FUNCTIONS

INTRODUCTION, DOMAIN AND RANGE (I)

Contents include:

- What are Functions?
- Range of a Function
- Domain of a Function
- Common Domain and Range Restrictions
- Finding the Intercepts of a Function

- What are Functions?

Put simply, functions are **rules or equations that define the relationship** between one variable (the independent variable) and another (the dependent variable.

Commonly, these functions are expressed like:

$$y = f(x)$$

In these situations:

- The $'x'$ is referred to as the independent variable
- The $'y'$ is referred to as the dependent variable

The equation will be given in terms of x or whatever the independent variable may be

For example, for the function $x = f(t)$, the independent variable in this case will be $'t'$ and the dependent variable this time will be $'x'$.

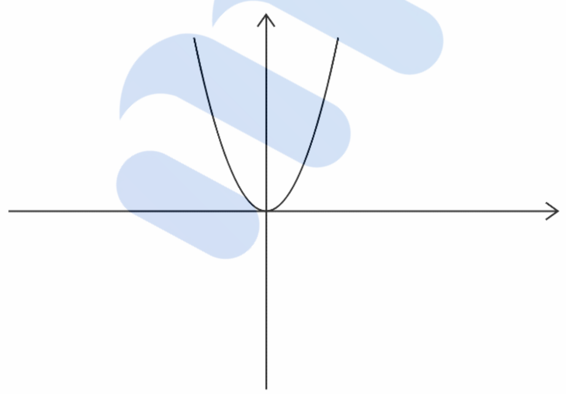
Think of it like whatever is in the bracket will be the independent variable!

- Range of a Function

The range of a function is defined as the set of all possible values y can take and is usually given as an **inequality** or 'for all real y'.

For example:

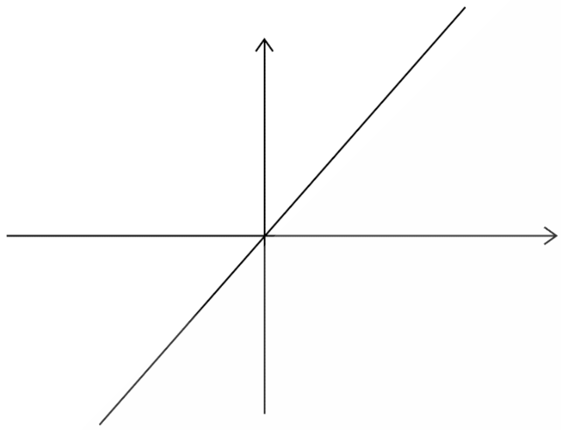
a) For the graph of $y = x^2$:



The range of the function would be:

$$y \geq 0$$

b) For the graph of $y = x$:



The range of the function would be:

$$all\ real\ y$$

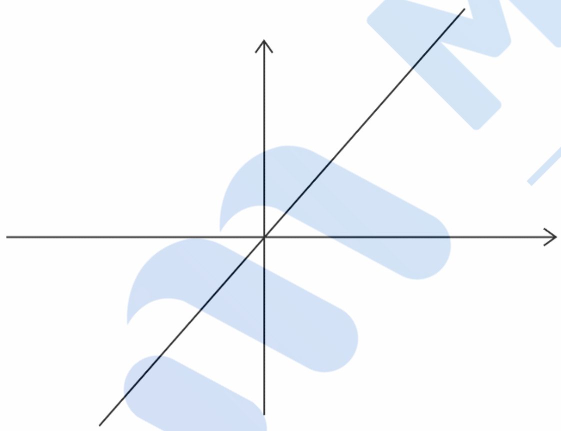
This basically just means all values of y!

- Domain of a Function

The domain of a function is defined as all the possible x values a function may have, and once again is given as an **inequality** or $'$for all real x'

For example:

a) For the graph of $y = x$:

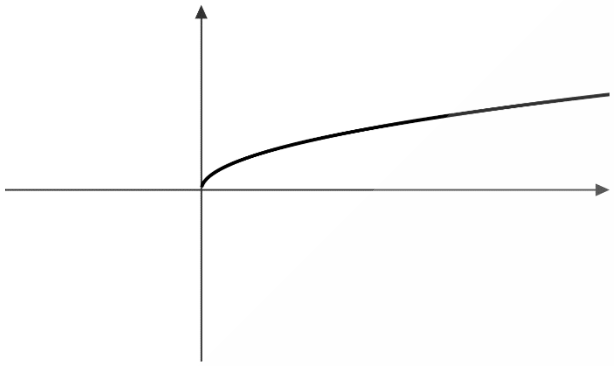


The domain of the function would be:

$$All\ real\ x$$

This basically just means all values of x!

b) For the graph of $y = \sqrt{x}$:



The domain of the function would be:

$$x \geq 0$$

- Common Domain and Range Restrictions

Some common functions in maths will have restricted domains, and it is important to always identify these when they occur.

