INTEGRATION

THE INDEFINITE INTEGRAL (II)

Contents include:

- Indefinite Integral Notation
- Finding the Value of Constant C

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- Indefinite integral notation

Finding the indefinite integral is the same thing as finding the primitive of a function $f(x)$

Notation commonly used for finding the primitive/indefinite integral of a function is the indefinite integral sign: $$\int f'(x)\,dx = f(x)$$

The general rule and previous functions for primitive functions previously defined in (I) still applies.

Example 1: Find the primitive function of $f(x) = 5x^4 + 6x^3 + 3x^2 + x - 26$

$$\int 5x^4 + 6x^3 + 3x^2 + x - 26\ dx = \frac{5x^5}{5} + \frac{6x^4}{4} + \frac{3x^3}{3} + \frac{x^2}{2} - 26x + C$$

$$= x^5 + \frac{3}{2}x^4 + x^3 + \frac{x^2}{2} - 26x + C$$

<u>Note:</u> The 'dx' in the integral is essential to always include whenever we write out our indefinite integrals, and means that we are 'Integrating with respect to x'

- Finding the value of the constant C

Sometimes we may have to leave our primitive expression with the "$+C$" at the end.

However, when we know the coordinates of **at least one point** on the primitive function $F(x)$, we can **find the value of C** by substituting a given point (x_1, y_1) into $F(x)$ and solving for the value of C

Example 2: The gradient function of a curve is $3x^2 - 2x$ and the curve passes through the point (2, 1). Find its equation.

$$Step\ 1: Find\ the\ primitive\ function\ F(x)$$

Since the gradient function is given:

$$f'(x) = 3x^2 - 2x$$

Then, using the general rules for finding a primitive:

$$f(x) = 3 \times \frac{x^3}{3} - 2 \times \frac{x^2}{2} + c$$
$$= x^3 - x^2 + c$$

$$Step\ 2: Substitute\ in\ the\ given\ point\ (2, 1)\ to\ find\ c$$

$$1 = 2^3 - 2^2 + c$$
$$\therefore c = 1 - 2^3 + 2^2$$
$$= 1 - 8 + 4 = -3$$

$Step\ 3: Rewrite\ f(x)\ now\ with\ the\ known\ value\ of\ c$

$$\therefore f(x) = x^3 - x^2 - 3$$

Example 2: Find $f(x)$ given that $f'(x) = 2x - 2\ and\ f(1) = 4$

$Step\ 1: Find\ the\ primitive\ expression\ f(x)$

Integrating $f'(x)$ to find $f(x)$:

$$f(x) = \int 2x - 2\ dx$$

$$\therefore f(x) = 2 \times \frac{x^2}{2} - 2x + C$$
$$= x^2 - 2x + C$$

$Step\ 2: Substitute\ in\ the\ given\ point\ (1, 4)\ to\ find\ C$

$$4 = 1^2 - 2(1) + C$$
$$4 = 1 - 2 + C$$

$$\therefore C = 4 - 1 + 2$$
$$= 5$$

$Step\ 3: Rewrite\ f(x)\ now\ with\ the\ known\ value\ for\ C$

$$\therefore f(x) = x^2 - 2x + 5$$

Example 3: Find $f(x)$ given that $f'(x) = 4x^2 - 3x + 1\ and\ f(-1) = 3$

$Step\ 1: Find\ the\ primitive\ expression\ f(x)$

Integrating $f'(x)$ to find $f(x)$:

$$f(x) = \int 4x^2 - 3x + 1\, dx$$

$$\therefore f(x) = 4 \times \frac{x^3}{3} - 3 \times \frac{x^2}{2} + x + C$$
$$= \frac{4}{3}x^3 - \frac{3}{2}x^2 + x + C$$

$Step\ 2: Substitute\ in\ the\ given\ point\ (-1, 3)\ to\ find\ C$

$$3 = \frac{4}{3}(-1)^3 - \frac{3}{2}(-1)^2 + (-1) + C$$

$$3 = -\frac{4}{3} - \frac{3}{2} - 1 + C$$
$$\therefore C = 3 + \frac{4}{3} + \frac{3}{2} + 1$$
$$= \frac{41}{6}$$

$$Step\ 3: Rewrite\ f(x)\ now\ with\ the\ known\ value\ for\ C$$

$$\therefore f(x) = \frac{4}{3}x^3 - \frac{3}{2}x^2 + x + \frac{41}{6}$$

Example 4: Find the equation of a curve, given that the gradient at any point $P(x, y)$ is $3x^2 - 2x + 3$ and that the point (3, 3) belongs to the curve.

$$Step\ 1: Find\ the\ primitive\ expression\ F(x)$$

For this question, the gradient function, $f'(x) = 3x^2 - 2x + 3$

Hence, integrating $f'(x)$ to get the original curve, $F(x)$:

$$F(x) = 3 \times \frac{x^3}{3} - 2 \times \frac{x^2}{2} + 3x + C$$
$$= x^3 - x^2 + 3x + C$$

$$Step\ 2: Substitute\ in\ the\ given\ point\ (3, 3)\ to\ find\ C$$

$$3 = 3^3 - 3^2 + 3 \times 3 + C$$
$$3 = 27 - 9 + 9 + C$$
$$3 = 27 + C$$
$$\therefore C = 3 - 27$$
$$= -24$$

$$Step\ 3: Rewrite\ F(x)\ now\ with\ the\ known\ value\ for\ C$$

$$\therefore F(x) = x^3 - x^2 + 3x - 24$$

Example 5: A curve contains the point (0, 4) and its gradient is $(x - 1)(x + 2)$ at any point on the curve. Find the equation of the curve.

$$Step\ 1: Expand\ your\ expression$$

$$(x - 1)(x + 2) = x^2 + x - 2$$

$$Step\ 2: Find\ the\ primitive\ expression\ F(x)$$

For this question, the gradient function, $f'(x) = x^2 + x - 2$

Hence, integrating $f'(x)$ to get the original curve, $F(x)$:

$$F(x) = \frac{x^3}{3} + \frac{x^2}{2} - 2x + C$$

$$Step\ 3{:}\ Substitute\ in\ the\ given\ point\ (0{,}4)\ to\ find\ C$$

$$4 = 0 + 0 - 0 + C$$
$$\therefore C = 4$$

$$Step\ 4{:}\ Rewrite\ F(x)\ now\ with\ the\ known\ value\ for\ C$$

$$\therefore F(x) = \frac{x^3}{3} + \frac{x^2}{2} - 2x + 4$$