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Solving hyperbolic partial differential equations using CUDA

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Abstract. Hyperbolic partial differential equations play a crucial role in modeling phenomena such as fluid dynamics, traffic flow, and wave interactions; due to the complexity of these equations, numerical methods are essential for obtaining solutions, particularly with the advent of parallel computing. Graphics Processing Units (GPUs), combined with NVIDIA's CUDA programming model, have enabled significant advancements in scientific computing. This work focuses on solving hyperbolic PDEs using CUDA to exploit the massive parallel processing power of GPUs. Specifically, we address Burgers' equation, the batch monodisperse sedimentation equation, and the polydisperse sedimentation model. Different numerical approaches are implemented, considering the architecture of NVIDIA GPUs and the parallel computing model. Numerical simulations are carried out to assess the efficiency and computational performance of each proposed method.

1. Introduction

Partial differential equations (PDEs) are fundamental for describing a wide range of phenomena in geophysics [1], biology, chemistry [2], and in the mining industry [3]. Hyperbolic PDEs, in particular, model complex processes such as mudflows, avalanches, and sedimentation, with diverse applications. However, their nonlinear nature and the complexity of the modeled phenomena make analytical solutions infeasible in most cases.

Technological advancements, especially in parallel computing [4], have revolutionized numerical methods for PDEs. GPUs, with their massive parallel processing capabilities, enable the efficient execution of thousands of threads simultaneously. Using programming environments like CUDA by NVIDIA [5], GPUs allow for significant reductions in computation times for high-dimensional dynamic systems.

This work extends classical numerical methods for solving hyperbolic PDEs using CUDA. It focuses on parallelizing methods for three problems: Burgers' equation, the monodisperse sedimentation model, and the polydisperse sedimentation model. These conservation law systems address scenarios involving particles of varying properties, offering realistic and scalable numerical solutions. Optimization strategies include efficient memory use, thread management, and minimization of high-latency global memory usage. Results are evaluated in terms of computational efficiency and scalability. Simulations demonstrate the superior performance of GPU implementations over traditional CPU-based methods for complex problems.