

Let $\mathbf{X} \sim N_2(\boldsymbol{\mu}, \Sigma)$ be a normally distributed random vector with

$$\mu = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad \Sigma = \frac{1}{2} \cdot \begin{pmatrix} 13 & 5 \\ 5 & 13 \end{pmatrix} = \begin{pmatrix} \frac{13}{2} & \frac{5}{2} \\ \frac{5}{2} & \frac{13}{2} \end{pmatrix}$$

The singular value decomposition (SVD) is given by $\Sigma = V\Lambda V'$ where $V = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ and $\Lambda = \begin{pmatrix} 9 & 0 \\ 0 & 4 \end{pmatrix}$ (no proof required).

For the tasks below, please give the numerical results in the fill in blanks.

3 von 4 Punkten

Determine the eigenvalues λ_1, λ_2 of the covariance matrix Σ where $\lambda_1 \geq \lambda_2$.

0.5 von 0.5 Punkten

$$\lambda_1 =$$

$$\lambda_2 =$$

0.5 von 0.5 Punkten

2

4 ✓

Let the matrix $B \in \mathbb{R}^{1 \times 2}$ be given by $B = \begin{pmatrix} 1 & 2 \end{pmatrix}$ and define $Y = 1 + B\mathbf{X}$. Then, the random variable $Y \sim N_1(\nu, \sigma^2)$ is also normally distributed with

1 von 1 Punkt

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414760 16:59:00

DYNEXITE

1 von 1 Punkt
Let the matrix $B \in \mathbb{R}^{1 \times 2}$ be given by $B = \begin{pmatrix} 1 & 2 \end{pmatrix}$ and define $Y = 1 + BX$. Then, the random variable $Y \sim N_1(\nu, \sigma^2)$ is also normally distributed with expectation $\nu =$

2 ✓

1 Punkt
variance $\sigma^2 =$

6.5 ✗ 42.5 🔑

0.5 von 0.5 Punkten
According to the assumptions, a matrix $A \in \mathbb{R}^{2 \times 2}$ exists such that

$Z = A(X - \mu) \sim N_2(\mathbf{0}, I_2),$

where I_2 denotes the (2-dimensional) identity matrix in $\mathbb{R}^{2 \times 2}$. Let $A = U\Delta U'$ be the SVD of A . Determine the diagonal entries δ_{11}, δ_{22} of the diagonal matrix Δ with $\delta_{11} \geq \delta_{22}$ with a precision of two decimals.

$\delta_{11} =$

0.5 ✓

$\delta_{22} =$

0.33 ✓

Nächste Aufgabe →

Nachkorrekturantrag anlegen?

ÜBERSICHT 1 2 3 4 5 6 7 EINSICHT BEENDEN

Vorherige Aufgabe ← Nächste → Aufgabe

Let $\mathbf{X} = (X_1, X_2)' \sim N_2(\boldsymbol{\mu}, \Sigma)$, where

$$\boldsymbol{\mu} = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad \Sigma = \frac{1}{2} \begin{pmatrix} 13 & 5 \\ 5 & 13 \end{pmatrix} = \begin{pmatrix} \frac{13}{2} & \frac{5}{2} \\ \frac{5}{2} & \frac{13}{2} \end{pmatrix}.$$

For the tasks below, please give the numerical results in the fill in blanks.

3.5 von 3.5 Punkten

1 von 1 Punkt

Due to the properties of a multivariate normal distribution, the distribution of X_1 conditionally on $X_2 = 1$ can be expressed as $X_1|X_2 = 1 \sim N_1(\nu, \sigma^2)$. Determine the conditional expectation $\nu = E(X_1 | X_2 = 1)$ and the (conditional) variance $\sigma^2 = \text{Var}(X_1 | X_2 = 1)$ with a precision of two decimals.

$\nu =$

1.38 ✓

1 von 1 Punkt

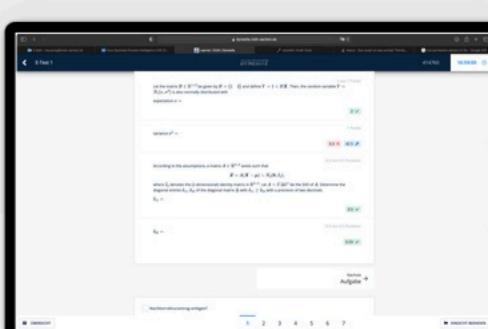
$\sigma^2 =$

5.54 ✓

1 von 1 Punkt

Check, whether an $x_2 \in \mathbb{R}$ exists with $X_1|X_2 = x_2 \sim N_1(1, \sigma^2)$. If such an $x_2 \in \mathbb{R}$ exists, then provide its value. If it does not exist, then type "NA" (without quotation marks) instead into the blank.

