ELP311
Communication Engineering Laboratory

Experiment 5
Noise Modelling using MATLAB

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Objective

Modeling additive white gaussian noise channel (AWGN)

Theory

AWGN channel: We simulate an additive white gaussian channel. The channel is additive since the output is the sum of input and noise, white since the power spectral density is uniform across all frequencies and gaussian since at each time instant, the noise sample comes from a Gaussian distribution.

The modulation schemes used in this experiment are: BPSK, ASK and FSK. Details about them is provided later in the report.

MATLAB code

```
%% AWGN analysis
mu = 0;
sd = 1;

% samples of N(0, 1)
samples = mu+sd*randn(1, 50);

r = (mu-10*sd)/sd:0.1/sd:(mu+10*sd)/sd;
% pdf of the normal distribution with mean mu and standard deviation sd, evaluated at the values in x
y = normpdf(r, mu, sd);
[ys, xs] = ksdensity(samples, r);
figure (1)
subplot(1, 2, 1)
plot(r, y);
title('analytical pdf of standard normal distribution');

subplot(1, 2, 2)
plot(xs, ys);
title('pdf of standard normal distribution using 50 samples');

% simulated pdf of n^2, n ~ N(0, 1)
t = 0:0.1/sd:(mu+10*sd)/sd;
n = mu+sd*randn(1, 1e6);
m = n.^2;
[f1, t] = ksdensity(m, t);

% analytical pdf of n^2, n ~ N(0, 1)
f2 = exp(-t/2)./sqrt(2*pi.*t);

figure (2)
subplot(1, 2, 1)
plot(f1, t);
title('simuated pdf of m = n^2, n ~ N(0, 1)');
```

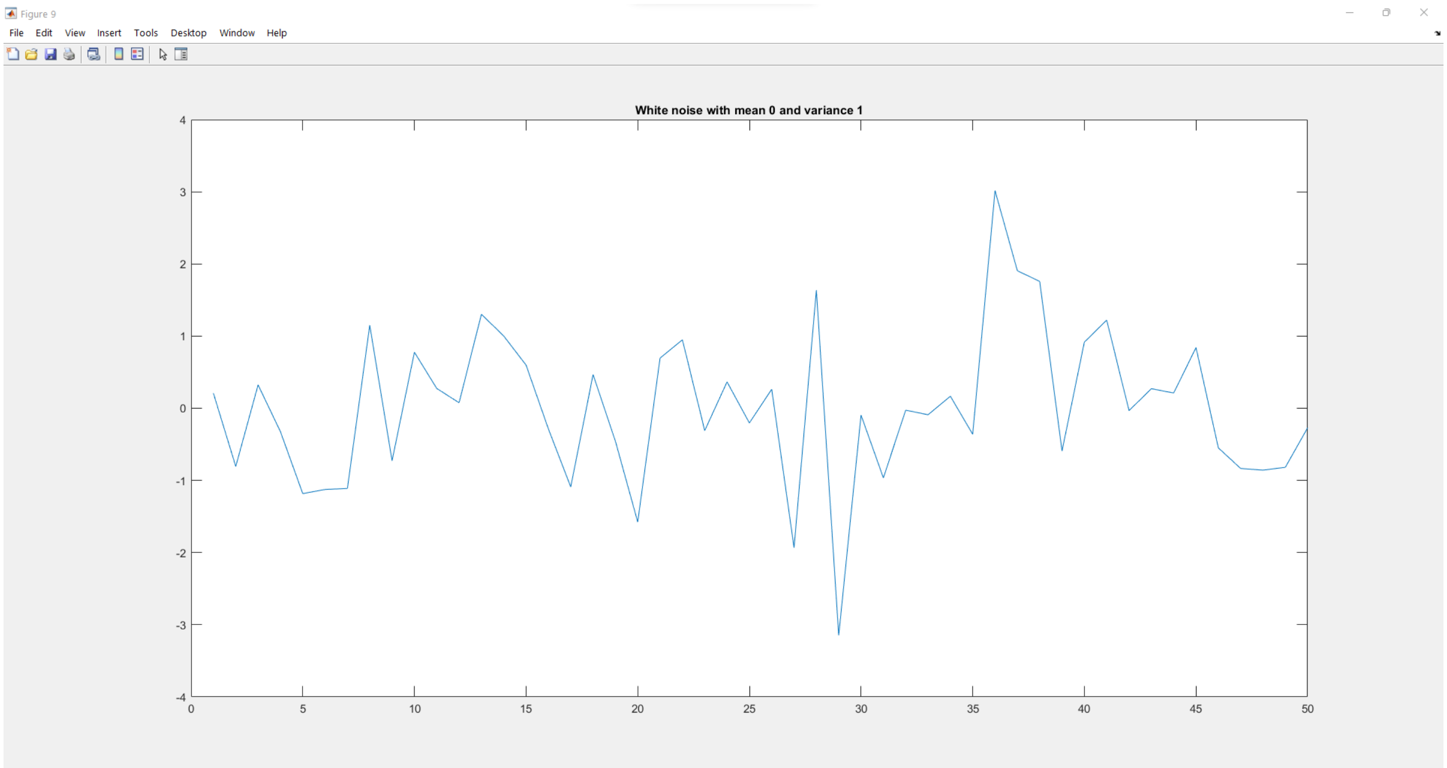
```matlab
subplot(1, 2, 2)
plot(f2, t);
title('analytical pdf of m = n^2, n ~ N(0, 1)');

%% Parameter Definition
Eb = 1;
snrdb = 0:1:10;
snrlen = length(snrdb);

snr = 10.^(snrdb/10);
sd = sqrt(Eb./snr);
amp = sqrt(Eb);

len = 10^6;
s0 = binornd(1, 0.5, len, 1);
s = repmat(s0, 1, snrlen);

n = randn(len, 1)*sd;

%% BER Analysis for BPSK
x1 = (2*s-1)*amp;

y1 = x1+n;

w1 = double(y1 > 0);
t1 = abs(w1-s);
ber_simulated1 = sum(t1)/len;

ber_analytical1 = qfunc(sqrt(snr));

figure (3)
semilogy(snrdb, ber_analytical1, 'b');
title('Analytical and Simulated Bit Error Rate for BPSK Modulation');
xlabel('SNR in dB');
ylabel('BER');

hold on
semilogy(snrdb, ber_simulated1, 'og');
legend('Analytical', 'Simulated');
hold off

P1 = y1(:, 1).^2;
[p1, k1] = ksdensity(P1);

figure (4)
plot(k1, p1);
title('simulated pdf of received power (BPSK)');
xlabel('Power');
ylabel('pdf');

%% BER analysis for ASK
x2 = s*amp;

y2 = x2+n;

w2 = double(y2 > amp/2);
t2 = abs(w2-s);
ber_simulated2 = sum(t2)/len;

ber_analytical2 = qfunc(sqrt(snr./4));
```

```
figure (5)
semilogy(snrdb, ber_analytical2, 'b');
title('Analytical and Simulated Bit Error Rate for ASK Modulation');
xlabel('SNR in dB');
ylabel('BER');

hold on
semilogy(snrdb, ber_simulated2, 'og');
legend('Analytical', 'Simulated');
hold off

P2 = y2(:, 1).^2;
[p2, k2] = ksdensity(P2);

figure (6)
plot(k2, p2);
title('simulated pdf of received power (ASK)');
xlabel('Power');
ylabel('pdf');

%% BER analysis for FSK
ber_analytical3 = qfunc(sqrt(snr/2));

f0 = 1;
f1 = 2;
t0 = 8;

t = 0 : (1 / t0) : 0.99;
t1 = kron(ones(1, len), t);

s0 = binornd(1, 0.5, 1, len);
f = f1 * s0 + (1 - s0) * f0;
ft = kron(f, ones(1, t0));
x = sqrt(2 / t0) * cos(2 * pi * ft.*t1);

n = (randn(1, len * t0) + 1i * randn(1, len * t0)) / sqrt(2);

ref0 = sqrt(2 / t0) * cos(2 * pi * f0 * t);
ref1 = sqrt(2 / t0) * cos(2 * pi * f1 * t);

ber0 = zeros(1, snrlen);
yi = zeros(1, len * t0);

for k = 1 : snrlen
   y = x + sd(k).*n;

   if(k == 1)
       yi = y;
   end

   y0 = conv(y, ref0);
   y1 = conv(y, ref1);

   r = double(real(y0((t0 + 1) : t0 : end)) < real(y1((t0 + 1) : t0 : end)));
   ber0(k) = sum(abs(s0 - r));
end

ber_simulated3 = ber0 / len;

figure (7)
semilogy(snrdb, ber_analytical3, 'b');
```

