Задача №1

$$Y = m(X) + \varepsilon\ , \quad X = (X_1, \dots, X_n)$$

$$Q(a,b) = \sum_{i=1}^{n} q_h(x)\,(X_i - x)\left[Y_i - a(x) - b(x)\cdot(X_i - x)\right]^2$$

$$\hat{m}(x) = \arg\min_{a(x),\, b(x)} Q(a,b)$$

$$\begin{cases} \dfrac{\partial Q}{\partial a} = 0 \\[2mm] \dfrac{\partial Q}{\partial b} = 0 \end{cases}$$

Далее, для краткости, $d_i := X_i - x$, $w_i := q_h(x)\cdot(X_i - x)$

$$\frac{\partial Q}{\partial a} = \left[\sum_{i=1}^{n} w_i\left(a^2 - 2Y_i a - 2Y_i b d_i + 2abd_i + Y_i^2 + b^2 d_i^2\right)\right]'_a =$$

$$= \sum_{i=1}^{n} w_i\left(\cancel{2}a - \cancel{2}Y_i + \cancel{2}bd_i\right) = 0 \quad \Rightarrow \quad \text{I} \quad a = \frac{\sum_{i=1}^{n} w_i(Y_i - bd_i)}{\sum_{i=1}^{n} w_i}$$

$$\frac{\partial Q}{\partial b} = \sum_{i=1}^{n} w_i\left(\cancel{2}bd_i^2 + \cancel{2}ad_i - \cancel{2}Y_i d_i\right) = 0 \quad \Rightarrow \quad b = \frac{\sum_{i=1}^{n} w_i d_i (Y_i - a)}{\sum_{i=1}^{n} w_i d_i^2}$$

$$\Downarrow$$

$$\text{II} \quad a = \frac{\sum_{i=1}^{n} w_i d_i (Y_i - bd_i)}{\sum_{i=1}^{n} w_i d_i}$$

$$a = \text{I} = \text{II}$$

$$\sum_{i=1}^{n} w_i d_i \cdot \sum_{i=1}^{n} w_i (Y_i - bd_i) = \sum_{i=1}^{n} w_i \cdot \sum_{i=1}^{n} w_i d_i (Y_i - bd_i)$$

$$\cancel{\sum_{i=1}^{n} w_i^2 d_i (Y_i - bd_i)} + \sum_{i \neq j} w_i w_j d_i (Y_j - bd_j) = \cancel{\sum_{i=1}^{n} w_i^2 d_i (Y_i - bd_i)} +$$

$$+ \sum_{i \neq j} w_i w_j d_j (Y_j - bd_j)$$

$$\sum_{i \neq j} w_i w_j \left(Y_j (d_i - d_j) - b(d_i d_j - d_j^2)\right) = 0$$

Ответ:

$$b = \frac{\sum_{i \neq j} w_i w_j Y_j (d_i - d_j)}{\sum_{i \neq j} w_i w_j d_j (d_i - d_j)}\ , \quad a = \frac{\sum_{i=1}^{n} w_i Y_i - \left(\sum_{i=1}^{n} w_i d_i\right)\cdot \underbrace{\dfrac{\sum_{i \neq j} w_i w_j Y_j (d_i - d_j)}{\sum_{i \neq j} w_i w_j d_j (d_i - d_j)}}_{b}}{\sum_{i=1}^{n} w_i}$$