

# Dark Trading, Market Quality and Price Discovery

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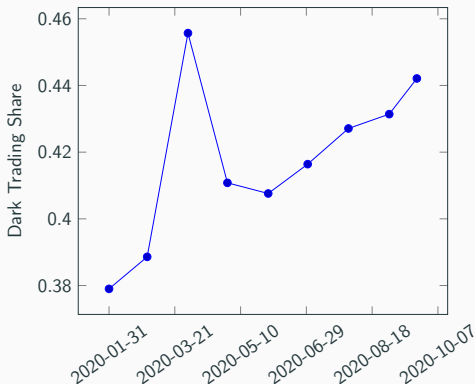
Vladimir Levin

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University of Luxembourg

## Motivation (1/2)

- ~ 700M euros worth of equity is changing hands within dark pools in Europe daily
- U.S. Equities dark trading share is around 42%

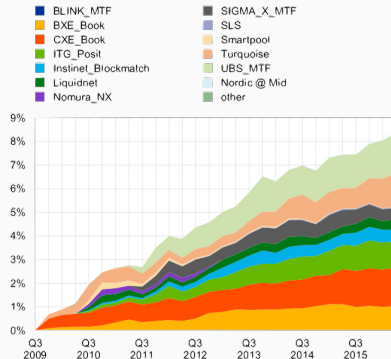


**Figure 1:** U.S. Equities dark trading. Source: Bats Global Markets.

## Motivation (2/2)

- Dark trading may adversely influence market quality, price discovery and the incentive for liquidity provision
- Market share of dark pools is growing

(percentage of total volume traded, by value)



Source : Bats Global Markets

**Figure 2:** Market share of dark pools in trading in european stocks.

- Order choice and venue choice problem of informed traders in limit order markets
- How are market quality and price discovery affected by dark pool?
- Optimal levels of informed trading and dark pool access

## Benchmark Model (no information asymmetry)

- Trading day is divided into 4 periods ( $t_1, t_2, t_3$ , and  $t_4$ )
- One asset: pays  $v$  in the end of the trading day
- LOB: set of prices and quantities  $\{p_i^z, q_i\}$  where  $z = \{A, B\}$  and  $i = \{1, \dots, 4\}$  is the level on a price grid
- Prices are defined relative to the common value of the asset:

$$p_1^z = v - 1.5\tau$$

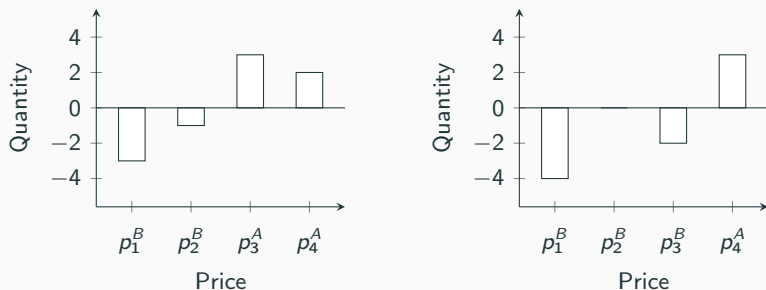
$$p_2^z = v - 0.5\tau$$

$$p_3^z = v + 0.5\tau$$

$$p_4^z = v + 1.5\tau$$

where,  $\tau$  is the minimum price increment (tick size)

# LOB states



**Figure 3:** Possible limit order book states.

- For bid-side  $q_i \leq 0$ , for ask side  $q_i \geq 0$
- Assume a crowd absorbs any amount at the highest ask and lowest bid ( $q_1 = -\infty$  and  $q_4 = \infty$ )
- The state of the book at each period is  $b_t = [q_2, q_3]$

- Fully rational, risk neutral, and trade because they wish to trade
- Only one agent trades at each period. Trader  $j$  selects an optimal order type upon arrival based on her valuation of the asset:  $v + \beta_j$ , where  $\beta_j \sim \mathbb{U}(\underline{a}, \bar{a})$  is a private valuation of the asset

**Table 1:** Order types.

Strategies	Notation
<i>Benchmark and Dark Pool</i>	
Market order	$\varphi_M(1, p_i^z) \quad i = 1, 2, 3, 4$
Limit order	$\varphi_L(1, p_i^z) \quad i = 2, 3$
No trading	$\varphi(0)$
<i>Dark Pool</i>	
Dark pool order	$\varphi_D(\pm 1, \tilde{p}_{Mid, t})$
IOC* on dark pool or market order	$\varphi_{IOC}(\pm 1, p_{Mid, t}, p_i^z)$

