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Item 1 **4 points**

Let $\mathbf{X} \sim \mathcal{N}(\boldsymbol{\mu}, \Sigma)$ be a normally distributed random vector with

$$\boldsymbol{\mu} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad \Sigma = \frac{1}{2}\begin{pmatrix} 13 & 5 \\ 5 & 13 \end{pmatrix} = \begin{pmatrix} \frac{13}{2} & \frac{5}{2} \\ \frac{5}{2} & \frac{13}{2} \end{pmatrix}.$$

The singular value decomposition (SVD) is given by $\Sigma = V \Lambda V^T$ where $V = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ and $\Lambda = \begin{pmatrix} 9 & 0 \\ 0 & 4 \end{pmatrix}$ (no proof required).

For the tasks below, please give the numerical results in the fill in blanks. 4 points

Determine the eigenvalues λ_1, λ_2 of the covariance matrix Σ where $\lambda_1 \geq \lambda_2$. 0.5 points

$\lambda_1 =$

$\lambda_2 =$ 0.5 points

Let the matrix $B \in \mathbb{R}^{1\times 2}$ be given by $B := (1 \quad 2)$ and define $Y = 1 + BX$. Then, the random variable $Y \sim \mathcal{N}(\nu, \sigma^2)$ is also normally distributed with 1 point

expectation $\nu =$

variance $\sigma^2 =$ 1 point

According to the assumptions, a matrix $A \in \mathbb{R}^{2\times 2}$ exists such that 0.5 points

$$\boldsymbol{Z} = A(\boldsymbol{X} - \boldsymbol{\mu}) \sim \mathcal{N}(0, I_2).$$

1 point

variance $\sigma^2 =$

0.5 points

According to the assumptions, a matrix $A \in \mathbb{R}^{2\times 2}$ exists such that

$$\boldsymbol{Z} = A(\boldsymbol{X} - \boldsymbol{\mu}) \sim N_2(\boldsymbol{0}, I_2),$$

where I_2 denotes the (2-dimensional) identity matrix in $\mathbb{R}^{2\times 2}$. Let $A = U\Delta U^T$ be the SVD of A . Determine the diagonal entries δ_{11}, δ_{22} of the diagonal matrix Δ with $\delta_{11} \geq \delta_{22}$ with a precision of two decimals.

$\delta_{11} =$

0.5 points

$\delta_{22} =$

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All answers have been saved!

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Item 2 **3.5 points**

Let $\boldsymbol{X} = (X_1, X_2)^T \sim N_2(\boldsymbol{\mu}, \Sigma)$, where

$$\boldsymbol{\mu} = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} = \begin{pmatrix} 11 \\ 10 \end{pmatrix} \quad \text{and} \quad \Sigma = \frac{1}{3}\begin{pmatrix} 13 & 4 \\ 4 & 13 \end{pmatrix} = \begin{pmatrix} \frac{13}{3} & \frac{4}{3} \\ \frac{4}{3} & \frac{13}{3} \end{pmatrix}.$$

For the tasks below, please give the numerical results in the fill in blanks. 3.5 points

Due to the properties of a multivariate normal distribution, the distribution of X_1 conditionally on $X_2 = 1$ can be expressed as $X_1|X_2 = 1 \sim N(\nu, \sigma^2)$. Determine the conditional expectation $\nu = E(X_1 \mid X_2 = 1)$ and the (conditional) variance $\sigma^2 = \text{Var}(X_1 \mid X_2 = 1)$ with a precision of two decimals.

1 point

$\nu =$

1 point

$\sigma^2 =$

Check, whether an $\nu_3 \in \mathbb{R}$ exists with $X_1|X_2 = \nu_2 \sim N_1(1, \nu^2)$. If such an $\nu_3 \in \mathbb{R}$ exists, then provide its value. If it does not exist, then type "NA" (without quotation marks) instead into the blank.

1 point

Check, whether an $\nu_1 \in \mathbb{R}$ exists with $X_2|X_1 = \nu_1 \sim N_1(\mathbb{R}, \mathbb{R})$. If such an $\nu_1 \in \mathbb{R}$ exists, then provide its value. If it does not exist, then type "NA" (without quotation marks) instead into the blank.

0.5 points

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All answers have been saved!

Item 3 **5.5 points**

Let $\boldsymbol{X} \sim N_2(\mu, I_2)$ for some $\mu \in \mathbb{R}^2$. Furthermore, define the matrices $A, B, C \in \mathbb{R}^{2\times 2}$ by

$$A = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & b \\ b & 1 \end{pmatrix}, \quad C = \begin{pmatrix} c_1 & c_2 \\ -1 & 1 \end{pmatrix}.$$

Give your answers to the tasks below by filling in the blanks. Results that are numerical values should, if necessary, be rounded to two decimals. In case of multiple solutions, please order your solutions from smallest to largest and separate them by " & " (without quotations marks but with spaces, e.g.: 3 & 5). If such a value **does not exist**, then type "NA" (without quotation marks) instead into the blank. If the value can be **chosen arbitrarily**, then type "R" (without quotation marks) instead into the blank. 5.5 points

1 point

Determine the value for $b \in \mathbb{R}$ such that $A\boldsymbol{X}$ and $B\boldsymbol{X}$ are independent.