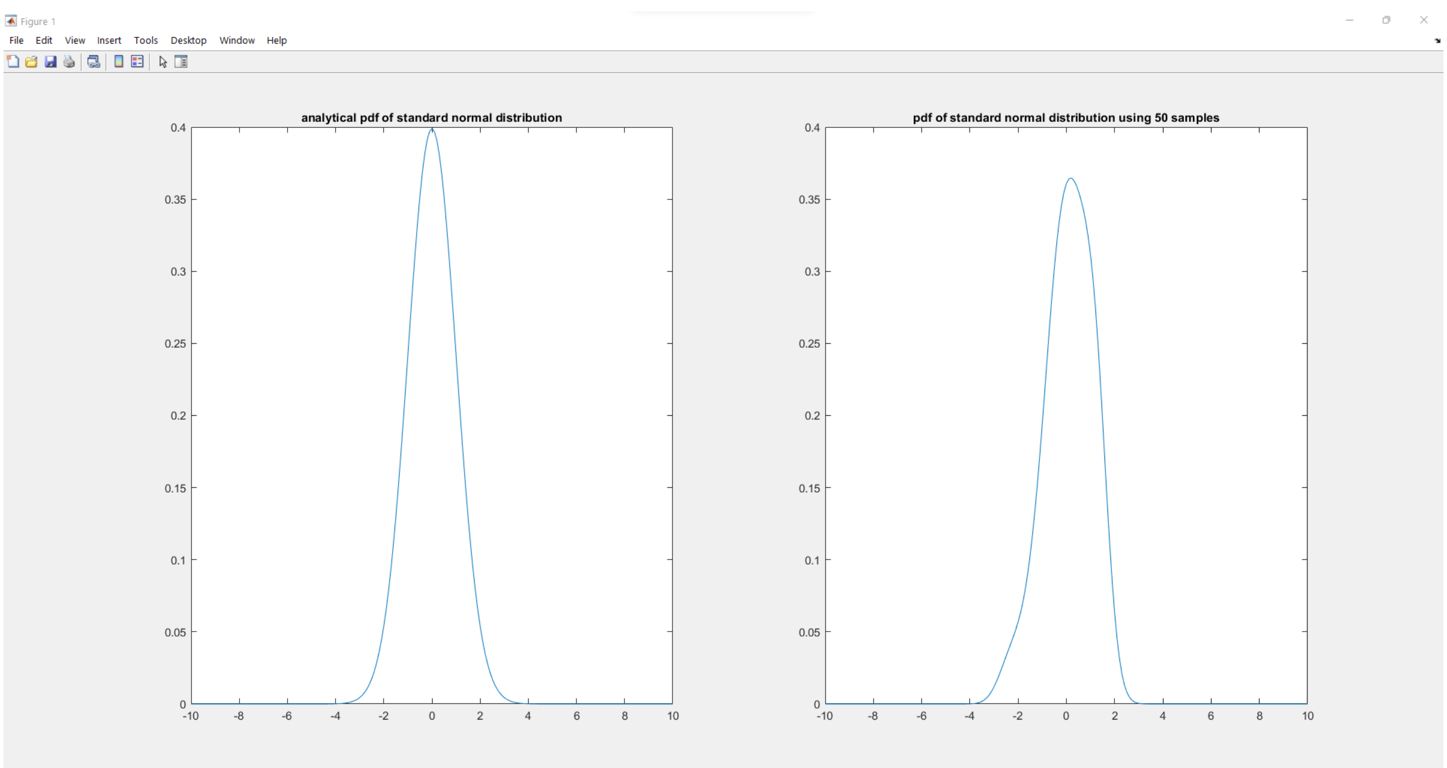
```
title('Analytical and Simulated Bit Error Rate for FSK Modulation');
xlabel('SNR in dB');
ylabel('BER');

hold on
semilogy(snrdb, ber_simulated3, 'og');
legend('Analytical', 'Simulated');
hold off

P3 = abs(yi).^2;
[p3, k3] = ksdensity(P3);

figure (8)
plot(k3, p3);
title('simulated pdf of received power (FSK)');
xlabel('Power');
ylabel('pdf');

%% comparision of different schemes

figure (9)
semilogy(snrdb, ber_analytical1);
xlabel('SNR in dB');
ylabel('BER');
hold on
semilogy(snrdb, ber_analytical2);
semilogy(snrdb, ber_analytical3);
legend('BPSK', 'ASK', 'FSK');
hold off
```

