Lab 8

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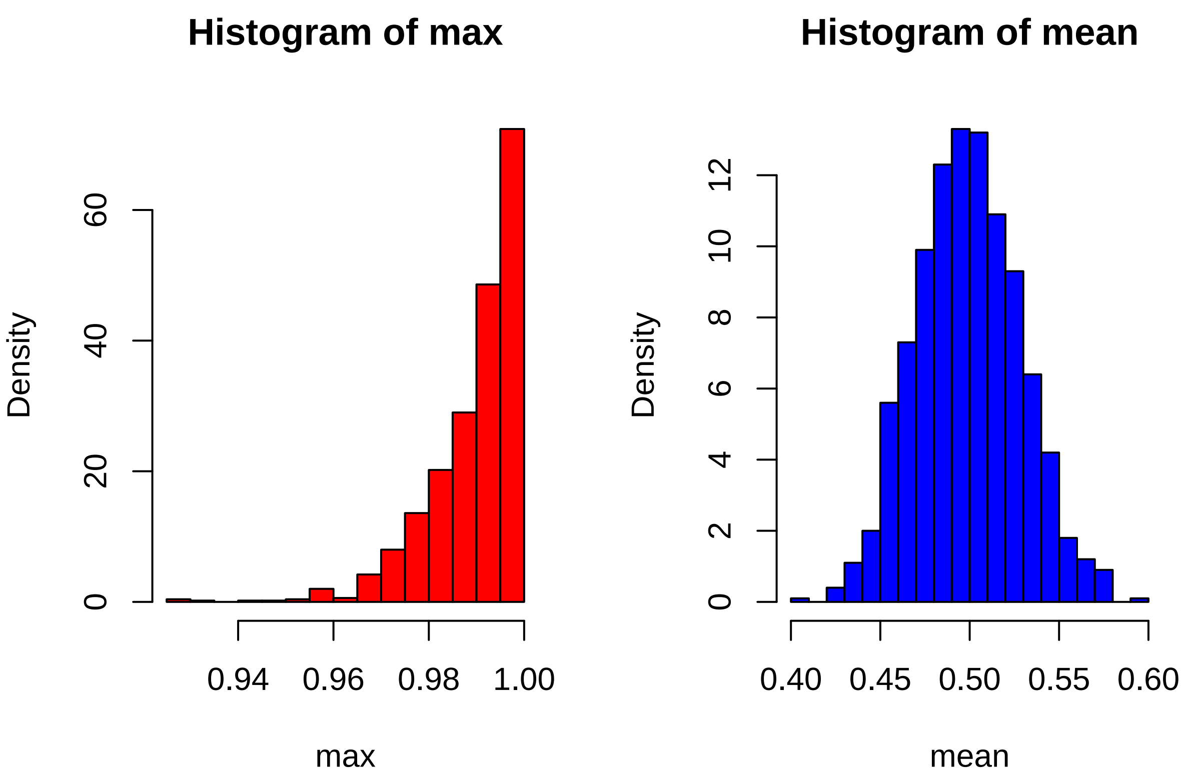
Problem 1)

1a

```r
mean = c()
max = c()
for (i in 1:1000){
  sample = runif(100,0,1)
  max = append(max,max(sample))
  mean = append(mean,mean(sample))
}
par(mfrow = c(1,2))
hist(max,prob=TRUE,col="RED",breaks=20)
hist(mean,prob=TRUE,col="BLUE",breaks=20)
```

