Lab3

李远航 PB20000137

一、实验内容

使用Coq证明如下命题:

```
Lemma ex1: forall A, ~~~ A -> ~ A.
Lemma ex2: forall A B, A \/ B -> ~ (~ A /\ ~ B).
Lemma ex3: forall T (P:T -> Prop),(~ exists x, P x) -> forall x, ~ P x.
```

二、实验思路

- $\forall A\ , \neg\neg\neg A \to \neg A$

  先将变量全部引入，通过假设推出矛盾的方式来证明

- $\forall A\ \forall B\ , A \vee B \to \neg(\neg A \wedge \neg B)$

  和前面相同，先引入变量，然后将$\neg(\neg A \wedge \neg B)$分解为 $\neg A$ 和 $\neg B$ 按情况分析，通过推出矛盾证明

- $\forall T\ (P : T -> Prop), (\neg \exists x, P\ x) \to \forall x\ , \neg P\ x$

  引入类型T，谓词P，以及后续的变量，利用存在性推出矛盾证明

三、实验结果

- $\forall A\ , \neg\neg\neg A \to \neg A$

```
1 goal
______________________________________(1/1)
forall A : Prop, ~ ~ ~ A -> ~ A

1 goal
A : Prop
H : ~ ~ ~ A
H0 : A
______________________________________(1/1)
False

1 goal
A : Prop
H : ~ ~ ~ A
H0 : A
______________________________________(1/1)
~ ~ A

1 goal
A : Prop
H : ~ ~ ~ A
H0 : A
```

```
H1 : ~ A
_____________________________________(1/1)
False

No more goals.
```

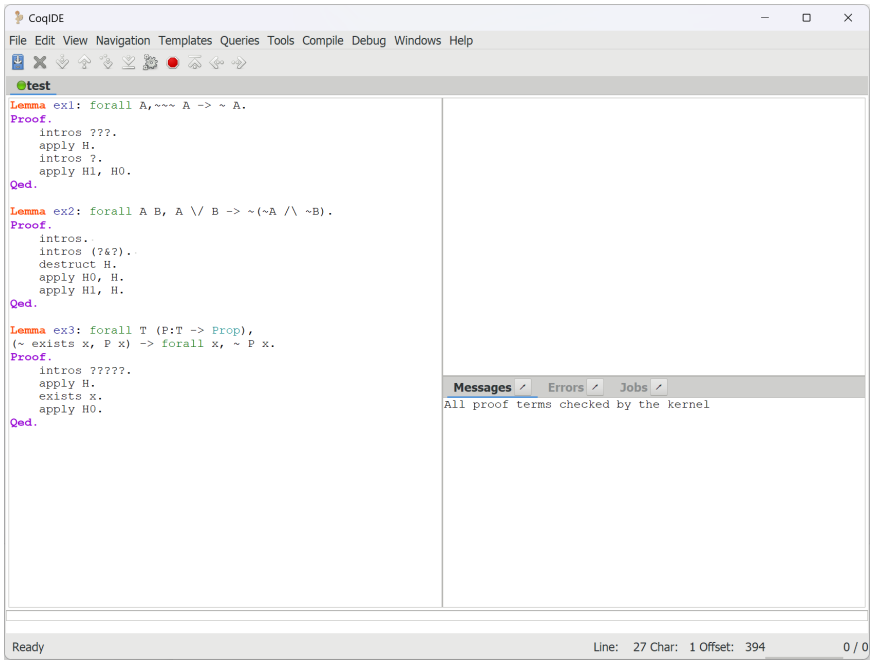
- $\forall A \, \forall B \, , A \vee B \rightarrow \neg (\neg A \wedge \neg B)$

```
1 goal
_____________________________________(1/1)
forall A B : Prop, A \/ B -> ~ (~ A /\ ~ B)

1 goal
A, B : Prop
H : A \/ B
_____________________________________(1/1)
~ (~ A /\ ~ B)

1 goal
A, B : Prop
H : A \/ B
H0 : ~ A
H1 : ~ B
_____________________________________(1/1)
False

2 goals
A, B : Prop
H : A
H0 : ~ A
H1 : ~ B
_____________________________________(1/2)
False
_____________________________________(2/2)
False

1 goal
A, B : Prop
H : B
H0 : ~ A
H1 : ~ B
_____________________________________(1/1)
False

No more goals.
```

- $\forall T \, (P : T -> Prop), (\neg \exists x, P \, x) \rightarrow \forall x \, , \neg P \, x$

```
1 goal
_____________________________________(1/1)
forall (T : Type) (P : T -> Prop),
~ (exists x : T, P x) -> forall x : T, ~ P x

```

```
1 goal
T : Type
P : T -> Prop
H : ~ (exists x : T, P x)
x : T
H0 : P x
______________________________________(1/1)
False

1 goal
T : Type
P : T -> Prop
H : ~ (exists x : T, P x)
x : T
H0 : P x
______________________________________(1/1)
exists x0 : T, P x0

1 goal
T : Type
P : T -> Prop
H : ~ (exists x : T, P x)
x : T
H0 : P x
______________________________________(1/1)
P x

No more goals.
```



四、实验收获

- 初步学习了Coq
- 对逻辑式的证明有更深的理解