- Logarithmic functions

Any logarithmic function $f(x) = \log\big(g(x)\big)$ has a domain restriction such that whatever is inside the log, in this case $g(x)$, must be **greater than 0.**

$$\therefore g(x) > 0$$

There is usually no range restriction for these functions

Example 1: What is the domain of the function $y = \log(x + 5)$?

Since we are dealing with a logarithm:

$$x + 5 > 0$$
$$\therefore x > -5 \text{ is the domain}$$

- Fractions

A common domain restriction with any function where a fraction is involved $f(x) = \frac{g(x)}{h(x)}$ has the restriction such that the denominator, in this case $h(x)$, cannot equal 0.

$$\therefore h(x) \neq 0$$

A common range restriction would be the horizontal asymptote of a function, though this varies between questions. This will be explored in further depth in booklet (V)

This is very often forgotten so make sure to prioritise this!

Example 2: What is the domain of the function $y = \frac{x^2+5}{x-3}$?

Since the denominator cannot equal 0:

$$x - 3 \neq 0$$

$$\therefore x \neq 3$$

Hence, the domain is:

$$for\ all\ real\ x, x \neq 3$$

- Square roots

Any function with a square root $f(x) = \sqrt{g(x)}$ has a domain restriction such that whatever is inside the root, in this case $g(x)$, must be greater than or equal to 0

$$\therefore g(x) \geq 0$$

There also exists a range restriction where:

$$f(x) \geq 0$$

Example 3: Identify the domain and range for the function $\sqrt{5x - 10}$

Since we have a square root function:

$$5x - 10 \geq 0$$
$$5x \geq 10$$

$$\therefore x \geq 2\ is\ the\ domain$$

The range would be:

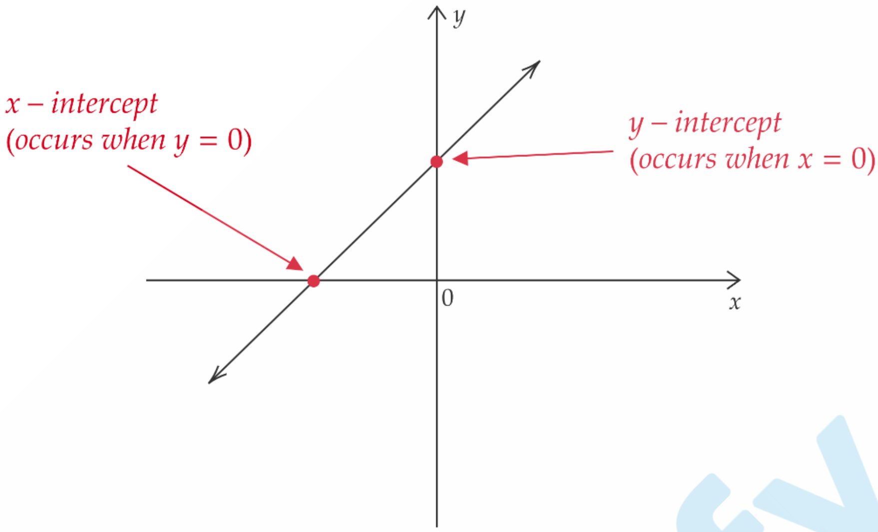
$$y \geq 0$$

- Finding the Intercepts of a Function

Recall that intercepts of a function are defined as the points where a function, $f(x)$, cross the x and the y axis.

- The point where the graph crosses the $x -$ axis is known as the $\boldsymbol{x - intercept}$. To find the $x -$ intercept, we **let our y equal to 0** and solve for x
- The point where the graph crosses the $y -$ axis is known as the $\boldsymbol{y - intercept}$. To find the $y -$ intercept, we **let our x equal to 0** and solve for y

The following is an illustration demonstrating this:



Tip: For questions where we must find both the x and y intercepts, find the y intercept first because it's easier to do!

Example 4: Find the intercepts of the function $f(x) = \frac{2}{3}x + 8$

Solution:

Finding the $y -$ intercept first:

$$f(0) = 8$$

Therefore, the $y -$ intercept is at $y = 8$

Finding the $x -$ intercept next:

$$0 = \frac{2}{3}x + 8$$

$$\therefore \frac{2}{3}x = -8$$

$$x = -12$$