PROBABILITY

INDEPENDENT EVENTS (VIII)

Contents include: independent events and mutually exclusive events

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- Independent Events

When two events A and B are independent, it means that the probability of A is unaffected even if we know the result of event B.

For example, if you flip a coin and roll a die, knowing the outcome of the dice roll (let's say it's a 6) doesn't change the probability of getting heads when you flip the coin.

Another way to think of it is how the probability of a flipping a head given the outcome of a dice roll is still 0.5 no matter what

Hence, this means that the probability of A given B is still equal to the probability of A, so for independent events, we say that:

$$P(A|B) = P(A)$$

Moreover, **another way to tell** if two events are independent is that:

If $P(A \cap B) = P(A) \times P(B)$, then events A and B are independent

Note: If events A and B are NOT independent, then $P(A \text{ and } B) \neq P(A) \times P(B)$

Example 1: If the probability of having blonde hair is 0.4 (event A), the probability of having blue eyes is 0.15 (event B) and the probability of having both blonde hair and blue eyes (event A and B) is 0.1, determine if events A and B are independent or not.

Writing out our known information:

$$P(A) = 0.4$$
$$P(B) = 0.15$$
$$P(A \text{ and } B) = 0.1$$

In this case:

$$\begin{aligned} P(A) \times P(B) &= 0.4 \times 0.15 \\ &= 0.06 \end{aligned}$$

$$\therefore P(A \text{ and } B) \neq P(A) \times P(B)$$

This means that events A and events B are NOT independent.

Exercise 2: If it is known that events A and B are independent:

a) Find $P(A \cap B)$ if $P(A) = 0.4$ and $P(B) = 0.5$
b) Find $P(A \cap B)$ if $P(A) = 0.25$ and $P(B) = 0.15$
c) Find $P(A|B)$ if $P(A) = 0.2$ and $P(B) = 0.7$

d) Find $P(B|A)$ if $P(A) = 0.15$ and $P(B) = 0.5$

Solutions:

a) Since it is known that events A and B are independent:

$$P(A \cap B) = P(A) \times P(B)$$

$$\therefore P(A \cap B) = 0.4 \times 0.5$$
$$= 0.2$$

b) Since it is known that events A and B are independent:

$$P(A \cap B) = P(A) \times P(B)$$

$$\therefore P(A \cap B) = 0.25 \times 0.15$$
$$= 0.0375$$

c) Since events A and B are independent:

$$P(A|B) = P(A)$$

$$\therefore P(A|B) = 0.2$$

d) Since events A and B are independent:

$$P(B|A) = P(B)$$

$$\therefore P(B|A) = 0.5$$

Example 3: For the following questions, determine if events A and B are independent or not if it is known that:

a) $P(A) = 0.5, P(B) = 0.3$ and $P(A \cap B) = 0.15$
b) $P(A) = 0.2, P(B) = 0.4$ and $P(A \cap B) = 0.1$
c) $P(A|B) = 0.5, P(B) = 0.5$ and $P(A) = 0.4$
d) $P(B|A) = 0.5, P(A) = 0.6$ and $P(B) = 0.5$

Solutions:

a) Considering $P(A) \times P(B)$:

$$\begin{aligned} P(A) \times P(B) &= 0.5 \times 0.3 \\ &= 0.15 \\ &= P(A \cap B) \end{aligned}$$

$$\therefore \mathit{Since}\ P(A \cap B) = P(A) \times P(B), A\ \mathit{and}\ B\ \mathit{are\ independent}$$

b) Considering $P(A) \times P(B)$:

$$\begin{aligned} P(A) \times P(B) &= 0.2 \times 0.4 \\ &= 0.08 \\ &\neq P(A \cap B) \end{aligned}$$

$$\therefore \mathit{Since}\ P(A \cap B) \neq P(A) \times P(B), A\ \mathit{and}\ B\ \mathit{are\ not\ independent}$$

c) Since it can be seen that:

$$P(A|B) \neq P(A)$$

This therefore means that A and B are not independent.

d) Since it can be seen that:

$$P(B|A) = P(B)$$

This therefore means that A and B are independent.

