FUNCTIONS

CIRCLES AND SEMI - CIRCLES (XIV)

Contents include:

- Equation of a Circle
- Equation of Semi - Circles

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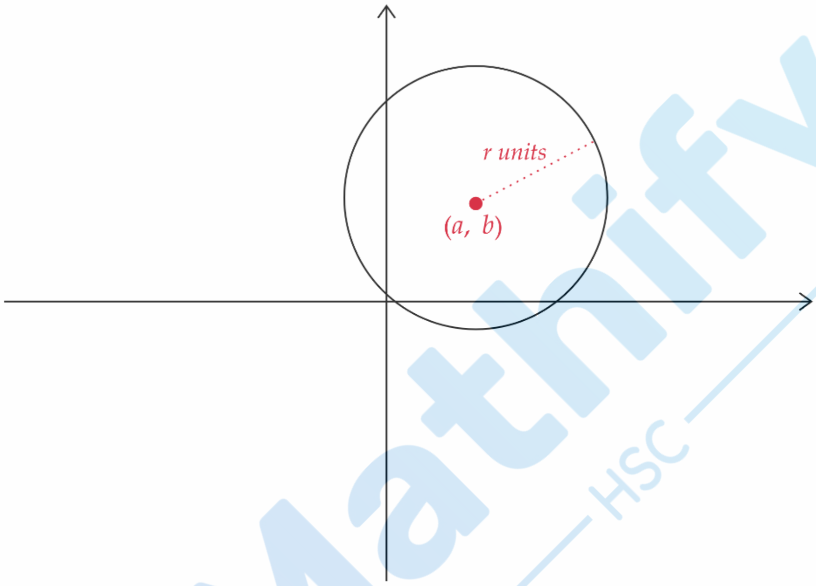
- Equation of a Circle

Given a circle with centre at point (a, b) and radius of r units, its equation will be:

$$(x-a)^2 + (y-b)^2 = r^2$$

Graphically, this would resemble:



Often though, questions won't be nice enough to provide the circle equation in factored form, thus meaning that students must **first** complete the square to factorise the equation before sketching. This is shown in the example below:

Example 1: Sketch the following equation:

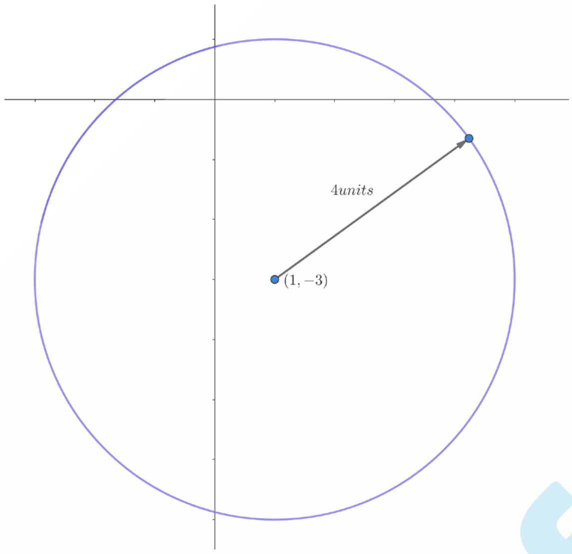
$$x^2 - 2x + y^2 + 6y = 6$$

Solution:

First, we must factorise by completing the square:

$$x^2 - 2x + \left(\frac{2}{2}\right)^2 + y^2 + 6y + \left(\frac{6}{2}\right)^2 = 6 + \left(\frac{2}{2}\right)^2 + \left(\frac{6}{2}\right)^2$$
$$x^2 - 2x + 1 + y^2 + 6y + 9 = 6 + 1 + 9$$
$$(x-1)^2 + (y+3)^2 = 16$$

Therefore, this is a circle with centre at $(1, -3)$ and radius 4, so sketching this below:



Notice here how we are not required to plot/find the x and y intercepts!

- Equation of Semi – Circles

Now that we know the basic circular equation:

$$x^2 + y^2 = r^2$$

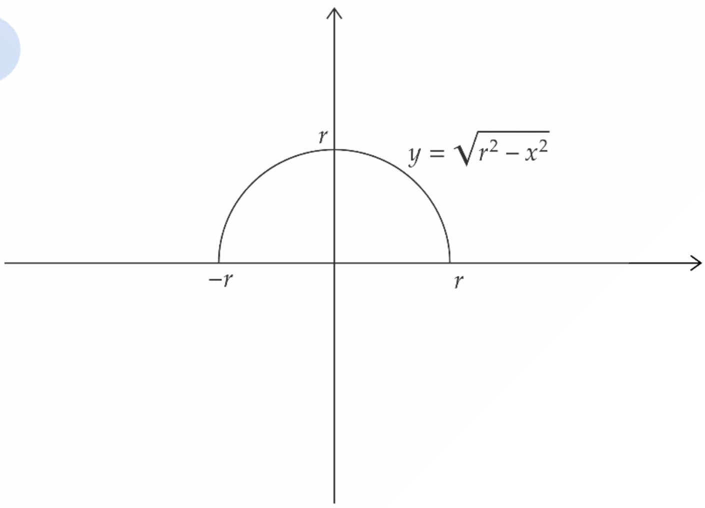
Has centre at $(0, 0)$ and radius of r, if we try to make y the subject:

$$y^2 = r^2 - x^2$$

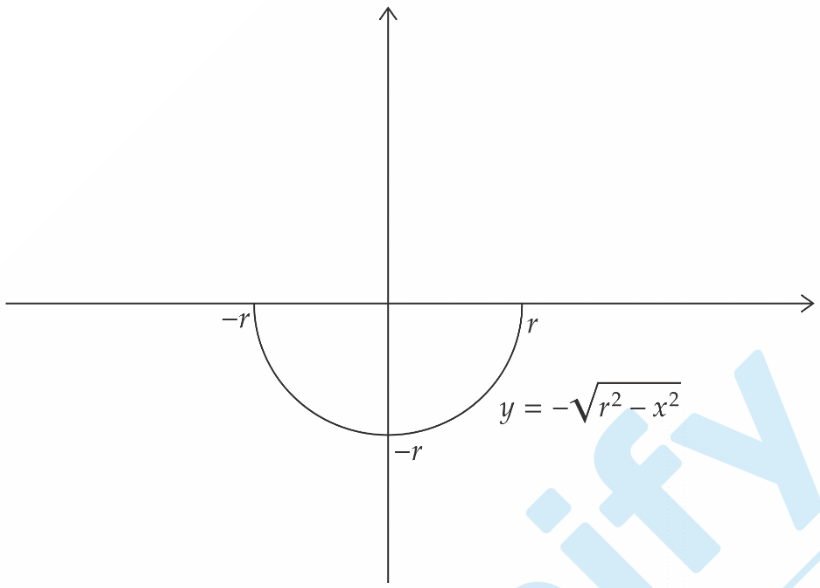
$$\therefore y = \pm\sqrt{r^2 - x^2}$$

As can be seen, there are two results here, and each represents a different semicircle.

- If $y = \sqrt{r^2 - x^2}$, this represents an upper semicircle, as shown below



- If $y = -\sqrt{r^2 - x^2}$, this represents a lower semicircle, as shown below



Now considering a more general circle with centre (a, b) and radius r, its equation would be:

$$(x - a)^2 + (y - b)^2 = r^2$$

Rearranging to make y the subject:

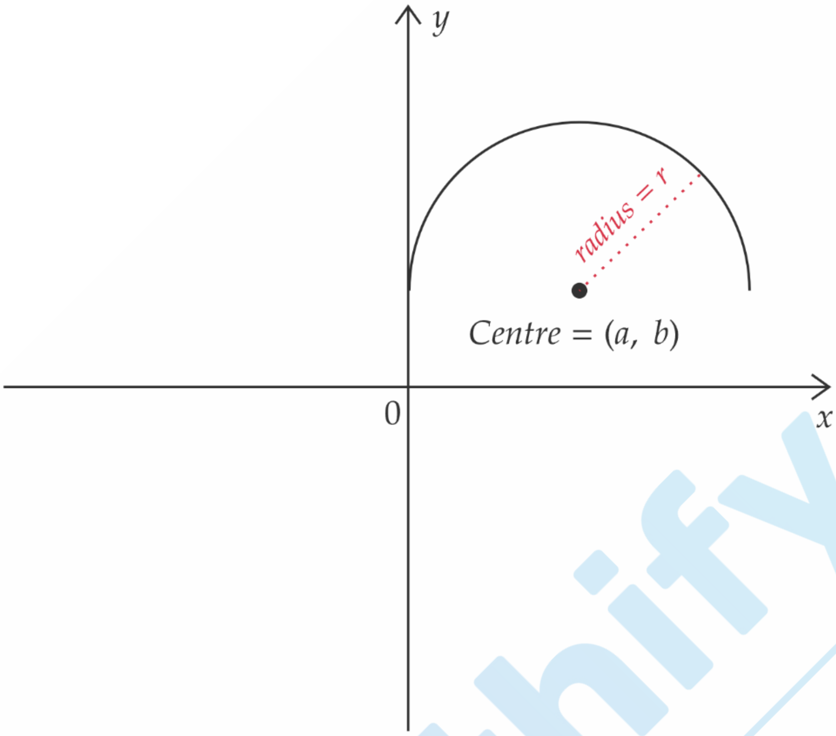
$$(y - b)^2 = r^2 - (x - a)^2$$

$$y - b = \pm\sqrt{r^2 - (x - a)^2}$$

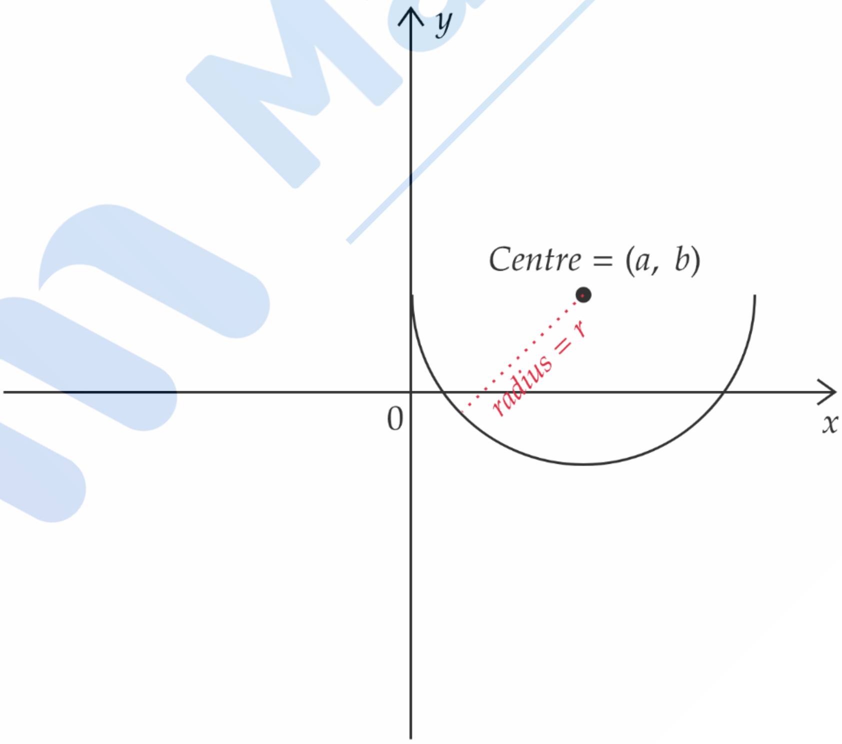
$$\therefore y = b \pm \sqrt{r^2 - (x - a)^2}$$

As can be seen here, there are two results here each representing a semicircle with centre at (a, b) and radius r.

- If $y = b + \sqrt{r^2 - (x - a)^2}$, this represents an upper semi – circle with centre (a, b) and radius r:



- If $y = b - \sqrt{r^2 - (x-a)^2}$, this represents a lower semi – circle with centre (a, b) and radius r:



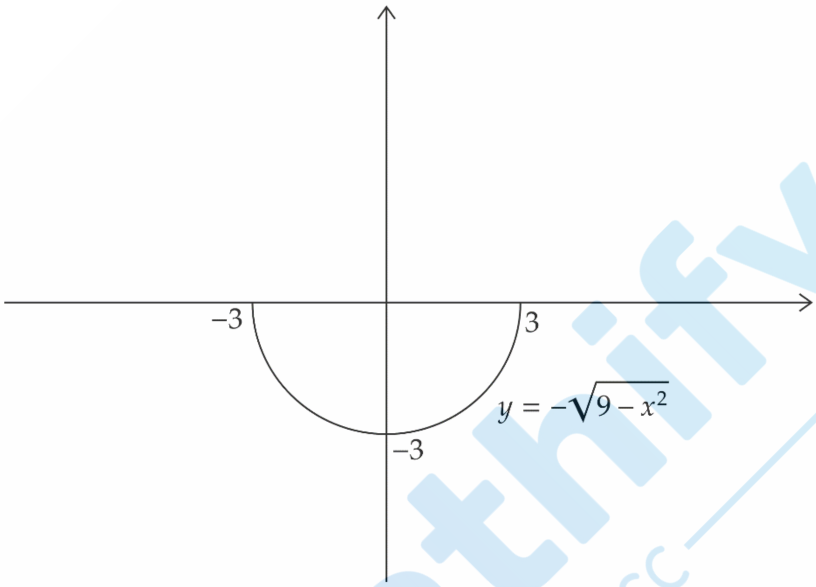
Example 2: Sketch the following equation, stating the domain and range:

$$y = -\sqrt{9 - x^2}$$

Solution:

This is a lower semicircle, with a domain of $-3 \leq x \leq 3$ and range of $-3 \leq y \leq 0$

Therefore, the sketch resembles:



Circles and Semi – Circles Exercises

1. Write down the centre and radius for each of the following circle equations:

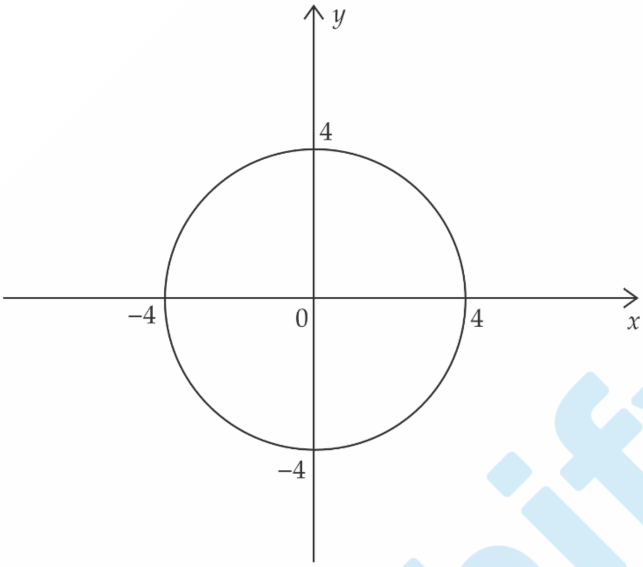
a) $x^2 + y^2 = 9$
b) $(x - 1)^2 + y^2 = 16$
c) $(x + 2)^2 + (y - 3)^2 = 25$
d) $\left(x - \frac{1}{2}\right)^2 + (y + 5)^2 = 30$
e) $(x + 2)^2 + \left(y - \sqrt{3}\right)^2 = 40$

2. Sketch graphs for each of the following circles, clearly labelling the centre point and radius (there is no need to find intercepts)

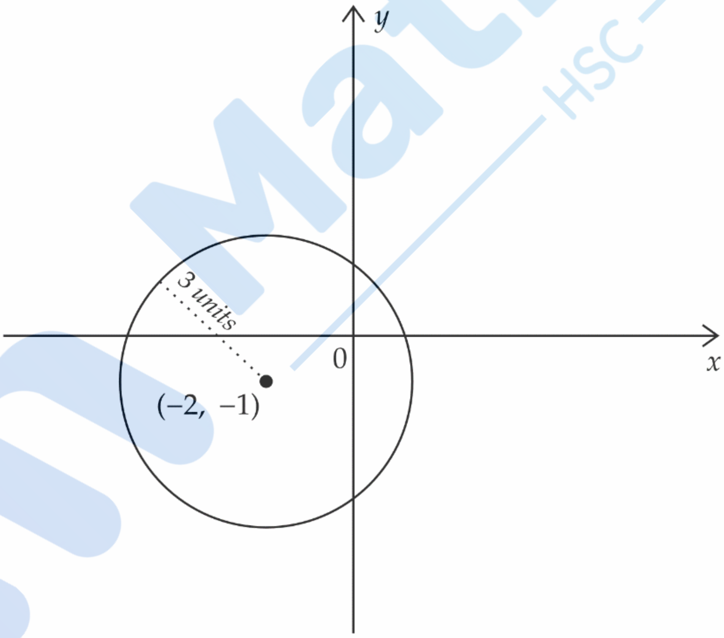
a) $x^2 + y^2 = 4$
b) $(x - 5)^2 + (y - 1)^2 = 9$
c) $(x + 2)^2 + (y - 3)^2 = 16$
d) $(x - 4)^2 + (y + 2)^2 = \frac{1}{4}$

3. Write down the equation of each of the following circles:

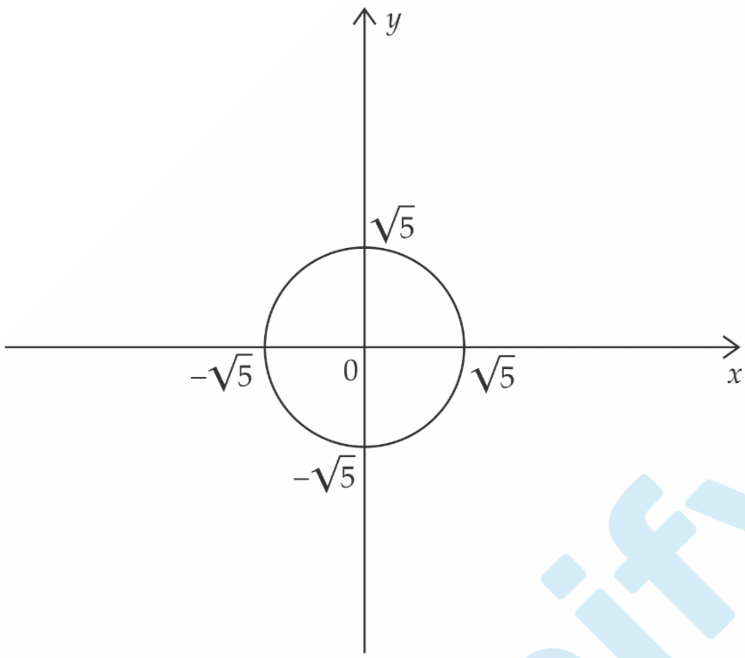
a)



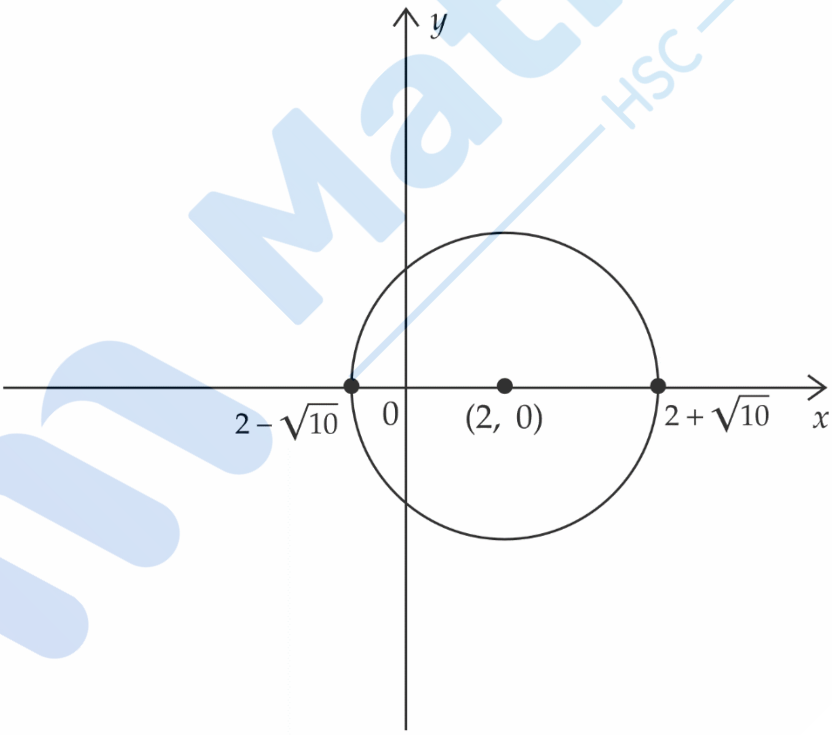
b)



c)



d)



