FUNCTIONS

INTERVAL NOTATION (III)

Contents include:

- Bracket Interval Notation

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Typically, when expressing the domain and range of a function, students may be accustomed to expressing these using the inequality signs $\leq$, $\geq$, $<$ and $>$.

For example: $-1 \leq x < 4$

An alternate method to express intervals is known as "Bracket Interval Notation", where the endpoints of our interval for x, $a \leq x \leq b$, are given in brackets like $[a, b]$. There are two types of brackets which can be used:

- A square bracket "[ ]" means that the endpoint is included. Therefore:

$$[a, b] \text{ will be the same as } a \leq x \leq b$$

- A round bracket "( )" means that the endpoint is excluded. Therefore:

$$(a, b) \text{ will be the same as } a < x < b$$

Example 1: Convert the following domains into interval notation

a) $-3 \leq x \leq 5$

$$[-3, 5]$$

b) $-1 \leq x < 4$

$$[-1, 4)$$

c) $-7 < x \leq 8$

$$(-7, 8]$$

When our interval for x is only bound to one endpoint, or no endpoints at all, we employ the use of ∞ or $-\infty$ with round brackets " () " to represent the unbounded side. Therefore:

$$\text{If } x \geq a, \text{then } [a, \infty)$$
$$\text{If } x \leq a, \text{then } (-\infty, a]$$

$$\text{If } x \text{ is any real number, then } (-\infty, \infty)$$

Example 2: Convert the following domains into interval notation

a) $x > 1$

$$(1, \infty)$$

b) $x \leq -7$

$$(-\infty, -7]$$

c) $x > -9$

$$(-9, \infty)$$

If an interval has two or more parts to it, it may be represented through bracket interval form using the set notation symbol " ∪ " which essentially means "or". Therefore:

$$If\ x > a\ or\ x < a, then\ (-\infty, -a) \cup (a, \infty)$$

Example 3: Convert the following domains into interval notation

a) $x \leq 3\ or\ x \geq 9$

$$(-\infty, 3] \cup [9, \infty)$$

b) $x < -8\ or\ x \geq 2$

$$(-\infty, -8) \cup [2, \infty)$$

c) $x \leq -26\ or\ x > -4$

$$(-\infty, -26] \cup (-4, \infty)$$

Interval Notation Exercises

1. Convert the following into inequality interval notation for the range of a function

a) $[-3, \infty)$
b) $(-1, 2]$
c) $(-\infty, 4)$
d) $(-\infty, -2] \cup (6, \infty)$

2. Convert the following into bracket interval notation

a) $-4 < x \leq 7$
b) $x \geq -8$
c) $x < -9\ or\ x \geq 8$
d) $x < -2\ or\ 1 < x \leq 3\ or\ x \geq 21$
e) $x \in \mathbb{R},\ x \neq 7$

3. Find the natural domain of the following functions, giving answers in interval notation

a) $f(x) = \frac{1}{2x+3}$
b) $g(x) = \sqrt{3 - x}$
c) $h(x) = \ln x + 3$
d) $f(x) = \ln(x^2 + 3x - 4)$

Interval Notation Exercise Answers

1.
a) $y \geq -3$
b) $-1 < y \leq 2$
c) $y < 4$
d) $y \leq -2 \text{ or } y > 6$

2.
a) $(-4, 7]$
b) $[-8, \infty)$
c) $(-\infty, -9) \cup [8, \infty)$
d) $(-\infty, -2) \cup (1, 3] \cup [21, \infty)$
e) $(-\infty, 7) \cup (7, \infty)$

3.
a) For these questions, it's always important to remember that the denominator cannot equal 0

$$\therefore 2x + 3 \neq 0$$
$$2x \neq -3$$
$$x \neq -\frac{3}{2}$$

Therefore, the domain is $x \in \mathbb{R}, x \neq -\frac{3}{2}$ which means that x is any real number except $-\frac{3}{2}$. In interval notation this may be written as:

$$\left(-\infty, -\frac{3}{2}\right) \cup \left(-\frac{3}{2}, \infty\right)$$

b) For these questions, always remember that whatever is inside a square root function can never be less than 0

$$\therefore 3 - x \geq 0$$
$$x \leq 3$$

The domain is that $x \leq 3$, in interval notation this may be written as $(-\infty, 3]$

c) For these questions, always remember that whatever is inside a ln function is greater than 0

$$\therefore x > 0$$

In interval notation this domain may be written as $(0, \infty)$

d) Once again, whatever is inside ln must be greater than 0

$$\therefore x^2 + 3x - 4 > 0$$
$$(x + 4)(x - 1) > 0$$

$$x < -4 \; or \; x > 1$$

The domain in interval notation may be written as $(-\infty, -4) \cup (1, \infty)$