Stereo and Haptic Cue Integration of Noisy and Non-Noisy Stimuli

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COGNITIVE SCIENCE HONORS THESIS

BROWN UNIVERSITY

March 26, 2021

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Submitted in partial fulfillment of requirements for the degree of Bachelors of Science with Honors in the Department of Cognitive, Linguistic, and Psychological Sciences.

Acknowledgements

First and foremost, I acknowledge that I designed the experiment with help from my advisor and developed the C++ code from a template provided by the graduate students of my lab.

I would like to thank Professor Fulvio Domini for supporting me as a principal investigator, research advisor, and concentration advisor. My time working in the lab has taught me valuable lessons about the nature of scientific research and the field of vision science. I cherish this opportunity to have a supportive space to develop, discuss, and run experiments at my own pace. Thank you for inviting me to check out the lab that tied various disparate parts of my college experience together into a thesis.

I would like to thank the graduate and undergraduate members of the 3D Information for Perception and Action Lab. You were a great help in fostering a learning environment for scientific research. I appreciate being allowed to have a space to share my ideas with lab meetings. I also appreciate the immense amount of help I was provided in setting up my experiment and my stimulus. Thank you to Ailin for providing me a working skeleton of the code that I could develop with. Thank you to Jovan for being available in the lab and giving helpful criticism and guidance towards my work.

Thank you to the housemates and friends that helped provide data for my experiment. COVID-19 surely made running experiments and recruiting participants more difficult but having you all and the lab members available to run in my experiment has been a huge relief.

Thank you to my loving parents for supporting me from afar. You made time to give me supplies despite the distance, and I am eternally grateful. I hope I make you proud.

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Abstract

When interacting with the world, vision and other senses combine in a certain way to produce an impression of shape, depth, and slant. Previous research has found that this combination is probabilistic, in that it follows a model that takes the reliabilities of each individual cue and produces a combined, weighted average as the final estimate. To examine whether the senses of vision and haptics combine according to this model for 3D slant, we tested subjects in a series of experiments involving visuo-haptic cue combination. In the first experiment, they judged non-noisy surface slants with visual, haptic, or a combination of visuo-haptic information. In a second experiment, visual and haptic noise were added to investigate how the variance would be influenced by the addition of noisy sensory cues. In both experiments, consistent with the MLE model predictions, the combined visuo-haptic cue was found to be a weighted average of the single-cue estimates and their variances. Inconsistent with the MLE model, variances were higher than predicted. Patterns of average haptic estimates as each experiment proceeded revealed that visual capture occurred, as the haptic cue recalibrated towards the visual cue by decreasing in estimate over time. Our results lead us to conclude that the method for interacting with the haptic slant, a two-finger touch, produced an unreliable estimate that would eventually lead towards visual capture. We did not show that vision and haptics combine in a statistically optimal fashion.

Introduction

Probabilistic Cue Combination

The field of vision science is interested in figuring out how the brain's visual system processes light energy transduced by the eyes. One of the problems in human vision is how the brain derives a three-dimensional (3D) scene from two-dimensional (2D) information. This 2D information may include visual depth cues such as binocular disparity, texture, and motion information. Furthermore, humans can interact with a 3D visual scene through more than just vision. For instance, haptic touch can be used to feel an object, and proprioception can be used to locate where an object is in relation to one's hand or body. When judging an object's properties, humans may use visual, haptic touch, and proprioceptive information in tandem. Properties such as size, mass, and material can be estimated to form a unified, holistic impression of an object.

Two types of models exist for visual depth cue combination: weak-fusion and strong-fusion models. Weak fusion models assume that each cue is processed independently and combined linearly to produce a perception of depth. Strong fusion models assume that cues may interact to form a non-linear combination for producing depth perception. Weak fusion models are often supported due to how difficult strong fusion is to prove. Strong fusion also requires combination rules that make the model arbitrarily complex (Landy et al., 1995).

It has been found that the combination of vision and haptics may be statistically optimal, in that it follows a maximum-likelihood estimation (MLE) model using Bayesian priors (Ernst & Banks, 2002). This is considered a probabilistic model, and one that is the mainstream approach to mapping a 2D signal to a 3D interpretation.

Ernst & Banks (2002) used a two-interval forced choice task(2IFC) containing visual and haptic cue combination to form the basis of their model. The visual stimulus consisted of a random-dot stereogram depicting a background plane and a raised bar. Subjects were strapped with force feedback devices for haptic feedback and were told to judge the height of raised planes. Two stimuli, with only visual, only haptic, or both visual and haptic information were presented in sequence and subjects selected the stimulus that had a taller raised bar. Within the visuo-haptic combined trials, a cue conflict between vision and haptics was introduced. There were four conditions of visual noise in all visual trials, that took the form of displacement of dots in depth.

The results of this experiment affirmed a statistically optimal MLE model. This model predicts that the combination of cues decreases the variance (squared standard deviation) of the combined visuo-haptic stimulus as compared to both the visual-alone and haptic-alone stimuli. In other terms, given a visual standard deviation, σ_{V}, and a haptic standard deviation, σ_{H}, the combined variance, σ^2_{VH} is:

$$\sigma^2_{\mathrm{VH}} = \frac{\sigma^2_{\mathrm{V}}\,\sigma^2_{\mathrm{H}}}{\sigma^2_{\mathrm{V}} + \sigma^2_{\mathrm{H}}} \qquad (1)$$

Furthermore, the optimal combination of individual estimates, such as vision, $\hat{\mathrm{S}}_{\mathrm{V}}$, and haptics, $\hat{\mathrm{S}}_{\mathrm{H}}$, is done through adding the individual estimates weighted by their variances:

$$\hat{\mathrm{S}} = w_1\hat{\mathrm{S}}_V + w_2\hat{\mathrm{S}}_H \qquad (2)$$

$$w_V = \frac{1/\sigma^2_{\mathrm{V}}}{1/\sigma^2_{\mathrm{V}} + 1/\sigma^2_{\mathrm{H}}} \text{ and } w_H = \frac{1/\sigma^2_{\mathrm{H}}}{1/\sigma^2_{\mathrm{V}} + 1/\sigma^2_{\mathrm{H}}} \qquad (3)$$

Thus, when the MLE model is followed, two individual cue estimates will produce a smaller variance in the combined cue. This combined cue's estimate will be a weighted average of the two individual estimates that is weighted depending on their variances.

As seen in Equation (3), these weights of visual and haptic cues change based on how reliable certain cues are in different situations. In the case of Ernst & Banks (2002), visual noise, in the form of random displacements of the dots in depth, was introduced to see how weights may change due to an unreliable cue. Higher visual noise did change the weights between vision and haptics, as subjects began to rely more on the haptic cue as vision became more unreliable.

A change in weight reflects a shift in perception towards the more reliable cue. The cue conflict within this experiment allowed the results to show this shift. The cue conflict consisted of differences between the heights of visual and haptic cues during the combined cue trials. In certain cases, the conflict was as high as a 6 mm difference, when the average bar height was 55 mm for both visual and haptic cues. When vision was noisy and unreliable, the judged height of the bar was on average closer to the haptic cue. When vision was reliable, the height of the bar was closer to the visual cue. This follows from Equation (2), where the reliability of the cue determines the final estimate based on a weighted average.

Helbig & Ernst (2007) showed similar results for perception of 3D shape. Subjects judged whether 3D elliptical ridges were vertically or horizontally elongated using vision, haptics, and combined vision and haptics. Visual noise was once again introduced through visual blurring of the shape, and there were cue conflicts between vision and haptics during combined trials. The authors concluded that humans optimally combined 3D shape information of real elliptical objects.

While statistically optimal cue combination and MLE can function in both within-modality cues, such as combining visual depth cues, and between-modality cues, such as combining visual and haptic cues, the ways in which it happens have been shown to differ. When cues from the same modality, such as texture and stereo vision, are combined, people lose access to the single-cue information. This means that texture and stereo would fuse into a singular percept, but subjects would not be able to access the texture or stereo estimates that make up that combination. In contrast, between-modality cues, such as vision and haptics, retain their single-cue estimates. It can be observed that within-modality cues usually exist within the same space, while between-modality cues can occur in different spaces, such as vision looking at one object while haptics is touching another (Hillis et. al., 2002). The fact that vision and haptics could specify different objects leads to the "binding" problem, where visual features must be correlated with other features such as location and haptic touch to provide a unified representation (von der Malsburg, 1981; Roskies, 1999). If vision and haptics retain their single cue estimates in combined cue conditions, certain conditions must be met for them to bind into unified stimulus.

Gepstein et. al. (2009) observed that humans performed statistically optimally when the visual and haptic cues are from the same location. Cues separated by at least 3 centimeters have poorer precision, on par with precision of vision or haptics alone. This suggests that the human visual system is tuned to bind with haptic if the two cues are close to one another. Attention may be a factor in binding of visual and haptic cues. If the cues are sufficiently separated, attention may need to be divided between both modalities rather than focused on their fusion. As shown by the phenomenon of illusory conjunctions, attention divided between different features can produce incorrect fusion of features (Treisman & Schmidt, 1982).

Knill & Saunders (2003) showed that the MLE model holds true for 3D slants with texture and stereo cues. In the present study, we are interested in these 3D slants and their interaction with vision and haptics. Slants are prone to many biases that may interfere with the combination of vision and haptic estimates within near space. The vertical tendency describes a bias where estimates of slants from horizontal tend to be overestimated. This means that as slants increase from horizontal, their slant value is reported higher than they are. Conversely, slants estimated from vertical are underestimated. This has been shown to be true for verbal reports, manual estimates using human hand gestures, and through reporting from haptic touch alone (Durgin et. al, 2010). This bias occurs within near space, in a space that affords interaction with objects through touch. Any slant farther away from near space is exaggerated and become more exaggerated as distance increases from the observer (Li & Durgin, 2010). Since both vision and haptics are affected by this bias (Li & Durgin, 2012), it is possible that optimal cue combination can still occur between them. For instance, the vertical tendency for a low slant from vertical may be stronger for vision than for haptic. When these cues combine, vision may be underestimated more than haptic, producing different slant estimates for each cue. The cue combination rule from Equation (2) would produce a value in between the visual and haptic cue that reflects the vertical bias's influence on each cue.

Another type of bias is scaling contrast, that involves visual texture information. Knill (1998a, 1998b) suggested the importance of texture foreshortening when interpreting slant from texture. The shape of slanted texture patches is foreshortened as one moves further in depth. The information about how distorted farther parts of the texture look compared to closer parts can be used to estimate a slant. In the same vein, the idea of scaling contrast describes the size relationship between closer and farther texture elements (Todd et. al., 2010). Together, these biases of slant from texture suggest that visual textures may be more reliable at higher slants from vertical, as the distortions of size and shape are more noticeable. This is another example of how slants can be prone to visual biases. These biases may not interfere with optimal cue integration but have been shown to change visual estimates and visual reliability of slant.