4. Write down the centre and radius for each of the following semi – circle equations

a) $y = \sqrt{9 - x^2}$

b) $y = 3 - \sqrt{4 - x^2}$

c) $y = 5 + \sqrt{10 - x^2}$

d) $y = 6 - \sqrt{\frac{25}{4} - (x-2)^2}$

5. For each of the semi-circle equations in the previous question, write down their domain and range in bracket – interval notation

6. Sketch each of the following semi – circles, clearly labelling the centre point and radius

a) $y = \sqrt{16 - x^2}$

b) $y = 3 - \sqrt{9 - x^2}$

c) $y = 1 + \sqrt{\frac{36}{25} - (x+2)^2}$

d) $y = 6 - \sqrt{\frac{49}{16} - (x-1)^2}$

Circles and Semi – Circle Exercise Answers

1.

a)

$$Centre = (0, 0)$$

$$Radius = \sqrt{9} = 3$$

b)

$$Centre = (1, 0)$$

$$Radius = \sqrt{16} = 4$$

c)

$$Centre = (-2, 3)$$

$$Radius = \sqrt{25} = 5$$

d)

$$Centre = \left(\frac{1}{2}, -5\right)$$

$$Radius = \sqrt{30}$$

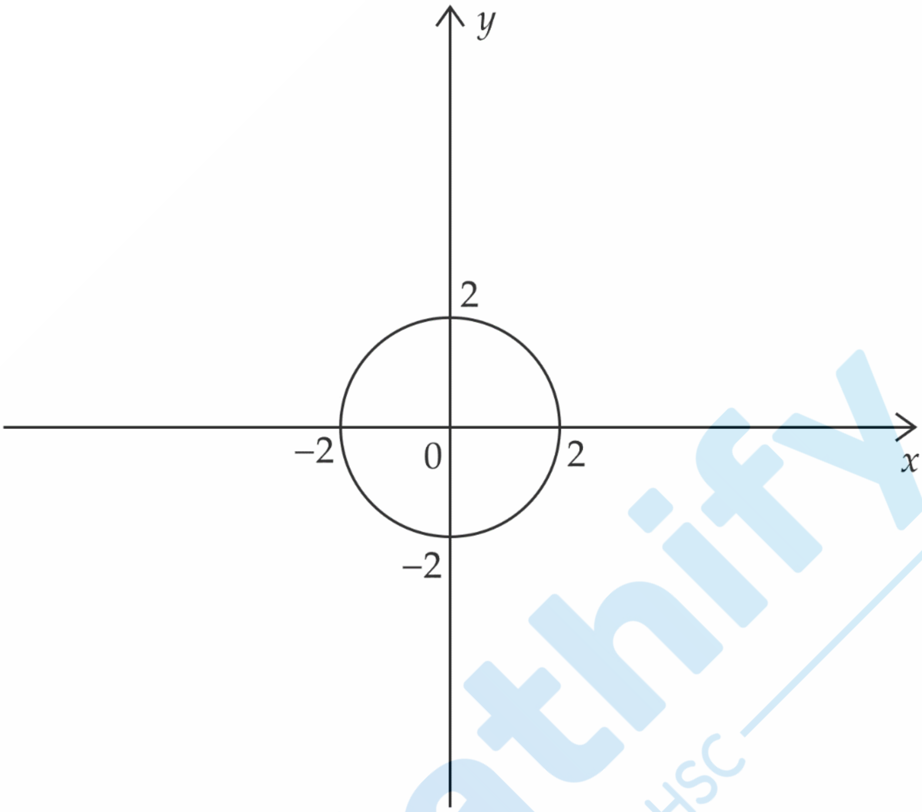
e)

