TRIGONOMETRY

THE SINE & COSINE RULES (III)

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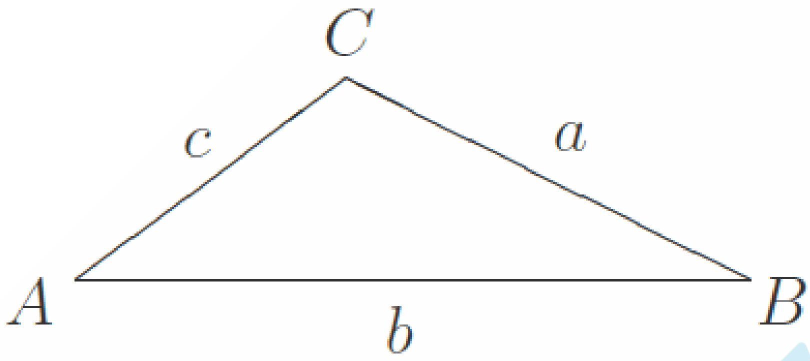
- The Sine Rule



For any given triangle with sides of length a, b and c along with angles whose magnitudes are A, B and C as shown above, the sine rule states that:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

When rearranged, the sine rule may also be written as:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

There are two applications of sine rule:

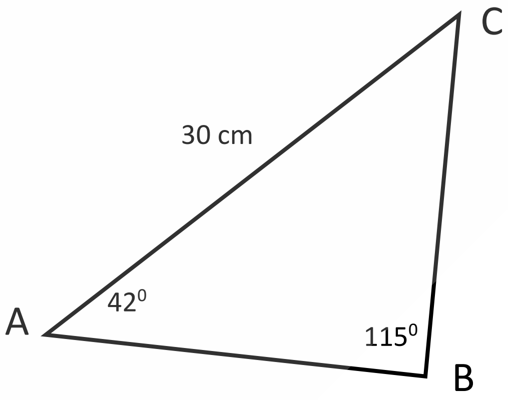
1. **To find the length of one unknown side of a triangle**

In order to do this, we must be given at least the value of 2 angles in my triangle, along with the length of another side.

For these applications, use the sine rule form of:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

Example 1: In triangle ABC the length of AC is 30 cm. Angle $\angle ABC = 115^0$ and angle $\angle BAC = 42^{0}$. Work out the length of BC.



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Solution:

First, we apply the sine rule formula

$$\therefore \frac{BC}{\sin 42°} = \frac{30}{\sin 115°} \; [sine\ rule]$$

Then, rearranging to make BC the subject:

$$\therefore BC = \frac{30}{\sin 115°} \times \sin 42°$$
$$= 22.49\ cm$$

In summary, we follow these steps:

$$\boldsymbol{Step\ 1: Write\ down\ your\ sine\ rule\ equation}$$

This should be in the form of:

$$\frac{unknown\ side}{opposite\ angle} = \frac{known\ side}{other\ opposite\ angle}$$

$$\boldsymbol{Step\ 2: Rearrange\ to\ make\ the\ unknown\ side\ the\ subject}$$

$$\boldsymbol{Step\ 3: Solve}$$

2. To find the value of an unknown angle in the triangle

To do this, we must be given at least the value of 2 sides in my triangle, along with the value of another angle

For these applications, use the sine rule form of:

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

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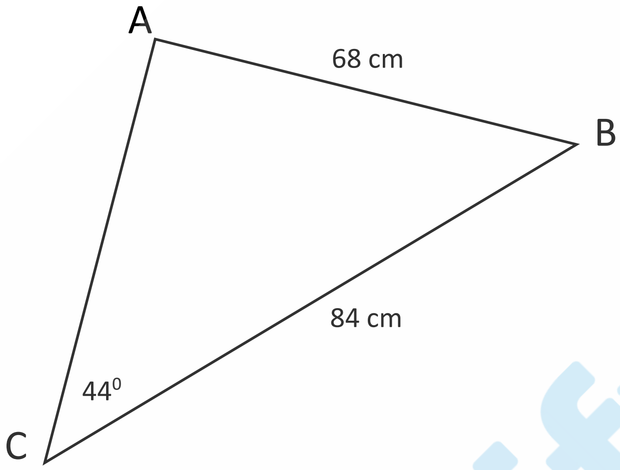
Example 2: In triangle ABC the length of BC is 84 cm. length of AB is 68 cm. Angle $\angle ACB = 44^0$. Find the angle A in triangle ABC to the nearest degree.



Solution:

First, we apply the sine rule formula:

$$\frac{\sin A}{84} = \frac{\sin 44}{68}$$

Then, rearranging to make $\sin A$ the subject:

$$\sin A = 84 \times \frac{\sin 44}{68}$$
$$= 0.858$$
$$\therefore A = 59°$$

In summary, we follow these steps:

$$\boldsymbol{Step\ 1: Write\ down\ your\ sine\ rule\ equation}$$

This should be in the form of:

$$\frac{\sin(unknown\ angle)}{opposite\ side} = \frac{\sin(known\ angle)}{other\ opposite\ side}$$

$$\boldsymbol{Step\ 2: Rearrange\ to\ make\ the\ unknown\ side\ the\ subject}$$

$$\boldsymbol{Step\ 3: Solve\ for\ the\ angle!}$$

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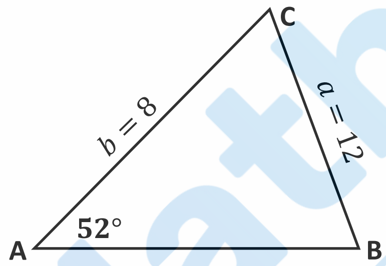
- The Ambiguous Case for Sine Rule

When using the Sine Rule to find a certain angle in a triangle, our working out will eventually reach a point where $\sin A = x$. Then, to find the value of A we plug it into our calculator using the inverse function. However, while the calculator gives us only one answer, θ, we must **ALWAYS** look out for $180 - \theta$ as that may also be a solution to the function.

