24025	3	0	0	1	0	-1,05
135,85	233,3	136,84	973,23	335,71	156	24336	4	0	0	0	1	1,01
136,01	237,66	137,23	947,13	337,64	157	24649	1	1	0	0	0	0,16
135,24	253,48	136,36	959,76	338,68	158	24964	2	0	1	0	0	-0,77
135,42	266,6	134,72	973,1	341,6	159	25281	3	0	0	1	0	0,18
134,94	244,45	127,49	989,06	348,17	160	25600	4	0	0	0	1	-0,48
135,86	232,22	122,92	923,27	351,91	161	25921	1	1	0	0	0	0,92
136,23	230,42	122,73	905,22	354,48	162	26244	2	0	1	0	0	0,37
135,52	239,26	127,49	893,5	357,33	163	26569	3	0	0	1	0	-0,71
139,13	243,5	134,76	894	364,27	164	26896	4	0	0	0	1	3,61
139,07	242,6	140,06	828,46	369,36	165	27225	1	1	0	0	0	-0,06
137,94	253,41	142,75	817,44	372,94	166	27556	2	0	1	0	0	-1,13
137,35	254,86	146,78	806,87	375,81	167	27889	3	0	0	1	0	-0,59
138,01	254,53	142,91	826,68	382,61	168	28224	4	0	0	0	1	0,66
139,37	265,11	143,19	779,57	384,89	169	28561	1	1	0	0	0	1,36
142,56	271,97	148,7	787,34	386,92	170	28900	2	0	1	0	0	3,19
143,47	272,34	154,53	793,43	388,28	171	29241	3	0	0	1	0	0,91
143,51	268,2	149,62	830,37	394,53	172	29584	4	0	0	0	1	0,04