The present study seeks to examine the relationship between visual and haptic slant perception. Specifically, we look at visual slant from binocular disparity through stereo goggles and haptic slant estimation from a two-finger touch of a slanted plexiglass board. The purpose of this method is to extend the findings of Ernst & Banks (2002) and Helbig & Ernst (2007) to slant perception. Given the biases affecting slant and the binding problem, it is possible that optimal cue combination may not occur.

Another aspect of this experiment describes the contribution of stimulus noise in determining optimal cue combination. In some cases (Serwe et al., 2009), noise may produce non-optimal cue combination.

Stimulus Noise

Noise, specifically stimulus noise, can be defined as an aspect of a stimulus that produces some ambiguity or lower reliability. Ernst & Banks (2002) showed that noise may influence how vision and haptic senses are weighted when perceiving an object. Under little to no noise, the phenomenon of visual dominance or visual capture over haptic cues occurs (Rock & Victor, 1964; Hay et. al, 1965). Classically, it was assumed that visual capture described the combination of vision and other cues. Different cues calibrate themselves towards a visual estimate. However, with the advent of statistically optimal cue combination and MLE, the reliability of vision and haptic are said to be relative to one another. When vision is unreliable, vision may reweight and adapt to match haptics. When haptics is unreliable, haptics adapts to vision (Burge et. al., 2010). Haptic touch, however, has been found to be less accurate than vision. When it is the only source of information, haptic touch is less precise, while vision can be just as precise as a combined visuo-haptic cue. When there is a cue conflict, the influence of haptic cues can be shown and manipulated through the addition of visual noise (Ernst & Banks, 2002; Pettypiece et. al., 2010).

Another reiteration of the binding problem involves how the nervous system determines if two cues belong together. Serwe et. al (2009) studied the combination of noisy visual and proprioceptive information. Their results show that for estimates of direction, both visual and proprioceptive cues are combined, but do not show an increase in reliability as predicted by MLE. Since the cues were redundant but unreliable on their own, their combination produced a stable visual-proprioceptive percept, but did not integrate single-cue information. The authors of this study suggest an alternative model, probabilistic cue switching, where within a combined cue trial, subjects choose either the vision or haptic cue depending on the reliability of both cues. In this model, between-modality cue integration does not occur, but some probabilistic mix of the two cues determines the estimate.

Serwe et. al. (2009) hypothesized that vision and haptics would likely integrate if each single-cue visual and haptic stimulus were unreliable and noisy. Under the MLE model, the combined visuo-haptic stimulus would be less noisy. Their results found that between-modality integration may not have occurred. The fact that each single-cue stimulus was unreliable alone casts doubt on their proposed model, as the single-cue estimates are, in fact, inaccessible or very noisy. If these cues were inaccessible, how did they form a probabilistic mix? In the current study, we intend to create visual and haptic stimuli with a smaller magnitude of noise than in

Serwe et al. (2009). If each cue is noisy but still reliable enough to produce single-cue estimates, then we can test if their model still holds. This will be achieved through visual displacement of random dots in depth like Ernst & Banks (2002), and a haptic displacement of a two-finger touch through touching soft foam instead of a plexiglass surface.

Current Study

The aim of this study is to test visual and haptic cue combination on a slanted stimulus. This will be performed in two experiments: cue combination with non-noisy cues, and cue combination on noisy cues. Previous experiments have shown that under certain conditions, the fusion of vision and haptic information will be statistically optimal. We would like to experiment on this effect using slant perception, which is prone to biases such as frontal tendency and scaling contrast from texture.

The design of the first experiment will be like Ernst & Banks (2002) and Helbig & Ernst (2007), where a visual stimulus defined by random dots is combined with haptic touch in some way. As both of those experiments measured the height of raised bars and the orientation of elliptical ridges respectively, this set of experiments will be an iteration of those experiments that measures the angles of slanted planes. Subjects will use a diagonal-line probe to judge the slant of visually or haptically-defined surface slants. This 2D probe corresponds to the 3D orientation of the slant. A range of slants for the subjects to judge will allow us to see how subjects' perception of slant changes as we increase the slant, and how vision and haptics interact with these changes in slant.

In a second experiment, visual and haptic noise will be added to the slants, and another probe task will be performed. Following the work of Serwe et. al. (2009), we will employ small amounts of noise applied to the visual and haptic cues. These take the form of displacement of dots in depth for vision and displacing a foam board as opposed to a flat board for haptics. We aim to quantify the effect of noise on the variance of the visual and haptic estimates of slant. Our goal is to see whether the visual and haptic cues integrate according to the MLE model, or whether an alternative model can explain why the cues do not integrate.

We predict that the non-noisy visual and haptic cues will combine in a statistically optimal fashion. Without any noise and with proper binding of vision and haptics, we could expect that the two cues are more reliable together than alone. When stimulus noise is added to these stimuli, our hypothesis is that in combined cue conditions, stereo noise should increase the variance of visual slant, and haptic noise will increase the variance of haptic slant. When the noisy cues are combined, it is predicted that the combined estimate will fall between the visual and haptic estimates, but the variance will not follow the MLE model. The variance will be higher than expected due to the presence of noisy stimuli, and optimal cue integration will not be observed.

Methods

Experiment 1: Non-Noisy Probe Adjustment

Experiment 1 aims to test the combination of non-noisy vision and haptics on a 3D slant. This experiment is split into three parts: 1a, 1b, and 1c. This is due to adjustments in the stimuli for each part, requiring separate data and analyses.

Participants

Experiment 1a: One undergraduate (the author, male) and 1 graduate student (female) at Brown University with normal-to-corrected vision participated in this experiment. All participants are right-handed and were tested for stereo vision.

Experiment 1b: Three undergraduate (2 male, 1 female) students at Brown University with normal-to-corrected vision participated in this experiment. All participants are right-handed and were tested for stereo vision. Another undergraduate (male) student was tested but was excluded due to being left-handed.

Experiment 1c: One undergraduate (female) and 2 graduate (2 male) students participated in this experiment. All participants are right-handed and were tested for stereo vision.

Materials

The apparatus used for this experiment is shown in Appendix A. Subjects sat in a height-adjustable chair resting their chin on a chinrest. Movement of the thumb and index finger were tracked using Optotrak Certus markers attached to subject’s thumb and index fingertips. A halfsilvered 45-degree oblique mirror reflected a 19’’ CRT monitor that displays visual feedback when necessary. A cardboard back panel was used to block vision of the haptic stimulus and hand. Velmex motors are mounted to the monitor and the structure holding the haptic stimulus. These motors could be moved to the proper distances at the start of the experiment. The monitor moved back to a 450 mm. viewing distance, and the haptic stimulus was moved so that the center of the stimulus rests at about eye height for the subject.

The apparatus holding the Phidgets motor controlling the haptic stimulus was mounted on a custom 3D printed structure. The haptic stimulus itself consists of two plexiglass boards. One board contained the non-noisy haptic stimulus, the plexiglass board itself. The other board contained the noisy haptic stimulus, a 45 mm. thick rectangle of polyurethane foam. These two boards were separated using a 3D printed block that was 15 mm. tall. The center of rotation was balanced between the two sides by adding another 3D printed block attached to the foam side of the stimulus. The stimulus subtends 22.62 degrees of visual angle from top to bottom when upright. Stereo cues were presented using liquid-crystal goggles synchronized to the frame rate of the monitor. Two Optotrak markers positioned on the 3D printed block tracked the slant of the stimulus.

A C++ program controlled the experiment, the finger tracking, and the movement of the motors. For the entirety of the experiment, the lights were turned off in the experiment room.

Procedure

Before the experiment began, subjects calibrated their fingers to the Optotrak. Then, subjects were presented with a pseudo-random set of trials containing either a “Look” or a “Touch” auditory cue, corresponding with trials involving vision and haptics, respectively. There were three total experimental conditions: a visual-alone task, a haptic-alone task, and a combined visuo-haptic task.

Visual Task: After a “Look” audio cue, both a random-dot slant and a diagonal-line probe were presented simultaneously. Subjects were asked to judge the slant angle of the 3D stimulus and match their perception of that slant to a 2D diagonal-line.

Haptic Task: After two motor rotations of the haptic stimulus, subjects heard a “Touch” audio cue, and a white dot target appeared on the monitor. Subjects were asked to move their fingers towards the white dot target. When the subject’s thumb moved into the range of the monitor, a red dot appeared on screen and followed the thumb to help guide the subject toward the target. When the subject contacted the board, coinciding with the red dot reaching the white dot, both the red and white dots disappeared, and a diagonal-line probe appeared. Subjects were asked to not move their fingers from the board once they touched it. Like in the visual task, subjects were asked to judge the angle of the board using their thumb and index fingers. Then, they adjusted the 2D line to match the angle of the board they perceive.

Visuo-haptic Task: The combined cue task is identical to the haptic task, but after the white and red dots disappeared, both a diagonal-line probe and the random-dot slant appeared simultaneously. Subjects once again kept their fingers on the board as they judged the slant of the board using their thumb and index fingers and the random dot pattern and adjusted the 2D line probe to match the angle perceived from the combination of both sources of information.

Subjects were asked to report on a numpad keyboard what slant they perceive. The keyboard allowed subjects to adjust the diagonal-line probe to be more flat or more upright, with coarse and fine adjustment options. Once they confirmed their response, they were asked to return to a comfortable home position away from the haptic stimulus and wait until the next auditory cue was given. Upon response, the angle of the diagonal-line probe from the y-axis is recorded. The reaction time and the final grip aperture, or distance between the fingers, at the moment where the white dot meets the red dot was recorded as well.

Subjects were informed to keep the markers on their fingers visible to the Optotrak as much as possible. If fingers were not visible at any time, a large letter “T” for the thumb and “X” for the index finger was shown on screen.

A total of 4 slant values were tested: 25, 35, 45, and 55 degrees. Across three different trial conditions and 10 repetitions, the total number of trials was 120. Only one session was administered for each experiment, 1a, 1b, and 1c.

Stimuli

The visual stimulus consisted of a random-dot stereogram on a plane that is rotated about the x-axis to the simulated slant condition for the current trial. This stereogram was constrained by a rectangular aperture that subtends a visual angle of 8.58 degrees of visual angle in both horizontal and vertical dimensions. The dot density of the random dot pattern differed between experiment 1a and experiments 1b and 1c.

The haptic stimulus consisted of a plexiglass plane mounted on a rotating motor. The plane was located 30 mm in depth forward from the center of rotation. For this experiment, rotations were constrained between 0-to-90-degree rotations. The haptic stimulus rotated twice to a new slant between trials, initially rotating to -45 degrees, then rotating to the proper trial slant. This method was used to prevent the sound of the motor to be used as a heuristic or additional cue to judge the slant.

The visual probe consisted of a 40 mm. diagonal line that could be adjusted using the numpad keyboard. The probe movements were also constrained between 0 and 90 degrees. Experiment 1c included a second line to help subjects judge the slant, a vertical reference line.

