Dark Trading, Market Quality and Price Discovery

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Motivation

- Dark trading may adversively influence market quality, price discovery and the incentive for liquidity provision
- $\approx$ 700M euros worth of equity is changing hands within dark pools in Europe daily
- Market share of dark pools is growing

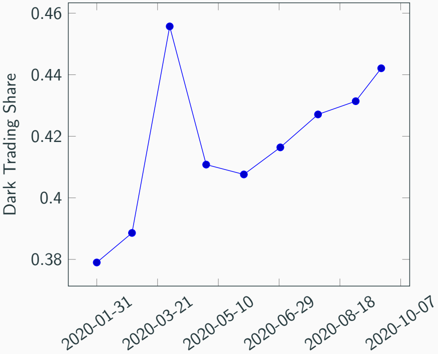

Figure 1: U.S. Equities dark trading. Source: Bats Global Markets.

Literature Review

- Trading venues with different level of transparency: Buti et al. [2017], Degryse et al. [2009], Zhu [2014], Ye [2012]
- Competition between venues: Kwan et al. [2015], Foucault and Menkveld [2008]
- Model of trading in limit order markets: Goettler et al. [2009], Dugast [2018], Roșu [2019]
- Dark trading and price discovery: Comerton-Forde and Putnins [2015]
- Dark trading and market quality: Hatheway et al. [2017]

Contributions

- Order choice and venue choice problem of informed traders in limit order markets
- How are market quality and price discovery affected by dark pool?
- Optimal levels of informed trading and dark pool access *(in a to-do list)*

Benchmark Model (no information asymmetry)

- Trading day is divided into 4 periods ($t_1, t_2, t_3,$ and t_4)
- One asset: pays v in the end of the trading day (common knowledge)
- LOB: set of prices and quantities $\{p_i^z, q_i\}$ where $z = \{A, B\}$ and $i = \{1, \ldots, 4\}$ is the level on a price grid
- Prices are defined relative to the common value of the asset:

$$p_1^z = v - 1.5\tau$$
$$p_2^z = v - 0.5\tau$$
$$p_3^z = v + 0.5\tau$$
$$p_4^z = v + 1.5\tau$$

where, τ is the minimum price increment (tick size)

LOB states

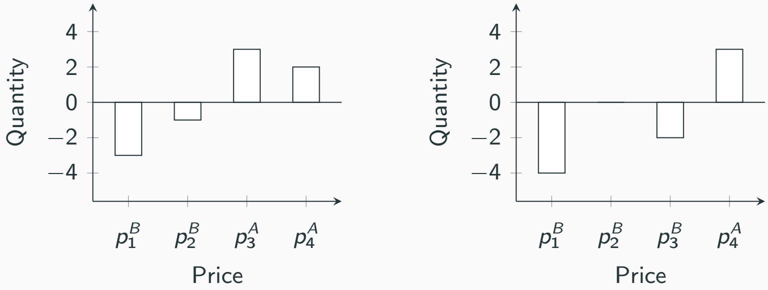

Figure 2: Some of the possible limit order book states.

- For bid-side $q_i \leq 0$, for ask side $q_i \geq 0$
- Assume a crowd absorbs any amount at the highest ask and lowest bid ($q_1 = -\infty$ and $q_4 = \infty$)
- The state of the book at each period is $b_t = [q_2, q_3]$

Agents

- Fully rational, risk neutral, and trade because they wish to trade
- Only one agent trades at each period. Trader j selects an optimal order type upon arrival based on her valuation of the asset: $v + \beta_j$, where $\beta_j \sim \mathbb{U}(\underline{a}, \bar{a})$ is a private valuation of the asset

Table 1: Order types.

