

FUNCTIONS

GRAPH REFLECTIONS (XII)

Contents include:

- Reflections About the y - axis
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- Reflections About Both Axes

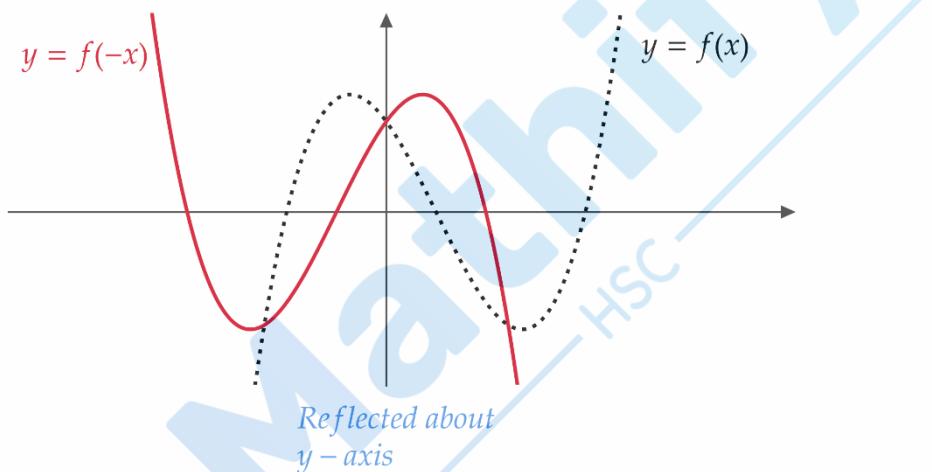
- Reflections About the $y - axis$

If we are given the graph of $y = f(x)$, and **asked to sketch $y = f(-x)$** , our answer would be the reflection of the graph $f(x)$ about the $y - axis$.

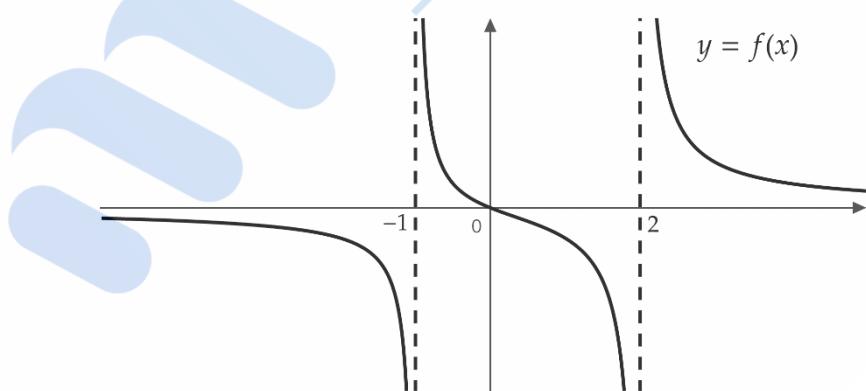
In other words, if x is replaced by $-x$ in the function, then the graph is reflected about the $y - axis$

This is because we essentially swap our negative x values with our positive x values, so the two vertical halves of my graph will flip!

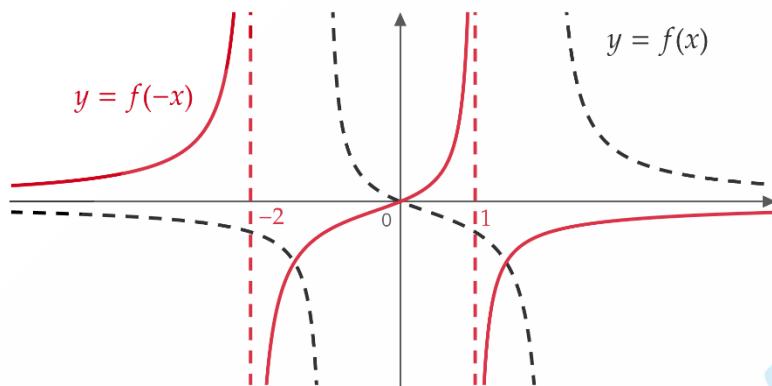
A visual representation is shown below:



Example 1: Given the graph of the function $y = f(x)$ below, sketch $y = f(-x)$



Solution:



As can be seen, the entire graph has been flipped about the y – axis.

