DERIVING LINEAR MODELS

Robust Control

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How do we develop linear models?

- Physical first-principle models (e.g., Newton, Ohm, Thermodynamics, etc.)
- Input-output data

PROCEDURE

- Formulate a (non-)linear state space model based on physical knowledge

$$\dot{x} = f(x, u) \qquad (1)$$

- Determine the steady state (nominal) operating point (or trajectory)

$$\dot{x}^* = f(x^*, u^*) \qquad (2)$$

- Introduce deviation variables (from the nominal operation)

$$\delta x\,(t) = x(t) - x^*(t)$$
$$\delta u\,(t) = u(t) - u^*(t)$$

- Define the dynamics of the state deviation (subtract (1) and (2))

$$\dot{\delta x} = f(x, u) - f(x^*, u^*)$$

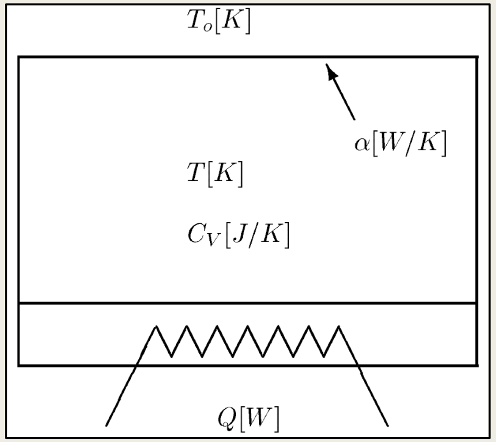
- Linearize using Taylor series expansion around the nominal operating point (x^*, u^*)

$$\dot{\delta x} = \left.\frac{\partial f}{\partial x}\right|_{x=x^*} \delta x + \left.\frac{\partial f}{\partial u}\right|_{u=u^*} \delta u + h.o.t.$$
$$\cong A\delta x + B\delta u \quad \xrightarrow{\text{Laplace}} \quad \delta x\,(s) = (sI - A)^{-1} B\,\delta u\,(s)$$

How do we develop linear models

EXAMPLE (room heating process)

Adjust the heat input $Q[W]$ to maintain fixed room temperature T ($\pm 1[K]$)



- Physical model from energy balance (already linear)

$$C_V \dot{T} = Q + a(T_o - T) \text{ with } C_V = 10^5[J/K], a = 100[W/K]$$

- Operating point

$$Q^* = 2000 \text{ and } T^* - T_o^* = 20$$

- Linear model in deviation variables

$$\delta T(t) = T(t) - T^*(t),\ \delta Q\ (t) = Q(t) - Q^*(t), \delta T_o(t) = T_o(t) - T_o^*(t)$$

$$C_V \dot{\delta T} = \delta Q(t) + a\big(\delta T_o(t) - \delta T(t)\big) \rightarrow \delta T(s) = \frac{1}{\tau s+1}\left(\frac{1}{a}\delta Q(s) + \delta T_o(s)\right) \text{ with } \tau = \frac{C_V}{a} = 1000[s]$$

- Linear model in scaled variables

$$\delta T_{max} = 1[K], \delta Q_{max} = 2000[W], \delta T_{o,max} = 10[K]$$

$$y(s) = \frac{\delta T(s)}{\delta T_{max}}, u(s) = \frac{\delta Q(s)}{\delta Q_{max}}, d(s) = \frac{\delta T_o(s)}{\delta T_{o,max}}$$

$y(s) = G(s)u + G_d(s)d(s)$ with

$$G(s) = \frac{1}{\tau s+1}\frac{1}{a}\frac{\delta Q_{max}}{\delta T_{max}} = \frac{20}{1000s+1}$$

$$G_d(s) = \frac{1}{\tau s+1}\frac{\delta T_{o,max}}{\delta T_{max}} = \frac{10}{1000s+1}$$