ELP311
Communication Engineering Laboratory

Experiment 3
Modelling of FM signal using MATLAB

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Objective

The experiment aims to achieve the following goals:

[1] Create an FM signal by modulating a waveform onto a carrier,
[2] Examine the spectrum of the modulated signal, and
[3] Evaluate the modulated carrier when the modulation index is varied.

Theory

Frequency modulation is a non-linear modulation.

Consider a message signal, $m(t)$ and carrier signal, $s(t) = A\cos(2\pi f_c t + \theta)$. The corresponding frequency modulated signal is given by:

$$s_{FM}(t) = A\cos[\varphi_{FM}(t)] = A\cos\left(2\pi f_c t + k_f\int_0^t m(\tau)d\tau + \theta\right)$$

where the instantaneous frequency of the FM signal is given as $\frac{d\varphi_{FM}(t)}{dt} = 2\pi f_c + k_f m(t)$. The frequency of the signal changes in accordance with the modulating signal $m(t)$

Let $m(t) = a\cos(2\pi f_m t)$. Therefore,

$$s_{FM}(t) = A\cos\left(2\pi f_c t + k_f\int_0^t m(\tau)d\tau + \theta\right) = A\cos(2\pi f_c t + \beta\sin(2\pi f_m t) + \theta)$$

The modulation index β is given as

$$\beta = \frac{k_f a}{2\pi f_m} = \frac{\Delta\omega}{\omega_m}$$

For $m(t) = a\cos(2\pi f_m t)$, the FM signal can be represented as a summation of Bessel functions

$$s_{FM}(t) = A\sum_{n=-\infty}^{\infty} J_n(\beta)\cos[2\pi(f_c + nf_m)t + \theta]$$

where $J_n(.)$ is the nth order Bessel function. In this case, the spectrum of the FM signal consists of an infinite sum of impulses. The bandwidth of the above signal for $n > \beta \Longrightarrow J_n(\beta) \approx 0$ is given by the Carson's rule.

$$BW = 2(\beta + 1)f_m$$

MATLAB code

We perform the experiment using $\beta = 0.01, 1.0, 2.4, 10, 50$, and plot the modulated signal, and their frequency spectra. We use the following formula for calculation of β:

$$\beta = \frac{k_f A_m}{\omega_m}$$

We consider ω_m to be the fundamental frequency of the message signal wave. To evaluate the modulated signals, in one case we vary $k_f \left(= {}^{\beta\omega_m}/_{A_m}\right)$ (ω_m is constant) and in the other case we vary $\omega_m \left(= {}^{k_f A_m}/_{\beta}\right)$ ($\Delta\omega$ is constant).

When ω_m is constant, we take $f_m = 1000\ Hz$ and when $\Delta\omega$ is constant, we take $k_f = 5000$

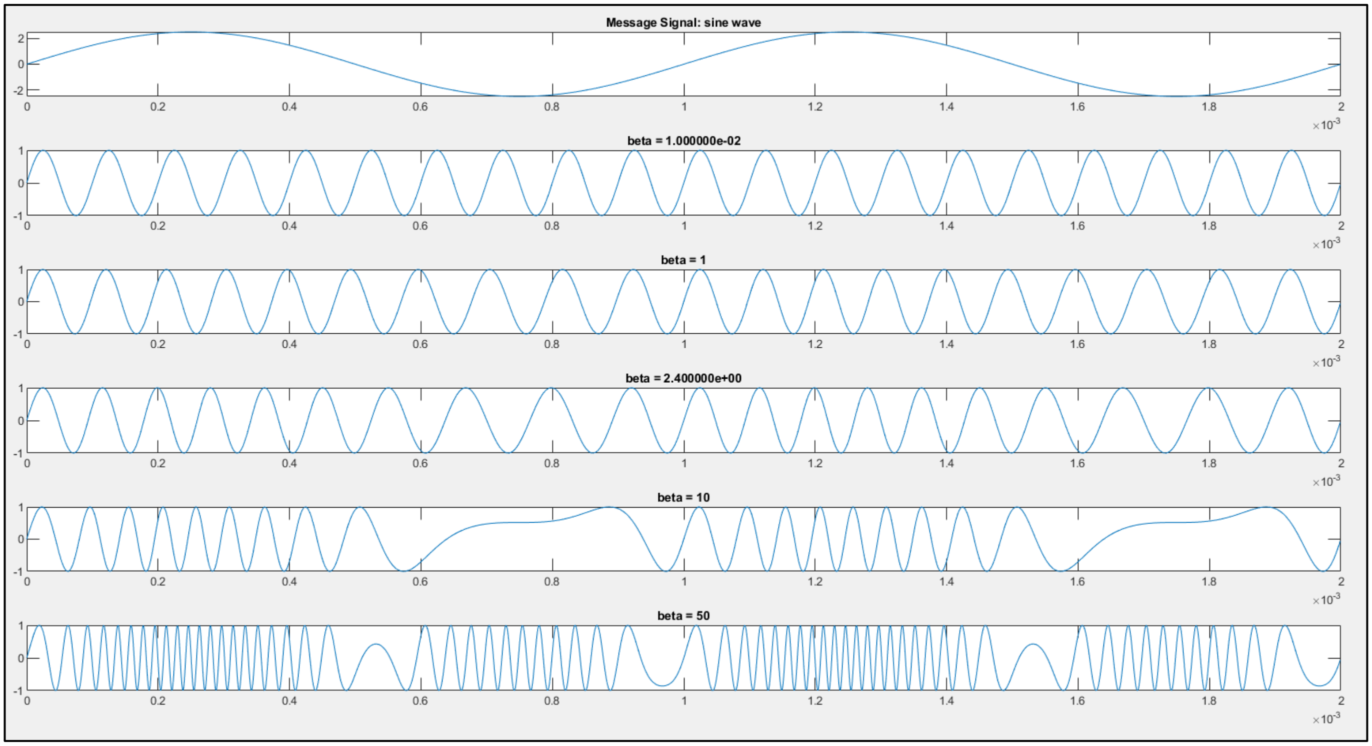
```
%%
% Modulating frequency (fixing fm for the case when Wm is constant)
fm=1000;

% Frequency sensitivity (fixing kf for the case when delW is constant)
kf = 5000;

% Carrier frequency
fc=10000;

% Sampling frequency
fs=100*fc;

% Time vector
t=0:1/fs:(0.002-1/fs);

% Amplitude of modulating signal
Am=2.5;

beta = [0.01, 1, 2.4, 10, 50];

%% Case 1: When Wm is constant, delW is varying
% using fundamental frequency for kf calculation
kfvar = 2*pi*fm*beta/Am;

%msg = Am * sin(2*pi*fm*t);
%msg = Am * square(2*pi*fm*t);
msg = Am * sawtooth(2*pi*fm*t, 1/2);

% Message Signal and FM signals
figure(1);
subplot(6,1,1);
plot(t,msg);
title("Message Signal: triangular wave");

for  i = 1:length(kfvar)

    cmsg = cumsum(msg)/fs;
    FM = sin(2*pi*fc*t + kfvar(i)*cmsg);

    subplot(6,1,i+1);
    plot(t,FM);
    title(sprintf('beta = %d', beta(i)));
end
```

```
% Frequency Spectrum of above signals
figure(2);
subplot(6,1,1);
plot(fftshift(abs(fft(msg))));
title("Frequency Spectrum of Message Signal");

for  i = 1:length(kfvar)

    cmsg = cumsum(msg)/fs;
    FM = sin(2*pi*fc*t + kfvar(i)*cmsg);

    subplot(6,1,i+1);
    plot(fftshift(abs(fft(FM))));
    title(sprintf('beta = %d', beta(i)));
end

%% Case 2: When delW is constant, Wm is varying
Wmvar = (kf * Am)./beta;

% FM signals
figure(3);

for  i = 1:length(Wmvar)

    msg = Am * sin(Wmvar(i)*t);
    cmsg = cumsum(msg)/fs;
    FM = sin(2*pi*fc*t + kf*cmsg);

    subplot(5,1,i);
    plot(t,FM);
    title(sprintf('beta = %d', beta(i)));
end

% Frequency Spectrum of above signals
figure(4);
subplot(5,1,1);

for  i = 1:length(Wmvar)

    msg = Am * sin(Wmvar(i)*t);
    cmsg = cumsum(msg)/fs;
    FM = sin(2*pi*fc*t + kf*cmsg);

    subplot(5,1,i);
    plot(fftshift(abs(fft(FM))));
    title(sprintf('beta = %d', beta(i)));
end
```

