FUNCTIONS

ABSOLUTE VALUES (XI)

Contents include:

- What are Absolute Values?
- Solving Absolute Value Equations Algebraically
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- Solving Absolute Value Equations Graphically

- What Are Absolute Values?

The absolute value of a number is denoted by the symbol $|\quad|$, and essentially refers to its magnitude. This is expressed as:

$$|a|, where\ a\ is\ a\ constant$$

Moreover, it may also be observed that:

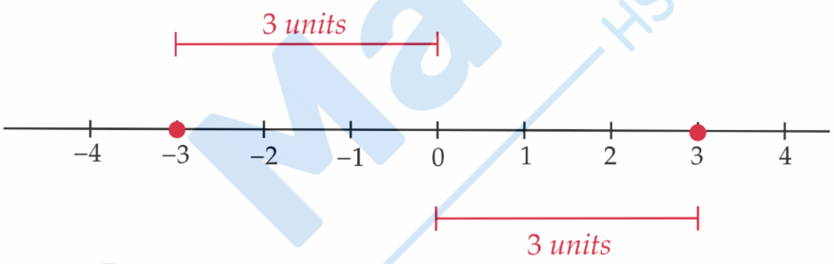
$$\sqrt{a^2} = |a|$$

Since a magnitude cannot be negative, when something is absolute valued, it is always positive (even if it was originally negative. In other words:

$$|a| \geq 0$$

For example, we say that $|-5| = 5$

We can imagine this concept of "magnitude" through a number line as well:



As can be seen, the 'magnitude' of -3 is the same as the 'magnitude' of 3, with both being 3 units. Therefore:

$$|-3| = |3|$$

- Solving Absolute Values Equations Algebraically

We now know that whenever we absolute value something, it is positive.

Now let's say you are told:

$$|a| = 2$$

The immediate thought would be to say that:

$$a = 2$$

However, we must also remember the negative case, where it is possible for:

$$a = -2$$

Hence, when solving for absolute values, we must remember that if:

$$|ax + b| = k$$

Where k is a constant

Then:

$$ax + b = \pm k$$

This means that **if** $ax + b > 0$ (positive), then $ax + b = k$

if $ax + b < 0$ (negative), then $ax + b = -k$

Remember: Whatever is inside the absolute value has 2 possibilities, either being positive or negative!

Example 1: Solve the equation $|x| = 5$ for x:

$$x = \pm 5$$

Example 2: Solve the equation $|x + 3| = 4$ for x:

Expanding the absolute value signs:

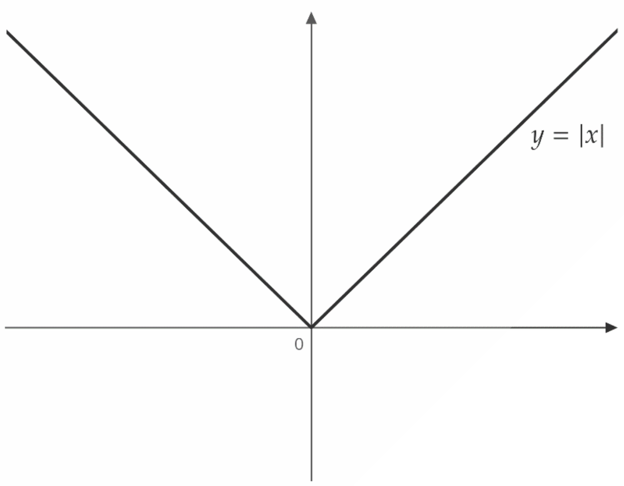
$$x + 3 = \pm 4$$

$$\therefore x = \pm 4 - 3$$
$$= 1 \; OR \; -7$$

- Sketching Absolute Value Graphs

Since whenever we absolute value something it cannot be negative, this means that an absolute value graph must lie above the $x -$ axis.

