EXPONENTIALS & LOGARITHMS

APPLICATIONS OF EXPONENTIALS AND LOGARITHMS (VI)

Contents include:

- Applications of Exponentials and Logarithms

- Applications to the Real World

The questions in this subtopic **involve no new theory**, but are essentially harder, wordier but more relevant questions that involve exponentials and logarithmic equations which apply to other topics such as physics and geology

Common applications of exponential equations include:

- Population Growth
- Moore's Law
- Nuclear Decay
- Newton's Law of Cooling

Common applications of logarithmic equations include:

- Earthquakes (The Richter Scale)
- Decibels
- PH
- Brightness in Physics

Overall, school exams and the HSC will focus on asking these sorts of questions rather than the previous "Textbook Questions" because these do tend to be harder and more complicated. Hence, it is essential to practice this!

Drown out the noise for these questions. They'll often be quite wordy since the question will explain the theory behind the equation, but we can just ignore this information and instead focus purely on the equation!

Applications of Exponentials and Logarithms Exercises

1. The population of a city doubles every 25 years and is modelled by the equation:

$$P = 1500000 \times 2^{\frac{t}{25}}$$

Where t is the number of years after 2020

a) Show that the population of the city in the year 2020 was 1.5 million
b) Find the population of the city to the nearest thousand in the year 2050
c) Find in which year the population of the city is expected to reach 10,000,000

2. According to Newton's Law of cooling, my heated lunch box when placed in the staff room fridge at $0°C$ will cool in a way that its temperature halves every 30 minutes. Its temperature is hence modelled by the equation:

$$T = 48 \times \left(\frac{1}{2}\right)^{2n}$$

Where n is the number of hours after my lunch box is placed in the fridge

a) Find the initial temperature of my lunch box
b) I put my lunchbox in the fridge just when I got to school at 10:30am
   i. What would the temperature of my lunchbox be at 1pm, when lunch time begins?
   ii. If I left my lunchbox in the fridge, what time to the nearest minute would it reach $1°C$?

3. Loudness, measured in decibels (dB), is modelled by the formula:

$$L = 10\log_{10}\left(\frac{I}{I_0}\right)$$

where I is the intensity level and I_0 is the threshold sound which is measured to be $10^{-12}\ W/m^2$

a) What decibel level corresponds to $6 \times 10^{-3}\ W/m^2$? Give your answer to the nearest decibel
b) Sounds louder than 85 dB can damage hearing. What is the minimum intensity of sound that would be loud enough to cause damage to a person's hearing?
c) The intensity of a vacuum cleaner is estimated to be 1000000 times the threshold level I_0. Would using a vacuum cleaner therefore damage your hearing based on this estimate?

4. Earthquake strengths are usually reported on the Richter scale, which defines the magnitude of an earthquake as:

$$M = \log_{10}\left(\frac{I}{S}\right)$$

Where M is the magnitude of the earthquake, I is the intensity of the earthquake wave and S is the intensity of the smallest detectable wave.

a) An earthquake of magnitude 8.0 is considered devastating. Find the value of $\left(\frac{I}{S}\right)$ for an earthquake of this magnitude
b) An earthquake that registered 6.6 in magnitude was followed by another which was 5 times more intense. Determine the magnitude of the second earthquake, accurate to 1 decimal place

<u>Applications of Exponentials and Logarithms Exercise Answers</u>

1.
a)

In the year 2020, $t = 0$. Subbing this into the equation for P:

$$P = 1500000 \times 2^0$$
$$= 1500000$$

Hence, the population was 1.5 million

b)

In the year 2050, 30 years has passed. Hence, letting $t = 30$:

$$P = 1500000 \times 2^{\frac{30}{25}}$$
$$\approx 3446000\ (nearest\ thousand)$$

c)

Letting $P = 10\,000\,000$ then solving the solution for t:

$$10000000 = 1500000 \times 2^{\frac{t}{25}}$$
$$\frac{10000000}{1500000} = 2^{\frac{t}{25}}$$
$$\frac{20}{3} = 2^{\frac{t}{25}}$$
$$\therefore \frac{t}{25} = \log_2 \frac{20}{3}$$
$$t = 25 \log_2 \frac{20}{3}$$

Using change of base laws:

$$t = 25 \times \frac{\ln \frac{20}{3}}{\ln 2}$$
$$\approx 68.424$$

Hence, we actually need 69 complete years to get the population to 10 million. The reason why we round up is because 68 years won't be enough.