0 ✓



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DYNEXITE

1 von 1 Punkt

Check, whether an $x_2 \in \mathbb{R}$ exists with $X_1|X_2 = x_2 \sim N_1(1, \sigma^2)$. If such an $x_2 \in \mathbb{R}$ exists, then provide its value. If it does not exist, then type "NA" (without quotation marks) instead into the blank.

0 ✓

0.5 von 0.5 Punkten

Check, whether an $x_1 \in \mathbb{R}$ exists with $X_2|X_1 = x_1 \sim N_1(0, 16)$. If such an $x_1 \in \mathbb{R}$ exists, then provide its value. If it does not exist, then type "NA" (without quotation marks) instead into the blank.

NA ✓

Vorherige Aufgabe ←

Nächste Aufgabe →

Nachkorrekturantrag anlegen?

Bitte beachte, dass dieses Dokument zu einem Teil deiner Prüfungsakte wird!

⚠

- Sei höflich und freundlich.
- Beschreibe deine Begründung so präzise wie möglich.

Dynexite, 21.05.2021

Antrag auf Nachkorrektur dieser Aufgabe

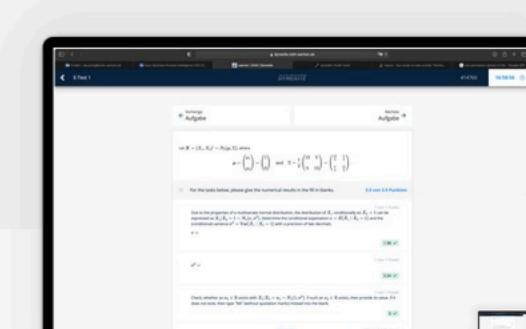
Sehr geehrte Damen und Herren, ...

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ÜBERSICHT

1 2 3 4 5 6 7

414760 16:58:52



Vorherige Aufgabe Nächste Aufgabe

Let $\mathbf{X} \sim N_2(\boldsymbol{\mu}, I_2)$ for some $\boldsymbol{\mu} \in \mathbb{R}^2$. Furthermore, define the matrices $A, B, C \in \mathbb{R}^{2 \times 2}$ by

$$A = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & b \\ b & 1 \end{pmatrix}, \quad C = \begin{pmatrix} c_1 & c_2 \\ -1 & 1 \end{pmatrix}.$$

Give your answers to the tasks below by filling in the blanks. Results that are numerical values should, if necessary, be rounded to two decimals. In case of multiple solutions, please order your solutions from smallest to largest and separate them by " & " (without quotations marks but with spaces, e.g.: 3 & 5). If such a value **does not exist**, then type "NA" (without quotation marks) instead into the blank. If the value can be **chosen arbitrarily**, then type "R" (without quotation marks) instead into the blank.

3.5 von 5.5 Punkten

1 von 1 Punkt

Determine the value for $b \in \mathbb{R}$ such that $A\mathbf{X}$ and $B\mathbf{X}$ are independent.

-1 ✓

0.5 Punkte

Assume that $c = c_1 = -c_2 \in \mathbb{R}$. Determine the value for $c \in \mathbb{R}$ such that $A\mathbf{X}$ and $C\mathbf{X}$ are independent.

1 ✗ R 🔑

0.5 von 0.5 Punkten

Assume that $c = c_1 = c_2 \in \mathbb{R} \setminus \{0\}$. Determine the value for $c \in \mathbb{R} \setminus \{0\}$ such that $A\mathbf{X}$ and $C\mathbf{X}$ are independent.

NA ✓

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Page header:

DYNEXITE

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Content area:

0.5 von 0.5 Punkten

Assume that $c = c_1 = c_2 \in \mathbb{R} \setminus \{0\}$. Determine the value for $c \in \mathbb{R} \setminus \{0\}$ such that $A\mathbf{X}$ and $C\mathbf{X}$ are independent.

NA ✓

1 von 1 Punkt

Assume that $c_1 = c$ and $c_2 = c^2 - 2$ for some $c \in \mathbb{R}$. Determine the values for $c \in \mathbb{R}$ such that $\mathbf{X}'A\mathbf{X}$ and $C\mathbf{X}$ are independent.