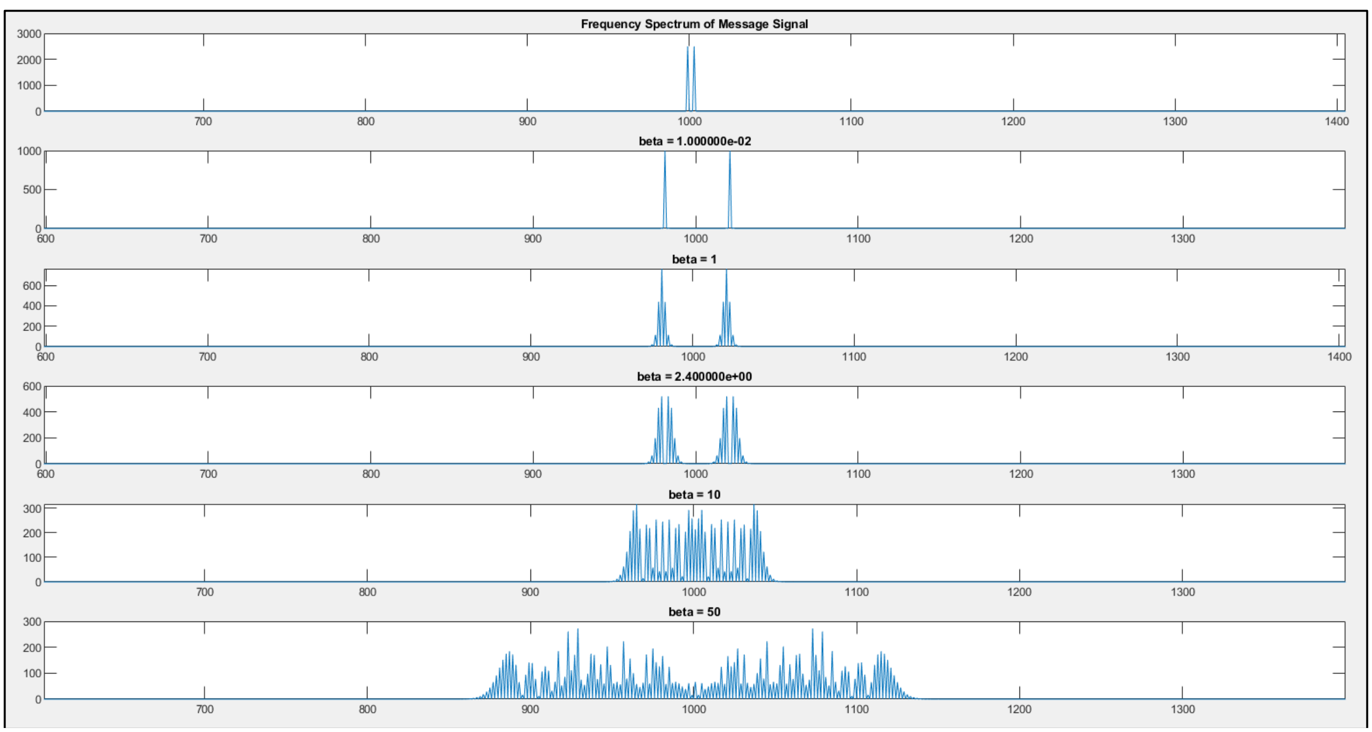
Plots

1. Message Signal: sine wave

Case 1: ω_m is constant

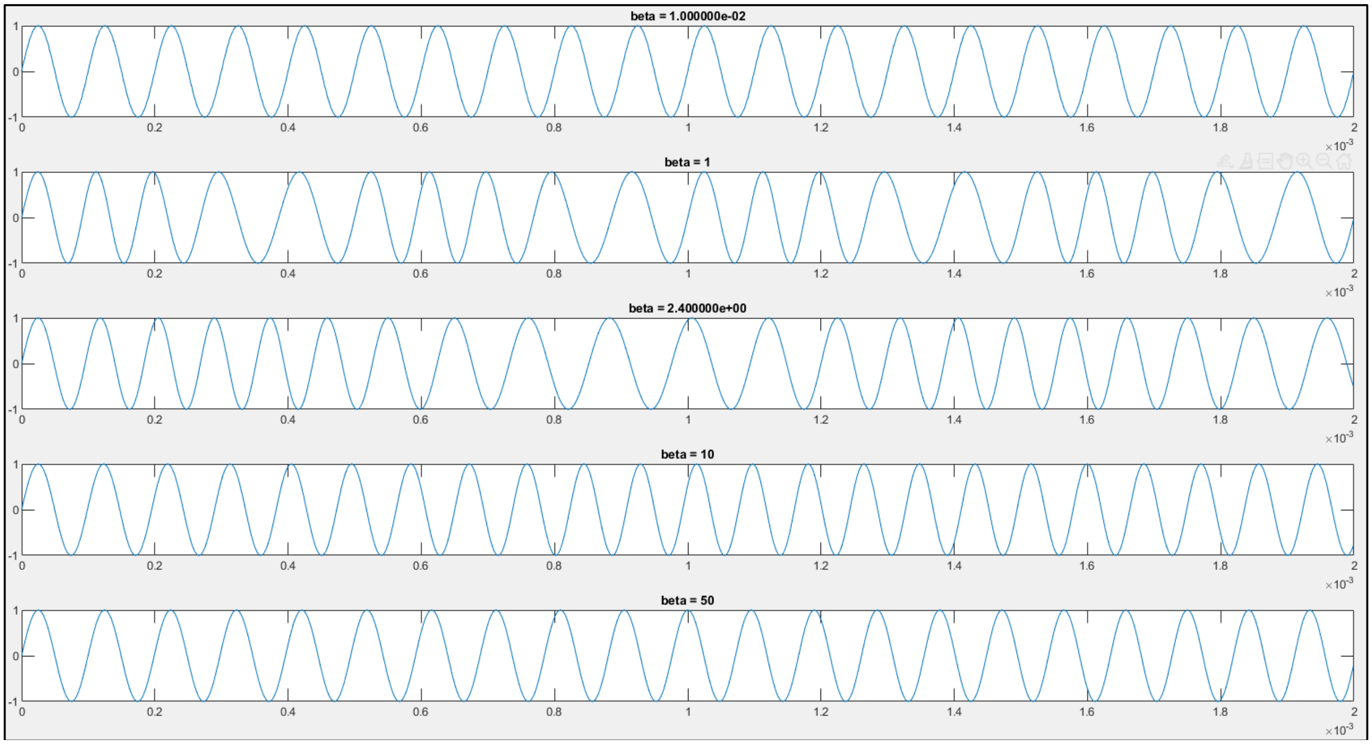


Message signal and the FM signals

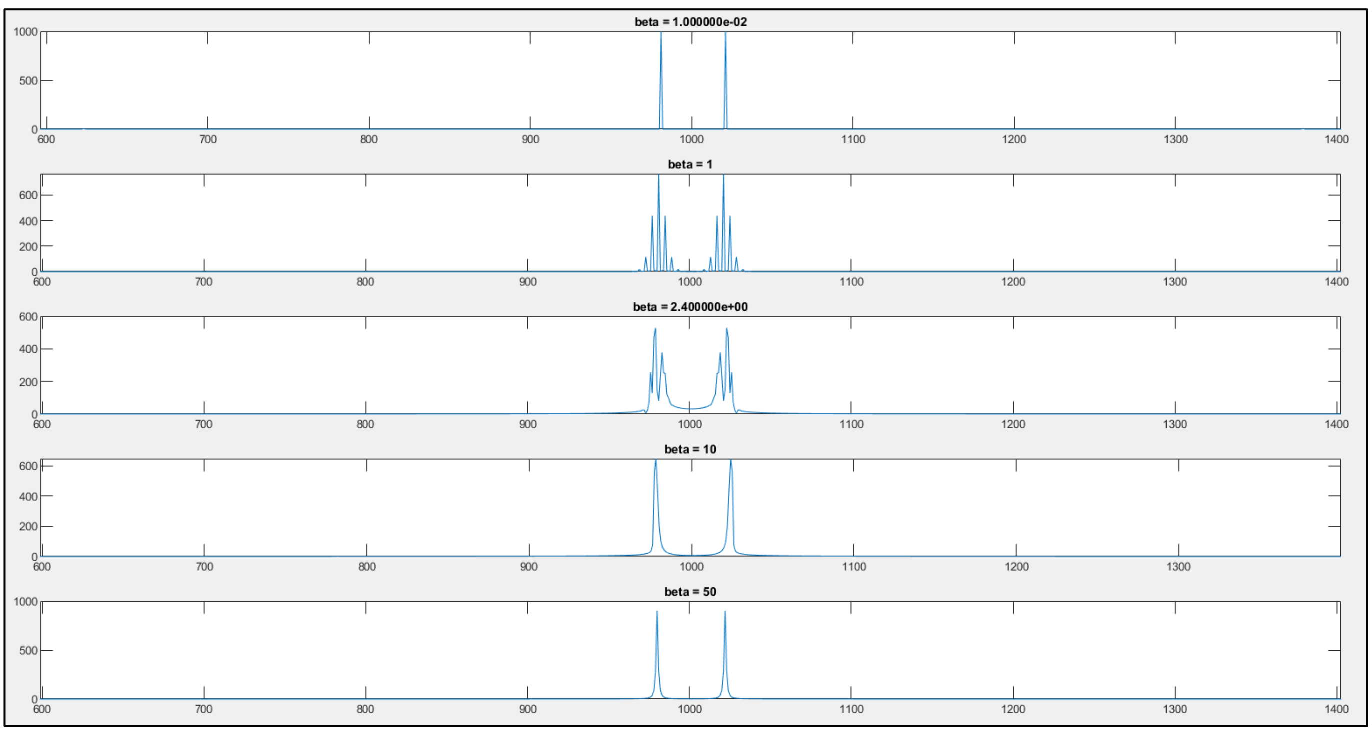


Frequency spectra of the message signal and the FM signals

Case 2: $\Delta\omega$ is constant



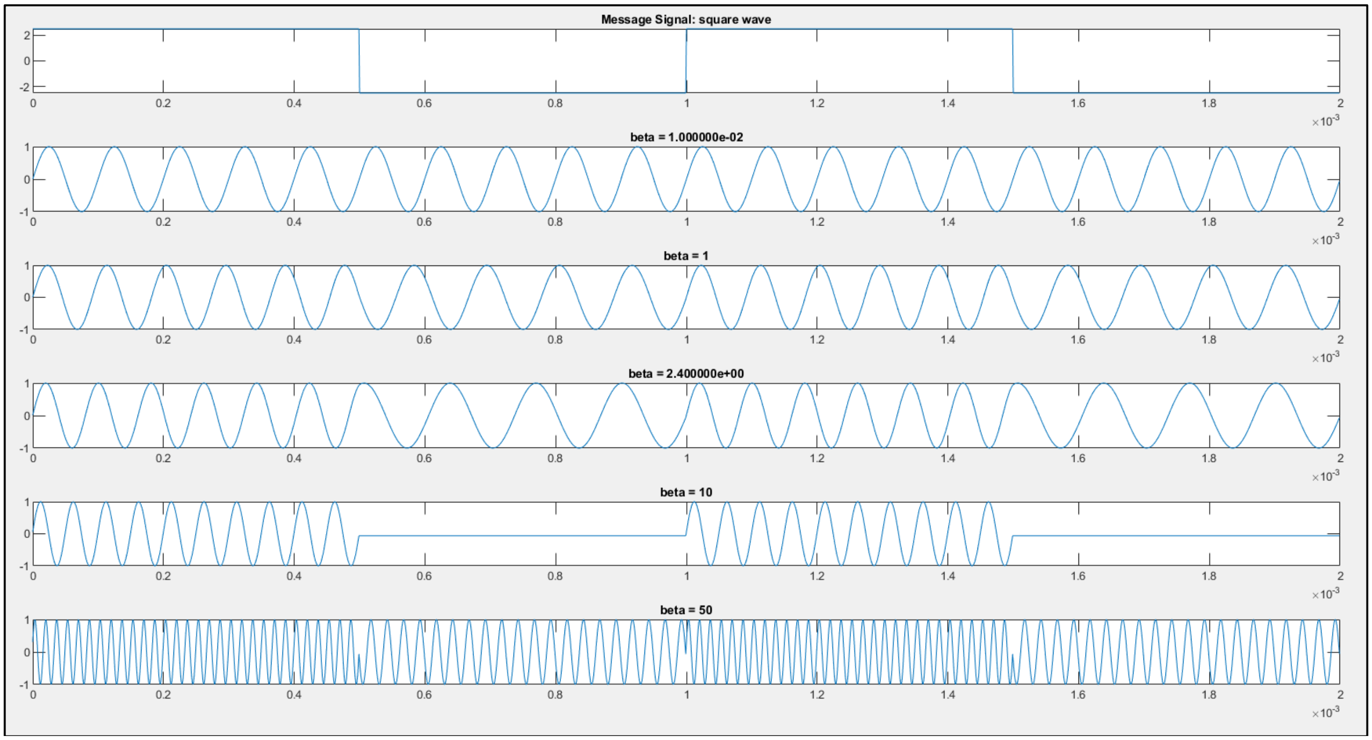
FM signals



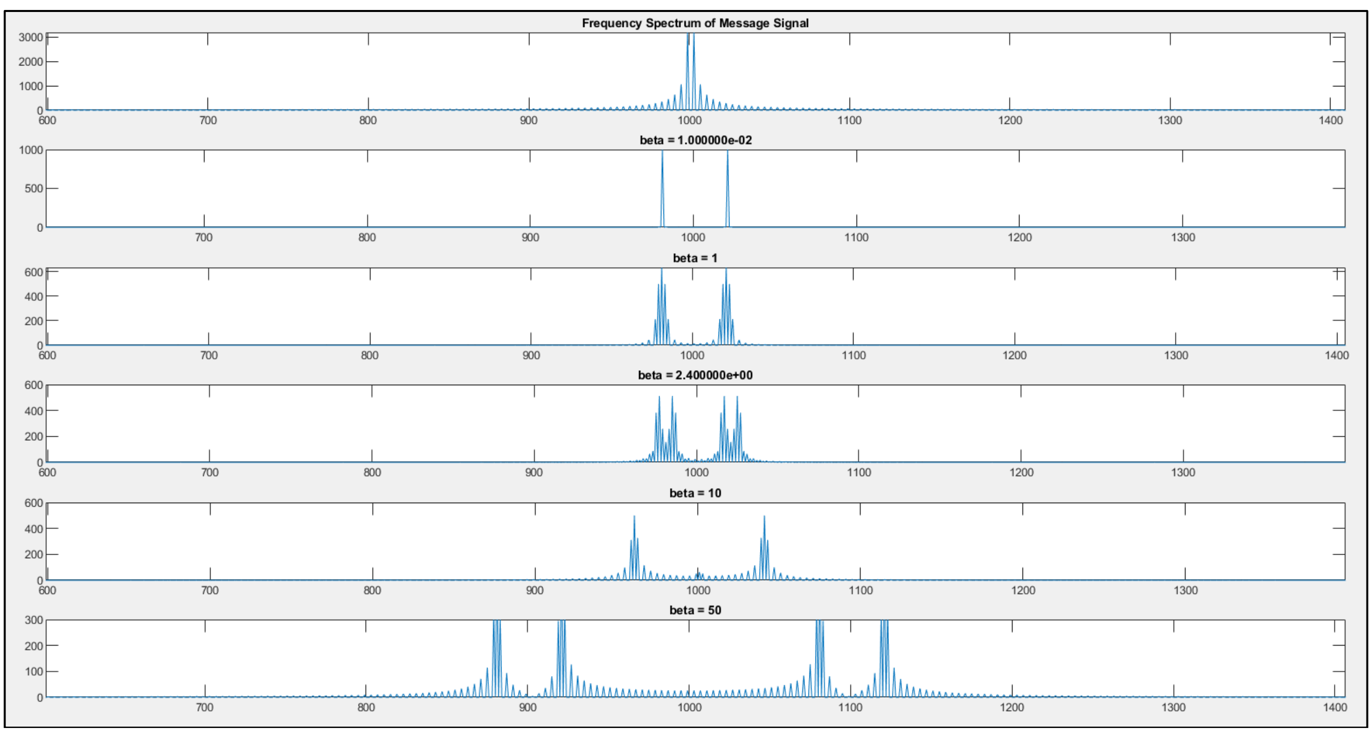