Experiment 1a. Low, non-uniform dot density: The random dot pattern was generated with a dot density inversely proportional to the slant. This method prevented detection of slants through noticing differences in dot density, as smaller slants seemed less dense than larger slants due to the aperture. For slant S, the total number dots, N, was:

$$N = 500/S \tag{4}$$

While this produced stimuli that looked similar in dot density across slants, the low dot density could decrease the believability of the visual stimulus. The haptic stimulus was a physical board that felt solid due to subjects' interactions with it. The visual stimulus, however, did not seem as solid as the haptic counterpart. The random dots specified a slant but may not have felt real enough to be associated with the haptic cue to promote integration. In a sense, two different stimuli could have been judged instead of a unified stimulus. Thus, the next experiment increased the dot density.

Experiment 1b. High, uniform dot density: The random dot pattern was generated with 1500 dots across all slants. The high number of dots allowed the dot density between slants to become unnoticeable, thus the density could be uniform.

Another important issue to address was the ambiguity presented by the diagonal-line probe. Subject reports during experiment debriefing revealed that the line probe may have been difficult to use in the dark without any spatial references. This may increase the variance of probe responses due to the ambiguity of the probe slant. To eliminate this extra variance, we added a vertical reference in experiment 1c to allow easier judgement of the probe slant.

Experiment 1c. Vertical reference addition: Experiment 1c included the high, uniform dot-density and a vertical reference added to the probe, intersecting the center of the diagonal-line probe. Subjects were informed that this reference probe represented the upright or fronto-parallel state of the visual and/or haptic stimuli, and thus could be used as a reference when adjusting the diagonal line.

Results

Data was analyzed through MATLAB. The mean and standard deviation probe slants for each repetition of conditions were analyzed. Then, to test the predictions of the MLE model, Equation (1) was used on the variance of the visual and haptic probe slants and compared against the results of the visuo-haptic condition. These results are all shown in Appendix B, Figures 1-3. Figure 1 showed the mean probe slants for each of the 4 slant conditions and 3 experimental conditions. Haptic estimates, across all three experiments, was often overestimated, and estimated higher than the visual or visuo-haptic estimates. The visuo-haptic estimate seemed to mostly lie somewhere between the visual and haptic estimates. In experiment 1b and 1c (Figure 1b and 1c), the visuo-haptic estimate seemed to closely coincide with the visual estimate. Figure 2 showed the standard deviations of the 4 slant conditions and 3 experimental conditions. The haptic standard deviation in experiment 1b and 1c (Figure 2b and 2c) were always higher than the visual and visuo-haptic standard deviations. Once again, the visuo-haptic standard deviation for these two experiments were close to the visual standard deviation. In experiment 1a, the visuo-haptic standard deviation was closer to the haptic standard deviation, or even higher in the case of the 55-degree slant. Experiment 1c (Figure 2c) showed that the visual and visuo-haptic standard deviations increase as the slant increases. Figure 3 shows the variance analysis performed. The variance of each condition was plotted, and the predictions from Equation (1) gotten from calculating the combined variance of the single-cue estimates was also plotted. It was found that for all experiments, the visuo-haptic variance (magenta line) was higher than the predicted values (black dotted line).

The mean of the combined cue was found to lie between vision and haptic cues for most of the trials, across all the slants. To test the weight and estimate predictions from the MLE model, the visual and haptic weights and weighted averages from vision and haptic means were calculated using Equation (3) and Equation (2) respectively. The weights are shown in Appendix B, Figure 4. For Experiment 1a, the weight of vision was higher than the weight of haptic at lower slant values, but eventually haptic was weighted higher at higher slant values. As experiment 1a contained a low dot density and this result only occurs in experiment 1a, this likely indicates that visual slants with low dot densities are unreliable at higher slants. Results from experiments 1b and 1c showed that vision stays relatively weighted higher than haptics across all slants, however the visual and haptic weights seem to get closer to 0.5 as the slant increases. The weighted averages were plotted alongside the subjects' visuo-haptic estimate means in Appendix B, Figure 5. The means from all three experiments were within the standard error of the predicted weighted averages. This suggests that some weighted average between vision and haptic cues is being taken for the combination of cues, consistent with the MLE model. However, the variances of these estimates were not consistent with the MLE model. Since all the variances are higher than predicted, as shown in Figure 3, the combination of cues is not optimal. It seemed that cue combination in the context of experiment 1 was only able to produce variances that are greater than or equal to the visual variance, as seen in experiment 1b and 1c (Figure 3b and 3c).

To test whether there was a significant difference between the means of visual, haptic, and visuo-haptic cues, a repeated-measures analysis of variances (ANOVA) between the sensory cue conditions and the slant conditions was performed. ANOVA analysis was performed in the statistics program JASP. The analysis concluded that there was a significant effect of sensory cue condition on probe estimates in experiment 1b with a large effect size ($F(2,4) = 29.051$, $p = 0.004$, $\omega^2 = 0.212$). This same effect was not significant in both experiments 1a and 1c (1a: $F(2,2) = 0.551$, $p = 0.645$, $\omega^2 = 0.000$; 1c: $F(2,4) = 0.305$, $p = 0.753$, $\omega^2 = 0.000$). A post hoc test on experiment 1b revealed that the haptic condition was significantly higher than the visual condition ($SE = 0.811$, $t = -6.358$, $p_{bonferroni} = 0.009$) and the visuo-haptic condition ($SE = 0.811$, $t = 6.820$, $p_{bonferroni} = 0.007$). To see if the changes to the stimuli between experiments 1a, 1b, and 1c affected the probe slant estimates, another repeated measures ANOVA was performed, between experiments 1a,1b, and 1c. It was found that there was not a statistically significant differences between all the experiments (1a & 1b: $SE = 3.310$, $p_{bonferroni} = 0.394$, Cohen's $d = 1.803$; 1a & 1c: $SE = 3.310$, $p_{bonferroni} = 0.325$, Cohen's $d = 1.952$; 1b & 1c: $SE = 2.961$, $p_{bonferroni} = 1.000$, Cohen's $d = 0.166$).

Taken together, the ANOVA results showed that in experiment 1b, the haptic cue was perceived as having a significantly larger slant than the visual or visuo-haptic cue. As seen from the visual and haptic weight analysis (Figure 4), vision was weighted higher than haptic in experiment 1b, thus the visuo-haptic cue estimate is closer to vision than haptic. This is likely the cause of the significant effect. As for experiment 1c, which included the same higher dot density as experiment 1b, it can be observed that the vertical reference aids both visual and haptic judgements. Thus, the insignificant effect of cue condition in experiment 1c despite having a high dot density may be due to an increase in haptic reliability and haptic weights, enough to create an insignificant difference. This was once again visible in the weight analysis, where experiment 1c (Figure 4c) shows a decrease in visual weight and an increase in haptic weight as slants increase.

We explored another possibility that could explain the results: recalibration of the visual cue towards the haptic cue. Given how different subjects seemed to estimate visual and haptic cues, these sensory cues may be inconsistent in perception to each other. Previous work has found that inconsistent visual and haptic cues may cause vision to recalibrate towards the haptic cue (Atkins et. al., 2003). This would mean that, as the trial numbers increase, the visual estimate may adapt to be closer to the haptic estimate. While there is no cue conflict in this experiment, the gap between the visual and haptic estimates in certain cases may provide the inconsistency of cues. Thus, the experiment was separated into three blocks for analysis, each 40 trials each. The average was taken for each slant value in each block. The results of this analysis can be seen in Appendix B, Figure 6. Across all experiments, as the block number increased, the estimates for vision, haptic, and visuo-haptic cues decrease. This was especially notable with the haptic estimate, as it often starts in block 1 as an overestimation and ends up as an underestimation or closer to veridical by block 3. Experiments 1b and 1c (Figure 6b and 6c) were characterized by the vision and visuo-haptic conditions having stable underestimates of the slant that change very little over the course of the experiment.

These results did not support a recalibration of vision towards haptic. However, it does support one of two conclusions: either the estimates of the slant become more veridical over time, or the estimates of slant decrease over time. In experiment 1a, block 1 showed that all the slant estimates are overestimated. Then, as the experiment proceeded, the slants decreased, eventually setting around the identity line, the point of veridicality, in block 3. This pattern was not repeated for experiments 1b and 1c however, as they started either veridical, in the case of 1c, or underestimated, in the case of 1b, and decreased in estimates from there. Experiment 1b showed that the haptic estimate decreases over time, while the visual and visuo-haptic estimates stay stable as underestimates. Experiment 1c showed a general decrease between all cue conditions. This pattern may suggest that experiment 1a is also a decrease, that happens to look like it is becoming more veridical by virtue of starting as an overestimate. Another possibility is that the opposite of what Atkins et. al (2003) proposes: haptic may be recalibrating to the vision cue. In experiments 1b and 1c, the haptic estimate seemed to be decreasing towards the visual estimate and is rarely below the visual estimate. It is possible that this is another case of visual capture, where the reliable visual cue produced by the high dot density causes the haptic estimate to be recalibrated towards the visual estimate.

The small subject number for each experiment presents another explanation for the pattern of results in experiment 1: that there is noise in the data from too small of a sample size. With a maximum of 3 participants in experiments 1b and 1c, it is possible that individual differences in slant perception influenced the data in one way or another.

Throughout experiment 1, we have shown that the combination of vision and haptics in perceiving 3D slants follows the MLE predictions of being a weighted average of individual cues but does not follow the MLE predictions of variance. These results are like Serwe et. al (2009), where cue combination also occurred according to the MLE model, but there was no improvement in reliability. We cannot judge the reliability of the cues in this experiment without first performing a 2IFC task and judging the just-noticeable-difference (JND) of slants. From this experiment, we can conclude that haptic information had a low weight when subjects interacted with the visuo-haptic stimuli. This could possibly be due to the design of the haptic stimulus itself. These issues are expanded in the Discussion section.

Experiment 1 did not manipulate the reliability of each cue. In experiment 2, noise is added to the stimulus to test how adding some unreliability to the stimulus will affect cue combination. It is predicted that these cues will be less reliable than their non-noisy counterparts, and that optimal cue integration will not occur between noisy visual and haptic cues.

Experiment 2: Noisy Probe Task

Participants

The three participants from experiment 1b participated in experiment 2.

Materials and Stimuli

The apparatus used for experiment 2 is identical to experiment 1. The motor was now allowed to rotate a full 360 degrees to present the other side of the haptic stimulus, the foam board. The foam board was parallel to the plexiglass board and slightly deforms to the touch. The visual stimulus included noise in the form of random displacements of dots in depth. The maximum displacement was 20 mm in depth parallel to the plane of the original stimulus. Both the haptic and visual noise could occur alone, alongside the normal visual or haptic stimulus from experiment 1, or together.

Procedure

This experiment has a similar procedure to experiment 1b, with various additions. After calibration, subjects were presented a set of pseudo-random trials where they either looked at a visual stimulus alone or touched some haptic stimulus with certain combinations of visual and haptic noise cues. There are a total of 5 trial conditions depending on the noise applied to the stimulus: visual noise alone (V1), haptic noise alone (H1), noisy vision with normal haptic (V1H0), normal vision with noisy haptic (V0H1), and both visual and haptic noise together (V1H1). For any noisy haptic stimulus, subjects were instructed to push lightly on the foam, to produce a small displacement in the foam. For the visual stimulus, the subjects were told that they would be seeing "fuzzier" stimuli from the provided dots. Each trial was cued by an auditory cue that said "Look" or "Touch" to specify whether the trial will only involve vision (V1) or have some haptic involvement (H1, V1H0, V10H1, V1H1).

