HW2

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a.

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q	p	(((p -> q) -> p) -> p)
0	0	true
0	1	true
1	0	true
1	1	true

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p	q	((p /\ q) -> (p \/ q))
0	0	true
0	1	true
1	0	true
1	1	true

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p	q	((p -> q) \/ (p -> (~q)))
0	0	true
0	1	true
1	0	true
1	1	true

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p	q	r	(((p \/ q) -> r) -> ((p -> r) \/ (q -> r)))
0	0	0	true
0	0	1	true
0	1	0	true
0	1	1	true
1	0	0	true
1	0	1	true
1	1	0	true
1	1	1	true

1.

- $\forall x(P(x) \to A(m,x))$
- $\exists x(P(x) \land A(x,m))$
- $A(m,m)$
- $\neg\exists x(S(x) \land \forall y(L(y) \to B(x,y))$
- $\neg\exists y(L(y) \land \forall x(S(x) \to B(x,y))$
- $\neg\exists y(L(y) \land \exists x(S(x) \land B(x,y)))$

2.

(a) 对于任意的x，可以取 $y=x+1$，$z=x-1$，此时满足

(b) 对于任意的x，可以取 $y=2x$，$z=x$，此时满足

(c) 对于任意的x，可以取 $y=x+4$，$z=x+2$，此时满足

1. ①. $(p \wedge q) \wedge r,\ s \wedge t \vdash q \wedge s$

1.	$(p \wedge q) \wedge r$	premise
2.	$p \wedge q$	$\wedge e_1$ 1
3	q	$\wedge e_2$ 2
4	$s \wedge t$	premise
5.	s	$\wedge e_1$ 1
6	$q \wedge s$	$\wedge i$ 3,5

② $q \to r \vdash (p \to q) \to (p \to r)$

1.	$p \to q$	assumption
2.	p	assumption
3	q	$\to e$ 1,2
4	$q \to r$	premise
5	r	$\to e$ 4,3
6.	$p \to r$	$\to i$ 2-5
7.	$(p \to q) \to (p \to r)$	$\to i$ 1-6

③ $\vdash q \to (p \to (p \to (q \to p)))$

1.	q	assumption
2.	p	assumption
3	p	assumption
4.	q	assumption
5	p	copy 3
6	$q \to p$	$\to i$ 4-5
7.	$p \to (q \to p)$	$\to i$ 3-6
8.	$p \to (p \to (q \to p))$	$\to i$ 2-7
9.	$q \to (p \to (p \to (q \to p)))$	$\to i$ 1-8

④. $p \to q \vee r \vdash (p \to q) \vee (p \to r)$

1.	p	assumption
2.	$p \to q \vee r$	premise
3.	$q \vee r$	$\to e$ 1,2
4.	q	$\vee e_1$ 1
5	$p \to q$	$\to i$ 1-4
6.	p	assumption
7.	$p \to q \vee r$	premise
8.	$q \vee r$	$\to e$ 1,7
9.	r	$\vee e_2$ 8
10	$p \to r$	$\to i$ 6-9
11.	$(p \to q) \vee (p \to r)$	$\vee i$ 5,10

⑤. $p \wedge \neg p \vdash \neg(r \to q) \wedge (r \to q)$

1.	$p \wedge \neg p$	premise
2.	p	$\wedge e_1$ 1
3.	$\neg p$	$\wedge e_2$ 2
4	$\bot$	$\neg e$ 2,3
5.	$\neg(r \to q) \wedge (r \to q)$	$\bot e$ 4.

2. ①. $\exists x(S \to Q(x)) \vdash S \to \exists x Q(x)$

1.	$\exists x(S \to Q(x))$	premise
2.	S	assumption
3.	$S \to Q(x_0)$	assumption
4.	$Q(x_0)$	$\to e$ 1,3
5.	$\exists x Q(x)$	$\exists i$ 4
6.	$S \to \exists x Q(x)$	$\to i$ 2-5

② $\forall x(P(x) \wedge Q(x)) \vdash \forall x P(x) \wedge \forall x Q(x)$

1.	$\forall x(P(x) \wedge Q(x))$	premise
2.	x_0. $P(x_0) \wedge Q(x_0)$	$\forall x$ e 1
3.	$P(x_0)$	$\wedge e_1$ 2
4.	$Q(x_0)$	$\wedge e_2$ 2
5.	$\forall x P(x)$	$\forall x$ i 2-3
6.	$\forall x Q(x)$	$\forall x$ i 2-4
7.	$\forall x P(x) \wedge \forall x Q(x)$	$\wedge i$ 5,6.

④. $\neg \forall x P(x) \vdash \exists x \neg P(x)$.

1.	$\neg \forall x P(x)$	premise
2.	$\neg \exists x \neg P(x)$	assumption
3.	x_0 $P(x_0)$.	assumption
4.	$\exists x P(x)$	$\exists$ i 3.
5	$\bot$	$\neg e$ 4,2.
6.	$\neg P(x_0)$.	$\neg i$ 3-5
7.	$\forall x \neg P(x)$	$\forall$ i 3-6
8.	$\bot$	$\neg e$ 1,7
9.	$\neg \neg \exists x P(x)$	$\neg i$ 2-8.
10.	$\exists x P(x)$	$\neg \neg e$ 9.

⑤. $\forall x \neg P(x) \vdash \neg \exists x P(x)$

1.	$\exists x P(x)$	assumption
2.	$\forall x \neg P(x)$	assumption
3.	x_0 $P(x_0)$	assumption
4.	$\neg P(x_0)$.	$\forall$ e 2
5.	$\bot$	$\neg e$ 3,4
6.	$\bot$	$\exists e$ 1,3-5
7.	$\neg \exists x P(x)$	$\neg i$ 1-6

(2). $\forall x P(x) \to S \vdash \exists x (P(x) \to S)$. 不会写.