PROBABILITY

SETS AND VENN DIAGRAMS (V)

Contents include: Set notation, intersections, unions, subsets, complements and Venn diagrams

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- Set Notation

A set is a collection of numbers or elements in a list inside curly brackets, for example:

$$S = \{2, 4, 6, 8, 10\}$$

The number of elements in a set is referred to as its **size**, with the notation for this being $|S|$. In this case, $|S| = 5$

If the size of a set is 0, $i.e. |S| = 0$, then this is referred to as an **empty set**. The notation for empty sets is $\emptyset$

Two sets are **equal** when they have exactly the same members. For **equal sets** though, there are two things to remember:

1. Order does **not** matter

$$i.e. \{1, 3, 5, 7, 9\} = \{3, 5, 9, 1, 7\}$$

2. Repetition does **not** matter

$$i.e. \{1, 3, 5, 7, 9\} = \{1, 1, 3, 5, 5, 7, 9, 9\}$$

- Intersections and Unions

The intersection between A and B refers to the set of elements that exist in set A **and** set B. **The set notation for this is given as $\boldsymbol{A \cap B}$**

In other words:

$$And = \cap$$

For example, if $A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 4, 6, 8, 10\}$, then $A \cap B = \{2, 4\}$

If $A \cap B = \emptyset$, then the sets A and B are called disjoint

The union between A and B refers to the set of elements that exist in **set A or in set B**. The set notation for this is given as $A \cup B$.

In other words:

$$Or = \cup$$

For example, if $A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 4, 6, 8, 10\}$, then $A \cup B = \{1, 2, 3, 4, 5, 6, 8, 10\}$

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- Subsets of Sets

Set A is called a **subset** of set B if every element of A is an element of B. The notation for this is given as $A \subset B$.

For example:

$$\{2,4,6\} \subset \{2,4,6,8\ 10\}$$

Two special cases to remember are that:

1. Every set is a subset of itself. For example:

$$\{1,4,5\} \subset \{4,5,1\}$$

2. The empty set is always a subset of any set.

$$\emptyset \subset \{1,2,3\}$$

- Universal Sets and Complements

A universal set is one that contains all elements under discussion, and will vary with each different question.

The **complement** of a set A contains all the elements that are **not** in A but are part of the universal set. The complement of a set A is written as $\bar{A}$ For example, if the universal set $E = \{1,2,3,4,5,6,7,8,9,10\}$ and $A = \{2,4,6,8,10\}$, then: $\bar{A} = \{1,3,5,7,9\}$

Note that the complement of a set and itself should never have any overlap, while the union of the two will equal the universal set.

Example 1: If $A = \{2,5,7,10\}$ and $B = \{1,2,4,5,7,8,10,11\}$, state whether the following are true or false:

a) $|A| = 4$
b) $A = B$
c) $|B| = 9$
d) $A \subset B$
e) $A \cap B = \{1,2,5,7,10\}$
f) $A \cup B = B$

Solution:

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a) Since there are 4 elements in set A, this means that $|A| = 4$

$$\therefore |A| = 4$$

$$\therefore True$$

b) Two sets are equal when they have the same elements. However, set B does not have the same elements as set A.

$$\therefore A \neq B$$

$$\therefore False$$

c) Since there are 8 elements in set B, this means that $|B| = 8$

$$\therefore |B| \neq 9$$

$$\therefore False$$

d) Since all the elements of set A $(2, 5, 7, 10)$ appear in set B, in can be said that set A is a subset of set B

$$\therefore A \subset B$$

$$\therefore True$$

e) When looking at which elements appear in set A and set B:

$$A \cap B = \{2, 5, 7, 10\}$$

$$\therefore A \cap B \neq \{1, 2, 5, 7, 10\}$$

$$\therefore False$$

f) Since it is known that A is a subset of B, anything that appears in A will appear in B

$$\begin{aligned} \therefore A \cup B &= \{1, 2, 4, 5, 7, 8, 10, 11\} \\ &= B \end{aligned}$$

$$\therefore True$$

Example 2: If $A = \{2, 5, 6, 8\}$, $B = \{1, 2, 5, 7\}$ and $E = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, find:

a) $|A|$
b) $|B|$
c) $A \cap B$
d) $|A \cap B|$
e) $A \cup B$
f) $\bar{A}$
g) $\bar{B}$

