TRIGONOMETRY

TRIGONOMETRIC RATIOS (I)

Contents include:

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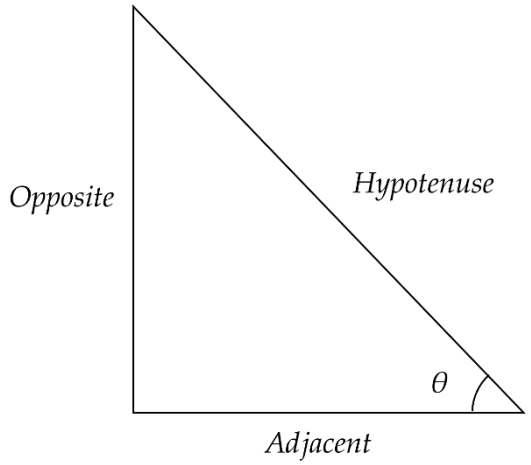
- Exact Values
- Degrees, Minutes and Seconds
- Reciprocal Trig Functions
- Finding the Unknown Side of a Triangle
- Finding the Unknown Angle of a Triangle

- Trigonometric Ratios in a Triangle



Recall that for a right-angled triangle with angle θ $(theta)$:

$$\sin\theta = \frac{opposite}{hypotenuse}$$

$$\cos\theta = \frac{adjacent}{hypotenuse}$$

$$\tan\theta = \frac{opposite}{adjacent}$$

- Exact Values

There are specific angles in trigonometry where the exact value must be remembered:

	$\theta = 30°$	$\theta = 45°$	$\theta = 60°$
$\sin\theta$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\tan\theta$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

The reason why these must be remembered is because the calculator cannot provide exact values, so if questions require us to give answers in exact values we **can't just rely on the calculator**

Example 1: Find the exact value of $\sin 60° \times \cos 45°$

Since it is known that:

$$\sin 60° = \frac{\sqrt{3}}{2}$$

$$\cos 45° = \frac{1}{\sqrt{2}}$$

$$\therefore \sin 60° \times \cos 45° = \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}}$$

- Degrees, Minutes and Seconds

In terms of units for angles, one degree can be further split into 60 minutes. In other words:

$$1° = 60'$$

Thus, when rounding to the nearest degree, make sure to remember that less than 30 minutes means **round down** and 30 or more is **round up**

Moreover, one minute can be further split into 60 seconds. In other words:

$$1' = 60"$$

Thus, when rounding to the nearest minute, make sure to remember that less than 30 seconds means **round down** and 30 or more is **round up**

For example:

a) $28° 44'$ rounded to the nearest degree is $29°$
b) $55°30'$ rounded to the nearest degree is $56°$
c) $67°25'34"$ rounded to the nearest minute is $67°26'$
d) $105°14'25"$ rounded to the nearest minute is $105°14'$

- Reciprocal Trig Functions

The reciprocal of each of the basic trigonometric functions are defined as:

$$\sec\theta = \frac{1}{\cos\theta} = \frac{hypotenuse}{adjacent}$$

$$cosec\ \theta = \frac{1}{\sin\theta} = \frac{hypotenuse}{opposite}$$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{adjacent}{opposite}$$

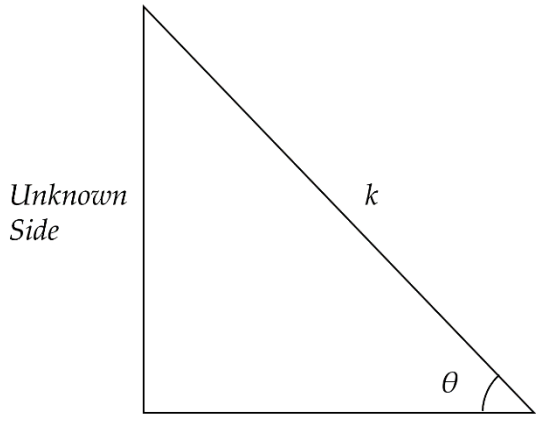
Example 2: Find the exact value of $\sec 60°$

Since $\sec\theta = \frac{1}{\cos\theta}$:

$$\therefore \sec 60° = \frac{1}{\cos 60°}$$
$$= \frac{1}{\frac{1}{2}}$$
$$= 2$$

- Finding the Unknown Side of a Triangle

In some questions we are provided with a triangle that has a known angle but has an unknown side length