Plots

Part 1: AWGN Analysis



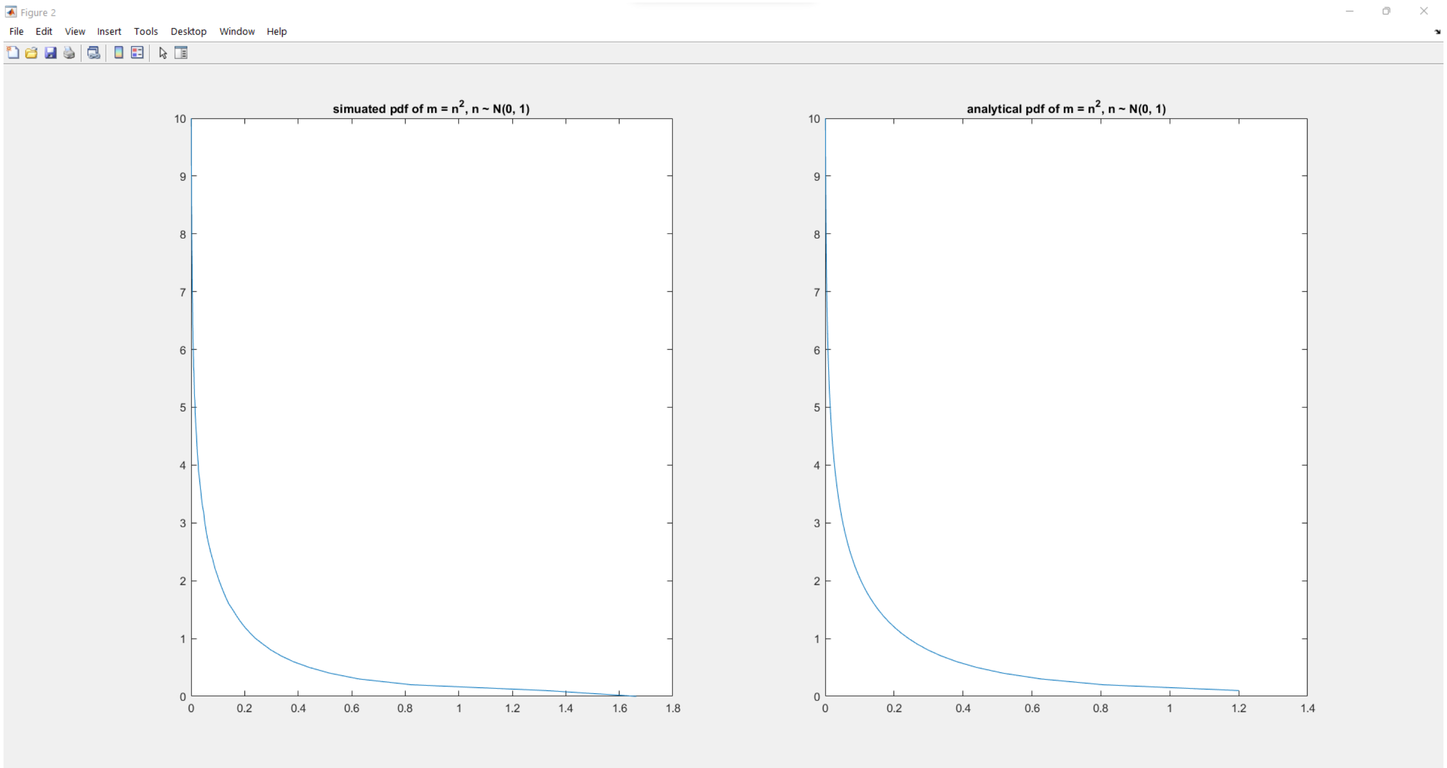
Samples of white noise, $n \sim N(0, 1)$



