

# FUNCTIONS

## FUNCTIONS AND RELATIONS (II)

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- The Four Types of Relations
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- Difference Between Functions and Relations

In general, both functions and relations are a set of ordered pairs/points  $(x, y)$ .

For functions:

No two  $y$  – values can have the same  $x$  – coordinate

However, for relations:

**Multiple**  $y$  – values can have the same  $x$  – coordinates.

Thus, all functions can be viewed as special types of relations but not all relations can be called functions.

- The Four Types of Relations

When we are given an independent variable  $x$  and a dependent variable  $y$ , there are four possible ways they can correlate to each other:

1. One  $x$  value corresponds to only one  $y$  value

We refer to this as **one to one**

2. One  $x$  value corresponds to many  $y$  values

We refer to this as **one to many**

3. Many  $x$  values correspond to one  $y$  value

We refer to this as **many to one**

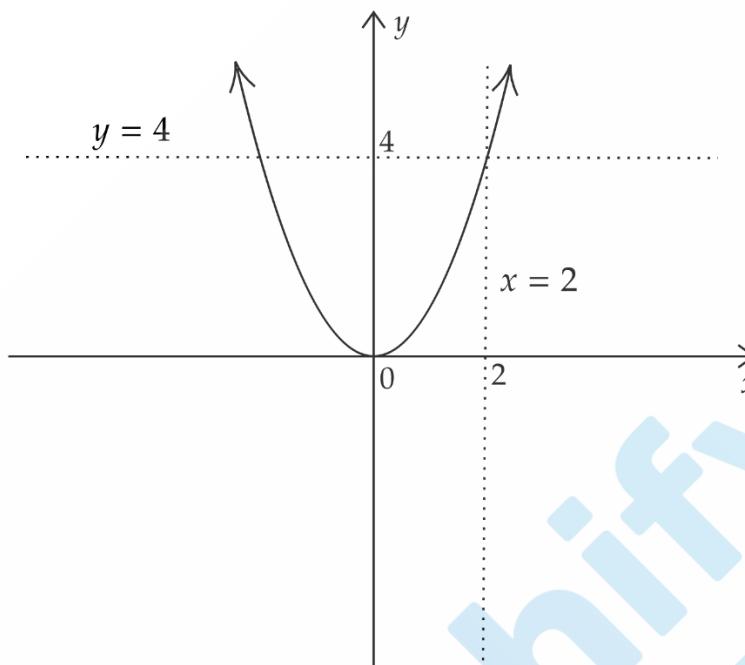
4. Many  $x$  values correspond to many  $y$  values

We refer to this as **many to many**

Functions are defined as either:

**one to one OR many to one**

For example:



$y = x^2$  is seen as *many to one* and is thus a function

- The Vertical and Horizontal Line Test

#### The vertical line test:

This can be done on any relation and essentially involves checking if any vertical line drawn on the graph intersects the graph more than once.

- If there are no vertical lines anywhere on the cartesian plane that intersect the graph more than once, **the graph passes the vertical line test**
- If there is any vertical line that intersects the graph more than once, the graph **fails the vertical line test**

#### The horizontal line test:

This is essentially the same as the vertical line test except the lines are horizontal.

## Uses of the line tests:

If a graph passes the vertical line test, it is a **function**

- If it also passes the horizontal line test, it is a **one-to-one relation**
- If it fails the horizontal line test, it is a **many-to-one relation**

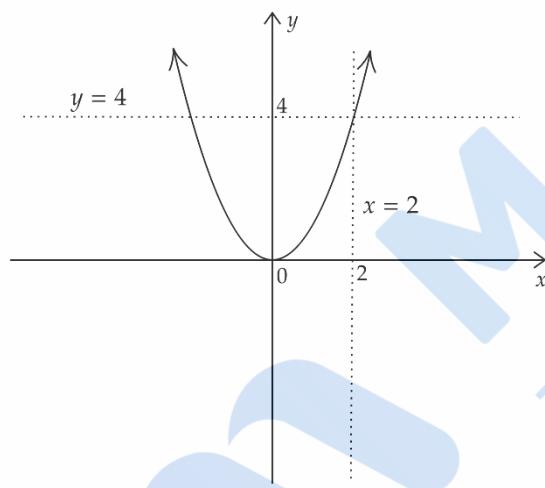
If a graph fails the vertical line test, it is **not a function**

