

Lab3

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一、实验内容

使用Coq证明如下命题:

```
1  Lemma ex1: forall A, ~~~ A -> ~ A.
2  Lemma ex2: forall A B, A \ / B -> ~ (~ A /\ ~ B).
3  Lemma ex3: forall T (P:T -> Prop), (~ exists x, P x) -> forall x, ~ P x.
```

二、实验思路

- $\forall A, \neg\neg\neg A \rightarrow \neg A$

先将变量全部引入, 通过假设推出矛盾的方式来证明

- $\forall A \forall B, A \vee B \rightarrow \neg(\neg A \wedge \neg B)$

和前面相同, 先引入变量, 然后将 $\neg(\neg A \wedge \neg B)$ 分解为 $\neg A$ 和 $\neg B$ 按情况分析, 通过推出矛盾证明

- $\forall T (P : T \rightarrow \text{Prop}), (\neg \exists x, P x) \rightarrow \forall x, \neg P x$

引入类型T, 谓词P, 以及后续变量, 利用存在性推出矛盾证明

三、实验结果

- $\forall A, \neg\neg\neg A \rightarrow \neg A$

```
1  1 goal
2  -----(1/1)
3  forall A : Prop, ~ ~ ~ A -> ~ A
4
5  1 goal
6  A : Prop
7  H : ~ ~ ~ A
8  H0 : A
9  -----(1/1)
10 False
11
12 1 goal
13 A : Prop
14 H : ~ ~ ~ A
15 H0 : A
16 -----(1/1)
17 ~ ~ A
18
19 1 goal
20 A : Prop
21 H : ~ ~ ~ A
22 H0 : A
```

```

23   H1 : ~ A
24   -----(1/1)
25   False
26
27   No more goals.

```

- $\forall A \forall B, A \vee B \rightarrow \neg(\neg A \wedge \neg B)$

```

1   1 goal
2   -----(1/1)
3   forall A B : Prop, A \ / B -> ~ (~ A /\ ~ B)
4
5   1 goal
6   A, B : Prop
7   H : A \ / B
8   -----(1/1)
9   ~ (~ A /\ ~ B)
10
11  1 goal
12  A, B : Prop
13  H : A \ / B
14  H0 : ~ A
15  H1 : ~ B
16  -----(1/1)
17  False
18
19  2 goals
20  A, B : Prop
21  H : A
22  H0 : ~ A
23  H1 : ~ B
24  -----(1/2)
25  False
26  -----(2/2)
27  False
28
29  1 goal
30  A, B : Prop
31  H : B
32  H0 : ~ A
33  H1 : ~ B
34  -----(1/1)
35  False
36
37  No more goals.

```

- $\forall T (P : T \rightarrow Prop), (\neg \exists x, P x) \rightarrow \forall x, \neg P x$

```

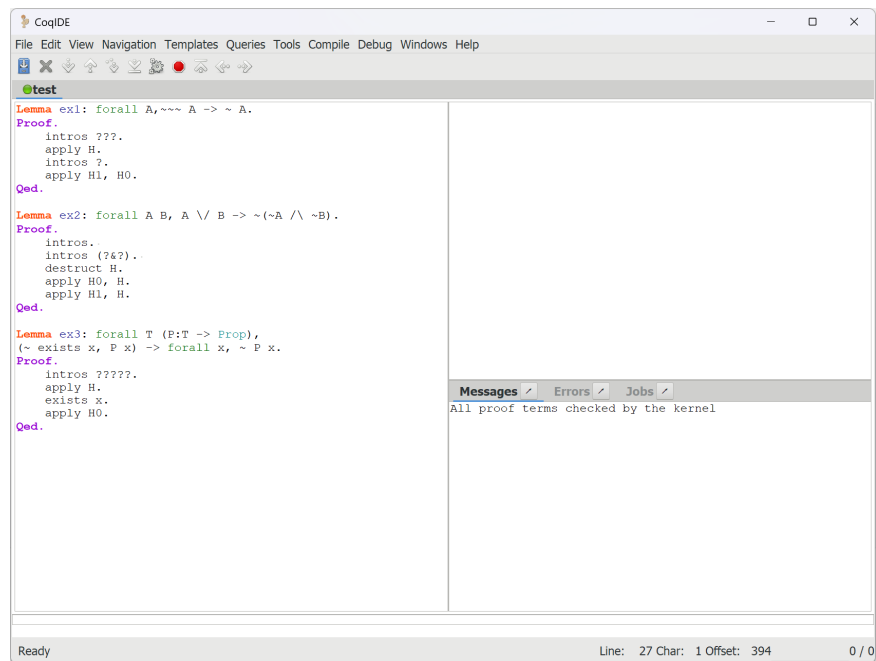
1   1 goal
2   -----(1/1)
3   forall (T : Type) (P : T -> Prop),
4   ~ (exists x : T, P x) -> forall x : T, ~ P x
5

```

```

6   1 goal
7   T : Type
8   P : T -> Prop
9   H : ~ (exists x : T, P x)
10  x : T
11  H0 : P x
12  -----(1/1)
13  False
14
15  1 goal
16  T : Type
17  P : T -> Prop
18  H : ~ (exists x : T, P x)
19  x : T
20  H0 : P x
21  -----(1/1)
22  exists x0 : T, P x0
23
24  1 goal
25  T : Type
26  P : T -> Prop
27  H : ~ (exists x : T, P x)
28  x : T
29  H0 : P x
30  -----(1/1)
31  P x
32
33  No more goals.

```



四、实验收获

- 初步学习了Coq
- 对逻辑式的证明有更深入的理解