> This is known as the ambiguous case.
>
> Usually, we can rule out the ambiguous case by remembering that the total angle sum of a triangle is 180°. Refer to example 3 for how to tackle these sorts of questions.

Example 3: Find the size of angle B in triangle ABC, given that $A = 52°$, $BC = 12cm$ and $AC = 8cm$:



By the sine rule:

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{12}{\sin 52°} = \frac{8}{\sin B}$$
$$\sin B = \frac{8 \sin 52°}{12}$$
$$= 0.5253$$

$$\therefore B = 31°41' \; OR \; 148°19'$$

However, since $A = 52°$, B therefore must only equal $31°41'$

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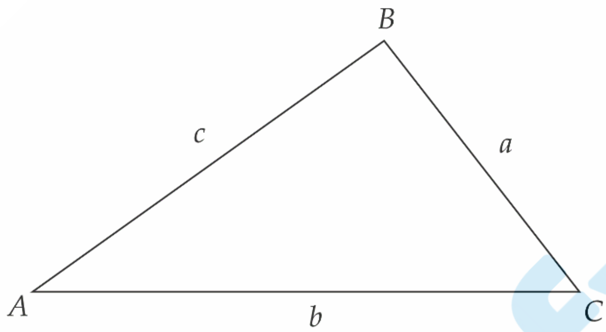
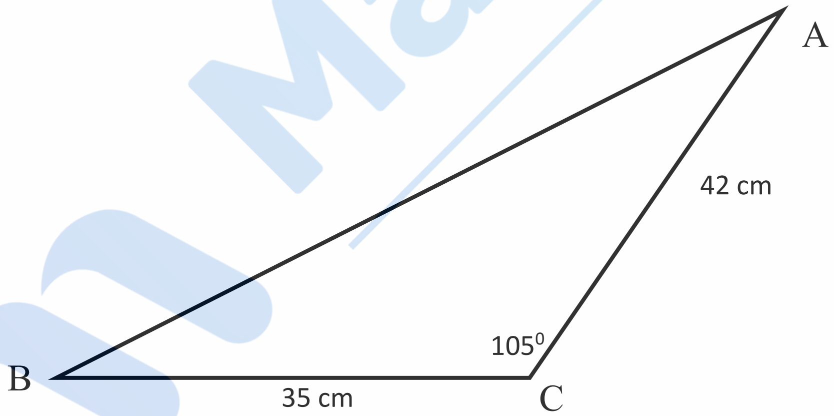
- The Cosine Rule

For any given triangle with sides of length a, b and c along with angles whose magnitudes are A, B and C, the cosine rule states that:



$$c^2 = a^2 + b^2 - 2ab\cos C$$

Example 4: In triangle ABC the length of BC is 35 cm. length of AC is 42 cm. Angle $\angle BCA = 105^0$. Find the length of AB in triangle ABC.



Using Cosine rule:

$$AB^2 = BC^2 + AC^2 - 2 \times BC \times AC \times \cos C$$

$$c^2 = 35^2 + 42^2 - 2 \times 35 \times 42 \cos 105°$$

$$c^2 = 1225 + 1764 - 2940 \times (-0.2588)$$
$$\therefore c = 61.23cm$$

Therefore, the length of AB is 61.23 cm

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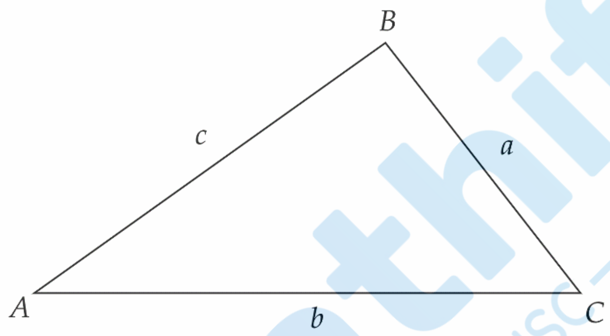
- Sine Area Formula

The sine area formula is essentially another method that can be utilised to calculate the area of a triangle.

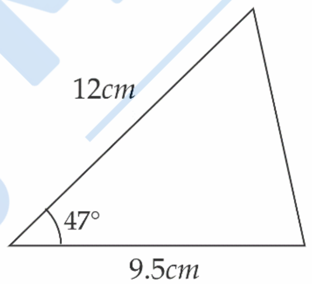
In any triangle ΔABC:

$$Area\ of\ \Delta ABC = \frac{1}{2}ab\sin C$$

Where angle A is the angle in between the sides a and b of the triangle



Example 5: Calculate the area of the triangle shown below, correct to the nearest 2 decimal places:



Solution:

Since we do not know the perpendicular height length, we must use the sine area formula:

$$\begin{aligned} Area\ of\ Triangle &= \frac{1}{2} \times 12 \times 9.5 \times \sin(47°) \\ &= 41.69\ cm^2\ (nearest\ 2\ dp) \end{aligned}$$

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