FUNCTIONS

QUADRATICS AND PARABOLAS (V)

Contents include:

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- General Form of a Quadratic Function

A quadratic function is any function that can be written in the form:

$$f(x) = ax^2 + bx + c$$

Where:

a, b *and* c are constants, and $a \neq 0$

If the coefficient of x^2 is equal to 1, i.e., $a = 1$, we call the quadratic function **monic.** Otherwise, it is referred to as **non – monic.**

For example:

a) $y = x^2 + 2x + 1$ is a monic quadratic
b) $y = -x^2 - 2x + 3$ is a non – monic quadratic
c) $y = x^3 - x^2 + 2$ is not a quadratic

- Two types of Quadratic Function Graphs

The graph of a quadratic function is known as a parabola. In general, there are 2 types:

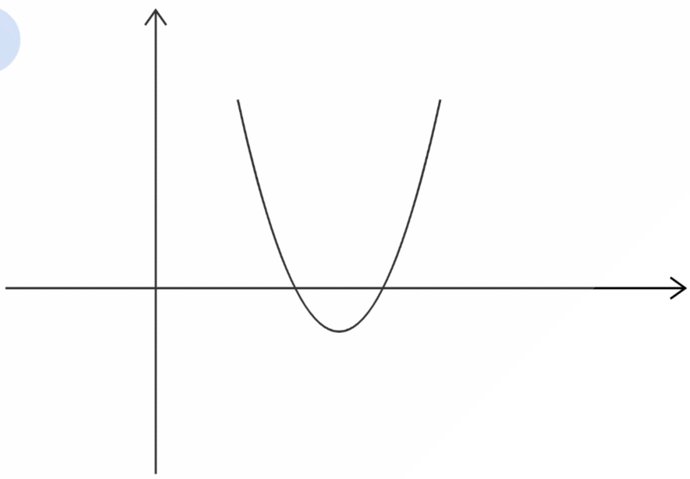
  ○ Concave up parabola

This one looks like a smiley face and occurs when the coefficient of x^2 is positive. In other words:

For a quadratic function $f(x) = ax^2 + bx + c$:

$$If\ a > 0, it\ is\ concave\ up$$

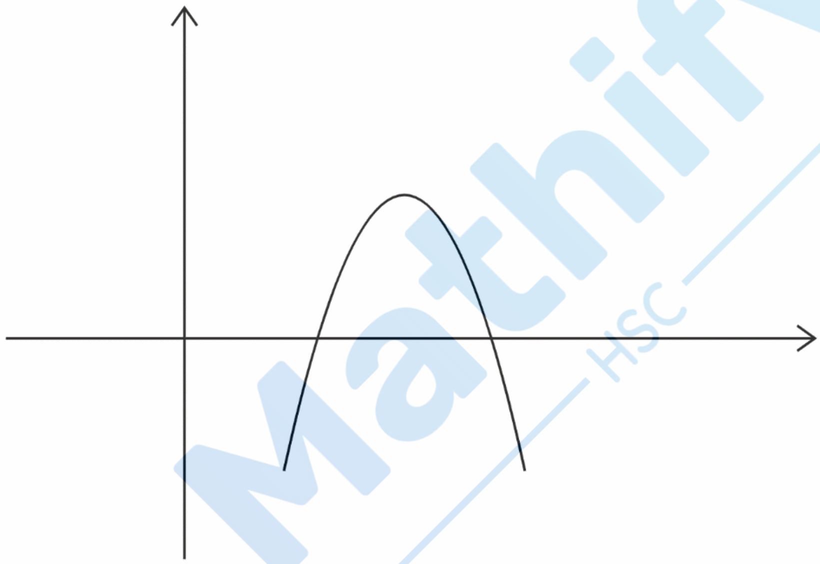
This is illustrated in the diagram below:



- ○ <u>Concave down parabola</u>

This one looks like a sad face and occurs when the coefficient of x^2 is negative. In other words:

> For a quadratic function $f(x) = ax^2 + bx + c$:
>
> $$If\ a < 0, it\ is\ concavve\ down$$
>
> This is illustrated in the diagram below:



- Zeroes and Roots of a Parabola

"*Zeroes*" and "*Roots*" generally mean the same thing, where although the definitions vary slightly, **markers typically don't care** which term is used.

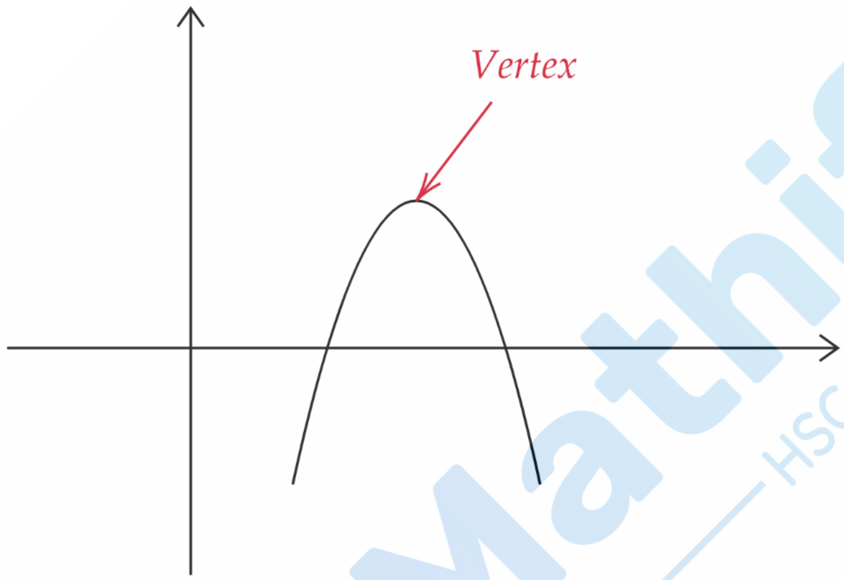
> **Roots:** are solutions of a quadratic **equation.** Solutions are $x -$ values for which $ax^2 + bx + c$ is equal to 0.
>
> **Zeroes:** are the $x - intercepts$ of a quadratic **function**
>
> A $x -$ intercept for a quadratic function occurs when $f(x) = 0$ such that $ax^2 + bx + c = 0$. This is known as a quadratic equation.