- If it passes the horizontal line test, it is a **one-to-many relation**
- If it fails the horizontal line test, it is a **many-to-many relation**

Summarised in a table format:

Type	Vertical Line Test	Horizontal Line Test
One – to – one	Pass (So it's a function)	Pass
Many – to – one	Pass (So it's a function)	Fail
One – to – many	Fail	Pass
Many – to – many	Fail	Fail

For example:

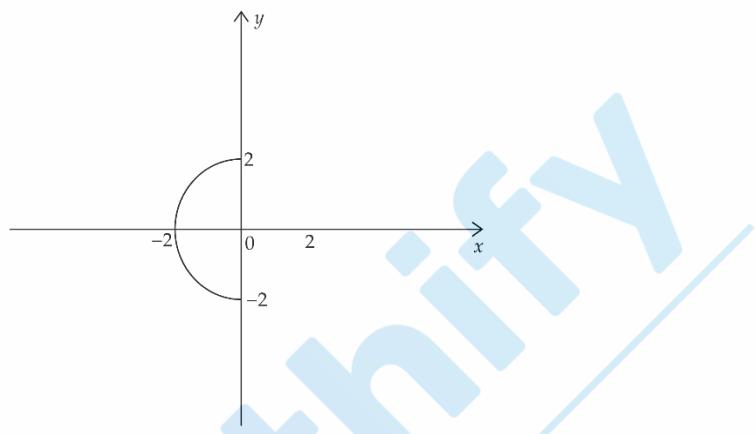


This graph passed the vertical line test (and is thus a function) however failed the horizontal line test. Hence it is **many to one**.

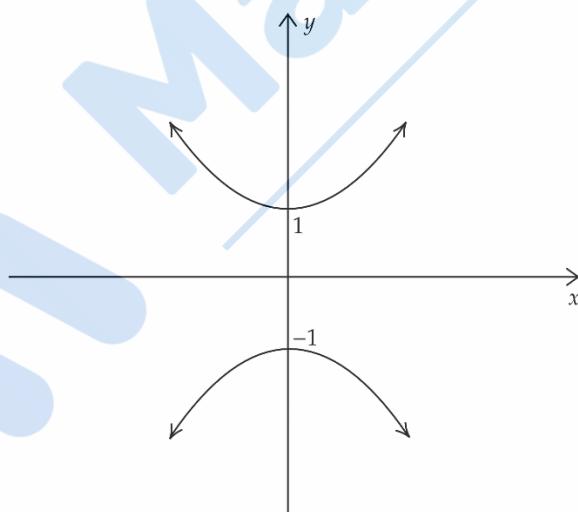
## **Functions and Relations Exercises**

1. State whether the following functions pass or fail the vertical line test, then whether they pass or fail the horizontal line test

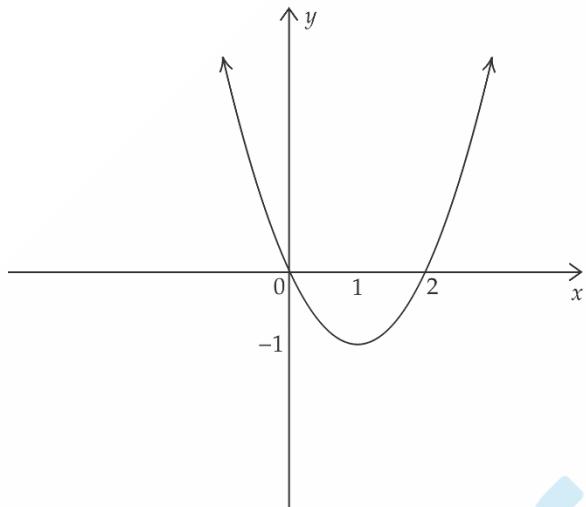
a)