Analytical and simulated pdf of normal distribution with mean 0 and variance 1

Consider $n \sim N(0,1)$ and $m = n^2$. Now the pdf of m is given as

$$f(m) = e^{\frac{-m}{2}} \Big/ \sqrt{2\pi m}$$



Analytical and simulated pdf of $m = n^2$, $n \sim N(0,1)$

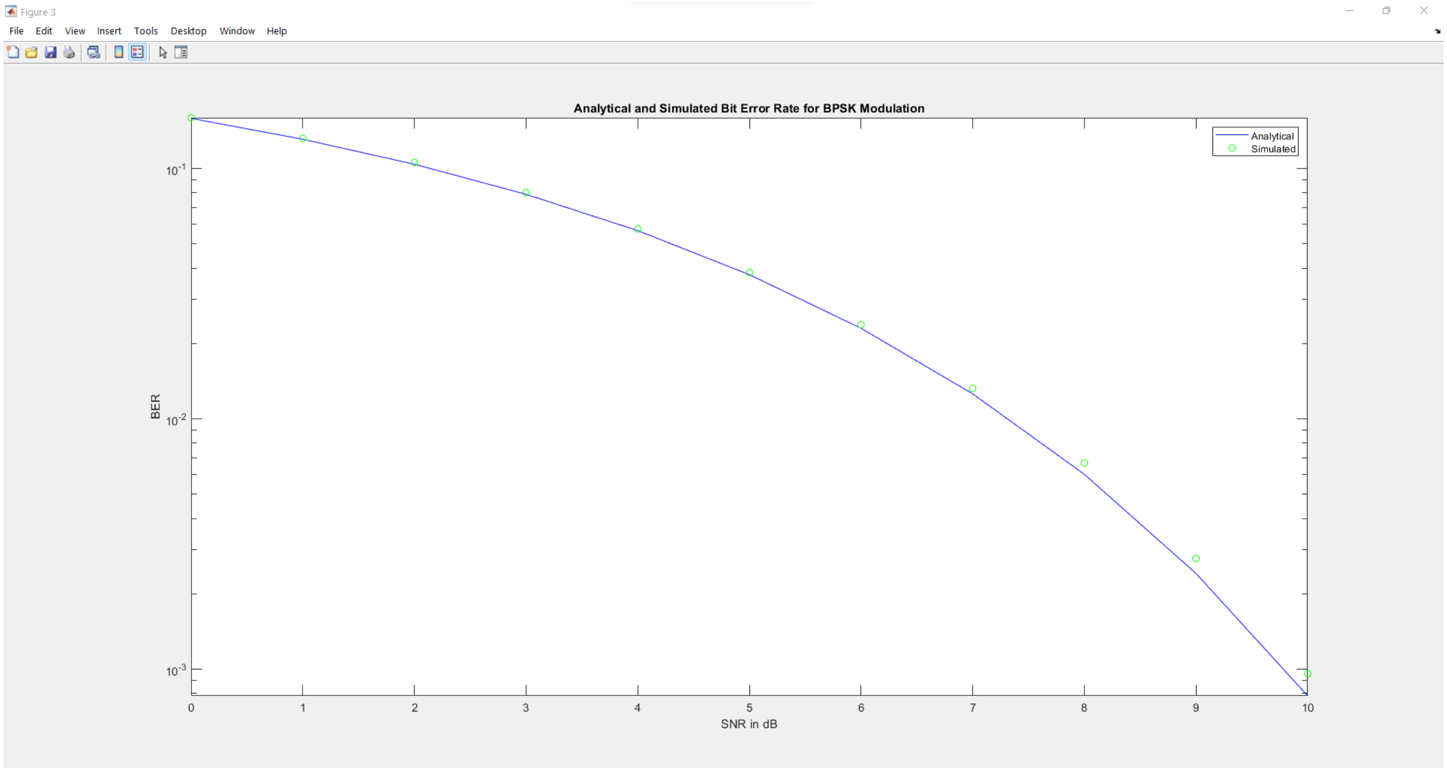
Part 2: BER Analysis for Binary Phase Shift Keying (BPSK)

The BPSK modulated signals $\{-Acos(\omega t), Acos(\omega t)\}$ are mapped to $x \epsilon \{-\sqrt{E_b}, \sqrt{E_b}\}$ (known as digital bits), where E_b is the bit energy. The bit stream is transmitted over AWGN channel. Therefore, the received signal is $y = x + n$, where $n \sim N(0, \sigma^2)$ is the AWGN random variable.