Strategies	Notation
	Benchmark
Market order	$\varphi_M(1,\ p_i^z) \quad i = 1, 2, 3, 4$
Limit order	$\varphi_L(1,\ p_i^z) \quad i = 2, 3$
No trading	$\varphi(0)$
	Dark Pool
Dark pool order	$\varphi_D(\pm 1,\ \tilde{p}_{Mid,\,t})$
IOC* on dark pool or market order	$\varphi_{IOC}(\pm 1,\ p_{Mid,\,t},\ p_i^z)$

* *IOC - Immediate or Cancel order type*

Optimal Order Choice

- Market orders pay the spread and execute with certainty:

$$\text{payoff}_{j,t}\left[\varphi_M\left(1, p_i^B\right)\right] = p_i^B - (v + \beta_j)$$

- Expected payoff of limit orders depends on the execution probability:

$$\text{payoff}^e_{j,t_3}\left[\varphi_L\left(1, p_2^B\right)|b_{t_3}\right] = \left(v + \beta_j - p_2^B\right) \cdot \Pr\left[\varphi_M\left(1, p_2^B\right)|b_{t_4}\right],$$

  where $b_{t_4} = b_{t_3} + [-1, 0]$ is state of the LOB in the next period

- No trading yields zero payoff: $\text{payoff}_{j,t}\left[\varphi(0)\right] = 0$

At each trading round a trader selects the optimal order submission strategy:

$$\max_{\varphi \in \left\{\varphi_M(1, p_i^z), \varphi_L(1, p_i^z), \varphi(0)\right\}} \text{payoff}^e_{j,t}\left[\varphi | b_t, \beta_j\right]$$

LOB Development Path

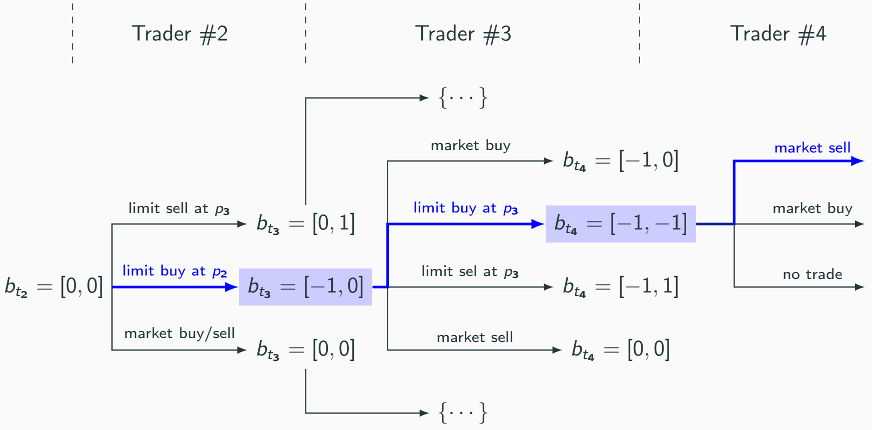

Figure 3: Benchmark model of limit order book. Extensive form of a trading game with equilibrium strategies.

Benchmark Model Solution

- Solution (numerical) by backward induction
- Assume $\tau = 0.08$, $v = 1$, $\beta_i \sim \mathbb{U}(-1, 1)$

Table 2: Order submission probabilities at $t_1, \ldots, t_4$ in the benchmark model.

	b_t	mb	ms	lb@p_2	lb@p_3	ls@p_2	ls@p_3	nt
$t = t_1$	$[0, 0]$	0.124	0.124	0.212	0.164	0.164	0.212	-
$t = t_2$	$[0, 0]$	0.277	0.277	0.223	-	-	0.223	-
$t = t_3$	$[-1, 0]$	0.397	0.443	-	0.083	-	0.077	-
$t = t_4$	$[-1, -1]$	0.440	0.520	-	-	-	-	0.040

Continuous Dark Pool (CDP)

- Opaque crossing network
- Continuous execution using a time priority rule
- CDP crosses orders at a prevailing midquote $p_{Mid,t}$ ▸ orders
- Only a fraction α of traders have access to the dark pool
- Traders infer the state by motinoring the LOB and Bayesian updating their expectations about the state of the CDP:

$$\Omega_t = \left\{ b_t,\ \tilde{CDP}_t \right\}$$

Optimal Order Choice with CDP

▸ Dark order ▸ IOC order ▸ Bayesian updating

- Dark order executes if there is an opposite side volume in the CDP or if next traders become a counterparty in the dark

