Dark Trading, Market Quality and Price Discovery

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Motivation (1/2)

- $\sim$ 700M euros worth of equity is changing hands within dark pools in Europe daily
- U.S. Equities dark trading share is around 42%

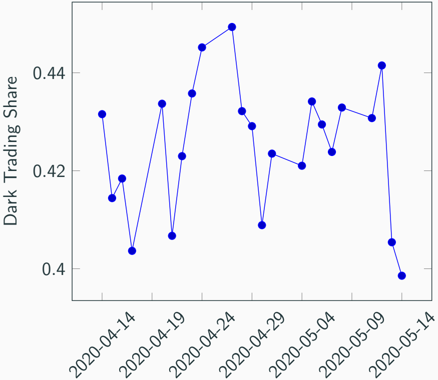

Figure 1: U.S. Equities dark trading. Source: Bats Global Markets.

Motivation (2/2)

- Dark trading may adversively influence market quality, price discovery and the incentive for liquidity provision
- Market share of dark pools is growing

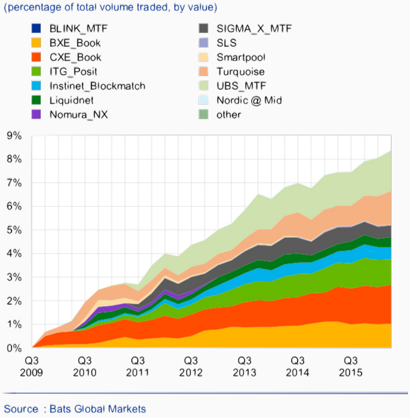

Figure 2: Market share of dark pools in trading in european stocks.

Contributions

- Order choice and venue choice problem of informed traders in limit order markets
- How are market quality and price discovery affected by dark pool?
- Optimal levels of informed trading and dark pool access

Benchmark Model (no information asymmetry)

- Trading day is divided into 4 periods ($t_1, t_2, t_3,$ and t_4)
- One asset: pays v in the end of the trading day
- LOB: set of prices and quantities $\{p_i^z, q_i\}$ where $z = \{A, B\}$ and $i = \{1, \ldots, 4\}$ is the level on a price grid
- Prices are defined relative to the common value of the asset:

$$p_1^z = v - 1.5\tau$$
$$p_2^z = v - 0.5\tau$$
$$p_3^z = v + 0.5\tau$$
$$p_4^z = v + 1.5\tau$$

where, τ is the minimum price increment (tick size)

LOB states

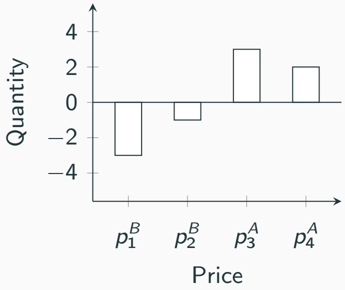

Figure 3: Possible limit order book states.

- For bid-side $q_i \leq 0$, for ask side $q_i \geq 0$
- Assume a crowd absorbs any amount at the highest ask and lowest bid ($q_1 = -\infty$ and $q_4 = \infty$)
- The state of the book at each period is $b_t = [q_2,\ q_3]$

Agents

- Fully rational, risk neutral, and trade because they wish to trade
- Only one agent trades at each period. Trader j selects an optimal order type upon arrival based on her valuation of the asset: $v + \beta_j$, where $\beta_j \sim \mathbb{U}(\underline{a}, \bar{a})$ is a private valuation of the asset

Table 1: Order types.

Strategies	Notation
	Benchmark and Dark Pool
Market order	$\varphi_M(1,\ p_i^z) \quad i = 1, 2, 3, 4$
Limit order	$\varphi_L(1,\ p_i^z) \quad i = 2, 3$
No trading	$\varphi(0)$
	Dark Pool
Dark pool order	$\varphi_D(\pm 1,\ \tilde{p}_{Mid,\,t})$
IOC* on dark pool or market order	$\varphi_{IOC}(\pm 1,\ p_{Mid,\,t},\ p_i^z)$

* *IOC - Immediate or Cancel order type*

Optimal Order Choice

- Market orders pay the spread and execute with certainty:

$$\pi_{j,t}\left[\varphi_M\left(1, p_2^B\right)\right] = p_2^B - (v + \beta_j)$$

- Expected payoff of limit orders depends on the execution probability:

$$\pi^e_{j,t_3}\left[\varphi_L\left(1, p_2^B\right)|b_{t_3}\right] = \left(v + \beta_j - p_2^B\right) \cdot \Pr\left[\varphi_M\left(1, p_2^B\right)|b_{t_4}\right],$$

  where $b_{t_4} = b_{t_3} + [-1, 0]$ is state of the LOB in the next period

- No trading yields zero payoff: $\pi_{j,t}\left[\varphi(0)\right] = 0$

At each trading round a trader selects the optimal order submission strategy:

$$\max_{\varphi \in \left\{\varphi_M(1, p_i^z), \varphi_L(1, p_i^z), \varphi(0)\right\}} \pi^e_{j,t}\left[\varphi | b_t, \beta_j\right]$$

LOB Development Path

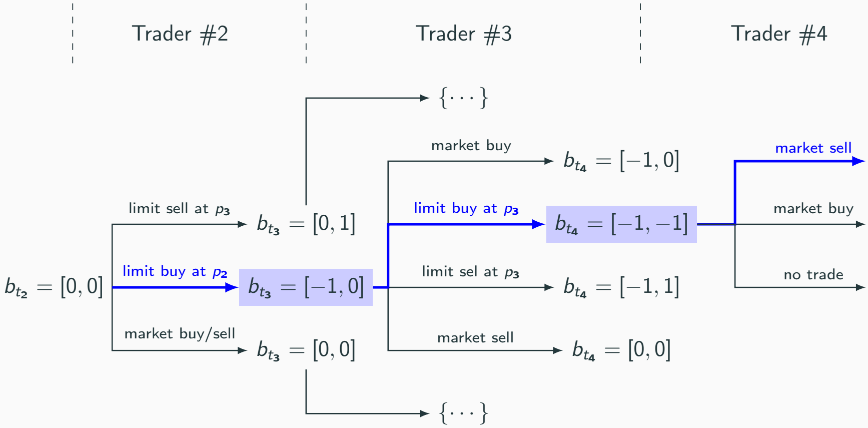

Figure 4: Benchmark model of limit order book. Extensive form of a trading game with equilibrium strategies.

Benchmark Model Solution

- Solution (numerical) by backward induction
- Assume $\tau = 0.08$, $v = 1$, $\beta_i \sim \mathbb{U}(-1, 1)$

Table 2: Order submission probabilities at $t_1, \ldots, t_4$ in the benchmark model.

	b_t	mb	ms	lb@p_2	lb@p_3	ls@p_2	ls@p_3	nt
$t = t_1$	$[0, 0]$	0.124	0.124	0.212	0.164	0.164	0.212	-
$t = t_2$	$[0, 0]$	0.277	0.277	0.223	-	-	0.223	-
$t = t_3$	$[-1, 0]$	0.397	0.443	-	0.083	-	0.077	-
$t = t_4$	$[-1, -1]$	0.440	0.520	-	-	-	-	0.040

Continuous Dark Pool (CDP)

- Opaque crossing network
- Continuous execution using a time priority rule
- CDP crosses orders at a prevailing midquote $p_{Mid,t}$ ▸ orders
- Only a fraction α of traders have access to the dark pool
- Traders infer the state by motinoring the LOB and Bayesian updating their expectations about the state of the CDP:

$$\Omega_t = \left\{ b_t,\ \tilde{CDP}_t \right\}$$

LOB Development Path with CDP

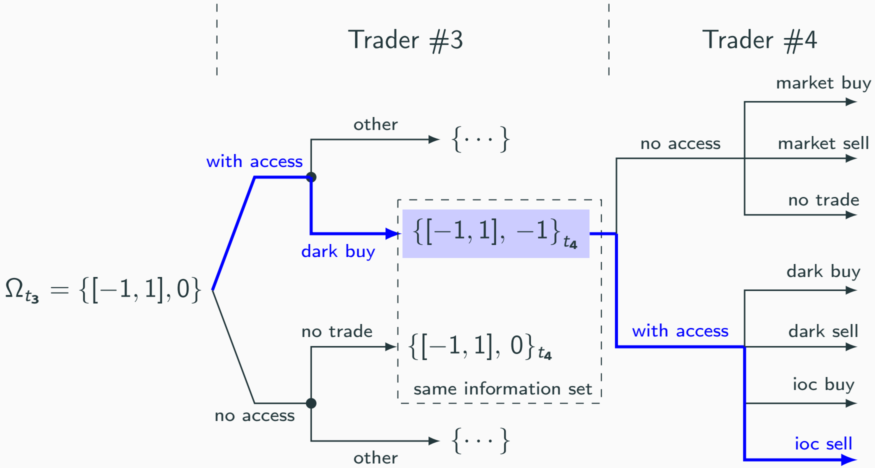

Figure 5: Limit order book and continuous dark pool. Extensive form of a trading game with equilibrium strategies.