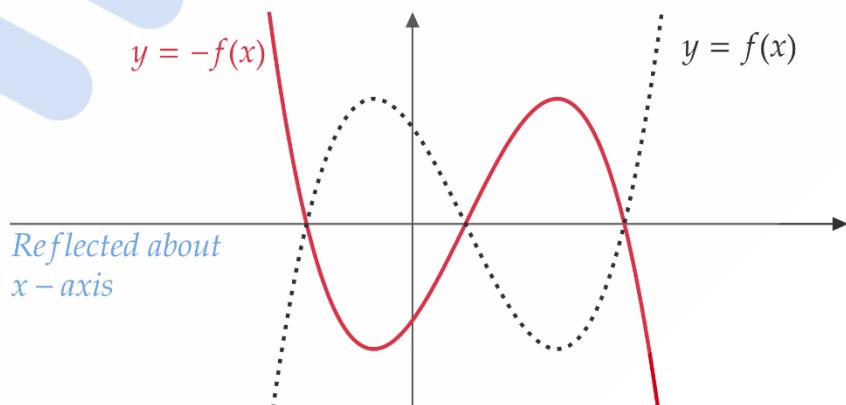
- Reflections About the x – axis

If we are now given the graph of $y = f(x)$, and **asked to sketch $y = -f(x)$** , our answer would be the reflection of the graph $f(x)$ about the x – axis.

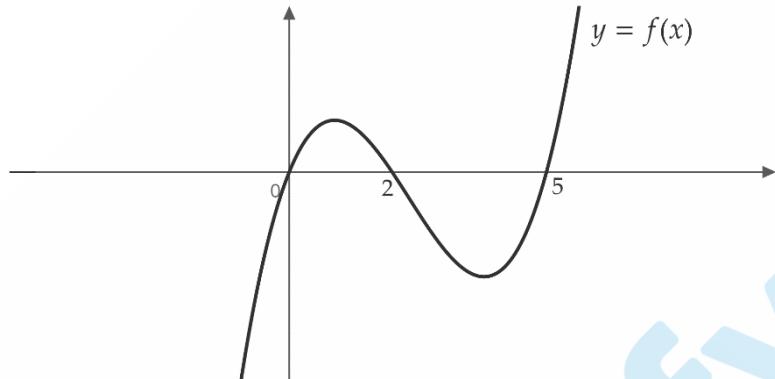
In other words, if $f(x)$ turns into $-f(x)$, the this means the graph is reflected about the x – axis

This is because previously, $y = f(x)$, so if we now make it $y = -f(x)$, all the positive y – values turn negative and all the negative y – values turn positive, so the two horizontal halves of my graph will flip!

A visual representation is shown below:



Example 2: Below, you are given the graph of $y = x(x - 2)(x - 5)$:



Sketch the graph of $y = -x(x - 2)(x - 5)$

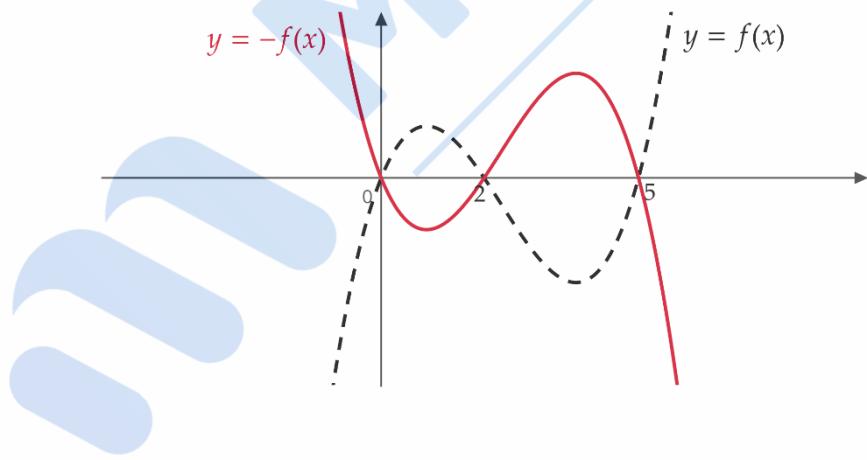
Solution:

Notice here that if $f(x) = x(x - 2)(x - 5)$, then:

$$-x(x - 2)(x - 5) = -f(x)$$

In other words, we are being asked to sketch $-f(x)$ after being given $f(x)$

∴ Reflecting the graph about the x axis



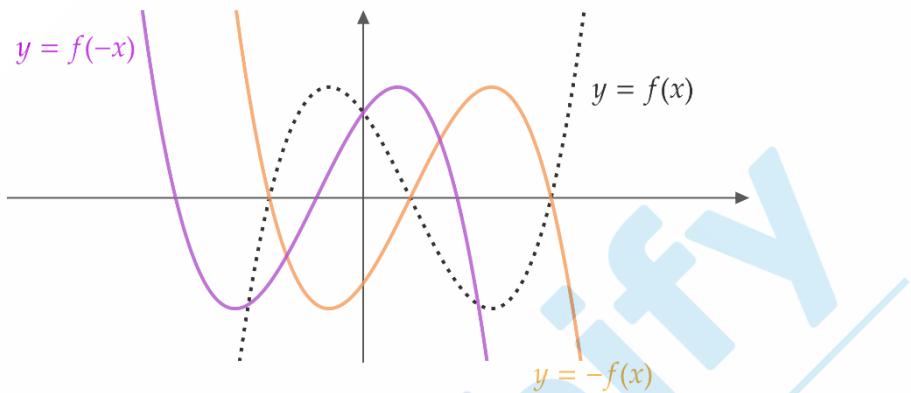
- Reflections About Both Axes

If we are now given the graph of $y = f(x)$ and asked to sketch $y = -f(-x)$, our answer would thus be the reflection of the graph about **both** the x and y axis. The order in which this is done does not matter!

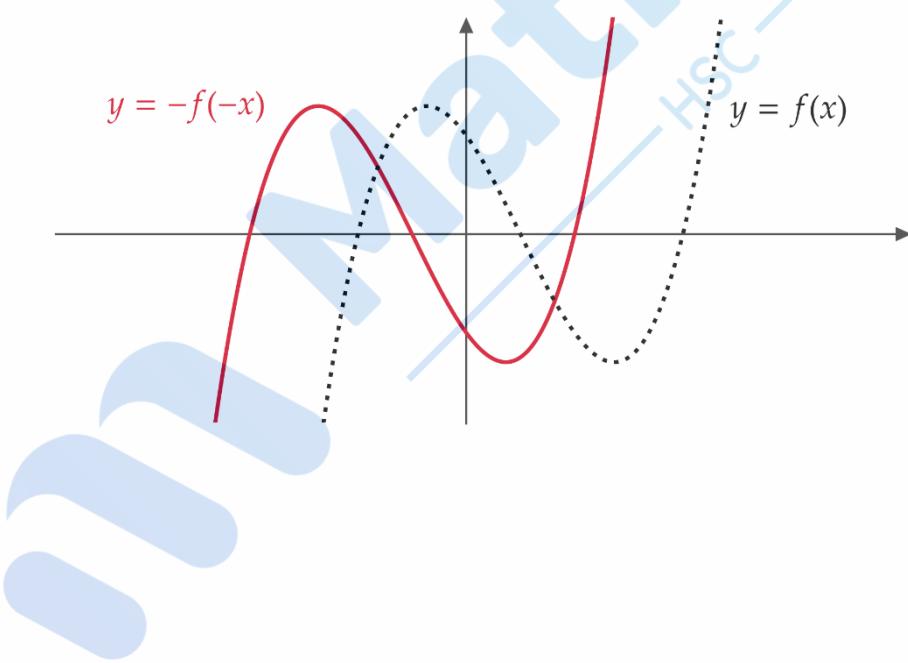
This is because we are combining both our transformations which we learnt previously! For these questions, you may need to split it into 2 parts as you do 1 graph reflection at a time!

A visual representation is shown below:

First flipping across either the x – axis (in orange) or the y – axis (in purple), it's your choice!