For example, the most basic sketch for absolute values is $y = |x|$:



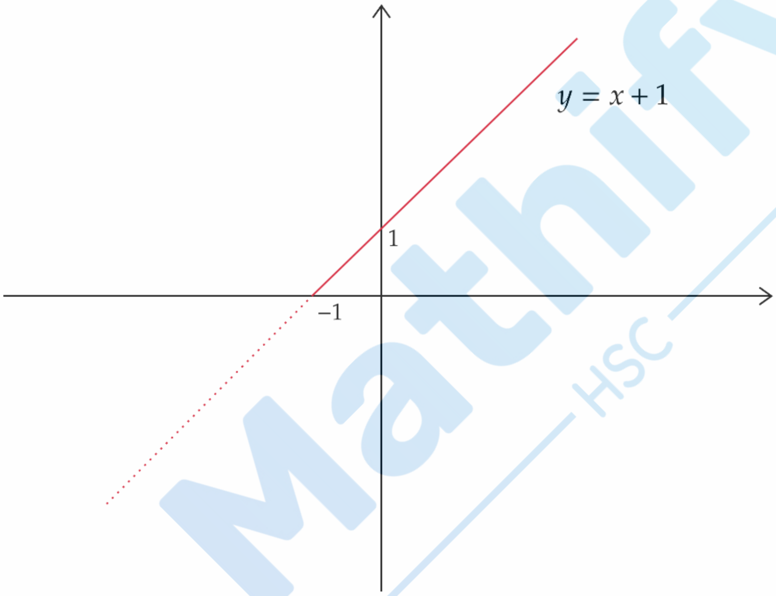
To sketch any absolute value graph given to you:

$$\textbf{\textit{Step 1: Imagine drawing the graph like normal}}$$

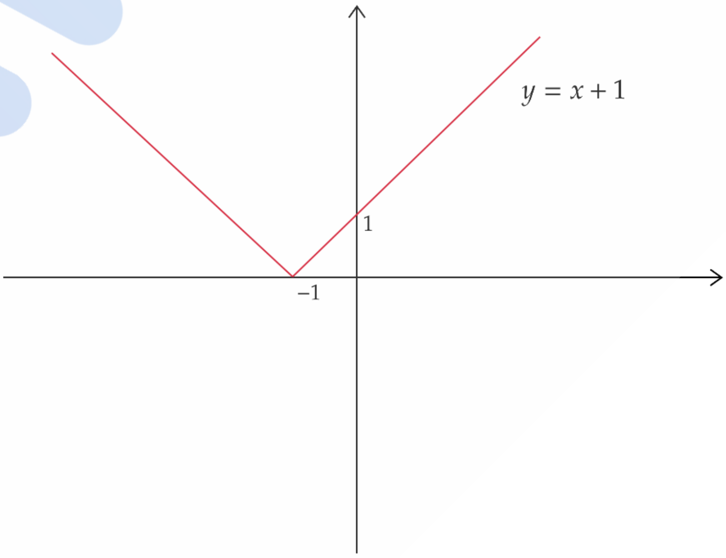
$$\textbf{\textit{Step 2: Reflect everything that's below the x − axis to above the x − axis}}$$

For example, let's say we are trying to draw the graph $y = |x + 1|$:

We would imagine its normal graph, $y = x + 1$ first:



Then, to get our final answer we would reflect its negative component (dotted part) about the $x -$ axis:



- Solving Absolute Value Equations Graphically

If we are asked to solve the absolute value equation:

$$|ax + b| = k$$

Apart from the algebraic method mentioned previously, we may also solve this graphically.

To solve $|ax + b| = k$:

$$\textbf{\textit{Step 1: Sketch the graph of }} |ax+b|$$

$$\textbf{\textit{Step 2: Sketch the horizontal line }} y = k$$

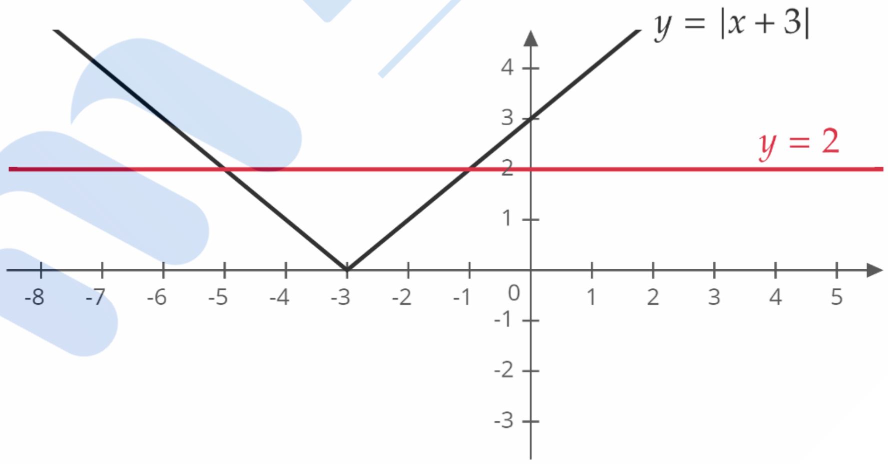
$$\textbf{\textit{Step 3: Find the points of intersections}}$$

The $x -$ values of the points of intersections will thus be the solutions to your equation!

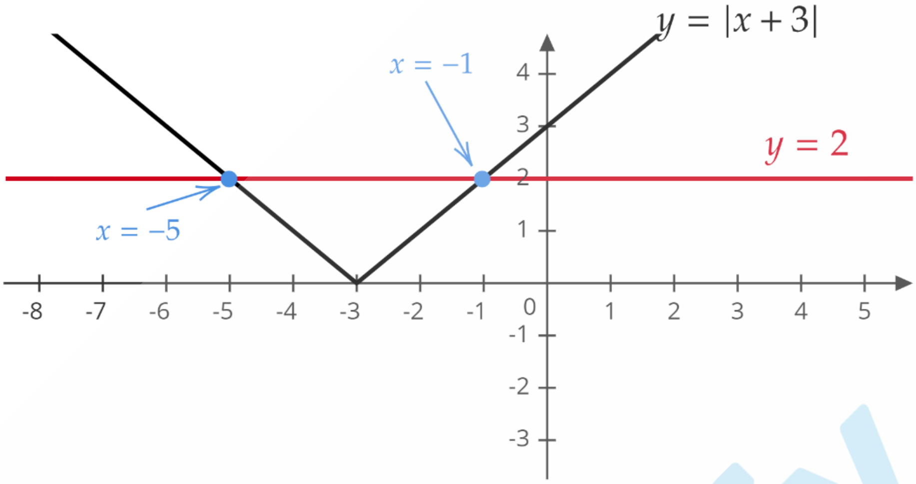
Note: When drawing your graphs, make sure to try draw them to scale so it's easier to find the points of intersections. Normally, it is recommended to solve algebraically rather than graphically as it is quicker, unless the question specifies which method to use.

Example 3: Solve the equation $|x + 3| = 2$ through graphical means

$$\textit{Step 1 \& 2: Sketching the two graphs}$$



$$\textit{Step 3: Finding the intersection points}$$



Therefore, $x = -1$ and $x = -5$ are our solutions!

Absolute Value Exercises

1. Write expressions for the following:

a) $\sqrt{x^2} + x$
b) $\sqrt{25} + \sqrt{(-5)^2}$
c) $|\sqrt{3} - 2|$
d) $\sqrt{(2x+5)^2}$

2. Solve the following equations for x algebraically:

a) $|x - 3| = 3$
b) $|2x - 5| = 5$
c) $|2 + x| = 5$
d) $|3x + 1| = 7$
e) $|5x + 1| = 4$

3. Sketch the following graphs:

a) $y = |x + 4|$
b) $y = |x - 5|$
c) $y = |3 - 2x|$
d) $y = |3x + 6|$

4. Solve the following equations for x through graphical methods:

a) $|3 - x| = 2$
b) $|2x + 1| = 3$

Absolute Value Exercise Solutions

1.