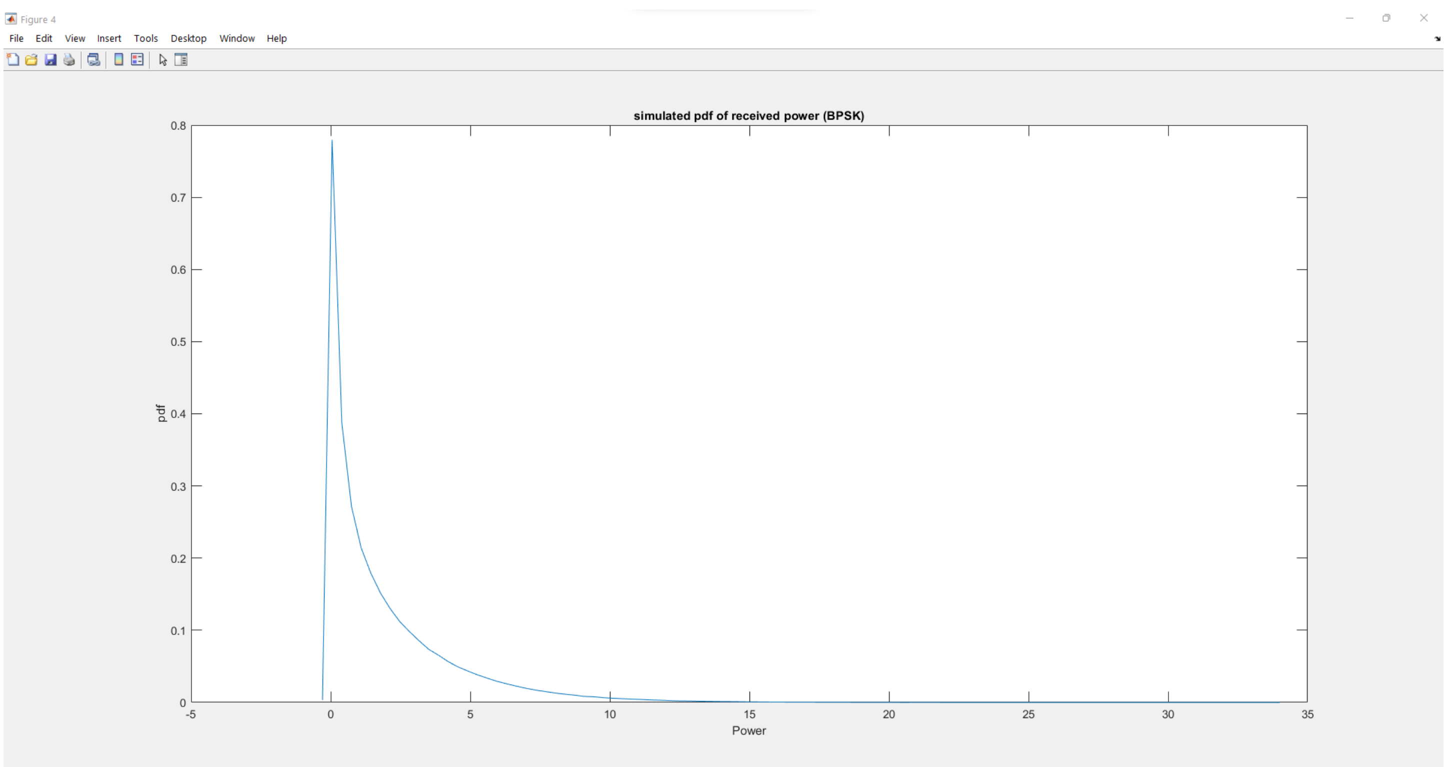
At the receiver, if $y > 0 \Longrightarrow x = 1$ was transmitted else $x = 0$ was transmitted.

