Mathify HSC

INTEGRATION

PRIMITIVE FUNCTIONS AND THE ANTI - DERIVATIVE (I)

Contents include:

- Introduction to Integral Calculus
- Calculating Anti - Derivatives

- Antidifferentiation

We will now consider the reverse problem of finding an unknown function $f(x)$ when its derivative $f'(x)$ is known. Thus, if we have a function $F(x)$ such that $F'(x) = f(x)$, then we call $\boldsymbol{F(x)}$ **an antiderivative or primitive of** $\boldsymbol{f(x)}$.

For example: $2x$ is the derivative of x^2 and so x^2 is a primitive of $2x$

- Primitive of x^n

$$The\ primitive\ of\ x^n = \frac{1}{n+1}x^{n+1} + C, n \neq -1$$

Where C is any constant

We can verify this formula by differentiating $\frac{1}{n+1}x^{n+1} + c$ to give x^n

Note: We must always include the "+$\boldsymbol{C}$" everytime we find the primitive expression!

Since whenever we differentiate a constant, it is equal to 0, when we find the primitive we thus do not know for certain what the constant is, so we must leave a "$+C$".

Some extra things to remember:

- Primitive of a constant k

$$If\ f(x) = k, then\ F(x) = kx$$

- Primitive of $kg(x)$

$$If\ f(x) = kg(x), then\ F(x) = kG(x)$$

This essentially means **you can factorise out a constant** when finding a primitive, just like how you can factorise out a constant when finding a derivative.

- Primitive of a sum or difference of functions

$$If\ f(x) = g(x) \pm h(x), then\ F(x) = G(x) \pm H(x)$$

This essentially means **you can split up a function** when finding its primitive, just like how you can split up a function when you differentiate.

Example 1: Find the primitive of $5x^2 - 3x + 4$

Using the general rules for finding a primitive:

$$F(x) = 5 \times \frac{x^3}{3} - 3 \times \frac{x^2}{2} + 4 \times x + C$$

$$= \frac{5x^3}{3} - \frac{3x^2}{2} + 4x + C$$

Example 2: Find the primitive of x^4

Using the general rules for finding a primitive:

$$F(x) = \frac{x^{4+1}}{4+1}$$
$$= \frac{x^5}{5}$$

Example 3: Find the primitive of $(x-1)(x-2)$

Before we find the primitive, we **first must expand our expression if it's factorised.** Hence:

$$Step\ 1: Expand\ your\ expression$$

$$(x-1)(x-2) = x^2 - 2x - x + 2$$
$$= x^2 - 3x + 2$$

$$Step\ 2: Find\ the\ primitive\ expression\ F(x)\ from\ this$$

Using the general rules for finding a primitive:

$$F(x) = \frac{x^3}{3} - 3 \times \frac{x^2}{2} + 2x + C$$
$$= \frac{x^3}{3} - \frac{3x^2}{2} + 2x + C$$

Example 4: Find the primitive of $(x+3)^2$

$$Step\ 1: Expand\ your\ expression$$

Before we find the primitive, we **first must expand our expression if it's factorised**

$$(x+3)^2 = x^2 + 6x + 9$$

$$Step\ 2: Find\ the\ primitive\ expression\ F(x)\ from\ this$$

Using the general rules for finding a primitive:

$$F(x) = \frac{x^3}{3} + 6 \times \frac{x^2}{2} + 9x + C$$
$$= \frac{x^3}{3} + 3x^2 + 9x + C$$