Mathematical methods of signal and image processing

Winter semester 2021/2022

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Exercise sheet 3 **Due: 12. November 2021**

General information

- Current information will be announced in RWTHmoodle.
- The due date only indicates in which exercise session the solution will be discussed.
- Office hours: By arrangement via Zoom.

Problem 1 (Rotational invariance of the Laplace operators)

Let $f \in C^2(\mathbb{R}^d)$ and $A \in O(d) := \left\{A \in \mathbb{R}^{d\times d} \,:\, A^T A = \mathbb{1}\right\}$. Show that

$$(\Delta f) \circ A = \Delta(f \circ A).$$

Problem 2 (The median minimizes the sum of absolute deviations)

Given $a_1, \ldots, a_n \in \mathbb{R}$, show that their median is a solution of the minimization problem

$$\min_{a\in\mathbb{R}} \sum_{i=1}^{n} |a - a_i|.$$

Problem 3 (Denoising filters)

Implement the filters from Remark 2.11 (Mean value filter, Gaussian filter, binomial filter, duto blur and median filter) and test the filters on the images from the first exercise sheet. Implement the discrete cross-correlation both by just using elementary functions and by using MATLAB's `xcorr2` or `scipy.signal.correlate2d`.