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Consider a linear model with design matrix

$$X = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & \frac{1}{\sqrt{3}} \\ 0 & -\frac{2}{\sqrt{3}} \\ -1 & \frac{1}{\sqrt{3}} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ 0 & -\frac{2}{\sqrt{6}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \end{pmatrix}$$

and parameter vector $\boldsymbol{\beta} = (\beta_1, \beta_2)'$. Denote the lasso and ridge regression estimators as solutions to the objective function

$$\min_{\beta \in \mathbb{R}^2} \left\{ \frac{1}{2} ||\boldsymbol{y} - X\boldsymbol{\beta}||^2 + \lambda ||\boldsymbol{\beta}||_q^q \right\}$$

for $q = 1$ or $q = 2$, respectively, as $\hat{\boldsymbol{\beta}}^{lasso}$ and $\hat{\boldsymbol{\beta}}^{ridge}$. Suppose we have observed

$$\boldsymbol{y} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}.$$

Hint: The following taks have to be solved theoretically (see related theoretical exercise). In particular, *do not* use `R`, since the objective function in `glmnet` - and thus the tuning paramater λ - are scaled differently than above.

Find the missing numerical values with a precision of two decimals. **3 of 5 points**

1 of 1 point

(a) Find the smallest value $\lambda_{\min}$, say, for the tuning parameter λ, such that *at least* one of the lasso estimates $\hat{\beta}_i^{lasso}$, $i \in \{1, 2\}$, equals exactly zero (with a precision of two decimal places).

$\lambda_{\min} =$

0.5 ✓

1 of 1 point

(b) Calculate the corresponding ridge regression estimates for the value of $\lambda_{\min}$ derived in (a) with a precision two decimal places.

(i)The corresponding ridge regression estimate of the first component is

$\hat{\beta}_1^{ridge} =$

0.33 ✓

1 of 1 point

(ii)The corresponding ridge regression estimate of the second component is

$\hat{\beta}_2^{ridge} =$

-0.58 ✓

1 point

(c) Suppose the underlying model is normal, that is

$$\boldsymbol{Y} = X\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

with $\boldsymbol{\varepsilon} \sim N_2(\mathbf{0}, 4 \cdot I_2)$, where I_2 denotes the 2-dimensional identity matrix.

Consider the mean squared error (MSE) of the ridge estimator of the first component β_1 of $\boldsymbol{\beta}$,

$$\mathrm{MSE}(\hat{\beta}_1^{ridge}) = \mathrm{bias}(\hat{\beta}_1^{ridge})^2 + \mathrm{Var}(\hat{\beta}_1^{ridge}).$$

Let $\lambda = 1$ and denote by

$$(\beta_1^{lower}, \beta_1^{upper})$$

the interval of values for β_1, for which

$$\mathrm{MSE}(\hat{\beta}_1^{ridge}) < \mathrm{Var}(\hat{\beta}_1^{LS}),$$

i.e., the MSE of $\hat{\beta}_1^{ridge}$ is smaller than the variance of the least squares estimator $\hat{\beta}_1^{LS}$ of β_1. Find this interval (with a precision of two decimal places).

Hint: Consult Exercise 27(a) and Theorem I.4.25 of the lecture to derive the distribution of the estimator $\hat{\beta}_1^{LS}$.

$\beta_1^{lower} =$

-3 ✗ -3.46

$\beta_1^{upper} =$

1 point

3 ✕ 3.46

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- Be polite and friendly.
- Describe your rationale as precisely as possible.

Dynexite, 30.07.2021

Request for correction of this item

Dear sir or madam, ...

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