Solutions:

a) Looking at the number of elements in set A:

$$|A| = 4$$

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b) Looking at the number of elements in set B:

$$|B| = 4$$

c) Looking at which elements appear in set A and set B:

$$A \cap B = \{2, 5\}$$

d) The number of elements in part c) is:

$$|A \cap B| = 2$$

e) Looking at which elements appear in set A OR set B:

$$A \cup B = \{1, 2, 5, 6, 7, 8\}$$

f) Considering the elements which aren't in set A, in other words the complement of A:

$$\bar{A} = \{1, 3, 4, 7, 9\}$$

g) Considering the elements which aren't in set B, in other words the complement of B:

$$\bar{B} = \{3, 4, 6, 8, 9\}$$

Example 3: If $A = \{1, 5, 8, 9\}$, $B = \{2, 5, 7, 8, 10\}$ and $E = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, find:

a) $\bar{A}$
b) $\bar{B}$
c) $A \cap B$
d) $\overline{A \cap B}$
e) $A \cup B$
f) $\overline{A \cup B}$
g) $\bar{A} \cap \bar{B}$
h) $\bar{A} \cup \bar{B}$

Solution:

a) Considering the complement of A, in other words the elements not in set A:

$$\bar{A} = \{2, 3, 4, 6, 7, 10\}$$

b) Considering the complement of B, in other words the elements not in set B:

$$\bar{B} = \{1, 3, 4, 6, 9\}$$

c) Looking at which elements appear in both set A and set B:

$$A \cap B = \{5, 8\}$$

d) Considering the complement of $A \cap B$:

$$\overline{A \cap B} = \{1, 2, 3, 4, 6, 7, 9, 10\}$$

e) Looking at which elements appear in set A or set B:

$$A \cup B = \{1, 2, 5, 7, 8, 9, 10\}$$

f) Considering the complement of $A \cup B$:

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$$\overline{A \cup B} = \{3, 4, 6\}$$

g) We are asked to find which elements appear in both $\bar{A}$ and $\bar{B}$, hence looking at parts a) and b):

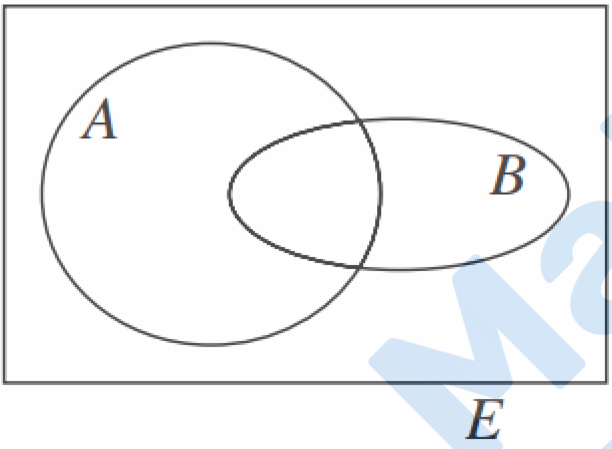
$$\bar{A} \cap \bar{B} = \{3, 4, 6\}$$

h) Which elements appear in $\bar{A}$ or $\bar{B}$:

$$\bar{A} \cup \bar{B} = \{1, 2, 3, 4, 6, 7, 9, 10\}$$

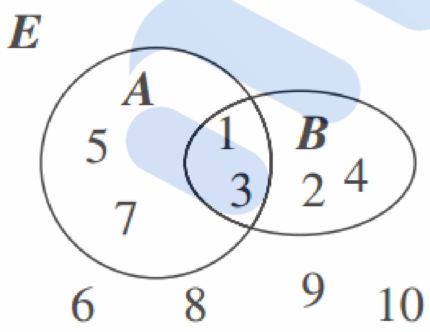
- Venn Diagrams

Venn diagrams are essentially **visual representation of sets**, where each area bounded by a line represents a single set.