Optimal Order Choice with CDP

▸ Dark order ▸ IOC order ▸ Bayesian updating

- Dark order executes if there is an opposite side volume in the CDP or if next traders become a counterparty in the dark

- IOC orders search liquidity in the dark pool to execute at midpoint, and if fail to find it they go to the LOB as market orders

- Traders infer the state of the CDP by Bayesian updating order submission probabilities given the state of the LOB and actions of previous traders

LOB and CDP Model Solution ($\alpha = 0.5, \tau = 0.08$)

Table 3: Order submission probabilities in the model with LOB and CDP.

	$\Omega_{t_1} = \{[0,0],0\}$		$\Omega_{t_3} = \{[-1,-1],0\}$		$\Omega_{t_4} = \{[-1,-1],h_{t_3}\}$	
	B	NA/WA	B/NA	WA	B/NA	WA
market buy	0.124	0.132	0.44	0.433	0.44	-
market sell	0.124	0.132	0.52	0.514	0.52	-
limit buy@p_2	0.212	0.217	-	-	-	-
limit buy@p_3	0.164	0.151	-	-	-	-
limit sell@p_2	0.164	0.151	-	-	-	-
limit sell@p_3	0.212	0.217	-	-	-	-
dark buy	-	-				
dark sell	-	-				
IOC buy	-	-				
IOC sell	-	-				
no trade	-	-	0.04	-	0.04	-

- order aggressivenes (increasing)

- dark order

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limit buy@p_2	0.212	0.217	-	-	-	-
limit buy@p_3	0.164	0.151	-	-	-	-
limit sell@p_2	0.164	0.151	-	-	-	-
limit sell@p_3	0.212	0.217	-	-	-	-
dark buy			-	0.027		
dark sell			-	0.026		
IOC buy			-	-		
IOC sell			-	-		
no trade	-	-	0.04	-	0.04	-

- order aggressivenes (increasing)

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market buy	0.124	0.132	0.44	0.433	0.44	-
market sell	0.124	0.132	0.52	0.514	0.52	-
limit buy@p_2	0.212	0.217	-	-	-	-
limit buy@p_3	0.164	0.151	-	-	-	-
limit sell@p_2	0.164	0.151	-	-	-	-
limit sell@p_3	0.212	0.217	-	-	-	-
dark buy					-	0.02
dark sell					-	0.02
IOC buy					-	0.44
IOC sell					-	0.52
no trade	-	-	0.04	-	0.04	-

- order aggressivenes (increasing)

- dark order

Order Migration

- $\text{OM} = \frac{1}{T}\sum_t \alpha \cdot \Pr(\varphi^d|\Omega_t)$, where $\varphi^d = \varphi_D + \varphi_{IOC}$

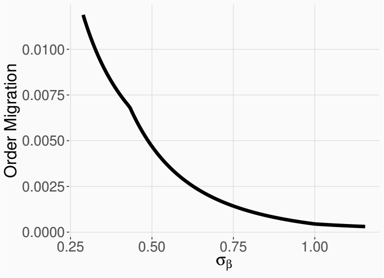

Figure 6: Order migration and private valuation of the asset.

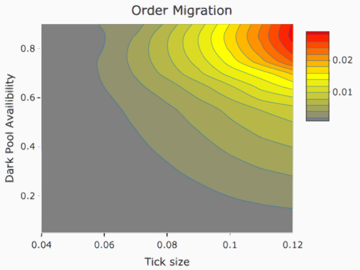

Figure 7: Order migration, tick size and dark pool availability.

Market Quality

- Compute the expected spread and depth in each period by weighting the realised values in the equilibrium states of the book by the corresponding order submission probabilities

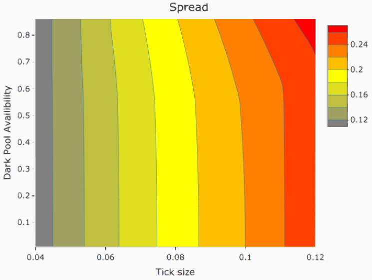

Figure 8: Spread, tick size and dark pool availability.