- IOC orders search liquidity in the dark pool to execute at midpoint, and if fail to find it they go to the LOB as market orders

- Traders infer the state of the CDP by Bayesian updating order submission probabilities given the state of the LOB and actions of previous traders

LOB Development Path with CDP

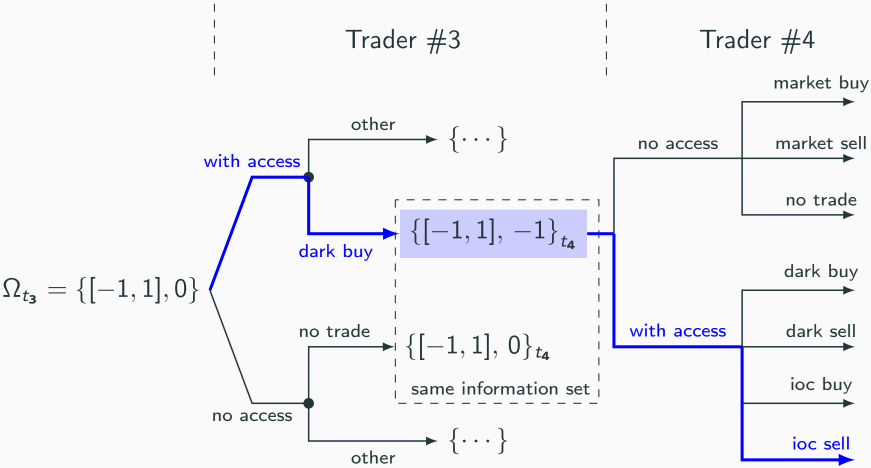

Figure 4: Limit order book and continuous dark pool. Extensive form of a trading game with equilibrium strategies.

LOB and CDP Model Solution ($\alpha = 0.5, \tau = 0.08$)

Table 3: Order submission probabilities in the model with LOB and CDP.

	$\Omega_{t_1} = \{[0,0],0\}$		$\Omega_{t_3} = \{[-1,-1],0\}$		$\Omega_{t_4} = \{[-1,-1],h_{t_3}\}$	
	B	NA/WA	B/NA	WA	B/NA	WA
market buy	0.124	0.132	0.44	0.433	0.44	-
market sell	0.124	0.132	0.52	0.514	0.52	-
limit buy@p_2	0.212	0.217	-	-	-	-
limit buy@p_3	0.164	0.151	-	-	-	-
limit sell@p_2	0.164	0.151	-	-	-	-
limit sell@p_3	0.212	0.217	-	-	-	-
dark buy	-	-				
dark sell	-	-				
IOC buy	-	-				
IOC sell	-	-				
no trade	-	-	0.04	-	0.04	-

- order aggressivenes (increasing) - dark order

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limit buy@p_3	0.164	0.151	-	-	-	-
limit sell@p_2	0.164	0.151	-	-	-	-
limit sell@p_3	0.212	0.217	-	-	-	-
dark buy			-	0.027		
dark sell			-	0.026		
IOC buy			-	-		
IOC sell			-	-		
no trade	-	-	0.04	-	0.04	-

- order aggressivenes (increasing)

- dark order

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limit buy@p_2	0.212	0.217	-	-	-	-
limit buy@p_3	0.164	0.151	-	-	-	-
limit sell@p_2	0.164	0.151	-	-	-	-
limit sell@p_3	0.212	0.217	-	-	-	-
dark buy					-	0.02
dark sell					-	0.02
IOC buy					-	0.44
IOC sell					-	0.52
no trade	-	-	0.04	-	0.04	-

- order aggressivenes (increasing)

- dark order

Order Migration

- $\text{OM} = \frac{1}{T}\sum_{t=1}^{T} \Pr(\varphi^d)$, where $\varphi^d = \varphi_D + \varphi_{IOC}$

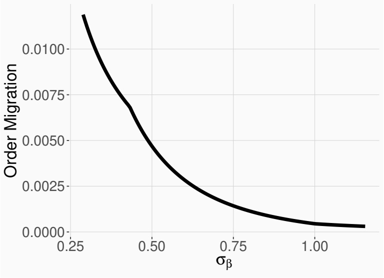

Figure 5: Order migration and private valuation of the asset.