a) Since it is known that:

$$\sqrt{x^2} = |x|$$

Therefore:

$$\sqrt{x^2} + x = |x| + x$$

b)

$$\begin{aligned}\sqrt{25} + \sqrt{(-5)^2} &= \sqrt{25} + \sqrt{25} \\ &= 5 + 5 \\ &= 10\end{aligned}$$

c) Since we know that:

$$\sqrt{3} < 2$$

This means that $\sqrt{3} - 2 < 0$, in other words, it's a negative number. Therefore:

$$\left|\sqrt{3} - 2\right| = 2 - \sqrt{3}$$

This is because $|-a| = a, if\ a\ is\ a\ positive\ constant$

d) Since it is known that:

$$\sqrt{x^2} = |x|$$

For this equation:

$$\therefore \sqrt{(2x+5)^2} = |2x + 5|$$

2.

a) Getting rid of the absolute value symbols:

$$\therefore x - 3 = \pm 3$$

Hence, rearranging for x:

$$\begin{aligned}x &= 3 + 3\ OR - 3 + 3 \\ &= 6\ OR\ 0\end{aligned}$$

b) Removing the absolute value symbols:

$$\therefore 2x - 5 = \pm 5$$

Rearranging to make x the subject:

$$2x = 5 + 5 \; OR - 5 + 5$$
$$2x = 10 \; OR \; 0$$

$$\therefore x = 5 \; OR \; 0$$

c) Removing the absolute value symbols:

$$\therefore 2 + x = \pm 5$$

Rearranging to make x the subject:

$$x = 5 - 2 \; OR \; x = -5 - 2$$
$$= 3 \; OR - 7$$

d) Removing the absolute value symbols:

$$\therefore 3x + 1 = \pm 7$$

Rearranging to make x the subject:

$$3x = 7 - 1 \; OR - 7 - 1$$
$$= 6 \; OR - 8$$

$$\therefore x = 2 \; OR - \frac{8}{3}$$

e) Removing the absolute value symbols:

$$\therefore 5x + 1 = \pm 4$$

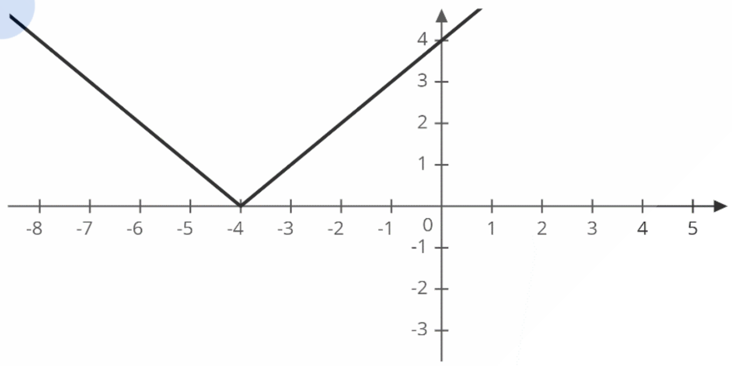
Rearranging to make x the subject:

$$5x = 4 - 1 \; OR - 4 - 1$$
$$= 3 \; OR - 5$$

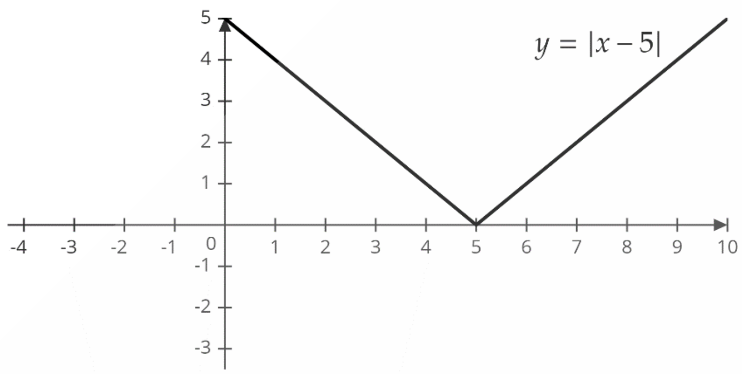
$$\therefore x = \frac{3}{5} \; OR - 1$$

3.