For example:

a) $x^2 - 2x + 1 = 0$ would have roots
b) $f(x) = x^2 - 2x + 1$ would have zeroes

- Finding the Vertex of Parabolas

The 'tip' or 'point' of a quadratic function's graph (a parabola) is known as the vertex. This is illustrated in the image below:



There are 2 methods we can use to find the $x-$coordinate of a parabola's **vertex:**

- **Method 1:** Find the average of the parabola's $x-$intercepts

The vertex always occurs midway between a parabola's $x-$intercepts, hence if α and β are roots of the parabola:

$$\boxed{x - coordinate\ of\ vertex = \frac{\alpha + \beta}{2}}$$

- **Method 2:** For a given quadratic $ax^2 + bx + c = 0$, use the equation:

$$\boxed{x - coordinate\ of\ vertex = -\frac{b}{2a}}$$

Example 1: Find the coordinates of the vertex for the parabola $y = x^2 - 3x + 2$

$$x - coordinate\ of\ vertex = -\frac{-3}{2(1)}$$

$$= \frac{3}{2}$$

When $x = \frac{3}{2}$:

$$\begin{aligned} y &= \left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) + 2 \\ &= \frac{9}{4} - \frac{9}{2} + 2 \\ &= -\frac{1}{4} \end{aligned}$$

Hence, the coordinate of the vertex is $\left(\frac{3}{2}, -\frac{1}{4}\right)$

Note: when asked to find the coordinate of something, always remember to also find the $y-$ coordinate!

- Finding $y-$ Intercept

To find the $y-$ intercept of a parabola:

$$Let\ x = 0 \text{ and solve for } y$$

Example 2: Find the $y-$ intercept of the parabola $y = x^2 - 3x + 2$

Solution:

To find the $y-$ intercept, let $x = 0$:

$$\begin{aligned} \therefore y &= 0 - 0 + 2 \\ &= 2 \end{aligned}$$

Hence, $y-$ intercept occurs at $(0, 2)$

- Finding $x-$ intercepts

To find the $x-$ intercepts of a parabola:

$$Let\ y = 0 \text{ and solve for } x$$

Afterwards, we will get an expression like:

$$ax^2 + bx + c = 0$$

Now there are two methods to solve for x in these scenarios:

Method 1: Factorise through cross multiplication or other methods

If we are able to factorise easily, let's say with roots α and β:

$$(x-\alpha)(x-\beta)=0$$

Then $x=\alpha$ and $x=\beta$ will be the $x-intercepts$

Method 2: Use the quadratic formula

The quadratic formula states that: $$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$ The solutions to this equation will be the $x-$ intercepts

We try to use this method for when we can't factorise quickly!

Example 2: Find the $x-$ intercepts of the quadratic $y=x^2-3x+2$

Using the factorisation method:

$$y=(x-2)(x-1)$$
$$\therefore x=2\ and\ x=1$$

Using the quadratic equation:

$$\therefore x=\frac{--3\pm\sqrt{(-3)^2-4(1)(2)}}{2(1)}$$
$$=\frac{3\pm\sqrt{1}}{2}$$
$$=2\ OR\ 1$$

- Sketching Quadratic Functions

Now combining what we've discussed so far into a simple step – by – step process for sketching quadratics:

Sketching parabolas involves 5 main steps:

$$\textbf{Step 1: Find the } \boldsymbol{x} \textbf{ intercepts}$$

$$\textbf{Step 2: Find the } \boldsymbol{y} \textbf{ intercept}$$

$$\textbf{Step 3: Determine which way up the parabola is}$$

$$\textbf{Step 4: Determine where the vertex is}$$

$$\textbf{Step 5: Sketch the parabola}$$

Example 3: Sketch the curve $y = x^2 - 6x + 5$

$$Step\ 1: Find\ x - intercepts$$

Factorising the quadratic through cross multiplication:

$$\begin{matrix} 1 & -1 \\ 1 & -5 \end{matrix}$$

$$\therefore y = (x-1)(x-5)$$

The $x -$ intercepts are therefore $x = 1$ and $x = 5$

$$Step\ 2: Find\ y - intercept$$

When $x = 0$:

$$\begin{aligned} y &= 0 - 0 + 5 \\ &= 5 \end{aligned}$$

$$Step\ 3: Determine\ concavity$$

Since the coefficient of x^2 is 1 and $1 > 0$, then therefore the parabola is concave up

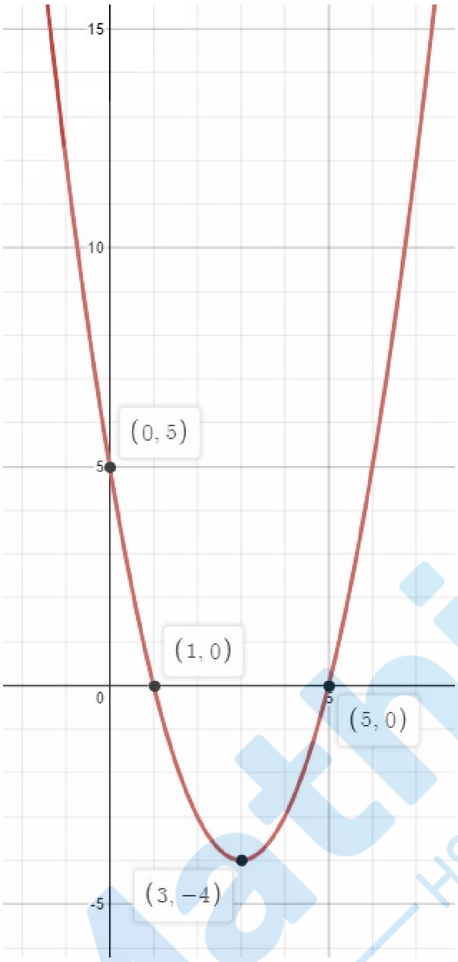
$$Step\ 4: Find\ the\ vertex$$

Since the $x - intercepts$ are $(1,0)$ and $(5,0)$. The $x -$ coordinate of the vertex will therefore be:

$$x = \frac{1+5}{2} = 3$$

$$\begin{aligned} \therefore y - coordinate &= 3^2 - 6(3) + 5 \\ &= -4 \end{aligned}$$

$$Step\ 5: Sketch$$

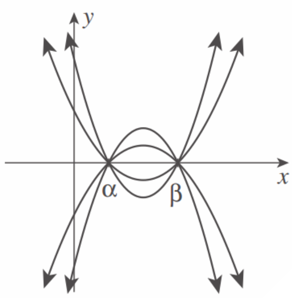


- Family of Quadratics with Given Zeroes

If we know that a quadratic $f(x)$ has the zeroes α and β, then we say that is equation must be in the form:

$$f(x) = a(x - \alpha)(x - \beta)$$

Where a is the coefficient of x^2. Hence, since we can take different values for a, ie. a can equal 1, 2, 3, etc., we say that this equation forms a ***family*** of quadratics that share common $x -$ intercepts. The below diagram represents a family of quadratics:



Example 4: The general form of a quadratic with zeroes $x = 3$ and $x = 5$ is $y = a(x-2)(x-8)$. Find the equation of such a quadratic for which:

a) The coefficient of x^2 is 3
b) The y −intercept is 32
c) The curve passes through the point $(3, -15)$
d) The vertex is $(5, 1)$

Solutions:

a)

$$\begin{aligned} y &= a(x-2)(x-8) \\ &= a(x^2 - 8x - 2x + 16) \\ &= a(x^2 - 10x + 16) \end{aligned}$$

Since we want the coefficient of x^2 to be 3:

$$\therefore a = 3$$

b)

$$\begin{aligned} y &= a(x^2 - 10x + 16) \\ &= ax^2 - 10ax + 16a \end{aligned}$$

Since we want $y -$ intercept to equal to 32:

$$16a = 32$$
$$\therefore a = 2$$

c)

$$y = a(x^2 - 10x + 16)$$

Since I want the curve to pass through $(3, -15)$, substituting these values in:

$$\begin{aligned} -15 &= a(3^2 - 10(3) + 16) \\ -15 &= 9a - 30a + 16a \\ -15 &= -5a \\ \therefore a &= 3 \end{aligned}$$

d)

$$\begin{aligned} y &= a(x^2 - 10x + 16) \\ &= ax^2 - 10ax + 16a \end{aligned}$$

Since I want the curve to pass through $(5, 1)$, substituting these values in:

$$\begin{aligned} 5 &= a(1)^2 - 10a(1) + 16a \\ 5 &= a - 10a + 16a \\ 5 &= 7a \end{aligned}$$

$$\therefore a = \frac{5}{7}$$

- Completing the Square for Monic Quadratics

Given a monic quadratic function $y = x^2 + bx + c$, we can change it into the form:

$$y = (x - h)^2 + k$$

This process is known as "completing the square".

The steps to complete the square when we are given a monic quadratic of the form $y = x^2 + bx + c$ are as follows:

$$\boldsymbol{Step\ 1: Add\ and\ subtract\ \left(\frac{b}{2}\right)^2}$$

In other words, we say that:

$$\begin{aligned} y &= x^2 + bx + c \\ &= x^2 + bx + \left(\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c \end{aligned}$$

$$\boldsymbol{Step\ 2: Make\ the\ squared\ expression}$$

$$\begin{aligned} y &= x^2 + bx + \left(\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c \\ &= \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c \end{aligned}$$

Note: Feel free to check your work by expanding your final expression and seeing if it eventually simplifies to the original quadratic!

Example 5: Complete the square for the quadratic expression $y = x^2 + 8x + 2$

Solution:

$$Step\ 1: Add\ and\ subtract\ \left(\frac{8}{2}\right)^2$$

$$\begin{aligned} y &= x^2 + 8x + \left(\frac{8}{2}\right)^2 - \left(\frac{8}{2}\right)^2 + 2 \\ &= x^2 + 8x + 16 - 16 + 2 \end{aligned}$$

$$Step\ 2: Make\ the\ squared\ expression$$

$$\therefore y = (x + 4)^2 - 14$$

- Completing the Square for Non – Monic Quadratics

Given a non – monic quadratic $y = ax^2 + bx + c$, the process of completing the square is much the same except for an extra step in the beginning factorising out a.

The steps to complete the square when we are given a non – monic quadratic of the form $y = ax^2 + bx + c$ are as follows:

$$\boldsymbol{Step\ 1: Factorise\ out\ a}$$

Therefore:

$$y = ax^2 + bx + c$$
$$= a\left(x^2 + \frac{b}{a}x\right) + c$$

Notice that there is no need to factorise our constant c!

$$\boldsymbol{Step\ 2: Add\ and\ subtract\ \left(\frac{b}{2a}\right)^2\ inside\ the\ bracket}$$

$$y = a\left(x^2 + \frac{b}{a}x\right) + c$$
$$= a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right) + c$$

$$\boldsymbol{Step\ 3: Make\ the\ squared\ expression\ insid\ the\ bracket}$$

$$y = a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right) - a \times \left(\frac{b}{2a}\right)^2 + c$$
$$= a\left(x + \frac{b}{2a}\right)^2 - a \times \frac{b^2}{4a^2} + c$$
$$= a\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a} + c$$

Example 6: Complete the square for the quadratic expression $y = 2x^2 + 16x - 3$

Solution:

$$Step\ 1: Factorise\ out\ 2$$

$$y = 2(x^2 + 8x) - 3$$

$$Step\ 2: Add\ and\ subtract\ \left(\frac{8}{2}\right)^2\ inside\ the\ bracket$$

$$y = 2\left(x^2 + 8x + \left(\frac{8}{2}\right)^2 - \left(\frac{8}{2}\right)^2\right) - 3$$
$$= 2(x^2 + 8x + 16 - 16) - 3$$

$$Step\ 3: Make\ the\ squared\ expression$$

$$\therefore y = 2(x^2 + 8x + 16) - 32 - 3$$
$$= 2(x + 4)^2 - 35$$

- Graphing a Quadratic by Completing the Square

Another way of sketching parabolas is through completing the square such that we get it in the form:

$$y = a(x - h)^2 + k$$

Moreover, if we get this form, then we know that the vertex is $V(h, k)$

The proof for this is as follows:

Since $(x - h)^2 \geq 0$, then the maximum point (if $k > 0$) or minimum point (if $k < 0$) will occur when $(x - h)^2 = 0$.

Thus:

Since $(x - h)^2 = 0$ when $x = h$, we say that the vertex V has $x - coordinate\ h$. Substituting this into the equation, then the $y - coordinate$ when $x = h$ will be $y = k$.