Therefore, the city will reach a population of 10 million in the year 2089

2.
a)

When finding initial conditions, let $n = 0$:

$$\therefore T = 48 \times \left(\frac{1}{2}\right)^0$$
$$= 48$$

Hence, the initial temperature was $48°C$

b)

i.

At 1pm, 2 and a half hours would have passed since I put my lunchbox in the fridge at $10{:}30$am. Therefore, letting $n = 4.5$ and solving for T:

$$\begin{aligned} T &= 48 \times \left(\frac{1}{2}\right)^{2(2.5)} \\ &= 48 \times \left(\frac{1}{2}\right)^{5} \\ &= 1.5°C \end{aligned}$$

Hence, at 1pm my lunchbox will be $1.5°C$

ii.

Letting $T = 1°C$ then solving for n:

$$1 = 48 \times \left(\frac{1}{2}\right)^{2n}$$

$$\frac{1}{48} = \left(\frac{1}{2}\right)^{2n}$$

Applying logarithms:

$$2n = \log_{\frac{1}{2}} \frac{1}{48}$$

$$\therefore n = \frac{1}{2} \log_{\frac{1}{2}} \frac{1}{48}$$

Using change of base laws:

$$\begin{aligned} \therefore n &= \frac{1}{2} \times \frac{\ln\left(\frac{1}{48}\right)}{\ln\left(\frac{1}{2}\right)} \\ &\approx 2.79248 \end{aligned}$$

Therefore, converting 2.79248 hours into hours and minutes:

$$\begin{aligned} 2.79248\ hours &\approx 167.55\ minutes \\ &= 2\ hours\ 47.55\ minutes \\ &\approx 2\ hours\ 48\ minutes\ (nearest\ minute) \end{aligned}$$

Hence, 2 hours and 48 minutes after 10:30am is 1:18pm

3.

a)

Letting $I = 6 \times 10^{-3}$:

$$L = 10\log_{10}\left(\frac{6\times 10^{-3}}{10^{-12}}\right)$$
$$= 97\ dB$$

b)

Letting $L = 85$ and solving for I:

$$\therefore 85 = 10\log_{10}\left(\frac{I}{10^{-12}}\right)$$

$$8.5 = \log_{10}\left(\frac{I}{10^{-12}}\right)$$

$$10^{8.5} = \frac{I}{10^{-12}}$$

$$\therefore I = 10^{8.5}\times 10^{-12}$$
$$= 10^{-3.5}$$

c)

If the intensity of a vacuum cleaner is 1000000 times the threshold, then that means:

$$I = 1000000I_0$$

Substituting this into the loudness equation:

$$L = 10\log_{10}\left(\frac{1000000I_0}{I_0}\right)$$
$$= 10\log_{10}(1000000)$$
$$= 10\times 6$$
$$= 60\ dB$$

Hence, since the vacuum cleaner is 60 dB, this will not damage hearing

4.
a)

If an earthquake has a magnitude of 8.0, let $M = 8$:

$$8 = \log_{10}\left(\frac{I}{S}\right)$$

$$\therefore \frac{I}{S} = 10^8$$

b)

Finding the intensity of the earthquake with magnitude 6.6 by letting $M = 6.6$:

$$6.6 = \log_{10}\left(\frac{I}{S}\right)$$

$$\therefore \frac{I}{S} = 10^{6.6}$$

$$I = 10^{6.6}S$$

Where S is a constant

The other earthquake is 5 times more intense, meaning that $I = 5 \times 10^{6.6}S$. Substituting this into our earthquake equation and finding the value of M:

$$\begin{aligned} M &= \log_{10}\left(\frac{5 \times 10^{6.6}S}{S}\right) \\ &= \log_{10}(5 \times 10^{6.6}) \\ &\approx 7.3\ (nearest\ 1\ d.p.) \end{aligned}$$