$$Centre = \left(-2, \sqrt{3}\right)$$

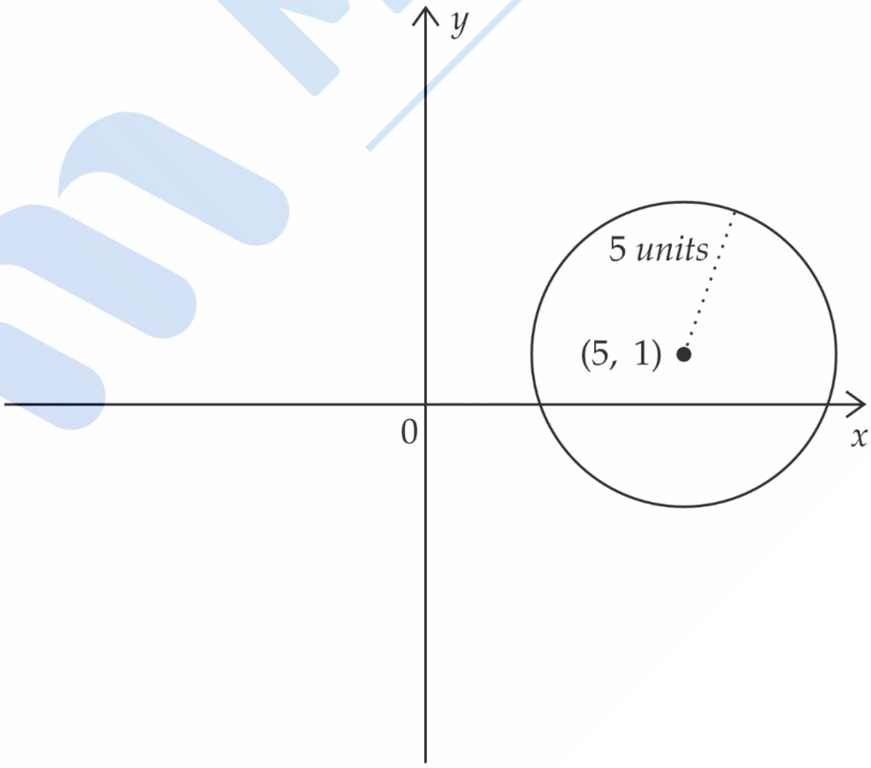
$$Radius = \sqrt{40}$$

2.

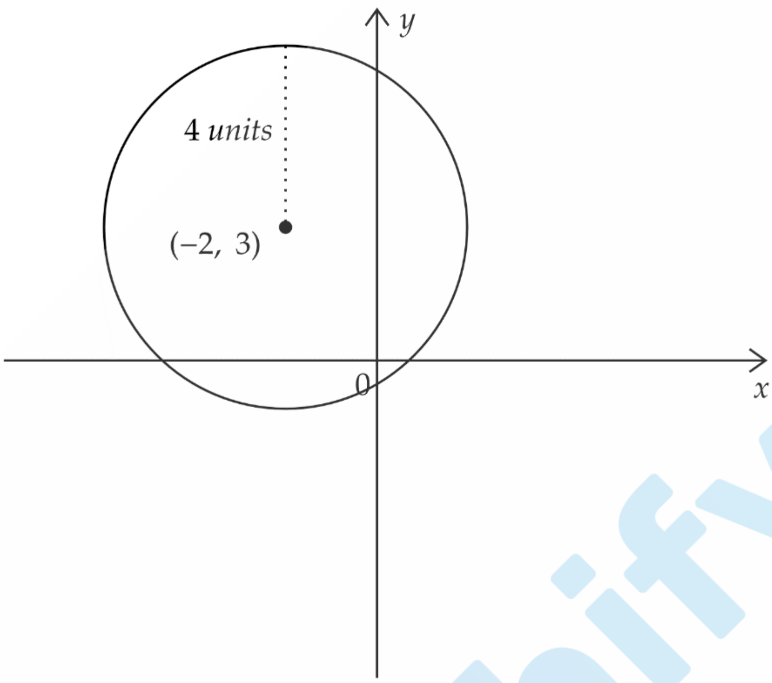
a)



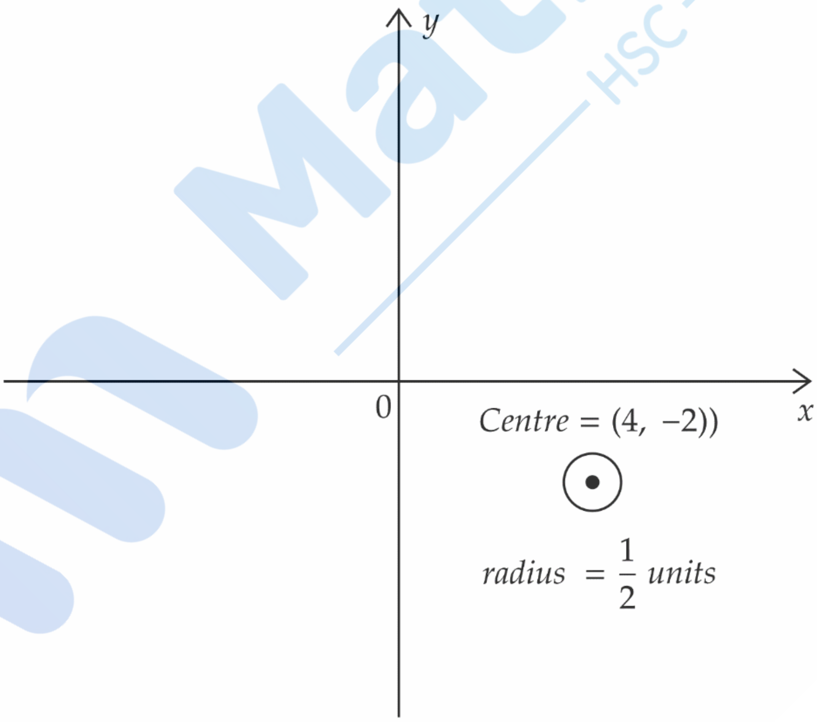
b)



c)



d)



3.

a) Since the centre is at $(0, 0)$ and the radius is 4 units:

$$x^2 + y^2 = 16$$

b) Since the centre is at $(-2, -1)$ and the radius is 3 units:

$$(x+2)^2 + (y+1)^2 = 9$$

c) Since the centre is at $(0, 0)$ and the radius is $\sqrt{5}$ units:

$$x^2 + y^2 = 5$$

d) Since the centre is at $(2, 0)$ and the radius is $\sqrt{10}$ units:

$$(x-2)^2 + y^2 = 10$$

4.

