PROBABILITY

THE MULTIPLICATION RULE (II)

Contents include: Multiplying probabilities and adding probabilities

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- Multiplying Probabilities

To calculate the probability of multiple events, e.g. event A **and** event B occurring, we need to **multiply** the probabilities of each event occurring together.

This is because the number of possibilities (sample space) will **expand** when we consider two events at once, and so to cater to this we must multiply them.

In other words, the multiplication rule states that:

Given events A and B:

$$P(A \text{ } and \text{ } B) = P(A) \times P(B)$$

Example 1: Find the probability of flipping a head **and** rolling a 3 on a die.

$$P(heads) = \frac{1}{2}$$

$$P(roll\ a\ 3) = \frac{1}{6}$$

$$\begin{aligned}\therefore P(heads\ and\ roll\ a\ 3) &= P(heads) \times P(roll\ a\ 3) \\ &= \frac{1}{2} \times \frac{1}{6} \\ &= \frac{1}{12}\end{aligned}$$

Note that this can apply to more than just two events occurring simultaneously

- Adding Probabilities

To calculate the probability of events A **or** B occurring, we need to **add** the probabilities of each event occurring together.

This is because we have expanded our event space now, so we add our probabilities to represent a greater chance of reaching a favourable outcome.

In other words:

Given events A and B:

$$P(A\ or\ B) = P(A) + P(B)$$

Example 2: Find the probability of rolling either a 1 or a 5 on a die:

$$P(rolling\ a\ 1) = \frac{1}{6}$$

$$P(rolling\ a\ 5) = \frac{1}{6}$$

$$\therefore P(rolling\ a\ 1\ or\ a\ 5) = P(rolling\ a\ 1) + P(rolling\ a\ 5)$$

$$= \frac{1}{6} + \frac{1}{6}$$
$$= \frac{1}{3}$$

Example 3: A bag contains 9 green balls, 7 red balls and 4 yellow balls. If one marble is drawn from the bag, find the probability that it is:

a) Red
b) Yellow
c) Green or Yellow

Solution:

a)

In total, there are $9 + 7 + 4 = 20$ balls, 7 of which are red

$$\therefore P(red) = \frac{7}{20}$$

b)

$$P(yellow) = \frac{4}{20}$$
$$= \frac{1}{5}$$

c) Green or yellow

$$P(Green\ or\ yellow) = P(green) + P(yellow)$$
$$= \frac{7}{20} + \frac{4}{20}$$
$$= \frac{11}{20}$$

Example 4: A letter is selected at random from the alphabet. Find the probability that:

a) It is a vowel
b) It is a consonant
c) It is either the letter J or the letter X
d) One of the letters of the word LEBRON

Solutions:

a) There are 5 vowels in the alphabet, along with 26 total letters:

$$\therefore P(vowel) = \frac{5}{26}$$

b) There are 21 consonants in the alphabet, along with 26 total letters:

$$\therefore P(consonant) = \frac{21}{26}$$

c)

$$\begin{aligned} P(J \text{ or } X) &= P(J) + P(X) \\ &= \frac{1}{26} + \frac{1}{26} \\ &= \frac{1}{13} \end{aligned}$$

d)

There are 6 letters in LEBRON, therefore:

$$\begin{aligned} P(LEBRON) &= \frac{6}{26} \\ &= \frac{3}{13} \end{aligned}$$

Example 5: A card is chosen from a standard deck of 52 cards. Find the probability that:

a) It is a heart
b) It is a diamond or clubs
c) It is a picture card (Jack, Queen or King)
d) It is a 5 of spades or a 7 of diamonds
e) It is red and a picture card
f) It is less than five

Solution:

a) $P(heart) = \frac{1}{4}$

b)

$$\begin{aligned} P(diamond \text{ or } clubs) &= P(diamond) + P(clubs) \\ &= \frac{1}{4} + \frac{1}{4} \\ &= \frac{1}{2} \end{aligned}$$

c) $P(picture\ card) = \frac{3}{13}$

d)

$$\begin{aligned} P(5 \text{ of } spades \text{ or } 7 \text{ of } diamonds) &= P(5 \text{ of } spades) + P(7 \text{ of } diamonds) \\ &= \frac{1}{52} + \frac{1}{52} \\ &= \frac{1}{26} \end{aligned}$$

e)

$$
\begin{aligned}
P(red\ and\ a\ picture\ card) &= P(red) \times P(picture\ card) \\
&= \frac{1}{2} \times \frac{3}{13} \\
&= \frac{3}{26}
\end{aligned}
$$

f)

$$
\begin{aligned}
P(less\ than\ 5) &= P(1) + P(2) + P(3) + P(4) \\
&= \frac{4}{13}
\end{aligned}
$$