ROBUST CONTROL

Exercise 4 – Performance limitations & Robust stability and performance for MIMO systems

1. A controller is to be designed for the system:

$$G(s) = \frac{1}{10s+1}\begin{pmatrix} 1 & -(s+1) \\ 1 & -1.1 \end{pmatrix}, G_d(s) = \frac{1}{(5s+1)(10s+1)}\begin{pmatrix} 1 & -2 \\ 1 & 2.5 \end{pmatrix}$$

   The control objective is to keep the control error smaller than 0.1 in both outputs for disturbances of magnitude up to 0.5 and 0.2 in d_1 and d_2, respectively. The deviations in each of the control inputs should be kept less than 10. Perform a controllability analysis in order to determine the feasibility of achieving the control objectives.

2. Compute μ and the corresponding perturbation Δ for

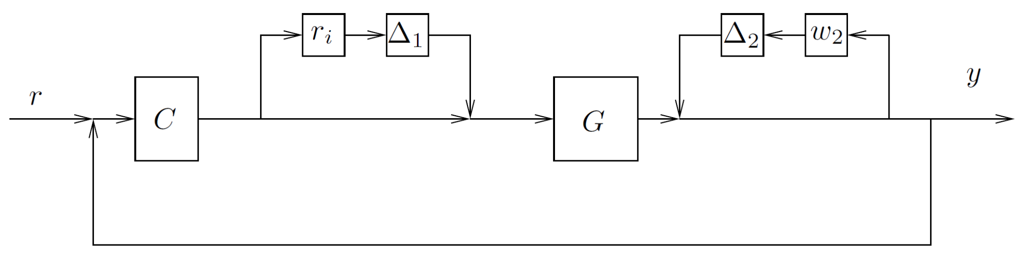
$$M = \begin{bmatrix} 1 & 0 \\ 10 & 10 \end{bmatrix}$$

when Δ is a:

   a. full matrix,
   b. diagonal matrix.

You should do this first by "hand-calculations" and then by using a Matlab tool for computing μ, such as *mussv*.

3. Consider a system with input and output uncertainty:



$$C(s) = \frac{k}{s} \; ; \quad G(s) = \frac{1}{(5s+1)^2} \; ; \quad w_2 = \frac{r_p}{s+1}$$

   with $\|\Delta_1\| < 1$ and $\|\Delta_2\| < 1$.

   a. Determine a robust stability condition.
   b. For what values of k is the system robustly stable with 10% input uncertainty (r_i = 0.1) and 10% pole uncertainty (r_p = 0.1).
   c. Repeat (b), but now with r_p = 2.

4. Consider a heat-exchanger with the nominal model:

$$\begin{pmatrix} T_c \\ T_H \end{pmatrix} = \underbrace{\frac{1}{100s+1} \begin{pmatrix} -1874 & 1785 \\ -1785 & 1874 \end{pmatrix}}_{G(s)} \begin{pmatrix} q_C \\ q_H \end{pmatrix}$$

The control performance objective is to keep the sensitivity in all plant directions below the bound:

$$f(s) = \frac{2s}{s+0.2}$$

The uncertainty has been modelled as independent input uncertainty with magnitude bounded by

$$w_i(s) = \frac{s+0.2}{0.5s+1}$$

corresponding to 20% uncertainty at low frequencies, and more than 100% uncertainty for frequencies above 1.

A controller has been designed using decoupling

$$C(s) = \frac{0.2}{s} G^{-1}(s)$$

Formulate and evaluate the conditions for nominal stability, nominal performance, robust stability and robust performance.