In this case, the set E represents the universal set which contains sets A and B. The overlap between the two ovals represents $A \cap B$, while the total area covered by A and B represents $A \cup B$.

Example 4: List out the elements in the sets $A, B, \bar{A}, \bar{B}, A \cap B, A \cup B$



Solution:

$$A = \{1, 3, 5, 7\} \text{ and } B = \{1, 2, 3, 4\}$$

$$\bar{A} = \{2, 4, 6, 8, 9, 10\} \text{ and } \bar{B} = \{5, 7, 6, 8, 9, 10\}$$

$$A \cap B = \{1, 3\} \text{ and } A \cup B = \{1, 3, 5, 7, 2, 4\}$$

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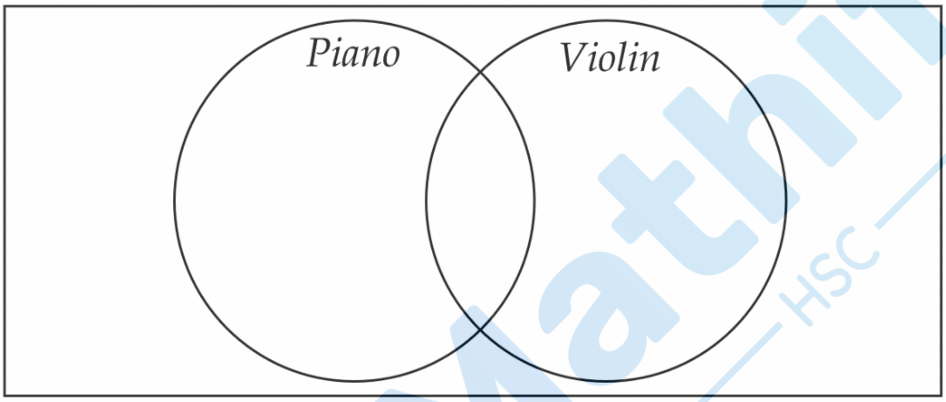
Example 5: Use Venn diagrams to solve the following:

a) In a group of 50 students, it is known that 12 play the violin, 8 play both the violin and the piano and 20 play the piano. If the rest don't play any instrument, find how many people don't play any instrument.

b) In a class of 60 students, it is known that 12 don't play any sport and 20 play soccer, with 11 playing only soccer. If there are only two sports; soccer and basketball, find the number of people playing only basketball in the class.

Solution:

a) We start with an empty Venn diagram labelled with Piano and Violin:



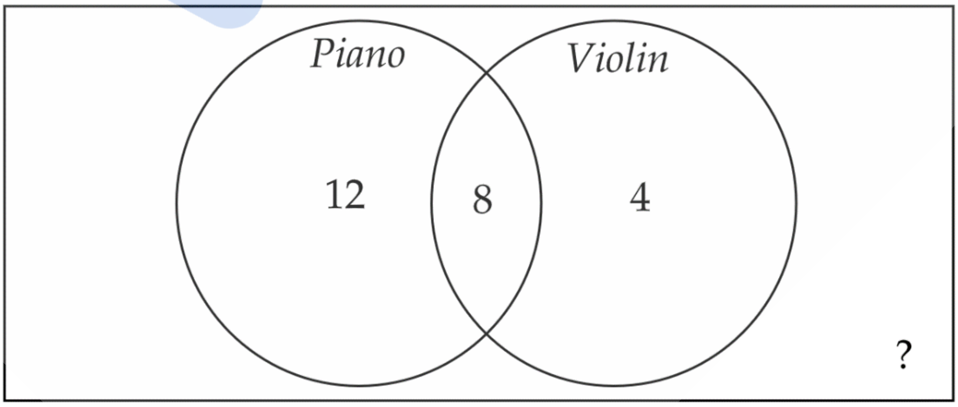
Before we fill in the Venn diagram, we first calculate:

$$Play\ both\ violin\ and\ piano = 8\ [given]$$

$$\therefore Play\ violin\ only = 12 - 8$$
$$= 4$$

$$\therefore Play\ Piano\ only = 20 - 8$$
$$= 12$$

Now sketching the Venn diagram with the information we know so far:



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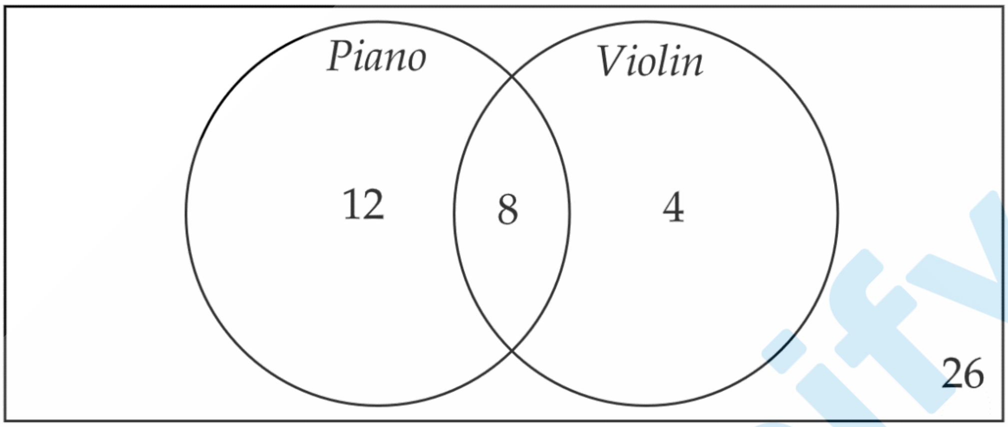
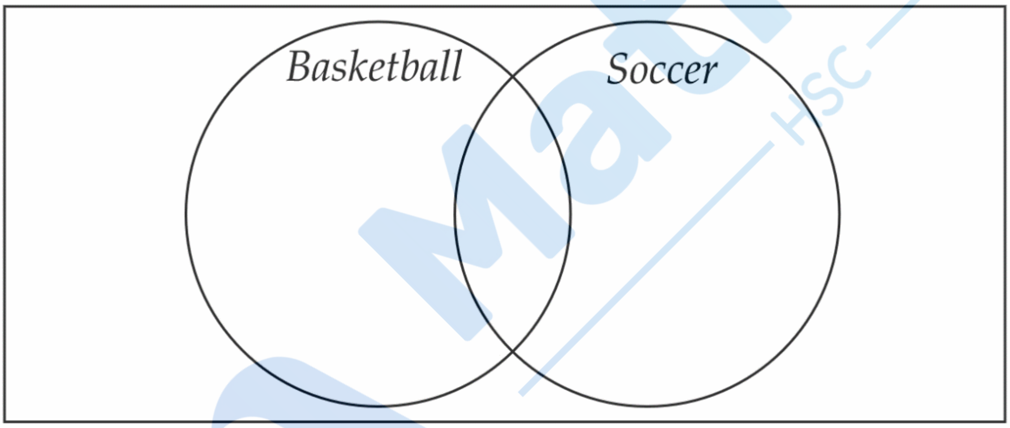
We so far have 24 students on my diagram. Since the rest don't play an instrument:

$$\begin{aligned} Number\ who\ don't play &= 50 - 24 \\ &= 26 \end{aligned}$$

Hence, our final Venn diagram is:



b) We start with an empty Venn diagram labelled with basketball and soccer:



Before we sketch the Venn diagram, we first calculate:

$$\begin{aligned} Play\ both\ basketball\ and\ soccer &= 20 - 11 \\ &= 9 \end{aligned}$$

$$Don't\ play\ either\ sport = 12\ [given]$$

Now sketching the Venn diagram with the information we know so far:

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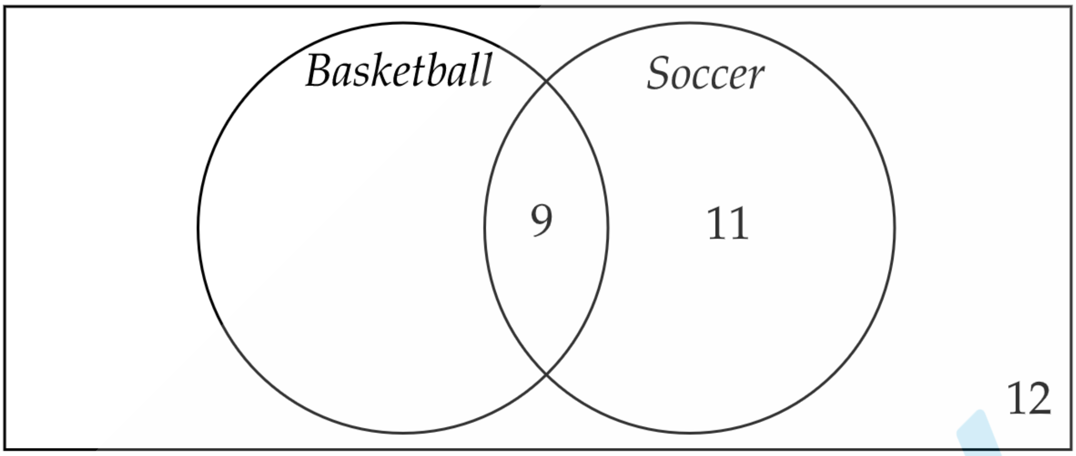
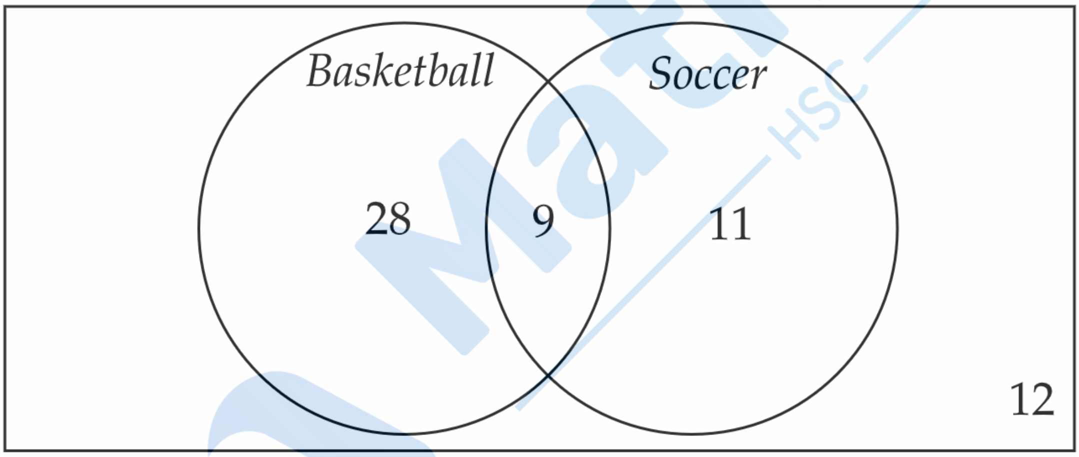


Therefore, the number of students who play basketball only may be calculated:

$$\begin{aligned} Play\ basketball\ only &= 60 - 9 - 11 - 12 \\ &= 28 \end{aligned}$$

Hence, our final Venn diagram is:



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