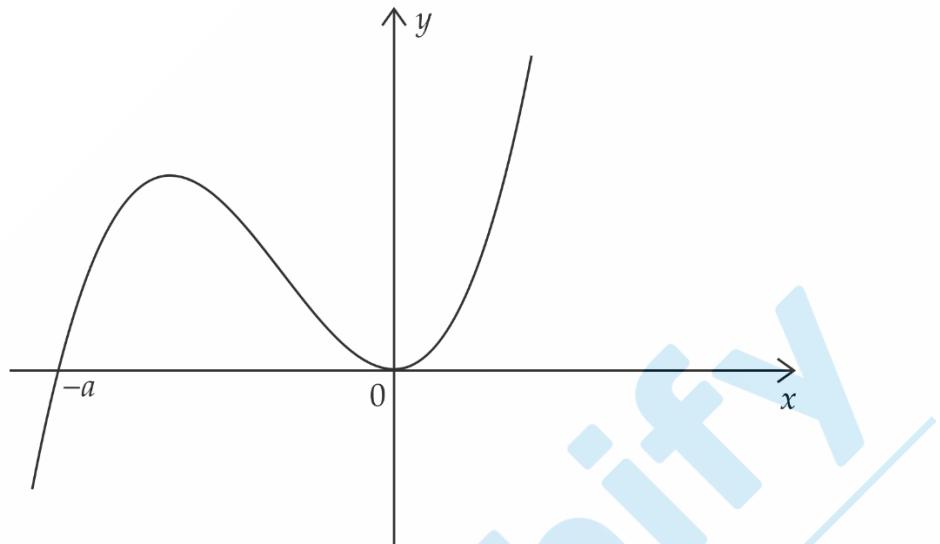


Then, we can flip across the other axes as appropriate, to get our final answer (in red):



Graph Reflection Exercises

1. The graph of $y = f(x)$ is given below:

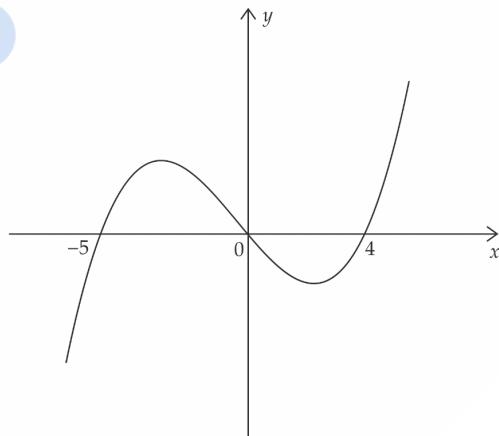


- a) Sketch $f(-x)$, the reflection of the graph about the $y -$ axis
b) Sketch $-f(x)$, the reflection of the graph about the $x -$ axis

2. Given that $g(x) = x^2 - x - 6$, draw the graphs of each of the following:

- a) $g(x)$
b) $g(-x)$
c) $-g(x)$
d) $-g(-x)$

3. Given the graph of $y = x^3 + x^2 - 20x$:

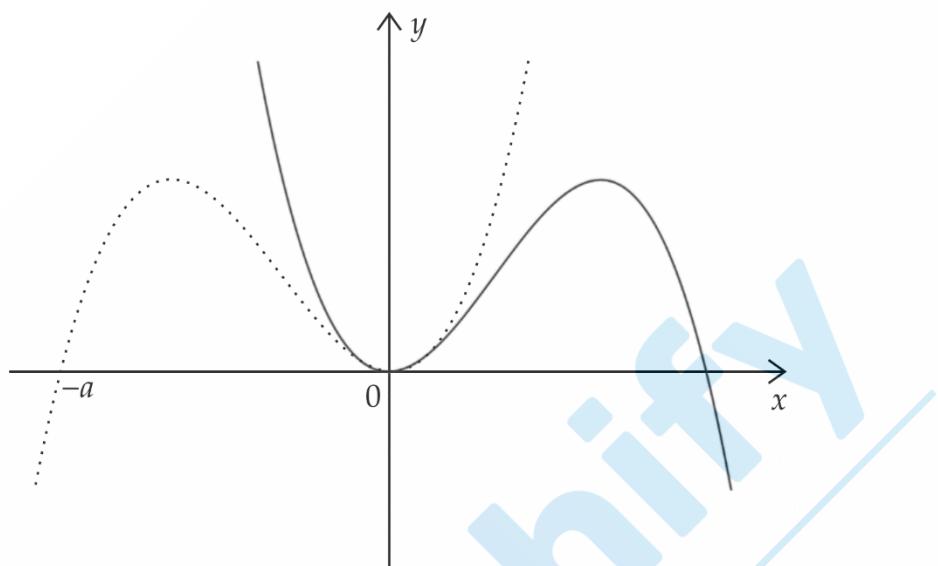


- a) Sketch the graph of $y = -x^3 + x^2 + 20x$
b) Sketch the graph of $y = -x^3 - x^2 + 20x$

