

A Modeling Approach to Stomatal Conductance under Different Vapor Pressure Conditions

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Abstract

Stomatal conductance, determined by the stomatal aperture of leaves, governs gas exchange between plants and the atmosphere. This mechanism is closely related to photosynthesis and the ability of plants to adapt to environmental changes, including those caused by global warming. This study proposes a two-dimensional nonlinear model based on ordinary differential equations to estimate stomatal conductance over time. The model integrates key environmental drivers, air temperature and relative humidity, through the vapor pressure deficit, and couples stomatal conductance with leaf water content. A modified Gompertz-type equation captures the asymmetric and time-delayed response of stomatal conductance towards a dynamic steady-state target. We perform both local and global sensitivity analyses to assess parameter influence, revealing vapor pressure deficit as the most significant factor influencing stomatal effects. The findings indicate that the model effectively forecasts trends in stomatal conductance,

providing information on plant water status and their possible regulation under different environmental conditions.

Keywords: transpiration, vapor pressure deficit, gas exchange, soil-plant-atmosphere continuum, stomatal behavior, dynamical modeling, global warming

1 Introduction

Stomatal conductance (g_s) governs the exchange of water vapor and carbon dioxide between leaves and the atmosphere along the soil-plant-atmosphere continuum (SPAC) and, therefore, plays a central role in plant responses to drought, heat, and other stressors associated with global warming (GW) [1, 2]. Accurately representing the temporal variability of g_s is essential for understanding how plants adjust to rapidly changing environmental conditions. Mediterranean-type climate regions, such as California, the Mediterranean Basin, South Africa, Australia, and Chile, are particularly affected by GW as warmer and drier summers intensify water limitations for both natural and agricultural vegetation [3–6]. In these ecosystems, g_s is widely used as a physiological indicator of plant water status and adaptation capacity [7]. Figure 1 illustrates the SPAC and the role of stomata in mediating gas exchange.

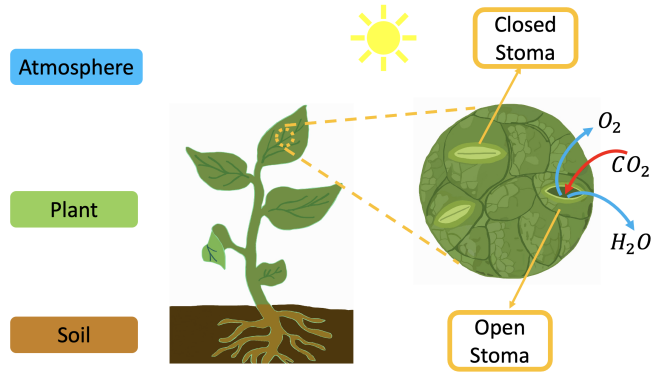


Fig. 1 Schematic representation of gas exchange within the soil-plant-atmosphere continuum. Photosynthesis facilitates the entry of CO_2 through open stomata, releasing oxygen (O_2), while transpiration releases water vapor (H_2O). (Source: Original Work).

Several modeling strategies have been developed to describe stomatal conductance. Empirical models, such as those of Jarvis [8] and Ball-Woodrow-Berry [9], relate g_s to environmental variables or photosynthesis and often yield strong correlations with measurements [10]. Optimization models [11, 12] frame stomatal behavior as a trade-off between carbon assimilation and water loss, while mechanistic models incorporate hydraulic and chemical signaling pathways [13, 14]. More recently, dynamic approaches have been proposed to capture the short-term temporal response of g_s under changing environmental conditions. This type of approach includes coupled differential equation

systems that describe guard-cell processes [15], irradiance-driven stomatal responses [16], and models integrating solute dynamics, mesophyll water content, and water flow [17]. Despite these advances, many existing models assume steady-state behavior, require biophysical parameters that are difficult to measure, or do not explicitly link stomatal conductance with leaf water content (W) in a tractable dynamic framework. These limitations reduce the applicability of current models under realistic environmental variability [2, 18].

To address this gap, the objective of this study is to develop and analyze a two-dimensional ordinary differential equation (ODE) model that explicitly couples the temporal evolution of stomatal conductance (g_s) and leaf water content (W). In our formulation, g_s evolves toward a steady-state target modulated by environmental drivers, represented through the parameter δ_e , which corresponds to the vapor pressure deficit (VPD) and reflects the interaction between air temperature (T_a) and relative humidity (RH) [16, 19]. This structure provides a simplified yet physiologically grounded representation of stomatal dynamics.

