Exercise sheet 5   **Due: 26. November 2021**

General information

- Current information will be announced in RWTHmoodle.

- The due date only indicates in which exercise session the solution will be discussed.

- Office hours: By arrangement via Zoom.

Problem 1 (Properties of the morphological operators)

Let $f, g : \mathbb{R}^d \to [0, 1]$ and $B \subset \mathbb{R}^d$ nonempty. Show the following properties of the morphological operators $\circ$ and $\bullet$ (cf. Remark 2.24):

$$
\begin{aligned}
\text{monotonicity} \quad & f \leq g \Rightarrow f \circ B \leq g \circ B \wedge f \bullet B \leq g \bullet B \\
\text{non-increasingness} \quad & f \circ B \leq f \\
\text{non-decreasingness} \quad & f \leq f \bullet B \\
\text{idempotence} \quad & (f \circ B) \circ B = f \circ B \\
& (f \bullet B) \bullet B = f \bullet B.
\end{aligned}
$$

Problem 2

Let X and Y be normed vector spaces, $T : X \to Y$ linear and $x_0 \in X$. Show that the following statements are equivalent:

1. T is continuous.

2. T is continuous in x_0.

3. $\displaystyle\sup_{\|x\|_X \leq 1} \|Tx\|_Y < \infty$

4. T is bounded, i. e. there exists a constant C with $\|Tx\|_Y \leq C \|x\|_X$ for all $x \in X$.

Hint: Recall

$$T : X \to Y \text{ continuous in } x_0 \Leftrightarrow \forall \epsilon > 0 \, \exists \delta > 0 : (\|x - x_0\|_X < \delta \Rightarrow \|Tx - Tx_0\|_Y < \epsilon).$$

Problem 3

Implement erosion $\ominus$ and dilation $\oplus$ of images with a square as structuring element. Test your implementations with squares of different sizes on the images from the first exercise sheet. Moreover, use erosion and dilation to implement opening $\circ$ and closing $\bullet$.

Extend the isodata algorithm (cf. Example 1.13 and Attendance Sheet 2) with a pre-processing step that does a background equalization of the input image as described in Example 2.23. Compare the segmentations of the image `tafel.png` (file available in RWTHmoodle) obtained with and without the background equalization pre-processing step.