Example 4: If the probability of having big hands is 0.3 (event A), the probability of having big feet is 0.2 (event B) and the probability of having both big hands and big feet is 0.06, determine if events A and B are independent or not

Solution:

It is given that:

$$P(A) = 0.3$$

$$P(B) = 0.2$$

$$P(A \cap B) = 0.06$$

Now calculating $P(A) \times P(B)$:

$$\begin{aligned} P(A) \times P(B) &= 0.3 \times 0.2 \\ &= 0.06 \end{aligned}$$

$\therefore\ Since\ P(A) \times P(B) = P(A \cap B)$:

$$Events\ A\ and\ B\ are\ Independent$$

- Mutually Exclusive

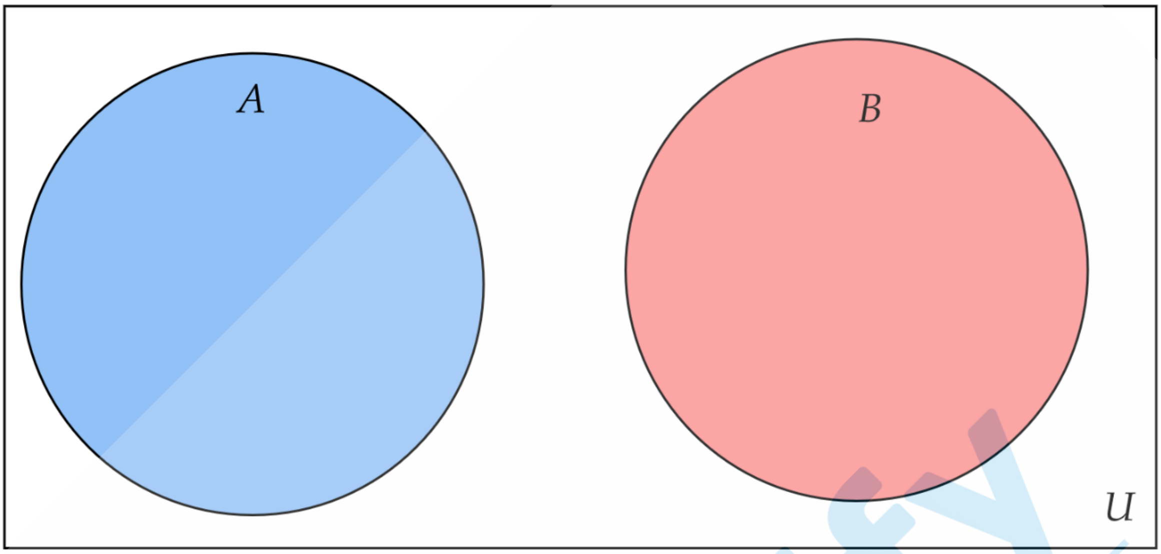
Two events are called mutually exclusive when they do not overlap. In other words, if we consider two events A and B:

$$If\ A\ and\ B\ are\ mutually\ exclusive, P(A \cap B) = 0$$

$$OR$$

$$P(A \cup B) = P(A) + P(B)$$

On the Venn diagram, mutually exclusive events A and B may resemble:



<u>Note:</u> Mutually exclusive events and independent events are NOT the same thing. Students often get the two confused between the two so watch out!

Example 5: From a set of 17 cards numbered $1, 2, 3, \ldots, 17$, one card is drawn at random. A is the event 'a multiple of 3', B is the event 'a multiple of 8', C is the event 'a multiple of 5'. Which of the events A, B and C are mutually exclusive?

Solution:

Events are defined as mutually exclusive when no overlap occurs. In this scenario, an overlap would occur between two events if a number is a multiple of both.

Hence, since no number from 1 – 17 is a multiple of 3 and 8, it can be said that:

$$Events\ A\ and\ B\ are\ mutually\ exclusive$$