Robust Control

(ECE_ΔK806)

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Course Content

Feedback control for SISO & MIMO LTI systems under model uncertainty

- Frequency domain analysis and design;
  - extension of classical SISO methods to MIMO systems
  - optimal control problems formulated in input-output space
- Input-output controllability; what can be achieved with feedback in a given system?
- Robustness: stability and performance under model uncertainty

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Course Goals

After completing the course you should be able to:

- Quantify the performance that can be achieved with feedback for a given system;
- Analyze feedback systems with respect to stability and performance in the presence of structured and unstructured model uncertainty;
- Design/synthesize controller for robust performance.



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Evaluation

- **Mandatory** Homeworks (3/10 points)
  [During the semester]
- **Mandatory** Lab Exercises (3/10 points)
  [During the semester]
- **Mandatory** Project (4/10 points)
  [End of semester]

PROJECT

https://janismac.github.io/ControlChallenges/

- Modeling
- Identification/Analysis
- Robust control design
- Implementation

```
var delta_t = 0.02; // The simulation time step
position_error_integral += delta_t * block.x;
monitor('var',[block.T,block.x,block.dx]);
const data = document.querySelector('#variableInfo').innerText;
console.log(data);
return -3*block.x -5*block.dx -2*position_error_integral;
```

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Outline

- Introduction & Mathematical Background
- Feedback Control
- Performance Limitations in SISO Feedback
- Uncertainty and Robustness in SISO Systems
- MIMO LTI systems
- Performance Limitations in MIMO Feedback
- Robust Stability and Performance Analysis for MIMO Systems
- Controller Synthesis

1 Introduction

1.1 The control problem

$$y = Gu + G_d d \tag{1.1}$$

y	:	output/controlled variable
u	:	input/manipulated variable
d	:	disturbance
r	:	reference/setpoint

Regulator problem	:	counteract d
Servo problem	:	let y follow r

Goal of control: make control error $e = y - r$

"small".

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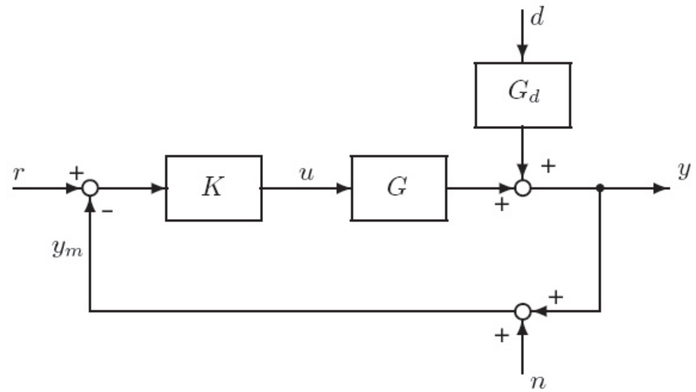
1.2 Why Feedback?



Why not $u = G^{-1}r - G^{-1}G_d d$?

- model uncertainty - uncertain knowledge of system behavior
- unmeasured disturbances
- instability

Cost of feedback:

- potentially induce instability
- feed measurement noise into process

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1.3 Fact 1: Feedback has limitations

Keep in mind:

- Feedback is a simple and potentially very powerful tool for tailoring the behavior of a dynamical system
- Power of control is limited.
- Control quality depends both on the controller **AND** on the plant/process.

Ziegler-Nichols (1943): *"In the application of automatic controllers, it is important to realize that controller and process form a unit; credit or discredit for results obtained are attributable to one as much as the other. A poor controller is often able to perform acceptably on a process which is easily controlled. The finest controller made, when applied to a miserably designed process, may not deliver the desired performance. True, on badly designed processes, advanced controllers are able to eke out better results than older models, but on these processes, there is a definite end point which can be approached by instrumentation and it falls short of perfection."*

$\Rightarrow$ Much of the course will be spent on input-output "controllability analysis" of the plant/process.