Where:

- k is a known side length
- θ is a known angle
-

Thus, to find the length of the unknown side using trigonometry, we use the following steps:

$$\textbf{\textit{Step 1: Determine which trig ratio you're going to use}}$$

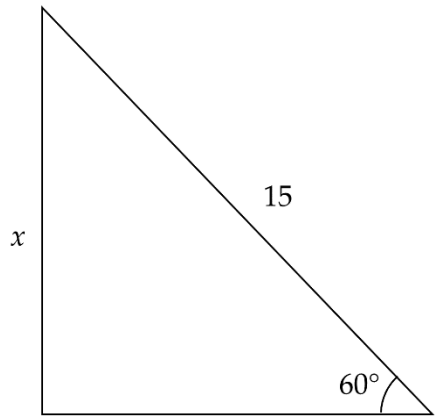
This is typically the ratio where:

$$trig\ ratio = \frac{unknown\ side}{known\ side}$$

$$\textbf{\textit{Step 2: Rearrange and find the unknown side}}$$

$$\therefore unknown\ side = trig\ ratio \times known\ side$$

Example 3: Find the value of x in the following right – angled triangle



Solution:

Notice for this question that:

$$\frac{x}{15} = \frac{opposite}{hypotenuse}$$

Therefore, we should use the sine function in this situation:

$$\sin 60° = \frac{x}{15}$$

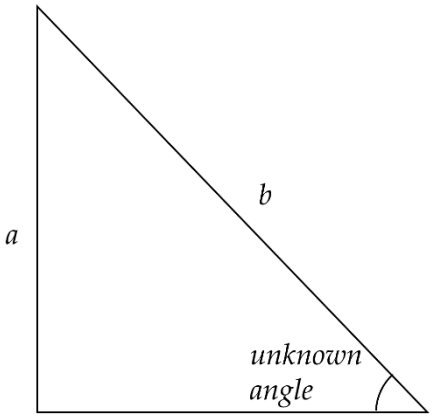
$$\therefore x = 15 \sin 60°$$

$$= 15 \times \frac{\sqrt{3}}{2}$$

$$= \frac{15\sqrt{3}}{2}$$

- Finding the Unknown Angle of a Triangle

In some questions we are provided with a triangle that has a known angle but has an unknown side length



Where:

- a, b are known side lengths of the triangle

Thus, to find the unknown angle using trigonometry, we use the following steps:

$$Step\ 1: Determine\ which\ trig\ ratio\ you\ want\ to\ use$$

This is typically the ratio where:

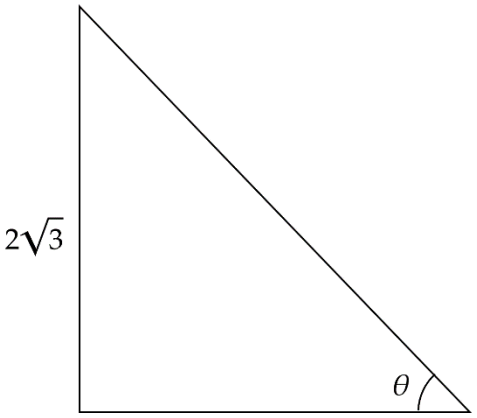
$$Trig\ ratio = \frac{a}{b}$$

$$Step\ 2: Calculate\ the\ unknown\ angle$$

We do this by using our calculator and any one of the following inverse trig functions:

$$\sin^{-1}\frac{a}{b}, \cos^{-1}\frac{a}{b}, OR\ \tan^{-1}\frac{a}{b}$$

Example 4: Find the value of θ in the following right – angled triangle:



Solution:

First determining the trig ratio, notice that:

$$\frac{2\sqrt{3}}{6} = \frac{opposite}{adjacent}$$

Therefore, we use the tan function:

$$\begin{aligned}
\tan\theta &= \frac{2\sqrt{3}}{6} \\
&= \frac{\sqrt{3}}{3} \\
&= \frac{1}{\sqrt{3}} \\
\therefore \theta &= \tan^{-1}\frac{1}{\sqrt{3}} \\
&= 30°
\end{aligned}$$