

TRIGONOMETRY

RADIANS (IV)

Contents include:

- Radians as a Measure of an Angle
- Converting between Degrees and Radians

- Radians as a Measure of an Angle

So far, we are aware of using degrees to measure angles. Another, more useful way we can measure angles is actually radians, which is typically given in terms of π . More specifically, comparing radians to degrees:

$$2\pi = 360^\circ$$

$$\therefore \pi = 180^\circ$$

Hence, from this we can determine the following conversions:

- To convert from radians to degrees:

$$\text{multiply radians by } \frac{180^\circ}{\pi}$$

- To convert from degrees to radians:

$$\text{multiply degrees by } \frac{\pi}{180}$$

Example 1: Convert 105° to radians

Solution:

$$\begin{aligned}105^\circ &= 105 \times \frac{\pi}{180} \\&= \frac{105\pi}{180} \\&= \frac{21\pi}{36} \\&= \frac{7\pi}{12}\end{aligned}$$

We should become more and more accustomed to dealing with radians in trigonometry as the course progresses. Some helpful conversions to remember are shown in the table below.

| degrees | 30° | 45° | 60° | 90° | 120° | 135° | 150° |
|---------|-----------------|-----------------|-----------------|-----------------|------------------|------------------|------------------|
| radians | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\frac{2\pi}{3}$ | $\frac{3\pi}{4}$ | $\frac{5\pi}{6}$ |

Note: Make sure you are aware **whether your calculator is set to radians or degrees!** It's very easy to make a silly mistake if you forget.

Example 2: Find the exact value of $\cos \frac{5\pi}{4}$

Since $\frac{5\pi}{4}$ is in the 3rd quadrant:

$$\begin{aligned}\therefore \cos \frac{5\pi}{4} &= -\cos \frac{\pi}{4} \\ &= -\frac{1}{\sqrt{2}}\end{aligned}$$

Example 3: Calculate the value of $\cos 1$ to the nearest 2 decimal places

Inputting this into our calculator:

$$\therefore \cos 1 \approx 0.54 \text{ (nearest 2 dp)}$$

Radians Exercises

1. Convert the following angles into radians in exact value:

- a) 30°
- b) 45°
- c) 60°
- d) 135°
- e) 240°
- f) 330°
- g) -85°
- h) 220°
- i) -18°
- j) 560°
- k) -195°

2. Convert the following angles into degrees:

- a) $\frac{\pi}{2}$
- b) $\frac{\pi}{6}$
- c) $\frac{\pi}{4}$
- d) $\frac{\pi}{3}$
- e) $\frac{2\pi}{3}$
- f) $\frac{5\pi}{4}$
- g) $\frac{11\pi}{6}$
- h) $-\frac{3\pi}{4}$
- i) $\frac{13\pi}{6}$

j) $-\frac{4}{9}\pi$

k) $-\frac{4\pi}{3}$

3. Use your calculator to evaluate the following to the nearest 2 decimal points:

- a) $\sin 1$
- b) $\sin 0.5$
- c) $\cos 3.12$
- d) $\cos -2.4$
- e) $\tan 1.2$
- f) $\tan -5.6$

4. Convert the following into radians, correct to the nearest 2 decimal points:

- a) 49°
- b) $77^\circ 33'$
- c) $223^\circ 54'$
- d) $305^\circ 21'$
- e) $-57^\circ 12'$
- f) $-406^\circ 20'$

5. Convert the following into degrees, correct to the nearest minute:

- a) 2 radians
- b) 0.4 radians
- c) 2.56 radians
- d) 3.09 radians
- e) -1.14 radians
- f) -4 radians

Radians Exercise Answers

1.

a)

$$\begin{aligned}30^\circ &= \frac{30}{180} \times \pi \\&= \frac{\pi}{6}\end{aligned}$$

b)

$$\begin{aligned}45^\circ &= \frac{45}{180} \times \pi \\&= \frac{\pi}{4}\end{aligned}$$

c)

$$\begin{aligned}60^\circ &= \frac{60}{180} \times \pi \\&= \frac{\pi}{3}\end{aligned}$$

d)

$$\begin{aligned}135^\circ &= \frac{135}{180} \times \pi \\&= \frac{3\pi}{4}\end{aligned}$$

e)

$$\begin{aligned}240^\circ &= \frac{240}{180} \times \pi \\&= \frac{4\pi}{3}\end{aligned}$$

f)

$$\begin{aligned}330^\circ &= \frac{330}{180} \times \pi \\&= \frac{11\pi}{6}\end{aligned}$$

g)

$$\begin{aligned}-85^\circ &= -\frac{85}{180} \times \pi \\&= -\frac{17\pi}{36}\end{aligned}$$

h)

$$\begin{aligned}220^\circ &= \frac{220}{180} \times \pi \\&= \frac{11\pi}{9}\end{aligned}$$

i)

$$\begin{aligned}-18^\circ &= -\frac{18}{180} \times \pi \\&= -\frac{\pi}{10}\end{aligned}$$

j)

$$\begin{aligned}560^\circ &= \frac{560}{180} \times \pi \\&= \frac{28}{9}\pi\end{aligned}$$

k)

$$\begin{aligned}-195^\circ &= -\frac{195}{180} \times \pi \\&= -\frac{13}{12}\pi\end{aligned}$$

2.

a)

$$\begin{aligned}\frac{\pi}{2} &= \frac{\pi}{\frac{2}{\pi}} \times 180^\circ \\&= \frac{1}{2} \times 180^\circ \\&= 90^\circ\end{aligned}$$

b)

$$\begin{aligned}\frac{\pi}{6} &= \frac{\pi}{\frac{6}{\pi}} \times 180^\circ \\&= \frac{1}{6} \times 180^\circ \\&= 30^\circ\end{aligned}$$

c)

$$\begin{aligned}\frac{\pi}{4} &= \frac{\pi}{\frac{4}{\pi}} \times 180^\circ \\&= \frac{1}{4} \times 180^\circ \\&= 45^\circ\end{aligned}$$

d)

$$\begin{aligned}\frac{\pi}{3} &= \frac{\pi}{\frac{3}{\pi}} \times 180^\circ \\&= \frac{1}{3} \times 180^\circ \\&= 60^\circ\end{aligned}$$

e)

$$\begin{aligned}\frac{2\pi}{3} &= \frac{2\pi}{\frac{3}{\pi}} \times 180^\circ \\&= \frac{2}{3} \times 180^\circ \\&= 120^\circ\end{aligned}$$

f)

$$\begin{aligned}\frac{5\pi}{4} &= \frac{\frac{5\pi}{4}}{\pi} \times 180^\circ \\ &= \frac{5}{4} \times 180^\circ \\ &= 225^\circ\end{aligned}$$

g)

$$\begin{aligned}\frac{11\pi}{6} &= \frac{\frac{11\pi}{6}}{\pi} \times 180^\circ \\ &= \frac{11}{6} \times 180^\circ \\ &= 330^\circ\end{aligned}$$

h)

$$\begin{aligned}-\frac{3\pi}{4} &= -\frac{\frac{3\pi}{4}}{\pi} \times 180^\circ \\ &= -\frac{3}{4} \times 180^\circ \\ &= -135^\circ\end{aligned}$$

i)

$$\begin{aligned}\frac{13\pi}{6} &= \frac{\frac{13\pi}{6}}{\pi} \times 180^\circ \\ &= \frac{13}{6} \times 180^\circ \\ &= 390^\circ\end{aligned}$$

3. Make sure your calculators are set to radians mode for these questions!

- a) $\sin 1 = 0.84$
- b) $\sin 0.5 = 0.48$
- c) $\cos 3.12 = 1.00$
- d) $\cos -2.4 = -0.74$
- e) $\tan 1.2 = 2.57$
- f) $\tan -5.6 = 0.81$

4.

a)

$$\begin{aligned}49^\circ &= \frac{49}{180} \times \pi \\ &= 0.86\end{aligned}$$

b)

$$77^\circ 33' = \frac{77^\circ 33'}{180} \times \pi \\ = 1.35$$

c)

$$223^\circ 54' = \frac{223^\circ 54'}{180} \times \pi \\ = 3.91$$

d)

$$305^\circ 21' = \frac{305^\circ 21'}{180} \times \pi \\ = 5.33$$

e)

$$-57^\circ 12' = \frac{-57^\circ 12'}{180} \times \pi \\ = -1.00$$

f)

$$-406^\circ 20' = \frac{-406^\circ 20'}{180} \times \pi \\ = -7.09$$

5.

a)

$$2 \text{ radians} = \frac{2}{\pi} \times 180^\circ \\ = 114^\circ 35'$$

b)

$$0.4 \text{ radians} = \frac{0.4}{\pi} \times 180^\circ \\ = 22^\circ 55'$$

c)

$$2.56 \text{ radians} = \frac{2.56}{\pi} \times 180^\circ \\ = 146^\circ 41'$$

d)

$$3.09 \text{ radians} = \frac{3.09}{\pi} \times 180^\circ \\ = 177^\circ 3'$$

e)

$$\begin{aligned}-1.14 \text{ radians} &= -\frac{1.14}{\pi} \times 180^\circ \\&= -65^\circ 19'\end{aligned}$$

f)

$$\begin{aligned}-4 \text{ radians} &= -\frac{4}{\pi} \times 180^\circ \\&= -229^\circ 11'\end{aligned}$$