1.4 Fact 2: Models are uncertain

Models (G, G_d) are inaccurate

$$\Rightarrow \text{Real Plant: } G_p = G + E \ ;$$

E = "uncertainty" or "perturbation" (unknown)

Definitions for closed loop

- **Nominal stability (NS)** : system is stable with no model uncertainty.
- **Nominal Performance (NP)** : system satisfies performance specifications with no model uncertainty.
- **Robust stability (RS)** : system stable for "all" perturbed plants
- **Robust performance (RP)** : system satisfies performance specifications for all perturbed plants

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1.5 A Brief History of Control

- **Classical, 30's-50's:** frequency domain methods (Bode, Nyquist, Nichols,...)
  + yields insight (loop shaping)
  + address model uncertainty (gain and phase margin)
  - applicable only to SISO systems
- **Modern, 60's-70's:** state space optimal control (Bellman, Pontryagin, Kalman,...)
  + control cast as time-domain optimization problem
  + applicable to MIMO systems (LQG)
  - can not accomodate for unmodeled dynamics
  - LQG has no guaranteed stability margins
  - no clear link to classical methods

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... A Brief History of Control

- **PostModern, 80's-90's:** robust control (Zames, Fransis, Doyle,...)
  - \+ frequency domain methods for MIMO systems
  - \+ explicitly address model uncertainty
  - \+ control casts as an optimization problem ($\mathcal{H}_2$, $\mathcal{H}_\infty$)
  - \+ links classical and modern approaches; formulate and analyze in input-output domain, compute in state-space
  - \- high order controllers, computational issues, ...

It was the introduction of norms in control, in particular the $\mathcal{H}_\infty$-norm, that paved the way for analyzing fundamental limitations and robustness in MIMO systems.

- **Post 90's:** analysis/synthesis using convex optimization (e.g., LMIs), combination of $\mathcal{H}_2$ for performance with $\mathcal{H}_\infty$ for robustness, beyond LTI systems

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1.6 System Representations

- State-space representation:

$$\begin{aligned} \dot{x} &= Ax + Bu,\ x \in \mathcal{R}^n,\ u \in \mathcal{R}^p \\ y &= Cx + Du,\ y \in \mathcal{R}^l \end{aligned}$$

- Transfer function:

$$\begin{aligned} Y(s) &= G(s)U(s), \\ G(s) &= C\left(sI - A\right)^{-1} B + D \end{aligned}$$

- Frequency response:

$$Y(j\omega) = G(j\omega)U(j\omega)$$

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Transfer functions

$$G(s) = \frac{\beta_{n_z} s^{n_z} + \cdots + \beta_1 s + \beta_0}{s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0} \tag{1.2}$$

For multivariable systems, $G(s)$ is a matrix of transfer functions.

n = order of denominator (or pole polynomial) or *order of the system*

n_z = order of numerator (or zero polynomial)

$n - n_z$ = pole excess or *relative order*.

Definition

- A system $G(s)$ is *strictly proper* if $G(s) \to 0$ as $s \to \infty$.
- A system $G(s)$ is *semi-proper* or *bi-proper* if $G(s) \to D \neq 0$ as $s \to \infty$.
- A system $G(s)$ which is strictly proper or semi-proper is *proper*.
- A system $G(s)$ is *improper* if $G(s) \to \infty$ as $s \to \infty$.

1.7 Scaling

Proper scaling simplifies controller design and performance analysis.

SISO :

unscaled:

$$\widehat{y} = \widehat{G}\widehat{u} + \widehat{G}_d\widehat{d}; \quad \widehat{e} = \widehat{y} - \widehat{r} \tag{1.3}$$

scaled:

$$d = \widehat{d}/\widehat{d}_{\max}, \quad u = \widehat{u}/\widehat{u}_{\max} \tag{1.4}$$

where:

- $\widehat{d}_{\max}$ — largest expected disturbance
- $\widehat{u}_{\max}$ — largest allowed input

Scale $\widehat{y}$, $\widehat{e}$ and $\widehat{r}$ by:

- $\widehat{e}_{\max}$ — largest allowed control error, or
- $\widehat{r}_{\max}$ — largest expected reference value

Usually:

$$y = \widehat{y}/\widehat{e}_{\max}, \quad r = \widehat{r}/\widehat{e}_{\max}, \quad e = \widehat{e}/\widehat{e}_{\max} \tag{1.5}$$

MIMO :

$$d = D_d^{-1}\widehat{d}, \quad u = D_u^{-1}\widehat{u}, \quad y = D_e^{-1}\widehat{y} \tag{1.6}$$

$$e = D_e^{-1}\widehat{e}, \quad r = D_e^{-1}\widehat{r} \tag{1.7}$$

where D_e, D_u, D_d and D_r are diagonal scaling *matrices*

Substituting (1.6) and (1.7) into (1.3):

$$D_e y = \widehat{G} D_u u + \widehat{G}_d D_d d; \quad D_e e = D_e y - D_e r$$

and introducing the scaled transfer functions

$$G = D_e^{-1}\widehat{G} D_u, \quad G_d = D_e^{-1}\widehat{G}_d D_d \tag{1.8}$$

Model in terms of scaled variables:

$$y = Gu + G_d d; \quad e = y - r \tag{1.9}$$

Often also:

$$\widetilde{r} = \widehat{r}/\widehat{r}_{\max} = D_r^{-1}\widehat{r} \tag{1.10}$$

so that:

$$r = R\widetilde{r} \quad \text{where} \quad R \stackrel{\Delta}{=} D_e^{-1} D_r = \widehat{r}_{\max}/\widehat{e}_{\max} \tag{1.11}$$

