Задача №1

$$g(x) = \frac{-3f(x) + 4f(x+h) - f(x+2h)}{2h} = \frac{1}{2h}\Big[-3f(x) + 4\Big(f(x) + hf'(x) + \frac{h^2}{2}f''(x) + \frac{h^3}{6}f'''(\xi_1)\Big) -$$

$$- \Big(f(x) + 2hf'(x) + \frac{4h^2}{2}f''(x) + \frac{8h^3}{6}f'''(\xi_2)\Big)\Big] =$$

$$= f'(x) - \frac{h^2}{3}\left(f'''(\xi_1) - 2f'''(\xi_2)\right) \qquad \xi_1, \xi_2 \in [x, x+2h]$$

Погрешность метода:

$$r(x) = |f'(x) - g(x)| \le \frac{h^2}{3}\cdot|f'''(\xi_1) - 2f'''(\xi_2)| \le M_3 h^2, \text{ где}$$

$$M_3 = \sup_{y\in[x,x+2h]} f'''(y)$$

Погрешность округления:

$$|g(x) - \hat{g}(x)| = \frac{1}{2h}\left(|f(x)|\varepsilon + 4|f(x+h)|\varepsilon + |f(x+2h)|\varepsilon\right) \le \frac{h^2}{3}\cdot 4N\varepsilon, \text{ где}$$

ε – ошибка округ f, $N = \max\{f(x), f(x+h), f(x+2h)\}$

Условия на ф-ию f:

Наличие производных порядка 1, 2, 3 на $(x, x+2h)$

Задача №2

$$(xy^*)_{ij} = (x\cdot\bar{y}^T)_{ij} = \left({}^{i\to}x\right)\cdot\left(\overset{j\downarrow}{\bar{y}}\right)$$

$$\|xy^*\|_F = \Big(\sum_i\sum_j |(xy^*)_{ij}|^2\Big)^{\frac12} = \Big(\sum_i\sum_j (x_i\bar{y}_j)^2\Big)^{\frac12} = \underbrace{\sqrt{\sum_i |x_i|^2}}_{\|x\|_2}\cdot\underbrace{\sqrt{\sum_j |\bar{y}_j|^2}}_{\|\bar{y}\|_2 = \|y\|_2} \checkmark$$

$$\|xy^*\|_2 = \sup_{\|z\|_2=1}\|xy^*z\|_2 = \sup_{\|z\|_2=1}\sqrt{\sum_i\Big|\sum_j (xy^*)_{ij} z_j\Big|^2} =$$

$$= \sup_{\|z\|_2=1}\sqrt{\sum_i\Big|\sum_j x_i\bar{y}_j z_j\Big|^2} = \sup_{\|z\|_2=1}\sqrt{\sum_i |x_i|^2\cdot\Big|\sum_j \bar{y}_j z_j\Big|^2} = \|x\|_2\cdot\sup_{\|z\|_2=1}\sqrt{\Big|\sum_j\bar{y}_j z_j\Big|^2} =$$

$$= \|x\|_2\cdot\sup_{\|z\|_2=1}\sqrt{\Big|\sum_j y_j\bar{z}_j\Big|} = \|x\|_2\cdot\sup_{\|z\|_2=1}|\langle y, z\rangle| = \|x\|_2\cdot\|y\|_2$$

$$\Big|\sum_j\bar{y}_j z_j\Big|^2 = \Big|\overline{\sum_j \bar{y}_j z_j}\Big|^2 = \Big|\sum_j y_j\bar{z}_j\Big|^2$$

$$\sup_{\|z\|_2=1}|\langle y, z\rangle| \underset{\text{КБШ}}{=} \sqrt{\|y\|_2^2\cdot\|z\|_2^2} = \|y\|_2$$

при $z = \dfrac{y}{\|y\|_2}$ ✓

■

Задача №3

$(QAV)^*(QAV) = V^*A^*Q^*QAV = V^*(A^*A)V$, но V - унитарная $\Rightarrow$

$\Rightarrow$ столбцы V образуют базис в унит. пр-ве

Тогда V^*A^*AV есть оператор A^*A в новом базисе

Тогда $\underbrace{tr(A^*A)}_{\|A\|_F} = tr[(QAV)^*(QAV)]$ и $\underbrace{\lambda_{max}(A^*A)}_{\|A\|_2} = \lambda_{max}[(QAV)^*(QAV)]$

■

Задача №4

$$\frac{\|\Delta x\|_2}{\|x\|_2} = \frac{\|A^{-1}(b+\Delta b) - A^{-1}b\|_2}{\|x\|_2} = \frac{\|A^{-1}\Delta b\|_2}{\|x\|_2} \leq \frac{\|A^{-1}\|_2 \cdot \|\Delta b\|_2}{\|x\|_2} = \frac{\|Ax\|_2 \cdot \|A^{-1}\|_2 \cdot \|\Delta b\|_2}{\|Ax\|_2 \cdot \|x\|_2} \leq$$

$\|Ax\|_2 \leq \|A\|_2 \cdot \|x\|_2$

$$\leq \frac{\|A\|_2 \cdot \|A^{-1}\|_2 \cdot \|\Delta b\|_2}{\|Ax\|_2} = \frac{\|A\|_2 \cdot \|A^{-1}\|_2 \cdot \|\Delta b\|_2}{\|b\|_2}$$

$diag(A) = [1, 2, \dots, 3000] \Rightarrow$

$3000 = |a_{nn}| > 2\sum_{i \neq n} |a_{in}|$

$$\|b\|_2 \geq \frac{\|b\|_1}{\sqrt{3000}} = \frac{2}{10\sqrt{30}}$$

$$\|A\|_2 = |\lambda_{max}(A)| \leq |a_{nn}| + \sum_{i \neq n} |a_{in}| \leq 4500$$

$$\|\Delta b\|_2 \leq 10\sqrt{30} \cdot \varepsilon = 10^{-4} \cdot \sqrt{30}$$

$$\|A^{-1}\|_2 = \frac{1}{|\lambda_{min}(A)|} \leq \frac{1}{\underbrace{a_{11}}_{1} - \sum_{i \neq 1} a_{1i}} \leq \frac{1}{\frac{1}{2}} = 2$$

Итого, $\dfrac{\|\Delta x\|_2}{\|x\|_2} \leq \dfrac{4500 \cdot 2 \cdot 10^{-4} \cdot \sqrt{30}}{\frac{2}{10\sqrt{30}}} = 4500 \cdot 30 \cdot 10^{-3} = 135$

$\boxed{\text{Ответ: } 135}$

Задача N5

Метод Гаусса

$\vec{z_k}$ k строка матрицы Z

$$A = \begin{pmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \dots & a_{nn} \end{pmatrix} \to \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ 0 & a_{22} - a_{12}\cdot\frac{a_{21}}{a_{11}} & & \vdots \\ \vdots & & & \vdots \\ 0 & a_{n2} - a_{12}\cdot\frac{a_{n1}}{a_{11}} & & \vdots \end{pmatrix} \vec{b_2} = B$$

т.е. $\vec{b_1} = \vec{a_1}$

$i \neq 1 \quad \vec{b_i} = \vec{a_i} - \vec{a_1} \cdot \frac{a_{i1}}{a_{11}}$

и т.д. "срезаем" 2 столбец т.е. $\vec{b_2} = \vec{u_2}$

$$\sum_{j=3}^{n} |b_{2j}| \le \sum_{j=3}^{n} |a_{2j}| + \frac{|a_{21}|}{|a_{11}|} \cdot \sum_{j=3}^{n} |a_{2j}| \underset{\uparrow}{<} (|a_{22}| - |a_{21}|) + \frac{|a_{21}|}{|a_{11}|} \cdot (|a_{11}| - |a_{12}|) \le$$

строг. диаг. преобладание в A

$$\le \left|a_{22} - a_{21} \cdot \frac{a_{12}}{a_{11}}\right| = |b_{22}|$$ и т.д. $\Rightarrow$ B и строгое диаг. преобладание

$$\sum_{j=2}^{n} |b_{2j}| \le \sum_{j=2}^{n} |a_{2j}| + \frac{|a_{21}|}{|a_{11}|} \cdot |a_{11}| = \sum_{j=1}^{n} |a_{2j}|$$

Аналогично, $\sum_j |u_{ij}| \le \sum_j |a_{ij}|$ при $\forall i$

стр. диаг. преобл. в A

$$|u_{ij}| \le \sum_{k=1}^{n} |u_{ik}| \le \sum_k |a_{ik}| = |a_{ii}| + \sum_{i \neq k} |a_{ik}| \le 2|a_{ii}|$$

Итого, $\max_{i,j} |u_{ij}| \le 2 \cdot \max_{i,j} |a_{ij}|$ ■

Задача N6.

