PROBABILITY

COMPLEMENTARY EVENTS (III)

Contents include: Maximum and minimum probabilities, Complementary events

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- Maximum and Minimum Probabilities

The maximum probability of an event occurring is 1. This is called a certain event.

The minimum probability of an event occurring is 0. This is called an impossible event.

Note: *The sum of all possibilities together should always add to 1!*

- Complementary Events

In some questions, it is more efficient to find the probability that an event does **not** occur rather than the probability it does occur. This is known as finding the **complement** of an event.

In terms of notation, the complement of an event, A is given as $\bar{A}$. In other words:

$$P(not\ A) = P(\bar{A})$$

Helpful tip: Whenever you see the word "**not**" in your question, you should consider calculating the complement as 99% of the time, that will be the most efficient method!

Since we know that the sum of all possibilities adds up to 1, therefore:

$$P(event\ occuring) = 1 - P(event\ \boldsymbol{not}\ occuring)$$

$$P(A) = 1 - P(\bar{A})$$

$$or$$

$$P(\bar{A}) = 1 - P(A)$$

Example 1: Taylor draws a card at random from a deck of playing cards. Find the probability that:

a) A club is not drawn

We know that there are 4 suits in total, so a quarter of the cards are clubs:

$$P(clubs) = \frac{1}{4}$$

Therefore, since we want to find the complement:

$$P(not\ clubs) = 1 - \frac{1}{4}$$
$$= \frac{3}{4}$$

b) That a picture card (Jack, Queen or King) is not drawn

We know that there are 13 different numbers in a deck of cards, 3 of which are pictures, so the probability of drawing a picture card is:

$$P(picture\ card) = \frac{3}{13}$$

Therefore, since we want to find the complement:

$$\begin{aligned} P(not\ picture\ card) &= 1 - \frac{3}{13} \\ &= \frac{10}{13} \end{aligned}$$

c) That it is neither a black 5 nor a red 10

Note that when it says 'black', it can be either clubs or spades, so each number has 2 such cards. The same applies for 'red', where it refers to either diamonds or hearts.

$$P(black\ 5) = \frac{2}{52} = \frac{1}{26}$$

$$P(red\ 10) = \frac{2}{52} = \frac{1}{26}$$

$$\begin{aligned} P(black\ 5\ or\ red\ 10) &= \frac{1}{26} + \frac{1}{26} \\ &= \frac{1}{13} \end{aligned}$$

$$\begin{aligned} P(neither\ black\ 5\ nor\ red\ 10) &= 1 - P(black\ 5\ or\ red\ 10) \\ &= 1 - \frac{1}{13} \\ &= \frac{12}{13} \end{aligned}$$

Example 2: A bag contains 10 balls of which 6 are red, 3 are yellow and 1 is white. A ball is drawn out at random. What is the probability that it is:

a) Red?
b) Not yellow?
c) White or yellow?
d) Neither red nor white?

Solution:

a) There are 10 balls in total, with 6 of these being red

$$\begin{aligned} \therefore P(red) &= \frac{6}{10} \\ &= \frac{3}{5} \end{aligned}$$

b) There are 3 yellow balls

$$\therefore P(Yellow) = \frac{3}{10}$$

Hence, considering the complement:

$$\begin{aligned} P(Not\ yellow) &= 1 - P(yellow) \\ &= 1 - \frac{3}{10} \\ &= \frac{7}{10} \end{aligned}$$

c) First finding the probability of white and yellow individually:

$$P(White) = \frac{1}{10}$$

$$P(Yellow) = \frac{3}{10}$$

$$\begin{aligned} \therefore P(White\ or\ Yellow) &= P(White) + P(Yellow) \\ &= \frac{1}{10} + \frac{3}{10} \\ &= \frac{4}{10} \\ &= \frac{2}{5} \end{aligned}$$

d) We are asked to find:

$$P(neither\ red\ nor\ white)$$

In other words, the ball chosen cannot be red or white. There's 2 ways/methods to think about this:

Method 1: Consider the complement

Since I cannot choose a ball that is red or white:

$$P(neither\ red\ nor\ white) = 1 - P(red\ or\ white)$$

$$\begin{aligned} P(red\ or\ white) &= P(red) + P(white) \\ &= \frac{6}{10} + \frac{1}{10} \\ &= \frac{7}{10} \end{aligned}$$

$$\begin{aligned} \therefore P(neither\ red\ nor\ white) &= 1 - \frac{7}{10} \\ &= \frac{3}{10} \end{aligned}$$

Method 2: Consider the probability of getting yellow

$$P(neither\ red\ nor\ white) = P(yellow)$$

$$= \frac{3}{10}$$

Example 3: A box has marbles of the same size but different colours – red, white and blue. If a marble is drawn out at random, the probability that it is red is the same as the probability that it is white and twice the probability that it is blue.

a) What is the smallest number of marbles the box could have?
b) If a marble is chosen at random, what is the probability that it is
   a. Red?
   b. White?
   c. Red or white?
   d. Not blue?

Solution:

a) Let the probability of drawing out a red marble be p

$$\therefore P(red) = p$$

Since this is the same as the probability of drawing out a white, and twice the probability of drawing out a blue:

$$\therefore P(white) = p$$

$$\therefore P(blue) = \frac{p}{2}$$

Since $P(red)$ and $P(white)$ is double $P(blue)$, this means that if there is only 1 blue marble (minimum), there must be 2 red and 2 white marbles

$$\begin{aligned}\therefore Minimum\ number\ of\ marbles &= 1 + 2 + 2 \\ &= 5\end{aligned}$$

b)
   a.

Since the sum of all probabilities must equal to 1:

$$\begin{aligned}\therefore P(red) + P(white) + P(blue) &= 1 \\ p + p + \frac{p}{2} &= 1\end{aligned}$$

Multiplying both sides by 2:

$$2p + 2p + p = 2$$
$$5p = 2$$
$$\therefore p = \frac{2}{5}$$

Hence, the probability of red is:

$$P(red) = \frac{2}{5}$$

b. The probability of white is:

$$P(white) = \frac{2}{5}$$

c. Finding the probability of red or white:

$$\begin{aligned} P(red\ or\ white) &= P(red) + P(white) \\ &= \frac{2}{5} + \frac{2}{5} \\ &= \frac{4}{5} \end{aligned}$$

d. Considering the complement:

$$P(not\ blue) = 1 - P(blue)$$

First finding $P(blue)$:

$$\begin{aligned} P(blue) &= \frac{1}{2} \times P(red) \\ &= \frac{1}{2} \times \frac{2}{5} \\ &= \frac{1}{5} \end{aligned}$$

$$\begin{aligned} \therefore P(not\ blue) &= 1 - P(blue) \\ &= 1 - \frac{1}{5} \\ &= \frac{4}{5} \end{aligned}$$