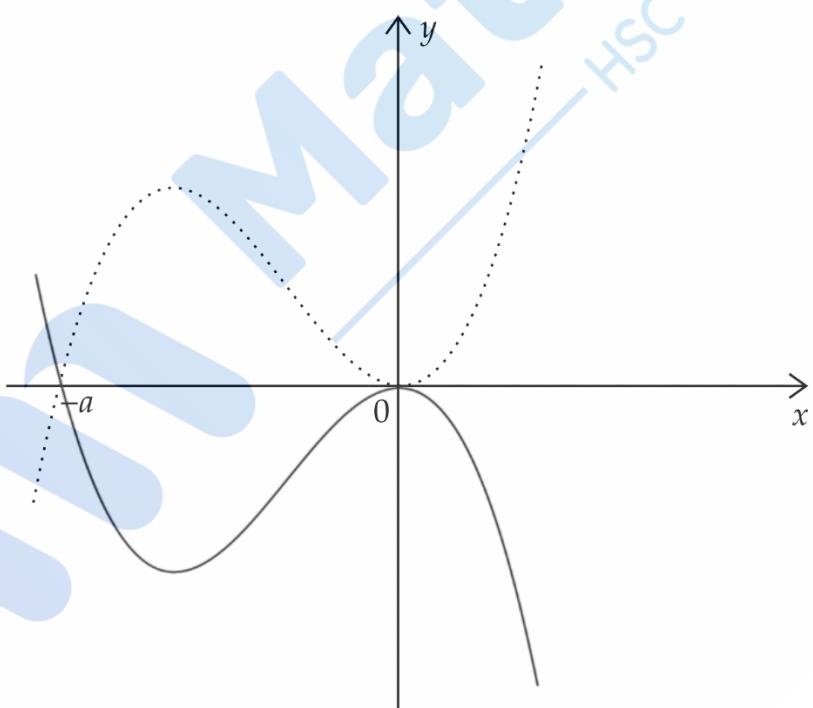
Graph Reflection Exercise Answers

1.

a)

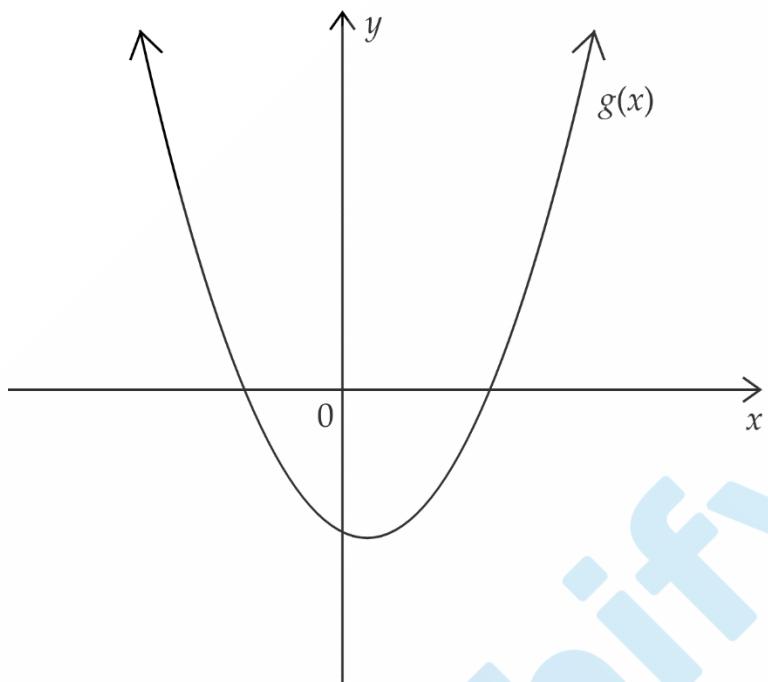


b)

