DIFFERENTIATION

GEOMETRICAL APPLICATIONS: DERIVATIVE FUNCTION GRAPHS (VI)

Contents include:

- Increasing and Decreasing Points
- Stationary Points
- Sketching Derivative Graphs

www.mathifyhsc.com.au

- Increasing and Decreasing Points

Since we now know that $f'(x)$ correlates to the gradient of a function when an $x-$ coordinate is subbed in, we can say that:

- When $f'(x) > 0$, the function is increasing (Since the gradient is positive)
- When $f'(x) < 0$, the function is decreasing (Since the gradient is negative)

Example 1: Determine whether the function $f(x) = x^2$ is increasing or decreasing at the point (2, 4).

Solution:

Differentiating $f(x)$:

$$f'(x) = 2x$$

Then, substituting in $x = 2$ into $f(x)$:

$$\begin{aligned} f'(2) &= 2 \times 2 \\ &= 4 \end{aligned}$$

$$\therefore Since\ f'(2) > 0, f(x)\ is\ increasing\ at\ the\ point\ (2, 4)$$

- The First Derivative and Stationary Points

Recall that the first derivative is used to determine the gradient of a function, $f(x)$ at any point on its graph.

When the gradient at any point is equal to 0, this is referred to as a **stationary point**. In other words:

$$Stationary\ points\ occur\ when\ f'(x) = 0$$

Example 2: Find the coordinate of the stationary point for the function $f(x) = 2 - 3x - x^2$

Solution:

First differentiating $f(x)$:

$$f'(x) = -3 - 2x$$

Since stationary points occur when $f'(x) = 0$, solving for x:

$$\begin{aligned} 0 &= -3 - 2x \\ 2x &= -3 \end{aligned}$$

$$\therefore x = -\frac{3}{2}$$

When $x = -\frac{3}{2}$:

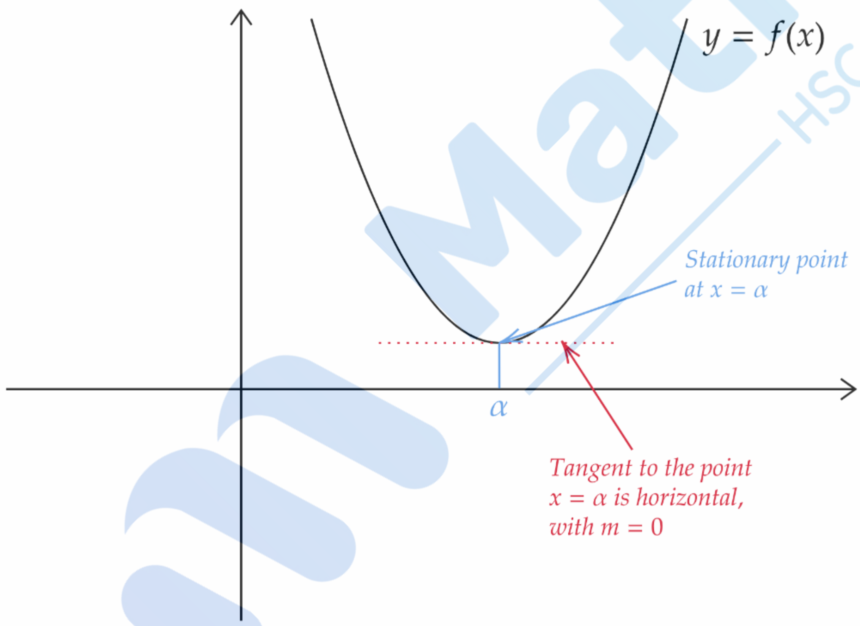
$$\begin{aligned} y = f\left(-\frac{3}{2}\right) &= 2 - 3\left(-\frac{3}{2}\right) - \left(-\frac{3}{2}\right)^2 \\ &= 2 + \frac{9}{2} - \frac{9}{4} \\ &= \frac{17}{4} \end{aligned}$$