The experiment was performed within 2 identical sessions of 100 trials. These 100 trials contained the same 4 slant values as experiment 1 (25, 35, 45, and 55 degrees from vertical), the 5 trial conditions detailed above, and 5 repetitions each. The two sessions were merged into 10 repetitions of the conditions for analysis. Subjects scheduled the 2nd session as soon as they finished the 1st session. There were at least 24 hours between each session.

Results

Analysis was performed in a similar manner as experiment 1. Means and standard deviations were analyzed for each of the 5 sensory cue conditions across the 4 slants. Data from both sessions was combined into a singular data set for each subject. Each variance of the 3 combined cue conditions (V1H0, V0H1, V1H1) were analyzed separately. For the predictions of variances that involved previous data (V1H0, haptic data, V0H1, visual data), the data was drawn from the subject's visual or haptic data from experiment 1b. Blocks were once again employed to investigate the possibility of recalibration. The blocks were split into 50 trials each, thus giving each session 2 blocks. This allows us to judge how probe slants may change from the start to the end of each session and allows us to see if learning occurred between the days of performing the experiment. The results of this analysis are detailed in Appendix C, Figures 7-8. The mean estimates showed that haptic estimates were higher than other estimates for 25 and 35-degree slants, but eventually converged towards the other estimates at higher slants. The V0H1 condition was noticeably estimated lower than the other combined cue conditions and had a standard deviation that changed little between slants. There were 3 points in which the variances fit the MLE predictions: V1H0 at 35 degrees, V1H1 at 25 degrees, and V1H0 at 45 degrees. Given that these do not have a pattern for their occurrence, it is likely that these variances could have been a result of noise in the data from a small sample size.

Once again, to test if the combined cue estimates fit the MLE model, we took the single cue data from this experiment and experiment 1b to estimate the weights and weighted averages from mean estimates using Equation (3) and Equation (2), respectively. Since the V1H0 and V0H1 conditions required single cue data about non-noisy visual or haptic conditions, single cue means were taken from experiment 1b, since the same subjects participated in this experiment. Since the procedure changed very little from experiment 1b to experiment 2, the mix of means from different experiments should be viable. The results of the weight calculations are plotted in Appendix C, Figure 8. The V1H0 and V0H1 conditions seemed to show that the weights converge on 0.5 as the slant increases. The difference between these two was that V1H0 has a high visual weight and a low haptic weight at the 25-degree slant, while V0H1 has a relatively low visual weight and high haptic weight at the same slant. The V1H0 condition displayed a reversal of weights between 35 and 45 degrees, as weights switch from a higher visual weight to a higher haptic weight. This reversal remains stable at 55 degrees. The results of the weighted averages are plotted on Appendix C, Figure 9. While the V1H0 weighted averages were all within the standard error of the means, the V0H1 and V1H1 conditions had certain values that were not within the standard error of the mean. In particular, the 25-, 35-, and 55-degree conditions from V0H1 and the 25-degree condition from V1H1 had lower means than predicted. It is possible that these results are once again influenced the noise in the data from a small sample size.

Block results are shown in Appendix C, Figure 10. Analysis of the block results showed that slant estimates once again decrease over time. The estimates from block 3 seemed to increase from block 2. This may be because block 3 signifies a new session, and learning did not occur between sessions. Thus, subjects may come into the second session with more experience with the task but may not retain their decreased calibration from the end of the previous session. It seemed like the haptic-alone estimate is most affected by the passing of trials. Blocks 2 and 4 showed that the haptic-alone estimate is often underestimated, while blocks 1 and 3 showed that it usually starts as an overestimate during a session.

A repeated measures ANOVA was once again performed between the 5 sensory cue conditions and the 4 slant conditions. The sensory cue condition was found to have a significant effect on the probe slant with a large effect size ($F(4,8) = 10.611$, $p = 0.003$, $\omega^2 = 0.107$). Given this data, post hoc analysis was performed on the conditions. A significant difference was found between V1 and H1 with a large effect size (SE = 0.860, t = -3.896, $p_{bonferroni} = 0.046$, Cohen's d = -2.249) and between H1 and V0H1 with a large effect size (SE = 0.860, t = 6.464, $p_{bonferroni} = 0.002$, Cohen's d = 3.732). This ANOVA data, synthesized with the mean data, suggests that H1 is estimated higher than V1, while V0H1 is estimated lower than H1. The noisy haptic cue alone seemed to only be estimated higher than vision, while the addition of a normal visual cue to the noisy haptic cue caused a decrease in slant estimates.

We had predicted that the addition of noise would increase the variance of a noisy cue as compared to a non-noisy cue. We took the variances of the noisy and non-noisy estimates from experiments 1b and 2 to compare how visual and haptic noise influenced the variances. The results are shown in in Appendix C, Figure 11. Contrary to our predictions, the haptic noise condition produced lower variances at all slant conditions. Visual noise produced just as much variance at low slants, and more variance at higher slants. There are two possible explanation for this trend in the data: subject experience with the task, and the strength of the noise presented. Since the subjects for experiment 2 were the subjects of experiment 1b, they had prior knowledge of the task and how they responded to each slant before. While using the same subjects allowed us to compare data between experiment 1b and 2, the results of experiment 2 could be influenced by prior participation in experiment 1b. This may result in lower variances than predicted, as seen in the data. The other explanation involves how strong the haptic noise and visual noise felt. The goal of experiment 2 was to increase the unreliability of the visual and haptic cues through noise. This was done at a small scale, to prevent each individual cue from being entirely unreliable on their own. The consequences of this scale may manifest in little to no change in variances. The foam may have been too stiff to produce enough ambiguity in slant, and the displacement of dots may have been too shallow to influence variance.

From these results, we can conclude that the combination of noisy visual and haptic cues follows the MLE model predictions of being a weighted average in most cases. The V0H1 condition displays significantly lower estimates than other conditions, and this causes it to fall below the predictions from the MLE model. Some of the combined cue variances are close to the predicted variances. It is possible that these results are due to subject experience, as the subjects for experiment 2 already participated in experiment 1b. The low subject count could have also contributed some noise to the data.

Discussion

Current Study

This study sought to test visual and haptic cue combination on a slanted surface with both noisy and non-noisy sensory conditions. Previous literature supports a Bayesian MLE model for statistically optimal cue combination, where there is probabilistic cue combination based on a weighted average of the cues to be combined. These cues are prone to changes in variance due to their reliability, thus the final combined estimate will be closer to the more reliable cue and have a smaller variance than both cues (Ernst & Banks, 2002; Helbig & Ernst, 2007). This effect had been tested on visual and haptic cues, but the interaction of those two cues on a slant perception had yet to be explored. Another study found that individually unreliable noisy cues could produce estimates following the MLE model without having improved reliability (Serwe et. al., 2009). Thus, our experiment was designed to tackle a smaller amount of noise to allow estimates of individual cues, to test the effects of cue combination and noise.

The results of this study found that both noisy and non-noisy estimates followed the MLE model for weighted averages in most cases and fell short of MLE predictions of variance. A combination of factors may be responsible for these results. The integration of vision and haptic may not have been strong; vision and haptic may have been regarded as cues to separate objects. Subjects may have seen a random-dot pattern that they do not associate with the haptic board, and subjects may have felt a board that they do not associate with the random-dot pattern they saw. Despite the two objects projecting to the same location, the fusion necessary for optimal cue integration may not have occurred. These results are another case of the binding problem.

Another possibility is visual capture. Throughout the course of experiment 1 and 2, the haptic-alone estimates decreased while the vision-alone estimates stayed relatively the same. This may be a sign that the haptic cue is recalibrating to be closer to the visual cue, which is a demonstration of visual capture. This change was more noticeable when vision was more reliable, as in experiment 1b and 1c, where the random-dot pattern has a large dot-density. The haptic cue in general may have been unreliable, as it shows high variances across every experiment and it reports significantly higher slants than many of the other sensory conditions. The method of touching the board, a two-finger touch, was meant to provide a way of measuring haptic slant. Subjects were told to keep their thumb and index fingers in a static position as they adjusted the probe, thus they were not allowed to explore the slant with their fingers. Many slant experiments allow subjects to explore the surface, thus providing a better impression of the slant through haptic touch (Klatzky & Lederman, 1993). Thus, the haptic cue is fundamentally unreliable, and vision comes to take over. In the absence of vision, such as in the haptic-only conditions of experiment 1 and 2, haptic is overestimated. The difference is visible in the normal vision/noisy haptic (V0H1) condition of experiment 2, where the presence of the only normal vision cue in experiment 2 produces smaller estimates than many of the other conditions.

Limitations and Future Studies

While this experiment shows that visual and haptic cues are being combined in a similar way to the MLE model, we cannot strongly conclude that the MLE model is correct without further tests. Specifically, we would need to perform a 2IFC task to get the JND measures to compare between the individual and combined tasks. A 2IFC task in the context of this study would involve the same visual, haptic, or visuo-haptic cues with or without noise. Two stimuli would be presented sequentially, and subjects would respond which stimulus felt more slanted away from them. The data from this experiment would be used to produce a psychometric curve in which the JND can be estimated. This JND represents the point in which a person would perceive one standard deviation of slant higher. A higher JND represents lower reliability. Thus, we would need to see if the combined visuo-haptic cue produces as lower JND to confirm the MLE model. In this case, it would be predicted that given the current state of the visual and haptic stimulus, the combined visuo-haptic cue would produce a similar or higher JND than the individual visual and haptic cues. The fact that visual capture occurred in the current study and the possible unreliability of the haptic stimulus may indicate that visuo-haptic integration does not occur, and their combination would not be statistically optimal.

A future experiment will need to re-evaluate the design of the visual and haptic noise, and the design of the haptic stimulus. Visual and haptic noise could be increased to increase their effect, and perhaps produce higher variances. Visual noise can be increased by increasing the maximum displacement of the dots in depth. Haptic noise could be influenced by the type of foam; thus, a softer, more pliant foam could be chosen to increase the noise. This more pliant foam would increase the ambiguity of a haptic estimate of slant as subjects' fingers displace the surface. On this note, two-finger touch could also be reconsidered. Allowing subjects to systematically explore the board with one finger is possible, and this method has been shown to produce accurate, yet biased estimates of haptic slant (Durgin & Li, 2012). A palm board is also possible, although this presents problems when paired with the foam board. A palm resting on foam may not change slant estimates, as the proprioception of the hand is influenced little by a displacement in the foam.

Conclusions

This study tested visual and haptic cue integration in slants. It found that the combined visuo-haptic estimates are a weighted average of the single-cue estimates, weighted based on variances. The MLE model predictions about variance, however, are not met. The haptic stimulus in this experiment may be fundamentally unreliable, thus visual capture occurred as subjects responded throughout the experiments. More data including the JND of the combined visuo-haptic cue would need to be gathered before the MLE model can be proven or disproven for visuo-haptic 3D slants.

