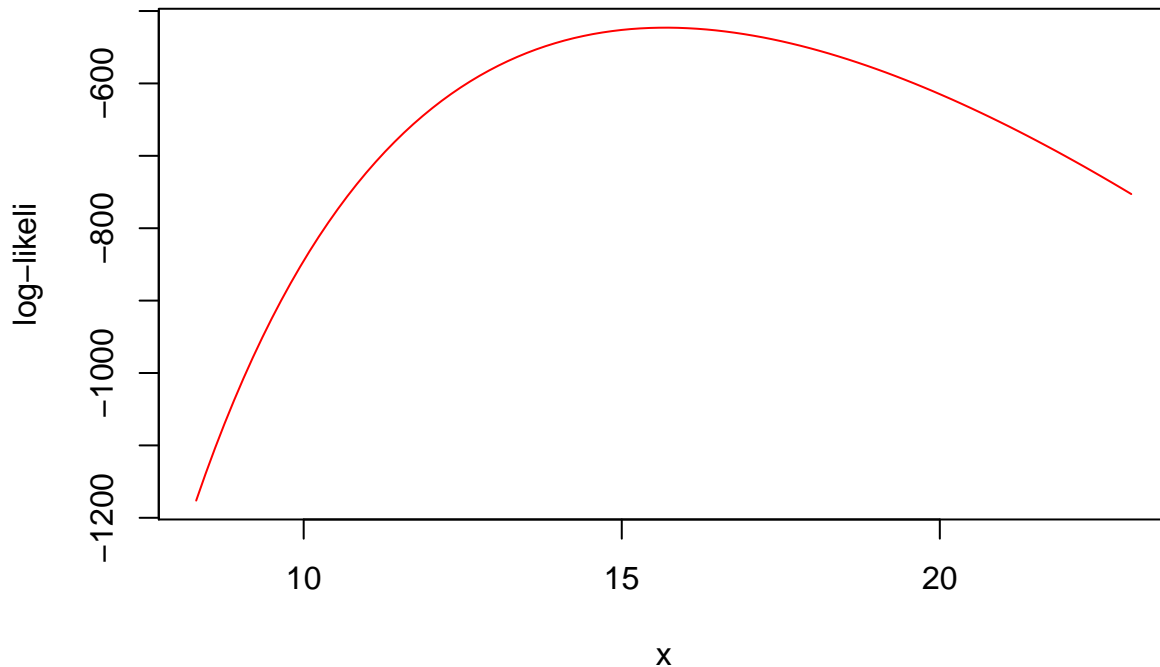# Test exam

Andreas Dahlberg

Problem 1

```r
#Problem 1a)
temperatures <- read.csv("zurichtemp.csv")$Temperature

log_likeli1 <- function(t,data){
  sum(dnorm(data,t,sqrt(t),log=TRUE))
}
x <- seq(min(temperatures),max(temperatures),length=100)
plot(x,sapply(x,function(t) log_likeli1(t,temperatures)),type="l",col="red",ylab="log-likeli")
```



```r
# Problem 1b)
ML1 = optim(15,function(t) -log_likeli1(t,temperatures),method="BFGS")$par

hist(temperatures,prob=TRUE,breaks=20,col="red")

x<-seq(5,30,length=300)
lines(x,dnorm(x,ML1,sqrt(ML1)),col="BLUE")

# Problem 1c)
log_likeli2 <- function(t){
    sum(dnorm(temperatures,t[1],sqrt(t[1]+t[2]),log=TRUE))
}

ML2 = optim(c(15,0), function(t) -log_likeli2(t))$par
lines(x,dnorm(x,ML2[1],sqrt(ML2[1]+ML2[2])),col="GREEN")
```
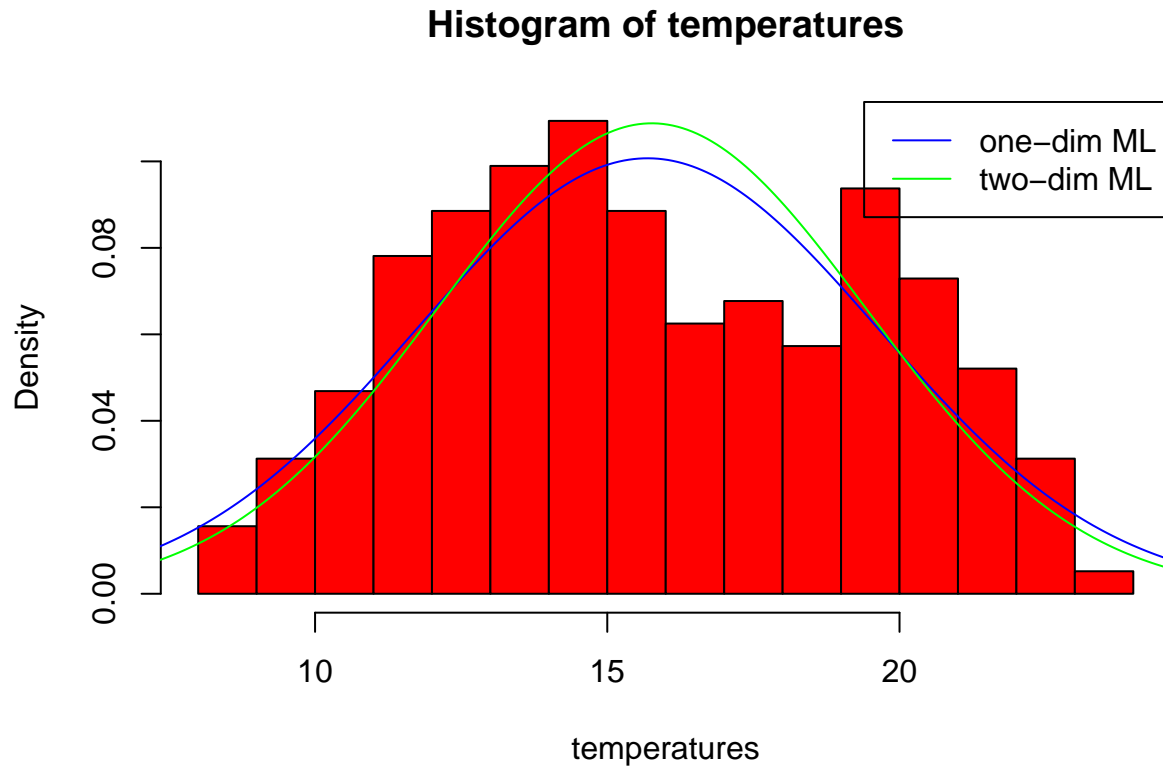
```r
legend("topright",legend=c("one-dim ML","two-dim ML"),col=c("BLUE","GREEN"),lty=c(1,1))
```