a)

$$Centre = (0, 0)$$

$$Radius = \sqrt{9} = 3$$

b)

$$Centre = (0, 3)$$

$$Radius = \sqrt{4} = 2$$

c)

$$Centre = (0, 5)$$

$$Radius = \sqrt{10}$$

d)

$$Centre = (2, 6)$$

$$Radius = \sqrt{\frac{25}{4}} = \frac{5}{2}$$

5.

a) For the domain:

$$[-3, 3]$$

For the range:

$$[0, 3]$$

b) For the domain:

$$[-2, 2]$$

For the range:

$$[1, 3]$$

c) For the domain:

$$\left[-\sqrt{10}, \sqrt{10}\right]$$

For the range:

$$\left[5, 5+\sqrt{10}\right]$$

d) For the domain:

$$\left[2-\frac{5}{2}, 2+\frac{5}{2}\right]$$

Simplifying this:

$$\left[-\frac{1}{2}, \frac{9}{2}\right]$$

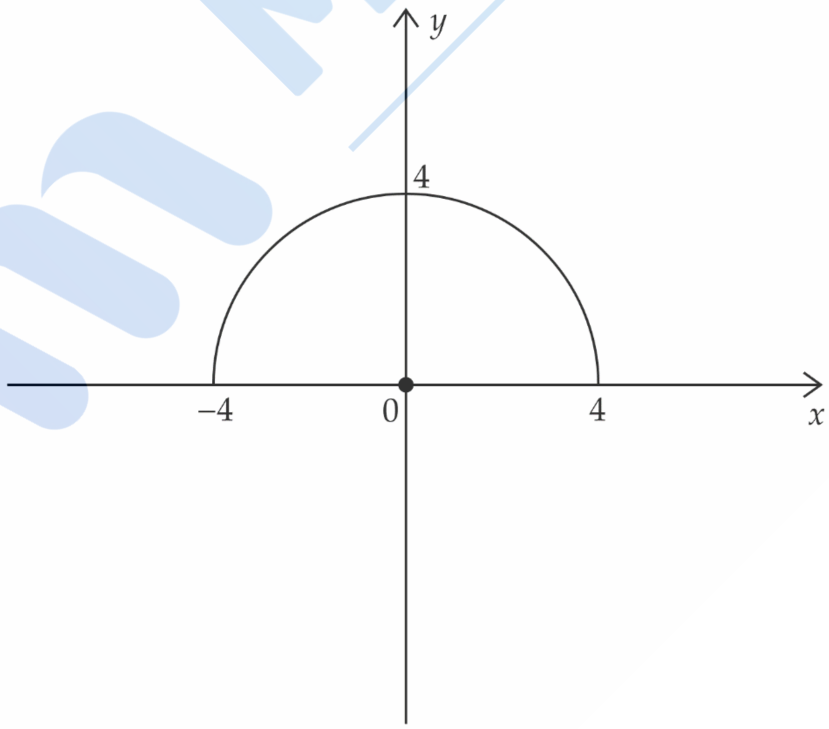
For the range:

$$\left[6-\frac{5}{2}, 6\right]$$

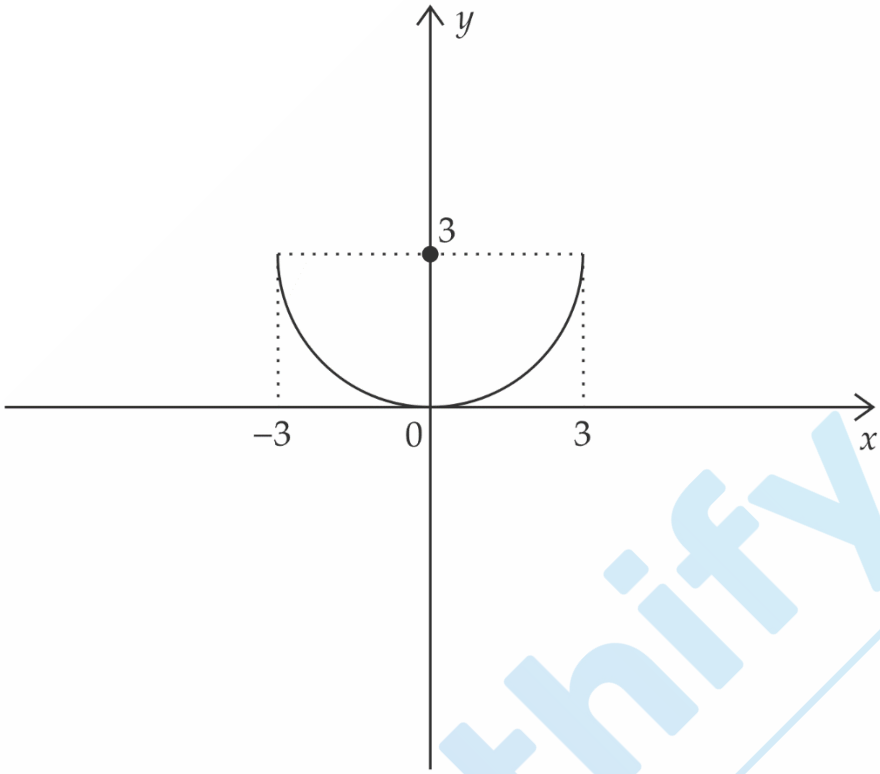
Simplifying this:

$$\left[\frac{7}{2}, 6\right]$$

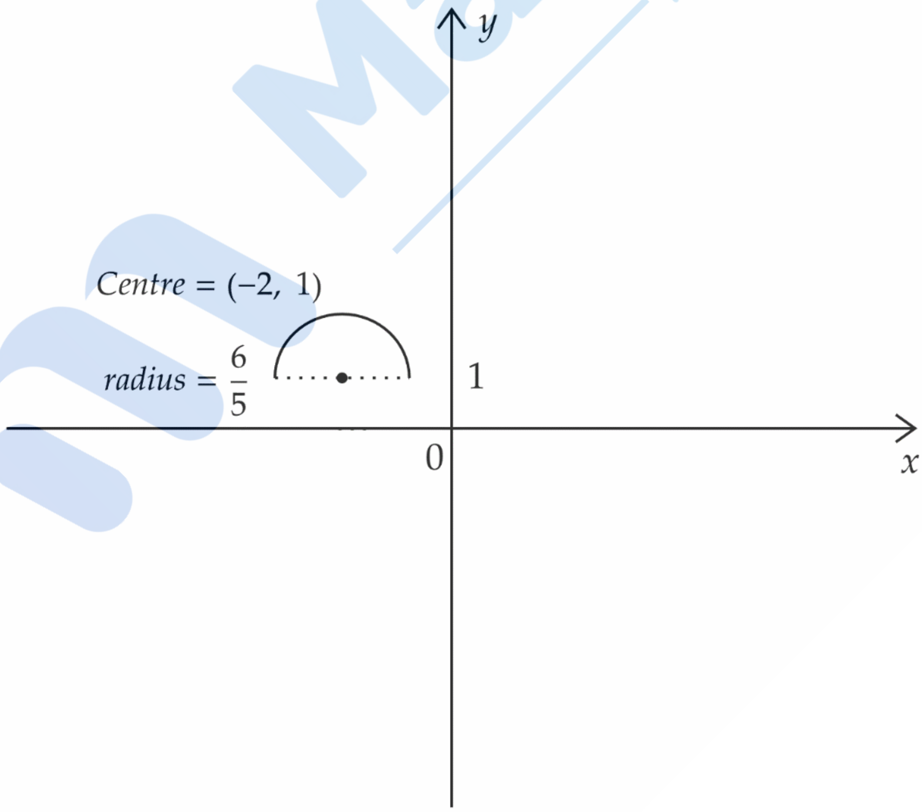
6.
a)



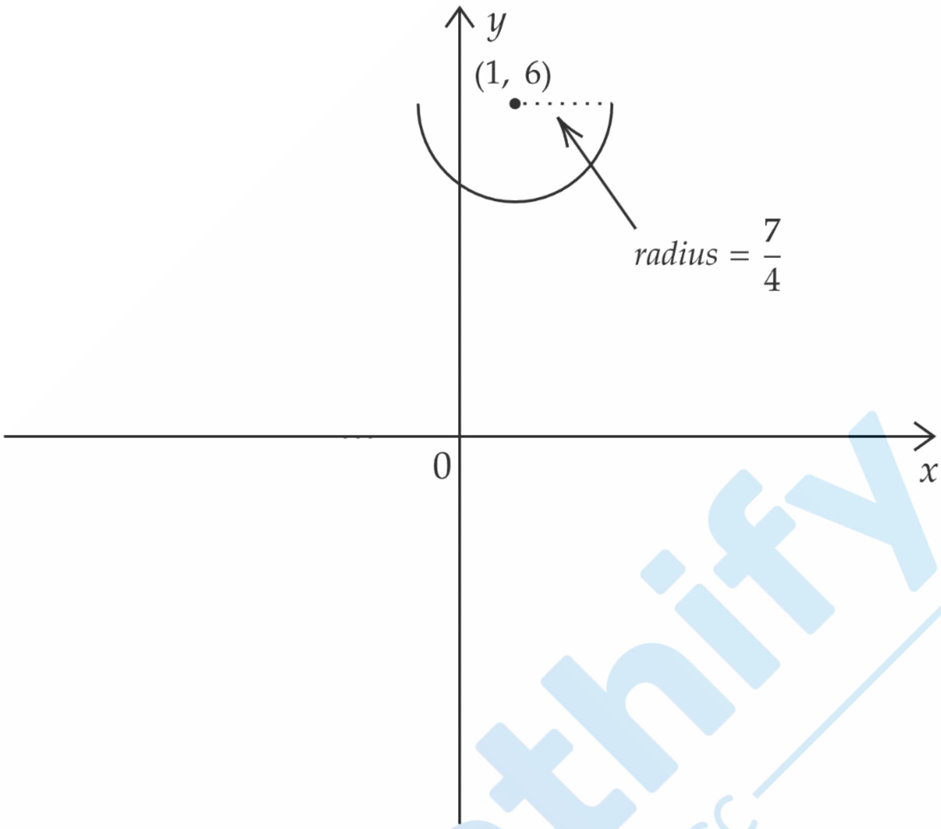
b)



c)



d)


y
(1, 6)
radius = $\frac{7}{4}$
0
x