Mathematical methods of signal and image processing

Winter semester 2021/2022

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Exercise sheet 6 **Due: 3. December 2021**

General information

- Current information will be announced in RWTHmoodle.
- The due date only indicates in which exercise session the solution will be discussed.
- Office hours: By arrangement via Zoom.

Problem 1

Let

$$\operatorname{sinc} : \mathbb{R} \to \mathbb{R}, x \mapsto \begin{cases} \frac{\sin(\pi x)}{\pi x} & x \neq 0 \\ 1 & x = 0 \end{cases}.$$

Show that

$$(\mathcal{F}\chi_{[-B,B]})(\omega) = \sqrt{\frac{2}{\pi}} B \operatorname{sinc}\left(\tfrac{B\omega}{\pi}\right).$$

Problem 2

Compute the Fourier transform of the triangle function

$$\Lambda : \mathbb{R} \to \mathbb{R}, x \mapsto \begin{cases} 1 - |x| & |x| \leq 1 \\ 0 & \text{else} \end{cases}.$$

Hint: While the Fourier transform of Λ can be computed directly, it is easier and more elegant to express Λ as convolution of two known functions and then to use the convolution theorem.

Problem 3

Show that

$g_\sigma \in \mathcal{S}(\mathbb{R}^d, \mathbb{C})$ for all $\sigma > 0$.