Hence, stationary point occurs at $\left(-\frac{3}{2}, \frac{17}{4}\right)$

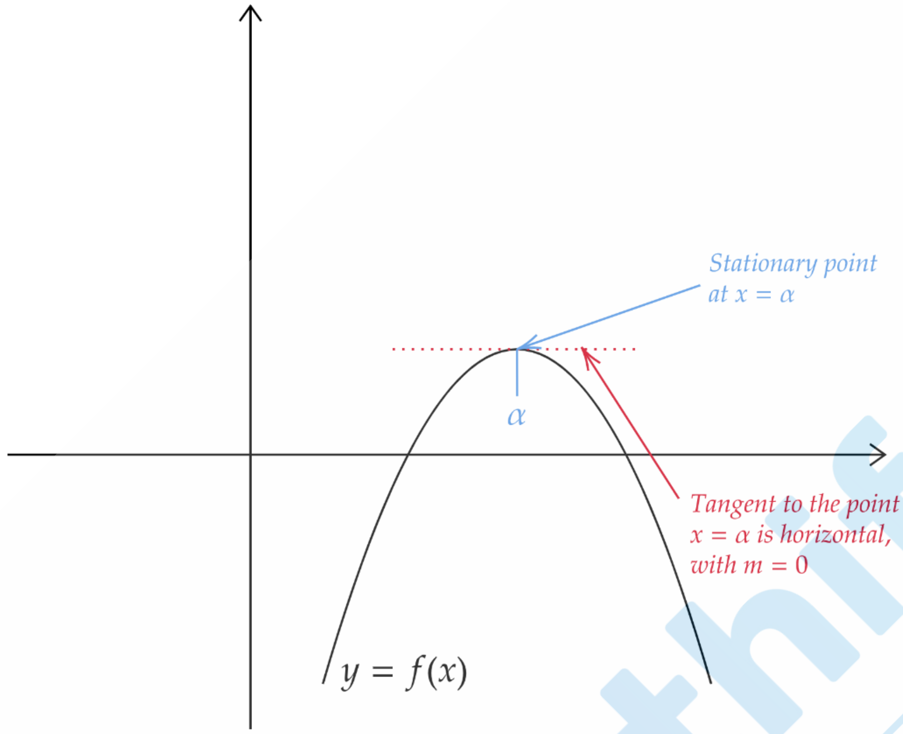
- Recognizing Stationary Points

If we try to visualise a stationary point, notice that there are two different ways we can do so:

Type 1:



Type 2:



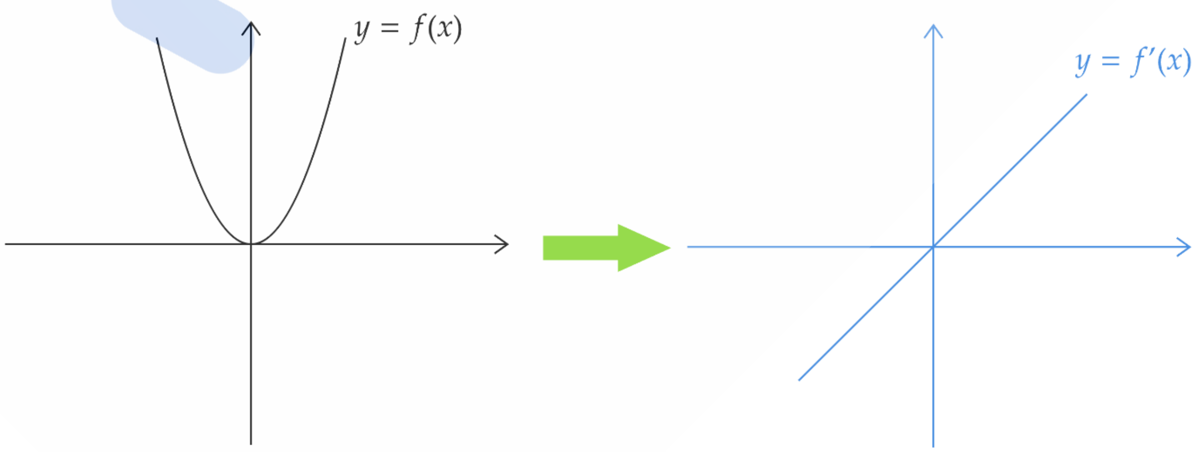
In other words, the "top" of a hill and the "bottom" of a valley can be seen as stationary points!