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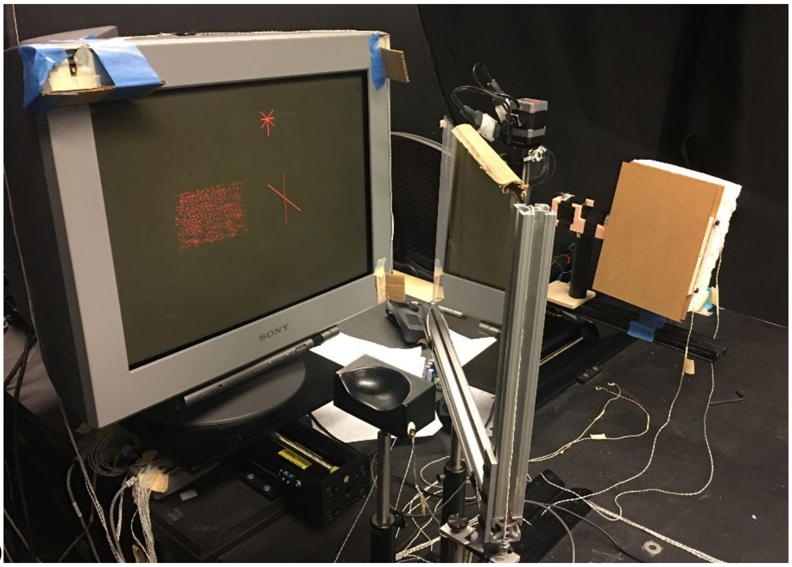
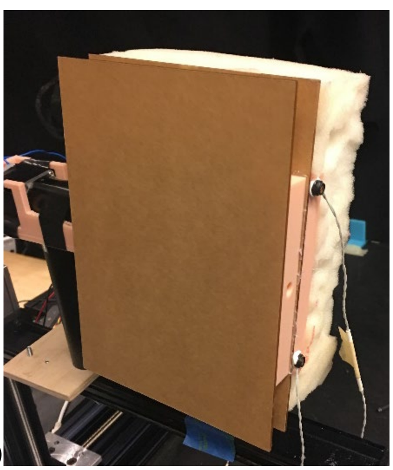
Appendix

Appendix A: Experiment Setup and Materials

a) 

b) 

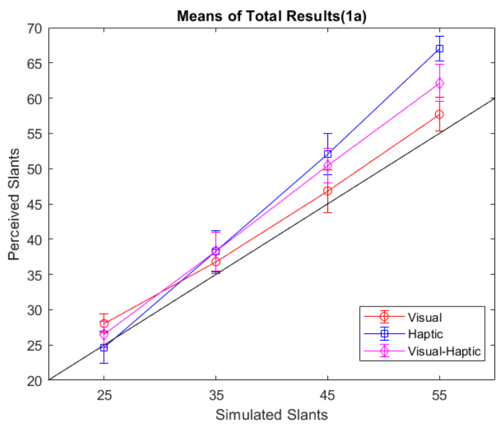
c) 

Experimental Materials used in both experiment 1 and experiment 2. ***a)*** *Organization of the experiment space. Subjects rested their chin in the chinrest and looked forward at the silvered 45-degree mirror reflecting the monitor.* ***b)*** *The haptic stimulus. Two plexiglass boards are separated by a 3D printed block. Another 3D printed block (obscured) is connected to the motor. A foam board is pasted on one side of the stimulus.* ***c)*** *The numpad used for responding to the* *stimuli during the experiment. Felt pads mark the usable keys: 1, 2, 4, 5, and +. Keys 1 and 2 are fine adjustment, and 4 and 5 are coarse adjustment. Keys 1 and 4 decrease the angle of the diagonal probe, while keys 2 and 5 increase the angle. The + key is used to move from trial to trial.*

Appendix B: Experiment 1 Results

Figure 1:

Experiment 1 Mean Probe Slants



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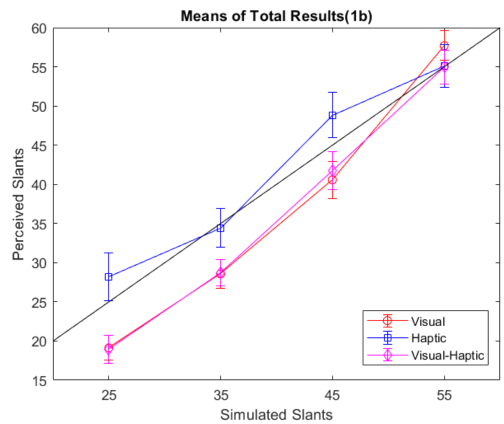

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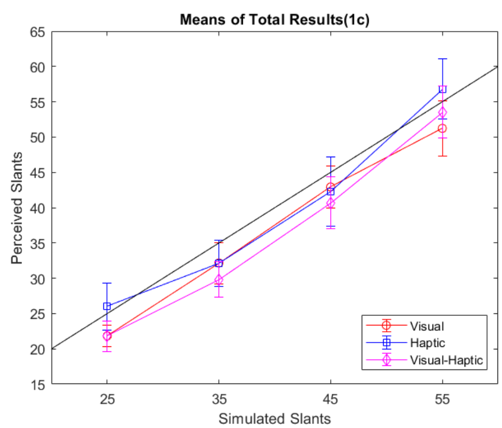
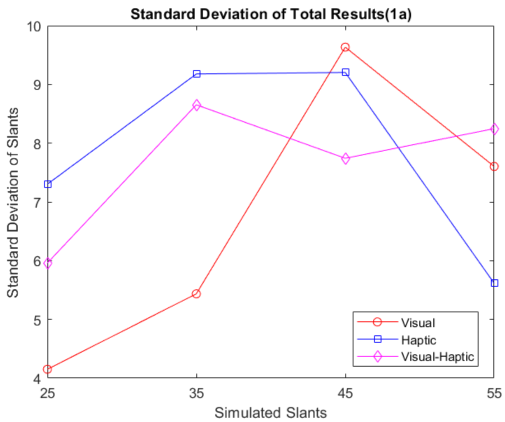


c)

Figure 1. The mean probe slants for the four stimulus slants for ***a)*** *experiment 1a,* ***b)*** *experiment 1b, and* ***c)*** *experiment 1c.*

Figure 2:

Experiment 1 Standard Deviations



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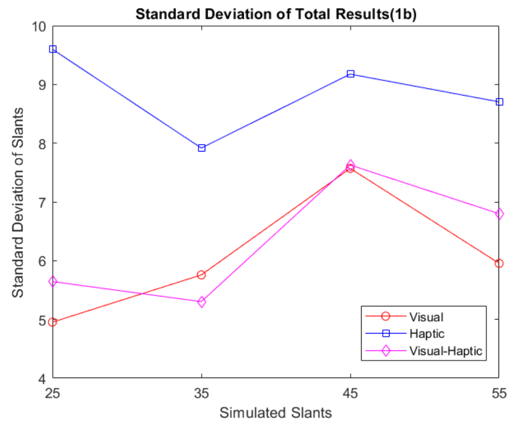

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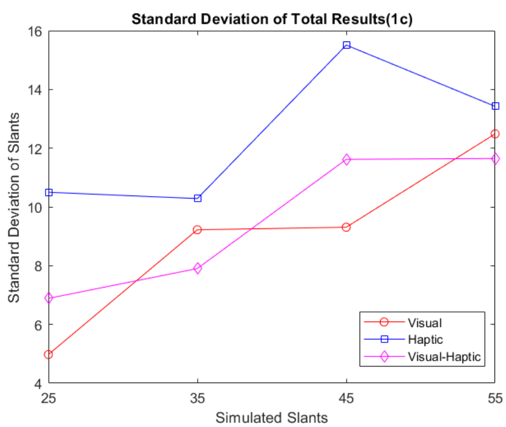
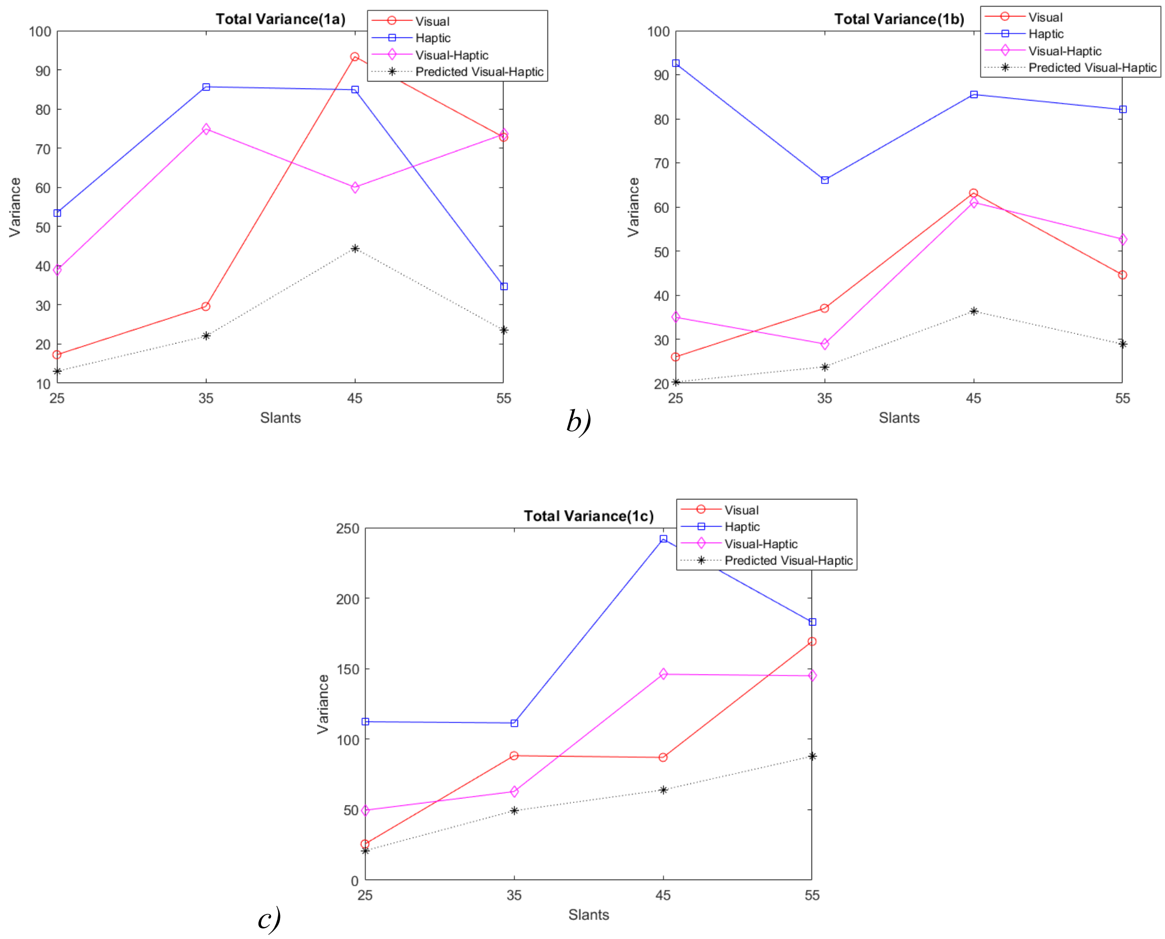


c)

