Mathematical methods of signal and image processing
Winter semester 2021/2022
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Presence exercise sheet 3

Problem 1 (Non-interchangeability of limits)

Find a bounded, continuous function

$$f : (0, \infty) \times (0, \infty) \to \mathbb{R}, (x, y) \mapsto f(x, y)$$

such that

- $\lim\limits_{x\to\infty} f(x, y)$ exists and is finite for all $y \in (0, \infty)$
- $\lim\limits_{y\to\infty} f(x, y)$ exists and is finite for all $x \in (0, \infty)$
- $\lim\limits_{x\to\infty} \lim\limits_{y\to\infty} f(x, y)$ and $\lim\limits_{y\to\infty} \lim\limits_{x\to\infty} f(x, y)$ exist
- $\lim\limits_{x\to\infty} \lim\limits_{y\to\infty} f(x, y) \neq \lim\limits_{y\to\infty} \lim\limits_{x\to\infty} f(x, y)$

Problem 2 (Derivative filters)

Show that the finite difference operators approximate the derivative, i.e.

$$\frac{f(x+h) - f(x)}{h} = f'(x) + \mathcal{O}(h) \quad \text{and} \quad \frac{f(x) - f(x-h)}{h} = f'(x) + \mathcal{O}(h)$$

for a sufficiently smooth $f : \mathbb{R} \to \mathbb{R}$. This is the 1D analytical form of $D^+_{x_1}$ and $D^+_{x_1}$ (Remark 2.12), respectively.

Proceed in the same way for $D^c_{x_1}$ and $D^2_{x_1}$. (Hint: Make use of the Taylor's theorem.)