Modeling epidemic and pest dynamics in plants

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Abstract

We propose and analyze a reaction diffusion partial differential equation (PDE) model to describe the spatio temporal dynamics of plant diseases and pest plant interactions. The model captures the interaction between susceptible and infected plant populations, incorporating local disease transmission mechanisms together with spatial diffusion processes. The well posedness of the resulting parabolic PDE system is established by proving the existence, uniqueness, and positivity of solutions under biologically meaningful assumptions. A disease free equilibrium and an endemic equilibrium are identified. The basic reproduction number is derived using the next generation matrix approach, providing a threshold parameter that governs disease extinction or persistence. In addition, the role of diffusion and spatial heterogeneity in the spread of infection is investigated. Numerical simulations are presented to illustrate the analytical results and to explore how parameter variations influence spatio temporal disease propagation patterns. The proposed model provides a flexible mathematical framework for understanding epidemic and pest dynamics in plant populations and may serve as a basis for the evaluation of control strategies in agricultural systems.

Key words: Population dynamics; reaction-diffusion model; infectious disease; plant disease; basic reproduction number; equilibrium; stability; next generation matrix; compound matrix.

Mathematics subject classifications (2000): XXXXX, 65N06, 76T20.

1 Introduction

1.1 Scope

Scientists worldwide are deeply committed to studying and analyzing infectious diseases and pests that affect a wide variety of living organisms, including animals, insects, plants, humans, and many others. This interest stems from the significant impact these diseases have on various aspects. On the one hand, they have a negative impact on human health, causing severe illnesses and posing a risk to public health. Additionally, these diseases can have a significant environmental impact by altering biodiversity and the health of natural ecosystems. Specifically, pests or infectious diseases that affect the food chain pose an imminent risk to the food industry by disrupting agricultural and livestock production, which could trigger significant economic and social consequences.

The increase in the world population poses several additional challenges. On one hand, there is the need to meet the demand for food, including grains, fruits, vegetables, and meat. On the other

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hand, there is the need to conserve food diversity, the health of ecosystems, and the sustainability of the agricultural industry (production, processing, and marketing of farm products) in general. This industry is heavily affected by various infectious diseases or pests that currently arise, impacting grain crops, fruit tree plantations, and vegetable cultivation intended for human consumption. These events reduce food production and quality [2]. Furthermore, the ongoing process of climate change exacerbates this situation by altering climatic patterns and the necessary environmental conditions for food production. Among the tangible negative effects is the increase in both the frequency and severity of plant diseases and pests, resulting in social, health, and economic impacts.

From an environmental standpoint, several countries constantly monitor and try to prevent the entry and spread of pests and diseases that affect fruit crops, native forests, and commercial plantations too, to protect and improve the conditions of these plantations and, at the same time, reduce high-cost economic and environmental impacts that would be generated if these events were to occur inevitably. Given the introduction of new species or diseases affecting plants, animals, fruit tree plantations, and others, along with their environmental impact, it becomes crucial to forecast potential disease or pest outbreaks.

From the perspective of mathematical modeling, it is known that the study and analysis of mathematical models describing the spread of infectious diseases or pests in plantations provide an important tool and support in decision-making and resource management associated with large plantations, as they allow for simulations, design of control strategies, and evaluation of potential epidemic outbreaks in the agricultural environment in case these events occur.

1.2 Related work

Currently, national and international researchers are continuously seeking to apply mathematical modeling and the design of numerical methods to understand, simulate, and predict the behavior of infectious diseases in humans, animals, on forest or fruit plantations, as well as to understand other events (geophysical, industrial, retail problems) of high social, environmental, and economic impact.

Such models are mathematically expressed as a system of partial differential equations (PDEs) dependent on space and time in the following form;

$$\partial_t \boldsymbol{u} + \partial_{x_1} \boldsymbol{f}_1(\boldsymbol{x}, \boldsymbol{u}) + \ldots + \partial_{x_d} \boldsymbol{f}_d(\boldsymbol{x}, \boldsymbol{u}) = \boldsymbol{\nabla} \cdot (\boldsymbol{B}(\boldsymbol{u}) \boldsymbol{\nabla} \boldsymbol{u}) + \boldsymbol{r}(\boldsymbol{u}), \quad \boldsymbol{x} \in \Omega \subset \mathbb{R}^d, \quad t > 0, \tag{1}$$

plus an appropriate initial and boundary condition. Here, t represents time, $\boldsymbol{x}$ is the spatial variable, the matrices $\boldsymbol{B}$ and $\boldsymbol{f}_1, \ldots, \boldsymbol{f}_d$ are given, and the vector of unknowns is $\boldsymbol{u} = (u_1, \ldots, u_N)^{\mathrm{T}}$, which typically represents the density of a species. If $\boldsymbol{B} = 0$ and $\boldsymbol{r} = 0$, a system of conservation laws in d dimensions is obtained, given by

$$\partial_t \boldsymbol{u} + \partial_{x_1} \boldsymbol{f}_1(\boldsymbol{x}, \boldsymbol{u}) + \ldots + \partial_{x_d} \boldsymbol{f}_d(\boldsymbol{x}, \boldsymbol{u}) = \boldsymbol{0}, \quad \boldsymbol{x} \in \Omega \subset \mathbb{R}^d, \quad t > 0. \tag{2}$$

The spatio-temporal spread of an infectious disease or pests on plantations can often be adequately described by reaction-diffusion systems of the form (1) in $d = 2$ dimensions, where convection is generally absent, i.e., $\boldsymbol{f}_i = \boldsymbol{0}$ for $i = 1, 2$, and the coordinates of the vector of unknowns $\boldsymbol{u}$ represent the densities of different epidemiological states or compartments of a population (see, for example [6, 10, 14, 17]).

These descriptions are generally based on models of known ordinary differential equations (ODEs) [3, 11, 15, 16, 24] (SIR, $SEIR$, $SEIRD$, $SVEIR$, among others). The $SEIR$ model distinguishes between compartments of susceptibles, exposed, infected, and recovered individuals, so in this case, the vector of unknowns is given by $\boldsymbol{u} = (S, E, I, R)^{\mathrm{T}}$, and the corresponding "reaction" term $\boldsymbol{r}(\boldsymbol{u})$ represents transmission rates, birth and death rates, and the proportion between different compartments. The diffusion term $\boldsymbol{\nabla} \cdot (\boldsymbol{B}(\boldsymbol{u}) \boldsymbol{\nabla} \boldsymbol{u})$ expresses that the movement of individuals is determined