PROBABILITY

INTRODUCTION AND RELATIVE FREQUENCY (I)

Contents include: Notation, logic, sample space and relative frequency

- Notation

If we want to express the probability of something occurring, we can use the notation:

$$\text{The probability of something occurring} = P(\text{something})$$

For example, if I want to write "The probability of an even number being rolled on a die is half", I can instead write:

$$P(even) = \frac{1}{2}$$

- The Logic of Probability

Suppose we conduct an experiment, where there are n possible results we can obtain, each being of equal chance. Then, we say that the probability of obtaining *one* of these results is given as $\frac{1}{n}$, since there are n possibilities, and we are picking one of these.

For example, looking at an experiment rolling a die, there are 6 sides and thus 6 possibilities. The chance of me rolling one of these results, say a 'five', will therefore be $\frac{1}{6}$.

Looking at another example, this time finding the probability of rolling an even number, there are still 6 total possibilities, but now we have 3 numbers we can roll that will be even, so the probability will therefore be $\frac{3}{6} = \frac{1}{2}$.

Hence:

$$P(event) = \frac{\text{number of favorable events}}{\text{total number of possibilities}}$$

- Sample and Event Space

The Sample Space: Is the set of all possible outcomes. We refer to the sample space as *uniform* if all the possible outcomes are equally likely to occur.

The Event Space: Is the set of outcomes that are favourable, in other words what we want to occur. The event space is a subset of the sample space.

Therefore, we can now represent the probability as:

$$P(event) = \frac{\text{event space}}{\text{sample space}}$$

Example 1: A card is drawn at random from a standard deck of 52 cards. Find the probability that:

a) An eight of clubs is drawn

The sample space in this question is 52, with only 1 possible card. Hence:

$$P(eight\ of\ clubs) = \frac{1}{52}$$

b) A six is drawn.

The sample space in this question is still 52, now with 4 possible cards (one for each suit). Hence:

$$\begin{aligned} P(6) &= \frac{4}{52} \\ &= \frac{1}{13} \end{aligned}$$

- Relative Frequencies

Relative frequency defined simply is: "**How often an event happens divided by total number of outcomes**"

Suppose an experiment is conducted multiple times, with the outcome being recorded each time. The relative frequency of an event A occurring would be:

$$Relative\ frequency\ of\ A = \frac{number\ of\ times\ A\ occurred}{total\ number\ of\ times\ experiment\ is\ conducted}$$

Notice that this looks rather familiar…

$$P(A) = \frac{event\ space}{sample\ space}$$

We can use the relative frequency of an event to **estimate** its probability, in other words, we can say that:

$$relative\ frequency = experimental\ probability$$

However:

$$experimental\ probability \neq theoretical\ probability$$

Theoretical probability MUST be 100% accurate. For example, a fair coin MUST have a 50% chance of landing on heads, this is theoretical probability. However, if I flip a coin twice, and obtain heads both times, its relative frequency may be $\frac{2}{2} = 1$ and its experimental probability is 100%, but that does not mean the theoretical probability of flipping a head is 100%.

Moreover, we must also realise that the more times an experiment is repeated, the more closer the relative frequency/experimental probability approaches the theoretical probability:

$$\therefore \textit{more trials} = \textit{more accurate}$$

Going off our previous example, if I instead flip the coin 100 times, and obtain heads 51 times, its experimental probability, AKA relative frequency $\frac{51}{100}$ now more accurately reflects its theoretical probability which is $\frac{1}{2}$.

Thus, in summary:

- Theoretical probability: How likely an event is going to occur (future tense)
- Experimental probability: How likely an event has actually occurred (past tense)

Example 5: A bottle is flipped 500 times, landing perfectly 54 times, and falling down 446 times.

a) Find the relative frequency of the bottle landing perfectly and falling down

Relative frequency of bottle landing: $\frac{54}{500} = \frac{27}{250}$

Relative frequency of bottle falling: $\frac{446}{500} = \frac{223}{250}$

b) The experiment was later repeated 500000 times, with the bottle landing perfectly 55000 times. Which experimental probability would more closely match the actual probability of landing the bottle perfectly?

When the experiment is repeated 500000 times, the experimental probability is:

$$P(successful) = \frac{55000}{500000} = \frac{11}{100}$$

Since this experiment has more repeats, its experimental probability will thus more closely resemble the actual probability compared to the first time when the bottle was flipped 500 times.

<u>Probability Exercises</u>

1. A magazine has 100 pages. Sandra randomly flips to a page, find the probability that the page number is:

a) A multiple of 10
b) A multiple of 3
c) A square number
d) Greater than 60
e) A multiple of 3 and 10

f) A multiple of 7 or 6

2. 200 deers were tagged and released into the forest. If later on 50 deers were caught and 18 of them were found to be a previously tagged deer, estimate the total number of deer in the forest

3. A rectangular field is 80 metres long and 50 metres wide. A horse wonders around the field at random. Find the probability that:
a) It is within 15 metres from the edge of the field
b) It is at least 20 metres away from the edge of the field

Probability Exercise Answers

1.
a) There are 10 multiple of 10s from $1 - 100$

$$\therefore P(multiple\ of\ 10) = \frac{10}{100}$$
$$= \frac{1}{10}$$

b) There are 33 multiples of 3 from $1 - 100$

$$\therefore P(muliple\ of\ 3) = \frac{33}{100}$$

c) The square numbers from $1 - 100$ are $1, 4, 9, 16, 25, 36, 49, 64, 81, 100$

$$\therefore P(square\ number) = \frac{10}{100}$$
$$= \frac{1}{10}$$

d) There are 40 numbers greater than 60 from $1 - 100$

$$\therefore P(number\ greater\ than\ 60) = \frac{40}{100}$$
$$= \frac{2}{5}$$

e) The numbers that are a multiple of 3 and 10 are: 30, 60 and 90.

$$P(multiple\ of\ 3\ and\ 10) = \frac{3}{100}$$

f)

The numbers that are a multiple of 7 or 6 are: 6, 7, 12, 14, 18, 21, 24, 28, 30, 35, 36, 42, 48, 49, 54, 56, 60, 63, 66, 70, 72, 77, 78, 84, 90, 91, 96, 98

$$\therefore P(multiple\ of\ 7\ or\ 6) = \frac{28}{100}$$
$$= \frac{7}{25}$$

2. Using relative frequency, we can estimate that:

$$P(deer\ caught\ is\ tagged) = \frac{18}{50}$$
$$= 36\%$$

This means that the 200 tagged deer represent 36% of the total population of deer, so:

$$0.36x = 200$$
$$x = \frac{200}{0.36}$$
$$= 555.56$$
$$\approx 556$$

3. These two questions use areas to determine their probabilities
a) If the horse must be within 15m from the edge, the area that it should be in is:

$$Area = (80 \times 50) - \big((80 - 15 - 15) \times (50 - 15 - 15)\big)$$
$$= 4000 - (50 \times 20)$$
$$= 4000 - 1000$$
$$= 3000\ m^2$$

$$\therefore Probability = \frac{3000}{80 \times 50}$$
$$= \frac{3}{40}$$

b) If the horse must be at least 20m away from the edge, the area that it should be in is:

$$Area = (80 - 20 - 20) \times (50 - 20 - 20)$$
$$= 40 \times 10 = 400\ m^2$$

$$\therefore Probability = \frac{400}{40000}$$
$$= \frac{1}{100}$$