Frequency spectra of the FM signals

2. Message Signal: square wave

Case 1: ω_m is constant

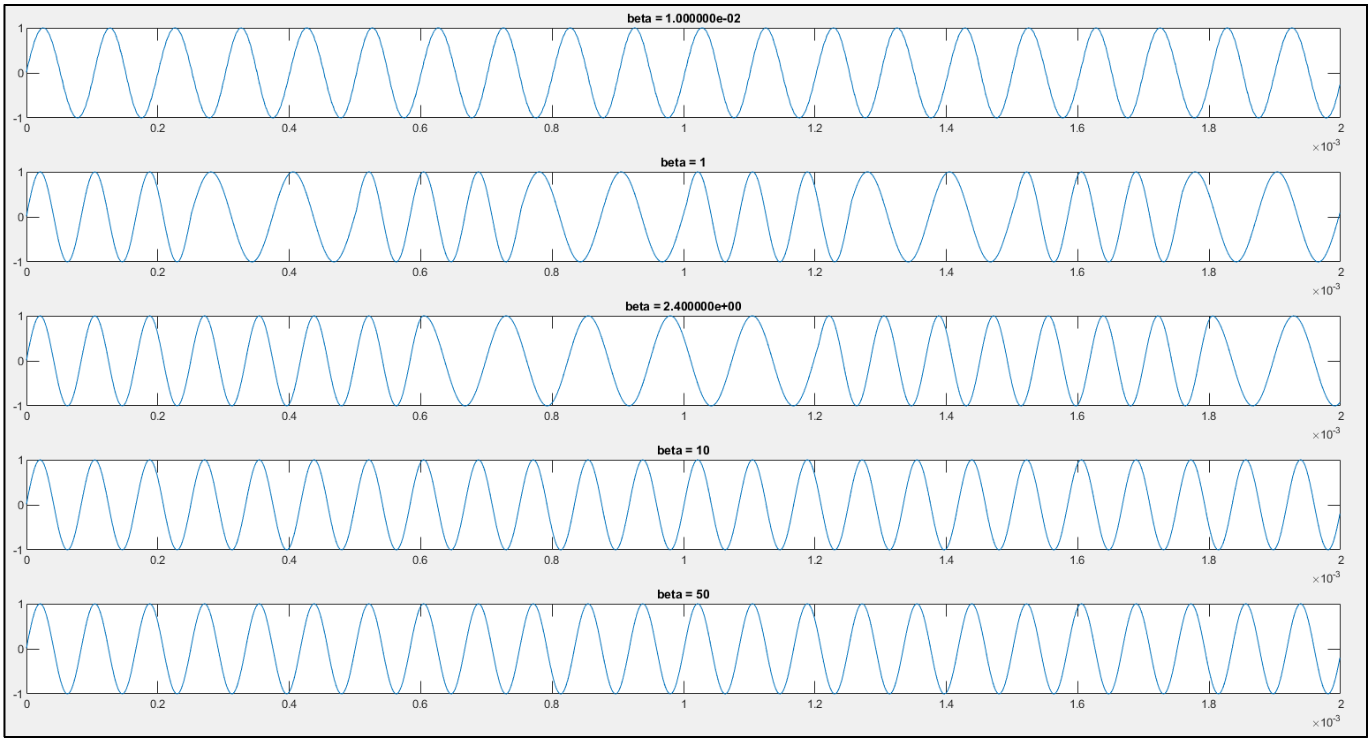


Message signal and the FM signals

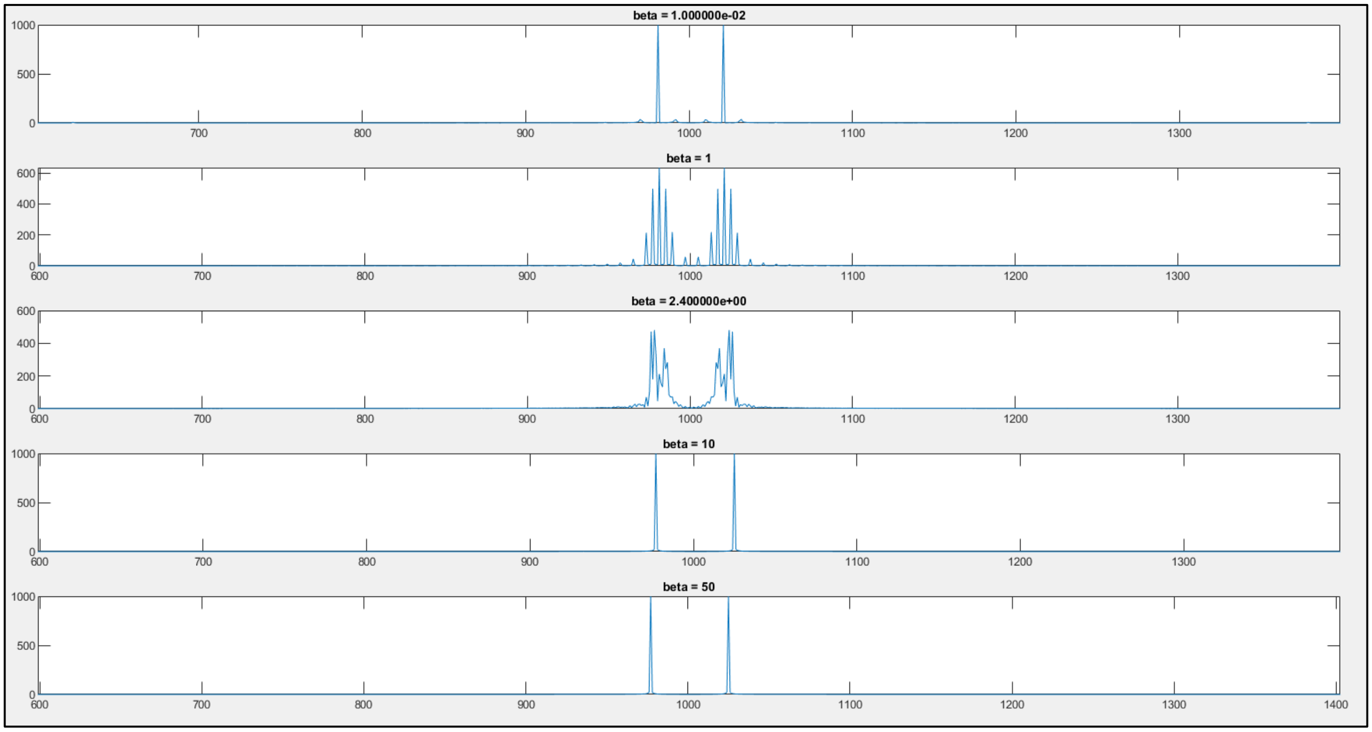


Frequency spectra of the message signal and the FM signals

Case 2: $\Delta\omega$ is constant



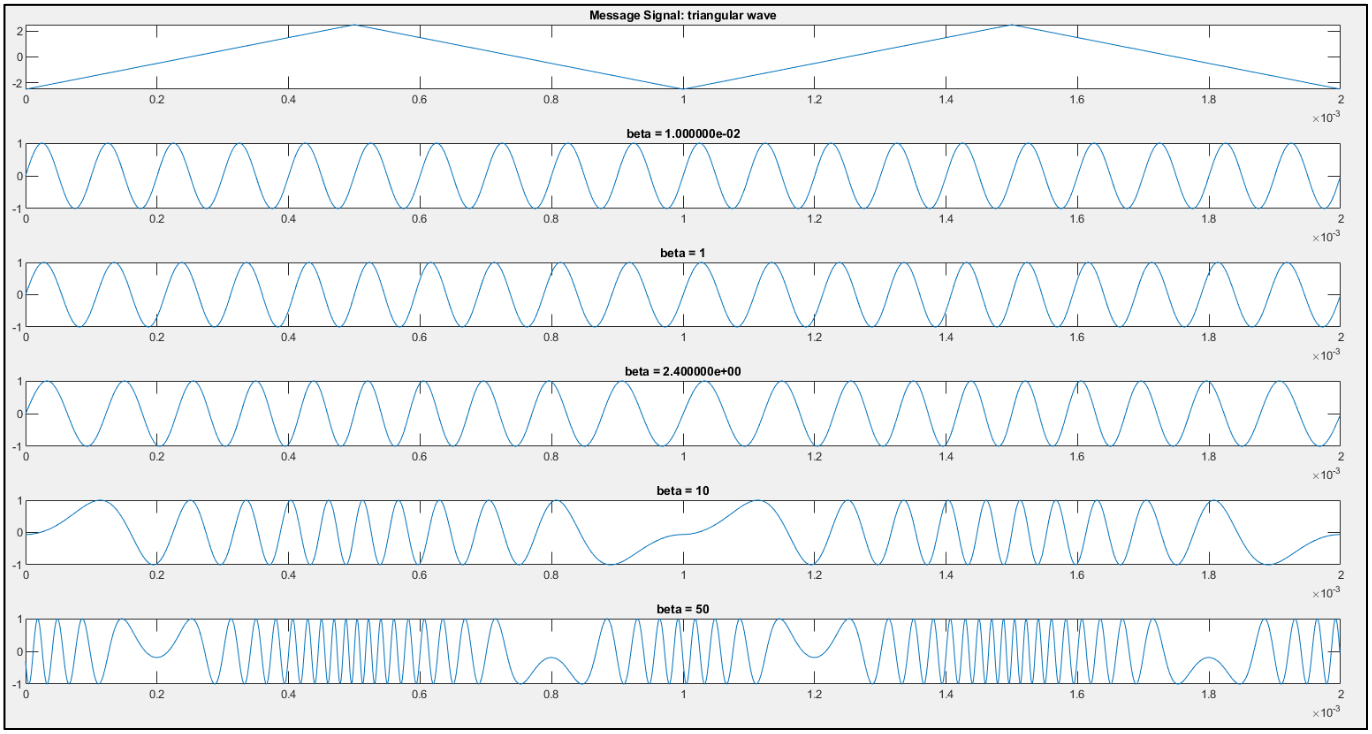
FM signals



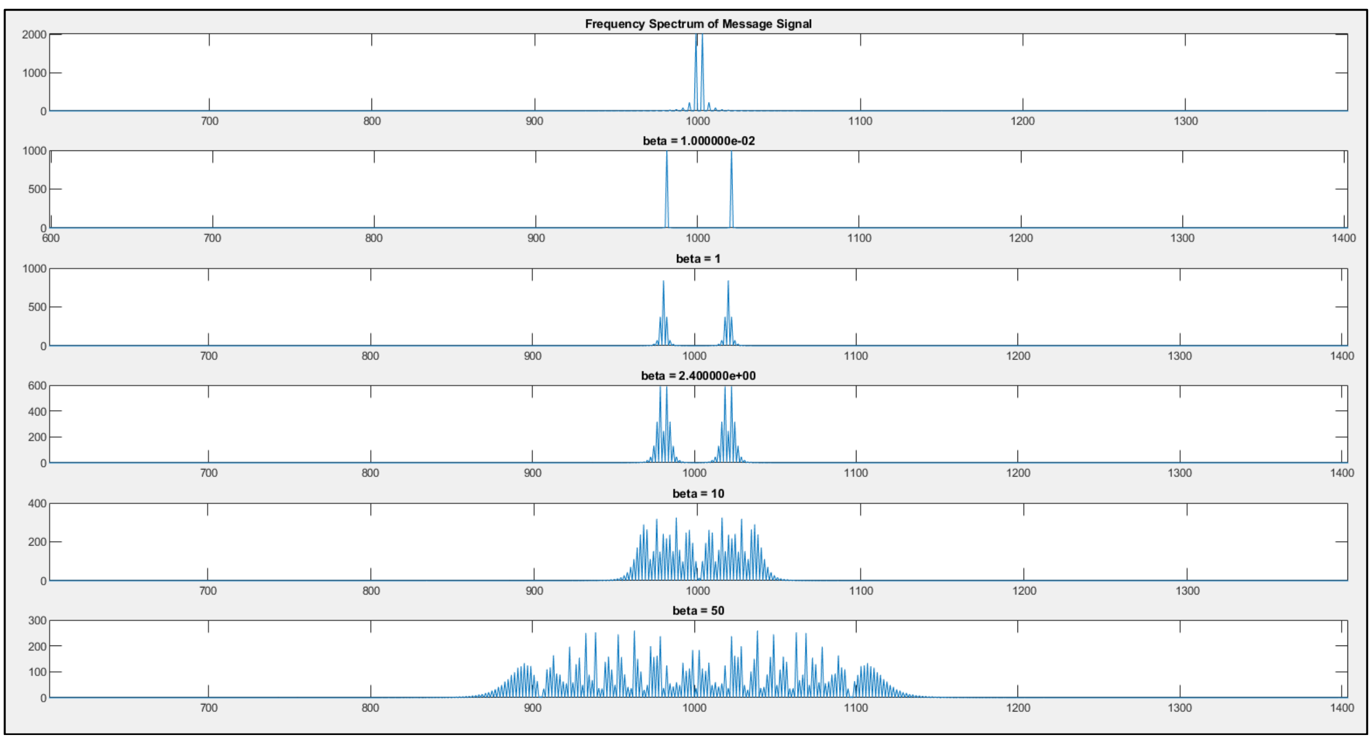
Frequency spectra of the FM signals

