If $f(x) = e^x$, then, integrating by parts the right-hand side, and putting $s = 1$, we have the result in the Question.

[For proofs of equation (A), and of the general case for n variables, see LIOUVILLE's *Journal de Mathématiques*, 2nd Series, Vol. IV., April 1859.]

QUESTIONS FOR SOLUTION.

6951. (By Prof. SYLVESTER, F.R.S.)—If F represents $x^3 - 3xy^2 + y^3$, where x, y are any positive or negative integers, prove that (1) every prime number divisor of F is 3, or else of the form $18i \pm 1$; (2) all the absolute integer values of F may be obtained by supposing x, y both positive and x less than y; (3) all the integer primes of the form $18i \pm 1$ up to 1009 inclusive (leaving out as doubtful the six primes 557, 593, 647, 773, 881, 991) are representable under the form F or $\frac{1}{3}$F.

[Prof. SYLVESTER finds the number of $18i - 1$ and $18i + 1$ primes not exceeding 1009 to be 42 and 39 respectively, all of the latter except one, viz. 991, being of the form F or $\frac{1}{3}$F; or, as it may be put more strikingly, the 38 primes of the form $18i + 1$ stopping at 991 are of the form F or $\frac{1}{3}$F, and he inquires whether the probability formula for the sun's continuing to rise would be applicable to calculate the probability that 991 is not an exception to the law.]

6952. (By Prof. TOWNSEND, F.R.S.)—Two systems of forces $\Sigma(F_1)$ and $\Sigma(F_2)$ being supposed to act in a common space; show that the complete locus of the entire system of points in the space for which their principal moments have similar or opposite directions consists of three right lines, one situated at infinity in the space, and all three lying in planes parallel to the two central axes of the systems.

6953. (By Prof. CAVALLIN, M.A.)—From the formula

$$\iint C^2 \, dp \, d\omega = 3\Omega^2$$

(WILLIAMSON's *Integral Calculus*, Art. 238), derive (1), by variation of contour, the formula $\int_0^{2\pi} p \, d\omega = L$; also (2) if, instead of passing from the given contour to a nearly situated equidistant contour ($PP' = \mu$ = constant), we go to a new contour defined by $PP' = \psi\mu$, where ψ is some function whatever of the coordinates of P, show that we get the more general formula $\int_0^{2\pi} p\,\psi\, d\omega = \int_0^{2\pi} \rho\,\psi\, d\omega$, provided that ψ be such that the derived contour also becomes convex, as is, in general, the case when $\psi = f(\rho)$; and (3), if we assume $\psi = \rho$, prove that $\int_0^{2\pi} p\,\rho\, d\omega = \int_0^{2\pi} \rho^2 \, d\omega$ = twice the area swept out by the radius of curvature.

[Many other theorems may be easily deduced from other integrals of Prof. CROFTON's respecting closed contours.]

6954. (By Prof. MINCHIN, M.A.)—1. For accelerations of any order there is in the uniplanar motion of a rigid body at each instant a centre or point of no acceleration of that order; and the accelerations of all particles are related, in magnitudes and directions, to this centre exactly as velocities and ordinary accelerations are related to the instantaneous centres of velocity and acceleration.

2. In the case in which two lines fixed in a lamina are guided through two fixed-space points, while the displacement of the body takes place with constant angular velocity, prove that the space and body acceleration centrodes are two circles.

6955. (By Prof. GENESE, M.A.)—If PSp be a focal chord of a conic, and PM, pm perpendiculars on the corresponding directrix; prove by Statics that parallels through P, p to Sm, SM respectively meet at the middle point of Mm.

6956. (By Prof. MALET, M.A.)—If A, B, C, H, &c., be the minors of the discriminant of the quadric of which the quadriplanar equation is

$$a\alpha^2 + b\beta^2 + c\gamma^2 + d\delta^2 + 2h\alpha\beta + 2g\alpha\gamma + 2f\beta\gamma + 2l\alpha\delta + 2m\beta\delta + 2n\gamma\delta = 0,$$

each taken with respect to the corresponding small letter, prove that the equations (1) of the quadric, (2) of the tangent line through $(\alpha', \beta', \gamma', \delta')$, (3) of the pair of tangent planes through the line joining the points $(\alpha', \beta', \gamma', \delta')$ and $(\alpha'', \beta'', \gamma'', \delta'')$, are given by equating to zero the respective determinants

$$\begin{vmatrix} A & H & G & L & \alpha \\ H & B & F & M & \beta \\ G & F & C & N & \gamma \\ L & M & N & D & \delta \\ \alpha & \beta & \gamma & \delta & 0 \end{vmatrix}, \qquad \begin{vmatrix} A & H & G & L & \alpha' & \alpha \\ H & B & F & M & \beta' & \beta \\ G & F & C & N & \gamma' & \gamma \\ L & M & N & D & \delta' & \delta \\ \alpha' & \beta' & \gamma' & \delta' & 0 & 0 \\ \alpha & \beta & \gamma & \delta & 0 & 0 \end{vmatrix},$$

$$\begin{vmatrix} A & H & G & L & \alpha' & \alpha'' & \alpha \\ H & B & F & M & \beta' & \beta'' & \beta \\ G & F & C & N & \gamma' & \gamma'' & \gamma \\ L & M & N & D & \delta' & \delta'' & \delta \\ \alpha' & \beta' & \gamma' & \delta' & 0 & 0 & 0 \\ \alpha'' & \beta'' & \gamma'' & \delta'' & 0 & 0 & 0 \\ \alpha & \beta & \gamma & \delta & 0 & 0 & 0 \end{vmatrix}.$$

