Mathify HSC

DIFFERENTIATIAL CALCULUS

GRAPH SKETCHING WITH DERIVATIVES (VII)

Contents include:

- Sketching Graphs With Derivatives

- Graph Sketches with the Derivative

Derivative graph questions involve students being asked to find intercepts, turning points and points of inflection in order to sketch a graph. In other words, this is just an application of all the theory we've learnt so far for differential calculus.

Since different questions will have different functions, they will often vary greatly. However, they do generally involve similar steps:

$$\textbf{Step 1}: \textit{Find the } x \textit{ and } y \textit{ intercepts}$$

$$\textbf{Step 2}: \textit{Find the stationary points}$$

$$\textbf{Step 3}: \textit{Find the nature of the stationary points, usually through using the second derivative}$$

$$\textbf{Step 4}: \textit{Find the points of inflection}$$

$$\textbf{Step 5}: \textit{Plot your points that you've found so far on the graph}$$

$$\textbf{Step 6}: \textit{Sketch by connecting the dots}$$

Practice makes perfect, especially for derivative graphs! They are also often work a lot of marks so get to work!

Example 1: Find the intercepts, turning points and point of inflection for $f(x) = x^2(3 - x)$. Then, sketch its graph

Solution:

Starting off finding the x intercepts by letting $f(x) = 0$ and solving:

$$0 = x^2(3 - x)$$

$$\therefore x = 0, x = 3 \textit{ are } x \textit{ intercepts}$$

Then, finding the y intercepts by letting $x = 0$:

$$\begin{aligned} f(0) &= 0(3 - 0) \\ &= 0 \end{aligned}$$

Hence, $y -$ intercept occurs at $(0, 0)$

Now differentiating:

$$\begin{aligned} f(x) &= 3x^2 - x^3 \\ f'(x) &= 6x - 3x^2 \end{aligned}$$

Letting $f'(x) = 0$ to find the stationary points:

$$\begin{aligned} 0 &= 6x - 3x^2 \\ &= 3x(2 - x) \end{aligned}$$

$$\therefore x = 0 \; and \; x = 2 \; are \; stationary \; points$$

When $x = 0, y = 0$

When $x = 2, y = 2^2(3 - 2) = 4$

Now finding the second derivative:

$$f''(x) = 6 - 6x$$

When $x = 0$:

$$f''(0) = 6, which \; is > 0$$

$$\therefore (0, 0) \; is \; a \; minimum \; TP$$

When $x = 2$:

$$f''(2) = -6, which \; is < 0$$

$$\therefore (2, 4) \; is \; a \; maximum \; TP$$

Now finding the point of inflection by letting $f''(x) = 0$:

$$0 = 6 - 6x$$
$$6x = 6$$
$$\therefore x = 1$$

When $x = 1, y = 1^2(3 - 1) = 2$

Checking for change in concavity by using a sign table:

x	0	1	2
$f''(x)$	6	0	-6

Therefore, since a change in concavity has occurred:

$$(1, 2) \; is \; a \; point \; of \; inflection$$

Finally, sketching the graph: