FUNCTIONS

HYPERBOLAS AND INVERSE VARIATION (XIII)

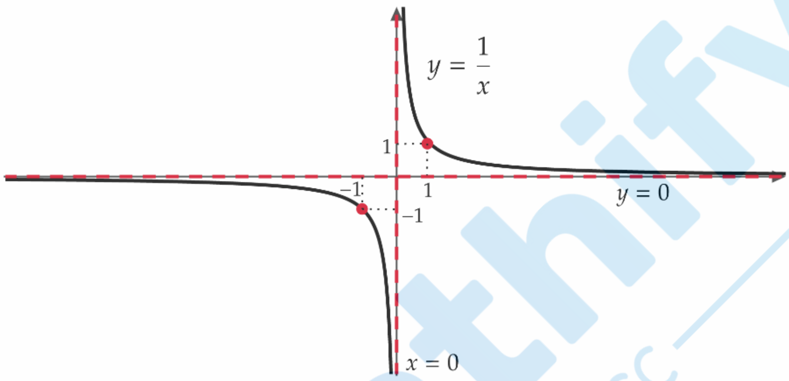
Contents include:

- Rectangular Hyperbolas
- Further Hyperbola Graphs
- Inversely Proportional

- Rectangular Hyperbolas

Rectangular hyperbolas, or hyperbolas for short, are special types of reciprocal functions, with the most basic function being $$y = \frac{1}{x}$$

The graph will resemble:



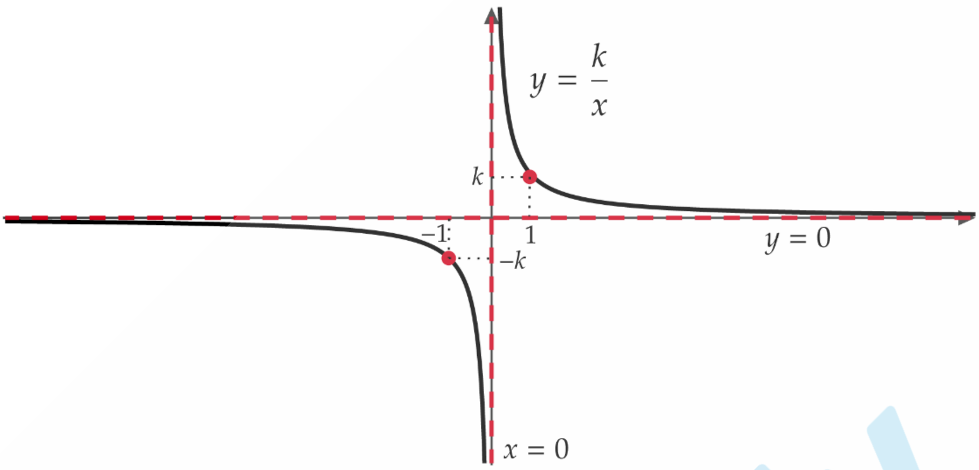
The important points to consider for these graphs:

- There are two "**branches**" of this graph
- There is a **vertical and horizontal asymptote**, at $x = 0$ $(y - axis)$ and $y = 0$ $(x - axis)$ respectively (hence, the dotted lines)
- Key points to plot whenever graphing hyperbolas include when $x = 1$ and $x = -1$. In this case, these two points have coordinate $(1, 1)$ and $(-1, -1)$

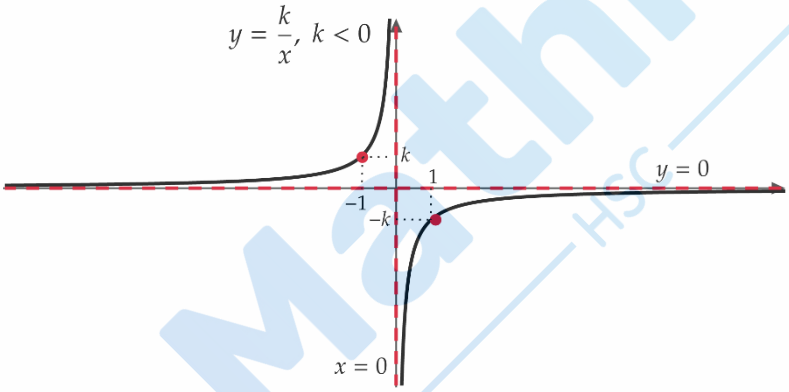
- Further Hyperbola Graphs

A more general hyperbola equation is: $$y = \frac{k}{x}$$ Where k is a constant

If $k > 0$, then its general graph will resemble:



If $k < 0$, then its general graph will resemble:



- Inversely Proportional

If we know that one variable (y) is inversely proportional to another variable (x), then:

$$y = \frac{k}{x}$$

Another way of expressing this is:

$$xy = k$$

For some $non - zero$ constant k, which is known as the *"constant of proportionality"*

The graph of an inverse proportion is thus a **rectangular hyperbola** with asymptotes at $x = 0$ $(y - axis)$ and $y = 0$ $(x - axis)$

Note: "Inversely proportional" and "inverse variation" mean the same thing! Questions may use these two terms interchangeably

Commonly, inverse variation questions will ask students to **find the equation** relating two variables x and y. To do so, we use the following steps:

$$\boldsymbol{Step\ 1: Express\ that\ y = \frac{k}{x}}$$

$$\boldsymbol{Step\ 2: Find\ the\ value\ of\ k}$$

We do this by substituting in known values into our equation for step 1, and rearrange to find k

$$\boldsymbol{Step\ 3: Write\ the\ final\ equation, with\ the\ correct\ k\ value}$$

Example 1:

i. The wavelength λ in metres of a soundwave is inversely proportional to the wave's frequency f in hertz. Write this algebraically

ii. It is known that when the frequency of the wave is 300 hertz, the wavelength is $1.43m$. Find the constant of proportionality

iii. Find the wavelength of a soundwave if it is known that its frequency is 390 hertz

Solution:

i. Since it is inversely proportional, we can say that:

$$\lambda = \frac{k}{f}$$

Where k is a constant of proportionality

ii. Letting $\lambda = 1.43$ and $f = 300$:

$$\therefore 1.43 = \frac{k}{300}$$
$$k = 1.43 \times 300$$
$$= 429$$

iii. Letting $f = 390$:

$$\therefore \lambda = \frac{429}{390}$$

Hyperbolas and Inverse Variation Exercises

1. Consider the hyperbola $y = \frac{3}{x}$

a) Copy and complete the following table of values for the hyperbola:

x	-3	-2	-1	1	2	3
y						

b) Hence, by plotting the points or otherwise, sketch the graph of the hyperbola $y = \frac{3}{x}$

c) Write down the domain and range of the hyperbola $y = \frac{3}{x}$ in bracket interval notation

2. Consider the hyperbola $y = -\frac{2}{x}$

a) Copy and complete the following table of values for the hyperbola:

x	-3	-2	-1	1	2	3
y						

b) Hence, by first plotting the points or otherwise, sketch the graph of the hyperbola $y = -\frac{2}{x}$

c) Write down the domain and range of the hyperbola $y = -\frac{2}{x}$ in bracket interval notation

3. A uniformly elastic demand curve is used to model the sales of Bob's shirts manufacturing, where if Bob decreases the price of each shirt, he will sell a greater quantity and if he increases the price he will sell a lesser quantity. Either way, no matter what price Bob sells each shirt for, the model predicts that he will obtain the same revenue regardless.

Thus, if the price of one shirt is P and the quantity sold per year is Q then the revenue R will always be $R = PQ$.

a) Last year, the total revenue of Bob's shirts was \$600 000 and each one was sold for \$75. Find how many shirts were sold

b) Under the same model as last year, find how many shirts Bob would expect to sell if the price of each shirt is reduced to \$50

c) Sketch a graph of the elastic demand curve with P on the horizontal axis and Q on the vertical axis

Hyperbolas and Inverse Variation Exercise Answers

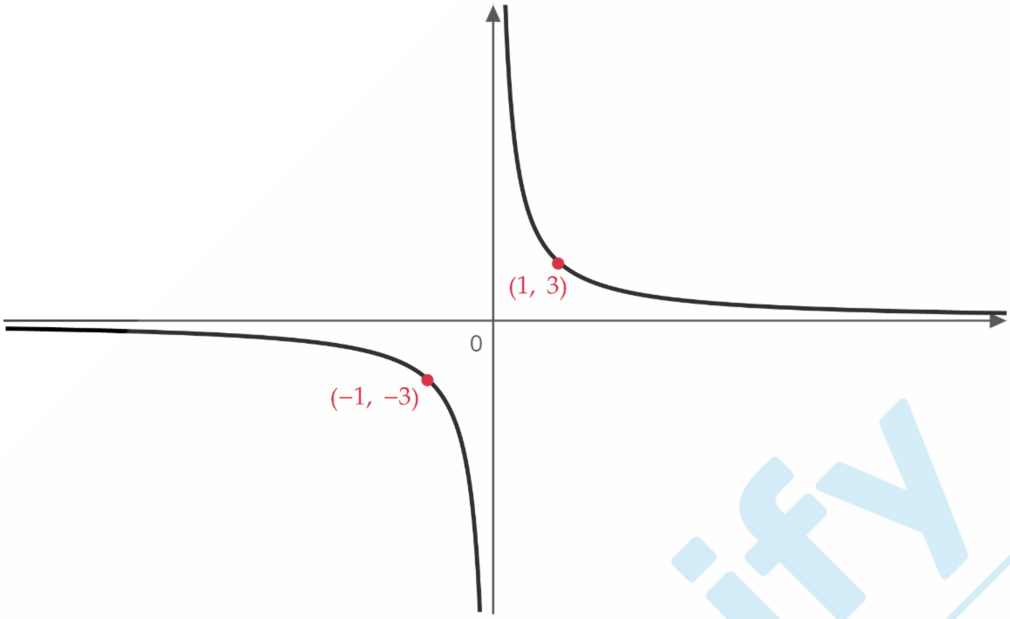
1.