-2 & 1 ✓

1 von 1 Punkt

Assume that $c_1 = c$ and $c_2 = c^2 - 2$ for some $c \in \mathbb{R}$. Determine the values for $c \in \mathbb{R}$ such that $\mathbf{X}'A\mathbf{X}$ and $\mathbf{X}'C\mathbf{X}$ are independent.

1 ✓

0.5 Punkte

Assume that $\mu = \mathbf{0} \in \mathbb{R}^2$. Find the value for $a \in \mathbb{R} \setminus \{0\}$ such that $a \cdot \mathbf{X}'A\mathbf{X} \sim \chi^2(2)$, where $\chi^2(d)$ denotes the χ^2 -distribution with $d \in \mathbb{N}$ degrees of freedom.

1 ✗ NA 🔑

1 Punkt

Assume that $\mu = \mathbf{0} \in \mathbb{R}^2$. Find the value for $a \in \mathbb{R} \setminus \{0\}$ such that $a \cdot \mathbf{X}'A\mathbf{X} \sim \chi^2(1)$.

0 ✗ 0.25 🔑

Vorherige Aufgabe ←

Nächste Aufgabe →

Nachkorrekturantrag anlegen?

Progress bar: 1 2 3 4 5 6 7

Bottom navigation:

ÜBERSICHT

Small screenshot in the bottom right corner shows a mobile device displaying a dashboard or application interface.

This task is based on **Task 7 of R-Lab 2**. The names of the data frames, variables etc. are the same as in the corresponding task and the corresponding solution in the RWTHmoodle space. **Please do not round your results. Notice that the decimal separator is "," (without quotation marks).** Remember that you have to transform **MeanScore** to **numeric** as in Task 7 (b).

What is the ratio of female participants in **data.survey.a**? 0.69 ✓

What is the mean value of **MeanScore** for the male participants in **data.survey.a**? 68.31 ✓

What is the variance of **MeanScore** for the male participants in **data.survey.a**? 116.67 ✓

3 von 3 Punkten

1 von 1 Punkt

1 von 1 Punkt

1 von 1 Punkt

Vorherige Aufgabe ← Nächste Aufgabe →

ÜBERSICHT

1 2 3 4 5 6 7

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Vorherige
[← Aufgabe](#)

This task is based on **Task 11 of R-Lab 3**. The names of the data frames, variables etc. are the same as in the corresponding task and the corresponding solution in the RWTHmoodle space. **Please do not round your results. Notice that the decimal separator is "," (without quotation marks).** **0 von 3 Punkten**

What is the mean of the fitted values in (c)? **1 Punkt**
15.15 × 15.15442

What is the variance of the fitted values in (c)? **1 Punkt**
3.52 × 3.524991

What is the variance of the residuals in (c)? **1 Punkt**
1.27 × 1.271082

Vorherige
[← Aufgabe](#)

Nachkorrekturantrag anlegen?

1 2 3 4 5 6 7

Vorherige AufgabeNächste Aufgabe

This task is based on **Task 10 of R-Lab 3**. The names of the data frames, variables etc. are the same as in the corresponding task and the corresponding solution in the RWTHmoodle space. **Please do not round your results. Notice that the decimal separator is "," (without quotation marks).**

1 von 3 Punkten

For the linear model fitted in (d) (ii), what is the resulting prediction for $P_{max} = 200$?

11.92 11.92083

1 von 1 Punkt

Compute the least squares estimates of the coefficients if you fit the linear model $Y = \beta_0 + \beta_1 X + \varepsilon$ only on modules from batch 3?

$\hat{\beta}_0$ (intercept)

3.75

$\hat{\beta}_1$

0.04 0.04129

Vorherige AufgabeNächste Aufgabe

Vorherige Aufgabe ← Nächste Aufgabe →

This task is based on **Task 8 of R-Lab 2**. The names of the data frames, variables etc. are the same as in the corresponding task and the corresponding solution in the RWTHmoodle space. **Please do not round your results. Notice that the decimal separator is "," (without quotation marks).**

3 von 3 Punkten

1.5 von 1.5 Punkten

What is the value of the sum of **savings** (in the data frame **credits.data**) for the subsample of observations with **repayment =1** ?

1603 ✓

1.5 von 1.5 Punkten

What is the proportion of observations satisfying **age > 40**?

0.27 ✓

Vorherige Aufgabe ← Nächste Aufgabe →

Nachkorrekturantrag anlegen?

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- Sei höflich und freundlich.
- Beschreibe deine Begründung so präzise wie möglich.