The BER expression of BPSK in AWGN is given as

$$P_b = Q(\sqrt{E_b/\sigma^2}) = Q(\sqrt{SNR})$$



Analytical and simulated BER

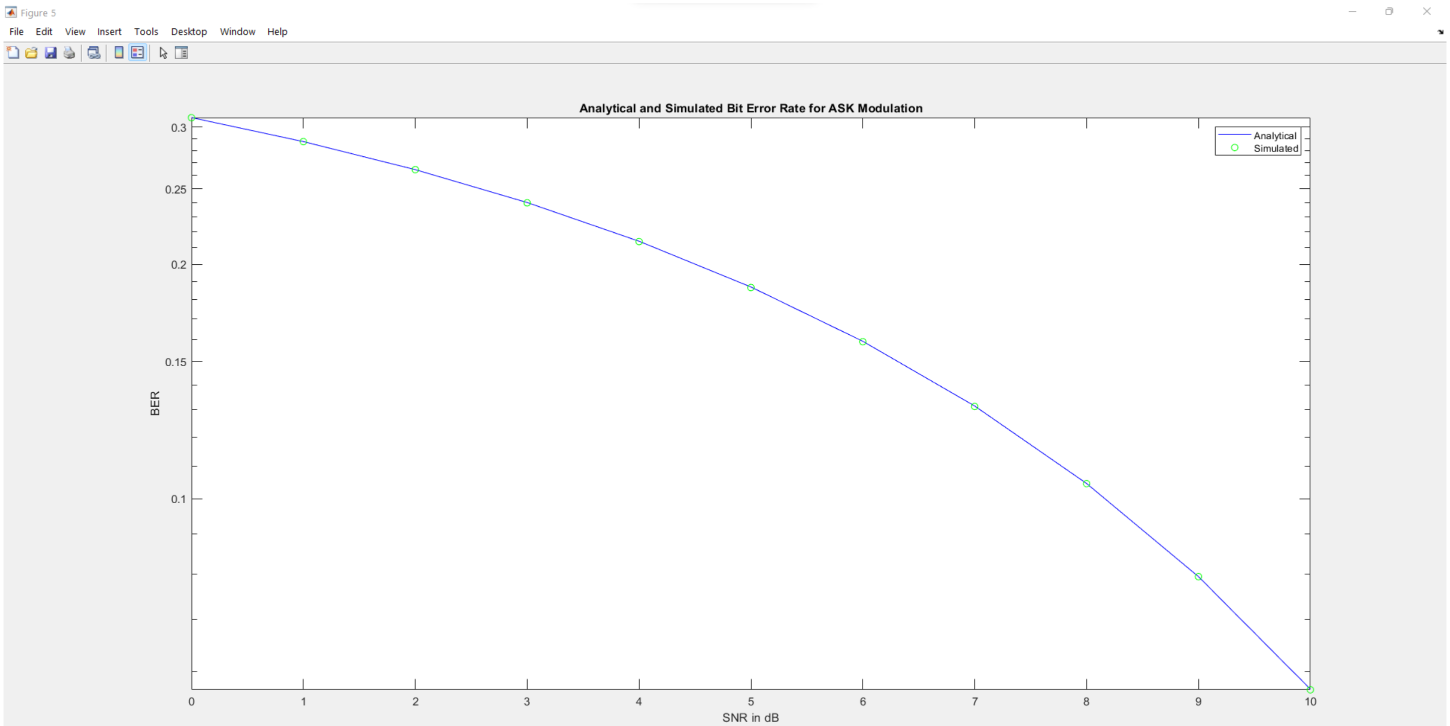


Simulated pdf of received power

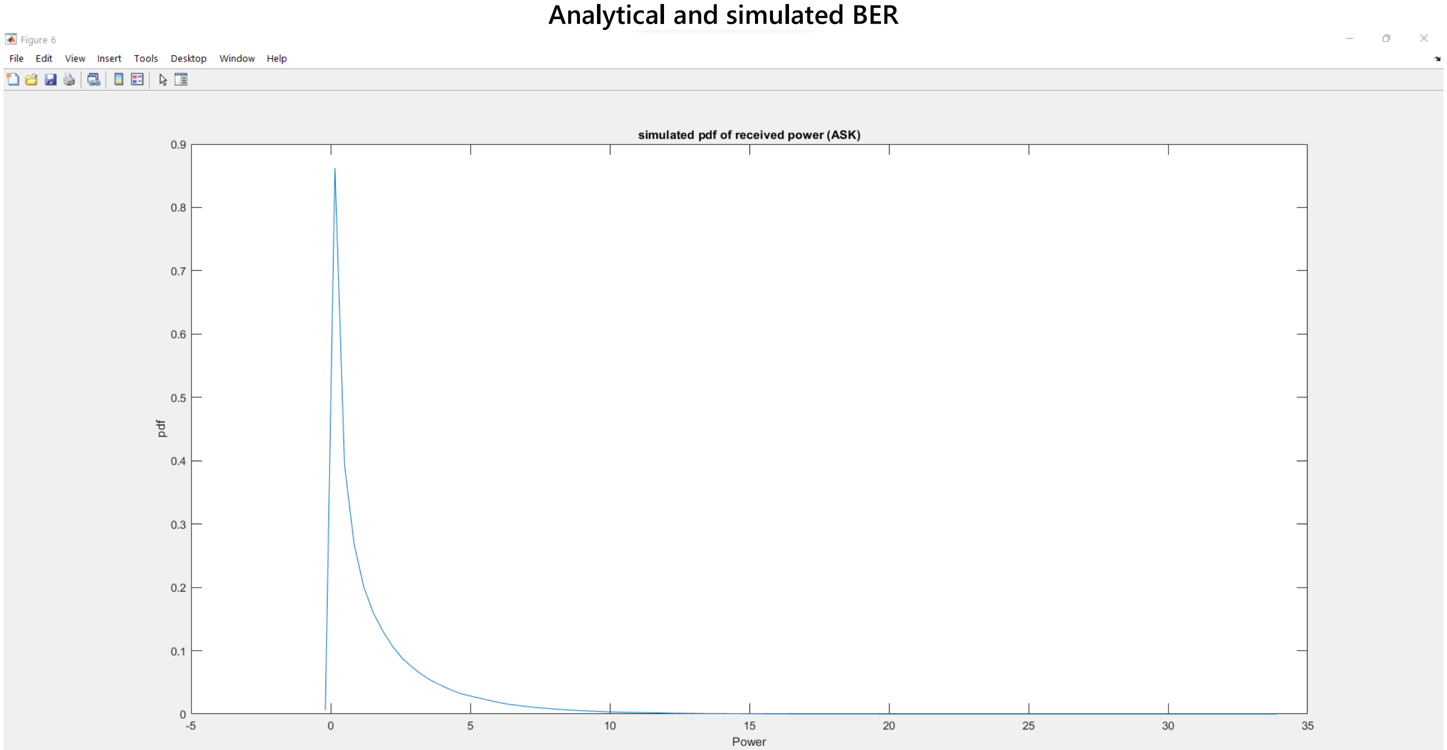
Part 3: BER Analysis for Amplitude Shift Keying (ASK)

In ASK bit 1 is transmitted as a sinusoid of amplitude A and energy $E_b = A^2$. 0 bit is transmitted using off-state and therefore, no energy is involved.

The received signal $y = x + n$ is mapped to 1 if $y > {}^{A}/_{2}$, otherwise mapped it is mapped to 0. The BER expression of ASK in AWGN is given as $P_b = Q(\sqrt{E_b/4\sigma^2}) = Q(\sqrt{\mathrm{SNR}/4})$