3-diag matrix

$$A = \begin{pmatrix} 4 & * & & & \\ * & 4 & * & & 0 \\ & * & \ddots & \ddots & \\ & & \ddots & \ddots & * \\ 0 & & & * & 4 \end{pmatrix} \qquad |*| \le 1$$

$\|x - x_0\|_\infty \le 10$

$$x^{(k+1)} = D^{-1}b - D^{-1}(L+U)x^{(k)} = \frac{b}{4} - \frac{(L+U)x^{(k)}}{4}$$

$$\|x - x^{(k+1)}\|_\infty = \left\| x - \frac{b}{4} - \frac{(L+U)x^{(k)}}{4} \right\|_\infty = \left\| x - \frac{b}{4} - \frac{(L+U)x^{(k)}}{4} + \frac{(L+U)x}{4} - \frac{(L+U)x}{4} \right\|_\infty =$$

$$= \left\| \frac{(L+U)}{4}(x - x^{(k)}) \right\|_\infty \le \frac{1}{2} \|x - x^{(k)}\|_\infty$$

$$\log_2\left(\frac{10}{10^{-6}}\right) = \log_2 10^7 = 7\log_2 10$$

$23 < 7\log_2 10 < 24$

$\boxed{\text{Ответ: 24 итерации}}$

Задача N 7

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$$

$$\lambda^2 - \frac{a_{12}a_{21}}{a_{11}a_{22}} = 0$$

Сходимость

$$|\lambda| = \sqrt{\left|\frac{a_{12}a_{21}}{a_{11}a_{22}}\right|} < 1 \Leftrightarrow |a_{12}a_{21}| < |a_{11}a_{22}|$$

Якоби $\quad S = -D^{-1}(L+U) = -\begin{pmatrix} a_{11}^{-1} & 0 \\ 0 & a_{22}^{-1} \end{pmatrix}\begin{pmatrix} 0 & a_{12} \\ a_{21} & 0 \end{pmatrix} = \begin{pmatrix} 0 & -\frac{a_{12}}{a_{11}} \\ -\frac{a_{21}}{a_{11}} & 0 \end{pmatrix}$

Гаусс - Зейдель $\quad S = (L+D)^{-1}U$

$$|(L+D)^{-1}U - \lambda E| = 0$$

$$|U - \lambda(L+D)| = 0$$

$$\begin{vmatrix} -\lambda a_{11} & a_{12} \\ -\lambda a_{21} & -\lambda a_{22} \end{vmatrix} = \lambda^2 a_{11}a_{22} + \lambda a_{12}a_{21} = 0$$

$$\lambda_1 = 0, \quad \lambda_2 = -\frac{a_{12}a_{21}}{a_{11}a_{22}}$$

Сходимость

$$|\lambda_2| = \left|\frac{a_{12}a_{21}}{a_{11}a_{22}}\right| < 1$$

Сходимость Якоби $\Leftrightarrow$ Сходимость Гаусса - Зейделя

Задача N8.

$$q^* = \frac{M-m}{M+m} = \frac{20-0.7}{20+0.7} \qquad x_0 = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}$$

$$\|x^{(k)} - x\|_2 \leq q^* \|x_0 - x\|_2 = q^* \|x\|_2$$

$$k \geq \frac{\log \frac{\|x^{(k)} - x\|_2}{\|x\|_2}}{\log q} \geq \frac{\log\left(\frac{10^{-6}}{\frac{25}{0.7}}\right)}{\log q^*}$$

$$\|b\|_2 \leq 25$$

$$Ax = b \Rightarrow x = A^{-1}b$$

$$\|x\|_2 \leq \|A^{-1}\|_2 \cdot \|b\|_2 \leq \frac{25}{0.7}$$

$$\|A^{-1}\| = \frac{1}{\lambda_{min}(A)} \qquad \lambda_{min}(A) = 0.7$$

$$k \geq \frac{\log \frac{7 \cdot 10^{-7}}{25}}{\log \frac{19.3}{20.7}} = \frac{\log 7 - 7\log 10 - \log 25}{\log 19.3 - \log 20.7} \in (248, 249)$$

Причём $\|\cdot\|_2 \geq \|\cdot\|_\infty \Rightarrow$ $\boxed{249 \text{ итераций}}$

Задача №9

$f(x) = e^x$ на $[0,1]$ интерполяция мн-ном 3 ст.