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What is the ratio of female participants in **data.survey.a**? 0.69 ✓

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What is the variance of **MeanScore** for the male participants in **data.survey.a**? 116.67 ✓

3 von 3 Punkten

1 von 1 Punkt

1 von 1 Punkt

1 von 1 Punkt

DYNEXITE

Vorherige Aufgabe ← Nächste Aufgabe →

ÜBERSICHT

1 2 3 4 5 6 7

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$$A = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} = 2 \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

$$X \sim N_2(0, I_2), X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, x_i \sim N(0, 1)$$

$$\alpha X' A X = \alpha (x_1, x_2) \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 2\alpha (x_1(x_1+x_2) + x_2(x_1+x_2)) \\ = 2\alpha (x_1^2 + 2x_1x_2 + x_2^2) = 2\alpha (x_1 + x_2)^2$$

I.2.13 $\boxed{x \sim N_p(\mu, \Sigma) \Rightarrow \sqrt{n} \frac{\bar{x} - \mu}{\sigma} \sim N(0, 1)}$

$$\bar{x} = \frac{1}{n} \sum_j x_j$$

$$= 2\alpha (x_1 + x_2)^2 = 2\alpha \cdot 4 ((x_1 + x_2) \frac{1}{2})^2 = 2\alpha \cdot 4 \bar{x}^2 = 2\alpha \cdot 4 \underbrace{\left(\frac{\bar{x}}{1} \right)^2}_{4 \cdot \alpha \left(\frac{\bar{x}}{1} \right)^2} = 4 \cdot \alpha \sum_{i=1}^4 \left(\frac{\bar{x}_i}{1} \right)^2 \sim N(0, 1)$$

$$\alpha = 4 \sum_{i=1}^4 \left(\frac{\bar{x}_i}{1} \right)^2 \sim \chi^2(4)$$

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E-Test 1 414760 16:58:43

DYNEXITE

Assume that $c = c_1 = c_2 \in \mathbb{R} \setminus \{0\}$. Determine the value for $c \in \mathbb{R} \setminus \{0\}$ such that $\mathbf{A}\mathbf{X}$ and $\mathbf{C}\mathbf{X}$ are independent.

NA ✓

Assume that $c_1 = c$ and $c_2 = c^2 - 2$ for some $c \in \mathbb{R}$. Determine the values for $c \in \mathbb{R}$ such that $\mathbf{X}'\mathbf{A}\mathbf{X}$ and $\mathbf{C}\mathbf{X}$ are independent.

-2 & 1 ✓

Assume that $c_1 = c$ and $c_2 = c^2 - 2$ for some $c \in \mathbb{R}$. Determine the values for $c \in \mathbb{R}$ such that $\mathbf{X}'\mathbf{A}\mathbf{X}$ and $\mathbf{X}'\mathbf{C}\mathbf{X}$ are independent.

1 ✓

Assume that $\boldsymbol{\mu} = \mathbf{0} \in \mathbb{R}^2$. Find the value for $a \in \mathbb{R} \setminus \{0\}$ such that $a \cdot \mathbf{X}'\mathbf{A}\mathbf{X} \sim \chi^2(2)$, where $\chi^2(d)$ denotes the χ^2 -distribution with $d \in \mathbb{N}$ degrees of freedom.

1 ✗ NA ↕

Assume that $\boldsymbol{\mu} = \mathbf{0} \in \mathbb{R}^2$. Find the value for $a \in \mathbb{R} \setminus \{0\}$ such that $a \cdot \mathbf{X}'\mathbf{A}\mathbf{X} \sim \chi^2(1)$.

0 ✗ 0.25 ↕

← Vorherige Aufgabe Nächste Aufgabe →

Nachkorrekturantrag anlegen?

1	2	3	4	5	6	7
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Vorherige Aufgabe
Nächste Aufgabe

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3.5 von 5.5 Punkten

1 von 1 Punkt

Determine the value for $b \in \mathbb{R}$ such that $A\mathbf{X}$ and $B\mathbf{X}$ are independent.

-1 ✓

0.5 Punkte

Assume that $c = c_1 = -c_2 \in \mathbb{R}$. Determine the value for $c \in \mathbb{R}$ such that $A\mathbf{X}$ and $C\mathbf{X}$ are independent.

1 ✗ R ↕

0.5 von 0.5 Punkten

Assume that $c = c_1 = c_2 \in \mathbb{R} \setminus \{0\}$. Determine the value for $c \in \mathbb{R} \setminus \{0\}$ such that $A\mathbf{X}$ and $C\mathbf{X}$ are independent.

NA ✓

1 2 3 4 5 6 7

ÜBERSICHT
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