Hence, the coordinate of the vertex is $V(h, k)$

Determining the intercepts is done the same way as before:

To find the $x -$ intercepts, substitute $y = 0$ and solve for x

To find the $y -$ intercepts, substitute $x = 0$ and solve for y

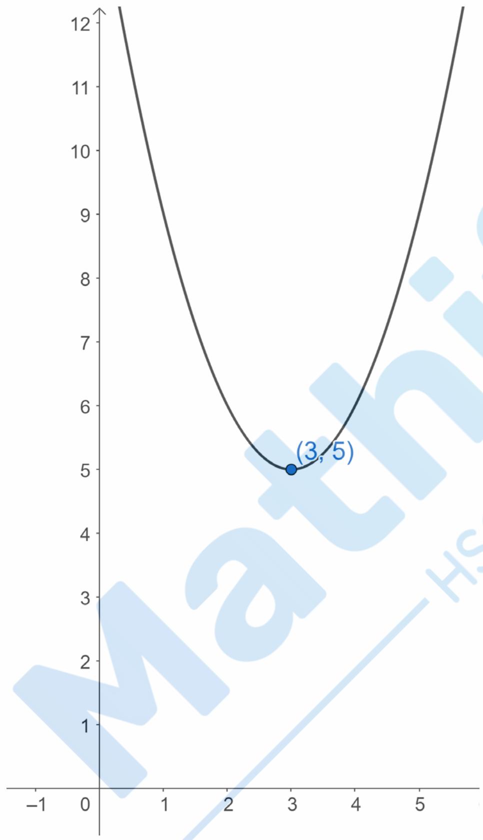
Example 7: Sketch the graph of $y = (x-3)^2 + 5$

Solution:

Using the vertex form, we can determine that the vertex of this parabola is:

$$V(3,5)$$

Therefore, sketching the parabola:



- The Discriminant and its Significance

For any quadratic expression $y = ax^2 + bx + c$, we can find the zeroes through using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The quantity $b^2 - 4ac$ inside the square root is referred to as the ***discriminant*** with the symbol Δ and is a particularly important concept to remember as the ***discriminant*** helps us to determine how many zeroes we have in a quadratic expression:

- If $\Delta > 0$ then there are two zeroes, since when we square root a positive number, there are two zeroes
- If $\Delta = 0$ then there is only one zero since the square root of 0 is 0. Moreover, this would also mean that the root is also the vertex
- If $\Delta < 0$ then there are no zeroes since we can't square root a negative value.

Example 8: Given that $x^2 - (k-2)x + 1 = 0$, find the value(s) of k such that:

a) The equation has two distinct roots

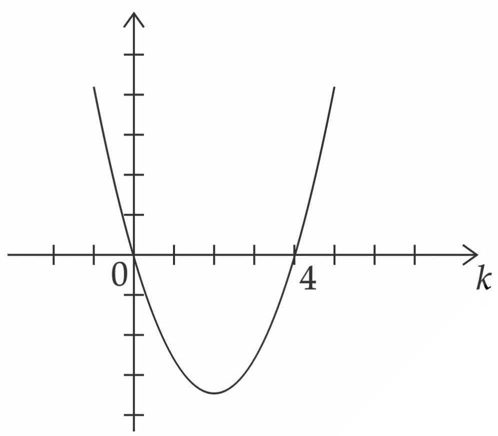
Considering the discriminant for this equation:

$$\begin{aligned} \Delta &= (-(k-2))^2 - 4 \times 1 \times 1 \\ &= (k-2)^2 - 4 \\ &= k^2 - 4k + 4 - 4 \\ &= k^2 - 4k \end{aligned}$$

To have two distinct roots, the discriminant must be greater than zero. Therefore:

$$\Delta > 0$$

Considering the graph of the discriminant $\Delta = k^2 - 4k$:



Therefore, to ensure that $\Delta > 0$:

$$k < 0 \; OR \; k > 4$$

b) The equation has one distinct root

An equation has one distinct root when:

$$\Delta = 0$$

Using the graph for the discriminant shown in part a):

$$\therefore k = 0 \; OR \; k = 4$$

c) No solutions

An equation has no solutions when:

$$\Delta < 0$$

Using the graph for the discriminant shown in part a):

$$0 < k < 4$$

Quadratics and Parabolas Exercises

1. Sketch the following parabolas, making sure to include any intercepts as part of your sketch:

a) $y = (x-2)^2$
b) $y = (x-1)(x+1)$
c) $y = -(x+3)(x-1)$
d) $y = (x-4)(x+2)$
e) $y = -\left(x - \frac{1}{2}\right)(x+3)$

2. Factorise each of the following quadratic equations:

a) $f(x) = x^2 - 9$
b) $f(x) = -x^2 - 5x$
c) $f(x) = x^2 - 5x + 6$
d) $f(x) = 8 - 2x - x^2$
e) $f(x) = -x^2 + 2x + 3$
f) $f(x) = 2x^2 + 7x + 5$
g) $f(x) = 3x^2 + 3x - 18$
h) $f(x) = -4x^2 + 7x - 3$

3. Sketch the following parabolas, making sure to label the vertex:

a) $y = (x - 1)^2 + 3$

b) $y = -(x + 3)^2 + 1$

c) $y = (x - 2)^2 + 5$

d) $y = 2(x - 5)^2 + 3$

e) $y = -3(x + 2)^2 - 1$

4. Complete the square for each of the following quadratic equations:

a) $f(x) = x^2 + 4x - 5$

b) $f(x) = x^2 + 6x + 2$

c) $f(x) = 2x^2 - 12x + 3$

d) $f(x) = -2x^2 - 2x + 7$

e) $f(x) = 3x^2 + 18x - 5$

f) $f(x) = -5x^2 - 20x + 12$

5. Find the discriminant for each of the following quadratic functions, then determine how many zeroes they have (you do not need to find the value of the zeroes)

a) $y = 3x^2 + 5x + 1$

b) $y = x^2 - 4x + 8$

c) $y = 2x^2 - 6x + 9$

d) $y = -x^2 - 2x + 3$

e) $y = 2x^2 - 4x + 2$

f) $y = 5x^2 - 2x - 3$

\

6. Use the quadratic formula to find the roots of the following quadratic equations, giving your answer to the nearest 2 decimal places:

a) $x^2 - 5x + 2 = 0$

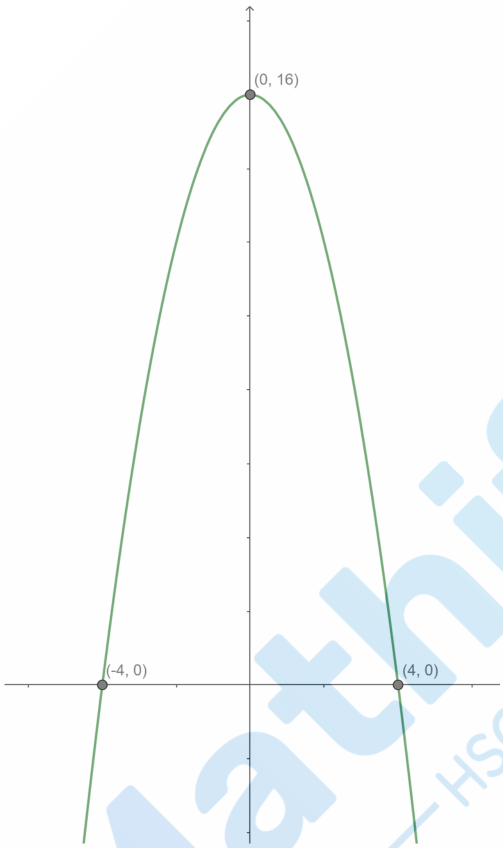
b) $-2x^2 - \frac{3}{2}x + 9 = 0$

c) $3x^2 + 10x - 11 = 0$

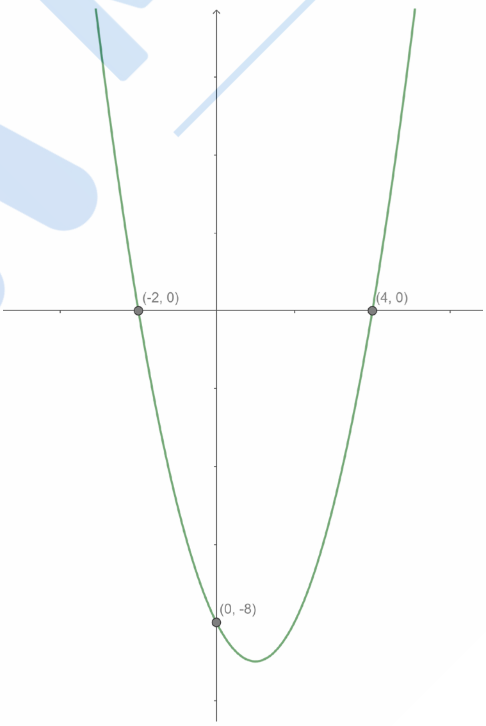
d) $-4x^2 + 5x + 3 = 0$

7. Find the equation of the parabolas sketched below:

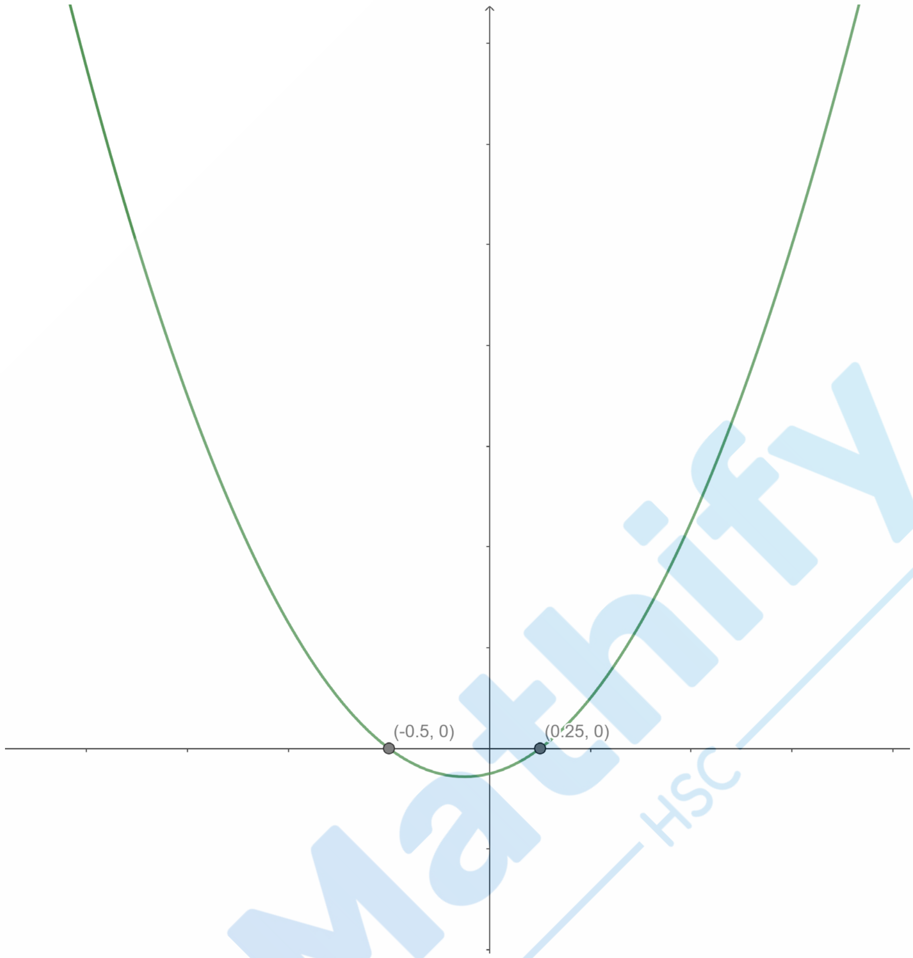
a)