Analytical and simulated BER



Simulated pdf of received power

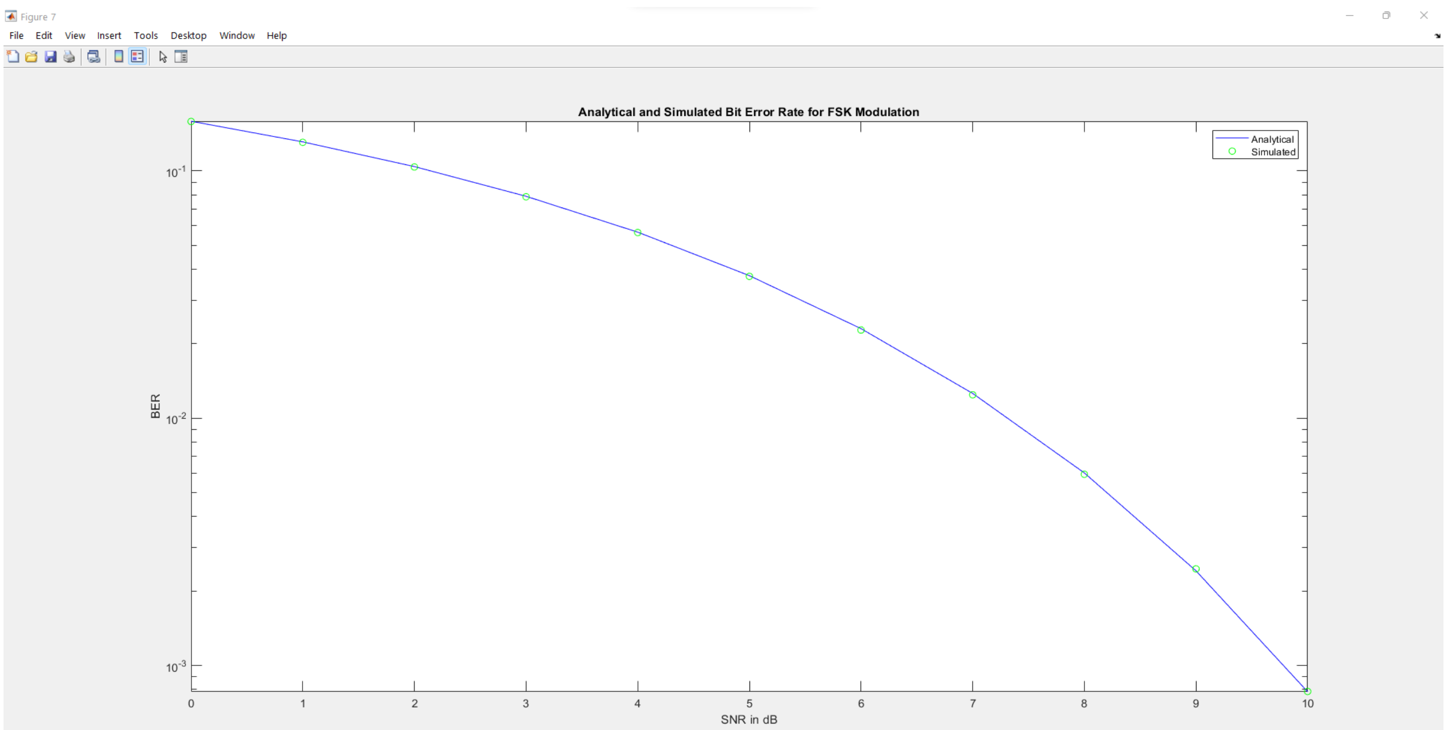
Part 4: BER Analysis for Frequency Shift Keying (FSK)

The bits 0 and 1 are transmitted as sinusoids of baseband frequencies $f_0 = 1\,Hz$ and $f_1 = 2\,Hz$ respectively, of amplitude A and energy $E_b = A^2$. The channel is assumed to add complex baseband noise n.

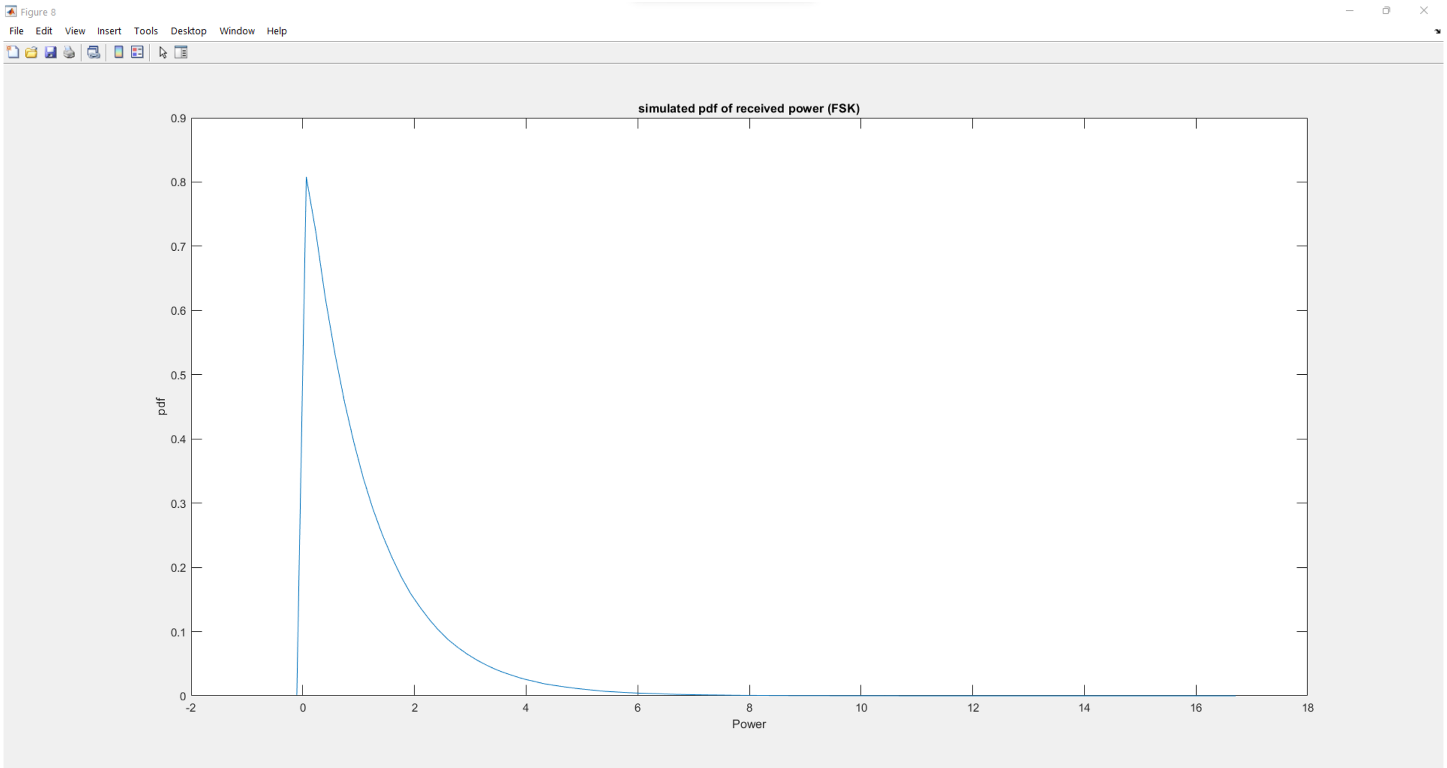
The received signal $y = x + n$ is convolved with the carriers at frequencies f_0 and f_1 separately and the products after a single bit duration is considered at intervals of bit duration. The bit whose carrier gives a higher convolution product is detected as the output at the end of each interval.