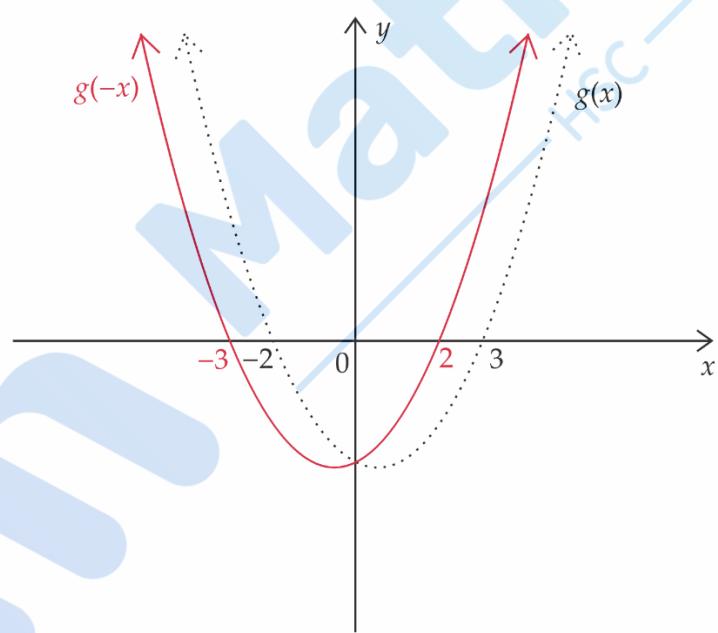


2.

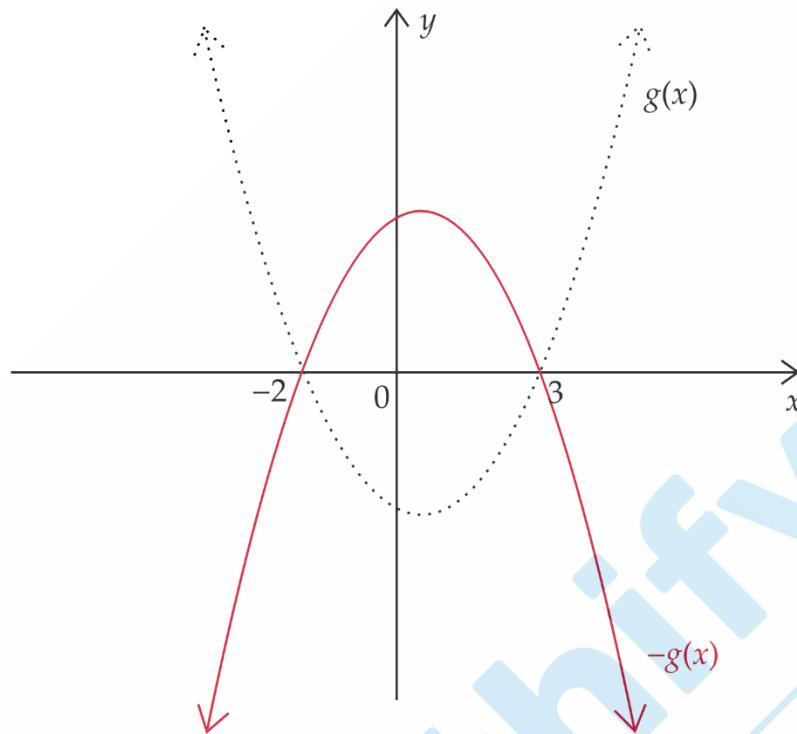
a)