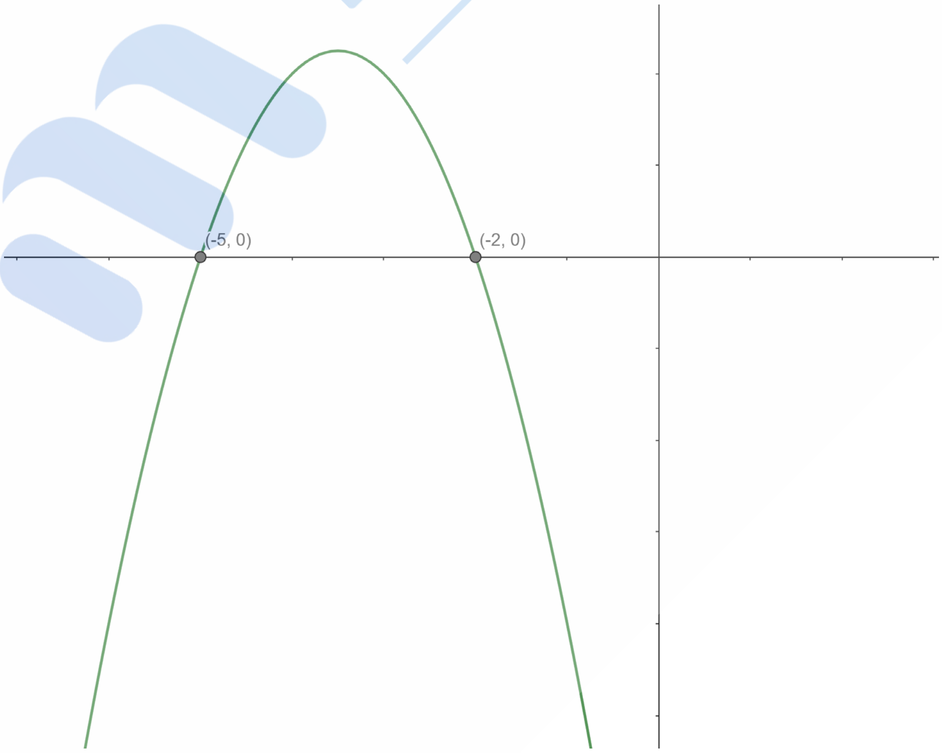
b)



c)



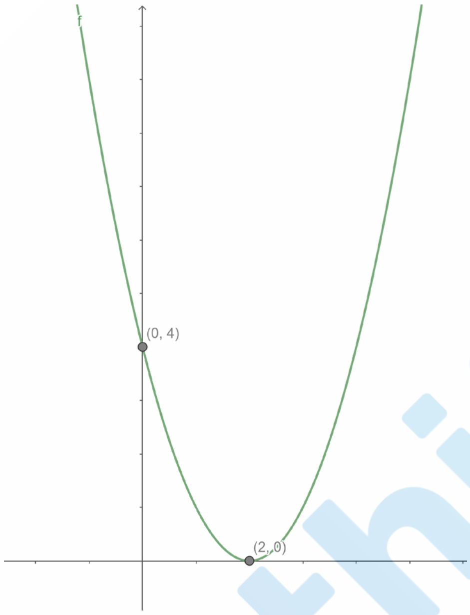
d)



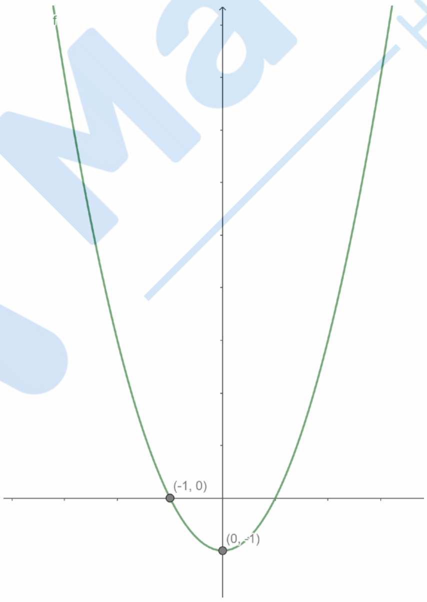
Quadratics and Parabolas Exercise Answers

1.

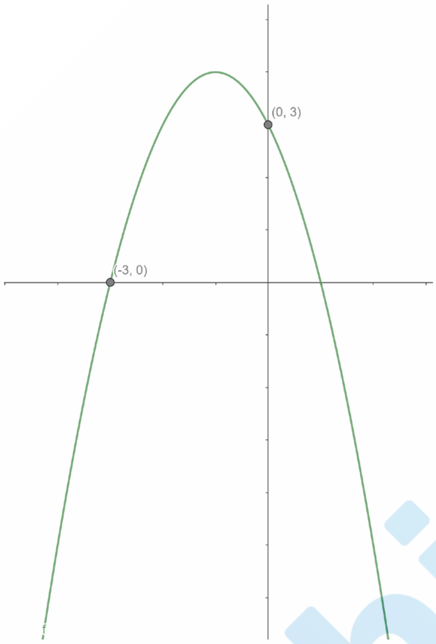
a)



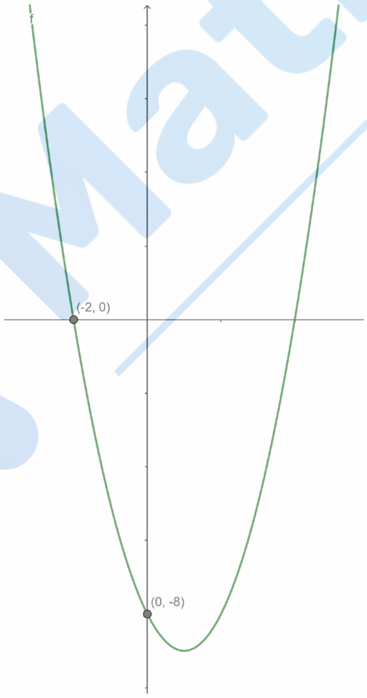
b)



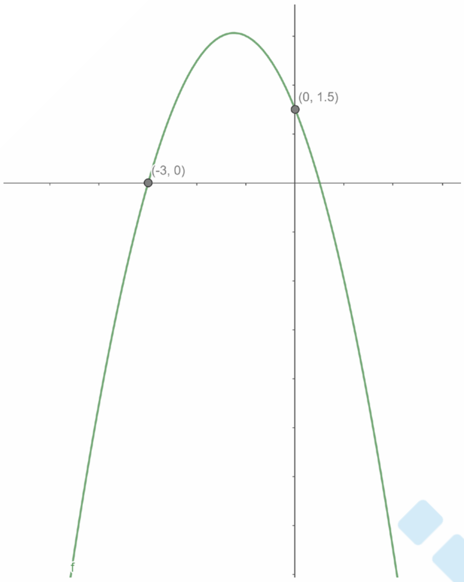
c)



d)



e)



2.
a)

$$f(x) = x^2 - 9$$
$$= (x+3)(x-3)\ [difference\ of\ squares]$$

b)

$$f(x) = -x^2 - 5x$$
$$= -x(x+5)$$

c)

$$f(x) = x^2 - 5x + 6$$

Cross multiplying:

$$\begin{array}{cc} 1 & -3 \\ 1 & -2 \end{array}$$

$$\therefore f(x) = (x-3)(x-2)$$

d)

$$f(x) = 8 - 2x - x^2$$
$$= -x^2 - 2x + 8$$

Cross multiplying:

$$\begin{array}{cc} -1 & 2 \\ 1 & 4 \end{array}$$

$$\therefore f(x) = (-x+2)(x+4)$$

e)

$$f(x) = -x^2 + 2x + 3$$

Cross multiplying:

$$\begin{matrix} -1 & 3 \\ 1 & 1 \end{matrix}$$

$$\therefore f(x) = (-x + 3)(x + 1)$$

f)

$$f(x) = 2x^2 + 7x + 5$$

Cross multiplying:

$$\begin{matrix} 2 & 5 \\ 1 & 1 \end{matrix}$$

$$\therefore f(x) = (2x + 5)(x + 1)$$

g)

$$f(x) = 3x^2 + 3x - 18$$

Cross multiplying:

$$\begin{matrix} 3 & 9 \\ 1 & -2 \end{matrix}$$

$$\therefore f(x) = (3x + 9)(x - 2)$$

h)

$$f(x) = -4x^2 + 7x - 3$$

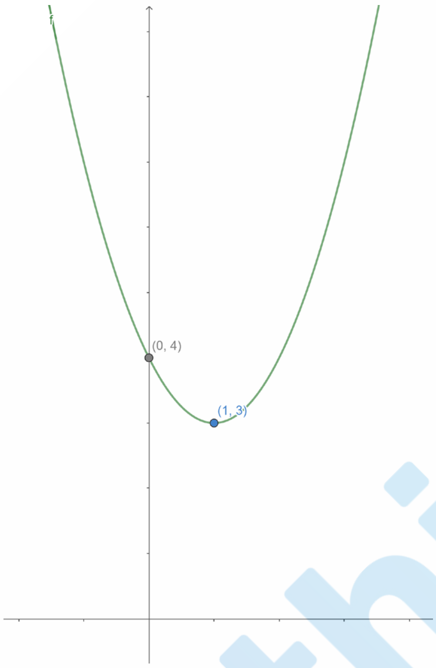
Cross multiplying:

$$\begin{matrix} -1 & 1 \\ 4 & -3 \end{matrix}$$

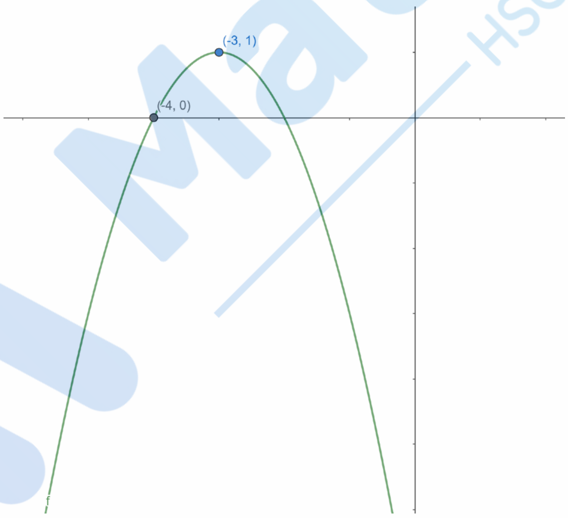
$$\therefore f(x) = (-x + 1)(4x - 3)$$

3.

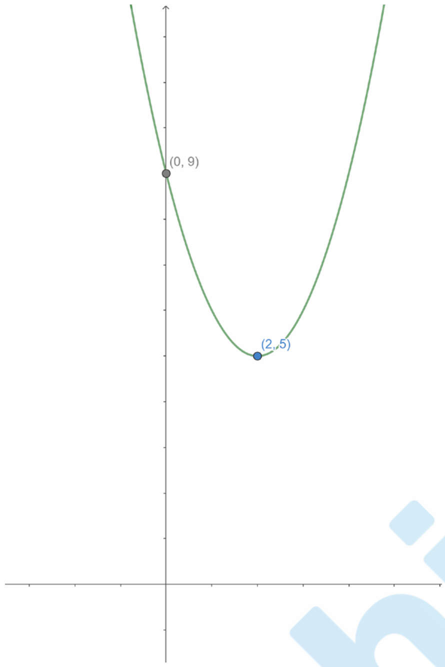
a)



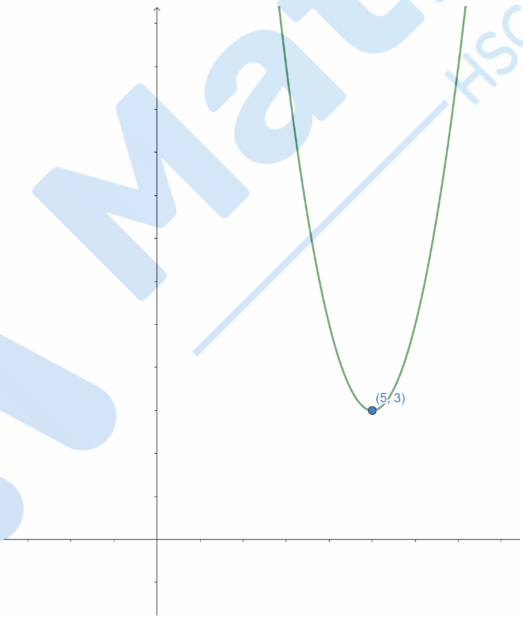
b)



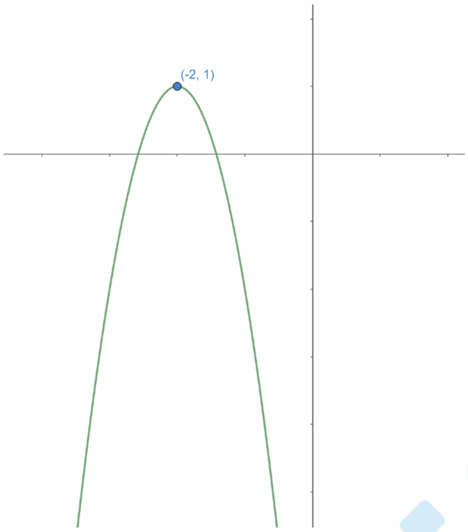
c)



d)



e)