- Sketching $f'(x)$, given the graph of $f(x)$

Now combining our knowledge of when a derivative function is increasing, decreasing or equal to 0, we can now tackle derivative function questions.

These typically involve the question giving a function, $f(x)$, and then asking students to sketch the subsequent $f'(x)$ graph.

An example is shown below:



The general steps involved to sketch these are:

$$\boldsymbol{Step\ 1: Note\ down\ stationary\ points}$$

If $x = \alpha$ is a stationary point for $f(x)$, then $x = \alpha$ will be an $x - intercept$ for $f'(x)$!

This is because since $x = \alpha$ is a stationary point, then $f'(\alpha) = 0$

$$\boldsymbol{Step\ 2: Note\ when\ the\ graph\ is\ increasing\ or\ decreasing}$$

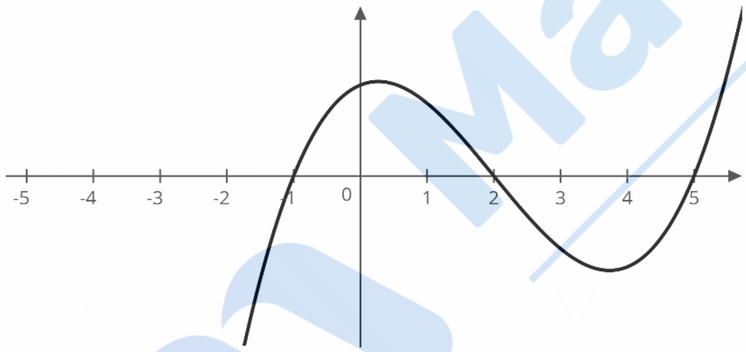
Whenever $f(x)$ is increasing, this means that $f'(x) > 0$, so the graph of $f'(x)$ is **above** the $x -$ axis

Whenever $f(x)$ is decreasing, this means that $f'(x) < 0$, so the graph of $f'(x)$ is **below** the $x -$ axis

$$\boldsymbol{Step\ 3: Sketch!}$$

Now you should be able to sketch the derivative graph, $f'(x)$, by simply connecting the dots of the stationary points

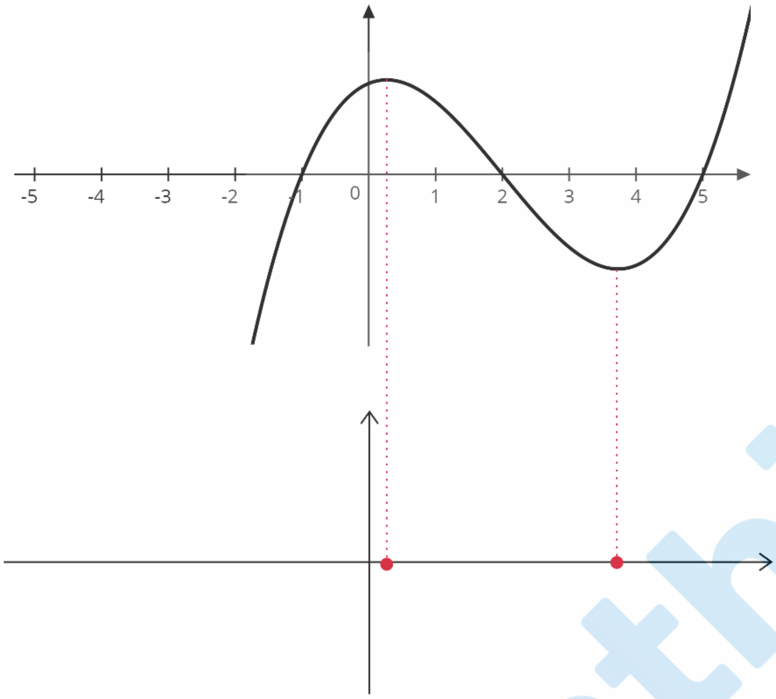
Example 3:



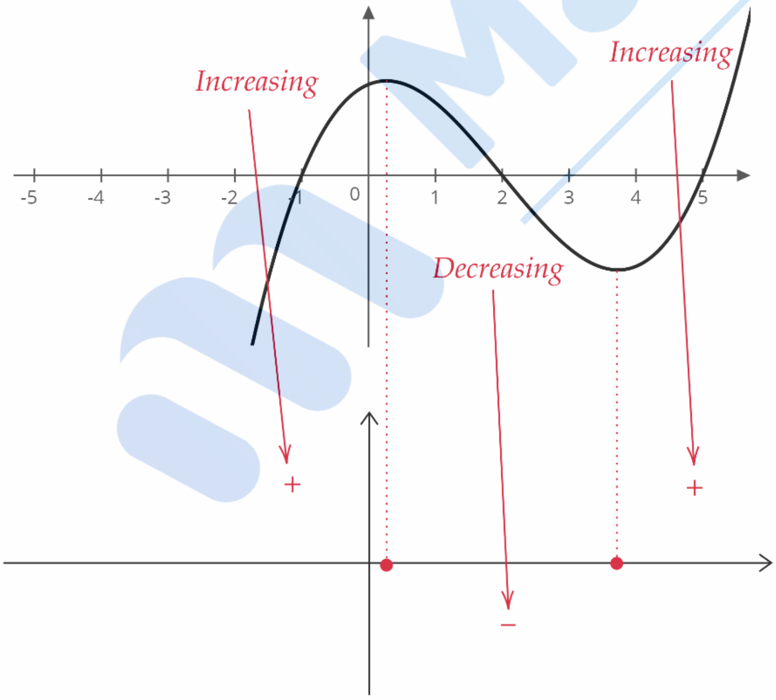
The graph above shows a function $y = f(x)$. Sketch a possible graph of $\frac{dy}{dx}$

Solutions:

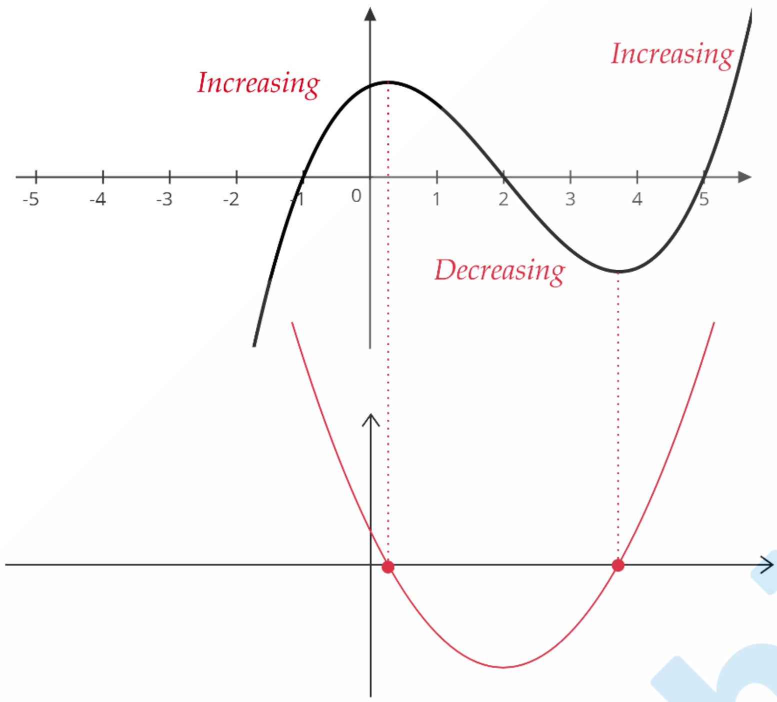
Following our steps, we first plot the stationary points:



Then, for our second step we determine where the graph is increasing or decreasing



Finally, we should sketch our answer:



And that's it! Obviously, for your working out in an actual exam, it does not need to be this detailed. This is purely for demonstration purposes to better help you understand.