6957. (By Prof. WOLSTENHOLME, M.A.)—Prove that, if n be positive and < 1,

$$\int_0^{\pi} \frac{\sin^2 nx}{\sin^2 (1-n)x}\, dx = n\int_0^{\pi} \frac{\sin^2 x}{\sin^2 (1-n)x}\, dx = \frac{n}{1-n}\int_0^{\pi} \frac{\sin(1+n)x}{\sin(1-n)x}\, dx,$$

$$\int_0^{\frac{\pi}{n}} \frac{\sin^2 nx}{\sin^2 (1-n)x}\, dx = n\int_0^{\frac{\pi}{n}} \frac{\sin^2 x}{\sin^2 (1-n)x}\, dx = \frac{n}{1-n}\int_0^{\frac{\pi}{n}} \frac{\sin(1+n)x}{\sin(1-n)x}\, dx,$$

[deduced by the PROPOSER only from geometrical considerations.]

6958. (By Prof. HADAMARD, M.A.)—Etant données deux circonférences O et O′ qui se coupent aux points A et B, on fait tourner autour du point A un angle de grandeur constante, dont les côtés coupent les circonférences en deux points M et N. On divide la droite MN dans un rapport donné $m : n$ par le point P, et on demande le lieu de ce point.

6959. (By the EDITOR.)—Find the average volume of all the spheres that can be drawn through three points within a given sphere, so as (1) to touch, (2) to lie wholly within, the given sphere.

6960. (By Dr. MCALISTER.)—Show from first principles that, if in any motion of a particle the tangential force be measured by the rate per second at which momentum is *increased*, the normal force will in the same units be measured by the rate per second at which momentum is *deflected*.

6961. (By T. MUIR, M.A., F.R.S.E.)—The Jacobian of the functions

$$\begin{array}{ll|ll} V_1 \equiv & \quad -ay - bz - cu - dv & V_4 \equiv cx + fy + hz & \quad -lv \\ V_2 \equiv ax & \quad -ez - fu - gv & V_5 \equiv dx + gy + kz + lu, & \\ V_3 \equiv bx + ey & \quad -hu - kv & & \end{array}$$

being a zero-axial skew determinant of odd order, vanishes; find the relation connecting the functions.

6962. (By J. J. WALKER, M.A.)—Show that the cosine of half the angle between tangents drawn from a point on an ellipse to an inner confocal ellipse, varies inversely as the diameter of the former parallel to its tangent at that point.

6963. (By the Rev. H. G. DAY, M.A.)—If three points be taken at random within a circle of radius c, prove that the area of the triangle whose corners are the random points is $\dfrac{35c^2}{96\pi}$.

6964. (By T. P. KIRKMAN, M.A., F.R.S.)—

When the Sultan of Borneo holds a review
Upon New Year's Day, the best of the splendour
Resides in a smoking cap, jewelled and new,
Which by olden law the Vizier must render.

The gift is a fez having twenty-four faces,
Counting the pentagon girding the head royal,
Silks in a rainbow; and gorgeous laces,
With diamonds ablaze on the lines polyedrial.

Forty-two provinces quake at his name;
Forty-two scutcheons blazon their fame.
Of shields pentagonal glitter a score,
Of heptagonal six, octagonal four,
And of heralding hexagons one dozen more.

In the twenty-four shields may be any of these,
With blazoning braudric as you please,
Aslant or askew,
If beaded true
They star all the angles in forty-four threes.

But pray, Say now, what is royalty more than a joke,
If it cannot be lord of the fashion in smoke?

All smoke, to be loyal and orthodox,
Must kiss a right rig of the knowledge-box,
With braiding after the Ring's.
Your smoke and your censers
Are bung ed pretence, sire.
With head-gear amiss, and those flaws in your frocks.
Who copies a cap not a three-year-old—swings;
The old, on big dummies, fill two palace wings.

Poor Vizier! he finds it a burden sore,
To con those numberless dummy-caps o'er,
In mortal fear and dread;
For thus the statute of caps is read:—
If any two, placed on their pentagon brimming,
And matched apart from the pranking and trimming,
Have shields in angles and order of limning
Alike o'er the base, off goes his head!

The last review is a horrible story:—
As side by side on a golden stand
Were mirror and cap, and the king was about
To unloose the storm of his thundering band
By donning that dazzle of glory—
Treason! they shout,
And the foe rushes out,
With a battered old fez in his hand.

As slow in the mirror its image turns,
Lo! lo! into fierce and fiercer flame
In the Sultan's sight the evidence burns,
That the braided shields in order and name
On the new cap and image are all the same.