The BER expression of FSK in AWGN is given as

$$P_b = Q(\sqrt{E_b/2\sigma^2}) = Q(\sqrt{\mathrm{SNR}/2})$$

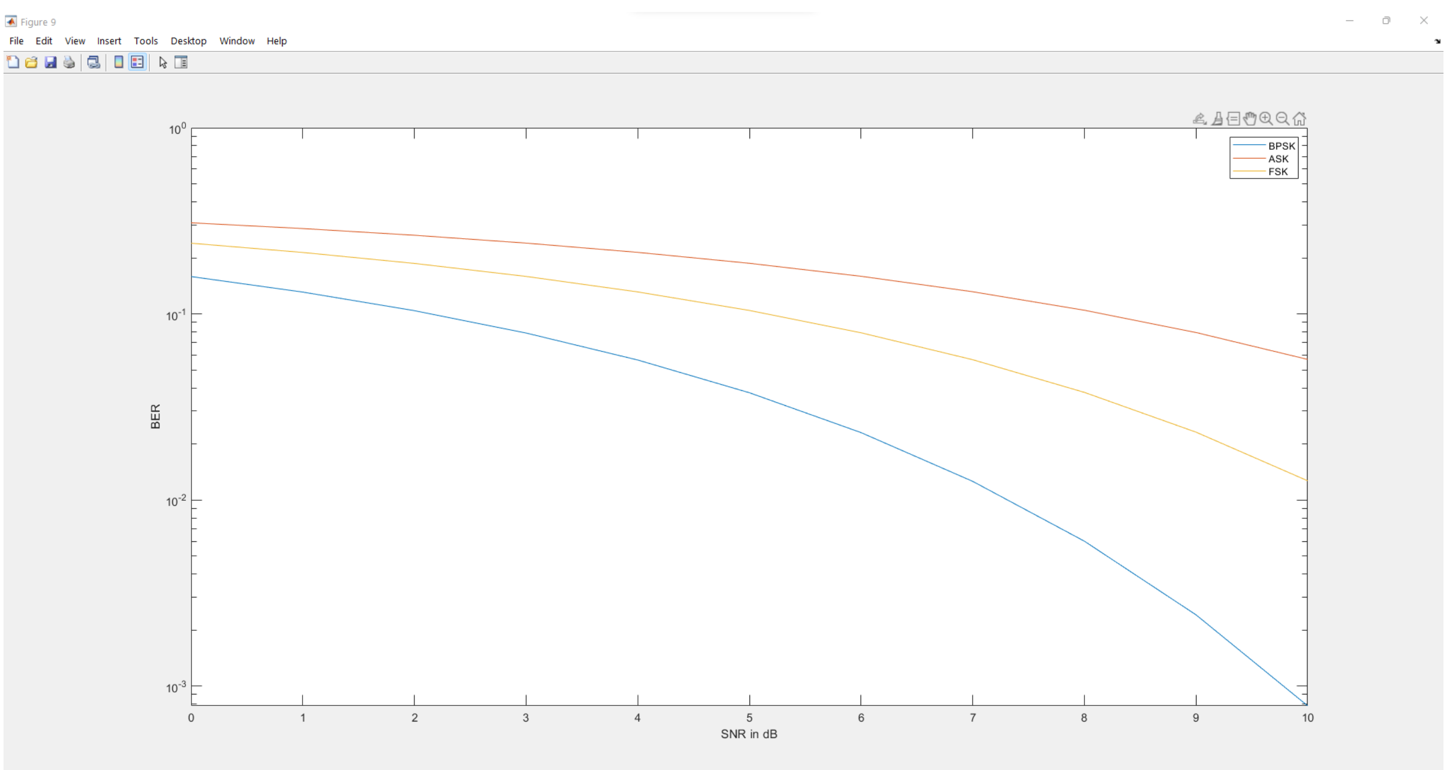


Analytical and simulated BER



Simulated pdf of received power

Part 5: Comparison of different modulation schemes based on BER



Observations and Conclusions

1. We observe that the pdf of normal distribution obtained using 50 samples of white Gaussian noise closely resemble the analytical pdf.
2. Simulated BER v/s SNR plots match the theoretical expectations, for each modulation scheme.
3. Based on BER v/s SNR plots, we can conclude the following order of performance for the modulation schemes used

$$\mathrm{BPSK} > \mathrm{FSK} > \mathrm{ASK}$$
