

HW2

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a.

q	p	$((p \rightarrow q) \rightarrow p) \rightarrow p$
0	0	true
0	1	true
1	0	true
1	1	true

p	q	$((p \wedge q) \rightarrow (p \vee q))$
0	0	true
0	1	true
1	0	true
1	1	true

p	q	$((p \rightarrow q) \vee (p \rightarrow (\sim q)))$
0	0	true
0	1	true
1	0	true
1	1	true

p	q	r	$((p \vee q) \rightarrow r) \rightarrow ((p \rightarrow r) \vee (q \rightarrow r))$
0	0	0	true
0	0	1	true
0	1	0	true
0	1	1	true
1	0	0	true
1	0	1	true
1	1	0	true
1	1	1	true

1.

- $\forall x(P(x) \rightarrow A(m, x))$
- $\exists x(P(x) \wedge A(x, m))$
- $A(m, m)$
- $\neg \exists x(S(x) \wedge \forall y(L(y) \rightarrow B(x, y)))$
- $\neg \exists y(L(y) \wedge \forall x(S(x) \rightarrow B(x, y)))$
- $\neg \exists y(L(y) \wedge \exists x(S(x) \wedge B(x, y)))$

2.

(a) 对于任意的 x , 可以取 $y = x + 1, z = x - 1$, 此时满足

(b) 对于任意的 x , 可以取 $y = 2x, z = x$, 此时满足

(c) 对于任意的 x , 可以取 $y = x + 4, z = x + 2$, 此时满足

1. ①. $(p \wedge q) \wedge r, s \wedge t \vdash q \wedge s$

1. $(p \wedge q) \wedge r$ premise
2. $p \wedge q$ $\wedge e_1$ 1
3. q $\wedge e_2$ 2
4. $s \wedge t$ premise
5. s $\wedge e_1$ 4
6. $q \wedge s$ $\wedge i$ 3,5

②. $q \rightarrow r \vdash (p \rightarrow q) \rightarrow (p \rightarrow r)$

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|----|---|---------------------|
| 1. | $p \rightarrow q$ | assumption |
| 2. | p | assumption |
| 3. | q | $\rightarrow e$ 1,2 |
| 4. | $q \rightarrow r$ | premise |
| 5. | r | $\rightarrow e$ 4,3 |
| 6. | $p \rightarrow r$ | $\rightarrow i$ 2-5 |
| 7. | $(p \rightarrow q) \rightarrow (p \rightarrow r) \rightarrow i$ 1-6 | |

③. $\vdash q \rightarrow (p \rightarrow (p \rightarrow (q \rightarrow p)))$

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|----|---|---------------------|
| 1. | q | assumption |
| 2. | p | assumption |
| 3. | p | assumption |
| 4. | q | assumption |
| 5. | p | copy 3 |
| 6. | $q \rightarrow p$ | $\rightarrow i$ 4-5 |
| 7. | $p \rightarrow (q \rightarrow p)$ | $\rightarrow i$ 2-6 |
| 8. | $p \rightarrow (p \rightarrow (q \rightarrow p))$ | $\rightarrow i$ 2-7 |
| 9. | $p \rightarrow (p \rightarrow (p \rightarrow (q \rightarrow p))) \rightarrow i$ 1-8 | |

④. $p \rightarrow q \wedge r \vdash (p \rightarrow q) \wedge (p \rightarrow r)$

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|-----|--|---------------------|
| 1. | p | assumption |
| 2. | $p \rightarrow q \wedge r$ | premise |
| 3. | $q \wedge r$ | $\rightarrow e$ 1,2 |
| 4. | q | $\wedge e_1$ 3 |
| 5. | $p \rightarrow q$ | $\rightarrow i$ 1-4 |
| 6. | p | assumption |
| 7. | $p \rightarrow q \wedge r$ | premise |
| 8. | $q \wedge r$ | $\rightarrow e$ 6,7 |
| 9. | r | $\wedge e_2$ 8 |
| 10. | $p \rightarrow r$ | $\rightarrow i$ 6-9 |
| 11. | $(p \rightarrow q) \wedge (p \rightarrow r) \wedge i$ 5,10 | |

⑤. $p \wedge \neg p \vdash \neg (r \rightarrow q) \wedge (r \rightarrow q)$

1. $p \wedge \neg p$ premise
2. p $\wedge e_1$ 1
3. $\neg p$ $\wedge e_2$ 1
4. \perp $\neg e$ 2,3
5. $\neg (r \rightarrow q) \wedge (r \rightarrow q)$ $\perp e$ 4.

