DIFFERENTIATION

BASIC DIFFERENTIATION (III)

Contents include:

- Rules for Differentiation

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When differentiating a function, the **first principles method should not be used unless specifically asked for** by the question.

This is because differentiating can be done in a much more efficient method by following the rules outlined:

- Derivative of ax^n

$$If\ f(x) = ax^n, then\ f'(x) = anx^{n-1}, where\ a\ is\ a\ constant$$

Example 1: Differentiate $f(x) = x^4$

$$\begin{aligned} f'(x) &= 4x^{4-1} \\ &= 4x^3 \end{aligned}$$

- Derivative of a constant

$$If\ f(x) = a, then\ f'(x) = 0$$

Example 2: Differentiate $f(x) = 5$

$$f(x) = 5x^0$$

$$\begin{aligned} \therefore f'(x) &= 0 \times 5x^{-1} \\ &= 0 \end{aligned}$$

- Derivative of a polynomial

$$\begin{aligned} &If\ h(x) = f(x) \pm g(x), \\ &Then\ h'(x) = f'(x) \pm g'(x) \end{aligned}$$

This means that we can essentially split up and differentiate each individual term

Example 3: Find the derivative of $f(x) = x^3 + x^2 + x + 3$

$$\begin{aligned} \frac{d(x^3 + x^2 + x + 1)}{dx} &= \frac{d(x^3)}{dx} + \frac{d(x^2)}{dx} + \frac{d(x)}{dx} + \frac{d(3)}{dx} \\ &= 3x^{3-1} + 2x^{2-1} + 1x^{1-1} + 0 \\ &= 3x^2 + 2x + 1 \end{aligned}$$

Example 4: Differentiate $h = \frac{1}{2}t^6 - \frac{1}{6}t^3$

$$\frac{dh}{dt} = \frac{1}{2} \times 6t^{6-1} - \frac{1}{6} \times 3t^{3-1}$$

$$= 3t^5 - \frac{1}{2}t^2$$

Example 5: If $f(x) = (3x - 2)^2$, find the expression for $f'(x)$

First we must expand our brackets to get:

$$f(x) = 9x^2 - 12x + 4$$

Then, differentiating our polynomial:

$$\begin{aligned} f'(x) &= 9 \times 2x^{2-1} - 12 \times 1x^{1-1} + 0 \\ &= 18x - 12 \end{aligned}$$

Differentiation Exercises

1. Differentiate $x^3 - x^2 - 6x + 1$

2. Differentiate $f(x) = 4x^2 - 3x + 2$

3. Differentiate $(x - 3)(x + 2)(3x + 1)$

4. Differentiate $\frac{3x^3+4x^2+5x}{x}$

5. If $f(x) = \frac{1}{3}x^3 - \frac{7}{2}x^2 + 2x - 1$, find the expression for $f'(x)$

Differentiation Exercise Answers

1. $f'(x) = 3x^2 - 2x - 6$

2.

$$\begin{aligned} f'(x) &= 4 \times 2x - 3 \\ &= 8x - 3 \end{aligned}$$

3. **Always try to expand your brackets first before differentiating!**

$$\begin{aligned} (x - 3)(x + 2)(3x + 1) &= (x^2 - x - 6)(3x + 1) \\ &= 3x^3 + x^2 - 3x^2 - x - 18x - 6 \\ &= 3x^3 - 2x^2 - 19x - 6 \end{aligned}$$

Then we can continue differentiating like normal

$$f'(x) = 9x^2 - 4x - 19$$

4. Whenever we can factorise or simplify or expression, do that first BEFORE differentiating!

$$3x^2 + 4x + 5$$

Then we can continue differentiating like normal

$$f'(x) = 6x + 4$$

5. $f'(x) = x^2 - 7x + 2$