TRIGONOMETRY

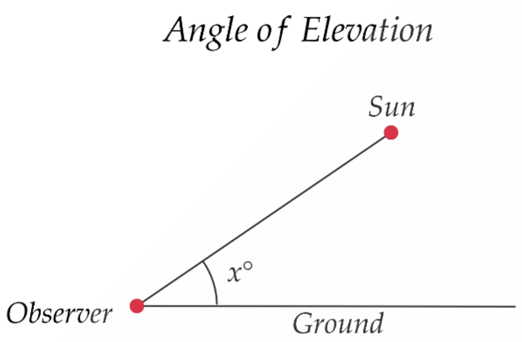
APPLICATIONS OF TRIGONOMETRY (IX)

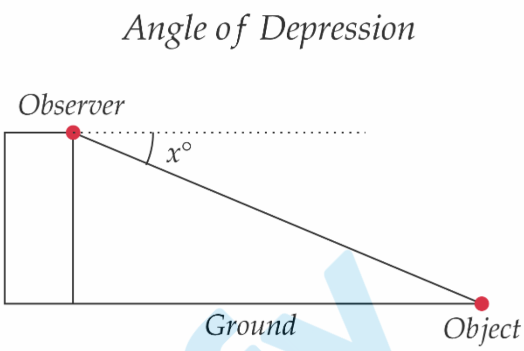
Contents include:

- Angles of Elevation and Depression
- Bearing Questions
    - True Bearings
    - Compass Bearings

- Angles of Elevation and Depression

Angles of elevation and depression are essentially the angle a line makes to the horizontal. An angle of elevation and depression $x°$ are both sketched in the diagrams below:



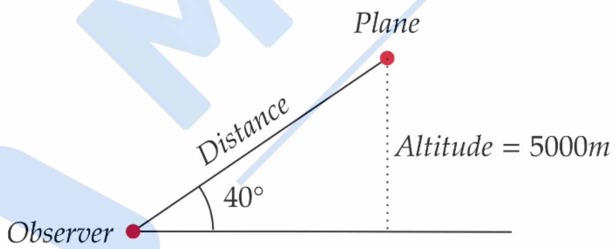


Note: The angle and elevation and depression are actually equal due to alternate angles in parallel lines!

Example 1: An observer on the ground notices a plane at an angle of elevation of 40°. The plane is said to be flying at a constant altitude of 5000m, how far away from the observer is the plane?

Solution:

Drawing our diagram, it would resemble something along the lines of:



Using our trigonometric ratios, we can see that:

$$\sin 40° = \frac{5000}{distance}$$

$$\therefore distance = \frac{5000}{\sin 40°}$$

$$\approx 7779m\ (nearest\ metre)$$

- Bearing Questions

There are two ways we may express bearings in Trigonometry:

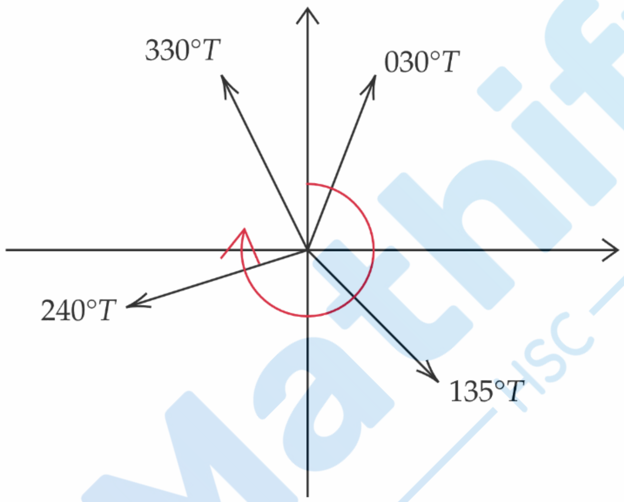
- True Bearings

The most common method, this essentially involves finding the anticlockwise angle as a **3-digit number** (add 0 in front if it's a 2 – digit or 1 – digit angle) and adding a "T" at the end.

For example:

$$095°T$$

This is better demonstrated in the diagram below:

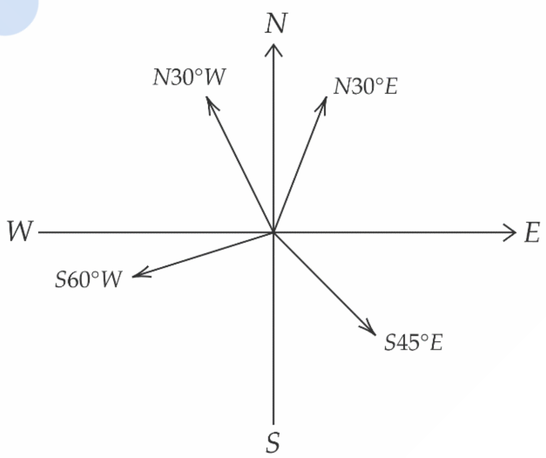


- Compass Bearings

Compass bearings essentially first tells us whether a direction is $North$ or $South$, then tells us how many degrees to the $East$ or $West$ it is. It will resemble something like:

$$Nx°E$$

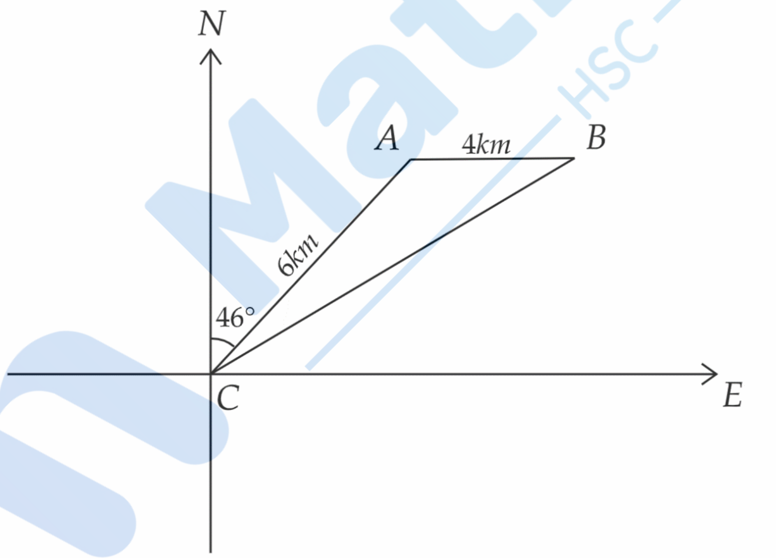
This is demonstrated in the diagram below:



Note: If the question asks for a bearing as an answer, you must give your final answer as a proper bearing, not just an angle!

Overall, bearing questions are quite difficult as you may be required to use the sine or cosine rule. Moreover, it is **often required** of students to use the following properties: $$alternate\ angles\ are\ equal$$ $$corresponding\ angles\ are\ equal$$ $$Co - interior\ angles\ are\ supplementary$$ This may involve us having to draw multiple rays that represent North. This is shown in the example problem below:

Example 2: A hiker walks 6km from camp (point C) to point A in a direction of $N46°E$. He then walks 4km due east to point B, as shown in the diagram below:



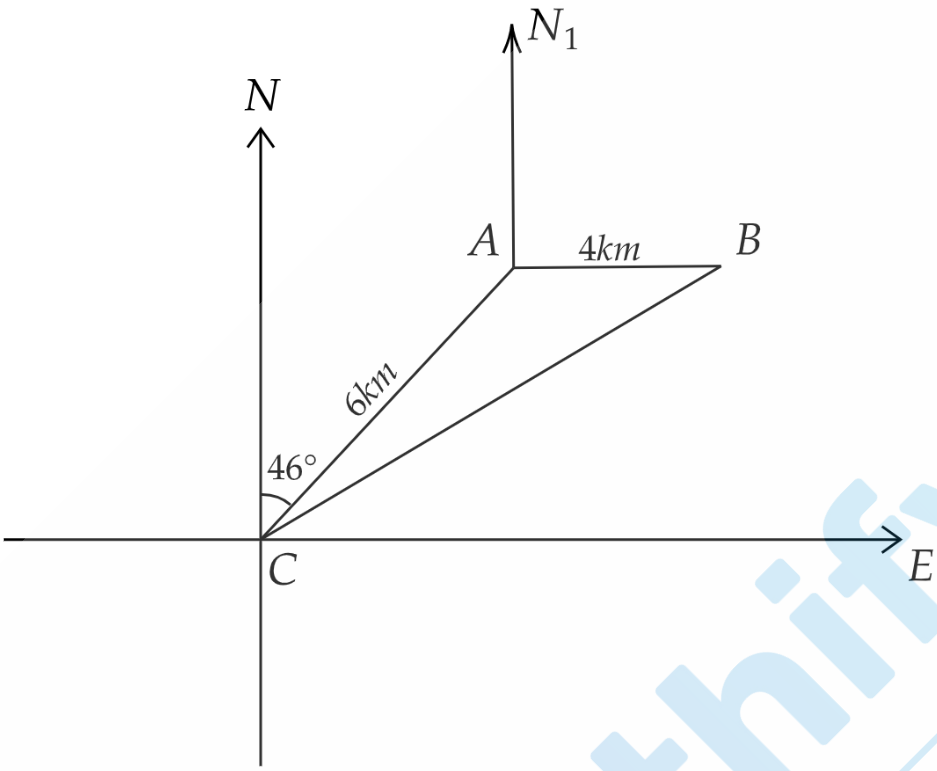
i. Calculate the shortest distance between point B and the camp, giving your answer to the nearest 3 significant figures [2 marks]

ii. Calculate the bearing of the hiker at point B from the camp, giving your answer to the nearest degree [2 marks]

Solutions:

i. To calculate the value of BC using cosine rule, we must first find the value of $\angle BAC$

To do so, construct interval AN_1 such that it points to North:



$$\therefore \angle CAN_1 = 180° - 46° \; [\because co - interior\ angles\ supplementary]$$
$$= 134°$$

Since B is due east of A, $\angle N_1AB = 90°$

$$\therefore \angle CAB = 360° - 134° - 90°$$
$$= 136°$$

Hence, now using cosine rule:

$$BC^2 = AC^2 + AB^2 - 2(AC)(AB)\cos\angle CAB$$
$$BC^2 = 6^2 + 4^2 - 2(6)(4)\cos 136°$$
$$BC^2 \approx 86.52831$$

$$\therefore BC \approx 9.30km \; [nearest\ 3\ sig\ fig]$$

ii. To find the bearing of B from C, we must first find $\angle ACB$

To do so, we will use cosine rule [sine rule may be used too, but must check for ambiguous case!]

$$\therefore \cos\angle ACB = \frac{AC^2 + BC^2 - AB^2}{2(AC)(BC)}$$
$$= \frac{6^2 + 9.3^2 - 4^2}{2(6)(9.3)}$$
$$\approx 0.9542$$

$$\therefore \angle ACB \approx 17.4055°$$

Note that it is good habit to avoid rounding too much during our working out to avoid getting an inaccurate answer

Hence, the bearing of B from C may be given as:

$$\begin{aligned} Bearing &= 46^\circ + 17.4055^\circ \\ &\approx 63^\circ \ [nearest\ degree] \end{aligned}$$