Mathematical methods of signal and image processing

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Presence exercise sheet 4

Problem 1

Let $\psi \in L^1(\mathbb{R}^d)$ such that $\psi \geq 0$ and $\int_{\mathbb{R}^d} \psi(x)\,\mathrm{d}x = 1$. Moreover, let $f \in L^\infty(\mathbb{R}^d)$ be continuous in $x_0 \in \mathbb{R}^d$ and $\psi_\epsilon(x) := \frac{1}{\epsilon^d}\psi(\frac{x}{\epsilon})$ for $\epsilon > 0$ and $x \in \mathbb{R}^d$. Then,

$$\lim_{\epsilon \to 0} (\psi_\epsilon \star f)(x_0) = f(x_0).$$

Hint: First show that

$$\lim_{\epsilon \to 0} \int_{\mathbb{R}^d \setminus B_R(0)} \psi_\epsilon(x)\,\mathrm{d}x = 0$$

and then continue as in the proof of $\lim_{t \to 0} (f * g_{\sqrt{2t}})(x) = f(x)$.

Problem 2

Implement the Bilateral filter from Remark 2.11 (vi) and test your implementation on the images from the first exercise sheet. Use the Gaussian kernel for both weight functions ψ_1 and ψ_2, and experiment with different filter widths σ for both Gaussian kernels. Compare the results with those of the normal Gaussian filter.