Figure 2. The standard deviation of the four stimulus slants for ***a)*** *experiment 1a,* ***b)*** *experiment 1b, and* ***c)*** *experiment 1c.*

Figure 3:

Experiment 1 Variance Analysis



a) *b)* *c)*

*Figure 3. The variances of the four stimulus slants for **a)** experiment 1a, **b)** experiment 1b, and **c)** experiment 1c. The stars mark the predicted visual and haptic predictions from Equation (1) using the single-cue variances.*

Figure 4:

Experiment 1 Weights

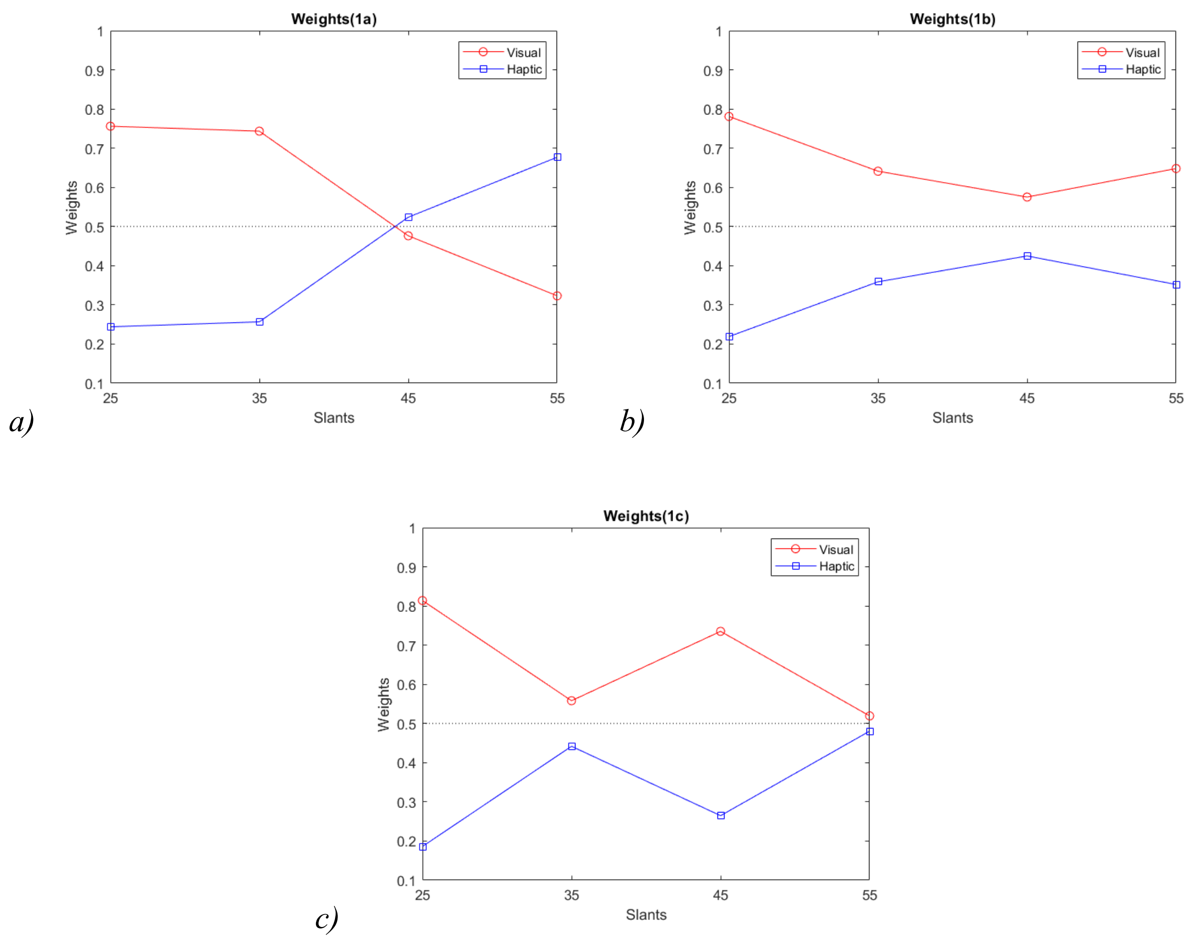
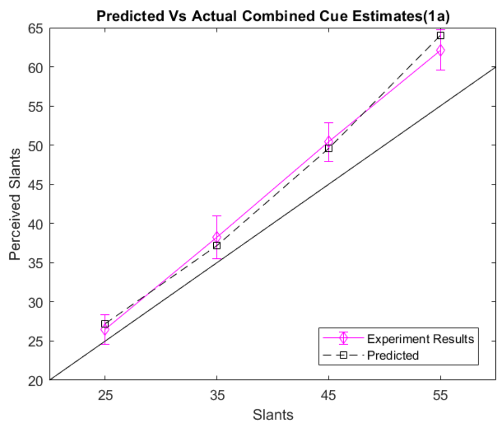


Figure 4. Calculated weights for the visual and haptic cue using Equation (3) with the visual and haptic variances for ***a)*** *experiment 1a,* ***b)*** *experiment 1b, and* ***c)*** *experiment 1c.*

Figure 5:

Experiment 1 Predicted Estimates



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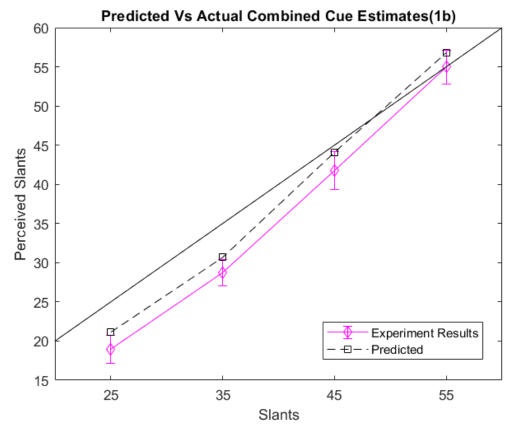

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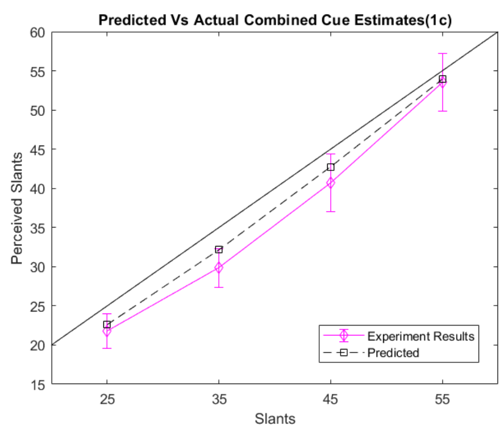
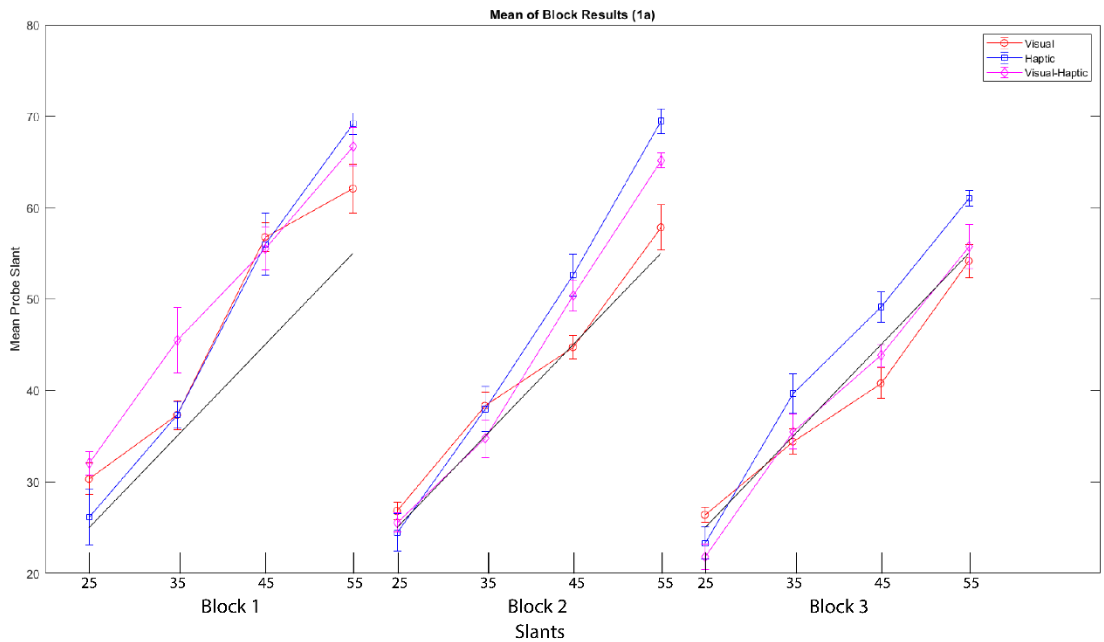


c)

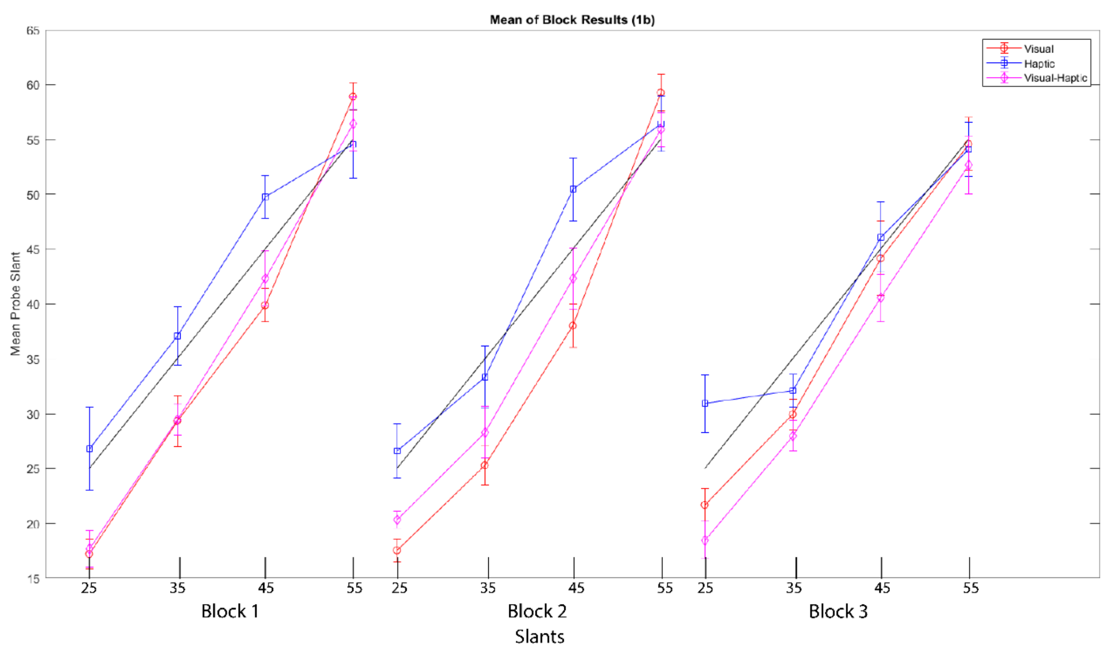
Figure 5. Calculated predicted combined cue estimates using Equation (2) plotted alongside the visuo-haptic experimental results for ***a)*** *experiment 1a,* ***b)*** *experiment 1b, and* ***c)*** *experiment 1c. Note that error bars denote standard error of the mean.*