3. Message Signal: triangular wave

Case 1: ω_m is constant

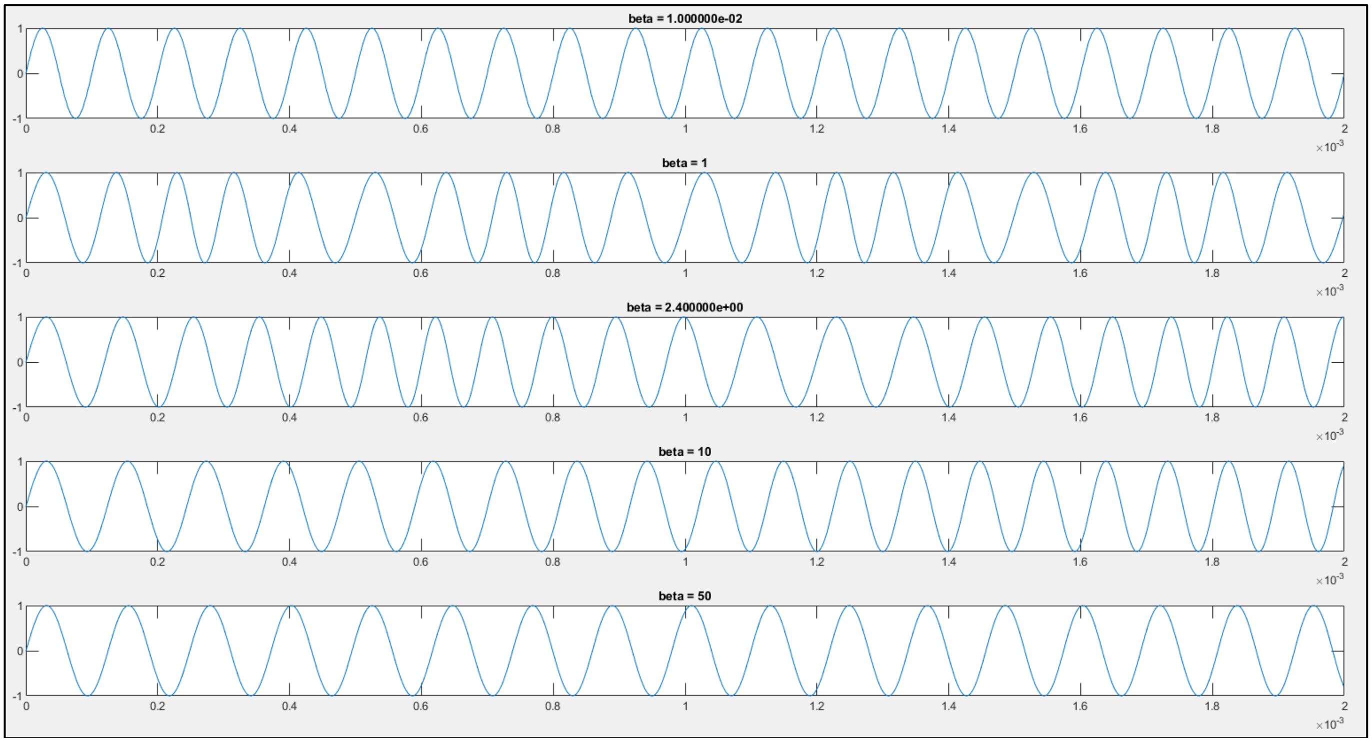


Message signal and the FM signals

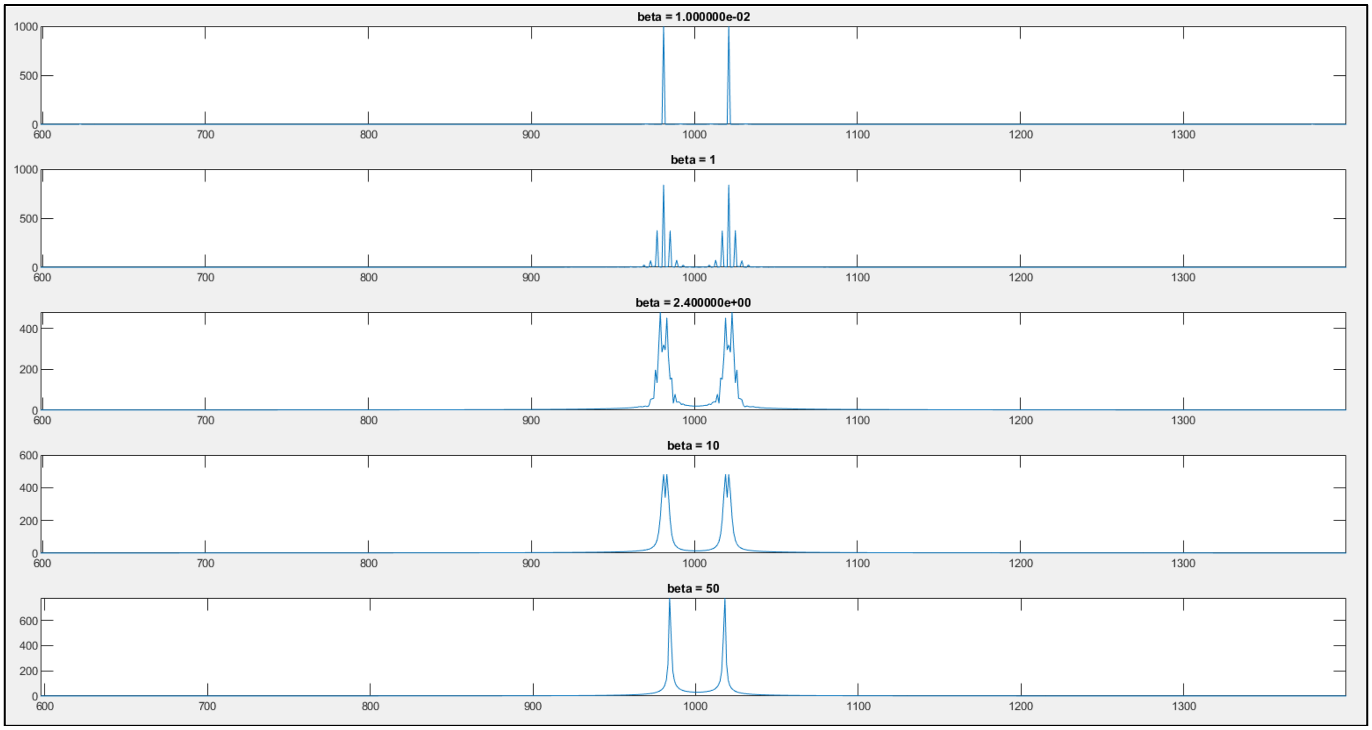


Frequency spectra of the message signal and the FM signals

Case 2: $\Delta\omega$ is constant



FM signals



Frequency spectra of FM signals

Observations and Conclusions

Part 1:

An FM signal is classified as NBFM if $\beta \ll 1$. Therefore, for each of the message signals the FM signal when $\beta = 0.01$ is classified as narrowband FM.

Part 2:

Case1: ω_m is constant

From the above time domain plots, we can observe that as β increases the frequency variation in the modulated signals increases. This is because as β varies, k_f varies.

As for the frequency domain plots, we can conclude that the FM signal is NBFM when $\beta = 0.01$. For other values of β, the FM signals are wideband FM. Further, the bandwidth of the FM signal increases as β increases since $BW = 2(\beta + 1)f_m$. Spectral nulls also increase in the frequency spectrum with increase in β.

Case 2: $\Delta\omega$ is constant

From the above time domain plots, we can observe that as β varies the frequency variation in the modulated signals remains almost the same since $\Delta\omega$ is constant.

As for the frequency domain, like the above case the FM signal is NBFM when $\beta = 0.01$. For other values of β, the FM signals are wideband FM. The bandwidth of FM signals in this case almost remains constant as β varies. The number of spectral nulls also remain the same in all the spectra. However, the amplitude of the frequency transform changes with β.

Part 3:

Case1: ω_m is constant

$$\Delta\omega = k_f A_m, \qquad k_f = {}^{\beta\omega_m}/_{A_m}$$

Here, $f_m = 1000\ Hz$, $A_m = 2.5$

Since $\beta = 0.01, 1.0, 2.4, 10, 50$, therefore we have

```
delW =

   1.0e+05 *

    0.0006    0.0628    0.1508    0.6283    3.1416
```

```
kf =

   1.0e+05 *

    0.0003    0.0251    0.0603    0.2513    1.2566
```

Case 2: $\Delta\omega$ is constant

In this case, we assumed $k_f = 5000$. This implies $\Delta\omega = k_f A_m = 12500$