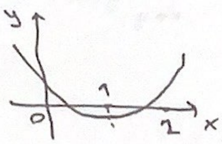
$T_4(t) = 0$ 4 чебыш узла

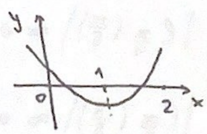
4 корня на $[-1,1]$: $\pm\cos\frac{\pi}{8}$, $\pm\cos\frac{3\pi}{8}$

$m: [-1,1] \mapsto [0,1]$ $m(t) = \frac{t+1}{2}$

c_1, c_2, c_3, c_4 новые узлы (на $[0,1]$) : $\frac{1\pm\cos\frac{\pi}{8}}{2}$, $\frac{1\pm\cos\frac{3\pi}{8}}{2}$

$$\left|\prod_{k=0}^{3}(x-c_k)\right| = \frac{1}{16}\cdot\left|(x-1)^2-\cos^2\frac{\pi}{8}\right|\cdot\left|(x-1)^2-\cos^2\frac{3\pi}{8}\right| \le \frac{1}{16}\cdot\sin^2\frac{\pi}{8}\cdot\sin^2\frac{3\pi}{8}$$





$$\sin\frac{\pi}{8}\cdot\sin\frac{3\pi}{8} = \frac{1}{2}\left(\cos\frac{\pi}{4}-\cos\frac{\pi}{2}\right) \qquad \cos\frac{\pi}{2}=0, \quad \cos\frac{\pi}{4}=\frac{\sqrt{2}}{2}$$

$$\cos\alpha = \frac{1+\cos(2\alpha)}{2}$$

$$\left(\frac{1}{2}\right)^2\left(\frac{\sqrt{2}}{2}\right)^2\cdot\frac{1}{16} = \frac{1}{2^7} = |w(x)|$$

$$|f(x)-L_3(x)| \le \frac{e \overset{\sup\limits_{[0,1]} e^x}{\leftarrow}}{4!}\cdot\frac{1}{2^7} = \frac{e}{2^{10}\cdot 3} < \frac{1}{2^{10}} < \frac{1}{10^3} \quad \blacksquare$$

Задача №10.

$f(x) = e^x$ $x_0 = 0$, $x_1 = 0.1$, $x_2 = 0.2$

$x_1' = 0.05$, $x_2' = 0.15$

$$f(x) - L_2(x) = \frac{\exp(\xi(x))}{(n+1)!}\,w(x) = \frac{\exp(\xi(x))}{6}\cdot x\cdot(x-0.1)(x-0.2) \qquad \xi(x)\in[0,0.2]$$

$$|f(x_1') - L_2(x_1')| = \left|\frac{\exp(\xi(x))}{6}\cdot 0.05^2\cdot 0.15\right| \le e^{0.2}\cdot\frac{0.05^2\cdot 0.15}{6} \approx 76\cdot 10^{-6}$$

$$|f(x_2') - L_2(x_2')| \le e^{0.2}\cdot\frac{0.15\cdot 0.05^2}{6} \approx 76\cdot 10^{-6}$$

$\boxed{\text{Ответ: } 76\cdot 10^{-6}}$

Задача №11.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\sin(x)$	0	0.5	0.71	0.87

Восстановить значение в $x = \frac{\pi}{5}$

L_n - интерпол. мн-н по точным знач., $\tilde{L}_n$ по апрокс. знач.

$$|L_n(x) - \tilde{L}_n(x)| \le \sum_{j=0}^{n} |f(x_j) - \tilde{f}(x_j)| \cdot |\ell_j(x)| \le \delta \cdot \sum_{j=1}^{n} \ell_j(x) \le$$

При $x = \frac{\pi}{5}$ $\qquad \ell_y(x) = \dfrac{\prod_{x' \ne y}(x - x')}{\prod_{x' \ne y}(y - x')}$

$|\ell_0(\frac{\pi}{5})| \approx 0.016$ $\qquad |\ell_{\frac{\pi}{4}}(\frac{\pi}{5})| \approx 0.512$