4.
a)

$$f(x) = x^2 + 4x - 5$$

Completing the square:

$$\begin{aligned} f(x) &= x^2 + 4x + \left(\frac{4}{2}\right)^2 - \left(\frac{4}{2}\right)^2 - 5 \\ &= x^2 + 4x + 4 - 4 - 5 \\ &= (x+2)^2 - 9 \end{aligned}$$

b)

$$f(x) = x^2 + 6x + 2$$

Completing the square:

$$\begin{aligned} f(x) &= x^2 + 6x + \left(\frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2 + 2 \\ &= x^2 + 6x + 9 - 9 + 2 \\ &= (x+3)^2 - 7 \end{aligned}$$

c)

$$f(x) = 2x^2 - 12x + 3$$

Completing the square:

$$\begin{aligned} f(x) &= 2(x^2 - 6x) + 3 \\ &= 2\left(x^2 - 6x + \left(\frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2\right) + 3 \\ &= 2(x^2 - 6x + 9) - 18 + 3 \\ &= 2(x-3)^2 - 15 \end{aligned}$$

d)

$$f(x) = -2x^2 - 2x + 7$$

Completing the square:

$$\begin{aligned} f(x) &= -2(x^2 + x) + 7 \\ &= -2\left(x^2 + x + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2\right) + 7 \\ &= -2\left(x^2 + x + \frac{1}{4}\right) + 2 \times \frac{1}{4} + 7 \\ &= -2\left(x + \frac{1}{2}\right)^2 + 7\frac{1}{2} \end{aligned}$$

e)

$$f(x) = 3x^2 + 18x - 5$$

Completing the square:

$$\begin{aligned} f(x) &= 3(x^2 + 6x) - 5 \\ &= 3\left(x^2 + 6x + \left(\frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2\right) - 5 \\ &= 3(x^2 + 6x + 9) - 3 \times 9 - 5 \\ &= 3(x + 3)^2 - 32 \end{aligned}$$

f)

$$f(x) = -5x^2 - 20x + 12$$

Completing the square:

$$\begin{aligned} f(x) &= -5(x^2 + 4x) + 12 \\ &= -5\left(x^2 + 4x + \left(\frac{4}{2}\right)^2 - \left(\frac{4}{2}\right)^2\right) + 12 \\ &= -5(x^2 + 4x + 4) + 5 \times 4 + 12 \\ &= -5(x + 2)^2 + 32 \end{aligned}$$

5.
a)

$$y = 3x^2 + 5x + 1$$

$$\begin{aligned} \Delta &= b^2 - 4ac \\ &= 5^2 - 4 \times 3 \times 1 \\ &= 25 - 12 \\ &= 13 \end{aligned}$$

$$\therefore \Delta > 0$$

Since $\Delta > 0$, this means that there are 2 zeroes!

b)

$$y = x^2 - 4x + 8$$

$$\begin{aligned} \Delta &= b^2 - 4ac \\ &= (-4)^2 - 4 \times 1 \times 8 \\ &= 16 - 32 \\ &= -16 \end{aligned}$$

$$\therefore \Delta < 0$$

Since $\Delta < 0$, this means that there are no zeroes!

c)

$$y = 2x^2 - 6x + 9$$

$$\begin{aligned} \Delta &= b^2 - 4ac \\ &= (-6)^2 - 4 \times 2 \times 9 \\ &= 36 - 72 \\ &= -36 \end{aligned}$$

$$\therefore \Delta < 0$$

Since $\Delta < 0$, this means that there are no zeroes!

d)

$$y = -x^2 - 2x + 3$$

$$\begin{aligned} \Delta &= b^2 - 4ac \\ &= (-2)^2 - 4 \times -1 \times 3 \\ &= 4 + 12 \\ &= 16 \end{aligned}$$

$$\therefore \Delta > 0$$

Since $\Delta > 0$, this means that there are two zeroes!

e)

$$y = 2x^2 - 4x + 2$$

$$\begin{aligned} \Delta &= b^2 - 4ac \\ &= (-4)^2 - 4 \times 2 \times 2 \\ &= 16 - 16 \\ &= 0 \end{aligned}$$

$$\therefore \Delta = 0$$

Since $\Delta = 0$, this means that there is one zero!

f)

$$y = 5x^2 - 2x - 3$$

$$\begin{aligned}\Delta &= b^2 - 4ac \\ &= (-2)^2 - 4 \times 5 \times -3 \\ &= 4 + 60 \\ &= 64\end{aligned}$$

$$\therefore \Delta > 0$$

Since $\Delta > 0$, this means that there are 2 zeroes!

6.

a)

$$x^2 - 5x + 2 = 0$$

Using the quadratic formula:

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{- -5 \pm \sqrt{(-5)^2 - 4(1)(2)}}{2(1)} \\ &= \frac{5 \pm \sqrt{25 - 8}}{2} \\ &= \frac{5 \pm \sqrt{17}}{2} \\ &\approx 4.56\ OR\ 0.44\ (nearest\ 2\ d.p.)\end{aligned}$$

Hence, the roots are 4.56 and 0.44

b)

$$-2x^2 - \frac{3}{2}x + 9 = 0$$

Using the quadratic formula:

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{- -\frac{3}{2} \pm \sqrt{\left(-\frac{3}{2}\right)^2 - 4(-2)(9)}}{2(-2)} \\ &= \frac{\frac{3}{2} \pm \sqrt{\frac{9}{4} + 72}}{-4} \\ &\approx -2.53\ OR\ 1.78\ (nearest\ 2\ d.p.)\end{aligned}$$

Hence, the roots are -2.53 and 1.78

c)

$$3x^2 + 10x - 11 = 0$$

Using the quadratic formula:

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-10 \pm \sqrt{10^2 - 4(3)(-11)}}{2(3)} \\ &= \frac{-10 \pm \sqrt{232}}{6} \\ &\approx 0.87\ OR - 4.21\ (nearest\ 2\ d.p.) \end{aligned}$$

Hence, the roots are 0.87 and -4.21

d)

$$-4x^2 + 5x + 3 = 0$$

Using the quadratic formula:

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-5 \pm \sqrt{(5)^2 - 4(-4)(3)}}{2(-4)} \\ &= \frac{-5 \pm \sqrt{73}}{-8} \\ &\approx -0.44\ OR\ 1.69\ (nearest\ 2\ d.p.) \end{aligned}$$

Hence, the roots are -0.44 and 1.69

7.

a)

$$y = (x+4)(x-4)$$

b)

$$y = (x+2)(x-4)$$

c)

$$y = \left(x + \frac{1}{2}\right)\left(x - \frac{1}{4}\right)$$

d)

$$y = (x+5)(x+2)$$