The novelty of our approach lies in the following contributions: (i) introducing a reduced dynamic framework that links g_s and W in a physiologically interpretable manner; (ii) incorporating environmental variation through the parameter δ_e , which reflects the interaction between air temperature and relative humidity; (iii) performing a sensitivity analysis to evaluate the influence of δ_e on model trajectories; and (iv) assessing the model’s robustness using experimental data through a cross-validation procedure. Together, these elements provide an efficient and interpretable tool for examining plant responses to environmental stressors, particularly under conditions intensified by GW in Mediterranean-type ecosystems.

This work is organized as follows. First, we present the complete two-dimensional system and the assumptions used to derive the model. Next, we perform a sensitivity analysis to assess the impact of δ_e on model solutions. Finally, we evaluate the robustness of the model through a cross-validation procedure using experimental data. Additional qualitative properties of the system, including equilibrium analysis and stability, are presented in the Appendix.

2 Model Formulation

2.1 Leaf water content

Taking into account the approach presented in [17], we study how δ_e influences the variation of leaf water content over time. The rate of change of $W = W(t) > 0$ (mol m^{-2}) varies over time (for notational simplicity, we omit the explicit time dependence of W) according to inflows and outflows described by:

$$\frac{dW}{dt} = F_{in}(t) - F_{out}(t). \quad (1)$$

92 The inflow $F_{in}(t)$ is determined by the leaf water potential ($\Psi_l < 0$) and the soil water
 93 potential ($\Psi_s < 0$). When Ψ_l is less than Ψ_s , water flows from the soil to the leaf:

$$F_{in}(t) = \frac{\Psi_s - \Psi_l}{R_{sl}}, \quad (2)$$

94 where $R_{sl} > 0$ ($MPa \cdot mol^{-1} \cdot m^2 \cdot s$) is the hydraulic resistance between the soil and the
 95 leaf. The Ψ_l is determined by:

$$\Psi_l = P_l - \pi_l, \quad (3)$$

96 where $P_l \geq 0$ (MPa) and $\pi_l > 0$ (MPa) represent the turgor pressure in the leaf and
 97 the osmotic pressure in the leaf, respectively, both determined by $W = W(t)$ (see [20]).
 98 As W increases, cell expansion occurs, stretching the cell walls and thereby increasing
 99 P_l . Conversely, a higher W leads to the dilution of osmotic solutes within the cells,
 100 resulting in a decrease in π_l . From these considerations, the following equations are
 101 derived:

$$\begin{aligned} P_l &= \pi_{lo} \cdot H(W) \\ \pi_l &= \pi_{lo} \cdot \frac{W_{max}}{W}, \end{aligned} \quad (4)$$

102 where W_{max} ($mol \cdot m^{-2}$) is the maximum leaf water content, π_{lo} (MPa) is the value
 103 of π_l when $W = W_{max}$, and $H(W)$ is the function defined as follows:

$$H(W) = \begin{cases} 0 & \text{if } W \leq W_{tlp} \\ \frac{W - W_{tlp}}{W_{max} - W_{tlp}} & \text{if } W > W_{tlp}, \end{cases} \quad (5)$$

104 where W_{tlp} ($mol \cdot m^{-2}$) represents the leaf water content at the turgor loss point (for
 105 more details on the turgor loss point, see [21]).

106 Thus, from (4) and (5), it is obtained that

$$\Psi_l = \pi_{lo} \left(H(W) - \frac{W_{max}}{W} \right), \quad (6)$$

107 and, consequently, the inflow is determined by

$$F_{in}(t) = \frac{1}{R_{sl}} \cdot \left[\Psi_s - \pi_{lo} \cdot \left(H(W) - \frac{W_{max}}{W} \right) \right]. \quad (7)$$

108 On the other hand, $F_{out}(t)$ is determined by the transpiration rate (E ,
 109 ($mol \cdot H_2O \cdot m^{-2} \cdot s^{-1}$)) according to the equation:

$$E = \left(\frac{g_s}{1 + r_b \cdot g_s} \right) \cdot \frac{\delta_e}{p_{atm}}. \quad (8)$$

110 As shown in [22, 23], p_{atm} represents the atmospheric pressure (MPa), r_b
 111 ($(mol \cdot m^{-2} \cdot s^{-1})^{-1}$) represents the boundary layer resistance and δ_e (MPa) is defined