$|\ell_{\frac{\pi}{6}}(\frac{\pi}{5})| \approx 0.576$ $\qquad |\ell_{\frac{\pi}{3}}(\frac{\pi}{5})| \approx 0.072$

$$\le \delta(0.016 + 0.576 + 0.512 + 0.072) \approx 0.0118$$

$$|L(\tfrac{\pi}{5}) - f(\tfrac{\pi}{5})| = \frac{\sin(\xi(\frac{\pi}{5}))}{4!} |\tfrac{\pi}{5} - 0| \cdot |\tfrac{\pi}{5} - \tfrac{\pi}{6}| \cdot |\tfrac{\pi}{5} - \tfrac{\pi}{4}| \cdot |\tfrac{\pi}{5} - \tfrac{\pi}{3}| \le$$

$$\le \frac{1}{24} \cdot \frac{\pi}{5} \cdot \frac{\pi}{30} \cdot \frac{\pi}{20} \cdot \frac{2\pi}{15} \approx 0.00018$$

Итого, $\boxed{\text{ошибка} \approx 0.012}$

Задача №12

$$\int_0^1 (\sin x - (ax^2 + bx + c))dx \to \min$$

$$\int_0^1 [-2\sin(x)(ax^2 + bx + c) + (ax^2 + bx + c)^2]dx \to \min$$

$$\int_0^1 (ax^2 + bx + c)^2 dx = \frac{a^2}{5} + \frac{ab}{2} + \frac{2ac}{3} + \frac{b^2}{3} + cb + c$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \underbrace{\begin{pmatrix} (1,1) & (1,x) & (1,x^2) \\ (x,1) & (x,x) & (x,x^2) \\ (x^2,1) & (x^2,x) & (x^2,x^2) \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} (1,\sin x) \\ (x,\sin x) \\ (x^2,\sin x) \end{pmatrix}}_{b}$$

$$A = \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{pmatrix}$$

$$b = \begin{pmatrix} \int_0^1 \sin x\, dx \\ \int_0^1 x \sin x\, dx \\ \int_0^1 x^2 \sin x\, dx \end{pmatrix} = \begin{pmatrix} 1 - \cos 1 \\ \sin 1 - \cos 1 \\ 2\sin 1 + \cos 1 - 2 \end{pmatrix}$$

Ответ:

$$\boxed{\sin x = (1 - \cos 1) + (\sin 1 - \cos 1)x + (2\sin 1 + \cos 1 - 2)x^2}$$

Задача №13

Рассмотрим приближение $x+c$

$g_c(x) = x^{\frac13} - x - c \qquad g(0) = -c \qquad g(1) = -c$

$g'(x) = \frac13 x^{-\frac23} - 1 = 0 \Rightarrow \max$ в $x = \frac{1}{3\sqrt3}$

При $g(x^*) = c$ | g хотя бы 3 т. альт. $\Rightarrow$ $x+c$ лучшее приближение

$\boxed{\text{Ответ: } x + \frac{1}{3\sqrt3}}$

Задача №14

$$\left|f'_M(x_2) - f(x_2)\right| = \left| \frac{f(x_2-2h) - 6f(x_2-h) + 3f(x_2) + 2f(x_2+h)}{6h}(1+\delta) - f'(x_2)\right| \le$$

$$\le \Big| f(x_2)\underbrace{(1-6+3+2)}_{0} + \underbrace{\left[f'(x_2)\cdot(-2h+6h+2h) - f'(x_2)\right]}_{0} + f''(x_2)\cdot\frac12\underbrace{(4h^2-6h^2+2h^2)}_{0} +$$

$$+ f'''(x_2)\cdot\frac16\underbrace{(-8h^3+6h^3+2h^3)}_{0} + f^{(4)}(x_2)\cdot\frac{1}{24}(16h^4-6h^4+2h^4) + o(h^5)\Big|\cdot\frac{1}{6h} + \frac{M_0(1+6+3+2)\delta}{6h} =$$

$$= \left|f^{(4)}(x_2)\right|\cdot\frac{h^4}{12h} + 2\frac{M_0\delta}{h} \le \frac{M_4h^3}{12} + \frac{2M_0\delta}{h}$$

$$h_{opt}: \quad \frac{3M_4h_{opt}^2}{12} - \frac{2M_0\delta}{h_{opt}^2} = 0 \Rightarrow \boxed{h_{opt} = \sqrt[4]{\frac{8M_0\delta}{M_4}}} \qquad M_i := \sup f^{(i)}(x)$$

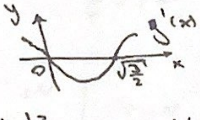
Задача №15

$$I = \int_0^1 e^{-x^2}dx \to f(x) = e^{-x^2} \qquad f'''(x) = 4x^2e^{-x^2} - 2e^{-x^2} = (4x^2-2)e^{-x^2}$$

$$f''(x) = 2xe^{-x^2}$$

$$M_2 = \sup_{[0,1]}\left|(-2xe^{-x^2})'\right| = \sup_{[0,1]}\Big|\overbrace{(4x^2-2)e^{-x^2}}^{g(x)}\Big|$$