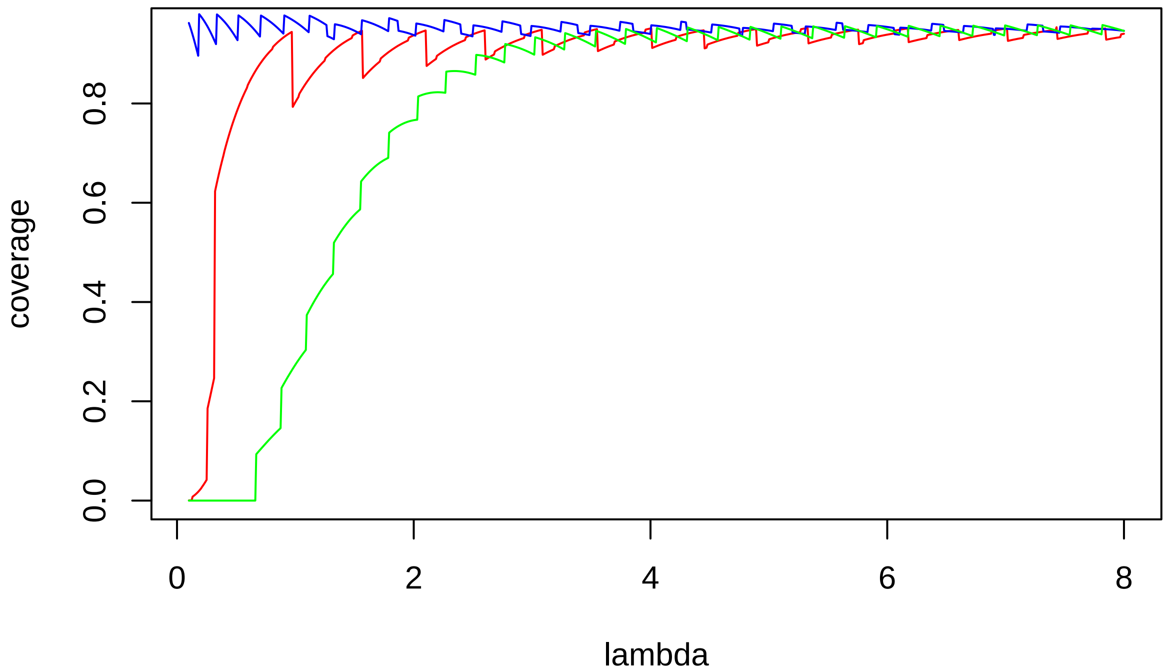


We can see that $\bar{x}$ is normally distributed (which also follows from the CLT) while the ML-estimate $X_{(n)}$ is not. We know that if FRC holds then the ML-estimate is always asymptotically normally distributed but since in this case FRC does not hold it is not certain that our ML-estimate is normally distribured. Since the Wald and likelihood ratio CI builds on that the ML-estimate is normally distributed, we see that it would be a bad idea to use those confidence intervals in this case. From the histogram we can also see that $\bar{x}$ is unbiased estimator of θ while $X_{(n)}$ is not, since it is always smaller than θ

Problem 2)**

```
ci1 <- function (x,e){
 ci = 1/e*(x+c(-1,1)*1.96*sqrt(x))
 if (ci[1] < 0)
    ci[1] = ci[1]*-1
 ci
}
ci2 <- function(x,e){
  ci = 1/2*(2*x/e+1.96*1.96/e)+c(-1,1)*sqrt(1/4*(2*x/e+1.96*1.96*1/e)^2-x^2/e^2)

  if (ci[1] < 0)
    ci[1] = ci[1]*-1
 ci
}
ci3 <- function(x,e){
 ci = x/(e*(1+c(1,-1)*1.96/sqrt(x)))
 if (ci[1] < 0)
    ci[1] = ci[1]*-1
 ci
}

e <- 3.04; alpha <- 0.05
lambda <- seq(0.1, to=8, length=1000)
x <- 0:100

cols = c("RED","BLUE","GREEN")
prob <- outer(x,lambda*e,dpois)
for (i in 1:3){

  if (i == 1)
    fun = ci1
  else if (i==2)
    fun = ci2
  else
    fun = ci3

  CImat <- sapply(x,function(x) { fun(x,e) } )
  ind <- outer(CImat[1,], lambda, "<") &
  outer(CImat[2,], lambda, ">")
  coverage <- apply(ind*prob, 2, sum)
  print(coverage[1])
  if (i == 1)
    plot(lambda,coverage,col=cols[i],type="l")
  else
    lines(lambda,coverage,col=cols[i],type="l")
}
```

```
## [1] 0.0002625775
```




```
## [1] 0.9621706
## [1] 0
```

It seems as if the blue one, which is the score CI when using $J(\lambda)$, seems to converge to the 95% the fastest.