0.5 points

Assume that $c = c_1 = -c_2 \in \mathbb{R}$. Determine the value for $c \in \mathbb{R}$ such that $A\boldsymbol{X}$ and $C\boldsymbol{X}$ are independent.

0.5 points

Assume that $c = c_1 = c_2 \in \mathbb{R}\setminus\{0\}$. Determine the value for $c \in \mathbb{R}\setminus\{0\}$ such that $A\boldsymbol{X}$ and $C\boldsymbol{X}$ are independent.

1 point

Assume that $c_1 = c$ and $c_2 = c^2 - 2$ for some $c \in \mathbb{R}$. Determine the values for $c \in \mathbb{R}$ such that $\boldsymbol{X}^T A\boldsymbol{X}$ and $C\boldsymbol{X}$ are independent.

1 point

Assume that $c_1 = c$ and $c_2 = c^2 - 2$ for some $c \in \mathbb{R}$. Determine the values for $c \in \mathbb{R}$ such that $\boldsymbol{X}^T A\boldsymbol{X}$ and $\boldsymbol{X}^T C\boldsymbol{X}$ are independent.

0.5 points

Assume that $\mu = 0 \in \mathbb{R}^2$. Find the value for $a \in \mathbb{R}\setminus\{0\}$ such that $a \cdot \boldsymbol{X}^T A\boldsymbol{X} \sim \chi^2(2)$, where $\chi^2(d)$ denotes the χ^2-distribution with $d \in \mathbb{N}$ degrees of freedom.

1 point

Assume that $c_1 = c$ and $c_2 = c^2 - 2$ for some $c \in \mathbb{R}$. Determine the values for $c \in \mathbb{R}$ such that $\boldsymbol{X}'\boldsymbol{A}\boldsymbol{X}$ and $\boldsymbol{X}'\boldsymbol{C}\boldsymbol{X}$ are independent.

0.5 points

Assume that $\boldsymbol{\mu} = \mathbf{0} \in \mathbb{R}^2$. Find the value for $a \in \mathbb{R}\backslash\{0\}$ such that $a \cdot \boldsymbol{X}'\boldsymbol{A}\boldsymbol{X} \sim \chi^2(2)$, where $\chi^2(d)$ denotes the χ^2-distribution with $d \in \mathbb{N}$ degrees of freedom.

1 point

Assume that $\boldsymbol{\mu} = \mathbf{0} \in \mathbb{R}^2$. Find the value for $a \in \mathbb{R}\backslash\{0\}$ such that $a \cdot \boldsymbol{X}'\boldsymbol{A}\boldsymbol{X} \sim \chi^2(1)$.

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Item 4 **3 points**

This task is based on **Task 7 of R-Lab 2**. The names of the data frames, variables etc. are the same as in the corresponding task and the corresponding solution in the RWTHmoodle space. **Please do not round your results. Notice that the decimal separator is "." (without quotation marks).** Remember that you have to transform `MeanScore` to *numeric* as in Task 7 (b). 3 points

What is the ratio of female participants in `data.survey.a`? 1 point

What is the mean value of `MeanScore` for the male participants in `data.survey.a`? 1 point

What is the variane of `MeanScore` for the male participants in `data.survey.a`? 1 point

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All answers have been saved!

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Item 5 **3 points**

This task is based on **Task 8 of R-Lab 2**. The names of the data frames, variables etc. are the same as in the corresponding task and the corresponding solution in the RWTHmoodle space. **Please do not round your results. Notice that the decimal separator is "." (without quotation marks).** 3 points

1.5 points

What is the value of the sum of `savings` (in the data frame `credits.data`) for the subsample of observations with `repayment =1` ?

1.5 points

What is the proportion of observations satisfying `age > 40`?

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Item 6 **3 points**

This task is based on **Task 10 of R-Lab 3**. The names of the data frames, variables etc. are the same as in the corresponding task and the corresponding solution in the RWTHmoodle space. **Please do not round your results. Notice that the decimal separator is "." (without quotation marks).** 3 points

1 point

For the linear model fitted in (d) (ii), what is the resulting prediction for $\texttt{Pmax} = 200$?

1 point

Compute the least squares estimates of the coefficients if you fit the linear model $Y = \beta_0 + \beta_1 X + \varepsilon$ only on modules from batch 3?

$\hat{\beta}_0$ (intercept)

1 point

$\hat{\beta}_1$

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Item 7 **3 points**

This task is based on **Task 11 of R-Lab 3**. The names of the data frames, variables etc. are the same as in the corresponding task and the corresponding solution in the RWTHmoodle space. **Please do not round your results. Notice that the decimal separator is "." (withhout quotation marks).** 3 points

What is the mean of the fitted values in (c) ? 1 point

What is the variance of the fitted values in (c)? 1 point

What is the variance of the residuals in (c)? 1 point

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All answers have been saved!