$g'(x) = 4xe^{-x^2}\cdot(2x^2-3) = 0$ при $x=0$ на $[0,1]$
при $x = \sqrt{\frac32}$



$g(0) = -2 \qquad g(1) = \frac2e$

Итого, $M_2 = 2 \qquad |I(f) - S(I)| \le \frac1{12}M_2h^3 \Rightarrow N \ge \sqrt[3]{\frac{M_2}{12\varepsilon}}$

Тогда $N \ge \sqrt[3]{\frac{2}{12\cdot10^{-4}}} = \sqrt[3]{\frac{10^4}{6}} \approx 11.85$

$\boxed{\text{Ответ: } 12}$

Задача №17

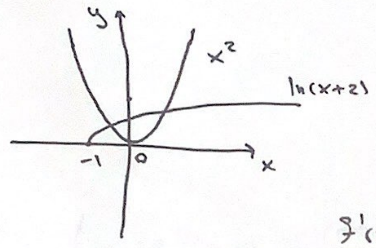
$x = e^{2x} - 1 \qquad x^{k+1} = \frac{\ln(x^k+1)}{2} = F(x^k) \Rightarrow F'(x) = \frac{1}{2(1+x)} < 1$ на $x \in [0, +\infty)$

Это сжим. отобр. на $[0, +\infty)$ с $q = \frac{1}{2}$

Скорость сходимости $|x^k - x| \leq \frac{q^k}{1-q}|x^0 - x| = \frac{1}{2^{k-1}}|x_0 - x|$

Задача №18

$f(x) = \ln(x+2) - x^2$ метод Ньютона для вычисления корня



При $x > 2$: $x^2 > 4$, $\ln(x+2) < \ln 4 < 4$

Смотрим на отрезок $x \in [-1, 2]$

$f'(x) = \frac{1}{x+2} - 2x \Rightarrow M_1 \leq \frac{15}{4}$

$f''(x) = \frac{-1}{(x+2)^2} - 2 \Rightarrow f''(x) < 0$ при $x > -2$, $f(-1) = \frac{1}{1} + 2 = 3$

$f(2) = \frac{1}{4} - 4 = -\frac{15}{4}$

$f''(x) \leq \frac{-1}{1^2} - 2 = -3$

$M_2 \leq 3$

$$x_{k+1} = x_k - \frac{\ln(x_k+2) - x_k^2}{\frac{1}{x+2} - 2x}$$

$$|\delta_{i+1}| = \frac{|f''(x_i)|}{2|f'(x_n)|} \cdot \delta_i^2$$

$$\gamma \leq \frac{M_2}{2M_1} = \frac{12}{30} = \frac{2}{5} = 0.4$$

$$10^{-6} \leq \gamma^{2^k - 1}$$

$$2^{k-1} \geq \frac{\ln 10^{-6}}{\ln \gamma} = \frac{-6 \cdot \ln 10}{\ln 0.4}$$

$$2^k \geq -\frac{12 \ln 10}{\ln 0.4} \approx 30.15$$

$\boxed{\text{Ответ: } 5}$

Задача №20

$$\begin{cases} y'' + 2xy' = 0 \\ y(0) = 0 \\ y'(0) = 0 \end{cases} \qquad u_1 = y, \quad u_2 = y' \qquad u = \begin{pmatrix} y \\ y' \end{pmatrix} \qquad \begin{cases} u_2' + 2xu_2 = 0 \\ u_1' = u_2 \end{cases}$$

$$u' = \begin{pmatrix} u_2 \\ -2xu_2 \end{pmatrix} = f(x, u)$$

Таблица Бутчера

$$\begin{array}{c|cc} 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ \hline & 0 & 1 \end{array}$$

$k_1 = -2t_n u_n$

$k_2 = f\left(\left(t_n + \frac{h}{2}, u_n + \frac{h}{2}k\right), t_n\right) = -2\left(t_n + \frac{h}{2}\right)\left(u_n - \frac{h}{2}t_n u_n\right) = 2u_n\left(t_n + \frac{h}{2}\right)(1 - t_n h)$

$u_{n+1} = u_n + hk_2 = u_n - 2hu_n(t+2)(1 - t_n h)$