EXPONENTIALS & LOGARITHMS

INTRODUCTION TO LOGARITHMS (II)

Contents include:
- Logarithms
- Log and Ln Functions

- Introduction to Logarithms

A logarithm is essentially the inverse of an index, where if $a^n = m$:

$$\therefore n = \log_a m$$

A good way to remember this is that:

$$\underset{\text{index}}{n} = \log_{\underset{\text{base}}{a}} m$$

$$\therefore a^n = m$$

It is also important to note:

$$m \text{ and } a \text{ must be positive}$$

$$i.e., a > 0 \text{ and } m > 0$$

For example:

$$The\ logarithm\ \log_{10} 100 = 2\ means\ that\ 10^2 = 100$$

If you ever forget which is which for logarithms, use your calculator! Enter into your calculator $\log 100$ and the result should equal to 2! This therefore tells you that 2 is your "index" and 10 is your "base".

Example 1: Given that $x = \log_2 4$, find the value of x

Solution:

Applying what we just learnt about logarithms, convert the equation $x = \log_2 4$ to index form:

$$\therefore 2^x = 4$$

Then, convert 4 into a base of 2 as well:

$$2^x = 2^2$$

Now equating powers:

$$\therefore x = 2$$

Example 2: Given that $4 = \log_x 256$, find the value of x

Solution:

Convert the equation $256 = \log_x 4$ to index form:

$$x^4 = 256$$

Now converting 256 into a base with power of 4:

$$x^4 = 4^4$$

Equating bases, we can therefore conclude that:

$$x = 4$$

Alternatively, since we know that $x^4 = 256$, we can also use our calculator to say that: $$x = \sqrt[4]{256}$$ This is effective, especially for harder questions!

Note that since x is a base of our logarithm, we do not need to consider any negative solutions for our expression!

- Log and ln

The two most common types of logarithms used are:

- "log" which stands for $\log_{10}$
- "ln" which stands for $\log_e$

These are the only two types of logarithms which we can calculate using NESA – approved calculators. Hence, if we wish to calculate logarithms involving other bases, we will have to change our base manually which will be discussed in the next booklet.

Example 3: Given that $3 = \ln x$, find the value of x

Solution:

Converting the equation to index form:

$$\therefore e^3 = x$$

Basic Logarithm Exercises

1. Rewrite each of the following logarithms in index form, then solve for x:

a) $x = \log 100$
b) $x = \log_2 64$
c) $x = \log_8 \frac{1}{8}$
d) $x = \log_{11} \frac{1}{121}$
e) $x = \log_4 8$
f) $x - 1 = \log_{64} 4$

g) $\frac{x}{2} = \log_3 \frac{1}{81}$

2. Rewrite each of the following logarithms in index form, then solve for x:

a) $\log_5 x = 4$

b) $\ln x = 5$

c) $\log_3 x = -3$

d) $\log_8 x = \frac{1}{3}$

e) $\log_{49} x = \frac{1}{2}$

f) $\log_9 x = -\frac{3}{2}$

g) $\log_{27} x = \frac{4}{3}$

3. Rewrite each of the following logarithms in index form, then solve for x:

a) $\log_x 27 = 3$

b) $\log_x 16 = 2$

c) $\log_x \frac{1}{125} = -3$

d) $\log_x \frac{1}{81} = -4$

e) $\log_x 625 = \frac{4}{3}$

f) $\log_x 8 = \frac{3}{2}$

g) $\log_x \frac{1}{243} = -\frac{5}{2}$

4. Given that k is a positive constant not equal to 1, evaluate the following expressions:

a) $\log_k k$

b) $\log_k k^4$

c) $\log_k 1$

d) $\log_k \sqrt{k}$

e) $\log_k \sqrt[5]{k}$

f) $\log_k \frac{1}{k}$

g) $\log_k \frac{1}{k^3}$

<u>**Basic Logarithm Exercise Answers**</u>

1.
a)

$$x = \log 100$$

Remember that this is a logarithm of base 10, so:

$$\begin{aligned} 10^x &= 100 \\ &= 10^2 \end{aligned}$$

$$\therefore x = 2$$

b)

$$x = \log_2 64$$

Converting to index form:

$$\begin{aligned} 2^x &= 64 \\ &= 2^6 \end{aligned}$$

$$\therefore x = 6$$

c)

$$x = \log_8 \frac{1}{8}$$

Converting to index form:

$$\begin{aligned} 8^x &= \frac{1}{8} \\ &= 8^{-1} \end{aligned}$$

$$\therefore x = -1$$

d)

$$x = \log_{11} \frac{1}{121}$$

Converting to index form:

$$\begin{aligned} 11^x &= \frac{1}{121} \\ &= 11^{-2} \end{aligned}$$

$$\therefore x = -2$$

e)

$$x = \log_4 8$$

Converting to index form:

$$4^x = 8$$

Now unfortunately, 8 cannot easily be expressed as a power of 4, but we can simplify both sides into a power of 2:

$$\therefore (2^2)^x = 2^3$$

$$2^{2x} = 2^3$$

$$\therefore 2x = 3$$

$$x = \frac{3}{2}$$

f)

$$x - 1 = \log_{64} 4$$

Converting to index form:

$$64^{x-1} = 4$$

Changing the LHS now into a power of 4:

$$(4^3)^{x-1} = 4$$

$$4^{3x-3} = 4^1$$

$$\therefore 3x - 3 = 1$$

$$3x = 4$$

$$x = \frac{4}{3}$$

g)