\* IOC - Immediate or Cancel order type

# Optimal Order Choice

- Market orders pay the spread and execute with certainty:

$$\pi_{j,t} [\varphi_M (1, p_2^B)] = p_2^B - (v + \beta_j)$$

- Expected payoff of limit orders depends on the execution probability:

$$\pi_{j,t_3}^e [\varphi_L (1, p_2^B) | b_{t_3}] = (v + \beta_j - p_2^B) \cdot \Pr [\varphi_M (1, p_2^B) | b_{t_4}],$$

where  $b_{t_4} = b_{t_3} + [-1, 0]$  is state of the LOB in the next period

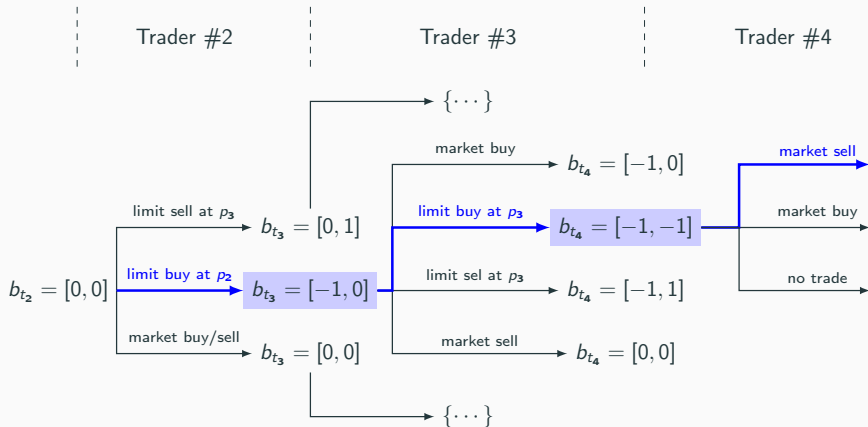
- No trading yields zero payoff:  $\pi_{j,t} [\varphi(0)] = 0$

At each trading round a trader selects the optimal order submission strategy:

$$\max_{\varphi \in \{\varphi_M(1, p_i^z), \varphi_L(1, p_i^z), \varphi(0)\}} \pi_{j,t}^e [\varphi | b_t, \beta_j]$$



# LOB Development Path



**Figure 4:** Benchmark model of limit order book. Extensive form of a trading game with equilibrium strategies.


# Benchmark Model Solution

- Solution (numerical) by backward induction
- Assume  $\tau = 0.08$ ,  $\nu = 1$ ,  $\beta_i \sim \mathbb{U}(-1, 1)$

**Table 2:** Order submission probabilities at  $t_1, \dots, t_4$  in the benchmark model.

	$b_t$	mb	ms	lb@ $p_2$	lb@ $p_3$	ls@ $p_2$	ls@ $p_3$	nt
$t = t_1$	$[0, 0]$	0.124	0.124	0.212	0.164	0.164	0.212	-
$t = t_2$	$[0, 0]$	0.277	0.277	0.223	-	-	0.223	-
$t = t_3$	$[-1, 0]$	0.397	0.443	-	0.083	-	0.077	-
$t = t_4$	$[-1, -1]$	0.440	0.520	-	-	-	-	0.040

# Continuous Dark Pool (CDP)

- Opaque crossing network
- Continuous execution using a time priority rule
- CDP crosses orders at a prevailing midquote  $p_{Mid,t}$  
- Only a fraction  $\alpha$  of traders have access to the dark pool
- Traders infer the state by monitoring the LOB and Bayesian updating their expectations about the state of the CDP:

$$\Omega_t = \{b_t, \tilde{CDP}_t\}$$

# Optimal Order Choice with CDP

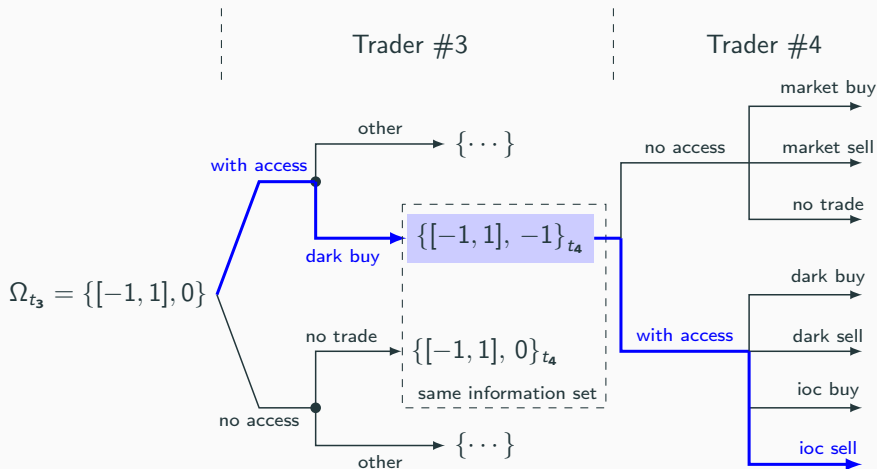
▸ Dark order

▸ IOC order

▸ Bayesian updating

- Dark order executes if there is an opposite side volume in the CDP or if next traders become a counterparty in the dark
- IOC orders search liquidity in the dark pool to execute at midpoint, and if fail to find it they go to the LOB as market orders
- Traders infer the state of the CDP by Bayesian updating order submission probabilities given the state of the LOB and actions of previous traders

# LOB Development Path with CDP

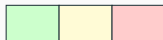


**Figure 5:** Limit order book and continuous dark pool. Extensive form of a trading game with equilibrium strategies.

# LOB and CDP Model Solution ( $\alpha = 0.5, \tau = 0.08$ )

**Table 3:** Order submission probabilities in the model with LOB and CDP.

	$\Omega_{t_1} = \{[0, 0], 0\}$		$\Omega_{t_3} = \{[-1, -1], 0\}$		$\Omega_{t_4} = \{[-1, -1], h_{t_3}\}$	
	B	NA/WA	B/NA	WA	B/NA	WA
market buy	0.124	0.132	0.44	0.433	0.44	-
market sell	0.124	0.132	0.52	0.514	0.52	-
limit buy@ $p_2$	0.212	0.217	-	-	-	-
limit buy@ $p_3$	0.164	0.151	-	-	-	-
limit sell@ $p_2$	0.164	0.151	-	-	-	-
limit sell@ $p_3$	0.212	0.217	-	-	-	-
dark buy	-	-	-	-	-	-
dark sell	-	-	-	-	-	-
IOC buy	-	-	-	-	-	-
IOC sell	-	-	-	-	-	-
no trade	-	-	0.04	-	0.04	-



- order aggressiveness (increasing)

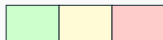


- dark order

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market buy	0.124	0.132	0.44	0.433	0.44	-
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limit buy@ $p_2$	0.212	0.217	-	-	-	-
limit buy@ $p_3$	0.164	0.151	-	-	-	-
limit sell@ $p_2$	0.164	0.151	-	-	-	-
limit sell@ $p_3$	0.212	0.217	-	-	-	-
dark buy			-	0.027		
dark sell			-	0.026		
IOC buy			-	-		
IOC sell			-	-		
no trade	-	-	0.04	-	0.04	-



- order aggressiveness (increasing)

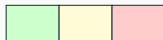


- dark order

# LOB and CDP Model Solution ( $\alpha = 0.5, \tau = 0.08$ )

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market buy	0.124	0.132	0.44	0.433	0.44	-
market sell	0.124	0.132	0.52	0.514	0.52	-
limit buy@ $p_2$	0.212	0.217	-	-	-	-
limit buy@ $p_3$	0.164	0.151	-	-	-	-
limit sell@ $p_2$	0.164	0.151	-	-	-	-
limit sell@ $p_3$	0.212	0.217	-	-	-	-
dark buy					-	0.02
dark sell					-	0.02
IOC buy					-	0.44
IOC sell					-	0.52
no trade	-	-	0.04	-	0.04	-



- order aggressiveness (increasing)