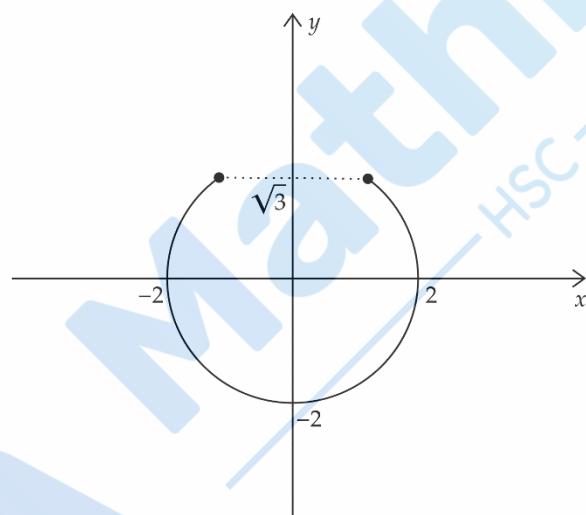
b)



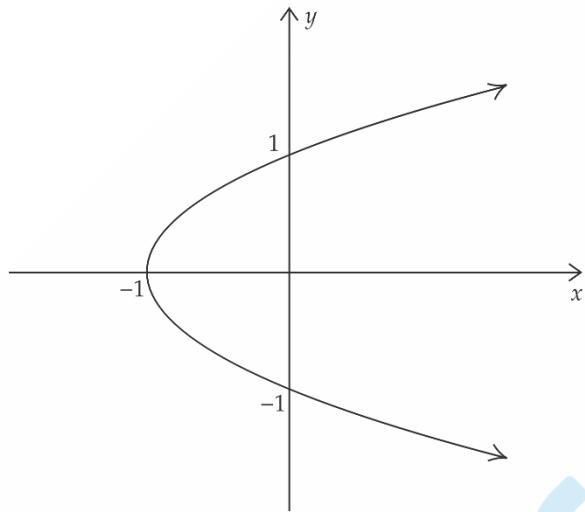
c)



d)

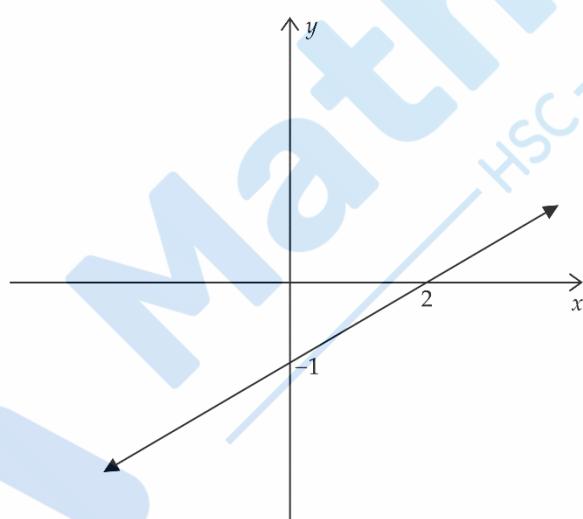


e)