$$\frac{x}{2} = \log_3 \frac{1}{81}$$

Converting to index form:

$$3^{\frac{x}{2}} = \frac{1}{81}$$

Changing the RHS now into a power of 3:

$$3^{\frac{x}{2}} = 3^{-4}$$

$$\therefore \frac{x}{2} = -4$$

$$x = -8$$

2.
a)

$$\log_5 x = 4$$

Converting to index form:

$$5^4 = x$$

$$\therefore x = 625$$

b)

$$\ln x = 5$$

Recall that $\ln x = \log_e x$. Hence, converting to index form:

$$e^5 = x$$

c)

$$\log_3 x = -3$$

Converting to index form:

$$3^{-3} = x$$

$$\therefore x = \frac{1}{3^3}$$

$$= \frac{1}{27}$$

d)

$$\log_8 x = \frac{1}{3}$$

Converting to index form:

$$8^{\frac{1}{3}} = x$$

$$\therefore x = 2$$

e)

$$\log_{49} x = \frac{1}{2}$$

Converting to index form:

$$49^{\frac{1}{2}} = x$$

$$\therefore x = 7$$

f)

$$\log_9 x = -\frac{3}{2}$$

Converting to index form:

$$9^{-\frac{3}{2}} = x$$

$$(3^2)^{-\frac{3}{2}} = x$$

$$\therefore x = 3^{-3}$$
$$= \frac{1}{27}$$

g)

$$\log_{27} x = \frac{4}{3}$$

Converting to index form:

$$27^{\frac{4}{3}} = x$$

$$(3^3)^{\frac{4}{3}} = x$$

$$\therefore x = 3^4$$
$$= 81$$

3.
a)

$$\log_x 27 = 3$$

Converting to index form:

$$x^3 = 27$$

Changing the RHS to have a base with power 3:

$$x^3 = 3^3$$

Equating bases:

$$\therefore x = 3$$

b)

$$\log_x 16 = 2$$

Converting to index form:

$$x^2 = 16$$

$$\therefore x = 4\ ONLY\ (\because x > 0\ for\ a\ logarithmic\ base)$$

c)

$$\log_x \frac{1}{125} = -3$$

Converting to index form:

$$x^{-3} = \frac{1}{125}$$
$$= 5^{-3}$$

Equating bases:

$$\therefore x = 5$$

d)

$$\log_x \frac{1}{81} = -4$$

Converting to index form:

$$x^{-4} = \frac{1}{81}$$
$$= 3^{-4}$$

Equating bases:

$$\therefore x = 3$$

e)

$$\log_x 625 = \frac{4}{3}$$

Converting to index form:

$$x^{\frac{4}{3}} = 625$$

Converting the RHS into base 5:

$$x^{\frac{4}{3}} = 5^4$$

Since we want the power to equal to $\frac{4}{3}$ for the RHS, we are going to cube then cube root it like so:

$$x^{\frac{4}{3}} = 5^4$$
$$= (5^3)^{\frac{4}{3}}$$
$$= 125^{\frac{4}{3}}$$

Hence, now equating the bases of both sides:

$$\therefore x = 125$$

Alternatively, this question may also be done by inputting the following into a calculator:

$$x = \sqrt[\frac{4}{3}]{5^4}$$
$$= 125$$

f)

$$\log_x 8 = \frac{3}{2}$$

Converting to index form:

$$x^{\frac{3}{2}} = 8$$

Converting the RHS into base 2:

$$x^{\frac{3}{2}} = 2^3$$

Since we want the power to equal to $\frac{3}{2}$ for the RHS, we are going to square then square root it like so:

$$\begin{aligned} x^{\frac{3}{2}} &= 2^3 \\ &= (2^2)^{\frac{3}{2}} \\ &= 4^{\frac{3}{2}} \end{aligned}$$

Hence, now equating the bases of both sides:

$$\therefore x = 4$$

Alternatively, this question may be done by inputting the following into our calculator:

$$\begin{aligned} x &= \sqrt[\frac{3}{2}]{2^3} \\ &= 4 \end{aligned}$$

g)

$$\log_x \frac{1}{243} = -\frac{5}{2}$$

Converting to index form:

$$x^{-\frac{5}{2}} = \frac{1}{243}$$

Converting the RHS into base 3:

$$x^{-\frac{5}{2}} = 3^{-5}$$

Since we want the power to equal to $-\frac{5}{2}$ for the RHS, we are going to square then square root it like so:

$$\begin{aligned} x^{-\frac{5}{2}} &= 3^{-5} \\ &= (3^2)^{-\frac{5}{2}} \\ &= 9^{-\frac{5}{2}} \end{aligned}$$

Hence, now equating the bases of both sides:

$$\therefore x = 9$$

Alternatively, this question may be done by inputting the following into our calculator:

$$\begin{aligned} x &= \sqrt[-\frac{5}{2}]{3^{-5}} \\ &= 9 \end{aligned}$$

4.

a)

$$\log_k k = 1$$

b)

$$\log_k k^4 = 4$$

c)

$$\log_k 1 = 0$$

d)

$$\log_k \sqrt{k} = \frac{1}{2}$$

e)

$$\log_k \sqrt[5]{k} = \frac{1}{5}$$

f)

$$\log_k \frac{1}{k} = -1$$

g)

$$\log_k \frac{1}{k^3} = -3$$