

PROBABILITY

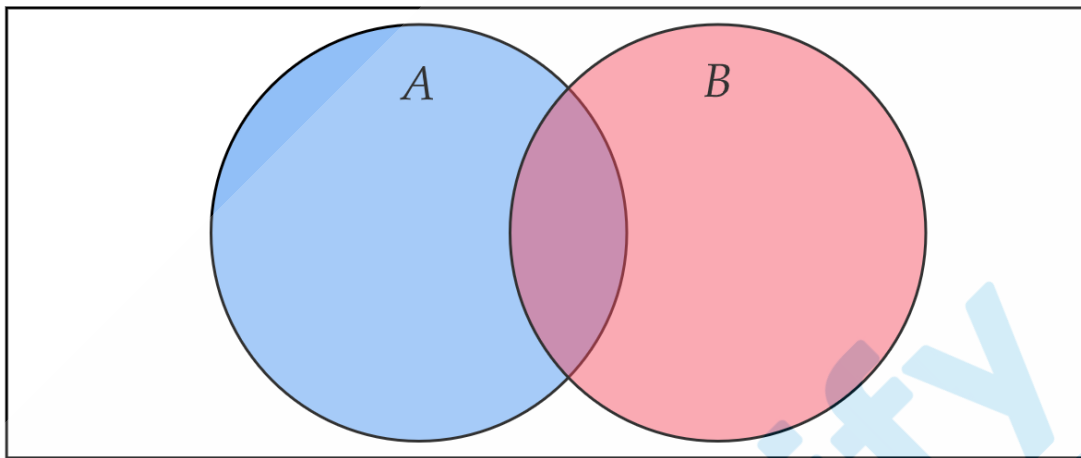
THE ADDITION RULE (VI)

Contents include: The addition rule

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- The Addition Rule for Venn Diagrams and Sets

Considering a Venn Diagram:



Notice from the above diagram that:

$$\text{Entire Shaded Area} = \text{Blue} + \text{Red} - \text{Purple}$$

We must subtract the purple area since we don't want to include the overlapped area twice!

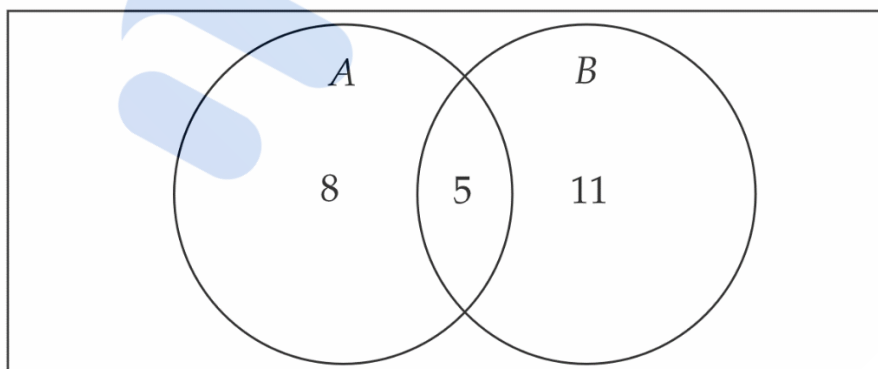
Hence, the **Addition rule** for Venn diagrams highlights that:

$$A \cup B = A + B - A \cap B$$

OR

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Example 1: Verify the addition rule for the below Venn diagram:



Solution:

For this Venn diagram, we can see that:

$$A = 8 + 5 = 13$$

$$B = 11 + 5 = 16$$

$$A \cap B = 5$$

$$A \cup B = 24$$

Now considering $A + B - A \cap B$:

$$\begin{aligned} A + B - A \cap B &= 13 + 16 - 5 \\ &= 24 \\ &= A \cup B \end{aligned}$$

Therefore, we have verified the addition rule to be correct!

Exercise 2: Use the addition rule to answer the following questions:

- a) If $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{12}$, find $P(A \cup B)$
- b) If $P(A) = \frac{2}{7}$, $P(B) = \frac{2}{3}$ and $P(A \cup B) = \frac{19}{21}$, find $P(A \cap B)$
- c) If $P(A \cup B) = \frac{9}{10}$, $P(A \cap B) = \frac{1}{5}$ and $P(A) = \frac{1}{2}$, find $P(B)$

Solution:

- a) Recalling the addition rule:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Hence, substituting in our values:

$$\begin{aligned} \therefore P(A \cup B) &= \frac{1}{4} + \frac{1}{3} - \frac{1}{12} \\ &= \frac{3}{12} + \frac{4}{12} - \frac{1}{12} \\ &= \frac{6}{12} \\ &= \frac{1}{2} \end{aligned}$$

- b) Recalling the addition rule once again:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Hence, substituting in our values:

$$\begin{aligned} \therefore \frac{19}{21} &= \frac{2}{7} + \frac{2}{3} - P(A \cap B) \\ \frac{19}{21} &= \frac{6}{21} + \frac{14}{21} - P(A \cap B) \end{aligned}$$

$$\begin{aligned}\therefore P(A \cap B) &= \frac{6}{21} + \frac{14}{21} - \frac{19}{21} \\ &= \frac{1}{21}\end{aligned}$$

c) Recalling the addition rule once again:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Hence, substituting in our values:

$$\begin{aligned}\frac{9}{10} &= \frac{1}{2} + P(B) - \frac{1}{5} \\ \therefore P(B) &= \frac{9}{10} - \frac{1}{2} + \frac{1}{5} \\ &= \frac{9}{10} - \frac{5}{10} + \frac{2}{10} \\ &= \frac{6}{10} \\ &= \frac{3}{5}\end{aligned}$$