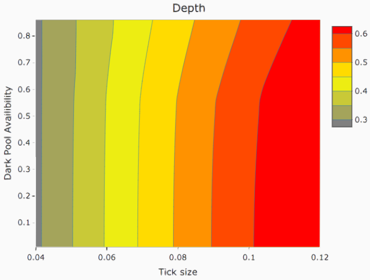

Figure 9: Depth, tick size and dark pool availability.

Impact of Time Horizon and LOB Liquidity

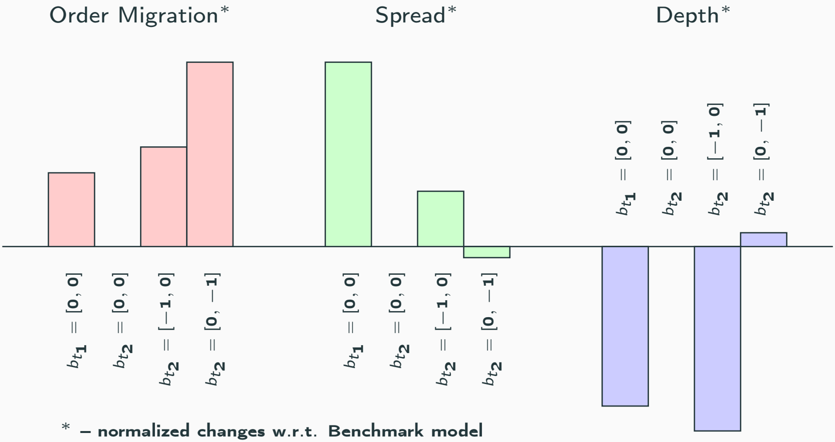

Figure 10: Changes in average market quality measures due to the introduction of the dark pool.

Extended Model (with information asymmetry)

- Same rules apply, but:
- The payoff of the asset in the end of the trading day is uncertain $V \in \{v^L, v^H\}$ with equal probabilities ($\mathrm{E}(V) = v$)
- A fraction π of the traders is informed about the future payoff
- Traders maximize their expected payoff by Bayesian updating the expected value of V, based on the actions of previous traders
- Assume $v^L = 0.5v$, $v^H = 1.5v$, $\pi = 0.2$

Table 4: Order submission probabilities at t_1 in the benchmark and extended model.

	mb	ms	lb@p_2	lb@p_3	ls@p_2	ls@p_3	nt
Benchmark	0.124	0.124	0.212	0.164	0.164	0.212	-
Uninformed	0.205	0.205	0.239	0.056	0.056	0.239	-
Informed ($V = v^H$)	0.511	-	0.199	0.038	-	0.251	-

Price discovery without dark pool

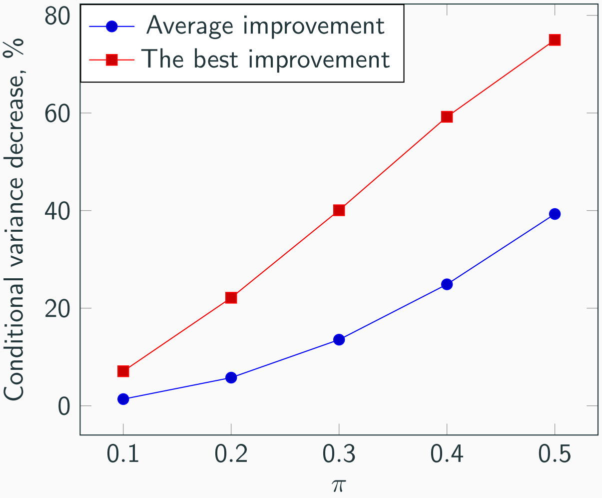

Figure 11: Price discovery through three rounds of trading in the limit order book.

Future Steps

Next Steps:

- Solve extended model with dark pool
- Endogenise α and π
- Which traders go to the dark?
- How is price discovery process influenced by the dark pool?

Empirical Implications:

- Introduction of a dark pool $\approx$ an increase in a dark pool activity
- α: measures of institutional ownership or number of active dark pools
- β: dispersion of analyst forecast, degree of analyst following, degree of fundamental uncertainty
- τ: changes in tick size regimes

Conclusions

- Theoretical model of limit order trading together with a dark pool
- Market and limit orders migrate to the dark venue
- Dark pool activity increases in the dispersion of trader valuations around the common value of an asset, the fraction of traders that may access the dark pool and the relative tick size
- Deterioration effects on market quality reduced if LOB opens with sufficient liquidity
- To be continued ...