01:14:18

Next Item

Item 1 **10 points**

Consider a linear model $\boldsymbol{Y} = B\boldsymbol{\beta} + \boldsymbol{\varepsilon}$ according to Definition I.4.2 with design matrix

$$B = \begin{pmatrix} \frac{1}{3} & 1 \\ \frac{2}{3} & b_{22} \\ \frac{2}{3} & b_{32} \end{pmatrix} \in \mathbb{R}^{3\times 2}$$

parameter vector $\boldsymbol{\beta} = (\beta_1, \beta_2)' \in \mathbb{R}^2$ and error term $\boldsymbol{\varepsilon}$ with $\mathrm{E}(\boldsymbol{\varepsilon}) = \mathbf{0}$ and $\mathrm{Cov}(\boldsymbol{\varepsilon}) = \sigma^2 I_3$, where $\sigma > 0$ is unknown.

For the following tasks, give your answers by filling the results into the blanks. If necessary, numerical values have to be rounded to three decimal places.

(a) Check if there exists $c \in \mathbb{R}$ with $b_{22} = c$ and $b_{32} = -c$, such that the corresponding least-squares estimator $\widehat{\boldsymbol{\beta}}$ of $\boldsymbol{\beta}$ is uniquely determined. **2 points**

2 points

If there exists a unique $c \in \mathbb{R}$, then give its value in the space below. If such an $c \in \mathbb{R}$ does not exist, then type "NA" (without quotation marks). If $\widehat{\boldsymbol{\beta}}$ is uniquely determined for all $c \in \mathbb{R}$, then type "R" (without quotation marks).

2

(b) Check if there exists $c \in \mathbb{R}$ with $b_{22} = b_{32} = c$, such that $\boldsymbol{\beta}$ is **not** identifiable. **2 points**

2 points

If there exists a unique $c \in \mathbb{R}$, then give its value in the space below. If such an $c \in \mathbb{R}$ does not exist, then type "NA" (without quotation marks). If $\boldsymbol{\beta}$ is identifiable for all $c \in \mathbb{R}$, then type "R" (without quotation marks).

2

For the following two parts (c) and (d), assume $b_{22} = 0$ and $b_{32} = 1$. Futher assume that the design matrix B possesses the following QR-decomposition:

$$B = \begin{pmatrix} \frac{1}{3} & 1 \\ \frac{2}{3} & 0 \\ \frac{2}{3} & 1 \end{pmatrix} = QR \quad \text{with} \quad Q = \frac{1}{3}\begin{pmatrix} 1 & 2 \\ 2 & -2 \\ 2 & 1 \end{pmatrix} \quad \text{and} \quad R = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

(c) Calculate the corresponding least-squares estimate $\widehat{\boldsymbol{\beta}} = (\widehat{\beta}_1, \widehat{\beta}_2)'$ of $\boldsymbol{\beta}$ for the observation $\boldsymbol{y} = (1, 1, 0)'$ of $\boldsymbol{Y}$. **3 points**

1.5 points

Give the value of $\widehat{\beta}_1$.

2

1.5 points

Give the value of $\widehat{\beta}_2$.

2

(d) For $\sigma = 1$, determine $\mathrm{Cov}(\widehat{\boldsymbol{\beta}}) = \begin{pmatrix} c_{11} & c_{12} \\ c_{12} & c_{22} \end{pmatrix}$. **3 points**

1 point

Give the value of c_{11}.

2

1 point

Give the value of c_{12}.

2

1 point

Give the value of c_{22}.

2

Next Item

All answers have been saved!