Наумкин Владимир СБ1-119 ДЗ 1

№1 $u=\sin(b_1+b_2)$, $b_1^*=\frac{\pi}{2}$, $b_2^*=\frac{\pi}{4}$, $\Delta(b_{1,2}^*)=\frac{1}{4}$

$u^*=\sin(\frac{\pi}{2}+\frac{\pi}{4})=1/\sqrt{2}$; $D(u^*)=\sup|\sin(b_1+b_2)-\frac{1}{\sqrt{2}}|=$

$|\frac{\pi}{2}-b_1|\le\frac{1}{4}$

$|\frac{\pi}{4}-b_2|\le\frac{1}{4}$

$=|\sin(\frac{\pi}{2}+\frac{1}{4}+\frac{\pi}{4}+\frac{1}{4})-\frac{1}{\sqrt{2}}|\approx 0{,}425567$

Ответ: 0,425567.

№2

$$A=\begin{pmatrix}1&2&0\\0&1&3\\0&3&4\end{pmatrix}\quad \Delta(a_{ij})=0{,}04$$

$\Delta(a+b)=\Delta(a)+\Delta(b)$

$\Delta(a\cdot b)=b\cdot\Delta(a)+a\cdot\Delta(b)$

$$\det(A-\lambda E)=\begin{vmatrix}1-\lambda&2&0\\0&1-\lambda&3\\0&3&4-\lambda\end{vmatrix}=(\underset{0{,}04}{1}-\underset{0}{\lambda})(\underset{0}{\lambda^2}-\underset{0{,}04+0{,}04=0{,}08}{5\lambda}+4-\overset{0{,}2+0{,}24=0{,}44}{9})=0$$

$3\cdot0{,}04+3\cdot0{,}04=0{,}24$

$1\cdot0{,}04+4\cdot0{,}04=0{,}2$

$$\Rightarrow(\overset{0}{\lambda^2}-\overset{0{,}08}{5\lambda}-\overset{0{,}44}{5})(\overset{0}{\lambda}-\overset{0{,}04}{1})=0$$

$$\lambda^3-1\lambda^2-5\lambda^2+35\lambda-5\lambda+35=0$$

$$1\cdot\lambda^3-12\lambda^2+30\lambda+35=0$$

0 $\quad 0{,}04+0{,}08=0{,}12$ $\quad 1\cdot0{,}08+5\cdot0{,}04+0{,}44=1{,}2$ $\quad 1\cdot0{,}44+5\cdot0{,}04=3{,}28$

$$\lambda^2-5\lambda-5=0$$

$$\lambda=\frac{5\pm\sqrt{25+20}}{2}=\frac{5\pm3\sqrt{5}}{2}$$

$a=1,\ b=-12,\ c=30,\ d=35$

Итак, $1\cdot\lambda^3 - 12\cdot\lambda^2 + 30\cdot\lambda + 35 = 0$

$\Delta(1)=0;\ \Delta(-12)=0{,}12;\ \Delta(30)=1{,}2;\ \Delta(35)=3{,}28$

$\lambda_1 = 7;\ \lambda_2 = \dfrac{5+3\sqrt5}{2};\ \lambda_3 = \dfrac{5-3\sqrt5}{2}$

По т-ме о неявной ф-ции для ур-я $f = a\lambda^3 + b\lambda^2 + c\lambda + d = 0$ получим: $\lambda(a,b,c,d)$

$$\frac{\partial\lambda}{\partial a} = -\frac{\partial f/\partial a}{\partial f/\partial\lambda} = -\frac{\lambda^3}{3a\lambda^2+2b\lambda+c};$$ аналогично

$$\frac{\partial\lambda}{\partial b} = -\frac{\lambda^2}{3a\lambda^2+2b\lambda+c},\quad \frac{\partial\lambda}{\partial c} = -\frac{\lambda}{3a\lambda^2+2b\lambda+c},\quad \frac{\partial\lambda}{\partial d} = -\frac{1}{3a\lambda^2+2b\lambda+c}$$

$$\Delta(\lambda^*) \approx \sum_j |\sigma_j(0)|\cdot\Delta(t_j^*),\ \text{где}\ \sigma_j(0) = \lambda'_{t_j}(t_1^*, t_2^*, \ldots)$$

$$\Delta(\lambda^*) \approx \frac{|-\lambda^3|\cdot 0 + |-\lambda^2|\cdot 0{,}12 + |-\lambda|\cdot 1{,}2 + |-1|\cdot 3{,}28}{|3\lambda^2 - 24\lambda + 30|} =$$

$$= \frac{0{,}12\cdot\lambda^2 + 1{,}2\cdot|\lambda| + 3{,}28}{|3\lambda^2 - 24\lambda + 30|}$$

Ответ: $\Delta(\lambda_1^*) \approx 1{,}95;\ \Delta(\lambda_2^*) \approx 1{,}88;\ \Delta(\lambda_3^*) \approx 0{,}08$

где $\lambda_1^* = 7$ $\quad \lambda_2^* = \dfrac{5+3\sqrt5}{2}$ $\quad \lambda_3^* = \dfrac{5-3\sqrt5}{2}$

№3