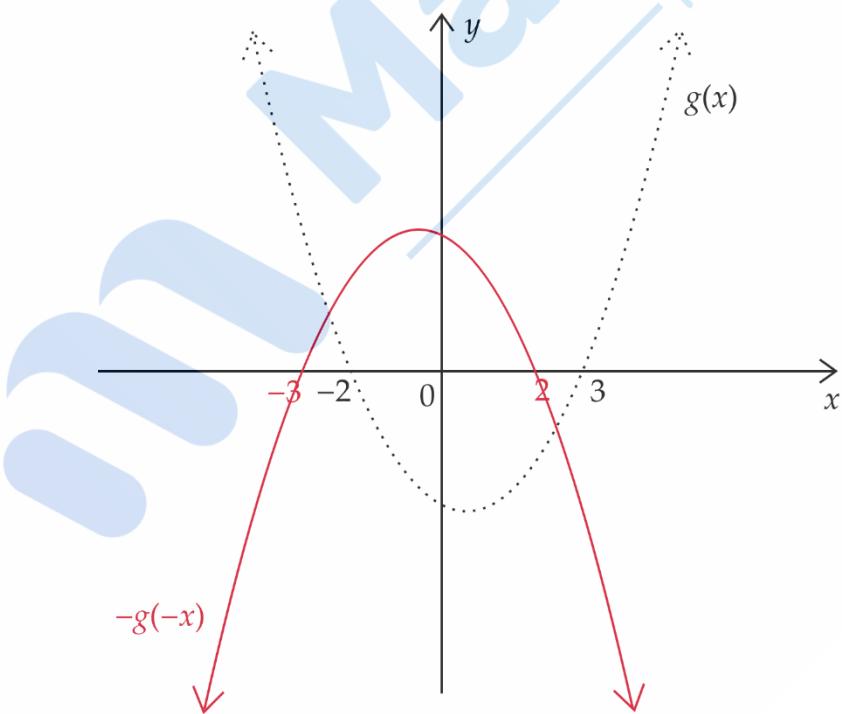
b)



c)



d)



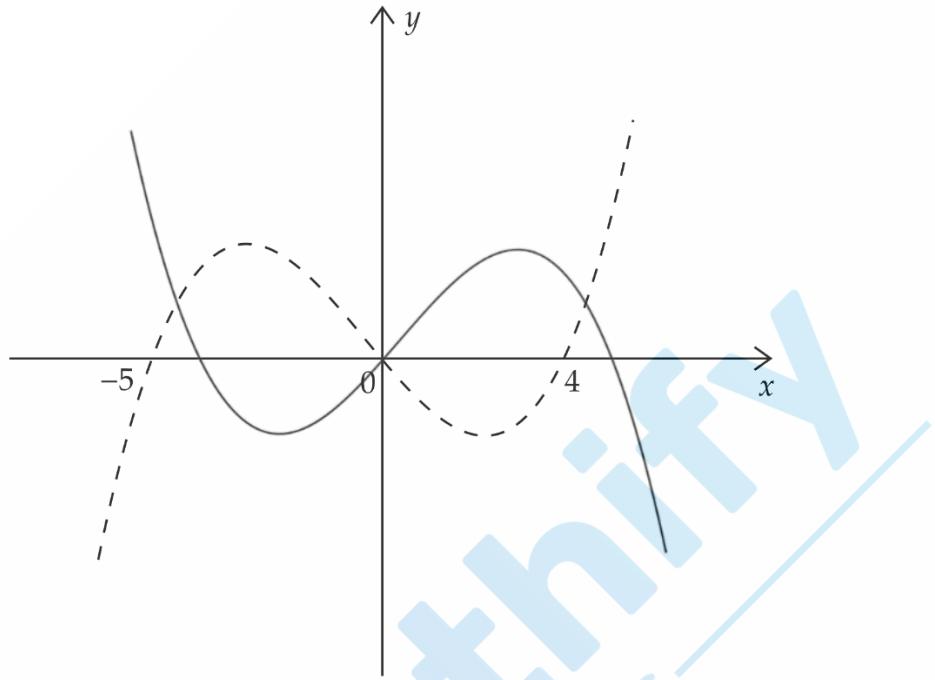
3.

- a) Notice here that if $f(x) = x^3 + x^2 - 20x$, then:

$$-x^3 + x^2 + 20x = (-x)^3 + (-x)^2 - 20(-x)$$

$$= f(-x)$$

In other words, $y = -x^3 + x^2 + 20x$ is the reflection of the given graph about the $y -$ axis



- b) You'll notice here that if $f(x) = x^3 + x^2 - 20x$, then:

$$\begin{aligned}-x^3 - x^2 + 20x &= -(x^3 + x^2 - 20x) \\&= -f(x)\end{aligned}$$

In other words, $y = -x^3 - x^2 + 20x$ is the reflection of the given graph about the $x -$ axis