Histogram of temperatures


```r
# Problem 1d)
cols=c("RED","BLUE","GREEN","PURPLE")
x <- seq(3,35,length=200)
for (i in 1:4){
  sample = rnorm(20*i^2,ML1,sqrt(ML1))
  ML = optim(15,function(t) -log_likeli1(t,sample),method="BFGS")$par
  if (i == 1)
    plot(x,sapply(x,function(t) log_likeli1(t,sample) - log_likeli1(ML,sample)),col=cols[i],type="l")
  else
    lines(x,sapply(x,function(t) log_likeli1(t,sample) -log_likeli1(ML,sample)),col=cols[i],type="l")
}
```
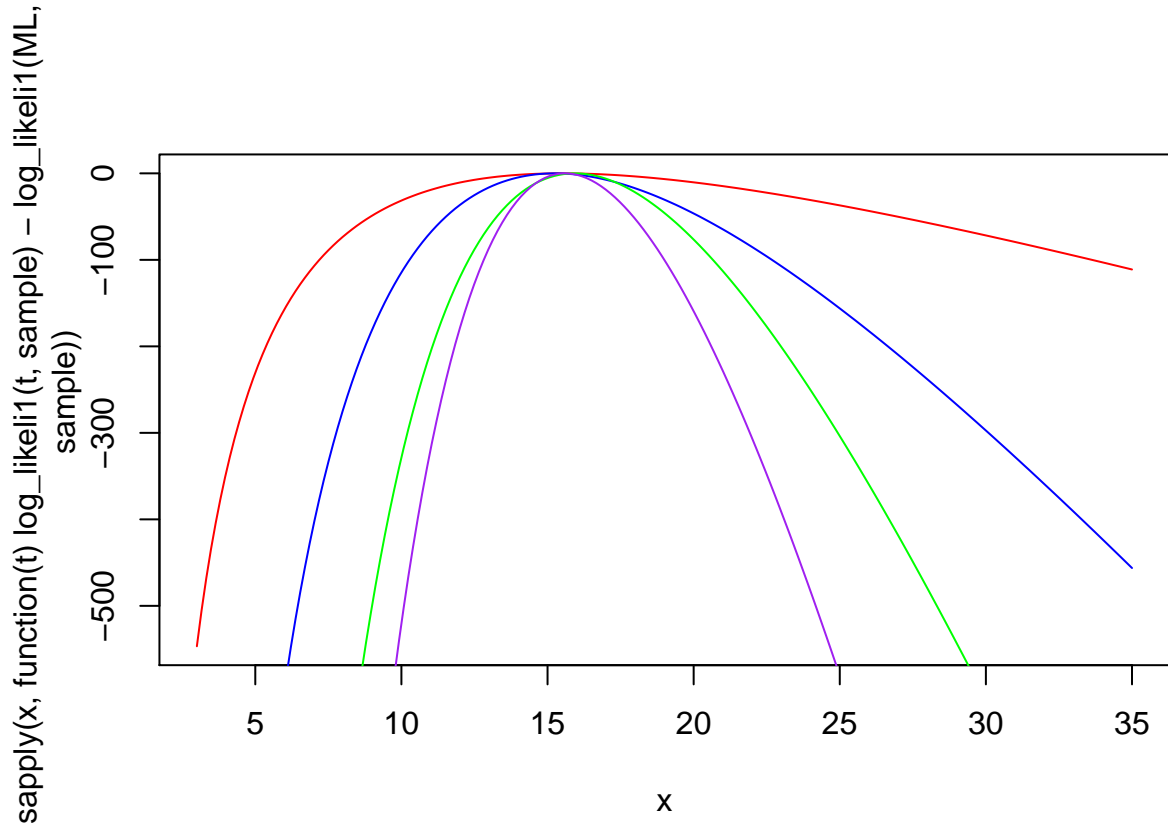
1c) Since there are two parameters θ and δ we have to do a two-dimensional optimization. This is seen in the code above

1e) In 1d) we plot the relative log-likelihood for different samples. It is clear that the relative log-likelihood, and hence also the log-likelihood, converges to a parabola that gets narrower and narrower for increasing sample sizes. This can also be seen by doing a Taylor expansion for the log-likelihood around the ML-estimate of θ. The relative log-likelihood can be interpreted as "the closer the value is to 0, the more likely it is". For large sample sizes we see that the only "likely" values of θ are the ones close to 15.