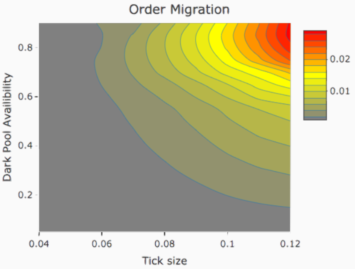

Figure 6: Order migration, tick size and dark pool availability.

Market Quality

- Compute the expected spread and depth in each period by weighting the realised values in the equilibrium states of the book by the corresponding order submission probabilities

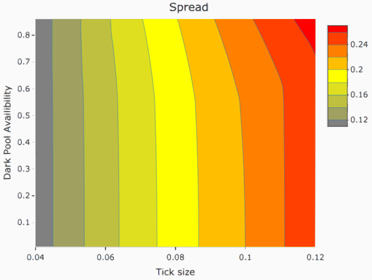

Figure 7: Spread, tick size and dark pool availability.

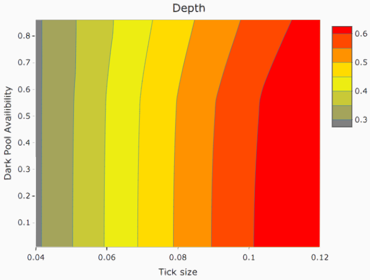

Figure 8: Depth, tick size and dark pool availability.

Impact of Time Horizon and LOB Liquidity

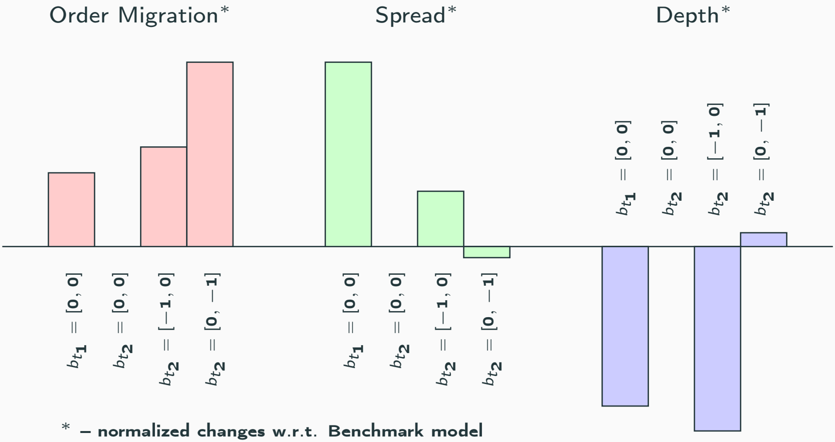

Figure 9: Changes in average market quality measures due to the introduction of the dark pool.

How to enrich the model

- Add information asymmetry (π – fraction of informed traders)
- Which traders go to the dark?
- How is price discovery process influenced by the dark pool?
- How aggregate welfare in the presence of informed traders is affected by dark pool?
- Endogenise α and π => equilibrium level of dark trading and information aquisition VS welfare

Extended Model (with information asymmetry)

- Same rules apply, but:
- The payoff of the asset in the end of the trading day is uncertain $V \in \{v^L, v^H\}$ with equal probabilities ($\mathrm{E}(V) = v$)
- A fraction π of the traders is informed about the future payoff
- Traders maximize their expected payoff by Bayesian updating the expected value of V, based on the actions of previous traders
- Assume $v^L = 0.5v$, $v^H = 1.5v$