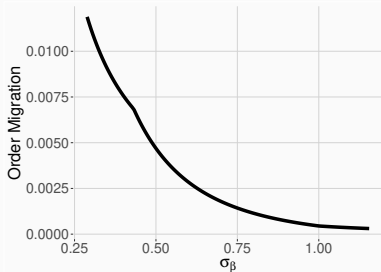


- dark order

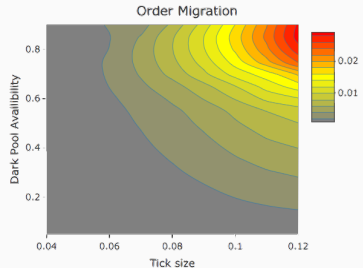


# Order Migration

- $OM = \frac{1}{T} \sum_t \alpha \cdot \Pr(\varphi^d | \Omega_t)$ , where  $\varphi^d = \varphi_D + \varphi_{IOC}$



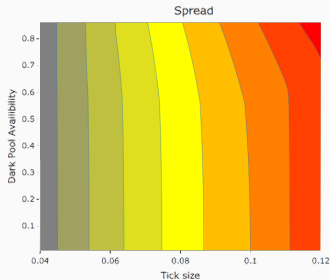
**Figure 6:** Order migration and private valuation of the asset.



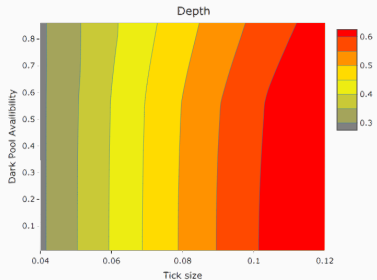
**Figure 7:** Order migration, tick size and dark pool availability.

# Market Quality

- Compute the expected spread and depth in each period by weighting the realised values in the equilibrium states of the book by the corresponding order submission probabilities



**Figure 8:** Spread, tick size and dark pool availability.



**Figure 9:** Depth, tick size and dark pool availability.

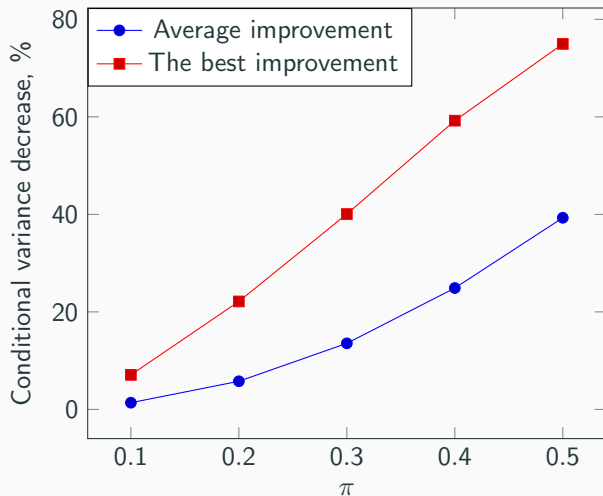
## Extended Model (with information asymmetry)

- Same rules apply, but:
- The payoff of the asset in the end of the trading day is uncertain  $V \in \{v^L, v^H\}$  with equal probabilities ( $E(V) = v$ )
- A fraction  $\pi$  of the traders is informed about the future payoff
- Traders maximize their expected payoff by Bayesian updating the expected value of  $V$ , based on the actions of previous traders
- Assume  $v^L = 0.5v$ ,  $v^H = 1.5v$ ,  $\pi = 0.2$

**Table 4:** Order submission probabilities at  $t_1$  in the benchmark and extended model.

	mb	ms	lb@ $p_2$	lb@ $p_3$	ls@ $p_2$	ls@ $p_3$	nt
Benchmark	0.124	0.124	0.212	0.164	0.164	0.212	-
Uninformed	0.245	0.245	0.255	-	-	0.255	-
Informed ( $V = v^H$ )	0.499	-	0.250	-	-	0.251	-

# Price discovery without dark pool



**Figure 10:** Price discovery through three rounds of trading in the limit order book.

## Next Steps:

- Solve extended model with dark pool
- Endogenise  $\alpha$  and  $\pi$
- Which traders go to the dark?
- How is price discovery process influenced by the dark pool?

- Theoretical model of limit order trading together with a dark pool
- Market and limit orders migrate to the dark venue
- Dark pool activity increases in the fraction of traders that may access the dark pool and the relative tick size, and decreases in the dispersion of trader valuations around the common value of an asset
- To be continued . . .