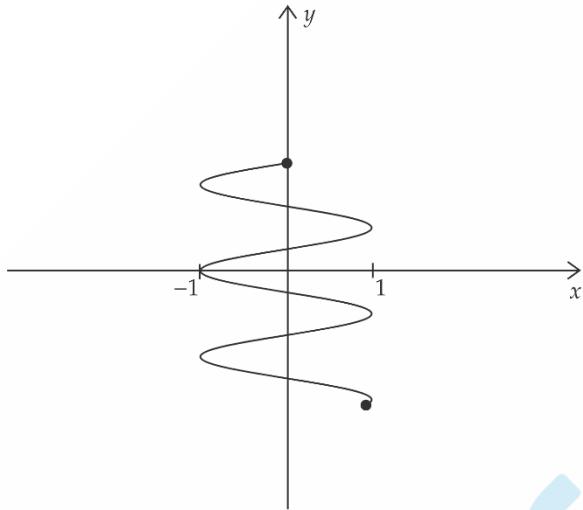


2. Determine which type of relation the following graphs are

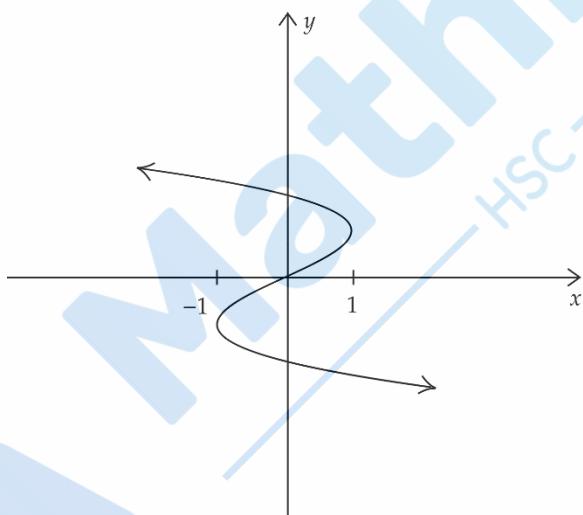
a)



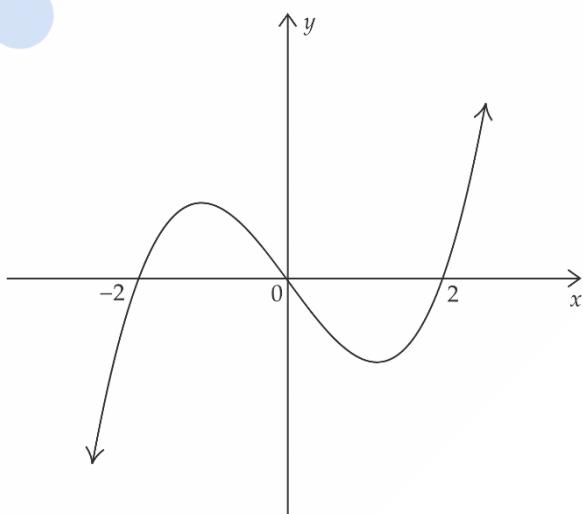
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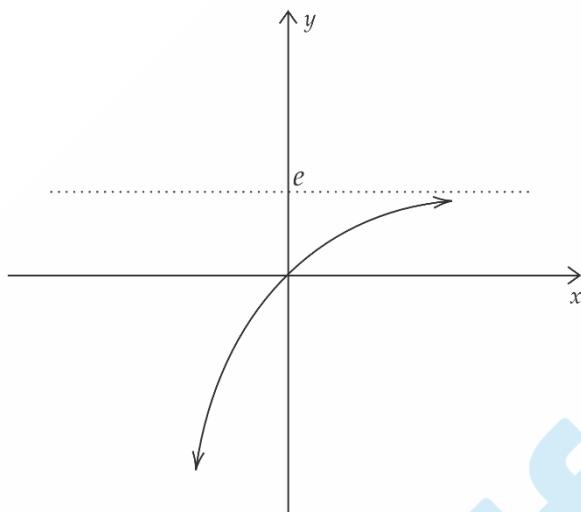
c)



d)



e)



3. Determine what type of relation the following relations are:

- a)  $(x - 3)^2 + (y + 1)^2 = 25$
- b)  $x = 2y + 3$
- c)  $y = \frac{3}{x}$
- d)  $y = x^2 - 2x + 1$
- e)  $x = y^2 - 4$

### **Functions and Relations Exercise Answers**

1.

- a) Vertical line test: fail, Horizontal line test: pass
- b) Vertical line test: fail, Horizontal line test: fail
- c) Vertical line test: pass, Horizontal line test: fail
- d) Vertical line test: fail, Horizontal line test: fail
- e) Vertical line test: fail, Horizontal line test: pass

2.

- a) Since it passes both the vertical and horizontal line test:

One – to – one

- b) Since it fails the vertical line test but passes the horizontal line test:

One – to - many

- c) Since it fails the vertical line test but passes the horizontal line test:

One – to - many

- d) Since it passes the vertical line test but fails the horizontal line test:

Many – to - one

- e) Since it passes both the vertical and horizontal line test:

One – to - one

3. For the following, a graph may be sketched to help visualise your answer

- a) Many – to – many
- b) One – to - one
- c) One – to - one
- d) Many – to - one
- e) One – to - many