2. ⑥. $\exists x (s \rightarrow Q(x)) \vdash s \rightarrow \exists x Q(x)$

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|----|--|---------------------|
| 1. | s | assumption |
| 2. | $\exists x (s \rightarrow Q(x))$ | premise |
| 3. | $s \rightarrow Q(x_0)$ | assumption |
| 4. | $Q(x_0)$ | $\rightarrow e$ 1,3 |
| 5. | $\exists x Q(x)$ | $\exists i$ 4 |
| 6. | $s \rightarrow \exists x Q(x) \rightarrow i$ 1-5 | |

⑦. $\forall x (p(x) \wedge Q(x)) \vdash \forall x p(x) \wedge \forall x Q(x)$

1. $\forall x (p(x) \wedge Q(x))$ premise
2. x_0 $p(x_0) \wedge Q(x_0)$ $\forall x e$ 1
3. $p(x_0)$ $\wedge e_1$ 2
4. $q(x_0)$ $\wedge e_2$ 2
5. $\forall x p(x)$ $\forall i$ 2-3
6. $\forall x q(x)$ $\forall i$ 2-4
7. $\forall x p(x) \wedge \forall x q(x)$ $\wedge i$ 5,6

⑧. $\neg \forall x \neg p(x) \vdash \exists x p(x)$

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|----|-------------------------------|-----------------|
| 1. | $\neg \forall x \neg p(x)$ | premise |
| 2. | $\neg \exists x p(x)$ | assumption |
| 3. | x_0 $\neg p(x_0)$ | assumption |
| 4. | $\exists x p(x)$ | $\exists i$ 3 |
| 5. | \perp | $\neg e$ 2,4 |
| 6. | $\neg \neg p(x_0)$ | $\neg i$ 3-5 |
| 7. | $\forall x \neg p(x)$ | $\forall i$ 3-6 |
| 8. | \perp | $\neg e$ 1,7 |
| 9. | $\exists x p(x)$ $\neg e$ 2-8 | |

⑨. $\forall x \neg p(x) \vdash \neg \exists x p(x)$

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|----|------------------------------------|---------------------|
| 1. | $\exists x p(x)$ | assumption |
| 2. | $\forall x \neg p(x)$ | assumption |
| 3. | x_0 $p(x_0)$ | assumption |
| 4. | $\neg \neg p(x_0)$ | $\forall x e$ 2 |
| 5. | \perp | $\neg e$ 3,4 |
| 6. | \perp | $\exists x e$ 1,3-5 |
| 7. | $\neg \exists x p(x)$ $\neg i$ 1-6 | |

Q. $\forall x (P(x) \rightarrow S) \vdash \exists x (P(x) \wedge S)$. 不会写.

1. $\forall x (P(x) \rightarrow S)$ premise
2. $\forall x (P(x))$ assumption
3. S $\rightarrow e$ 1, 2
4. $x_0 \ P(x_0)$ assumption
5. ~~$P(x_0) \wedge S$~~ \wedge copy 3
6. $P(x_0) \rightarrow S$ $\rightarrow i$ 4-5.
7. $\exists x (P(x) \rightarrow S)$ $\exists x$ 6.
8. $\neg \forall x (P(x))$ assumption
9. $\neg \exists x \neg P(x)$ assumption
10. $x_0 \ \neg P(x_0)$ ~~assumption~~
11. $\exists x \neg P(x)$ $\exists x$ 10.
12. \bot $\neg e$ 9, 11.
13. $P(x_0)$ PBC 10-12
14. $\forall x (P(x))$ $\forall x$ i 10-13.
15. \bot $\neg e$ 14, 8.
16. $\exists x \neg P(x)$ PBC 9-15.
17. $x_0 \ \neg P(x_0)$ assumption
18. $P(x_0)$ assumption
19. \bot $\neg e$ 17, 18
20. S $\bot e$ 19
21. $P(x_0) \rightarrow S$ $\rightarrow i$ 18-20.
22. $\exists x (P(x) \rightarrow S)$ $\exists x$ 21, 17-21.
23. $\forall x (P(x)) \vee \neg \forall x (P(x))$ LEM
24. $\exists x (P(x) \rightarrow S)$ $\vee e$ 21, 10, 11-26.