a) Completing the table of values:

x	-3	-2	-1	1	2	3
y	-1	$-\frac{3}{2}$	-3	3	$\frac{3}{2}$	1

b)

Sketching the graph of $y = \frac{3}{x}$:



c)

For the domain of the hyperbolas, we must remember that $x \neq 0$. Hence, in bracket interval notation for the domain:

$$(-\infty, 0) \cup (0, \infty)$$

For the range of the hyperbolas, we must also remember that $y \neq 0$. Hence, in bracket interval notation for the range:

$$(-\infty, 0) \cup (0, \infty)$$

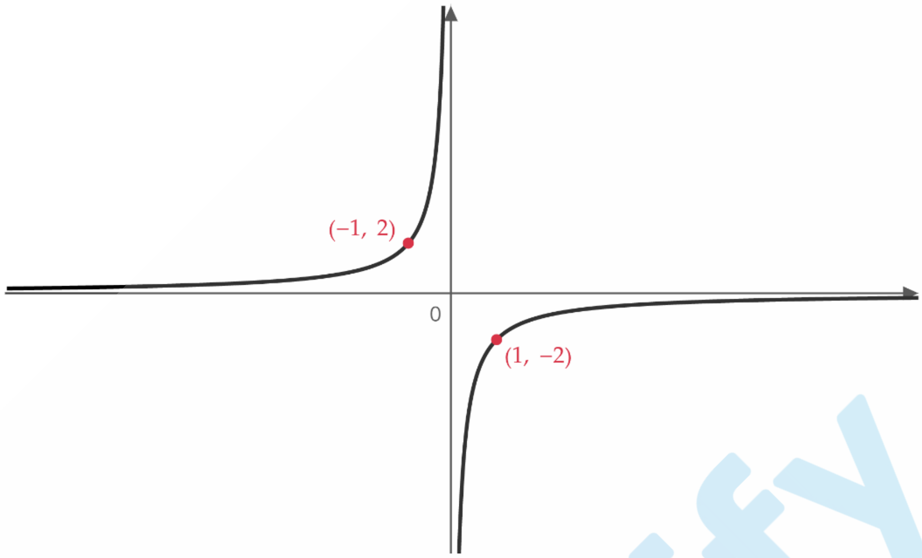
2.

a) Completing the table of values:

x	-3	-2	-1	1	2	3
y	$\frac{2}{3}$	1	2	-2	-1	$-\frac{2}{3}$

b)

Sketching the graph of $y = -\frac{2}{x}$:



c)

For the domain of the hyperbolas, we must remember that $x \neq 0$. Hence, in bracket interval notation for the domain:

$$(-\infty, 0) \cup (0, \infty)$$

For the range of the hyperbolas, we must also remember that $y \neq 0$. Hence, in bracket interval notation for the range:

$$(-\infty, 0) \cup (0, \infty)$$

3.

a) Since it is known that $R = PQ$:

$$Q = \frac{R}{P}$$

Substituting $R = 600000$ and $P = 75$:

$$\therefore Q = \frac{600000}{75}$$
$$= 8000$$

b) Now since revenue is constant, it is known that $R = 600000$. Hence:

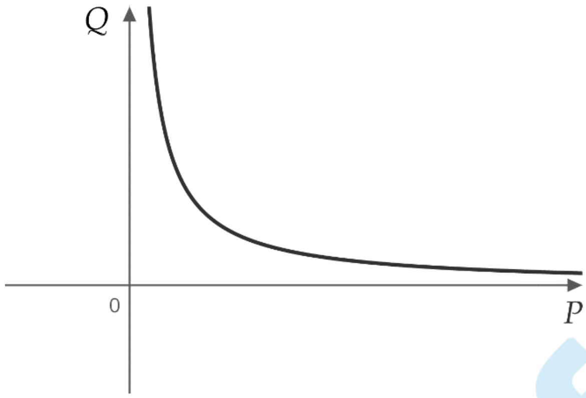
$$Q = \frac{R}{P}$$
$$= \frac{600000}{P}$$

Therefore, if price is reduced to $P = 50$:

$$Q = \frac{600000}{50}$$
$$= 12000$$

Bob will therefore sell 12000 shirts.

c) Sketching the formula for $Q = \frac{600000}{P}$:



Notice here that since price can only be positive, $P > 0$ is our domain