Results: Order Migration

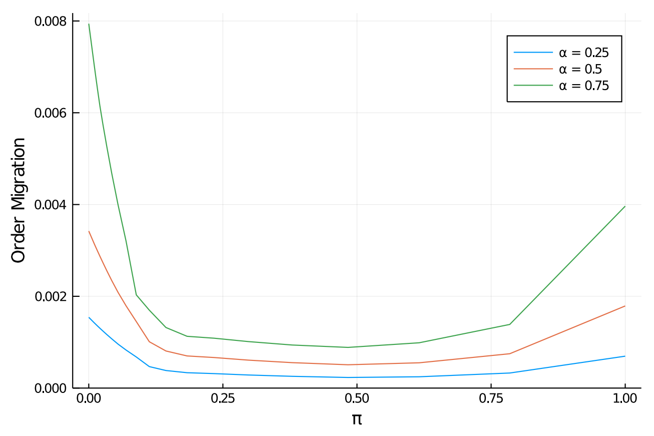

Figure 10: Dark pool availability, informed trading and **order migration**.

Results: Who goes to the dark?

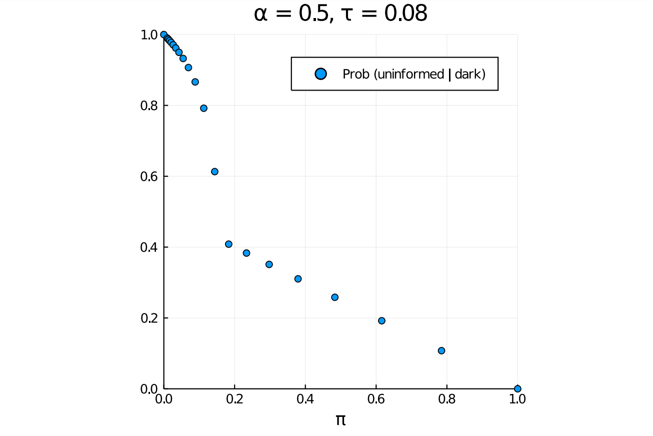

Figure 11: Dark pool availability, informed trading and **uninformed order migration**.

Results: Price Discovery

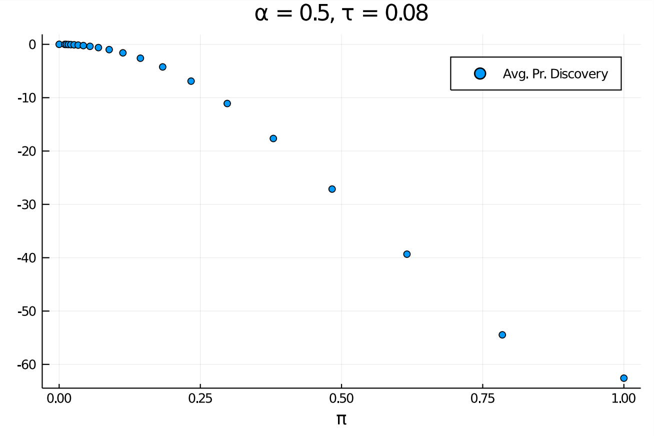

Figure 12: Dark pool availability, informed trading and average price discovery.

Additionly: Spread and Depth

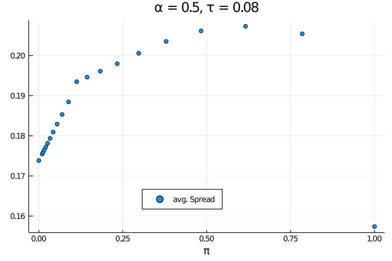

Figure 13: Dark pool availability, informed trading and **average spread**.

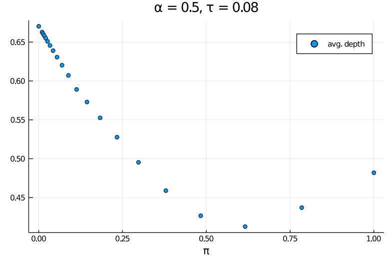

Figure 14: Dark pool availability, informed trading and **average depth**.

Future Steps

- ($\checkmark$) Add information asymmetry
- ($\checkmark$) Which traders go to the dark? => **infrormed** (mostly)
- ($\checkmark$) How is price discovery process influenced by the dark pool? => **no effect**

- How aggregate welfare in the presence of informed traders is affected by dark pool?
- Endogenise α and π => equilibrium level of dark trading and information aquisition VS welfare

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