INTEGRATION

PROPERTIES OF DEFINITE INTEGRALS (X)

Contents include:

- Properties of Definite Integrals

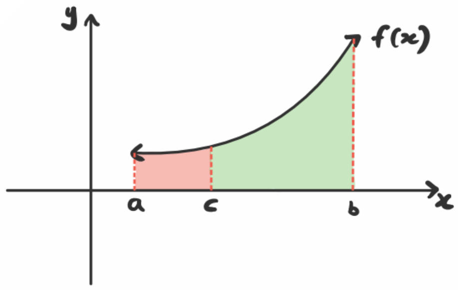
- Definite Integral Properties

The following are properties to keep in mind and understand. Knowing these will be helpful for the next booklet where we look at harder area questions in integration!

Dissection of the Interval:

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

This can be visualised in the diagram below:



The total area underneath the curve for the closed interval $[a, b]$ is equal to the red area plus the green area.

Intervals of Zero Width:

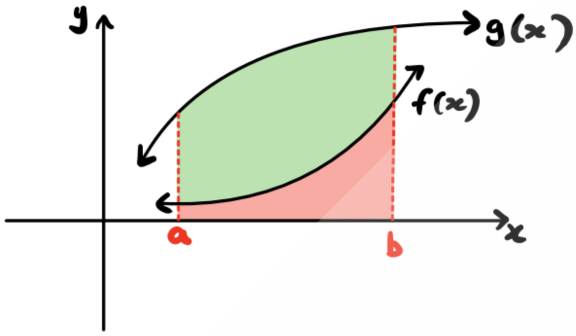
$$\int_a^a f(x)\,dx = 0$$

Inequalities with Definite Integrals:

$$If\ f(x) \leq g(x)\ in\ the\ interval\ a \leq x \leq b, then:$$

$$\int_a^b f(x)\,dx \leq \int_a^b g(x)\,dx$$

This can be visualised in the diagram below:



The area underneath $f(x)$ is given as the red region and the area underneath $g(x)$ is given as the **green and red region.**

Reversing Intervals:

$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$$

This is because:

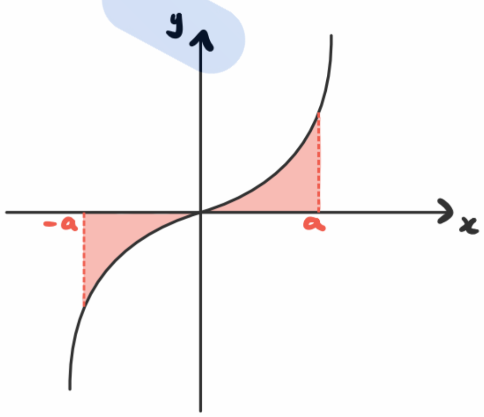
$$F(a) - F(b) = -\big(F(b) - F(a)\big)$$

Odd function Areas:

$$If\ f(x)\ is\ odd, i.e.\ f(-x) = -f(x),$$

$$\int_{-a}^{a} f(x)\,dx = 0$$

This can be visualised in the diagram below:



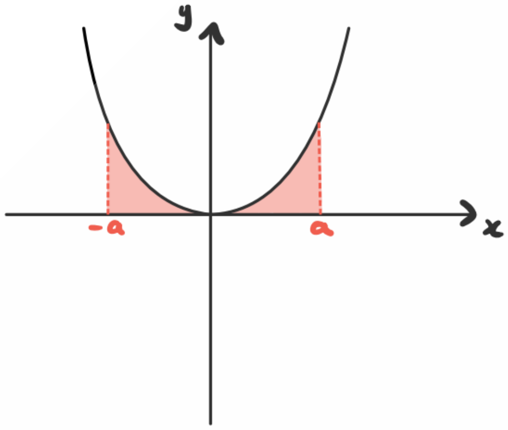
As can be seen, the region on both sides of the $y-$ axis are equal in size. However, one is below the $x-$ axis while the other is above, so their integrals cancel each other out.

Even Function Areas:

$$If\ f(x)\ is\ even, i.e. f(-x) = f(x),$$

$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$$

This can be visualised in the diagram below:



The region on both sides of the $y-$axis are equal in size and both above the $x-$axis.