Let a family of distributions be given by their pdfs (probability density functions) defined for $\alpha > 0, \gamma > 0$ as

$$f(x;\alpha,\gamma) = \sqrt{\frac{\gamma}{2\pi x^2}} \exp\left(-\frac{\gamma(x-\alpha)^2}{2\alpha^2 x}\right), \quad x > 0. \qquad (1)$$

For fixed (known) $\gamma > 0$, $f_\gamma(x;\alpha) = f(x;\alpha,\gamma)$ defines a subfamily of the exponential dispersion family (EDF) of distributions with

$$c(x,\phi) = \frac{1}{2}\left(\ln(\gamma) - \ln(2\pi x^2) - \frac{\gamma}{x}\right).$$

Find the missing numerical values. **For all numerical results the exact values have to be provided without any rounding.** 3 of 6 points

1 of 1 point

(a) Let $\gamma = 2$ and $X \sim f_2(\,\cdot\,;\alpha)$.

(i) Determine the values of the natural parameter θ and the dispersion $a(\phi)$, when $\alpha = 1$.

$\theta =$

-0.5 ✓

1 of 1 point

$a(\phi) =$

0.5 ✓

1 point

(ii) Calculate the expectation $\mathrm{E}(X)$, when $\alpha = 1$.

$\mathrm{E}(X) =$

-1 ✗ 1

1 of 1 point

(iii) Calculate the variance $\mathrm{Var}(X)$, when $\alpha = 1$.

$\mathrm{Var}(X) =$

0.5 ✓

1 point

(b) Further assume that Y is a binary response variable and let

$$\pi(x) = P(Y = 1 \mid X = x).$$

Suppose that

$$(X|Y = j) \sim f(\,\cdot\,|\alpha_j, \gamma_j),$$

that is, conditionally on $Y = j$ the explanatory variable X has pdf (1) with parameters $\alpha_j, \gamma_j > 0$, $j \in \{0, 1\}$, and consider the model

$$\mathrm{logit}(\pi(x)) = \log\left(\frac{\pi(x)}{1-\pi(x)}\right) = \beta_0 + \beta_1 x^{-1} + \beta_2 x.$$

Assume that $\alpha_0 = \gamma_0 = 2$ and $\alpha_1 = \gamma_1 = 1$. Calculate the values of β_1 and β_2.

Hint: Use Bayes' Theorem applied to probability distributions: For random variables X_1, X_2 with pdfs or pmfs f^{X_1}, f^{X_2}, respectively, it holds

$$f^{X_1|X_2=x_2}(x_1) = \frac{f^{X_2|X_1=x_1}(x_2) f^{X_1}(x_1)}{f^{X_2}(x_2)} I_{\mathrm{supp}(X_2)}(x_2), \quad x_1 \in \mathbb{R},$$

where $f^{X_i|X_j}, i \neq j, i, j \in \{1, 2\}$ is the conditional probability density or mass function of X_i given X_j and I_A is the indicator function on a set A.

$\beta_1 =$

--- ✗ -0.5

1 point

$\beta_2 =$

--- ✗ 0.25

Next
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