

Mathematical methods of signal and image processing

Winter semester 2021/2022

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Exercise sheet 1

Due: 29. October 2021

General information

- Current information will be announced in RWTHmoodle.
- The due date only indicates in which exercise session the solution will be discussed.
- Office hours: By arrangement via Zoom.

Problem 1 (Properties of the histogram)

Let $f \in L^2(\Omega, [0, 1]) := \{f : \Omega \rightarrow [0, 1] : f \in L^2(\Omega)\}$. Moreover, let G_f be the cumulative distribution function of f , i.e.

$$G_f(s) = \text{Vol}(\{f \leq s\})$$

and H_f the histogram of f , i.e.

$$H_f : \mathcal{B}(\mathbb{R}) \rightarrow \mathbb{R}, A \mapsto \text{Vol}(\{f \in A\}).$$

Prove the following statements:

1. $G_f : \mathbb{R} \rightarrow [0, \text{Vol}(\Omega)]$ is monotonically increasing and $G_f(s) = 0$ for $s < \text{ess inf } f$, as well as $G_f(s) = \text{Vol}(\Omega)$ for $s > \text{ess sup } f$.
2. $G'_f = H_f$ holds in the sense of distributions, i.e. one has to prove

$$-\int_{\mathbb{R}} G_f(s) \psi'(s) \, ds = \int_{\mathbb{R}} \psi(s) dH_f(s) \text{ for all } \psi \in C_c^\infty(\mathbb{R}).$$

Hints: Exploit that the histogram is the push-forward measure (or image measure) of the Lebesgue measure under f , i.e.

$$\int_{\mathbb{R}} \psi(s) dH_f(s) = \int_{\Omega} (\psi \circ f)(x) \, dx.$$

Problem 2

Let $\Omega \subset \mathbb{R}^d$, $x \in \Omega$ and δ_x the Dirac measure at x . Moreover, let $f \in C(\Omega)$. Show that

$$\int_{\Omega} f d\delta_x = f(x)$$

using the definition of the Lebesgue integral on a general measure space based on step functions.

Problem 3 (Histogram equalization)

Implement histogram equalization for grayscale images. Do not use any MATLAB or Python histogram functions, but implement an algorithm that computes the histogram using binning. Test your implementation with the following images:

<http://www.ece.rice.edu/~wakin/images/lenaTest3.jpg>

<http://www.engineering.uiowa.edu/~dip/homework/peppers.png>

http://www.mathworks.com/matlabcentral/answers/uploaded_files/741/cameraman.jpg