Figure 6:

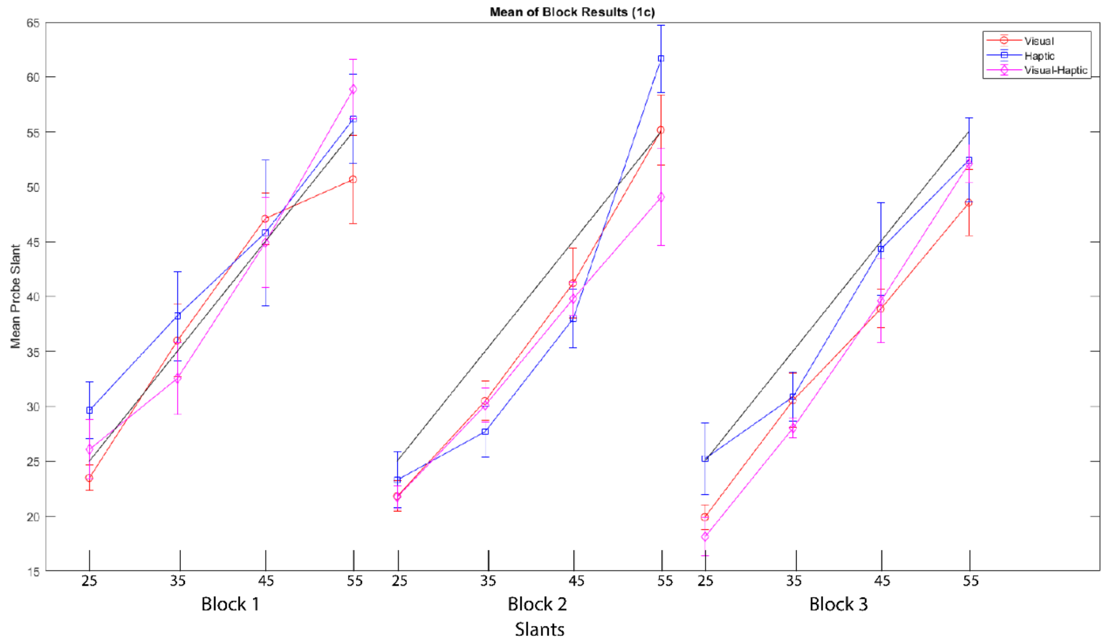
Experiment 1 Block Analysis



a)



b)



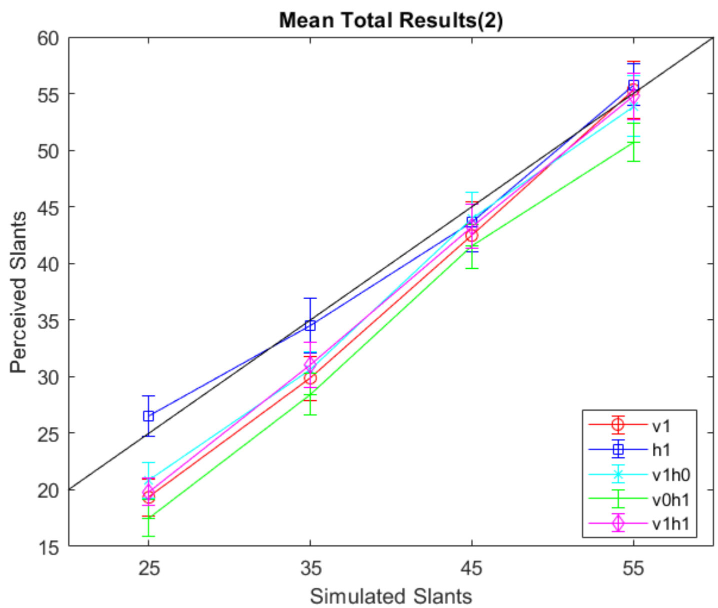
c)

*Figure 6. Analysis of the block means for **a)** experiment 1a, **b)** experiment 1b, and **c)** experiment 1c. Blocks consist of sets of 40 trials with non-uniform distributions of conditions between each block. The identity line, the solid black line, remains the same between each block to allow visual comparison between blocks.*

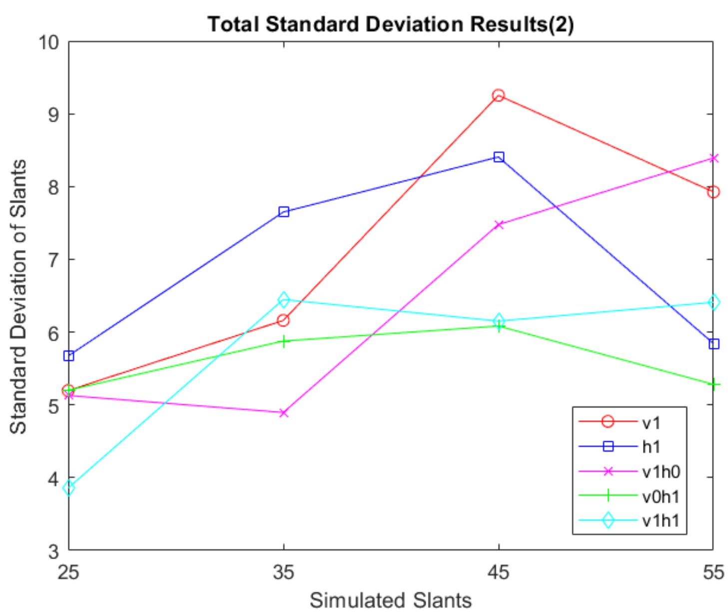
Appendix C: Experiment 2 Results

Figure 7:

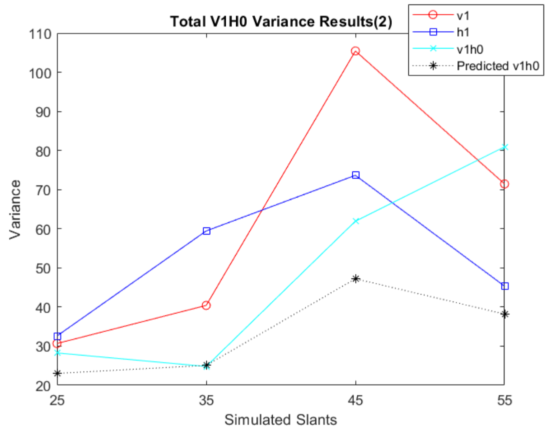
Experiment 2 Summary



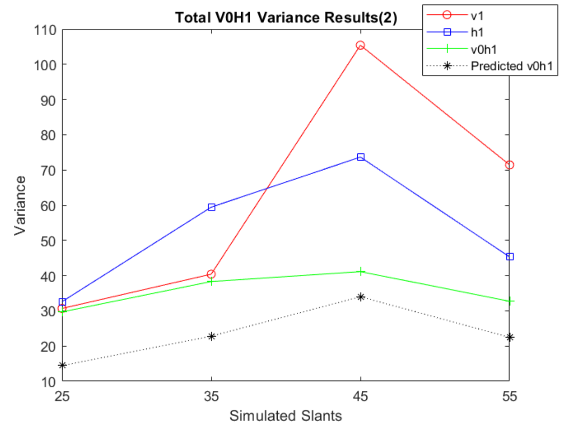
a)



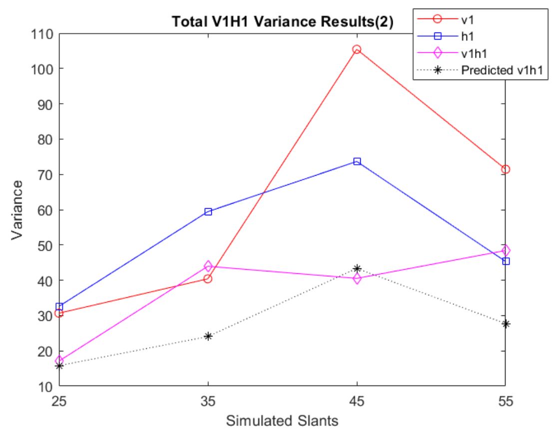
b)



c)



d)



e)

*Figure 7. The **a)** mean, **b)** standard deviation, and variances of the four stimulus slants from experiment 2. Each combined cue condition, **c)** V1H0, **d)** V0H1, and **e)** V1H1 has separate variance analyses.*

Figure 8:

Experiment 2 Weights

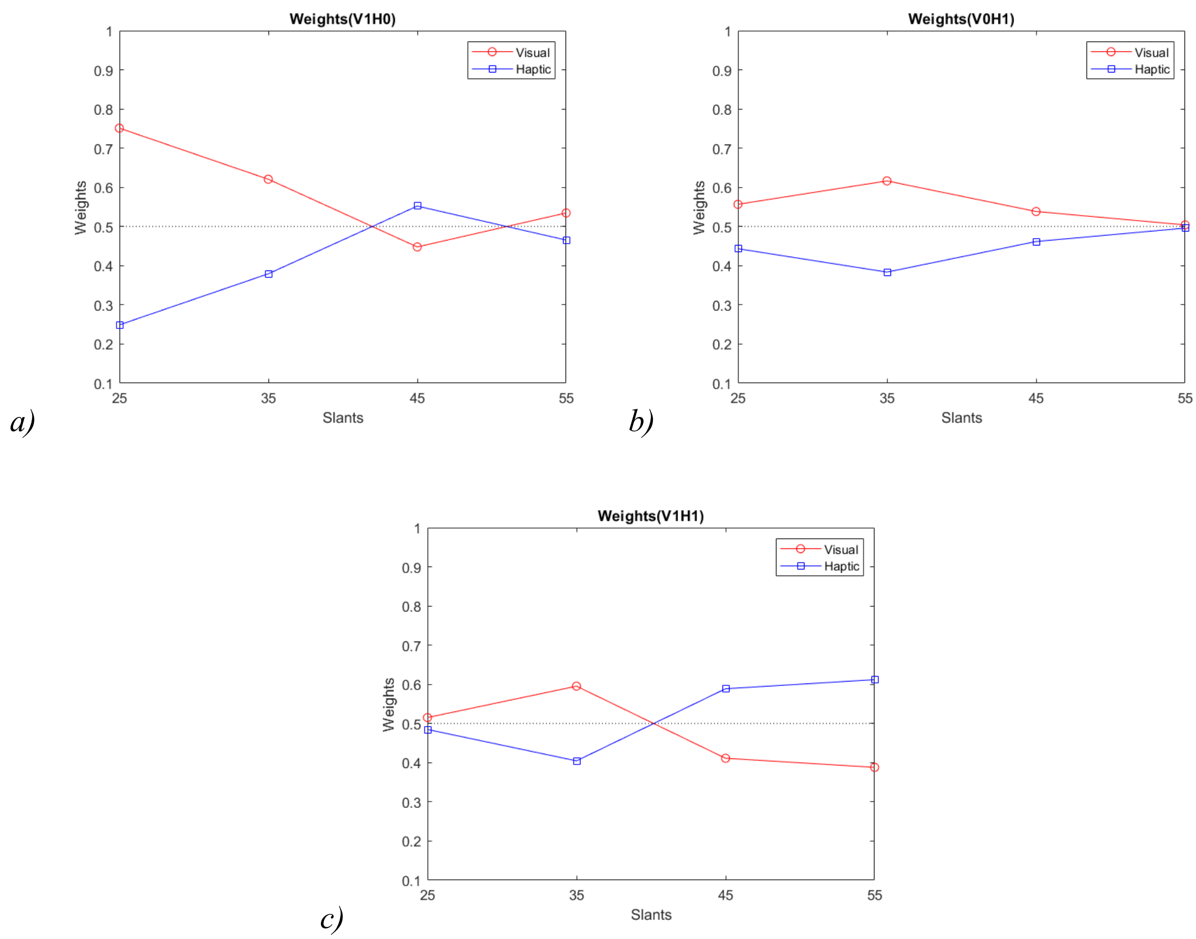


Figure 8. Calculated weights for the visual and haptic cue using Equation (3) with the visual and haptic variances for each of the combined cue conditions, ***a)*** *V1H0,* ***b)*** *V0H1, and* ***c)*** *V1H1.*

Figure 9:

Experiment 2 Predicted Estimates

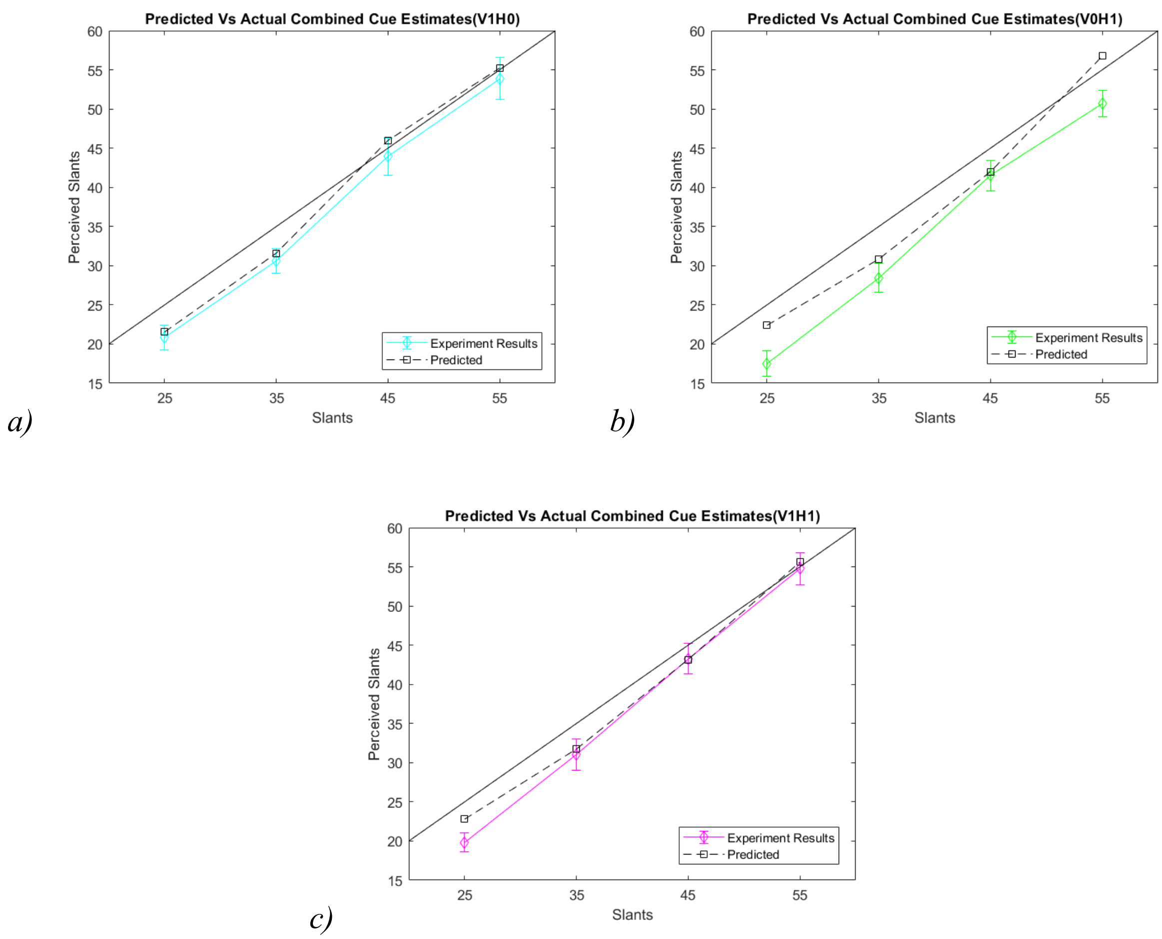


Figure 9. Calculated predicted combined cue estimates using Equation (2) plotted alongside the corresponding combined cue conditions, ***a)*** *V1H0,* ***b)*** *V0H1, and* ***c)*** *V1H1.*

Figure 10:

Experiment 2 Block Analysis

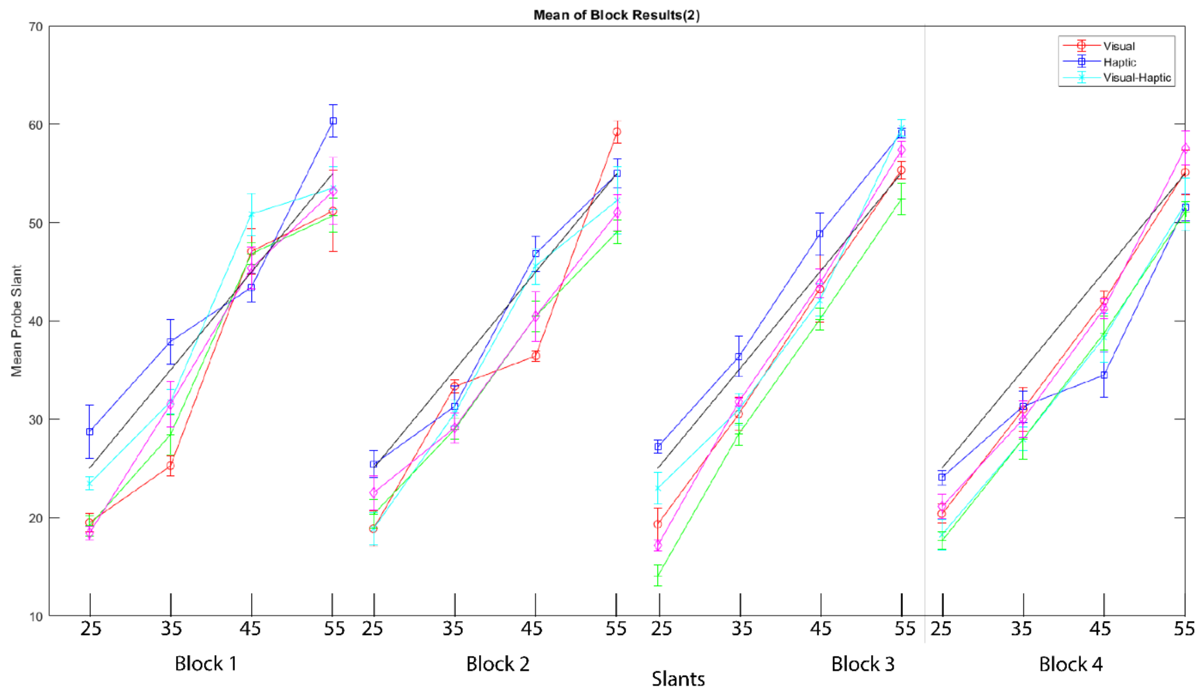


Figure 10. Analysis of the block means experiment 2. Blocks consist of sets of 50 trials with non-uniform distributions of conditions between each block. Blocks 1 and 2 make up the first session of the experiment, while blocks 3 and 4 make up the second session. The identity line, the solid black line, remains the same between each block to allow visual comparison between blocks.

Figure 11:

Single-Cue Variance Comparison between Experiments

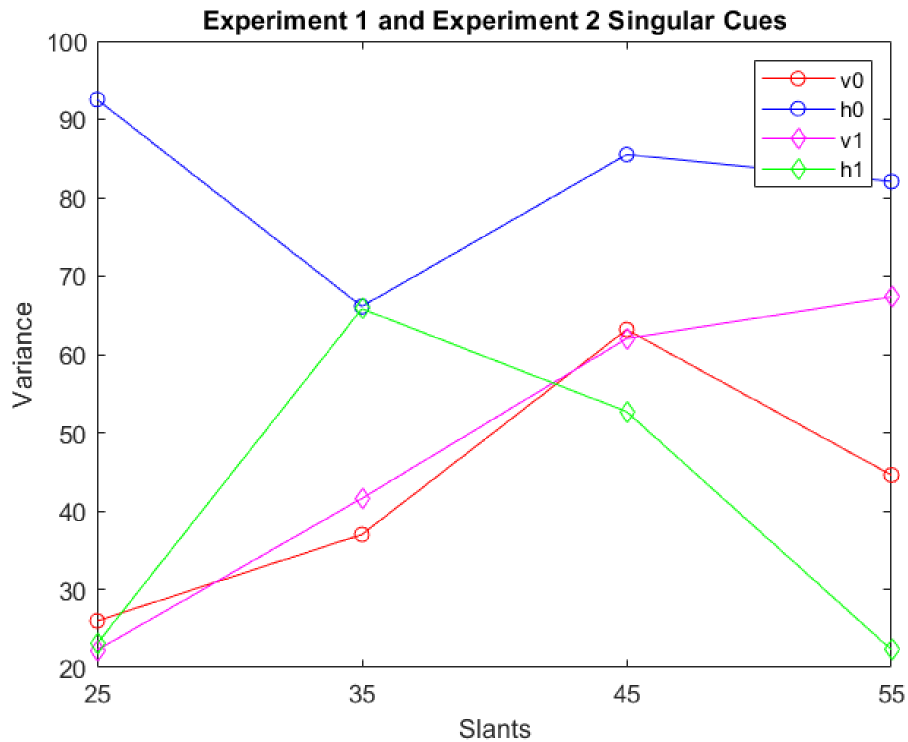


Figure 11. Means of the single cue variances of experiment 1 and 2. V0 and H0 are the single cue variances from experiment 1, and V1 and H1 are the single cue variances from experiment 2.