TRIGONOMETRY

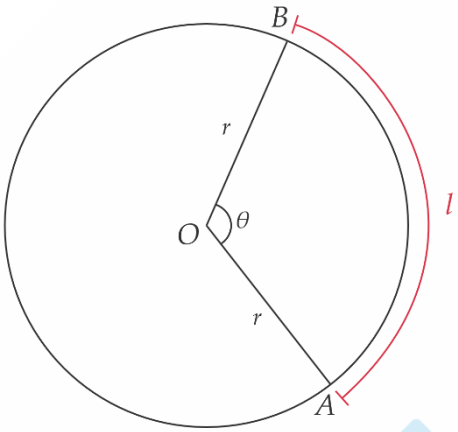
ARC LENGTH AND SECTOR AREA (VI)

Contents include:

- Arc Length
- Sector Area
- Area of a Segment

- Arc Length

The length of an arc, l, is shown in the diagram below:



To calculate the arc length l, we use the following formula:

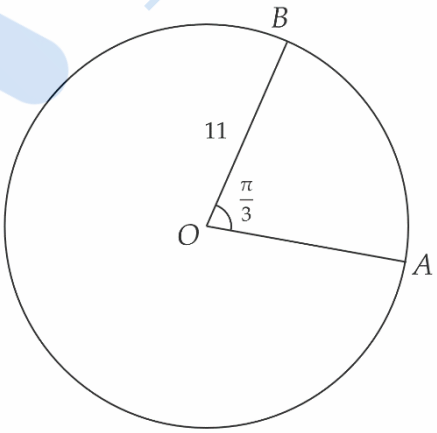
$$l = r\theta$$

Where θ is the angle in **radians**

Note that l here is the **minor arc** since $0 < \theta < \pi$. To find the length of the major arc AB, we would do:

$$major\ arc\ AB = circumference - minor\ arc\ AB$$

Example 1: Find the length of the minor arc AB in the following diagram:



Solution:

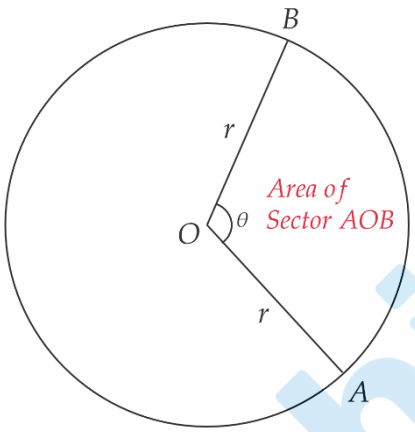
Using the arc length formula:

$$\therefore minor\ length\ AB = r\theta$$
$$= 11 \times \frac{\pi}{3}$$

$$= \frac{11\pi}{3}\ units$$

- Sector Area

The area of the sector AOB is shown in the diagram below:



To calculate the area of the sector AOB, we use the following formula:

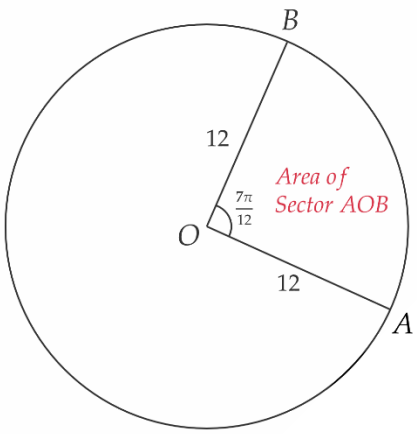
$$Area = \frac{1}{2}r^2\theta$$

Where θ is the angle of the sector in **radians**

Note that sector AOB here is the **minor sector** since $0 < \theta < \pi$. To find the area of the major sector AOB, we would do:

$$major\ sector\ AOB = whole\ circle\ area - minor\ sector\ AOB$$

Example 2: Find the area of the sector AOB in the following diagram:



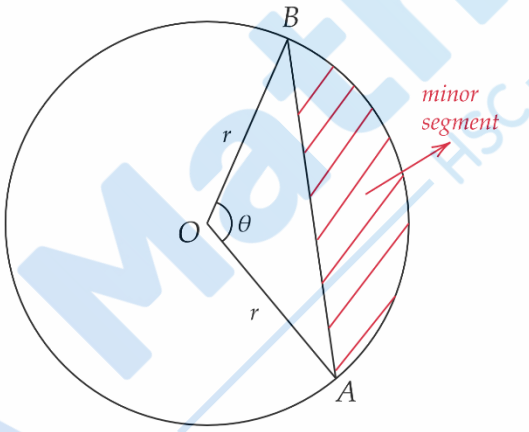
Solution:

Following the formula of a sector:

$$\begin{aligned} Area\ of\ sector\ AOB &= \frac{1}{2}r^2\theta \\ &= \frac{1}{2}(12)^2 \times \frac{7\pi}{12} \\ &= \frac{144}{2} \times \frac{7\pi}{12} \\ &= 72 \times \frac{7\pi}{12} \\ &= 6 \times 7\pi \\ &= 42\pi\ units^2 \end{aligned}$$

- Area of a Segment

The area of a minor segment is shown in the diagram below:



The rest of the circle, excluding the minor segment, is called the major segment.

To calculate the area of the minor segment AOB, we use the following formula:

$$Area = \frac{1}{2}r^2\theta - \frac{1}{2}r^2 \sin\theta$$

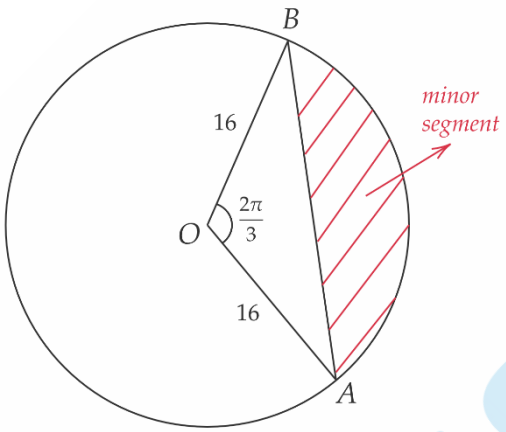
$Area\ of\ sector\ AOB$ — $Area\ of\ \Delta\ AOB$

Where θ is the angle in **radians**

Note that to find the area of the major segment, we would do:

$$major\ segment = whole\ circle\ area - minor\ segment$$

Example 3: Calculate the area of the minor segment in the following diagram:



Solution:

$$\therefore minor\ segment\ area = \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$$
$$= \frac{1}{2}(16)^2 \times \frac{2\pi}{3} - \frac{1}{2}(16)^2 \sin\frac{2\pi}{3}$$
$$= \frac{256}{2} \times \frac{2\pi}{3} - \frac{1}{2} \times 256 \times \frac{\sqrt{3}}{2}$$
$$= \frac{256\pi}{3} - 64\sqrt{3}\ units^2$$