a) $y = |x + 4|$

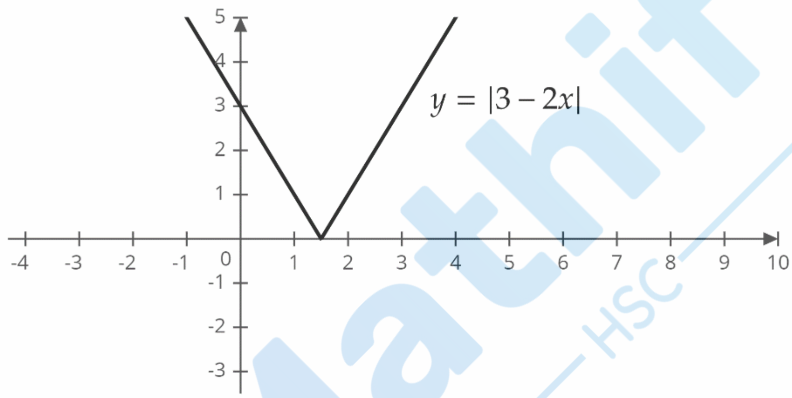


b) $y = |x - 5|$

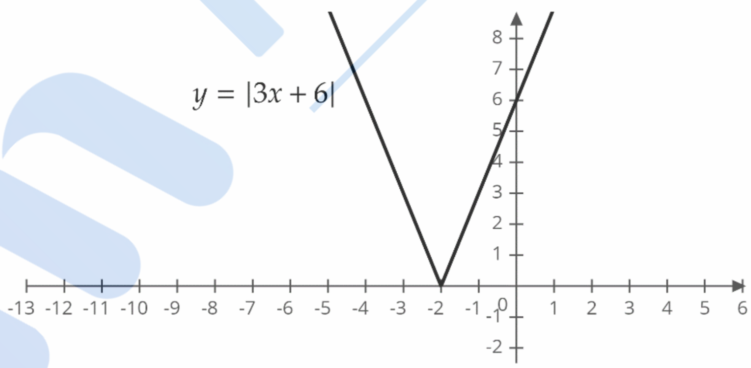


c) $y = |3 - 2x|$

This can be thought of as sketching the gram $y = |-2x + 3|$ instead to make it a bit easier:

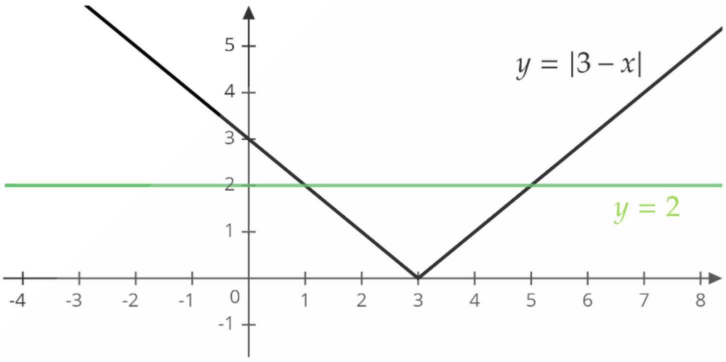


d) $y = |3x + 6|$



4.

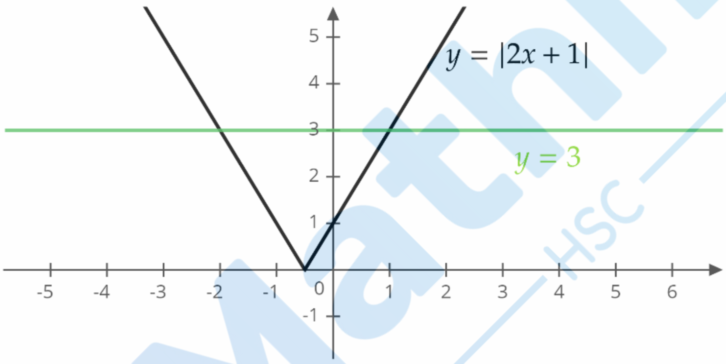
a) Since we are solving through graphical methods, sketching the two lines $y = |3 - x|$ and $y = 2$ on the same axes:



Therefore, looking at the intersection points, the solutions will be:

$$x = 1\ OR\ x = 5$$

b) Sketching the two lines $y = |2x + 1|$ and $y = 3$ on the same axes:



Therefore, looking at the intersection points, the solutions will be:

$$x = -2\ OR\ x = 1$$