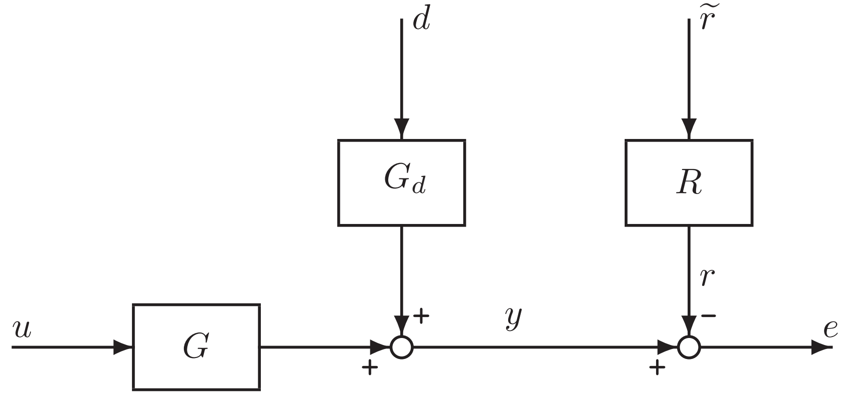

Figure 1: Model in terms of scaled variables

Objective:

for $|d(t)| \leq 1$ and $|\widetilde{r}(t)| \leq 1$,

manipulate u with $|u(t)| \leq 1$

such that $|e(t)| = |y(t) - r(t)| \leq 1$.

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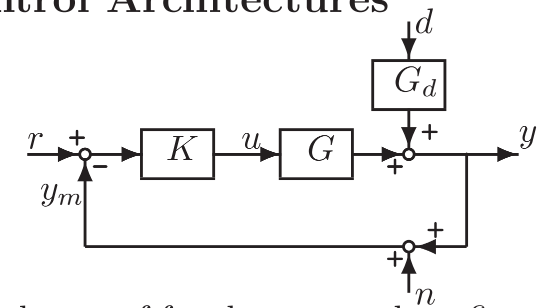
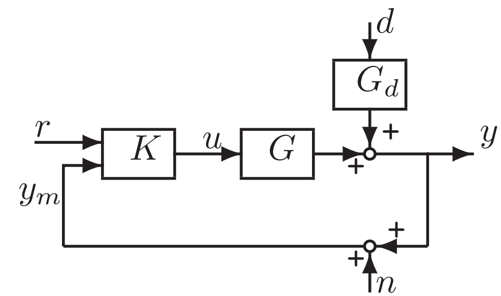
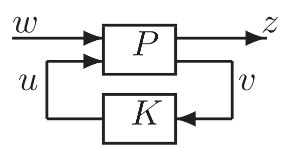
1.8 Control Architectures



(a) One degree-of-freedom control configuration



(b) Two degrees-of-freedom control configuration



(c) General control configuration

Figure 2: Control configurations

Table 1: Nomenclature

K	controller, in whatever configuration. Sometimes broken down into parts. For example, in Figure 2(b), $K = [\,K_r \quad K_y\,]$ where K_r is a pre-filter and K_y is the feedback controller.

Conventional configurations (Fig 2(a), 2(b)):

G	plant model
G_d	disturbance model
r	reference inputs (commands, setpoints)
d	disturbances (process noise)
n	measurement noise
y	plant outputs. (include the variables to be controlled ("primary" outputs with reference values r) and possibly additional "secondary" measurements to improve control)
y_m	measured y
u	control signals (manipulated plant inputs)

General configuration (Fig 2(c)):

P	generalized plant model. Includes G and G_d and the interconnection structure between the plant and the controller. May also include weighting functions.
w	exogenous inputs: commands, disturbances and noise
z	exogenous outputs; "error" signals to be minimized, e.g. $y - r$
v	controller inputs for the general configuration, e.g. commands, measured plant outputs, measured disturbances, etc. For the special case of a one degree-of-freedom controller with perfect measurements we have $v = r - y$.
u	control signals